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ACTIVE CONTROLS FOR FLUTTER SUPPRESSION AND GUST ALLEVIATION IN SUPERSONIC AIRCRAFT

by

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FINAL REPORT
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ACTIVE CONTROLS FOR FLUTTER SUPPRESSION AND GUST
ALLEVIATION IN SUPERSONIC AIRCRAFT

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FINAL REPORT

GRANT NSG 7373 monitored by Mr. I. Abel of NASA,
Langley Research Center
General Outline of the Report

Most of the results pertaining to the work performed under the above grant had already been published. The list of these publications is given in the following (copies of which are included in the present report):


There is no intention in the present report to repeat results appearing in the above-mentioned publications.

During the course of the grant, its scope had been extended to cover some work done on active controls on the modified YF-17 flutter model. The results of this effort are summarized in two attached reports. The first report relates to the basic derivation of a suitable control law. The second report relates to the discrepancies found between analysis and wind tunnel tests and shows that they originate from the lack of proper implementation of the desired control law. These reports which are attached herein are the following:
Active Controls for Flutter Suppression and Gust Alleviation in Supersonic Aircraft

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Application is made to the present paper of the recently developed relaxed aerodynamic energy concept and synthesis techniques to the definition of active control systems for the low-speed flutter model of the B-707-300 supersonic cruise airplane. The effectiveness of the resulting activated systems is analytically tested for flutter suppression, wing root bending moment alleviation, and ride control (fuselage accelerations). The results obtained indicate that considerable increase in flutter speeds can be obtained by the various control systems, using a single trailing-edge control. In all cases, the flutter suppression control system led to a substantial reduction in both wing root bending moments and in fuselage and wing accelerations.

Introduction

Theoretical analyses and wind tunnel tests of a low-speed flutter model (1/20 scale) of the B-707-300 airplane (Fig 1), were conducted under the supersonic transport (SST) Follow-on Program—Phase II.1 Reference 1 states that "two constraints of the airplane made a flutter-free design unusually difficult: 1) the relatively low payload/total weight ratio made additional structural weight or mass balance particularly distasteful, and 2) any arrangement of lifting surface planform, thickness, or major mass relocation (e.g., fuel cells) degraded the delicate cruise economy or e.g. balance." Because of this flutter dilemma, considerable efforts were directed towards the development of an active flutter suppression system with the objective of improving the flutter speeds of the SST airplane.

Reference 1 shows that the developed flutter suppression system yields only minor improvements in flutter speeds (9.4% increase with activated inboard ailerons, 3.5% increase with activated outboard ailerons, and 11.3% increase with activated inboard and outboard ailerons). The purpose of the present work is to apply the recently developed relaxed energy concept and synthesis techniques to the definition of an appropriate active control system for flutter suppression. The effectiveness of the resulting activated system is then analytically tested for flutter suppression, root bending moment alleviation, and ride control (fuselage and wing accelerations).

Previous analytical applications of the relaxed energy concept for flutter suppression involved the BQM-34E/F drone aircraft2 (with a research supercritical wing) and the YF-17 fighter aircraft3 (suppression of three different configurations of wing store flutter). The present work supplements the applications to include supersonic type cruise aircraft and is also the first one to investigate the effectiveness of the flutter suppression system (as obtained through the use of the relaxed energy concept), not only for flutter suppression but also for gust alleviation and ride control.

Description of SST Model

The general form of the control law employed in this work was established in Ref 2 using the relaxed energy approach.

Equations of Motion and Their Solution

The equations of motion are formulated and solved (for both flutter suppression and gust alleviation problems) following identical lines as outlined in Refs. 3-5, 7. The flutter results are presented by root locus type plots taking the dynamic pressure Qp as a parameter. The gust alleviation and ride control results are obtained from a continuous gust program, using unit rms gust input based on a Von Kármán gust spectrum.

Control Law

The general form of the control law employed in this work was established in Ref 2 using the relaxed energy approach.
The control law for the t.e. control surface is given by the following general form:

$$\delta = -1.86(\alpha - \alpha_r) + R_T [4, 2.8] \left\{ \begin{array}{l} h - h_r \\ \frac{b}{b - h_r} \end{array} \right\}$$

where $\delta$ is the deflection of the t.e. control surface (see Fig. 2) and $h$, $\alpha$ denote the translation and rotation of the 30% chord point of the control surface mid-span section, respectively (see Fig. 2). The parameters $h$, and $\alpha$, similarly denote the translation and rotation of a reference point located along the center line of the fuselage and $b$ denotes the semi-chord length at the control surface mid-span section (see Fig. 5). $R_T$ is defined by the following expression (see also Refs. 2, 3):

$$R_T = \frac{a_s S^2}{S^2 + 2\gamma \omega_s \eta S + \omega_s^2} + \frac{a_s S^2}{S^2 + 2\gamma \omega_s \eta S + \omega_s^2}$$

The parameters $\alpha$, $\gamma$, $\omega_s$, are all positive and their values determined through an optimization program based on the gust response of the aircraft under consideration following the method of Ref. 3.

Mathematical Model

The equations of motion, included two rigid-body modes (plunge and pitch) and nine symmetric elastic modes. The generalized aerodynamic forces were computed using the Doublet-Lattice method. The generalized inertia and elastic matrices for the flutter model were supplied by the aircraft manufacturing company together with the mode shapes. The t.e. control was assumed to be mass balanced.

Objectives

The following objectives were set for the present work:

1) To define control systems for different values of assumed maximum flight dynamic pressure (with $M = 0.2$) to determine whether an upper bound exists for flutter speed (while activating a single t.e. control).

2) To check the effectiveness of the resulting flutter suppression systems in reducing the wing root bending moments (b.m.) and in reducing the accelerations of the aircraft due to continuous gust inputs.

3) To spot check the effectiveness for flutter suppression of a control system, defined as in objective 1, above, at a higher Mach number, such as $M = 0.9$.

Presentation and Discussion of Results

The presentation and discussion of results will be grouped under three major headings involving flutter suppression, gust alleviation and ride control characteristics.

Flutter Suppression Systems

The effectiveness of the activated t.e. control system can only be assessed by comparison with the open-loop system. The open-loop root locus plots for $M = 0.2$ and $M = 0.9$ are presented in Figs. 3 and 4. It can be seen that for $M = 0.2$, two flutter dynamic pressures ($Q_{DF}$) exist: the first with $Q_{DF} = 32$ psf for zero structural damping $g$ and $\omega_p = 89.7$ rad/s, and the second with $Q_{DF} = 82.5$ psf (for $g = 0$) and $\omega_p = 25.1$ rad/s. Similarly, for $M = 0.9$, three flutter speeds exist with the following values (for $g = 0$): $Q_{DF} = 33$ psf with $\omega_p = 81.8$ rad/s, $Q_{DF} = 74.5$ psf with $\omega_p = 21.8$ rad/s, and $Q_{DF} = 78.5$ psf with $\omega_p = 68.7$ rad/s. Since some of the above flutter branches represent mild flutter instabilities the values of $Q_{DF}$ for the cases where $g = 0.015$ and $g = 0.03$ are included in a summarizing table (Table 1). It is interesting to note that the lowest value of $Q_{DF}$ increases from $Q_{DF} = 32$ psf at $M = 0.2$ and $g = 0$ to $Q_{DF} = 31$ psf at $M = 0.2$ and $g = 0.03$.

The optimization was performed at two different flight dynamic pressures: at $Q_D = 75$ psf and at $Q_D = 89$ psf, while maintaining $M = 0.2$. The optimization procedure yields the following optimal control laws:

For $Q_D = 75$ psf

$$\delta = \left[ \begin{array}{l} 0 \\ S^2 + 2 \times 1 \times 34.4S + (34.4)^2 \end{array} \right] \left[ \begin{array}{c} 0.5S^2 \\ S^2 + 2 \times 0.5 \times 100.6S + (100.6)^2 \end{array} \right] \left[ \begin{array}{c} 4, 2.8 \end{array} \right]$$

$$\left\{ \begin{array}{l} h - h_r \\ \frac{b}{b - h_r} \end{array} \right\}$$

with

$$\delta_{ms} = 14.37 \text{ deg/s/ft/s}$$

$$\delta_{ms} = 0.236 \text{ deg/s/ft/s}$$
For $Q_D = 89$ psi

$$a = \frac{1}{\sqrt{\frac{s^2 + 2 \times 0.5 \times 205 + (20)^2}{s^2 + 2 \times 0.5 \times 100 + (100)^2}}}, h = 0.85^\circ$$

with

$$\delta_{in} = 23.81 \text{ deg/s ft/s}, \delta_{on} = 0.789 \text{ deg/s ft/s}$$

The control law given by Eq. (3) will be referred to as control law I, whereas the one given by Eq. (4) will be referred to as control law II. The meaning of the different parameters of the control laws, Eqs. (3) and (4), is explained in Refs. 2 and 3. There is no intention to repeat the various details herein except for the statement that the above results show that for minimum control rates, maximum damping is introduced around the frequency of 100 rad/s whereas the minimum flutter frequency is around 90 rad/s. A secondary damping concentration is introduced by the above control laws at frequencies which vary with the optimization $Q_D$. For $Q_D = 75$ psi the frequency is around 34 rad/s whereas for $Q_D = 89$ psi the frequency is around 20 rad/s. Both frequencies relating to the secondary damping concentration are in the neighborhood of the frequency relating to the second open-loop flutter branch located around 25 rad/s.

**Closed Loop Performance**

The effectiveness of the above control laws in flutter suppression at $M = 0.2$ is shown in Figs. 5 and 6. As can be seen, the flutter branch relating to $Q_D = 32$ psi and $w = 89.7$ rad/s (for the open-loop case) is suppressed and yields no flutter up to the maximum dynamic pressure used for the root-locus plots (that is up to $Q_D = 120$ psi). On the other hand, the flutter branch associated with the open-loop values of $Q_D = 82.5$ psi and $w = 25.1$ rad/s is only slightly affected by the activated e system. For control law I, the value of $Q_D$ associated with this branch is increased to $Q_D = 84$ psi and for control law II to $Q_D = 89$ psi. The attempts to increase the values of this flutter branch beyond $Q_D = 90$ psi were not successful. This result is in interesting since the relaxed energy approach does not ensure the suppression of flutter in all cases, due to the fact that it does not turn all the aerodynamic energy eigenvalues positive (in the case of the activated e system). For a i.e.-ff system the suppression of flutter is ensured since all the aerodynamic energy eigenvalues assume positive values. Table 1 supplements the above mentioned results to include the effects of structural damping on the flutter speeds.

![Table 1 Summary of flutter results](image)

![Fig. 4 Open-loop root locus plot at $M = 0.9$](image)

* * *
Comparison between various results for the SST model at $M=0.2$.

Wing Root Bending Moment Alleviation

The quantitative effect of the activated t.e. system on the rms wing root bending moment (b.m.) is meaningful only for flight speeds which are below the open-loop flutter speeds. For speeds above the open-loop flutter speed the nonactivated rms wing root b.m. must clearly assume infinite values. Therefore, for flight speeds which lie between the open- and closed-loop flutter speeds the alleviation must therefore be infinite since the closed-loop system clearly yields finite rms values of b.m. The results to be presented herein will therefore relate to a range of dynamic pressures up to 32 psf which represents the open-loop value of $Q_{DF}$ for $g=0$ and $M=0.2$.

No attempt will be made to change the above value of $g$ or the above value of $M$. Figure 13 shows the variation of flight dynamic pressure of the rms bending moment ratio, defined as the ratio between the closed-loop and open-loop rms b.m. [denoted as $(b.m.)_c/(b.m.)_o$] for the abovementioned two control laws. Figure 14 shows the variation of $Q_{DF}$ of the ratio between the peak open- and closed-loop values of the b.m. as obtained from a PSD plot, an example of which is shown in Fig. 15 (with $Q_{DF} = 26$ psf using control law I). As can be seen, the alleviation in peak bending moments is much larger than the alleviation in rms b.m. at comparable values of $Q_{DF}$. It is also interesting to note that control law II is more effective in reducing peak values of b.m. and relatively ineffective (that is, yields only minor improvements over the results obtained from control law I) in reducing rms b.m. values. Hence, the increase in the control activity associated with control law II, although ineffective for flutter suppression appears to be effective for peak b.m. alleviation.

Control Surface Activity

The activity of the t.e. control (due to the different control laws) at the various flight dynamic pressures is shown in Figs. 10 and 11 for various values of $g$. It can be seen that control law II requires about 3.3 times as large rms control deflections as control law I, whereas rms control rates are larger by about 66% compared with control law I. Hence control law I appears to be better especially when considering that the difference between the overall flutter speeds due to those two control laws is small. Figure 12 shows that the control surface activity of control law I at $M=0.9$ is smaller than the activity at $M=0.2$.

substantially more effective than those reported in Ref. 1 for the same SST flutter model with $g=0.03$ and $M=0.2$.

Fig. 7 Variation of flutter dynamic pressure with dynamic pressure at which optimization is performed at $M=0.2$ ($g=0$).

Fig. 9 Comparison between various results for the SST model at $M=0.2$. 

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The quantitative effect of the activated t.e. system on the rms wing root bending moment (b.m.) is meaningful only for flight speeds which are below the open-loop flutter speeds. For speeds above the open-loop flutter speed the nonactivated rms wing root b.m. must clearly assume infinite values. Therefore, for flight speeds which lie between the open- and closed-loop flutter speeds the alleviation must therefore be infinite since the closed-loop system clearly yields finite rms values of b.m. The results to be presented herein will therefore relate to a range of dynamic pressures up to 32 psf which represents the open-loop value of $Q_{DF}$ for $g=0$ and $M=0.2$. No attempt will be made to change the above value of $g$ or the above value of $M$. Figure 13 shows the variation of flight dynamic pressure of the rms bending moment ratio, defined as the ratio between the closed-loop and open-loop rms b.m. [denoted as $(b.m.)_c/(b.m.)_o$] for the abovementioned two control laws. Figure 14 shows the variation of $Q_{DF}$ of the ratio between the peak open- and closed-loop values of the b.m. as obtained from a PSD plot, an example of which is shown in Fig. 15 (with $Q_{DF} = 26$ psf using control law I). As can be seen, the alleviation in peak bending moments is much larger than the alleviation in rms b.m. at comparable values of $Q_{DF}$. It is also interesting to note that control law II is more effective in reducing peak values of b.m. and relatively ineffective (that is, yields only minor improvements over the results obtained from control law I) in reducing rms b.m. values. Hence, the increase in the control activity associated with control law II, although ineffective for flutter suppression appears to be effective for peak b.m. alleviation.
Fig. 10 Variation of control surface activity with dynamic pressure (for various values of structural damping) at \( M = 0.2 \), using control law I (\( g = 0 \)): a) control surface deflection, b) control surface rate.

Fig. 11 Variation of control surface activity with dynamic pressure (for various values of structural damping) at \( M = 0.2 \), using control law II (\( g = 0 \)): a) control surface deflection, b) control surface rate.

Fig. 12 Variation of control surface activity with dynamic pressure (for various values of structural damping) at \( M = 0.5 \), using control law II (\( g = 0 \)): a) control surface deflection, b) control surface rate.

Fig. 13 Variation of rms wing root bending moment ratio with flight dynamic pressure at \( M = 0.2 \) (\( g = 0 \)).

Fig. 14 Variation of peak wing root bending moment ratio with flight dynamic pressure at \( M = 0.2 \) (\( g = 0 \)).
Acceleration Alleviation

For reasons similar to those given in the case of the b.m. alleviation, the acceleration alleviation is relevant for flight speeds up to the open-loop flutter speed. Here again, only the case relating to \( g = 0 \) and \( M = 0.2 \) will be treated. Figure 16 shows the variation of the rms acceleration ratio (defined as the ratio between the closed-loop and open-loop rms accelerations) at the center of gravity (c.g.) of the SST model. Figure 17 shows the variation of the peak acceleration ratio at the c.g. The results here are similar to those obtained for the b.m. ratio, that is, the activated systems are much more effective in reducing peak c.g. acceleration than in reducing rms accelerations at c.g. Similarly, control law II is relatively more effective in reducing peak c.g. accelerations than in reducing rms accelerations (compared with control law I).

The variation with \( Q_D \) of the rms acceleration ratio for a point on the wing located at the midchord of the midspan section of the outboard control surface (to be referred to as the wing point) is shown in Fig. 18. The ratio between the peak accelerations at the above point with and without activation of the control surface is shown in Fig. 19 as a function of \( Q_D \). As can be seen, the activated i.e. system is effective in reducing both the rms and the peak values of the accelerations at the above wing point. Here, control law II is substantially more effective than control law I in reducing both the peak acceleration ratio (similar to previously discussed cases) and the rms acceleration ratio (unlike previous cases where the difference was small).

Conclusions

The application of the relaxed aerodynamic energy method coupled with the previously developed synthesis techniques yields effective flutter suppression systems when applied to the SST flutter model. The effectiveness of the control laws obtained herein substantially exceed the effectiveness of
similar systems designed by classical methods as reported in Phase II of the SST Technology Follow-on Program. The application treated in this work follows two successful applications relating to the BQM-34E/F drone aircraft (DAST Program) and to the YF-17 external store flutter suppression program, as mentioned earlier in this paper. The beneficial effects of the flutter suppression system on both gust alleviation and ride control problems are in agreement with a previous work involving combined i.e.-i.e. control systems based on the original formulation of the aerodynamic energy method.

Cases can be envisaged where the effectiveness of the i.e. control system, based on the relaxed energy method, will be of doubtful nature. Such a case was encountered in this work when trying to increase the flutter speed associated with the second flutter branch. It is however felt that when such cases do arise, an alternative location of the activated control surface might prove to overcome this difficulty. Alternatively, a combined i.e.-i.e. control system might be attempted. Finally, it might be worth noting that the control surface activity as obtained from the derived control laws in the present application is within present-day technology capability.

Acknowledgments

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References


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REMOTE SENSING OF EARTH FROM SPACE: ROLE OF "SMART SENSORS"—v. 67

Edited by Roger A. Breckenridge, NASA Langley Research Center

The technology of remote sensing of Earth from orbiting spacecraft has advanced rapidly from the time two decades ago when the first Earth satellites returned simple radio transmissions and simple photographic information to Earth receivers. The advance has been largely the result of greatly improved detection sensitivity, signal discrimination, and response time of the sensors, as well as the introduction of new and diverse sensors for different physical and chemical functions. But the systems for such remote sensing have until now remained essentially unaltered: raw signals are radioed to ground receivers where the electrical quantities are recorded, converted, zero-adjusted, computed, and tabulated by specially designed electronic apparatus and large main frame computers. The recent emergence of efficient detector arrays, microprocessors, integrated electronics, and specialized computer circuitry has sparked a revolution in sensor system technology, the so-called smart sensor. By incorporating many or all of the processing functions within the sensor device itself, a smart sensor can, with greater versatility, extract much more useful information from the received physical signals than a simple sensor, and it can handle a much larger volume of data. Smart sensor systems are expected to find application for remote data collection not only in spacecraft but in terrestrial systems as well, in order to circumvent the cumbersome methods associated with limited on-site sensing.

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On Single-Degree-of-Freedom Flutter Induced by Activated Controls

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It is shown that activation of the trailing-edge control of an airfoil leads to single-degree-of-freedom type instabilities which span over a very wide region of reduced frequencies $\alpha$, including high values of $\alpha$ (unlike the nonactivated system). These instabilities are shown to be sensitive to changes in pitching axis location, control deflection phase angle, and values of the reduced frequency. These sensitivities of the single-degree-of-freedom system cause the activated airfoil to be potentially sensitive to changes in flight conditions, and may be the source of the many difficulties encountered in suppressing classical multi-degree-of-freedom flutter by means of active controls. The results presented herein relate to zero Mach number and to a 20% trailing-edge control surface.

Nomenclature

- $h$: semichord length
- $C$: control gain (see Eqs. (1) and (8))
- $\dot{\alpha}$: displacement of the quarter chord point, positive downward
- $l$: torsional moment of inertia per unit span
- $\text{Im} (\alpha)$: the imaginary part of the complex frequency $\alpha$
- $k$: reduced frequency $= \omega/\omega_0$
- $K$: structural stiffness
- $L$: aerodynamic lift force, positive downward
- $\alpha$: aerodynamic pitching moment, positive nose up
- $L_{\alpha}, L_{\alpha^2}$: oscillatory lift and moment coefficients due to plunging oscillation
- $L_{\alpha}, L_{\alpha^2}$: oscillatory lift and moment coefficients due to pitching oscillation
- $L_{\alpha}, L_{\alpha^2}$: oscillatory lift and moment coefficients due to control oscillation
- $\alpha^*$: cross moment of inertia (between torsional and control rotation degree of freedom)
- $\alpha^c$: flight velocity
- $\alpha, \beta$: angle of attack
- $\delta$: deflection of the trailing edge control surface, positive downward
- $\phi$: phase angle between $\alpha$ and $\beta$ or $h$ and $\beta$ [see Eqs. (1) and (8)]
- $\rho$: air density

Introduction

The classical aerelastic dynamics instability, known as flutter, is a result of the interaction of two or more structural degrees of freedom. Each of the fluttering degrees of freedom is stable in the absence of the remaining degrees of freedom. Single degree of freedom flutter instabilities are normally associated with either nonlinear aerodynamics or separated flows. There exists, however, a single degree of freedom type of flutter instability which is based on linear aerodynamics, and comes about when an airfoil oscillates in pitch around an axis located at the vicinity of its leading edge at very low values of reduced frequencies $\alpha$. This instability is known to originate from a negative aerodynamic damping term caused by the unsteady nature of the oscillating flow. This flutter pitching instability is, however, of academic nature only, due to the very low values of reduced frequency required for its existence. In all other cases (having somewhat higher value of $\alpha$), the aerodynamic damping term due to structural oscillations, are known to be always of positive definite nature.

Recent technological advances in automatic control technology have promoted a considerable number of investigations regarding the effects of active controls on problems of flutter suppression and gust alleviation. An active control system on a lifting surface such as a wing is designed to actuate a control surface in response to oscillations of the wing in a manner which stabilizes the system. Hence the activated control surface introduces considerable changes in the aerodynamic forces acting on the system. Since the determination of stability boundaries for multiple degree of freedom fluttering system using active controls can be carried out numerically for specific examples only, and since the results obtained normally lack in generality, it is proposed to investigate in the present paper the existence of instability boundaries involving activated single degree of freedom systems. Such single degree of freedom stability boundaries can serve to indicate the regions of definite instabilities in any activated multiple degree of freedom system, and they clearly fail to indicate the regions of stability for such systems. These simplified instability boundaries can therefore, help to define possible regions of stability in complex systems and promote some physical understanding of a complex problem.

