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GRAVITATIONAL FIELD MODELS
FOR STUDY OF EARTH
MANTLE DYNAMICS

FINAL REPORT

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Prepared for:

Dr. H.S. Liu
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20771

Prepared by:

Systems and Applied Sciences Corporation
6811 Kenilworth Avenue, Suite 500
Riverdale, Maryland 20840

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SECTION 1 INTRODUCTION

O'Keefe (1959) tested the basic hypothesis of geodesy by using the harmonics of the earth's gravity field from satellite measurements. He pointed out that these harmonics may be supported by hydrodynamic forces resulting from mantle convection. Runcorn (1964, 1967) developed a theory connecting mantle convection and the geoid.

Although numerical calculations of thermal convection models of the mantle have been carried out by Turcotte et al. (1973) and by McKenzie et al. (1974), the non-linear nature of mantle convection and the wide range of viscosities present formidable obstacles to the thorough knowledge of convection patterns. It may well be that unless simple models of mantle flow are first understood, complex cases would not be interpretable in terms of the contribution to the dynamics that each case provides. Therefore, it seems necessary, and perhaps even desirable, to apply the relatively simple model of mantle convection developed by Runcorn (1964, 1967). In this paper, we propose to detect and determine the tectonic forces or stresses due to the small-scale mantle flow under the South American plate by utilizing the harmonics of the geopotential field model.

SECTION 2 THEORY OF MANTLE CONVECTION AND THE GEOID

Given a set of fully normalized harmonic coefficients, $C_{n,m}$ and $S_{n,m}$, determined from satellite measurements and surface gravity observations, the relationship between the mantle convection pattern at the upper boundary and the geoid may be described by (Runcorn, 1964, 1967):

$$\left(\frac{d^2W_n}{dr^2} + \frac{2}{a} \cdot \frac{dW_n}{dr}\right)_{r=a} = \sum_{n=2}^{\infty} \sum_{m=0}^{m=n} \frac{Mg}{4\pi\mu a^2}\left(\frac{a'}{a}\right)^{n+1}$$

$$\frac{2n+1}{n+1} \bar{P}_n^m (\cos\zeta) \left[\bar{C}_{n,m} \cos(m\lambda) + \bar{S}_{n,m} \sin(m\lambda)\right] \tag{1}$$

where

M = mass of the earth,

μ = coefficient of viscosity,

a = radius of outer spherical surface of the flowing region,

$\bar{P}_n^m (\cos\zeta)$ = associated Legendre polynomial of degree n, order m and argument cos ,

λ = the longitude,

$\bar{C}_{2,0} = C_{2,0} - C'_{2,0}$,

$\bar{S}_{n,m} = S_{n,m}$

$C'_{2,0}$ and

$C'_{4,0}$ = coefficients characterizing the ellipsoid of revolution,

W = a poloidal function of convection currents

g = gravitational acceleration,

a' = radius of the earth,

ζ = the co-latitude,

$$\bar{C}_{4,0} = C_{4,0} - C'_{4,0},$$

$$\bar{C}_{n,m} = C_{n,m} \text{ for } n \neq 2 \text{ or } 4 \text{ and } m = 0.$$

In the derivation of eq. 1, a Newtonian viscosity is attributed to the mantle and laminar viscous flow is assumed.

SECTION 3 SUBCRUSTAL STRESSES EXERTED BY CONVECTION CURRENTS

If the crust above the flowing mantle is assumed to be an elastic shell, the tangential component of the velocity vanishes at r=a, i.e., $(dW_n/dr)|_{r=a} = 0$. Then the stress components exerted by the convection flow on the crust in the eastward and northward directions are determined by:

$$\sigma_E(\zeta,\lambda) = -\frac{\mu}{\sin\zeta} \cdot \frac{d}{d\lambda}\left(\frac{d^2W_n}{dr^2}\right)_{r=a}$$

$$= \sum_{n=2}^{\infty}\sum_{m=0}^{m=n} \frac{Mg}{4\pi a^2}\left(\frac{a'}{a}\right)^{n+1} \cdot \frac{2n+1}{n+1}\,\frac{1}{\sin\zeta}\,\bar{P}_n^m(\cos\zeta)$$

$$\cdot\,[m\bar{C}_{n,m}\sin(m\lambda) - m\bar{S}_{n,m}\cos(m\lambda)] \qquad (2)$$

and

$$\sigma_N(\zeta,\lambda) = \mu\frac{d}{d\zeta}\left[\frac{1}{r}\cdot\frac{d}{d\zeta}\cdot\frac{d}{dr}(rW_n)\right]_{r=a}$$