Mathematical Model

The airfoil is assumed to oscillate in pitch around an axis located at $\alpha, \beta$ from its midchord point (positive direction of displacements and forces are shown in Fig. 1). The trailing edge control surface deflection is assumed to be driven by a control law of the form

\[ \delta = C_{\alpha} \cos \omega t \]

\[ \delta = C_{\beta} \sin \omega t \]
where $C$ denotes the control gain and $\psi$ the phase angle between $\alpha$ and $\delta$. In the absence of structural damping, the equation of motion in the pitching degree of freedom assumes the form

$$I\ddot{\alpha} + K\alpha + R\delta = -L(x_4 + 0.5)b + M$$

where $I$, $K$, $R$ are inertia, cross-inertia, and stiffness terms, respectively, and $L$, $M$ are given by

$$L = \pi pb^2 \omega^2 \left[ L_b h + L_a x + L_\delta \right]$$

$$M = \pi pb^2 \omega^2 \left[ M h + M_a x + M_\delta \right]$$

The coordinate $h$ refers to the displacement of the quarter-chord point (positive downward). $L_b$, $M_b$, $L_a$, $M_a$, $L_\delta$, and $M_\delta$ are complex aerodynamic coefficients which depend on $k$ and on the Mach number. For further definition of the notation, see Fig. 1.

Ignoring the inertia coupling with the control surface ($R_\delta$), and substituting Eqs. (1) and (3) into Eq. (2) and rearranging, the following equation is obtained using the relation $h/b = -(x+0.50x)$:

$$I\ddot{\alpha} + K\alpha + R\delta = -L(x_4 + 0.5)b + M$$

Remembering that the values of the various aerodynamic derivatives are complex, that $I$, $K$ are real and positive, and assuming the system to be statically stable, we obtain [from Eq. (4)] the following condition for dynamic instability:

$$\text{Im} \{ (x_4 + 0.5)L_b - (x_4 + 0.5) \} \alpha = 0$$

where $\text{Im}$ denotes "imaginary part of." It is interesting to note that Eq. (5) contains aerodynamic terms only. For any constant value of Mach number, instability boundaries can therefore be plotted using Eq. (5), for various values of reduced frequency $k$, of pitching axis locations $x_4$, of control gains $C$, and of phase angle $\psi$.

For the limiting case of pure bending oscillations of an activated control system (with mass balanced control surface), the following equation of motion is obtained:

$$Bh + kh = l.$$ 

Assuming the control law

$$\delta = C(h/b)e^{\psi}$$

and substituting Eqs. (7) and (8) into Eq. (6), the following condition for dynamic instability in pure bending is obtained:

$$\text{Im} \{ L_a + L_\delta e^{\psi} \} > 0$$

In this case, the instability boundaries (Eqs. 9) are functions of $k$, $C$, and $\psi$ only (for any given constant value of Mach number).

**Presentation and Discussion of Results**

The instability boundaries for the single-degree-of-freedom, nonactived system will first be presented for purposes of subsequent comparison with the activated system. The pure bending instability boundaries of the activated system will then be presented in the form of $C$ vs $1/k$ for various values of $\psi$. Finally, the pitching instability boundaries of the activated system will be presented in a series of graphs. Each graph relates to a constant value of $C$ and the boundaries are presented in the form $x_4$ vs $1/k$ for various values of $\psi$. The Mach number is kept equal to zero throughout this work. The system is assumed to have a 20% chord trailing-edge control surface. The aerodynamic derivatives are computed using analytical expression following the method of Ref. 7.

**Instability Boundary for the Unactived System**

Figure 2 shows the unstable region caused by pitching oscillation as a function of the pitching axis location $x_4$ and $1/k$. It can be seen that instability starts around the value of $1/k > 25$ or $k < 0.04$. Furthermore, the critical location of the pitching axis is around the leading edge (that is, $x_4 = -1$). The instability boundary in Fig. 2 has been known for many years and it has little practical value due to the very low values of $k$ associated with this instability.
Instability Boundaries for the Activated, Pure Bending Oscillation

Figure 3 shows the instability boundaries $C$ vs $1/k$ for various values of phase angle $\psi$. The unstable region lies above the various curves, whereas the stable region lies below them. The gain $C$ is made to vary between 0 and 2 and the angle $\psi$ is varied between 0 and $\pm 180$ deg. For negative values of $C$ (i.e., $-2 < C < 0$), the instability curves shown in Fig. 3 have the form of their reflection (about the abscissa) with the values of $\psi$ changed to $(\psi + 180$ deg). This point will be discussed further in a subsequent section of this paper.

It is very interesting to note that:

1) Activated single-degree-of-freedom bending instability occurs over a very wide range of values of $k$ (not necessarily low values of $k$).

2) Phase angle changes between 0 and 90 deg promote the instability, with $\psi = 90$ deg as the most critical angle. The instability subsides as $\psi$ is further changed toward $\psi = 180$ deg. Positive values of phase angles (0 deg $< \psi < 180$ deg) do not show any instabilities within the positive range of values of $C$, as shown in Fig. 3.

Instability Boundaries for the Activated Pitching Oscillation

Figures 4-11 present the instability boundaries of the activated system. Each figure relates to a different fixed value of gain $C$ and shows the effects of the pitching axis location $x_A$ and the reduced frequency $k$ on the instability boundaries. A careful study of the figures shows that:

1) The instability boundaries cover a very wide range of $k$ values, including a high value of $k$.

2) The instability regions increase as the gain $C$ is increased.

3) The largest instability regions are obtained for phase angle of $\psi = \pm 90$ deg, with instabilities for both values ranging with $C = 0.5$.

4) The least unstable location of the pitching axis lies around the midchord region (i.e., $x_A = 0$).

5) The phase angles $\psi$ which maintain stability throughout the various values of $C$ and $x_A$ lie in the first quadrant within $0 < \psi < 15$ deg (that is, in the region of $\psi = 15$ deg).

6) A second range of values of $\psi$ which maintains stability, except for large values of $C$ (that is, $C > 1.8$), lies in the third quadrant around $\psi = 180$ deg. For $C = 2$ and $\psi = 180$ deg, the region of instability is very narrow (around the midchord region).

7) For $0 < \psi < 180$ deg, two distinct regions of instability often occur (see, for example, Figs. 7-11), with one region located at very high values of $k$ (that is, at very low values of $1/k$).

8) The shapes of the instability regions vary considerably with the reduced frequency $k$. Hence, the employment of unsteady aerodynamics is essential for activated flutter analysis.

Closed-Form Expressions of the Effects of Control Surface on the Stability Boundaries

It has been shown that in the absence of control surface rotation, single-degree-of-freedom instability can only occur for pitching oscillations provided $1/k > 25$ (see Fig. 2). Since the remaining figures presented in this paper (i.e., Figs. 3-11) cover the range of $0 < 1/k < 25$, it follows that instabilities within this latter region must be brought about by the detrimental effects of control surface rotation. These detrimental effects can be isolated from Eqs. (5) and (9) to yield closed-form expressions. A study of these expressions
where the added subscripts R and I denote, respectively, the real and imaginary parts of the associated parameters (i.e., $L$ in the preceding case).

The value of $L_{4R}$ is about one order of magnitude larger than $L_{4I}$ over most of the $1/k$ range $\psi > 0.1$ $k > 1.5$. Hence, instability is largest around $\psi = 90$ $deg$ for the preceding $1/k$ range. Equation (10) also shows how the real and imaginary parts of the control surface lift coefficient are turned into a pure bending damping coefficient through the phase angle $\psi$ and control gain $C$. It is worth noting that the following identity:

$$C[L_{4R} \sin \psi + L_{4I} \cos \psi] = -C[L_{4R} \sin(\psi + 180 deg) + L_{4I} \cos(\psi + 180 deg)] \quad (11)$$

implies that instability boundaries with positive gain values may be replaced by identical boundaries with negative gain values, provided the corresponding values of the phase angle $\psi$ are increased by 180 $deg$ (as already noted earlier in this subsection). It may also be observed that the destabilizing effect of the control surface rotation is directly proportional to the control gain $C$.

### Control Surface Effects in Pure Pitching Oscillations

The destabilizing effects of the control surface during pitching oscillations can easily be isolated from Eq. (5) to yield

$$\text{Im}(C \psi [M_{4} + (x_{4} + 0.5) L_{4R}]) > 0$$

or, alternatively,

$$C[L_{4R} \sin \psi + L_{4I} \cos \psi] > 0 \quad (12)$$

Here again the control surface aerodynamic coefficients are transformed into main surface damping coefficients through the phase angle $\psi$ and control gain $C$. The inequality expressed by Eq. (12) depends not only on the relative values of $L_{4R}$, $L_{4I}$, $M_{4R}$, $M_{4I}$ (which vary with $k$) but also on the pitching axis location $x_{4}$ which, in turn, affects the damping of the main surface through the remaining terms in Eq. (5). Hence, the effects of the various parameters on the instability boundaries are of complex nature. Even the dominance of $M_{4R}$ over $M_{4I}$ is limited to a lower reduced frequency range ($1/k$ greater than about 4) than the corresponding one associated with the $L_{4}$ coefficient. Hence, for $1/k > 4$, the...
Some Remarks on Flutter Suppression of Activated Systems

It can be seen that for almost any chosen phase angle, there exists a region of pitching axis locations for which single degree of freedom instability exists. This implies that an activated trailing edge control may stabilize a mode of the pitching axis located outside the unstable region and yet may lead to a severe instability of another mode whose pitching axis falls within the unstable region. Similar sensitivities to changes in phase angles can also be observed (keeping the pitching axis constant), especially in the low region of 1/4 to 90 deg. These facts make stabilization both difficult and also very sensitive to modal and phase angle changes. It is well known that activated flutter suppression systems have a tendency to be sensitive to changes in flight conditions and flight configurations, in addition to their possible adverse effects on initially stable modes. It is, therefore, very possible that this sensitivity essentially originates from the aforementioned single degree of freedom instabilities rather than from the more complex multidegree of freedom flutter. It is also well known that the classical bending-torsion type of flutter is caused by the skew symmetric components of the real part of the generalized aerodynamic matrices. It can be shown that symmetries in the preceding matrix can be avoided if at 180 deg for 1 and 0.5 if at 0.5. Therefore, classical flutter will not occur for values of c equal to those just specified (dependent on k). Hence, from classical flutter point of view, there is a third quadrant, around 0.5, 180 deg. As already noted, the region of 0.5, 180 deg leads to single degree of freedom instability for values of c>1.8 and is inferior to the first quadrant values from the point of view of the single degree of freedom type instability. Hence, if c is limited to a value of c<1.8 and 0.5, 180 deg, no single degree of freedom flutter will occur, but classical flutter may occur. If, on the other hand, c is given the value of 1.8 or larger depending on k, no classical bending-torsion flutter can occur, but a single degree of freedom instability will take place. Hence, a system may exist (having c<1 around the midchord region), for which stabilization by means of activated trailing edge control is impossible. The stabilization of such systems can only be achieved if modal changes are introduced that cause the pitching axis to shift from the midchord region. These results are in agreement with those obtained by the use of the aerodynamic energy concept.

Conclusions

It has been shown that activation of the trailing edge control of an airfoil leads to single degree of freedom type instabilities which span a very wide region of reduced frequencies k, including high values of k (unlike the nonactivated system). The origin of these instabilities lies in the introduction by the control surface of negative aerodynamic damping forces. This implies that aerodynamic damping forces must never be neglected while performing flutter analysis of activated control systems (unlike many instances of nonactivated flutter problems). Furthermore, since the instability boundaries vary considerably with the reduced frequency k, oscillatory aerodynamic coefficients must always be used in active control flutter analysis. The sensitivities of the activated single degree of freedom system to changes in pitching axis location, control deflection phase angle, and values of the reduced frequency cause the activated airfoil to be potentially sensitive to changes in flight conditions and may be the source of the many difficulties encountered in suppressing flutter by means of active controls. Some incompressible flow has been assumed throughout this paper, it is left that further work is required to determine the possible effects of compressibility on the single degree of freedom instability reported herein.

Acknowledgments

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References


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The following award will be presented during the AIAA Guidance and Control Conference, August 13, 1980, Danvers, Mass. If you wish to submit a nomination, please contact Roberta Shapero, Director, Honors and Awards, AIAA, 1200 Avenue of the Americas, N.Y., N.Y. 10019 (212) 861-3300. The deadline for submission of nominations is January 3, 1980.

Mechanics and Control of Flight Award

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Abstract

The paper presents the current state-of-the-art of the aerodynamic energy concept. The latest applications of the relaxed energy concept, most of which are as yet unpublished, are also presented in this paper. These applications include the suppression of flutter in three different configurations of the YF-17 flight model, using single trailing-edge (T.E.) control surface activated by single, fixed gain, control law. Also included are some initial results regarding the suppression of flutter on the 1/20 scale, low speed wind-tunnel model, of the Boeing 2707-300 super-sonic transport, using an activated T.E. control surface. Additional results regarding comparative study between activated leading-edge - T.E. and T.E. systems are also presented together with a review of previously published formulations and applications.

Introduction

The ability of the aerodynamic control surfaces to promote flutter instabilities has been known for many decades. Classical books in the field of Aeroelasticity (1) include considerable material to this effect under such headings as "bending-aileron flutter" or "torsion-aileron flutter". These control surface induced flutter instabilities are traditionally overcome by reducing the deflections of the control surfaces by mass balancing of the control surfaces. It seems therefore reasonable to assume that this ability of the aerodynamic control surfaces to promote flutter could be reversed by appropriate control of their deflection, so as to combat the main lifting surface flutter instability, such as the wing bending-torsion flutter. Indeed, to put it differently, the origin of flutter lies in the nature of the oscillatory aerodynamic forces which permit the transfer of energy from the airstream to the wing. This flow of energy could be controlled, in principle, by modifying the aerodynamic forces through appropriate deflections of the control surfaces. The implementation of this approach requires, therefore, a rapidly responding control system which is activated by the motion of the main surface and which leads to an appropriate deflection of the control surface.

The introduction of such activated control surfaces is not limited to problems of flutter suppression. Their potential applications span over a wide class of problems related to the improvement of performance of aircraft. The recent technological advances made in the field of control systems and the increased reliability of control system components, brought about by the space program, have paved the way for the incorporation of increasingly sophisticated control systems in aircraft. In his AIAA Von Karman Lecture (2), I.E. Garruck states: 'A major current trend which will play a dominant role in research, development, and practice during the years ahead is the union of modern control technology and aeroelasticity; for example, in control configured vehicles (CCV).'

Although aeroelasticians and control specialists have in the past usually gone their separate ways and both fields have become quite sophisticated, in the last few years there have been attempts at real cooperation and adaptation to each other's methods so that important information has been published. Among the numerous proposed applications in CCV are: relaxed aerodynamic stability, gust and maneuver load alleviation (with fatigue damage reduction through modal suppression), ride quality control, flutter suppression, taxi load alleviation and automatic control of variable geometry. As could be expected some of the proposed applications have recently come to fruition: An active control system has been installed on the B-52 aircraft (3,4) to control the response of the rigid body mode and one elastic mode (first aft body bending) to gust inputs. Flutter suppression by active controls has been demonstrated in flight (5) on the B-52 airplane (the mild flutter instability was induced by an added ballast tank). Other applications relating to the control of the rigid body modes have been incorporated in several military development areas, including the YF-16 aircraft. Applications relating to the suppression of external store flutter are currently under way for the F-4 airplane. (6,7) In addition, a number of feasibility studies have been made to assess the merits (in terms of weight saving and of performance increase) of applications of active control technology to aircraft (8-13). Some of these studies were supplemented by comprehensive wind-tunnel validation programs. (10,15)

As can be seen, the use of active controls spans a wide class of problems. However, one of the main difficulties which characterizes the introduction of active control systems into existing structures lies in the tendency of the activated systems to be very sensitive to system changes caused by the different flight conditions (such as flight speed, flight altitude, flight duration and type of mission). This sensitivity implies that a control system which is optimized at one flight condition may either show considerable degradation, or even give rise to adverse effects at other flight conditions.

The aerodynamic energy concept was formulated (16) in an attempt to define active control systems which do not exhibit such sensitivities to changing flight conditions. This is no intention presented herein a review of the extensive literature in the field of active control of aeroelastic response, nor is there any intention to review the different approaches and methods available for synthesis. Attempt will only be made in the present paper to review the developments of the aerodynamic
energy approach, together with its applications, to problems of flutter suppression and gust alleviation (with emphasis on flutter suppression problems). Whenever possible, comparisons will be made between results obtained by the aerodynamic energy method and those obtained by other methods such as classical or modern control theory.

**The Aerodynamic Energy Approach**

**Basic Concept**

The aerodynamic energy concept was developed primarily for problems of flutter suppression using active controls. It hinges on the idea that since flutter instabilities originate from the nature of the aerodynamic forces, the roots of their suppression should clearly lie in the ability to modify these forces. The above idea can be implemented provided the following problem can successfully be treated: given a fluttering system and a control surface which can be activated, what should be the relationship between the oscillation of the system and the deflection of the control surface (normally referred to as “control law”) that will ensure the necessary changes in the aerodynamic forces. This problem has been treated in refs. 10, 17. Major points relating to analysis and results are presented in the following section.

**The Energy Analysis**

Let the n equations

\[ \dot{q}^T = -\left( B + k b^b a^b(A_R + i A_T) \right) (q) + (E) (q) \]  

represent the equations of motion of n structural modes with r activated controls, where at flutter

\[ (F) = 0 \]

and where \( \omega \) represents the frequency of oscillation; \( B \), the mass matrix; \( A_R \) and \( A_T \), the real and imaginary parts of the aerodynamic matrix, respectively; \( E \), the stiffness matrix; \( p \), the density of the fluid; \( s \), reference length; \( b \), a reference semichord length; and \( q \), the response vector. The matrices in equation (1) can be partitioned into square matrices \( (n \times n) \) relating to the structural modes (subscripted by \( s \)) and rectangular matrices \( (n \times r) \) relating to control surface couplings (subscripted by \( c \)). After partitioning the matrices, equation (1) becomes

\[ (F) = \omega^2 \left[ B + k b^b a^b q^s + \left( A_R s^s \right)^T + \left( A_T c^c \right)^T \right] q^s + 2 \left( A_R s^s \right)^T \left( A_T c^c \right)^T \]

Assume a control law of the form

\[ (q) = [q_R + i q_T] \]

where \( [q] \) is a \( (r \times n) \) matrix representing the transfer functions of the control law, and assume that no elastic couplings exist between structural modes and control deflections, thus causing \( E_R = 0 \). It can be shown that the work \( P \) done by the system on its surrounding per cycle can be written as

\[ P = \frac{1}{2} \left( [A_R s^s] + [A_T c^c] \right)^T \left( B + k b^b a^b \right) - \frac{1}{2} \left( [A_T c^c] \right)^T \left( B + k b^b a^b \right) [A_R s^s] + \frac{1}{2} \left( [A_T c^c] \right)^T \left( B + k b^b a^b \right) [A_T c^c] \]

and where

\[ [A_R s^s] + i [A_T c^c] = \left( q_R + i q_T \right) \]

The sign \( \omega \) determines stability, and therefore it is necessary to convert equation (4) to a more convenient form. It can be shown that \( P \) can be reduced to the form

**Discussion of Energy Concept**

The work per cycle \( P \) done by the system on its surroundings has a direct bearing on the stability of the system. If \( P \) is positive, the system is dissipative, and therefore stable. If \( P \) is negative, the system is unstable because work is done by the surroundings on the system. Equation (8) shows that if all the eigenvalues \( \lambda_i \) are necessarily real, of the Hermitian matrix \( [U] \) (as given by eqn (5)), and where the vectors \( [q_R] \) and \( [q_T] \) are defined by the transformation

\[ (q) = [q_R + i q_T] \]

The matrix \( [q_R + i q_T] \) is a square modal matrix of the principal eigenvectors.

**For mass-balanced control surfaces \( (B_R) = 0 \), the eigenvalues \( \lambda \) obtained from \( [U] \) (Eq. (5)) are dependent only on the aerodynamic properties of the system and the activated control law (matrix \( [T] \). In the case of mass-balanced surfaces, the eigenvalues are referred to as aerodynamic eigenvalues. These latter eigenvalues are, in general, functions of the reduced frequency \( k \) and Mach number \( M \). If mass unbalance is a fixed quantity in the system, the eigenvalues \( \lambda_i \) also depend on the fluid density \( p \) in addition to their dependence on \( k \) and \( M \). Note that instability at zero airspeed can be brought about only through these mass unbalance...
terms. All the results presented in this paper relate to mass-balanced control systems only and therefore, aerodynamic eigenvalues are obtained from the following [U] matrix

\[
[U] = -[A_{1,s}]^{T} + [A_{1,s}]^{T}[T] + [T]^{T}[A_{1,s}]^{T} + [A_{1,s}]^{T}[T]^{T}[A_{1,s}]^{T} + [A_{1,s}]^{T}[T]^{T}[A_{1,s}]^{T}
\]

It may be recalled that the energy approach, in its original development (16), sought to determine the matrix [T] to render all the aerodynamic eigenvalues (of matrix [U] eq. (10)) large and positive. This requirement regarding the aerodynamic eigenvalues insures both the stability of the system (since \( P \) is always positive) and its insensitivity to various flight conditions (which manifest themselves in the form of changing values of \( \lambda \) and changing values of the system responses \( T \)).

**Generalized Model**

The energy approach has been formulated for a general n degree of freedom system. Therefore, the energy concept can be applied to any problem. The results of such application, however, will be specific for the system considered since the generalized aerodynamic forces depend not only on the system geometry but also on its structural natural model \( P \). Thus, however, the energy concept as applied to a two-dimensional strip, the aerodynamic matrix are independent of geometry and span strips. As a result, the aerodynamic eigenvalues are independent of any specific aerodynamic Eq. of functions of \( k \), \( M \), and the transfer function matrix \([T]\). Therefore, if \([T]\) is obtained using a two-dimensional strip as a model, these \([T]\) values are applicable to any three-dimensional wing within the limitations of strip theory. In general, the model is generally applicable. Sketch (a) illustrates the generalized model considered, and the arrows indicate positive displacements and rotations.