$$= \mu\frac{d}{d\zeta}\left(\frac{d^2W_n}{dr^2}\right)_{r=a} \qquad (3)$$

$$= \sum_{n=2}^{\infty}\sum_{m=0}^{m=n} \frac{Mg}{4\pi a^2}\left(\frac{a'}{a}\right)^{n+1}\frac{2n+1}{n+1}\cdot\frac{d}{d\zeta}[\bar{P}_n^m(\cos\zeta)]$$

$$\cdot\,[\bar{C}_{n,m}\cos(m\lambda) + \bar{S}_{n,m}\sin(m\lambda)]$$

SECTION 4 DATA SETS

The harmonic coefficients $C_{n,m}$ and $S_{n,m}$ of the geopotential field model in eqs. 2 and 3 have been significantly improved in recent years, principally because of the improved satellite observations and surface gravity measurements (Smith et. al., 1976). The low-degree harmonics for $n < 12$ may reflect a large-scale mantle flow system (Runcorn 1967; Richter and Parsons, 1975). In this paper, the high-degree harmonics, $n > 13$, are assumed to describe the small-scale mantle convection patterns. The GEM-10 gravity model used here is based on the work by Lerch et. al. (1977). The input data used in the derivation of this model is made up of 840,000 optical, electronic and laser observations and 1,656 $5° \times 5°$ mean free-air anomalies. Although there remain some statistically questionable aspects of the high-degree harmonics, it seems appropriate now to explore their implications for the tectonic forces or stress field under the crust.

SECTION 5 CALCULATIONS AND RESULTS

In order to compute the magnitudes and directions of stresses exerted by the convection currents on the crust, the following recursive relationships among the Legendre and associated Legendre functions were employed (Arfken, 1966):

$$(2n+1)\cos\zeta P_n^0(\cos\zeta) = (n+1)P_{n+1}^0(\cos\zeta) + nP_{n-1}^0(\cos\zeta) \tag{4}$$

$$P_{n+1}^{m+1}(\cos\zeta) - P_{n-1}^{m+1}(\cos\zeta) = (2n+1)\sin\zeta\, P_n^m(\cos\zeta) \tag{5}$$

$$(2n+1)\cos\zeta \frac{d}{d\zeta} P_n^0(\cos\zeta) - (2n+1)\sin\zeta\, P_n^0(\cos\zeta) = (n+1)\frac{d}{d\zeta}P_{n+1}^0(\cos\zeta) + n\frac{d}{d\zeta}P_{n-1}^0(\cos\zeta) \tag{6}$$

$$\frac{d}{d\zeta}P_{n+1}^{m+1}(\cos\zeta) - \frac{d}{d\zeta}P_{n-1}^{m+1}(\cos\zeta) = (2n+1)\sin\zeta\frac{d}{d\zeta}P_n^m(\cos\zeta) + (2n+1)\cos\zeta\, P_n^m(\cos\zeta) \tag{7}$$

As each function was computed, it was normalized using (Mather, 1971):

$$\bar{P}_n^m(u) = \left(\frac{(2-\delta_{0m})(2n+1)(n-m)!}{(n+m)!}\right)^{1/2} P_n^m(u)$$

where

$$\delta_{0m} = \begin{cases} 0 \text{ if } m\neq 0 \\ 1 \text{ if } m=0 \end{cases}$$

Due to the slow computer job turn-around at the GSFC 360/95, the program SCP that computes the stress components as a function of latitude and longitude was coded so that each job computed the stress components for a small region and the results from each job were later merged. This merged result was then used as input to the stress plotting program SPP which generated the necessary data for producing a picture display on the SD 4060.

The figure shows the stress pattern corresponding to the mantle flow system under the South American Plate. As shown in the figure, in the vicinity of Bolivia and off the coast of Bolivia, these regions are under tension. These results are in excellent agreement with the geoidal highs that have been concluded by Marsh and Chang (1979).