**Analysis of the Generalized Model**

The motion of the generalized two-dimensional model is defined by two parameters: the displacement \( h \) of the 30 per cent chord point and the rotation \( \alpha \) about this point. Two control surfaces are assumed to be available for activation: a 20 per cent chord trailing-edge (T.E.) control and a 20 per cent chord leading-edge (L.E.) control. All the results presented in this paper relate to mass-balanced control systems only and therefore, aerodynamic eigenvalues are obtained from the following [U] matrix

\[
[U] = -[A_{1,s}]^{T} + [A_{1,s}]^{T}[T]^{T}[A_{1,s}]^{T} + [A_{1,s}]^{T}[T]^{T}[A_{1,s}]^{T}
\]

The matrices \([C]\) and \([G]\) were assumed to have constant values (in eqn (11)) thus making the subsequent mechanization of the control law difficult. The matrix \([C]\) was determined numerically by an optimization program which required \( \lambda_{\text{min}} \) to be positive and large over a wide range of \( k \) values. This was achieved by maximizing the area under the curve \( \lambda_{\text{min}} \) vs \( 1/k \) using the \( C \) and \( G \) terms as parameters.

It should be stressed at this stage that the generalized two-dimensional model adopted herein serves only to indicate, on the basis of the strip theory, whether energy is dissipated or absorbed by the partial span strip where the activated controls are installed. Therefore, in order to suppress flutter with a minimum number of activated partial span strips, one should aim at dissipating enough energy in the activated strip, so as to compensate for any energy input by the nonactivated portions of the wing. One should therefore attempt not only to turn \( \lambda_{\text{min}} \) positive but also to cause \( \lambda_{\text{min}} \) to assume large (and positive) values.

**Results of the Original Formulation of the Energy Concept**

Typical results obtained with \( M=0 \) using the procedure just described (16) are presented in Fig. 1 for the unactivated system, in Fig. 2 for the activated T.E. control and in Fig. 3 for the activated combined L.E.-T.E. control system (for further details see ref. 16). The optimized values of the transfer functions \([C]\) and \([G]\) for these two types of activated systems are given by

\[
[C]_{\text{opt}} = \begin{bmatrix} 0 & 0 \\ -0.35 & -1.9 \end{bmatrix} \quad [G]_{\text{opt}} = \begin{bmatrix} 0.35 & 0.1 \\ 0 & 0 \end{bmatrix}
\]

\[
[C]_{\text{opt}} = \begin{bmatrix} 0.5 & 1.0 \\ -0.45 & -1.7 \end{bmatrix} \quad [G]_{\text{opt}} = \begin{bmatrix} 0.45 & 0.2 \\ -0.5 & 1.0 \end{bmatrix}
\]

The following points emerging from these figures are worth noting:

1) The value of \( \lambda_{\text{min}} \) for the in-activated system (Fig. 1) is negative throughout the range of \( k (0.0128 < k < 19.5) \) and the value of \( \lambda_{\text{max}} \) is positive throughout this same range. Furthermore, the absolute values of \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) are of the same order of magnitude.

2) The values of \( \lambda_{\text{min}} \) for the T.E. system (Fig. 2) is only marginally positive (except at high \( k \) values) and is highly sensitive to off-design values. The values of \( C_{22} \) which improve \( \lambda_{\text{min}} \) cause \( \lambda_{\text{max}} \) to deteriorate appreciably.
3) The optimum values of $\lambda_{\text{min}}$ for the combined L.E.-T.E. control system (Fig. 3) is large and positive over the whole range of $1/k$. The off-design sensitivity is greatly reduced as compared with the T.E. control system. Here again, the values of $C_{22}$ which improve $\lambda_{\text{max}}$ cause $\lambda_{\text{min}}$ to deteriorate.

The results presented in ref. 16 indicate the following additional important points:

4) Systems having two sensors (to determine both $h$ and $\alpha$) are superior to any single-sensor system.

5) Mach number effects are beneficial for the whole $k$ range for the L.E.-T.E. system (Fig. 4) whereas the T.E. system shows minor improvements except for the very low range of $k$ values where some deterioration takes place.

6) The values of $\lambda_{\text{min}}$ (and $\lambda_{\text{max}}$) for the L.E.-T.E. control system could be increased considerably by the simultaneous increase of all the $G_{ij}$ terms by a constant factor $\omega/\omega_r > 1$ (see fig. 5). The T.E. control system showed a deterioration in $\lambda_{\text{min}}$ accompanied by a considerable improvement in $\lambda_{\text{max}}$ when such an increase in its $G_{ij}$ terms was attempted (see ref. 16). Thus, the control law for the L.E.-T.E. control system could be brought to the following convenient form

\[ (\delta) = [C] \left\{ \begin{array}{c} h/b \\ \alpha \end{array} \right\} + \frac{1}{\omega_r} \left[ G \right] \left\{ \begin{array}{c} \dot{h}/b \\ \dot{\alpha} \end{array} \right\} \]  

(12)

where $\omega_r$ is a reference frequency which maintains the non-dimensional nature of eqn. (12). Clearly, the mechanization of this latter control law is much simpler than the one given by eqn. (11).

The above results led to the conclusion that the L.E.-T.E. control system, driven by two sensors, is the most effective system for purposes of flutter suppression. For this reason the L.E.-T.E. system was chosen for testing the effectiveness of active controls in the early applications of the energy method. However, before proceeding to these applications, a few points should be mentioned regarding the physical significance of the optimized control laws (see sketches (b) and (c)). The optimized L.E.-T.E. control law will be chosen for this purpose since it includes the essential features of the two control surfaces employed by the generalized model.

It is interesting to note that the main effect of the in-phase deflections of the control surfaces is to counteract any lift building up; that is, the lift increase due to the angle of attack $\alpha$ is opposed by the forces created by the deflections of the L.E. and T.E. control surfaces. Furthermore, the out-of-phase control deflections increase the damping forces. It can therefore be seen that flutter suppression is achieved by both reducing the energy input into the system and increasing the dissipation of energy.

**Early Applications of the Aerodynamic Energy Concept**

The first application of the results produced by the aerodynamic energy concept was made using a SST-type wing for which detailed analysis using at least 10 degrees of freedom already existed (18). The application was carried out by members of the Boeing Wichita division under contract to the Langley Research Center. The wing configuration is indicated in Fig. 6. Flutter control was achieved using several independent stripwise units each of which consisted of combined L.E.-T.E. control surfaces having 20% chord each and activated by sensors located at 30% and 70% chord locations, (using a control law as given by eqn. (11)). The results, employing $M=0.9$ lifting surface aerodynamics, supplemented by strip theory for the control strips, indicated that the use of T.E. controls alone would increase the flutter speed by only a few percent (≈ 5%) while the use of the combined L.E.-T.E. systems yielded with outboard segment A alone an 11% increase, with mid segment B alone 28% increase, and with inboard segment C alone 21% increase in the flutter speed. The combined use of B and C led to an increase in flutter speed not specifically determined but noted to be in excess of 41% of the original speed. A root locus plot corresponding to this case is shown in Fig. 7. A corresponding experimental exploratory study (19) was undertaken in the Langley Transonic Dynamics Tunnel using a simplified version of a proposed supersonic transport wing design (Fig. 8). The active flutter suppression method, based on the aerodynamic energy criterion, was verified experimentally using three different control laws (as defined by eqn (11)). The first two control laws utilized both leading edge and trailing-edge active control surfaces, whereas the third control law required only a single...
T.E. control surface. At Mach number 0.9 the experimental results demonstrated increases in flutter dynamic pressure from 12.5 per cent with a L.E.-T.E. active control system to 30 per cent with active T.E. control. The mechanisation of the L.E. control has met with great difficulties due to what has been believed to be a control induced instability caused by the mass unbalanced L.E. control. As a result of this instability the L.E. control which was present even at zero air speeds activation of the L.E. control could only be attained at Mach 0.9. Nevertheless, two important points follow this essentially experimental study (15).

1) An active flutter suppression system was demonstrated successfully, using L.E. and T.E. control surfaces, to suppress flutter on a model in a wind tunnel.

2) Irrespective of the difficulties encountered in the mechanization of the L.E. control, it is still significant to note that a single L.E. control yielded satisfactory results in suppressing flutter over the entire range of Mach numbers tested.

Several different analytical applications of the 1/20th scale model in the horizontal and vertical tail sections of the test model were considered. Two types of tail sections were considered: an adiabatic approach (20) based on aerodynamic similarity. These aircraft are the twin-boom, twin-tail Arava SDI transport (maximum mass 9400 Kg., Fig. 9) and the Westwind, twinjet transport (maximum mass 9400 Kg.) which is a modified version of the Rockwell Jet Commander.

The wing on each aircraft was divided into equal spans containing a pair of active control strips. The inboard strip could accommodate a pair of active control strips (20% chord L.E.-T.E. controls), the active strip located along the horizontal tail were the span equal to one third and one tenth of the root region of the wing but inboard of the Arava and Westwind respectively. The best locations for a single activated system along the span of the wing was determined for bending moment alleviation, flutter reduction, and fuselage acceleration, and elastic suppression. Reference 19 deals with the active strip location for the ordinary control (see Fig. 11). The optimum strip location for maximum bending-moment reductions is in the tip region of the wing but inboard of the tip strip.

2) The optimum strip location for maximum alleviation of flutter speeds in the tip strip. Furthermore, the effectiveness of the activated strip is greatly increased by the introduction of the extended control law. Flutter speeds could easily be increased by more than 70% of the open loop flutter speeds.

3) The optimum strip location for maximum reductions in fuselage accelerations is at the root strip location for the ordinary control law (Fig. 13). The extended control law yields better results with optimum strip location at the inboard region of the wing (not clearly on the reference strip, see Fig. 13).

In summarizing the results of the above applications (19), it may be stated that the extended control law, which is based on the wing elastic deformations, presents a major step forward in the alleviation of flutter suppression and gust alleviation. It leads to almost complete decoupling between the rigid body responses, elastic responses, and the activated control forces. As a result, lower improvements in performance are obtained. For this reason, free flying wind tunnel models might show greatly reduced performance as compared with clamped models unless some form of extended control law is used.

The above applications have shown that the energy concept produces effective activated systems. There were indications, however, that the derived control laws could be improved and that the mechanization of the L.E. control was more involved than that of the T.E. control. Furthermore, some of the control laws (such as the one defined by eqn (11)) were difficult to realize. This led to an investigation aimed at avoiding the use of the L.E. control while maintaining the effectiveness of the activated system. The results of this investigation are described in the following section.
**Active Flutter Suppression Using Trailing-Edge and Tab Control Surfaces**

As already stated earlier in this paper, the L.E. control may present some control problems as it carries relatively large aerodynamic hinge moments. Furthermore, there has been some reluctance to introduce a L.E. control due to its possible detrimental effects on the general aerodynamic characteristics of the wing. The activated T.E.-tab combination, if effective for flutter suppression, could alleviate the difficulties associated with the L.E.-T.E. system. It is shown (21) that an 82 chord tab should be chosen for a 20X chord T.E. control. The results obtained (21) for the variations of $\lambda_{\text{min}}$ with $1/k$ show that the T.E.-tab system, activated by both linear and rotational sensors, has a flutter suppression performance comparable to the L.E.-T.E. system. The main advantage of the T.E.-tab system over the L.E.-T.E. system lies in its lower actuator torque requirements, whereas its main disadvantage lies in its relatively higher control surface rotational requirements. Applications pertaining to the T.E.-tab system were not further pursued in view of the progress made regarding the activation of T.E. alone control systems. Some details regarding these developments are presented in the following section.

**Relaxation of the Energy Concept**

**Objective and formulation of relaxed conditions.**

The energy approach, in its original development (10), sought to determine the matrix $[T]$ so as to render all the aerodynamic eigenvalues large and positive. This requirement regarding the aerodynamic eigenvalues ensures both the stability of the system (since $P$ will always be positive) and its insensitivity to the various flight conditions. Since the derived control laws are of general nature and do not take into consideration any specific property of the analyzed system, it is possible to argue that the limitations concerning the potentialities of the T.E. control system to perform effectively as a flutter suppressor is inherent in the above formulation of the problem. Assume that other methods of stabilization exist, or can be devised, and that all we wish to ensure is the insensitivity of the stabilized system to changes in flight conditions. The implications of such an approach on the energy concept involve the relaxation of the requirement that all the aerodynamic eigenvalues must be large and positive. Assume, therefore, that such a relaxation is now introduced which permits some of the aerodynamic eigenvalues to be negative. Stability can only be achieved under these conditions by modifying the responses of the system so as to render the responses associated with the positive eigenvalues to be the dominant ones. This latter requirement forms a necessary condition for stability but does not ensure, in itself, the insensitivity of the resulting stabilized system to the various flight conditions. In order to ensure that this relaxed stability requirement yields a system which shows only small sensitivities to the changing flight conditions the absolute values of the negative eigenvalues must always be made much smaller than those eigenvalues associated with the dominant responses of the stabilized system. For the generalized two-dimensional model adopted in this work, two aerodynamic eigenvalues, $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ are obtained. In the original derivation of the aerodynamic energy concept, $\lambda_{\text{min}}$ was required to be positive and large. In the relaxed energy approach, $\lambda_{\text{min}}$ is permitted to be negative provided

$$\lambda_{\text{min}} = \text{near maximum value (may be negative)}$$

$$\lambda_{\text{max}} \gg \lambda_{\text{min}}$$

and provided that these relations are maintained for all flight conditions. The above two requirements regarding $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ will be referred to as the "relaxed energy requirements". As can be noted, the above relaxation is made possible by abandoning the sufficiency condition for stability in the original formulation while maintaining its insensitivity to changes in flight conditions. It is worth noting that since the dissipation of energy by the activated strip depends both on $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$, the importance of $\lambda_{\text{max}}$ should not be overlooked even when $\lambda_{\text{min}}$ is positive and large. Considerable improvements in the potential performance of the activated control system may result, it changes in the control gains are permitted which lead to small degradations in $\lambda_{\text{min}}$, provided these degradations are accompanied by large increases in $\lambda_{\text{max}}$. This implies that while determining the optimum values of the transfer function matrix $[T]$ we seek to optimize not only the area under the $\lambda_{\text{min}}$ vs $1/k$ curve but also the weighted addition of the area under the $\lambda_{\text{max}}$ vs $1/k$ curve, so as to satisfy eqn (13). Convenient ways of performing the above optimization of the $[T]$ matrix are described in ref. 17.

In addition to the above relaxation of the energy concept, two other major changes were introduced in ref. 17:

1) Unlike the original derivation, only realizable transfer functions were considered.

2) The influence on the target function of the very low frequency portion of the $\lambda_{\text{min}}$ vs $1/k$ curve was reduced by both an appropriate redefinition of the aerodynamic eigenvalues and the reduction of the $k$ range from 0.0128 to $k < 19.5$

as used during the original derivation, to

$$0.04 \leq k \leq 3.5$$

The redefinition of the aerodynamic eigenvalues involves the inclusion of the frequency effects into these aerodynamic eigenvalues. Hence, eqn (6) was modified to the form

$$p = 4\pi^2 \nu_0 S^2 \left[ \lambda_{1} \left( \frac{r_{x,1}}{r_{n,1}} + \frac{r_{z,1}}{r_{n,1}} \right) + \lambda_{2} \left( \frac{r_{x,2}}{r_{n,2}} + \frac{r_{z,2}}{r_{n,2}} \right) + \ldots \ldots \lambda_{n} \left( \frac{r_{x,n}}{r_{n,n}} + \frac{r_{z,n}}{r_{n,n}} \right) \right]$$

yielding the following relation between the $\lambda_{i}$s

$$\lambda_{i} = k^{2} \lambda_{i}$$

Hence, at the low range of $k$ values, the newly
defined eigenvalues are smaller than the originally defined eigenvalues by a factor of \( k^2 \). These changes result in the gaining of more weight to the intermediate frequencies during the optimization process.

Optimization Results (17).

The portion of the non-activated \( Y \)‘s with \( 1/k \) is shown in Fig. 17. It is interesting to compare these \( Y \) with their \( \lambda \) counterparts in Fig. 1 and to note the large changes in the shape of the curves.

Two types of optimized transfer functions were derived (17). The first type is referred to as the damping type transfer function (D.T.T.F.) and it assumes the following optimum values for \([T]\):

\[
[T] = \begin{bmatrix}
0 & 0 \\
0 & -1.86
\end{bmatrix} + k \begin{bmatrix}
\sigma_t & 0 \\
0 & \sigma_T
\end{bmatrix}
\]

(16)

where \( \sigma_t \) and \( \sigma_T \) are positive free parameters. These free parameters were introduced as a result of the unbounded nature of the target function with respect to increase of these parameters. The transfer function for the T.E. alone system is obtained from eqn (16) by letting \( \sigma_t = 0 \).

The second type of optimized transfer function is referred to as the localized damping type transfer function (L.D.T.T.F.) and it assumes the following optimum values for \([T]\):

\[
[T] = \begin{bmatrix}
0 & 0 \\
0 & -1.86
\end{bmatrix} + R \begin{bmatrix}
\sigma_t & 0 \\
0 & \sigma_T
\end{bmatrix}
\]

(17)

where once again \( \sigma_t \) and \( \sigma_T \) are positive free parameters (which follow the unbounded nature of the target function with increase of these parameters) and \( R \) is given by

\[
R = \frac{(1k)^2}{(1k)^2 + 2k(1k) + k^2}
\]

(18)

where \( \sigma_t \) and \( \sigma_T \) are positive constants.

Fig. 18 shows the variation of \( \sigma_t \) vs 1/k and \( \sigma_T \) vs 1/k at various Mach numbers using the optimized D.T.T.F., as defined by eqn (16) with \( \sigma_t = 0 \) (i.e., T.E. only control system) and \( \sigma_T = 25 \).

The corresponding curves using the L.D.T.T.F. derived by eqns (17,18) are shown in Fig. 19 using the values of \( \sigma_t = 0 \), \( \sigma_T = 25 \) and \( \sigma_T = 0.5 \) and \( k = 0.2 \). It can be seen that the results corresponding to the L.D.T.T.F. of Fig. 18 satisfy the relaxed energy requirements as expressed by eqn (13) over the whole range of \( k \) investigated. The L.D.T.T.F. yields results (Fig. 19) which satisfy the relaxed energy requirements only around the peak region of the curves. The location of this peak region (along the 1/k axis) is around 1/km and the width of the curve (in addition to their height) are controlled by the parameter \( \zeta \). In addition, stiffness terms are introduced as \( R \) varies with \( k \). These terms vanish when \( k = 0 \) and therefore do not affect the static behaviour of the system.

They, however, can be used to change the response of the system, if necessary, so as to ensure stabilization. In general, several \( R \) values can be used, having different values of \( k \) at different \( \lambda \) distributions, with \( k \) is required while using the L.D.T.T.F.

(see ref. 22). For the L.E.-T.E. systems, large improvements in the values of \( \sigma_T \) are obtained (see ref. 17) with almost negligible effects on the values of \( \sigma_T \) (as compared with the T.E. alone control system).

The working forms of the above transfer functions are simplified to the following forms for purposes of subsequent applications:

For the D.T.T.F. matrix \([T]\) is given by

\[
[T] = \begin{bmatrix}
0 & 0 \\
0 & -1.86
\end{bmatrix} + \begin{bmatrix}
\sigma_t & 0 \\
0 & \sigma_T
\end{bmatrix}
\]

(19)

where \( \omega_n \) is a reference frequency, normally chosen as the open-loop flutter frequency. For the L.D.T.T.F., matrix \([T]\) is given by

\[
[T] = \begin{bmatrix}
0 & 0 \\
0 & -1.86
\end{bmatrix} + \begin{bmatrix}
R_L,1 \sigma_L,1 R_L,2 \sigma_L,2 \\
0 & \sigma_T,1 \sigma_T,2 \sigma_T,2
\end{bmatrix}
\]

(20)

where

\[
R_L = (L_1)^2 + 2 \zeta L_1 \zeta L_1 (\omega) + (\omega L_1)^2
\]

(21)

It can be seen that both transfer functions include parameters which can only be determined in connection with the system considered. The L.D.T.T.F. has more parameters for determination and has more potential regarding possible changes in the responses of the system. It is generally considered to be preferable to the D.T.T.F. On the other hand, the D.T.T.F. has less such parameters and, therefore, their values are much easier to determine.

Analytical Applications of the Relaxed Energy Approach

An optimization procedure was developed (22) for the determination of the various free parameters (that exist in the above transfer functions) so as to minimize control surface response to continuous gust inputs over a wide range of flight conditions. Most applications relate to T.E. alone control systems in an attempt to determine their effectiveness for flutter suppression. Extended type control laws (driven by the elastic responses of the system) were exclusively employed in all applications.

The first application of the above optimization procedure using the newly defined transfer functions was made to a violent wing flutter case of a drone aircraft (23) selected by the National Aeronautics and Space Administration for flight research programs aimed at validating active control
A plan view drawing of the flight vehicle-research wing combination is shown in Fig. 20. Guided by previous results (20), the T.E. control surface was placed as near to the tip of each wing as was structurally possible (Fig. 21). All the aerodynamic forces were computed using unsteady lifting surface doublet lattice method. The design objective of the flutter suppression system was to provide a 20% increase in flutter speed (to be demonstrated in flight) above that of the basic wing. Although detailed results regarding this case appear in ref. 22, preference will be given here to the results appearing in refs. 23 since they include comparisons with results obtained using classical control system synthesis.

Table 1 presents a summary of the calculated flutter characteristics. It can be seen that both the classical and the energy methods achieve the objective set for the flutter suppression system (with somewhat higher flutter speed values using the energy method). Figure 22 shows comparisons of control surface rates and displacements. As can be seen, the maximum values for the rates (and displacements) using the energy method are around 20% lower than those produced by the classical method.

In their discussion of results the authors state (23): “Two major differences result in the application of these methods. The first is the manner in which the control surfaces in the L.E.-T.E. control system are used. In the classical method the fixed form of the shaping filter is given with free parameters available to fit this form to the dynamic characteristics of the system being considered. The second difference is the manner in which the gust analysis is used. In the classical method the gust is used to evaluate rates and deflections of the control system after preliminary design of the shaping filter is complete. If the rates or deflections are beyond the capability of the control system then an iterative process including changes to the shaping filter and possibly the control surface size is begun. This process is continued until both the stability and gust response requirements are met. In the energy method, the fixed form of the shaping filter allows the gust to act as a driver in establishing the free parameters which in turn permits the minimization of control surface activity while maintaining stability.”