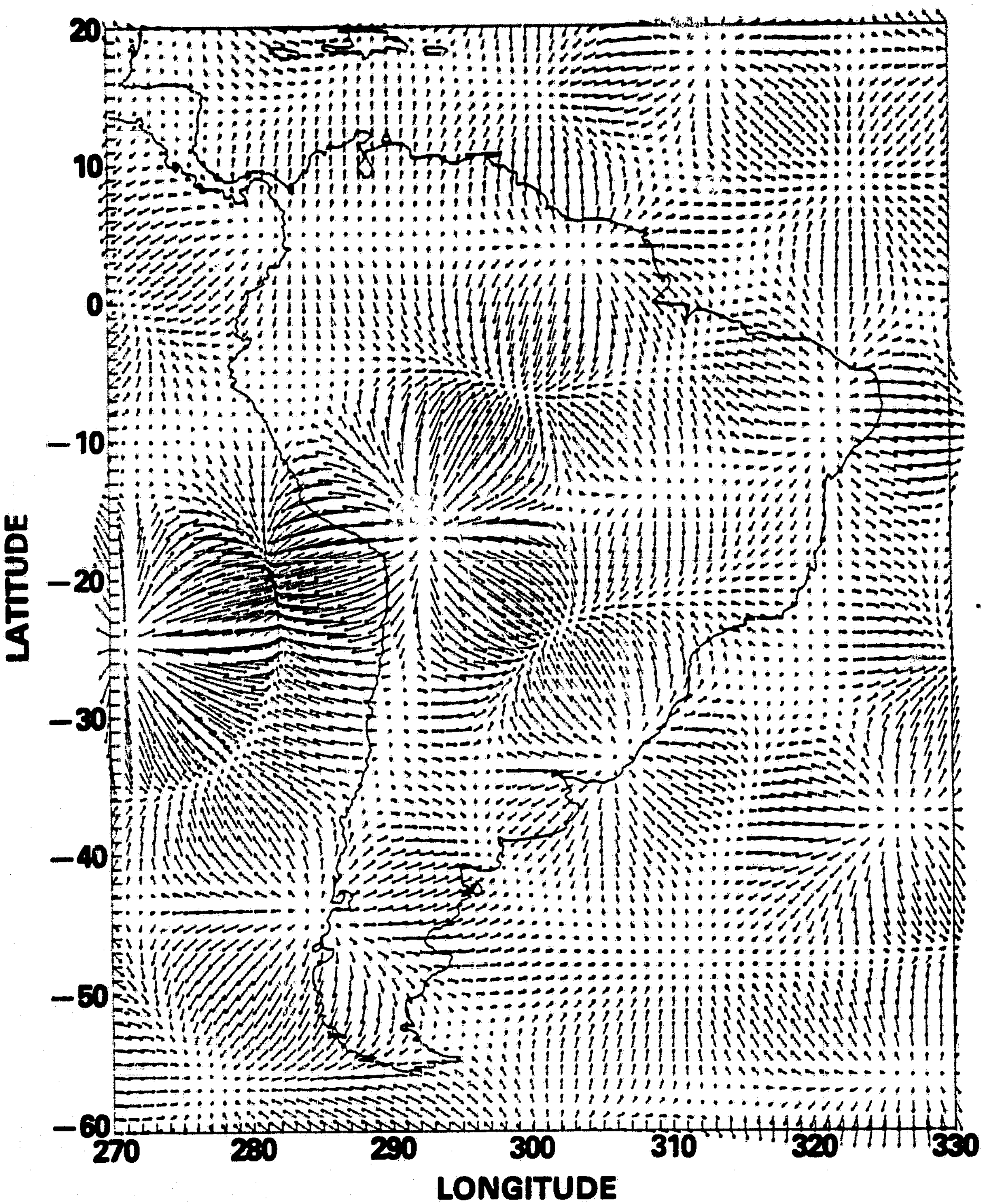

Figure 5-1. Stress Pattern for South America

SECTION 6 REFERENCES

Arfken, G., 1966. Mathematical Methods for Physicists. Academic Press, New York, N.Y., pp. 423 and 436.

Lerch, F.J., Klosko, S.M., Laubscher, R.E. and Wagner, C.A., 1977. Gravity Model Improvement Using GEOS-3 (GZM9 & GZM10) GSFC, Rep. TMX-921-77-246, NASA, Goddard Space Flight Center, Greenbelt, Md.

March, J.G. and Chang, E.S., 1979. Global Detailed Gravimetric Geoid. Marine Geodesy, Vol. 2, pp. 145-159.

McKenzie, D.P., Roberts , J.M. and Weiss, N.O., 1974. Toward the Numerical Solution of Convection in the Earth's Mantle. J. Fluid Mech., 62: 465-538.

O'Keefe, J.A., 1959. Zonal Harmonics of the Earth's Gravitational Field and the Basic Hypothesis of Geodesy. J. Geophys.

Richter, F.M. and Parsons, B., 1975. On the Interaction of Two Scales of Convection in the Mantle. J. Geophys. Res., 80: 2529-2541.

Runcorn, S.K., 1964. Satellite Gravity Measurements and Laminar Viscous Flow Model of the Earth's Mantle. J. Geophys. Res., 69: 4389-4394.

Smith, D.E., Lerch, F.J., Marsh, J.G., Wagner, C.A., Kolenkiewicz, R. and Khan, M.A., 1976. Contribution to the National Geodetic Satellite Program by Goddard Space Flight Center. J. Geophys. Res., 81: 1006-1026.

Turcotte, D.L., Torrance, K.E. and Hsui, A.T., 1973. Convection in the Earth's Mantle. In: B.A. Bolt (Editor), Methods in Computation Physics, Vol. 13, Academic Press, New York, N.Y., pp. 431-454.