A second application has recently been made to the YF-17 flutter model (26) with the object of suppressing the external store flutter of three different store configurations using a T.E. alone control surface. The geometrical description of the active control system is shown in Fig. 23. Note that the T.E. control surface spans only 7% of the wing. The description of the three configurations is given in Table 2 and the results of the optimization process are given in Table 3. These latter results relate to M = 0 and V = 965 m/s and were obtained using a dynamic pressure Qp which is twice the value (determined arbitrarily in the absence of a definition of the desired flight envelope) of the minimum flutter dynamic pressure, corresponding to configuration B. A L.D.T.T.F. was employed and its free parameters were determined using configuration B. The resulting control law was maintained fixed during applications to configurations A and C. The significance of these results is threefold:

1) A single control law with fixed gains is employed for all configurations

2) Very large increases in flutter dynamic pressures are obtained for all configurations

3) The effectiveness of the activated control system is maintained over the whole range of flight conditions (thus providing yet another confirmation regarding the potential of the relaxed energy concept).

It may also be worth noting that although the open loop configuration B is most critical from flutter considerations, the largest control surface activity corresponds to configuration C. This activity can be reduced by increasing the span of the control surface (% 7%) employed in this application.

A single application of a L.E.-T.E. control system has recently been made using the previously described drone aircraft (25). It is shown that the L.E.-T.E. control system yields a closed loop system with flutter speeds which are higher than those of the T.E. alone system. In addition the activity of each of the control surfaces in the L.E.-T.E. system is much lower than that corresponding to the T.E. alone system. If, however, the performance of the two systems is judged on the basis of the maximum control surface activity (corresponding to the desired 40% increase in the flutter dynamic pressure) rather than on the maximum flutter speed, and if we further require that the performance of a system with two control surfaces be compared only with systems having two control surfaces (in this case a comparison between L.E.-T.E. and T.E.-T.E. systems) one finds that the performance of the L.E.-T.E. control system is comparable to the performance of the T.E. alone system, with slight advantage to the latter system. Although this finding may be of specific nature and need not necessarily hold true for other applications, it is of importance since it shows that a T.E. alone control system can yield results which compare favourably with a L.E.-T.E. control system.

It is not unintentional that we choose to close the circle off applications by returning to the first example which served to test the potentials of the aerodynamic energy method — that is the application relating to the Boeing's supersonic transport. Comparison is now made between the results reported in reference 20, and which forms Phase II of the SST technology follow-on program, and those obtained through the use of the relaxed energy concept (23). These results relate to the full span 1/20 scale low-speed model of the Boeing 2707-300 supersonic transport. Figure 24 shows the general configuration of the model. It can be seen that two T.E. control surfaces were activated, one on each control surface. The application based on classical control methods (26) attempted the activation of both control surfaces whereas the application based on the energy approach (23) attempted the activation of the outboard aileron only (based on experience gained from previous applications (20)). These results, which were obtained using lifting surface unsteady aerodynamics, are presented in Fig. 25. As can be seen, the energy method yields an increase in flutter speed.
of 33% using the outboard aileron only (and L.D.T.T.F.) whereas the classical method yields an increase in flutter speed of 11.5X only, using both outboard and inboard ailerons. Furthermore, the energy method yields the following control surface activity of the outboard aileron, at a speed which is 16% above the inactivated flutter speed

\[ \dot{\theta}_{\text{MS}} = 25.1 \text{ deg/s/m/s} \]

\[ \dot{\theta}_{\text{MS}} = 0.33 \text{ deg/s/m/s} \]

These activities are not considered to be excessive. It should be noted that flutter speeds could further be increased by specifying higher flight dynamic pressures when using the gust optimization program.

### Remarks on Applications using Modern Control Theory

The author of this paper is unaware of any major comparative studies between designs based on the aerodynamic energy method and those based on modern control theory. Some use has, however, been made of the aerodynamic energy control law (eqn (11b), (12)) as derived for the L.E.-T.E. system in the original formulation of the energy concept in connection with some work which employed optimal control methods [28]. The above control law was applied [28] to a two dimensional subsonic strip, with specified aerodynamic dynamics included in the analysis. The results showed that the plunge and pitch modes were stabilized throughout the range of parameter investigated whereas the leading-edge control wing was unstable throughout this range. Such a condition arises if one considers the control laws in the form given by eqn (3) to correspond to the constant reactions rather than to the actual deviations. It should therefore be stressed that control surface dynamics should be compensated in all applications employing the energy control law and therefore the transfer function matrix \([A]\) to relate between the structural oscillations and the actual control surface deflections. It is worth mentioning the results which correspond to the above mentioned two dimensional strip as obtained through the use of optimal control theory [28]. It will be appropriate, however, to make a brief illustration of the method used.

The linear optimal control theory requires [29] the equations of motion of the system to be brought to the following form

\[ \dot{X} = [A]X + [B]u \]  

where \(X\) represents the \(N\) state variables, \([A]\) a \(N \times N\) the plant (or system) matrix, \([B]\) a \(N \times m\) the control distribution matrix; and \(u\) of order \(m\) the control input vector. Both the matrices \([A]\) and \([B]\) (eqn 22) are constant for a given Mach number, given flight velocity and given flight altitude. Optimal control theory requires the minimization of the performance index \((P_I)\), with equations (22) used as constraints, where \(P_I\) is given by

\[ P_I = \int ((X,[Q]X) + [u][P][u])dt \]

and where \([u]\) is either positive definite or positive semidefinite, and \([P]\) is always required to be positive definite. The problem now remains of selecting the weighting matrices \([Q]\) and \([P]\).

For the minimization of \([u],[Q]\) is chosen as \([Q]=0\). The resulting optimal control law, which is of the form

\[ u = \{T\}X \]

where the \(T_{ij}\) terms are constants, causes all the stable open-loop eigenvalues to remain unchanged while the open-loop unstable eigenvalues are reflected about the \(\pm \omega\) axis (that is, the sign of the real part of the unstable roots is reversed). This result (see also ref. 31) permits application of the "pole placement" method for the determination of the matrix \([T]\). Application of the above optimal control method was made to the two dimensional strip example using a l.T.E. only control system [28]. The stabilized closed-loop system was found to become unstable below the open loop flutter speed, thus showing the importance of the sensitivity of the activated system to off-design conditions. The above system with two control surfaces was eventually stabilized by reflecting the unstable flutter eigenvalue about a line parallel to the \(\pm \omega\) axis and crossing the real axis of the root locus plot at a value of 5 rad/sec. Such a reflection is arbitrary and is not, in itself, a result of application of optimal control considerations. It can, thus, be seen that off-design considerations forces the designer to compromise for a suboptimal system. The aerodynamic energy concept introduces these compromises in a consistent manner whereas other methods deal with this problem in an ad hoc arbitrary fashion.

An additional point which is worth noting relates to the inclusion of the actuator dynamics in the plant equations (22). It is felt that such inclusion [28,30] is limiting since parameters relating to control surface dynamics can be changed if necessary so as to reduce control surface activity. The exclusion of control surface dynamics from the energy synthesis considerations should therefore be viewed as promoting efficiency rather than as a limitation. The form of the various \(R_i\)'s (eqn (18)) associated with the L.D.T.T.F. have the form of an actuator transfer function. It is therefore possible to view the values of the optimised \(R_i\) as representing the desired actuator dynamics. These latter values clearly indicate the changes that need to be introduced into the existing actuator.

As a final remark, it is interesting to note that the determination of the control law using the energy concept meets none of the difficulties which characterize the optimal control approach such as problems associated with aerodynamic modeling, state augmentation and eventually, the state vector identification for purposes of implementation of the control law. The use of the continuous gust program for the minimization of the control surface activity using the energy method presents absolutely no aerodynamic modeling or state augmentation problems. Similarly, the relationship between the control surface deflection and the response of the wing at a specified location (see eqn (12) as an example) presents no need for state vector identification (this is similar to the L.E.A.T. concept developed in reference 8).

### Concluding Remarks

The paper presents the current state-of-the-art...
of the aerodynamic energy concept. Many of the applications relating to the relaxed energy method have not yet been published. It is felt that the relaxed energy method, coupled with the gust response optimization procedure yields effective control systems for the suppression of flutter. These systems may consist of either L.R.-T.R. or T.R. alone control surfaces. These activated systems may also be used for gust load alleviation and ride control (if appropriately located) as shown in one of the early applications. There remains to extend the method to the supersonic flight regime and to test the possible advantages of deriving control laws based on the system's generalized matrices (somewhat along the lines of ref. [11] using the relaxed energy approach) rather than on the generalized two-dimensional strip model.

Further substantiation of results is needed using both wind tunnel models and flight test programs before attempting to incorporate some flutter suppression systems in either existing or future aircraft.

Acknowledgement

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27. Missile, E.: Study of Active Control Systems for Application to Supersonic Cruise Aircraft To be published.


Table 1: Summary of Calculated Flutter Characteristics of Drone Research Vehicle.

<table>
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<tr>
<th>Config.</th>
<th>A</th>
<th>B</th>
<th>C</th>
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</thead>
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<tr>
<td>Description</td>
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<td>Aim-9E(flex)</td>
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<td>Empty</td>
<td>Aim-7(rig)</td>
<td>Not install.</td>
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<td>11.9221</td>
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<td>14.5104</td>
<td>14.9097</td>
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<td>&quot;n7&quot;</td>
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<td>41.0960</td>
<td>46.9919</td>
</tr>
</tbody>
</table>

* No flutter to sea level dynamic pressure

Table 2: Description of the Three Wing/Store Configurations of YF-17

<table>
<thead>
<tr>
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<th>A</th>
<th>B</th>
<th>C</th>
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<td>&quot;n7&quot;</td>
<td>44.4797</td>
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</tbody>
</table>

* natural frequency of the 1st elastic mode (Hz).

Table 3: Summary of Results: Three wing/store configurations of YF-17 with activated outboard T.E. control using 1.D.T.T.F. and V = 98 m/s

<table>
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<tr>
<th>CONFIG.</th>
<th>Basl-Wing</th>
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<td>g = 0</td>
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<td>Flutter Dyn. Press.</td>
<td>Flutter Dyn. Press.</td>
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<td>B</td>
<td>2.63</td>
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</tr>
<tr>
<td>C</td>
<td>4.31</td>
<td>65</td>
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</table>

* Values relate to flights up to dyn. press. of 5.26 kPa.
Fig. 1. $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ vs $1/k$. Wing strip with no control surfaces.

Fig. 2. $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ vs $1/k$ for various values of $C_{22}$. Wing strip with T.E. control using eqns (11), (11a).

Fig. 3. $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ vs $1/k$ for various values of $C_{22}$. Wing strip with L.E.-T.E. controls using eqns (11), (11b).
Fig. 4. \( \lambda_{\text{min}} vs \frac{1}{k} \) at various Mach numbers. Wing strip with L.E.-T.E. controls using eqns (11), (11b).

Fig. 5. \( \lambda_{\text{min}} vs \frac{1}{k} \) for various values of \( \frac{w}{w_0} \). Wing strip with L.E.-T.E. controls using eqns (11b), (12).

Fig. 6. Effectiveness of L.E.-T.E. system as flutter suppressor for SST type wing with engines.

Fig. 7. Root locus plot comparing the unmodified airplane with modified one for combined case B and C of Fig. 6.

Fig. 8. Experimental wing for flutter suppression shown mounted in the Langley transonic dynamic tunnel.

Fig. 9. Plan view of Arava STOL Transport.
Fig. 10. Plan view of Westwind business jet transport.

Fig. 11. Strip allocations along wing and horizontal tail of Araya and Westwind aircraft.

Fig. 12. Variation with time of wing root bending moment. Westwind transport with activated L.E.-T.E. system at strip 4 and with $\delta_{\text{max}}=0.5$ rad.

Fig. 13. Variation with time of wing root bending moment. Westwind transport with activated L.E.-T.E. system at strip 10 and with $\delta_{\text{max}}=0.5$ rad.

Fig. 14. Variation with time of wing root bending moment. Westwind transport with activated L.E.-T.E. system at strip 4 and with $\delta_{\text{max}}=0.5$ rad (using extended control law).

Fig. 15. Variation with time of linear acceleration at center of gravity. Westwind transport with activated L.E.-T.E. system at strip 10 and with $\delta_{\text{max}}=0.5$ rad.
Fig. 16. Variation with time of linear acceleration at center of gravity. Westwind transport with activated L.E.-T.E. system at strip 6 and with \( \delta_{\text{max}} = 0.5 \text{ rad.} \) (using extended control law).

Fig. 17. \( \bar{\lambda}_{\text{min}} \) and \( \bar{\lambda}_{\text{max}} \) vs \( 1/k \) at various Mach numbers. Wing strip with no control surfaces.

Fig. 18. \( \bar{\lambda}_{\text{min}} \) and \( \bar{\lambda}_{\text{max}} \) vs \( 1/k \) at various Mach numbers. Wing strip with T.E. control using D.T.T.F.

Fig. 19. \( \bar{\lambda}_{\text{min}} \) and \( \bar{\lambda}_{\text{max}} \) vs \( 1/k \) at various Mach numbers. Wing strip with T.E. control using I.D.T.T.F.
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ACTIVE EXTERNAL STORE FLUTTER SUPPRESSION
IN THE MODIFIED YF-17 FLUTTER MODEL

by

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INTRODUCTION

The investigation reported in this work relates to the suppression of external store flutter in the YF-17 flutter model. Configuration B was specified for the above purpose with the objective of enabling the activated model to be tested in a wind tunnel at Mach number $M = 0.8$ and at dynamic pressures up to 69% above open loop flutter dynamic pressure. A schematic plan view of the model is shown in Fig. 1. Two control surfaces are available for activation: A leading-edge (L.E.) control and a trailing edge (T.E.) control. Control laws are defined in an attempt to meet the above mentioned objectives. No attempt is made, however, to get into the details associated with the mechanization of the control laws obtained.

Mathematical Model

The dynamic characteristics of the model were supplied by NASA. They included generalized masses, natural frequencies and mode shapes for 10 symmetric structural modes in addition to two rigid body modes. The generalized aerodynamic forces were computed using the Doublet-Lattice method with 126 boxes on each wing and 32 boxes on each half horizontal tail.

The formulation of the equations of motion and synthesis techniques (1) are based on the relaxed aerodynamic energy approach (2). The general form of the control law employed was established in Ref. (2) and is given by the following expressions
\[
\begin{bmatrix}
\mathbf{h} \\
\mathbf{a}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1.86
\end{bmatrix}
+ 
\begin{bmatrix}
K_{\text{E}} & 0 \\
0 & K_{\text{T.E.}}
\end{bmatrix}
\begin{bmatrix}
-4 & 4 \\
4 & 2.8
\end{bmatrix}
\begin{bmatrix}
h_1 - h_r \\
b/a_1 - a_r
\end{bmatrix}
\] (1a)

where \( h_1 \) and \( a_1 \) are the deflections of the L.E. and T.E. control surfaces, respectively, and where \( h_{\text{T.E.}} \) denote the translation and rotation of the 30 per cent chord point at the control surface and span section respectively (see Fig. 1). The parameters \( h_{\text{T.E.}} \) and \( a_{\text{T.E.}} \) similarly denote the translation and rotation of a reference point located along the center line of the fuselage and \( b \) denotes the semi-chord length at the control surface and span section. The \( K \)'s are given by the following expressions:

\[
K = \frac{d_1}{s^2 + \frac{1}{\omega_1^2} s + \omega_1^2} + \frac{d_2}{s^2 + \frac{1}{\omega_2^2} s + \omega_2^2}
\] (1b)

where \( s = \omega t \) and where \( \omega \) represents the frequency of oscillation.

The parameters \( d_1, \omega_1, \omega_2 \) are all positive and their values determined by an optimization program based on the gust response of the model following the method of Ref. 1. The parameters in Eq. (1) will be substituted by either L.E. or T.E. coordinate reference to either L.E. or T.E. control law transfer functions, respectively.
Presentation and Discussion of Results

The root locus plot for the open loop system, with the dynamic pressure \( Q_D \) acting as a parameter, is shown in Fig. 2. As can be seen the value of the dynamic pressure at flutter \( (Q_{DF}) \) is equal to \( Q_{DF} = 84 \) psf. with frequency \( \omega = 36.6 \) rad/s. Activation of the T.E. alone yielded only marginal results, indicating the need to relocate the control surface (see also Ref. 3). The L.E. alone yielded better results but since these results originate from changes in the responses associated with the energy eigenvectors and not from changes in the energy eigenvalues (as required by the relaxed energy approach), the work based on a L.E. alone system was not pursued. Hence, the work to be reported herein will relate to a combined L.E. - T.E. system (at \( M = 0.8 \)).

The control laws derived from the energy approach neglected the effects of control surface mass unbalance in an attempt to obtain generalized results. An activated system with mass-balanced control surfaces was therefore tested first. The synthesis technique yielded the following control law by specifying that the model should fly at a maximum dynamic pressure \( (Q_{D_{\text{max}}}) \) of 143 psf., and by attempting to minimize the control surface rates of the system:

\[
R_{\text{T.E.}} = \frac{1.62 s^2}{s^2 + 2 \times 1 \times 4 \times s + (4)^2} + \frac{15. s^2}{s^2 + 2 \times 0.5 \times 57 \times s + (57)^2}
\]

\[
R_{\text{L.E.}} = \frac{4.07 s^2}{s^2 + 2 \times 1 \times 41.5 \times s + (41.5)^2}
\]

with \( (g = 0.\) , structural damping)
\[
\begin{align*}
\beta_{\text{rms}} &= 16.04 \text{ deg/s/ft/s} \\
\beta'_{\text{rms}} &= 0.31 \text{ deg/ft/s} \\
\delta_{\text{rms}} &= 15.21 \text{ deg/s/ft/s} \\
\delta'_{\text{rms}} &= 0.29 \text{ deg/ft/s}
\end{align*}
\]

The optimization was constrained to yield control surfaces with nearly equal values of control rates. The closed loop root locus plot for the above activated system is shown in Fig. 3. As can be seen, flutter has completely been suppressed up to a dynamic pressure of 200 psf (maximum value used in plotting all the root locus plots to be presented herein).

The introduction of control surface mass unbalance has modified the root-locus plot (Fig. 4) to such an extent that instabilities cover most of the flight dynamic pressures. A careful examination of the variation of R with frequency (Fig. 5) and its effects on the flutter speed has shown that the aerodynamic and inertial stability effects are not compatible. The gust optimization program was constrained to yield maximum aerodynamic damping around the flutter frequency only while minimizing the control activity at higher frequencies. This approach yields the following values for the control law parameters:

\[
R_{\text{T.E.}} = \frac{1.88 \ s^2}{s^2 + 2 \times 0.16 \times 39.1 \times s + (39.1)^2}
\]

\[
R_{\text{L.E.}} = \frac{1.26 \ s^2}{s^2 + 2 \times 0.29 \times 38.8 \times s + (38.8)^2}
\]

(3)

with \( g = 0 \), structural damping)
The root locus plot associated with the above control system is shown in Fig. 6. As can be seen, except for a small region of instability at very low values of $Q_D$ (which is counteracted by normal structural damping) no flutter exists up to $Q_D = 200$ psf. The above control law will be referred to as control law I. The variation of the control surfaces activity with $Q_D$ is shown in Fig. 7 and a sensitivity variation of the T.E. control rate activity (as an example) with the control parameters is shown in Fig. 8. Cancellation of the parameter $C_2 = -1.86$ (eq. 1a) simplifies the control law and shows no effect on stability (figures not included).

A second alternative control law (to be referred to as control law II) was attempted by trying to match the flutter and inertial stability requirements at the various regions of frequency. This was done by using the synthesis technique\(^{(1)}\) in the presence of a filter \(\frac{-300}{s + 300}\) which multiplies the transfer functions shown in Eqr. (1a). The results for the control parameters are given by

\begin{align*}
R_{T.E.} &= \frac{4s^2}{s^2 + 2 \times 0.43 \times 57.4 \times s + (57.4)^2} \\
R_{L.E.} &= \frac{2.07s^2}{s^2 + 2 \times 0.5 \times 41.5 \times s + (41.5)^2} \\
\end{align*}

(4)

with ($g = 0$, structural damping)
\[ \delta_{\text{rms}} = 21.38 \text{ deg/s/ft/s} \]
\[ \delta_{\text{rms}} = .51 \text{ deg/ft/s} \]
\[ \beta_{\text{rms}} = 19.35 \text{ deg/s/ft/s} \]
\[ \beta_{\text{rms}} = .52 \text{ deg/ft/s} \]

The closed loop root locus plot is shown in Fig. 9. As can be seen, there is no flutter up to \( Q_D = 200 \text{ psf} \). The variation of the control surface activities with \( Q_D \) is shown, for control law II, in Fig. 10. A sensitivity variation of the T.E. control rate (as an example) with the control parameters is shown in Fig. 11. The cancellation of \( C_{21} = -1.86 \) introduces in this case a flutter instability, at \( Q_D = 145 \text{ psf} \) (see Fig. 12). Therefore \( C_{21} = -1.86 \) has to be retained. This implies that the acceleration signals have to be integrated. Integrations of the form \( \frac{1}{s + \varepsilon} \) and \( \frac{1}{(s + \varepsilon)^2} \) had been tested in the region of \( 0.1 < \varepsilon < 1 \), and no visible effects could be detected on the root locus plots (figures not included).

The block diagrams for the above two control laws are presented in Figs. 13, 14.

Acknowledgment

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REFERENCES


Figure 1: Plan view of YF-17 flutter model and geometrical description of the active control system.
Figure 3: Closed loop root locus plot using control law obtained with mass balanced control surfaces (n = 0.8)
Figure 4: Closed loop root locus plot after introduction of control surfaces mass unbalance (using control law obtained for mass balanced system) at $M = 0.8$. 
Figure 6: Closed loop root locus plot using control law I with unbalanced control surfaces ($M = 0.8$)
Figure 7a: T.E. control rate

Figure 7: Variation with $Q_D$ of control surface activity, using control law I with unbalanced control surfaces ($M = 0.8$)
Figure 7b: T.E. control deflection
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Figure 8a: Variation with $Q_{T.E.}$.

Figure 8: Variation of T.E. control rate with control law parameters using control law 1 with $Q_D = 1.0$ psf (H = 0.5, unbalanced control surface).
Figure 8b: Variation with $\omega_{n_{T.E.}}$
Figure 8e: Variation with \( \omega_{nLE} \)
Figure 10a: T.E. control rate

Figure 10: Variation with $Q_D$ of control surface activity, using control law II with unbalanced control surfaces ($\kappa = 0.3$).
Figure 10b: Z.F. control deflector.
Figure 11a: Variation with $Q_{T.E.}$.

Figure 11: Variation of T.E. control rate with control law parameters using control law II with $Q_D = 143$ psf ($M = 0.8$, unbalanced control surfaces).
Figure 11: Variation with $\omega_{n_{T,E}}$. 

$\dot{\phi}_{T,E}$

deg/s/ft/s

$0$ $20$ $40$ $60$

$16$ $36$ $56$ $76$ $96$

$\omega_{n_{T,E}}$
Figure 11d: Variation with C, F.
Figure 11e: Variation with $\omega n_{L.E.}$.
Figure 12: Closed loop root locus plot using control law II with unblanced control surfaces and with $Q_x = 0$ ($M = 0.8$).
Figure 14: Block diagram representation of control law.
Figure 14: Block diagram representation of control law, \( h_1 - h_r \) with \( f = 0 \)
ACTIVE EXTERNAL STORE FLUTTER SUPPRESSION IN THE MODIFIED
YF-17 FLUTTER MODEL:
ANALYSIS VS. WIND TUNNEL TESTS

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INTRODUCTION

The investigation reported in this work relates to the suppression of external store flutter in the YF-17 flutter model. Configuration B was specified for the above purpose with the objective of enabling the activated model to be tested in a wind tunnel at Mach number $M = 0.8$ and at dynamic pressures up to 69% above the open loop flutter dynamic pressure. Two control laws were derived at an earlier stage of this work\(^1\), and were shown to yield the desired flutter suppression capability through the activation of a combined leading-edge (L.E.) - Trailing-edge (T.E.) control system.

The mechanization of one of the derived control laws was carried out by Northrop and subsequently, the flutter stability augmented YF-17 model was tested in the Langley 16 ft transonic dynamic tunnel. The test results, as reported to the authors of the present work, showed no correlation with the analysis and the tunnel tests were discontinued at a dynamic pressure which was below the open loop flutter dynamic pressure.

The object of the present paper is to present a critical review of the analysis versus the test results and to indicate the sources of the discrepancies obtained. For convenience, some of the major results reported in Reference 1 will be presented herein once again.
ANALYTICAL RESULTS\textsuperscript{1} - INITIAL MODEL

Background

The analytical results reported in Ref. 1 were based on a dynamic model supplied by NASA. It included generalized masses, natural frequencies and mode shapes for 10 symmetric structural modes and two rigid body modes. The generalized aerodynamic forces were computed using the Doublet-Lattice method with 126 boxes on each wing and 32 boxes on each half horizontal tail. The box allocation was identical to the one appearing in the Northrop report supplied to the authors of this paper.

The general form of the control laws is given by the following expression

\[
\begin{bmatrix}
\delta
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & C_{22}
\end{bmatrix} + \begin{bmatrix}
R_{\text{L.E.}} & 0 \\
0 & R_{\text{T.E.}}
\end{bmatrix} \begin{bmatrix}
-4 & 0 \\
4 & 2.8
\end{bmatrix} \begin{bmatrix}
h_1 - h_r \\
\alpha_1 - \alpha_r
\end{bmatrix}
\]

(1)

where $\delta$ and $\delta$ are the deflections of the L.E. and T.E. control surfaces, respectively, and where $h_1$, $\alpha_1$ denote the translation and rotation of the 30 percent chord point at the control surface mid-span section, respectively (see Fig. 1). The parameters $h_r$ and $\alpha_r$ similarly denote the translation and rotation of a reference point located along the center line of the fuselage and $b$ denotes the semi-chord length at the control surface mid-span section. The $R$'s represent transfer functions which are dependent on $S$ where $S = i\omega$. 


**Open Loop Results:**

The root locus plot for the open loop system, with the dynamic pressure $Q_D$ acting as a parameter, is shown in Fig. 2. As can be seen, the value of the dynamic pressure at flutter ($Q_{DF}$) is equal to $Q_{DF} = 84 \text{ PSF}$, with frequency $\omega = 36.6 \text{ rad/s}$.

**Closed Loop Results:**

Activation of the T.E. alone yielded only marginal improvements in $Q_{DF}$, indicating the need to relocate the control surface (see also refs. 2, 3). The L.E. alone yielded better results but since these results originated from changes in responses associated with the energy eigenvectors and not from changes in the energy eigenvalues (as required by the energy approach), the work based on a L.E. alone system was not pursued. Hence, the work reported herein relates to a combined L.E.-T.E. system (at $M = 0.8$).

Two closed loop L.E.-T.E. control laws were derived and presented in Ref. 1. They are presented once again in the following for sake of completeness.

**Control Law 1:**

In this control law $C_{22} = 0$ and the $R$'s appearing in Eq. (1) are given by

$$R_{T.E.} = \frac{1.88 S^2}{S^2 + 2 \times 0.16 \times 39.1 \times S + (39.1)^2} \tag{2}$$

$$R_{L.E.} = \frac{1.26 S^2}{S^2 + 2 \times 0.29 \times 38.8 \times S + (38.8)^2} \tag{2}$$
The root locus plot associated with the above control system is shown in Fig. 3, (assuming zero structural damping, \( g = 0 \)). As can be seen, except for a small region of instability at very low values of \( Q_1 \) (which is counteracted by normal structural damping), no flutter exists up to \( Q_1 = 200 \) psf. The maximum control activity (at \( Q_1 = 143 \) psf, corresponding to the highest specified \( Q_1 \)) is given by (for \( g=0 \))

\[
\delta_{\text{rms}} = 14.86 \text{ deg/s/ft/s}
\]

\[
\delta_{\text{rms}} = 0.44 \text{ deg/ft/s}
\]

\[
\dot{\delta}_{\text{rms}} = 14.26 \text{ deg/s/ft/s}
\]

\[
\dot{\delta}_{\text{rms}} = 0.4 \text{ deg/ft/s}
\]

The variation of the control activity with \( Q_1 \) for various values of \( g \) is shown in Fig. 4. The block diagram for the control system associated with control law 1 is shown in Fig. 5.

At this stage it may be observed that the R's presented in Eq. (2) represent transfer functions of second order systems. Since actuators often have the form of third order system, it was decided to increase the order of the R's to yield three poles, so that normal actuators may be compensated through the newly derived control law. This point will be made clear in the following section.
Control Law II:

This control law was derived by using the synthesis technique in the presence of a filter \( \frac{300}{(S+300)} \) which multiplies the transfer functions shown in Eq.(1). The above value of 300 was determined following a parametric study in conjunction with the synthesis technique mentioned earlier. The values obtained for the \( R \)'s appearing in Eq.(1) are given by

\[
R_{T.E.} = \frac{4S^2}{S^2 + 2 \times 0.43 \times 57.4 \times S + (57.4)^2}
\]

\[
R_{L.E.} = \frac{2.07S^2}{S^2 + 2 \times 0.5 \times 41.5 \times S + (41.5)^2}
\]

with \( C_{22} = -1.86 \) and \( g = 0 \).

The closed loop root locus plot (with \( g = 0 \)) is shown in Fig. 6. As can be seen, there is no flutter up to \( Q_\ell = 200 \) psf. The maximum control activity (for \( g = 0 \)) is given at \( Q_D = 143 \) psf by the following values:

\[
\delta_{\text{rms}} = 21.38 \text{ deg/s/ft/s}
\]

\[
\hat{\beta}_{\text{rms}} = 0.51 \text{ deg/ft/s}
\]

\[
\tilde{\delta}_{\text{rms}} = 19.35 \text{ deg/s/ft/s}
\]

\[
\tilde{\beta}_{\text{rms}} = 0.52 \text{ deg/ft/s}
\]

The variation of the control activity with \( Q_\ell \) for various values of \( g \) is shown in Fig. 7. Since \( C_{22} \neq 0 \) in this case, (cancellation of \( C_{22} \))
leads to flutter at $Q_D = 145$ psf), this implies that acceleration signals have to be integrated. Integrations of the form $1/(S+\varepsilon)$ and $1/(S+\varepsilon)^2$ had been tested in the region of $0.1 < \varepsilon < 1$ and no visible effects could be detected on the root locus plots.

The block diagram for the above control law is presented in Fig. 8. The transfer functions representing third order systems can clearly be seen in Fig. 8. Furthermore, a third order actuator can readily be compensated. This can be illustrated for the T.L. control surface having an actuator transfer function $T(S)$ of the form

$$T(S) = \frac{\omega_n^2 d}{(S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2)(S + \omega_n^2)}$$

(4)

The following compensation procedure (see Fig. 9)

$$\frac{300}{(S + 300)(S^2 + 2 \cdot 0.43 \cdot 57.4 \cdot S + (57.4)^2)} \cdot T(S)$$

$$= \frac{300 \cdot (S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2)(S + \omega_n^2)}{(S + 300)(S^2 + 2 \cdot 0.43 \cdot 57.4 \cdot S + (57.4)^2)} \cdot \omega_n^2 d \cdot T(S)$$

can be seen to yield the same effective control law.

Summary of Analysis:

Two control laws were derived. Control Law I, suitable for second order actuators and Control Law II suitable for third order actuators.
Control law II was chosen for the mechanization performed by Northrop. Fig. 9 represents the block diagram of the L.E.-T.E. control system. The control surface actuator transfer functions are denoted by $G_{s,L.E.}$ and $G_{s,T.E.}$ and are defined by the following expressions:

$$G_{s,L.E.} = \frac{(S+24)(S + 260)}{24} \frac{(S + 260)}{28} \frac{(S + 94)}{94} \frac{(S^2 + 204S + 28,900)}{28,900} \frac{(S^2 + 6165 + 193,600)}{193,600}$$

$$G_{s,T.E.} = \frac{S + 260}{260} \frac{(S + 124)}{124} \frac{(S^2 + 138S + 19,044)}{19,044} \frac{(S^2 + 4405 + 98,596)}{98,596}$$

These expressions were supplied to the present authors long after control laws I and II were determined and presented at NASA.

As can be seen, the above actuator transfer functions include some built-in filters which were introduced by Northrop. As a result, the effective expressions for the transfer functions in the mechanized system are given by

$$T(S)_{L.E.,E,F.} = \frac{(S + 24)(S + 260)}{24} \frac{(S + 260)}{28} \frac{(S^2 + 204S + 28,900)}{28,900} \frac{(S^2 + 6165 + 193,600)}{193,600} \frac{T(S)_{L.E.}}{T(S)_{L.E.}}$$

$$T(S)_{T.E.,E,F.} = \frac{(S + 260)}{260} \frac{(S^2 + 4405 + 98,596)}{98,596} \frac{T(S)_{T.E.}}{T(S)_{T.E.}}$$

where $T(S)_{L.E.}$, $T(S)_{T.E.}$ denote the desired transfer functions.
As can be seen, the effective control law had been varied by a considerable factor representing an additional transfer function. As a result, the mechanized control law represents a different control law than the original control law II. Furthermore, the integration in Fig. 10 was performed by \( \frac{1}{S+c} \) (instead of \( \frac{1}{S+S} \), with \( 0.1 \leq c \leq 1 \)) without checking its possible effects. It is also tacitly assumed that proper account had been taken of the accelerometers and actuators' sensitivities (does not appear in the block diagram in Fig. 9). It is further assumed that the changes between the assumed accelerometer locations and the actual locations are too small to have any significant effects on the gains of control law II.

At this stage, the authors of this paper decided to rederive the control law, in the presence of \( G_{S,L,E.,T,E.} \) and some additional filters used by Northrop (denoted by \( H(S) \)). The results of this latter analysis are presented in Appendix 1, and are based on an updated dynamic model and a refined calculation of the aerodynamic forces. This latter model was supplied by Northrop, through NASA. It arrived too late to be included in the derivation of control laws I and II.

Unfortunately, the results appearing in Appendix 1 arrived Northrop at too late a stage to be incorporated into the tunnel model. Consequently, the tunnel tests were performed using the \( T(S)_{L,E.,E_{L,E.}} \) and \( T(S)_{T,E.,E_{T,E.}} \) (see Eq.(6)), which are different from \( T(S)_{L,E.} \) and \( T(S)_{T,E.} \) of control law II (based on an older mathematical model).
WIND TUNNEL TEST RESULTS

The wind tunnel test results as reported to the authors of this paper, reveal the following picture:

"Because of high frequency problems associated with the control law, and a lack of knowledge concerning this law, testing could not be continued above a dynamic pressure of 70 psf. This was a condition below passive flutter \( q_p < 75 \text{ psf} \). Attachments 1, 2, and 3 are included to assist in describing the problems encountered in the tunnel. The first attachment presents zero airspeed transfer functions for the control law using either the leading or trailing edge surface as input. As can be seen, the gains are quite high across the frequency range. This is particularly true for the T.E. surface. Attachment 2 presents peak hold data taken during the test, while attachment 3 provides model response data at the various test points.

Initial tests indicated significant wing response near 30 Hz. Response data for test point 419 with the expanded time scale illustrates the problem which is particularly noticeable in the wing bending response. For test point 414, the control law was turned off while a Northrop leading edge law was activated. This also shows the significant frequency content of the command signals.

Since there was no one available at the test who could offer guidance in modifying the control law, 30 Hz notch filters were incorporated into the control law. With this change, test point 475 still shows some high frequency content and significant L.E. commands. As a result, the
global gain was reduced 25%. While increasing dynamic pressure from 65 to 70 psf, a divergent oscillation was encountered and the control law was deactivated. The frequency of the divergent oscillation was about 14 Hz. Further modifications to the control law were not attempted. All high frequency modifications affect the performance of the overall control law and without guidance it was not practical to compensate for these changes in the flutter frequency range."

Part of the attachment 3, relating to test point 419, is presented herein as Fig. 10.

ANALYSIS OF WIND-TUNNEL TEST RESULTS

It was found impossible even to attempt any correlation between analysis and test results, since the control laws used in each case were widely different. The changes introduced in control law II (see Eq.(6)) include high frequency transfer functions which, as noted in the previous section, "affect the performance of the overall control law." Consequently, it was decided to analyze the control law, as mechanized by Northrop, and compare the analytical results with those obtained during the wind-tunnel tests.

The new analysis reported herein, is based on the updated mathematical model and the refined aerodynamic coefficients. Since the wind tunnel problems reported above relate to high frequency regions, no attempts are
made to investigate the possible effects of the \( \frac{1}{s+5} \) integration.

The effective control law tested in the wind tunnel is given by the following expressions

\[
\begin{bmatrix}
\beta \\
\delta \\
\end{bmatrix}
= \frac{300}{s+300}
\begin{bmatrix}
Q_{L.E.} & 0 \\
0 & Q_{T.E.} \\
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
0 & -1.86 \\
\end{bmatrix}
\begin{bmatrix}
R_{L.E.} \\
0 & R_{T.E.} \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-4 & 4 \\
4 & 2.8 \\
\end{bmatrix}
\begin{bmatrix}
\frac{h_1 - h_r}{b} \\
\alpha_1 - \alpha_r \\
\end{bmatrix}
\]

(7)

where \( Q_{L.E.} \) and \( Q_{T.E.} \) are the transfer functions transforming the original control law II into the mechanized control law II, and where \( R_{L.E.} \) and \( R_{T.E.} \) are defined in Eq.(3). Using Eq.(6) the following relations can be written

\[
Q_{L.E.}(s) = \frac{(s + 24)(s + 260)}{(s + 28)(s^2 + 204s + 28,900)(s^2 + 616s + 193,600)}
\]

(8)

\[
Q_{T.E.}(s) = \frac{s + 260}{s^2 + 440s + 98,596}
\]

(9)

The control law defined in Eqs.(3), (7), (8), will be referred to as the Northrop modified control law II.
The root locus plot for the closed loop system is shown in Fig. 11 (with g = 0). Similarly, for comparison purposes, the closed loop root locus plot for the original control law II, but with the updated dynamical model and aerodynamics, is shown in Fig. 12. As can be seen, the changes in the mathematical model degrade the root locus plot for the original control law II (Fig. 12). For g = 0 flutter occurs at $Q_{DF} = 128$ psf, whereas for $g = 0.015$, flutter occurs at $Q_{DF} = 152$ psf. In addition, there is a lower frequency flutter branch yielding $Q_{DF} = 143$ psf ($g = 0$) with $\omega_f = 16$ rad/s, and a high frequency negative damping mode at around $\omega = 270$ rad/s. This latter high frequency mode becomes stable for values of $g > 0.015$. It can be concluded that the updating of the mathematical model, especially the changes introduced in the control surface aerodynamic coefficients, degrades the closed loop performance of control law II (see for comparison Fig. 6) to the extent which warrants its modification.

The root locus plot for the Northrop modified control law II (Fig. 11) shows flutter at $Q_{DF} = 68$ psf ($g = 0$) or $Q_{DF} = 82$ psf with $g = 0.015$. The flutter frequency lies around 110-115 rad/s. In addition, some high frequency modes show low damping when compared to the $g = 0.015$ line shown in Fig. 11. Hence, there is no wonder that the wind tunnel tests could not proceed beyond $Q_D = 70$ psf. Furthermore, at $Q_D = 60$ psf (relating to TP 419, see also Fig. 10) low damping modes can be observed at $\omega = 160$ rad/s and around $\omega = 260$ rad/s, thus explaining the high frequency content of the responses of the system and of the control signals.

An example of PSD representation for control surface deflections, using the Northrop modified control law II, is shown in Figs. 13, 14 (with values...
of $g$ as defined by ground resonance tests (GRT)). Fig. 13 shows the
PSD representation for $\beta_{\text{out}}$ and $\delta_{\text{out}}$ (at $Q_D = 60$ psf) and Fig. 14
shows a similar representation for $\beta_{\text{in}}$ and $\delta_{\text{in}}$ (also at $Q_D = 60$ psf).
These latter figures were computed for comparison purposes with the test
recordings, shown in Fig. 10.

Three main points emerge from the above comparison: First -- both
Figs. 10, 13, show correlation with respect to the low frequency content
(around 15-17 Hz) of the $\delta_{\text{out}}$ signal, and with respect to the lack of
any significant high frequency signal. Second -- both Figs. 10, 14 show
that $\beta_{\text{in}}$ has the largest high frequency content (around 40-50 Hz).
Third -- $\beta_{\text{out}}$ in both Figs. 10, 14 show lower amplitudes in the high
frequency content of the signal. The superposition of two signals with
frequencies of order 15 and 40 Hz can be seen in both figures.

The analytical simulation of the wind tunnel test results relating to
the 34 Hz notch filter was found impossible since no data regarding the
notch filter was supplied to the authors of this work.

The control surface activity, with values of $g$ as determined by
GRT of the model, at various values of $Q_D$, are shown in Fig. 15 for the
Northrop modified control law II. The control activities can be seen to
be much larger than those relating to the original control law II (by a factor
of about 3) and presented in Fig. 7.
CONCLUDING REMARKS

The control law, as mechanized by Northrop and tested in the wind tunnel, bears no analytical resemblance to the original control law II. The main deviation lies in the form of the effective control law used, which does not compensate for the actuator transfer functions (part of them could have easily been compensated). A second, smaller deviation, originates from the fact that control law II was derived using the older mathematical model (the updated model was sent too late to be included in the original analysis). The control surface aerodynamic derivatives in the updated model were computed by Northrop using a more rational box allocation over the control surfaces than in the older model (both computations used the Doublet Lattice method).

The analytical simulation of the flutter suppression performance of the YF-17 model (using control law II, as mechanized by Northrop) shows good correlation with the wind tunnel tests both with respect to flutter dynamic pressure and to the response characteristics of the model.
REFERENCES


APPENDIX 1: OPTIMIZATION OF A CONTROL LAW IN THE PRESENCE OF TRANSFER FUNCTIONS REPRESENTING ACTUATORS AND FILTERS.

The control law for the YF-17 flutter model has been recomputed using the following data:

(a) The new mode shapes and dynamic data and the new aerodynamic coefficients.

(b) The new sensor locations (at W.S. 51.45 instead of W.S. 44.85 previously used).

(c) Incorporation of the following filters for both the L.E. and T.E. control surfaces, following a specific request.

\[ H(S) = \frac{S^2 + 21S + (213)^2}{S^2 + 299S + (213)^2} \cdot \frac{S^2 + (552.6)^2}{S^2 + 552S + (552.6)^2} \cdot \frac{(264)^2}{S^2 + 264S + (264)^2} \]

(d) Incorporation of the following actuator transfer functions taken from Northrop’s papers attached to the above mentioned letter:

\[ G_{S,T.E.} = \frac{S + 260}{260} \cdot \frac{124}{S + 124} \cdot \frac{(138)^2}{S^2 + 138S + (138)^2} \cdot \frac{(314)^2}{S^2 + 440S + (314)^2} \]

\[ G_{S,L.E.} = \frac{S + 24}{24} \cdot \frac{S + 260}{260} \cdot \frac{28}{S + 28} \cdot \frac{94}{S + 94} \cdot \frac{(170)^2}{S^2 + 204S + (170)^2} \cdot \frac{(440)^2}{S^2 + 616S + (440)^2} \]

The results presented earlier employ an older set of dynamic data and were computed using the doublet lattice box distribution used by Northrop at an earlier stage of the work. None of the filters \( H(S) \) and \( G(S) \) were
then used, although $G(S)$ could have partially been accounted for by a simple transfer function compensation.

No attempt was made to rederive the previous control laws, using the new information included in the above paragraphs (a) and (b). Instead, the recomputation includes all the new elements mentioned in the above paragraphs (a) through (d).

Before presenting the new results it should be stressed that the constraints imposed by having to use the filters denoted by $H(S)$ and the form of $G(S)$ which appear to include compensation filters, do not seem to be justified. These filters represent an integral part of the control law developed by Northrop and they were required for stabilization of their resulting closed loop system. It is difficult to see the need for their introduction herein since if it is assumed that the mathematical representation is satisfactory, why is it not possible to rely on the control laws previously derived, which stabilize the closed loop system and have to resort to the statement that "based on previous testing experience, Northrop has found it necessary to insert filters in all the feedback signals to prevent system instability?" If, on the other hand, the mathematical model is not satisfactory, then there is no value to the present results and there is very little trust one can put in them.

As already mentioned, the above constraints were adopted in the new computations (some of these constraints were eventually compensated by the introduction of appropriate transfer functions in the control laws).

It was found possible to stabilize the system by using different control laws which yielded reasonably high flutter margins. The chosen
control law gives the smallest flutter margin but shows the best behaviour at lower values of dynamic pressure and at lower values of structural damping \((g)\). The results include a root locus computer run which includes the values of \(g\) defined during GRT of the model. To cut down labour, the results are brought to the form used by Northrop (degrees per \(g\)) and their sign convention is used (in this Appendix only).

Finally, before presenting the results, attention should be drawn to the fact that the control law requires that a free flying model (that is, having plunge and pitch degrees of freedom) should be fitted with reference accelerometers located along the center line of the fuselage, or near it. In the present results, the location used for these accelerometers is denoted.
Sensor Location, Units and Sign Convention:

Four accelerometers are used, \( a_1, a_2, a_{r1}, a_{r2} \). The accelerometers \( a_1, a_2 \) are located at W.S. 51.45, F.S. 145.18 (25% C) and F.S. 158.00 (76% C) respectively. The accelerometers \( a_{r1} \) and \( a_{r2} \) are located along the fuselage centerline, at F.S. 131.85 and F.S. 165.5 respectively. The accelerations are positive downwards and the units are assumed to be given in "g" units. The deflection of the control surfaces is given in degrees with positive rotations obtained by deflecting the T.E. control downwards \( (\delta_{T.E.}) \) and the L.E. control upwards \( (\beta_{L.E.}) \).

Suggested Control Law:

The suggested control law involves the activation of a combined L.E.-T.E. system. The block diagram for the activation of the L.E.-T.E. control is shown in Fig. A.1 and the expressions for the different transfer functions are given in Table 1.

Flutter Results:

Figure A.2 shows a root locus plot using the above control laws with zero structural damping. Fig. A.3 shows a similar root locus plot using the values of structural damping as measured by Northrop. The parameter of variation in the root locus plots is the dynamic pressure \( Q \). The spacing between adjacent points along each branch represents a change in \( Q \) of 10 psf. The plots were obtained by varying \( Q \) between 0 and 200 psf.
It can be seen that the flutter dynamic pressure is around 158 psf when structural damping is present and 147 psf with zero structural damping. Figs. A.4-A.7 show the variations of the activities of both L.E. and T.E. control surfaces (due to unit RMS gust input with dynamic pressure Q). It can be seen that both the deflections and the rates are relatively small. Structural damping (Northrop's measurement) was assumed to be present in deriving Figs. A.4-A.7.
### Table 1. Expressions for the Various Transfer Functions Used in Activating the Suggested L.E.-T.E. System

<table>
<thead>
<tr>
<th>Transfer Function</th>
<th>Expression</th>
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| H(S)              | \[
\frac{S^2 + 21S + (213)^2}{S^2 + 299S + (213)^2} \cdot \frac{S^2 + (552.6)^2}{S^2 + 552S + (552.6)^2} \cdot \frac{(264)^2}{S^2 + 264S + (264)^2}
\] |
| G_{S,T.E.}(S)     | \[
\frac{S + 260}{260} \cdot \frac{124}{S + 24} \cdot \frac{S + 28}{S + 24} \cdot \frac{(138)^2}{S^2 + 138S + (138)^2} \cdot \frac{(314)^2}{S^2 + 440S + (314)^2}
\] |
| G_{S,L.E.}(S)     | \[
\frac{S + 260}{260} \cdot \frac{94}{S + 28} \cdot \frac{S + 24}{S + 24} \cdot \frac{28}{S + 24} \cdot \frac{(170)^2}{S^2 + 204S + (170)^2} \cdot \frac{(440)^2}{S^2 + 616S + (440)^2}
\] |
| T_{T.E.}(S)       | \[
R(S) \cdot \frac{S + 124}{124} \cdot \frac{S^2 + 138S + (138)^2}{(138)^2} \cdot K_{T.E.}(S)
\] |
| R(S)              | \[
\frac{S^2 + 264S + (264)^2}{(264)^2} \cdot \frac{260}{S + 260} \cdot \frac{S + 60}{60} \cdot \frac{225}{S + 125} \cdot \frac{(347.9)^2}{S^2 + 492S + (347.9)^2}
\] |
| T_{L.E.}(S)       | \[
R(S) \cdot \frac{S + 94}{94} \cdot \frac{S + 28}{28} \cdot \frac{S^2 + 204S + (170)^2}{(170)^2} \cdot K_{L.E.}(S)
\] |
| K_{T.E.}(S)       | \[
3.09 \cdot \frac{[S^2 + 43.3S + (94.3)^2]}{[S^2 + 21.6S + (45)^2][S^2 + 88.3S + (152.3)^2]}
\] |
| K_{L.E.}(S)       | \[
1.874 \cdot \frac{[S^2 + 291.1S + (168.6)^2]}{[S^2 + 39.8S + (41.5)^2][S^2 + 400S + (200)^2]}
\] |
Figure 1: Plan view of YF-17 flutter model and geometrical description of the active control system.
Figure 2: Open loop root locus plot (M = 0.8)
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Figure 4a: L.E. control deflection

Figure 4: Variation with $Q_D$ of control surface activity, using control law I with unbalanced control surfaces ($N = 0.6$)
Figure 4b: T.E. control deflection
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Figure 7a: L.R. control deflection

Figure 7: Variation with $Q_D$ of control surface activity, using control law II with unbalanced control surfaces ($M = 0.8$).
Figure 7b: T.E. control deflection
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and $Q_o = 60.6$ psp (TP 419)

ORIGINAL PAGE IS OF POOR QUALITY
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Figure 12: Closed Loop Root Locus Plot Using the Original Control Law II in conjunction with the New Dynamic Model ($g = 0.015$, $M = 0.8$)
Figure 13a: PSD of Control Surface Response $\beta_{LE}^{out}$

Figure 13: PSD of Control Surface Responses $\beta_{LE,\text{out}}, \delta_{\text{TE,\text{out}}}$

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$\beta_{LE}, \delta_{TE}$ deg/ft/s

$0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$ $0.7$

$10$ $20$ $30$ $40$ $50$ $60$ $70$

$\beta_{LE}$ $\delta_{TE}$
Figure A1: Block Diagram Representation of the Suggested L.E.-T.E. Control System
Figure A2: Root Locus Plot-Closed Loop System (g = 0, H = 0.8)
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Figure A5: Variation with $Q_D$ of rms T.E. Control Rate due to Unit rms Gust Input ($g = 0$)
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Figure A7: Variation with $Q_D$ of rms L.E. Control Rate Due to Unit rms Gust Input ($\xi = 0$)

$\gamma$ (deg/sec/ft/sec) vs $Q_D$ (psf)
## FLUTTER AND CONTINUOUS GUST COMPUTATIONS
### WITH MULTI-ACTIVE CONTROLS

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1. THE EQUATIONS OF MOTION FOR FLUTTER ANALYSIS WITH
MULTI-ACTIVE CONTROLS

A simplified method of formulation of the equations of motion for
flutter analysis with any number of active control systems is presented
in this work. The suggested method combines computational economy and
programming simplicity with generality of formulation. It enables the
treatment of multi-active control systems with no limitations on the form
of the activated control laws. By way of introduction, two current
methods of analysis will first be described and their limitations will be
discussed. Following the presentation of these current methods, the new
proposed method will be presented and its special features will be
described.

The Equations of Motion

Let the $n_s$ equations

$$([M]s^2 + \frac{1}{2} \rho V^2 [A] + [K])\ddot{q} = 0$$

represent the equations of motion of $n_s$ structural modes (including
rigid body modes) with $n_c$ activated controls where $[M]$ represents the
mass matrix; $[A]$, the complex aerodynamic matrix; $[K]$, the stiffness
matrix; $\rho$, the density of the surrounding fluid; $V$, the velocity of the
fluid; and $\ddot{q}$, the response vector. All the matrices in equation (1)
are of size $n_s \times (n_s + n_c)$, that is, $n_s$ structural modes + $n_c$
active controls. The response vector $\ddot{q}$ can be expressed in terms of
$n_s$ structural responses and $n_c$ control deflections, that is,

$$\ddot{q} = \begin{bmatrix} q_s \\ q_c \end{bmatrix}$$
Equation (1) can therefore be written as

\[
([M_s \quad M_c]s^2 + \frac{1}{2} \rho V^2 [A_s \quad A_c] + [K_s \quad K_c]) \begin{cases}
q_s \\
q_c
\end{cases} = 0
\]  

(3)

where subscript \(s\) denotes a structural quantity and \(c\), a control quantity. Assume now a control law of the form

\[
c_c = [T] q_s
\]

(4)

where \([T]\) is a \(n_c \times n_s\) matrix representing the transfer functions of the control law. Substitution of equation (4) into equation (3) yields

\[
([M_s] + [M_c][T])s^2 + \frac{\rho V^2}{2}([A_s] + [A_c][T]) + [K_s] + [K_c][T]) q_s = 0
\]

(5)

Typically, the elements of the aerodynamics matrices \(A_s\) and \(A_c\) are available as functions of the reduced frequency \(k\) and the Mach number \(M\) whereas the transfer function matrix \([T]\) is a function of the Laplace variable \(s\), normally expressed in terms of rational polynomials in \(s\).

**Flutter Analysis Based on the Common Denominator Method (CDM)**

This method of analysis is described in ref. 1. It is based on the representation of the matrix \([T]\) by

\[
[T] = \frac{1}{Q(s)} [T_N]
\]

(6)

where \(Q(s)\) is a scalar polynomial representing the common denominator of all the \(T_{ij}\) terms and where \([T_N]\) is a matrix involving the resulting numerators (as a function of \(s\)).

The variation with \(s\) of the aerodynamic matrix \([A_s \quad A_c]\) can be approximated by the following Pade representation

\[
[A] = [A_0] + [A_1] \left(\frac{b}{V}\right)s + [A_2] \left(\frac{b}{V}\right)^2 s^2 + \sum_{j=1}^{r} \frac{[A_(2+j)]}{s + \frac{V}{b} \theta_j} s
\]

(7)
where all the matrix coefficients and the q_j values are real and
constants and where r normally varies between 1 ≤ r ≤ 4. Substitution of
equations (6) and (7) into equation (5) yields a rational matrix equation
in s. The common denominator of the equation of motion is given by the
scalar D(s) defined by

\[ D(s) = Q(s) \prod_{j=1}^{r} \left[ s + \frac{V}{b} q_j \right] \]  

To solve the above rational equation of motion, it is multiplied by D(s)
where D(s) is assumed to be of order s^{(p-2)}. Hence equation (5) which is
of order s^2 turns to be of order s^p and assumes the form of a matrix
polynomial expression

\[ ([F_0] + [F_1]s + [F_2]s^2 + \ldots + [F_p]s^p) q_s = 0 \]  

where the matrix coefficients [F_j] are functions of M, V, and dynamic
pressure q_n(\frac{1}{2}\rho V^2). Equation (9) can be reduced to the following
canonical form for eigenvalue solution

\[ s \times = [U] \times \]  

where [U] is of size (p x n_s) x (p x n_s) defined by

\[
[U] = \begin{bmatrix}
    [-F_p^{-1}_{F_1}] & [-F_p^{-1}_{F_2}] & \ldots & [-F_p^{-1}_{F_{p-1}}] & [-F_p^{-1}_{F_{p-2}}] & \ldots & [-F_p^{-1}_{F_{p}}] \\
    [I] & 0 & \ldots & 0 & 0 & \ldots & 0 \\
    0 & [I] & \ldots & 0 & 0 & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \ldots & [I] & 0 & \ldots & 0 \\
\end{bmatrix}  
\]  

and X is given by

\[
X = \begin{bmatrix}
    s^{(p-1)} q_s \\
    s^{(p-2)} q_s \\
    \vdots \\
    s q_s \\
    s^0 q_s \\
\end{bmatrix}  
\]
It can thus be seen that the original \( n \) structural equations of motion end up with \((p \times n)\) equations which need to be solved for their eigenvalues.

The main disadvantage of this method lies in the very rapid expansion with control law transfer function of the order of the eigenvalue problem. For illustration purposes, consider a 10 degree of freedom flutter problem \((n=10)\) with aerodynamics approximated using 4 lag terms \((r=4)\) and with two active control surfaces driven by control laws having four poles each. Hence \( Q(S) \) will be of order \( S^8 \) and \( D(S) \) of order \( S^{12} \) (see eq. (8)). The value of \( p \) will therefore be equal to \( p=14 \). It can therefore be seen that the original 10 degree of freedom flutter problem turns into an eigenvalue problem of order \((14 \times 10)\), that is, of order 140.

**FLUTTER ANALYSIS BASED ON OPTIMAL CONTROL FORM OF TRANSFER FUNCTIONS (OCF) (Ref.2)**

Consider equation (3), substitute equation (7) and multiply by the common denominator of the lag terms to obtain a matrix polynomial equation of the form

\[
([F_0 S F_0 C] + [F_1 S F_1 C] S + [F_2 S F_2 C] S^2 + \ldots + [F(r+2) S F(r+2) C] S^{(r+2)})\{q_s\} = 0
\]

(13)

where \( r \) represents the number of lag terms in equation (7) and where the matrix coefficients \([F_j]\) are functions of \( M \), \( V \) and \( q_d \). As in the previous case treated above, equation (13) can be brought to the form

\[
s X_s = [A_s] X_s + [B_c] X_c
\]

(14)

where
\[ X_s = \begin{bmatrix} s^{(r+1)} q_s \\ s^r q_s \\ s^r q_s \\ \vdots \\ s q_r \\ s^0 q_s \end{bmatrix} \]  
\[ X_c = \begin{bmatrix} s^{(r+2)} q_c \\ s^{(r+1)} q_c \\ s^{(r+1)} q_c \\ \vdots \\ s q_c \\ s^0 q_c \end{bmatrix} \]  
\[ \begin{bmatrix} G_s \ F(r+1)_s \\ G_s \ F_{r_s} \\ \vdots \\ G_s \ F_1 \end{bmatrix} \begin{bmatrix} B_s \\ F_o_s \end{bmatrix} \]
\[ [\bar{A}_s] = \begin{bmatrix} [I] & 0 & \cdots & 0 & 0 \\ 0 & [I] & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & [I] & 0 \end{bmatrix} \]  
where
\[ [G_s] = [F(r+2)_s]^{-1} \]
and where
\[ [\bar{B}_c] = \begin{bmatrix} [G_s \ F(r+2)_c] \\ [G_s \ F(r+1)_c] \\ \vdots \\ [G_s \ F_1_c] \\ [G_s \ F_o_c] \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \]

To include the effects of actuator dynamics using optimal control form of transfer functions, the actuator model is described in state-space form. For simplicity of illustration, consider the case of a single actuator that is, when \( q_c \) is a scalar rational polynomial quantity. Assume the following form for the actuator transfer function.
\[
\frac{q_c(s)}{q_c, I(s)} = \frac{b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}
\]  

(20)

where \(q_c, I(s)\) represents the input signal to the actuator.

Equation (12) can be brought to the form

\[
(s^n + a_{n-1}s^{n-1} + \ldots + a_0)q_c = b_0q_c, I
\]  

(21)

which, in turn, can be represented by

\[
s \begin{bmatrix}
X_{a,c}
\end{bmatrix} = [A_{a,c}] \begin{bmatrix}
X_{a,c}
\end{bmatrix} + [B_{a,c}] \begin{bmatrix}
u_{a,c}
\end{bmatrix}
\]  

(22)

where

\[
\begin{bmatrix}
\begin{bmatrix} s^{n-1} & q_c \\ s^{n-2} & q_c \\ \vdots \\ s & q_c \\ s^0 & q_c
\end{bmatrix}
\end{bmatrix}
\]  

(73)

\[
u_{a,c} = q_c, I
\]  

(24)

\[
[A_{a,c}] = \begin{bmatrix}
-a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\]  

(25)

\[
[B_{a,c}] = \begin{bmatrix}
b_0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]  

(26)

For a number of control surfaces, an equation similar to equation (22) is obtained.

Denote by \(\bar{X}_c\) the longest of the vectors \(X_c\) and \(X_{a,c}\) and modify either \([A_{a,c}]\) or \([B_{c}]\) accordingly (denoted by adding an additional bar to these matrices). In so doing, it is possible to merge equations (14) and (22) into a single equation of the form
The matrix $A_s$ is of order $[n_s x (r+2)] x [n_s x (r+2)]$, whereas $[A_{a,c}]$ is of order $n_{a,c} x n_{a,c}$ where

$$n_{a,c} = \max \left[ \sum_{i=1}^{n_c} n_i : (r+3) x n_c \right]$$

where $n_i$ denotes the value of $n$ for the $i$th control.

Optimal control analysis yields control laws of the following form

$$u_{a,c} = [E] \begin{bmatrix} x_s \\ X_c \end{bmatrix}$$

where $[E]$ is a matrix of constants. Substitution of equation (29) into eq. (27) yields the following eigenvalue equation which forms the basic equation for flutter analysis

$$s \begin{bmatrix} X_s \\ X_c \end{bmatrix} = \begin{bmatrix} A_s & B_c \\ 0 & A_{a,c} \end{bmatrix} \begin{bmatrix} X_s \\ X_c \end{bmatrix} + \begin{bmatrix} 0 \\ B_{a,c} \end{bmatrix} [E] \begin{bmatrix} X_s \\ X_c \end{bmatrix}$$

The order of this eigenvalue equation is therefore $[n_s x (r+2) + n_{a,c}] x [n_s x (r+2) + n_{a,c}]$.

For comparison purposes, consider the example treated earlier, that is, the case where

$$n_s = 10, \quad n_c = 2, \quad r = 4, \quad n = 4 \quad \text{(for each control)}$$

Hence, the order of the eigenvalue equation (30) will be, in this case,

$$10 x (4+2) + 7 x 2 = 74$$

which is almost half the order obtained by using the CDM method ($= 140$).

The main disadvantage of this method involves the limitation brought about by the use of a control law defined by equation (29). In this latter equation, the control law transfer function is linear with $X_s$.
and is therefore limited to derivatives of $q_s$ not exceeding the order of $(r+1)$ whereas a general transfer function may employ any order of $q_s$ derivatives provided it is smaller than the order of its denominator.

In the following section, a different method is presented which is very similar to the method just described but which avoids the use of the limiting forms of control laws, such as the one described by equation (29).

**THE PROPOSED METHOD**

Consider equations (3) and (4) and represent the matrix $[T]$ in equation (4) by

$$[T] = \left[ \frac{1}{Q(s)} \right] [P(s)]$$

where $\left[ \frac{1}{Q(s)} \right]$ is a diagonal matrix consisting of the common denominators of each of the rows of matrix $[T]$ and where $[P(s)]$ represents the remaining numerator polynomial (in $s$) of matrix $[T]$. Substituting equation (31) into equation (4) and combining it with equation (3) we obtain

$$\left[ \left[ M_S - M_C \right] s^2 + q_o [A_S - A_C] + [K_S - K_C] \right] \begin{pmatrix} q_s \\ q_c \end{pmatrix} = 0$$

or, after some rearrangement

$$\left[ \left[ M_S - M_C \right] s^2 + q_o [A_S - A_C] + [K_S - K_C] \right] \begin{pmatrix} q_s \\ q_c \end{pmatrix} = 0$$

Substitute equation (1) into equation (33) and multiply the structural equations by the common denominator of the lag terms to obtain after some rearrangements

$$\begin{bmatrix} E(s) & G(s) \\ -P(s) & Q(s) \end{bmatrix} \begin{pmatrix} q_s \\ q_c \end{pmatrix} = 0$$

where $E(s)$ and $G(s)$ are matrix polynomials of order $s^{(r+2)}$. 
Define the following matrices

\[ R(s) = \begin{bmatrix} E(s) \\ P(s) \end{bmatrix} \quad (35) \]

\[ D(s) = \begin{bmatrix} G(s) \\ Q(s) \end{bmatrix} \quad (36) \]

where \( R(s) \) and \( D(s) \) can be written in the following matrix polynomial form.

\[ R(s) = R_0 + R_1 s + R_2 s^2 + \ldots + R_m s^m \quad (37) \]

\[ D(s) = D_0 + D_1 s + D_2 s^2 + \ldots + D_n s^n \quad (38) \]

The value of \( m \) is \((r+2)\) unless the order of the numerators \( P(s) \) is larger than \((r+2)\). In this latter case, \( m \) assumes the maximum value of the power \((in s)\) of the numerators. Similarly, the value of \( n \) is equal to the largest value of the powers of \( Q(s) \) (which represents the denominators of the control laws transfer functions), provided it is larger than \((r+2)\). Otherwise, \( m \) will assume the value of \((r+2)\).

It should be stated at this stage that the representation of \( D(s) \) by equation (38) is convenient for mathematical representation and for programming, but is somewhat wasteful regarding the final order of the eigenvalue problem. However, these changes in the order of the eigenvalue problem are generally small, and do not, therefore, warrant a different, more cumbersome formulation. It should also be observed that the highest powers in \( s \) of both \( E(s) \) and \( G(s) \) are of order \((r+2)\) and that the highest powers in \( s \) of \( P(s) \) are either equal or smaller than the highest powers in \( s \) of \( Q(s) \) (since \( P(s) \) appears in the numerator of the transfer functions whereas \( Q(s) \) appears in the denominator). Hence it can be stated that

\[ m \leq n \quad (39) \]
substituting equations (35) - (38) into equation (34) and rearranging yields the following equation

\[
\begin{bmatrix}
[R_m & D_n] & [R_{m-1} & D_{n-1}] & [R_{m-2} & D_{n-2}] & \ldots & [R_0 & D_{n-m}] & [D_{n-m-1}] & \ldots \\
\end{bmatrix}
\begin{bmatrix}
Z \\
\end{bmatrix} = 0
\]

(40)

where

\[
Y = \begin{bmatrix}
 s_{m-1} & q_s \\
 s_{n-1} & q_c \\
 s_{m-2} & q_s \\
 s_{n-2} & q_c \\
 \vdots \\
 s_{n-m} & q_s \\
 s_{n-m-1} & q_c \\
 \vdots \\
 s_0 & q_c
\end{bmatrix}
\]

\[
Z = \begin{bmatrix}
 s_{m-1} & q_s \\
 s_{n-1} & q_c
\end{bmatrix}
\]

(41)

For the case where \( n = m \), all the terms appearing in equations (40), (41) which involve powers or indices smaller than \( (n-m) \), should be omitted from the equations.

Premultiplying equation (4) by \([R_m & D_n]^{-1}\) and defining

\[
[K] = -[R_m & D_n]^{-1} [R_j & D_i]
\]

(42)

we obtain the following equation

\[
[[I]s [R_{m-1} & D_{n-1}] [R_{m-2} & D_{n-2}] \ldots [R_0 & D_{n-m}] [D_{n-m-1}] \ldots [D_0]] [Y] = 0
\]

(43)

Finally, equation (43) can be written in the form

\[
\begin{bmatrix}
[R_{m-1} & D_{n-1}] & [R_{m-2} & D_{n-2}] & \ldots & [R_1 & D_1] & [R_0 & D_{n-m}] & [D_{n-m-1}] & \ldots & [D_1] & [D_0]
\end{bmatrix}
\begin{bmatrix}
[I] \\
0 \\
[1] \\
\vdots \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
Y \\
\end{bmatrix}
\]

(44)
where \([I]\) is a unit matrix of order \((n_s+n_c)\) and \([I^*]\) is a unit matrix of order \(n_c\). Equation (44) represents therefore an eigenvalue problem of order \((m \times n_s + n \times n_c)\).

For illustration purposes, consider the example treated earlier in this work, that is

\[
\begin{align*}
  n_s &= 10, \quad n_c = 2, \quad r = 4
\end{align*}
\]

with control surfaces transfer function with 4 poles each. In this case

\[
\begin{align*}
  m &= 6; \quad n = 6
\end{align*}
\]

Hence, the order of the eigenvalue equation will be

\[
6 \times 10 + 6 \times 2 = 72
\]

This is about the same order as the OCF method (= 74) and is of considerably smaller order than the CDM method (= 140). Hence, the method proposed herein, enjoys the compactness of the OCF method while maintaining the utmost generality in the form of the control law used for activation. It should be mentioned at this stage that care must be exercised while setting up equation (40) so as to ensure that the matrix

\[
\begin{bmatrix}
  R_m & D_n
\end{bmatrix}
\]

is non-singular (since it needs to be inverted). This point is important while programming equation (40).
2. THE EQUATIONS OF MOTION FOR GUST RESPONSE OPTIMIZATION
ANALYSIS WITH MULTI-ACTIVE CONTROLS

The equations of motion represented by eq. (1) now assume the following form

\[
([M]s^2 + \frac{1}{2} \rho v^2 [A] + [K]) q = F_G
\]  

(46)

where \( F_G \) represents the gust force acting on the system due to a sinusoidal gust velocity of unit amplitude at a specified Mach number and a specified dynamic pressure. Following identical steps represented by eqs (2-4), the following equivalent form of eq. (5) is obtained

\[
\left(\left([M_s] + [M_c][T]\right)s^2 + \frac{\rho v^2}{2}([A_s] + [A_c][T]) + ([K_s] + [K_c][T])\right) q_s = F_G
\]  

(47)

Eq. (47) yields

\[
q_s = [B] F_G
\]  

(48)

where

\[
[B] = \left(\left([M_s] + [M_c][T]\right)s^2 + \frac{\rho v^2}{2}([A_s] + [A_c][T]) + ([K_s] + [K_c][T])\right)^{-1}
\]  

(49)

Using eqs. (4), (48), the control response can be computed

\[
q_c = [T][B] F_G
\]  

(50)

The control rates can similarly be represented by

\[
\dot{q}_c = s[T][B] F_G
\]  

(51)

The 1st root-mean-square (rms) control deflection or the 1st rms control surface rate per unit rms gust input is then computed using the following relations for the 1st control surface

\[
(q_{c_1})_{\text{rms}} = \left( \int_{-\infty}^{\infty} q_{c_1}^2 \phi(\omega) d\omega \right)^{1/2}
\]  

(52)
or

$$(\delta C_i)_{\text{rms}} = \left( \int_{\omega_1}^{\omega_2} \delta C_i^2 \phi(\omega) d\omega \right)^{1/2}$$  \hspace{1cm} (53)$$

where $\phi(\omega)$ represents the Von-Karman gust spectrum.

The gust optimization program seeks to minimize a target function consisting of weighted rms responses or weighted rms response rates of the control surfaces by varying the various specified control gains available in matrix $[T]$. For the optimization results to yield sensible values it is absolutely necessary that the initial values of $[T]$ (for the specified flight dynamic pressure and the specified Mach number) be such as to yield a stable system. Under this condition, stability is maintained during the optimization process while the control surface rms responses are reduced.

Further details regarding the gust optimization method for flutter suppression are presented in ref. 1.
REFERENCES


The program computes the eigenvalues of the flutter equations of motion with active controls. The dimensions assigned to the different arrays permit the simultaneous activation of up to 6 controls (of leading-edge (L.E.), and/or trailing edge (T.E.) types) with resulting augmented eigenvalue problem of up to 100 values (the basic unaugmented system is limited to 15 modes, including rigid body modes). The input data is organized on file 5, with the aerodynamic data (defined by array AERO(I,J,K)) located on file 2. The printed output is located on file 6. The control law transfer function matrix is computed in subroutine CONTRL. The program includes two versions for CONTRL based on the concept of aerodynamic energy. It is imperative to extract one of these two versions of CONTRL before running the program. For other types of control laws, subroutine CONTRL needs to be reprogrammed. To ease this task, details relating to subroutine CONTRL are given in Appendix C.

The output of the program consists of the input data together with the system's eigenvalues over a selected range of dynamic pressures. The package includes all the subroutines used by the program except for the plotting subroutines (which are installation oriented) and the eigenvalue routines (IMSL routines). To ease the substitution of these eigenvalue routines by other ones (should the IMSL library be unavailable) a full description of the COMMON parameters of these routines is given in Appendix D. A root-locus plot (with dynamic pressure as variable) may form a part of the output when desired.

The program is written in FORTRAN and was developed on an IBM 370/168 computer. Double precision is used throughout the program due to the
shorter IBM word length relative to the CDC computers. For CDC installations, it is recommended to convert the program to a single precision version. An example of an input and an output is included herein.

**INPUT OF DATA**

In the following, the data required for the operation of the flutter program is described. For sake of clarity and brevity READ statements are reproduced here together with the specified FORMAT and with the full explanation of the various parameters.

READ (FORMAT (15A4)), (HDR(I), I=1,15)

HDR     an alphanumeric header for the job (up to 60 characters, including spaces).

READ (5, CASE)

where 5 designates the input file and CASE is a namelist defined by

NAMELIST/CASE/NM, NC, NAER, B, NG, NL

where

NM     Integer specifying the number of modes (<15)
NC     Integer specifying the number of controls (<6)
NAER - 1 Input aerodynamics will be introduced by means of PADE interpolation coefficients
- 0     Input aerodynamics will be introduced by means of aerodynamic coefficients at different values of reduced frequency k.
B     Array of values of lag terms to be used during the PADE interpolation (<4)
If gust aerodynamic coefficients are included in the aerodynamic data.
- 0 If gust aerodynamic coefficients are not included in the aerodynamic data.

NL Integer specifying the number of lag terms to be used during the PADE interpolation (<4)

The aerodynamic data is then introduced as follows:

If NAER = 1 then
   DO 1 I = 1, NM
       DO 1 J = 1, (NM+NC)
       READ (FORMAT (6X, 7E10.4)), AO(I,J), A1(I,J), A2(I,J), A3(I,J), A4(I,J), A5(I,J), A6(I,J)
   1 CONTINUE

where the aerodynamic matrix A (see eq. (1)) is assumed to be expressed by

\[
[A] = [A0] + [A1](ik) + [A2](ik)^2 + \sum_{L=1}^{NL} [AL](ik)k^L
\]

and k denotes the reduced frequency. The aerodynamic matrix [A] should be arranged so that control coefficients are located in the last columns with the gust coefficients at the very last column.

If NAER = 0 then

READ (5, FT)
   DO 1 K = 1, NK
       DO 1 J = 1, (NM + NC + NG)
       DO 1 I = 1, NM
       READ (2, FORMAT (2E15.5)) AERO (I,J,K)
   1 CONTINUE
where FT is a namelist defined by

```
NAMELIST/FT/NK, AK, MAXNK, NPRINT, NPUNCH, IRIGID, JRIGID
```

and where 2 designates the file in which the aerodynamic data is located. The various parameters are defined as follows:

- **NK** Number of reduced frequencies \( k \) used for the interpolation of the aerodynamic coefficients.
- **AK** Array \( \leq 20 \) containing the values of \( k \) corresponding to the aerodynamic coefficients. The first value of \( k \) must be zero. The order of the frequency values must correspond to the order of the aero coefficients AERO \( (I,J,K) \) - see below.
- **MAXNK** Maximum value of NK (\( = 20 \) in present program).
- **NPRINT**
  - 0 No printed output from the Pade interpolation routine (named subroutine FIT).
  - 1 Printed output is available.
- **NPUNCH**
  - 0 No punched output from subroutine FIT.
- **IRIGID, JRIGID**

Interpolation coefficients for the aerodynamic coefficients \( \text{PADE representation} \) of the first IRIGID rows and first JRIGID columns are determined using the first few values of reduced frequency \( k \) (assumed to be the lowest) without resorting to a least squares procedure. In this case the rigid body modes must be located so as to be the first modes. This is done in order to increase the accuracy of the aerodynamic coefficients at low \( k \) values where steady state stiffness and damping terms are zero (the least square routine may render them negative).
AERO(I,J,K) Array containing the values of the aerodynamic matrix A
(see eq. (1)) - that is, the \((I,J)^{th}\) coefficient at the
\(K^{th}\) reduced frequency. The order at which the
different \(K\) values are arranged must correspond to the \(A_K\)
values. The first \(k\) value must correspond to \(k=U\). For
order of columns in \([Aj\) see remark for case NAER\#U.

The program proceeds to the construction of the equations of motion
(in subroutine FLUTCA) in first order form, as explained in the
theoretical section of this work. The data required for this purpose
is the following:

\[
\text{READ (5, FLUT)}
\]

where FLUT is a namelist defined by

\[
\text{NAMELIST /FLUT/MASS, OMEGAN, QBEGIN, QEND, NO, VEL, BTRAN, CTRAN,}
\text{CREF, ZW, ZREF, IPOINT, CLF, CTR, NCACT.}
\]

and the parameters are as follows:

\[
\text{MASS \quad MASS matrix (< (15 x 15)}
\]

\[
\text{OMEGAN \quad Array containing the values of the natural frequencies (in HZ). Stiffness is computed from MASS and OMEGAN and is therefore correct for diagonal mass matrices only. For nondiagonal mass matrices the stiffness computation in cards 331-333 (in FLUTCA) must be replaced by an appropriate READ statement.}
\]

\[
\text{QBEGIN, QEND, NO}
\]

The flutter eigenvalue equations are solved for \((NO + 1)\)
values of dynamic pressure \(Q\), starting with the value of
\(Q=QBEGIN\) and ending with the value of \(Q=QEND\).

\[
\text{VEL \quad Flight velocity.}
\]
BTRAN

Array of semichord lengths of wing (and/or tail) sections where the different controls are located (at mid-span of control sections) - (≤ 6).

CTRAN

Array of distances between the two transducers at each control surface mid-section (used to compute the angle of deformation) - (≤ 6).

CREF

Reference semi-chord length (normally wing root semi-chord length) - should be consistent with the reference length used in computing the reduced frequency k (in aero program).

ZW

Matrix where Zw (I,J) indicates the displacement (positive down) of the Ith transducer due to the Jth mode. For each section, two transducers are allowed. The fore transducer should be placed (in the data) ahead of the aft transducer. The present subroutines CONTRL assume the fore transducer to be located at 30 chord from leading edge (L.E.) and these sets of transducers should be arranged in the same order as the controls - (≤ (12 x 15)). For other types of subroutines CONTRL see Appendix C.

ZREF

Values like Zw of reference transducers are used to detect the rigid body motion of the aircraft. They are used in this program to determine the elastic deformation of the wing. If not needed, use zero values for ZREF - (≤ (2 x 15)).

IPL0T - 1

A root locus plot will be made.

- 0 No plotted output.
CLR — Array of X distances (positive aft) between the fore reference transducer and the fore control transducer - (≤6).

CTR — Distance between the two transducers at the reference section.

NCACT — Number of active controls starting from control No. 1. No intermediate controls can be assumed to be inactive (control gains can, in this case, be made equal to zero).

Remark: The transducer data as indicated above is tailored fit to the control laws employed using the aerodynamic energy concept. The form of control law assumed is as follows:

\[
q_c = \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_{\text{NC}}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{\eta_1(s)} & 0 & \cdots & 0 \\
0 & \frac{1}{\eta_2(s)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{\eta_{\text{NC}}(s)}
\end{bmatrix} \begin{bmatrix}
P_{1,1}(s) & \cdots & P_{1,\text{NA}}(s) \\
P_{\text{NC},1}(s) & \cdots & P_{\text{NC},\text{NA}}(s)
\end{bmatrix} \begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_{\text{NA}}
\end{bmatrix}
\]

where the vector \([h_1, h_2, \ldots, h_{\text{NA}}]^T\) denotes relative displacements and/or relative rotations. The aerodynamic energy control law assume that \(\delta_1\) is driven by \(h_1\) and \(h_2\), that \(\delta_2\) is driven by \(h_3\) and \(h_4\) and so forth, so that \(\text{NA} = 2\times\text{NC}\). The matrices \([-\frac{1}{Q(s)}]\) and \([P(s)]\) are computed in subroutine CONTRL (see Appendix C). The above form, however, is very general and can be readily used for other types of control laws which are driven by any number of either relative or absolute (or both) displacements (and/or rotations) at any chordwise location. The cards 473-482 in FLUTCA process the transformations matrix \([H]\) (or order \(\text{NA}\times\text{NM}\)) connecting the vector \([h_1, h_2, \ldots, h_{\text{NA}}]^T\) with the generalized coordinates

\[
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_{\text{NA}}
\end{bmatrix} = [H] \begin{bmatrix}
q_s
\end{bmatrix}
\]

(A.2)
and computes the product \( P(s) \) (see eq. (31)) denoted by \( PH \) in the program), that is
\[
[P(s)] = [\bar{P}(s)][H]
\] (A3)
where \( [P(s)] \) is of order \((NC \times NM)\).

In summary, subroutine CNTRL provides the matrices \( \frac{1}{Q(s)} \) and \( [\bar{P}(s)] \) whereas the matrix \( (P(s)) \) is computed in cards 473-482 (in FLUTCA).

If and only if the parameter \( IPLUT = 1 \) the program then reads the namelist PLUTPA

\[\text{READ (5, PLUTPA)}\]

defined by

\[\text{NAMELIST /PLUTPA/XI, YI, XSCALE, YSCALE, XL, YL, ISYM, IENTRY}\]

where

- \( XI \) Left hand limit of real part of root locus.
- \( YI = 0 \)
- \( XSCALE \) Abscissa scale (value per inch).
- \( YSCALE \) Ordinate scale (value per inch).
- \( XL \) Length of abscissa in inches.
- \( YL \) Length of ordinate in inches.
- \( ISYM \) Integer defining symbol during root locus plot (\#3 is recommended).
- \( IENTRY = 1 \)

The program then reads the namelist MXSIZE

\[\text{READ (5, MXSIZE)}\]

defined by

\[\text{NAMELIST/MXSIZE/MAXC, MAXNM, MAXK, MAXT}\]

where

- \( MAXC \) Maximum number of controls (6 in this program).
- \( MAXNM \) Maximum number of modes (15 in this program).
MAXK Maximum number of polynomial terms per element in the transfer function numerator and denominator matrices (-10 herein).

MAXT Maximum order of final matrix \([U]\) (where \(dY/dt = [U] Y\) - assigned the value 100 in this program.

If, and only if, NCACT ≠ 0 the program reads the namelist CONC

```
READ (5, CONC)
```

defined by

```
NAMELIST/CONC/WR,NTE,X
```

where:

WR Reference frequency (rad/sec) used only for the D.T.T.F. control law (aerodynamic energy) - the value chosen is normally around the value of the flutter frequency.

NTE Integer array (following the order of the controls) which identifies between L.E. and T.E. controls.

- 1, T.E. control
- 0, L.E. control.

Note that whenever a control is not active, put NTE = 0 when using aerodynamic energy versions for CONTRL.

\(X\) Array of gains. There are 6 gains per control surface for the L.D.T.T.F. and 1 gain per control surface for the D.T.T.F. (≤ 36). The values of \(X(1)\) for the L.D.T.T.F. should be used considering the following basic form for the \(i^{th}\) control surface transfer function

\[
F_i = \frac{X(1+6*(1-1))s^2}{s^2 + 2*X(2+6*(1-1))*X(3+6*(1-1))*s + (X(2+6*(1-1))^2} + \frac{X(4+6*(1-1))s^2}{s^2 + 2*X(5+6*(1-1))*X(6+6*(1-1))*s + (X(5+6*(1-1))^2} \]
For L.E. control
\[ \delta_1 = F_1 L - 4 J \left\{ \frac{(h/b)}{U.3C} \right\} \]

For T.E. control
\[ \delta_1 = F_1 L - 4 J \left\{ \frac{(h/b)}{U.3C} \right\} - 1.8 \alpha \]

For further details see Ref. 3.
4. APPENDIX B

OPERATION INSTRUCTIONS FOR THE GUST OPTIMIZATION/GUST SENSITIVITY PROGRAM

The gust package permits the computation of the spectral responses of an aircraft due to a continuous gust environment. The effects of active controls (up to 6 controls) on the gust response can be accounted for. Furthermore, the basic gust program is coupled, in the present package, with an optimization routine which enables the determination of the various control gains which minimize the control responses to gust. Sensitivity studies (with plotted output) around the given or optimal control gains can also be made.

The input data is organized on file 5, with aerodynamic data (defined by array AERO (I,J,K)) located on file 2. Most of the printed output is located on file 6 with some additional output (arising from the optimization stage) located on file 4. File 13 is used by the package for labelling of plots and needs to be declared by the programmer.

The control law transfer function is computed in subroutine CUNTRL. The program includes two versions for CUNTRL based on the concept of aerodynamic energy. It is imperative to extract one of these two versions of CUNTRL before running the program. For other types of control laws, subroutine control needs to be reprogrammed. To ease this task, details relating to subroutine CUNTRL are given in Appendix C.

The output of the program consists of the input data together with the optimal control gains and the power spectral density (PSD) plots of the control responses, when used in its gust optimization version. When used as a control gain sensitivity program, the output is supplemented by sensitivity plots showing the variation of the rms control responses with the various control law gains. The package includes all the subroutines...
used by the program except for the plotting subroutines (which are installation oriented).

The program is written in FORTRAN and was developed on an IBM 370/168 computer. Double precision is used throughout the program due to the shorter IBM word length relative to the CDC computers. For CDC installations, it is recommended to convert the program to a single precision version. Input/output examples are included herein.

When using the program in its gust optimization version it is advisable to extract subroutines GUSPLT and PLT from the package. The input data for the gust optimization version will first be presented. The changes required in the data and in the program when running the program in its gust sensitivity version will then be presented.

**INPUT OF DATA - GUST OPTIMIZATION VERSION**

In the following, the data required for the operation of the gust optimization program is described. Here again READ statements will be reproduced together with the specified FORMAT and with the full explanation of the various parameters.

```fortran
READ (F377 (15A4)), (HDR(I), 1 = 1, 15)

HDR          An alphanumeric header for the job (up to 60 characters, including spaces).

READ (5, CASE)

where 5 designates the input file and CASE is a namelist defined by
```

```fortran
NAMELIST/CASE/NM, NC, NAER, B NG, NL
```

where

- **NM** Integer specifying the number of modes (\(\leq 15\)).
- **NC** Integer specifying the number of controls (\(\leq 6\)).
- **NAER** Integer specifying the aerodynamics will be introduced by means of PADE interpolation coefficients.
- 0  Input aerodynamics will be introduced by means of
    aerodynamic coefficients at different values of reduced
    frequency $k$.

- 0  Array of values of lag terms to be used during the PADE
    interpolation ($\leq 4$).

- 0  If gust aerodynamic coefficient are included in the
    aerodynamic data.

- 0  If gust aerodynamic coefficients are not included in the
    aerodynamic data.

NL  Integer specifying the number of lag terms to be used
    during the PADE interpolation ($\leq 4$).

The aerodynamic data is then introduced as follows:

If NAER $\neq 0$ then

DO 1 I = 1, NEI

DO 1 J = 1, (NM + NC + NG)

READ (FORMAT(6X,7E10.4)), AO(I,J), A1(I,J), A2(I,J), A3(I,J), A4(I,J),
A5(I,J), A6(I,J)

1 CONTINUE

Where the aerodynamic matrix $A$ is assumed to be expressed by

$$A = AO + A1(ik) + A2(ik)^2 + \sum_{L=1}^{NL} A(L) \frac{AL(ik)}{TK+B(L)}$$

and $k$ denotes the reduced frequency. The aerodynamic matrix $[A]$ should be arranged so that control coefficients are located in the last columns with the gust coefficients at the very last column.

The program proceeds to read the namelist GST defined by

NAMELIST/GST/RMASS, OMEGAN, VEL, BTRAN, CTEAN, CREF, ZW, ZREF, Q,
CLR, CTR, WR, NTE, NCACT

where
RMASS

Mass matrix ($< 15 \times 15$)

OMEGAN

Array containing the values of the natural frequencies (in HZ). Stiffness is computed from RMASS and OMEGAN and is therefore correct for diagonal mass matrices only.

For non-diagonal mass matrices the stiffness computation in card 437 (in subroutine SOLGST) should be replaced by an appropriate READ statement. It is important to note that 1.5 structural damping is assumed in the program. Modify card 438 if other values are desired.

VEL

Flight velocity.

BTRAN

Array of semichord lengths of wing (and/or tail) sections where the different controls are located (at mid-span of control sections) - ($\leq 6$).

CTRAN

Array of distances between the two transducers at each control surface mid-section (used to compute the angle of deformation - ($\leq 6$)).

CREF

Reference semichord length (normally wing root semichord length) - should be consistent with the reference length used in computing the reduced frequency $k$ (in aero program).

ZW

Matrix where ZW (I,J) indicates the displacement (positive down) of the I$^{th}$ transducer due to the J$^{th}$ mode. For each section, two transducers are allowed - the fore transducer should be placed (in the data) ahead of the aft transducer. The present subroutines CONTRL assume the fore transducer to be located at 30 chord from leading-edge (L.E.) and these sets of transducers should be arranged in the same order as the controls - ($\leq (12 \times 15$)). For other types of subroutines CONTRL see Appendix C.
ZREF  Values like ZW of reference transducers used to detect the rigid body motion of the aircraft. Used in this program to determine the elastic deformation of the wing. If not needed, use zero values for ZREF – (< (2 x 15)).

Q  Flight dynamic pressure

CLR  Array of X distances (positive aft) between the fore reference transducer and the fore control transducer - (<5).

CTR  Distance between the two transducers at the reference section.

WR  Reference frequency (rad/sec), used only for the D.T.T.F. control laws (aerodynamic energy) – the value chosen is normally around the value of the flutter frequency.

NTE  Integer array following the order of the controls which identifies between L.E. and T.E. controls.

- 1  T.E. control
- 0  L.E. control

NCACT  Number of active controls

Note that whenever a control is not active, put NTE = 0 when using aerodynamic energy versions for CONTRL.

Remark: The transducer data as indicated above is tailored fit to the control laws employed using the aerodynamic energy concept. The form of control law assumed is as follows:

\[ q_c = \left[ \begin{array}{c} q_1 \\ q_2 \\ \vdots \\ q_{NC} \end{array} \right] = \left[ \begin{array}{cccc} 1 \\ \frac{1}{Q_1(s)} \\ \frac{1}{Q_2(s)} & \cdots & \frac{1}{Q_{NC}(s)} \end{array} \right] \left[ \begin{array}{cccc} \bar{p}_{1,1}(s) & \cdots & \bar{p}_{1,NA(s)} & h_1 \\ \vdots & \vdots & \vdots & \vdots \\ \bar{p}_{NC,1}(s) & \bar{p}_{NC,NA(s)} & h_{NA} \end{array} \right] \]

\[ \left[ \frac{1}{P(s)} \right] \]

(B1)
where the vector $[h_1, h_2, \ldots, h_{NA}]^T$ denotes relative displacements and/or relative rotations. The aerodynamic energy control laws assume that $s_1$ is driven by $h_1$ and $h_2$, that $s_2$ is driven by $h_3$ and $h_4$ and so forth so that $NA = 2*NC$. The matrices $\frac{1}{Q(s)}$ and $[P(s)]$ are computed in subroutine CONTRL (See Appendix C). The above form, however, is very general and can be readily used for other types of control laws which are driven by any number of either relative or absolute (or both) displacements (and/or rotations) at any chordwise locations. The cards 282-288 in the main program process the transformation matrix $[H]$ (of order $NA \times NM$) connecting the vector $[h_1, h_2, \ldots, h_{NA}]^T$ with the generalized coordinates

$$
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_{NA}
\end{bmatrix} = [H] \begin{bmatrix}
q_s
\end{bmatrix}
$$

(B2)

so that the matrix $[P(s)]$ in eq. 31 can be computed by

$$[P(s)] = [P(s)][H]
$$

(B3)

In summary, subroutine CONTRL provides the matrices $\frac{1}{Q(s)}$ and $[P(s)]$ whereas the matrix $[H]$ is computed in cards 282-288 (in MAIN).

If $NAER = 0$ then

READ (5,FT)

DO 1 K = 1, NK

DO 1 J = 1, (NM + NC + NG)

DO 1 I = 1, NM

READ (2, FORMAT(2E15.5)) AERO (I,J,K)

1 CONTINUE

where FT is a namelist defined by
and where 2 designates the file in which the aerodynamic data is located. The various parameters are defined as follows:

NK

Number of reduced frequencies used for the interpolation of the aerodynamic coefficients.

AK

Array (\leq 20) containing the values of \( k \) corresponding to the aerodynamic coefficients. The first value of \( k \) must be zero. The order of the frequency value must correspond to the order of the aerodynamic coefficients.

AERO (I,J,K) - see below.

MAXNK

Maximum value of NK (\leq 20 in present program).

NPRINT - 0 No printed output from the PAADS interpolation routine (called FIT).

- 1 Printed output is available.

NPUNCH - 0 No punched output from subroutine FIT.

- 1 Interpolation coefficients are punched.

IRIGID, JRIGID

Interpolation coefficients for the aerodynamic coefficients (PADES representation) for the first IRIGID rows and first JRIGID are determined using the first few values of reduced frequency \( k \) (assumed to be the lowest) without resorting to a least squares procedure. In this case the rigid body modes must be located so as to be the first modes. This is done in order to increase the accuracy of the aero-coefficients at low \( k \) values where steady state stiffness and damping terms are zero (the least square routine may render them negative).
AERO(I,J,K) Array containing the values of the aerodynamic matrix A (see eq. (1)) - that is, the \((I,J)\)th coefficient at the \(K\)th reduced frequency. The order at which the different \(K\) values are arranged must correspond to the \(AK\) values. The first \(K\) value must correspond to \(K=0\). For order of columns in \([A]\) see the remark above for case NAER40.

READ (FORMAT (4E10.0)), ETA1, PHI

ETA1  Accuracy of computer relative to 1 (on I.B.M. double precision = 5.E-13). Absolute accuracy = \(X*ETA1\) (value unimportant for gust sensitivity-version).

PHI  Relative size of "suction zone" within which the optimized parameter is "sucked" to the constraint in order to avoid false convergence. Absolute size of zone = \(XI(I)*PHI\) or \(X2(I)*PHI\) depending on whether near lower or upper constraints (value unimportant for gust sensitivity version).

READ (FORMAT (5I5), NV, NPR, NDR

NV  Number of independent control gains in the control laws (\(\leq 36\)).

NPR = 0

NDR = 0  Optimization is based on the minimization of the RMS responses of controls.

- 1  Optimization is based on the minimization of the RMS response rates of controls.

READ (FORMAT (5I5), NONACT

NONACT  Number of non-active optimization parameters (that is, number of control gains kept fixed during optimization).
READ (FORMAT (515), (NA(I), I = 1, NONACT))

**NA**
Integer Array containing the location of the non-active parameters in the X array (see below). If NONACT = 0, a blank card should be placed here.

READ (FORMAT (4E10.0), WL, WT)

**WL, WT**
Two weights for emphasizing the contributions of any of the control responses in the target function expression (defined as FUNCTN in subroutine SOLGST, cards 461 and 467). More details regarding the target function FUNCTN will be given below at the end of the data description.

DO 200 I = 1, NV
READ (FUNMAT (4E10.U)), XI(I), X(I), X2(I), EPS(I)
200 CONTINUE

**X1(I)**
Value of the lowest bound of the Ith control parameter (during optimization).

**X(I)**
Initial value of the Ith control parameter (at the onset of the optimization process). There are 6 gains per control surface for the L.D.T.T.F. and 1 gain per control surface for the D.T.T.F. (< 36). The values of X(I) for the L.D.T.T.F. should be used considering the following basic form for the Ith control surface transfer function

\[
F_I = \frac{x(1+6*(I-1))*s^2}{s^2 + 2*x(2+6*(I-1))*x(3+6*(I-1))*s + (x(2+6*(I-1)))^2} + \frac{x(4+6*(I-1))*s^2}{s^2 + 2*x(5+6*(I-1))*x(6+6*(I-1))*s + (x(5+6*(I-1)))^2}
\]

For L.E. control

\[
s_1 = F_I \cdot L - 4 \cdot \left\{ \begin{array}{l}
(h/b) \\
0.3C \\
\alpha
\end{array} \right. \]


For T.E. control

\[ \alpha_1 = f_1 L 4 2.8 \left\{ \frac{(h/b)}{a} \right\}^{0.3C/\theta} - 1.8a \]

For further details see Ref. 3.

X2(I) Value of the upper bound of the I\textsuperscript{th} control parameter (during optimization).

EPS(I) The desired absolute accuracy of the optimal final X(I) value.

READ (FORMAT (4E10.0)), FMIN, ETA

FMIN Parameter containing an approximate value to the minimum of the target function FUNCTN (see remark at the end of this section). If unknown, use FMIN = 0.

ETA Parameter containing an estimate of the relative accuracy of the rms response computations. Used to determine the type of difference approximation to the gradient (value unimportant for the gust sensitivity version).

READ (FORMAT (515)), ITMAX, IW

ITMAX An input/output integer. On input, ITMAX contains the maximum allowable number of optimization iterations. On output, ITMAX contains the number of iterations used (value unimportant for the gust sensitivity version).

IW An integer code for printing during computation (value unimportant for the gust sensitivity version).

- 0 No printing.
- 1 Print gradient vector, direction of each linear minimization and function value before and after each linear minimization.
- 2 In addition to the above, print function values calculated during the course of linear minimizations.
3 In addition to the above, print function values calculated in evaluating the gradients.

READ (FORMAT (11U, 2E10.0)), NF FBEGIN, FEND

NF Number of frequency intervals used in computing the spectral response (Total number of frequencies used = NFT = NF+1, should be ≤ 100).

FBEGIN Lower value of frequency (in HZ) in computing the spectral response.

FEND Upper value of frequency (in HZ) in computing the spectral response.

READ (FORMAT (4E10.0)), LENGTH

LENGTH Gust scale length. Used to determine the Von Karman gust spectrum.

READ (FORMAT (4E10.0)), EM

EM Flight Mach number.

Remark: The definition of the target function FUNCTN (in subroutine SOLGST, card 461 for function based on rms control deflections and card 467 for function based on rms rates of control deflections) is left open to the user. It can be defined for example as a weighted sum of the rms responses, that is

\[
\text{FUNCTN} = \sum_{i=1}^{NC} W_i (q_{c_i})_{\text{rms}}
\]

or

\[
\text{FUNCTN} = \sum_{i=1}^{NC} W_i (\dot{q}_{c_i})_{\text{rms}}
\]

where \( W_i \) represents the \( i \)\textsuperscript{th} weight.

In some cases it may be of interest to keep the various rms control responses equal and a penalty function may be introduced into the
target function. Note the following equivalence relations between
the notations used in eqs. (52), (53) and those used in the program:

\[(q_{c_i})_{\text{rms}} = DRMS(I)\]
\[(\dot{q}_{c_i})_{\text{rms}} = DRRMS(I)\]

Important: Do not forget to check whether the definition of FUNCTN
in the program (cards 461, 467) is applicable.

INPUT OF DATA - GUST SENSITIVITY VERSION

As already mentioned earlier, the gust sensitivity version of this
program yields plots showing the sensitivity of the rms responses of
the controls with respect to variations of the various \(X(I)\) gain
parameters. To accomplish this, the following modifications should be
made to the program:

1) Replace cards 299-309 by the following

\[
\text{IFINAL = 1}\]
\[
\text{CALL GUSPLT (XX, XI\text{ACT}, X2\text{ACT}, EPS\text{ACT}, QQ, EM)}
\]

2) Delete cards 320-321.

3) Delete cards 472-526.

4) Delete one of the two subroutines CONT\text{RI} present in the package
or replace both of them by a new one.

One should make sure that both subroutines (GUSPLT and PLT) are
included in the source program.

The data required is identical to the one outlined in the above gust
optimization version except for the following change in the meaning
of the following data:

\[
\text{DU 200 I = 1, N}
\]
READ (FORMAT (4E10.0)), XL(1), X(1), X2(1), EPS(1)

200 CONTINUE

XL(1) Value of the lowest bound of the $i^{th}$ control parameter (during sensitivity variations of this parameter).

X(1) Initial value of the $i^{th}$ control parameter (at the onset of the sensitivity variation).

X2(1) Value of the upper bound of the $i^{th}$ control parameter (during sensitivity variations of this parameter).

EPS(1) The step size used in moving from X(1) to both XL(1) and X2(1).

Furthermore, some of the data needed for the gust optimization version is still read but the values are irrelevant for the gust-sensitivity version since they are not used. These parameters had been indicated while explaining their meaning in the gust optimization version of the program.
5. APPENDIX C
SUBROUTINE CONTRL
DETAILS ON THE COMMON PARAMETERS

Subroutine CONTRL computes the control laws used for either the flutter package or the gust package (including gust optimization program, or control response sensitivity to control law parameter variation program). The subroutines included in the above packages relate to aerodynamic energy control laws of the D.T.T.F and of the L.D.T.T.F. Whenever other types of control laws are required, subroutine CONTRL has to be reprogrammed (the same subroutine CONTRL can be used for both packages mentioned above). In the following, some explanations regarding the COMMON parameters employed by subroutine CONTRL, will be given in order to facilitate the reprogramming of subroutine CONTRL whenever deemed necessary. The subroutine is defined by

SUBROUTINE CONTRL(NP, P, ND, QD, NC, WR, NTE, X)

where

NP Two-dimensional integer output array. NP(I,J) contains the number of polynomial terms (as function of s, starting from s°) in the numerator control law element (I,J) of matrix $[\overline{F}(s)]$ (see eqs. (31), (B3) above) - (1≤6).

P Three-dimensional output array representing the numerator control law matrix $[\overline{F}(s)]$. P(I,J,K) represents the coefficient of $s^{(k-1)}$ in the numerator polynomial located at position (I,J) in $[\overline{F}(s)]$. 
NO One-dimensional integer output array. ND(I) represents the number of polynomial terms (as function of s, starting from s*) in the denominator of the Ith element in the diagonal matrix \( \frac{1}{Q(s)} \) which forms a part of the control law transfer function matrix [T].

QD Two-dimensional output array representing the denominator control law diagonal matrix \( \frac{1}{Q(s)} \). QD(I,K) represents the coefficient of \( s^{(k-1)} \) in the denominator of the Ith element in the diagonal matrix \( \frac{1}{Q(s)} \).

NC Number of controls.

WR An input parameter. Used in present program for the aerodynamic energy control law of the D.T.T.F. to represent reference frequency (rad/sec). The value chosen is normally around the value of the flutter frequency.

NTE One dimensional input array used to distinguish between L.E. and T.E. control surfaces.

\[ = 1, \text{T.E. control.} \]
\[ = 0, \text{L.E. control.} \]

X One-dimensional input array of control gains used for computing both \( P(s) \) and \( \frac{1}{Q(s)} \).
Note that matrix \( P(s) \) (see eq. (31)) is not computed in subroutine CONTRL \( (P(s) = P(s) \times [H]) \) where \([H]\) is the modal matrix connecting the deflection at the different sensor locations with the generalized coordinates of the system, (see also eq. (B3)).
APPENDIX D

EIGENVALUE SUBROUTINES

DETAILS ON THE COMMON PARAMETERS

The subroutines described in the following pages belong to the IMSL library. They can easily be used in installations enjoying access to the IMSL library. Their replacement by other routines, if necessary, involves little effort and can be easily accomplished using the information included herein.
SUBROUTINE BALAF (A,N,IA,D,E,K,L)

C FUNCTION
  BALANCE A REAL MATRIX A.

C USAGE
  CALL BALAF(A,N,IA,D,E,K,L)

C PARAMETERS
  A - THE A X A MATRIX TO BE BALANCED. THE INPUT A IS
  REPLACED BY THE BALANCED MATRIX.
  N - THE ORDER OF THE A X A MATRIX A.
  IA - HOW MANY OF A IN CALLING PROGRAM.
  D - THE OUTPUT ALGORITHM OF LENGTH N WHICH CONTAINS
  INFORMATIONS DETERMINING THE PERMUTATIONS AND THE
  SCALING FACTORS.
  K - A AND L ARE THE OUTPUT INTEGERS SUCH THAT
  A(I,J) = D(I), IF
    1) I IS GREATER THAN J AND J = I + 1 + L*K
  2) J = L + 1 + L*K
  L - AN INTEGER. IF L = 0 THE ORIGINAL MATRIX A IS
  IN HESSNERLING FORM.

C PRECISION
  SINGLE/DOUBLE

C LANGUAGE
  FORTRAN

SUBROUTINE CHESSF (A,K,L,IA,D,U)

C FUNCTION
  REDUCE A N X N SYMMETRIC MATRIX TO UPPER
  HESSNERLING FORM BY ORTHOGONAL TRANSFORMATIONS.

C USAGE
  CALL CHESSF(A,K,L,IA,D,U)

C PARAMETERS
  A - AN N X N SYMMETRIC MATRIX TO BE REDUCED TO
  UPPER HESSNERLING FORM. (INPUT)
  K - AN INTEGER. (INPUT)
  L - THE INDEX OF THE LAST ELEMENT. (INPUT)
  IA - HOW MANY OF A IN CALLING PROGRAM. (INPUT)
  D - THE OUTPUT FORTRAN OF LENGTH N CONTAINING THE
  UPPER HESSNERLING FORM. (INPUT)
  U - AN N X N SYMMETRIC MATRIX. (INPUT)

C PRECISION
  SINGLE/DOUBLE

C LANGUAGE
  FORTRAN

SUBROUTINE EQRHF (M,N,K,L,II,IA,U)

C FUNCTION
  REDUCE A M X N SYMMETRIC MATRIX TO UPPER
  RHOF FORM.

C USAGE
  CALL EQRHF(M,N,K,L,II,IA,U).

C PARAMETERS
  M - THE ORDER OF A SYMMETRIC MATRIX. (INPUT)
  N - THE ORDER OF A SYMMETRIC MATRIX. (INPUT)
  K - HOW MANY OF A IN CALLING PROGRAM. (INPUT)
  L - HOW MANY OF A IN CALLING PROGRAM. (INPUT)
  II - AN INTEGER. (INPUT)
  IA - AN INTEGER. (INPUT)
  U - AN M X N SYMMETRIC MATRIX. (INPUT)

C PRECISION
  SINGLE/DOUBLE

C LANGUAGE
  FORTRAN

ORIGINAL PAGE IS OF POOR QUALITY.
FUNCTION

- FIND THE EIGENVALUES AND (OPTIONALLY) EIGENVECTORS OF A REAL UPPER HESSENBerg MATRIX.

PARAMETERS

- M - AN INPUT, M CONTAINS THE NUMBER OF EIGENVALUES AND, IF REQUESTED, THE EIGENVECTORS. THE EIGENVALUES ARE SORTED IN DECREASING ORDER.

USAGE

- CALL EGHJF (M, H, K, L, M, 12, 12, 1ER).

DESCRIPTION

- H CONTAINS THE REAL UPPER HESSENBerg MATRIX.
- K CONTAINS THE DLCOMPLETED TRIANGLE UNDER M MAY BE USED TO STORE THE HESSENBerg FORM OF THE MATRIX.
- L UNEEDS TO BE STORED.
- M CONTAINS THE NUMBER OF EIGENVALUES AND, IF REQUESTED, THE EIGENVECTORS. THE EIGENVALUES ARE SORTED IN DECREASING ORDER.

NOTES

- THE EIGENVALUES ARE UNORDERED.

INPUT

- M - AN INPUT, M CONTAINS THE NUMBER OF EIGENVALUES AND, IF REQUESTED, THE EIGENVECTORS. THE EIGENVALUES ARE SORTED IN DECREASING ORDER.
- H - A REAL HESSENBerg MATRIX.
- K - AN L UNEEDS TO BE STORED.
- L - AN UNEEDS TO BE STORED.

OUTPUT

- M - AN OUTPUT, M CONTAINS THE REAL AND IMAGINARY PARTS OF THE EIGENVALUES, RESPECTIVELY.
- H - A REAL HESSENBerg MATRIX. THE EIGENVALUES ARE UNORDERED.
- K - AN L UNEEDS TO BE STORED.
- L - AN UNEEDS TO BE STORED.

ERRORS

- EIGHJF CALLS EGHJF IF THE EIGENVECTORS OF THE UPPER HESSENBerg MATRIX ARE DESIRED.
- EIGHJF CALLS EGHJF IF THE EIGENVECTORS OF A REAL GENERAL MATRIX ARE DESIRED.

EXAMPLE

- EIGHJF (M, H, K, L, M, 12, 12, 1ER).

END
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
</table>
| J1, J2, ..., Jn | Eigenvalues i, ..., J are set to zero if iz >= n. 

- **precision**: - single/double
- **routines**: - gentst
- **language**: - fortran

---

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APPENDIX E

SOURCE LISTING AND INPUT/OUPUT EXAMPLE FOR FLUTTER PROGRAM

The first part of the Appendix consists of the source listing of the program and is followed by an input/output example. The example chosen relates to the DAST configuration at M=0.9 with one active T.E. control surface based on the L.D.T.T.F. The output of the computer run includes a root-locus plot together with all the data required by the program. The aerodynamic coefficients AERO (I,J,K) used by the program are listed for convenience (this aerodynamic data is retrieved by the program from file 2).

It is recommended to use the plotting symbol '+' in the root locus plot. The symbol used in the present example is a result of some transient difficulties encountered using a new plotter.
IMPLICIT REAL*8(A-H,O-Z)

REAL*8

CENTER TITLES

FLUTTER SUPPRESSION PACKAGE (WITH OR WITHOUT ACTIVE CONTROLS)

USING ROUT LOCUS TECHNIQUES. THE FOLLOWING INPUT DATA IS REQUIRED.

HDR - HEADER (FORMAT 15A4)

NAMELIST/CASE -

NM - NUMBER OF NODES (15 MAX)

NC - NUMBER OF CONTROLS (6 MAX)

NAER - 1 INPUT AERO IN TERMS OF INTERPOLATION COEFFICIENTS OF K

- 0 INPUT AERO FOR DIFFERENT VALUES OF K - INTERPOLATION

CUE COEFFICIENTS TO BE COMPUTED IN SUBROUTINE FIT.

O - ARRAY OF LAG TERMS USED DURING INTERPOLATION (4 MAX)

NG - 1 IF JUST AERO IS SUPPLIED

- 0 IF JUST AERO IS NOT SUPPLIED.

NL - NUMBER OF LAG TERMS TO BE USED DURING INTERPOLATION.

IF NAER=1 THEN AERO COEFFICIENTS ARE READ (FORMAT 6X.7E10.4)

IF NAER=0 NEXT INPUTS ARE READ IN SUBROUTINE FLUTC.

SUBSEQUENT INPUTS ARE READ IN SUBROUTINE FLUTC.

EXTERNAL DREAL, DAIMAG

COMMON/AERF/A0(15,22), A1(15,22), A2(15,22), A3(15,22), A4(15,22),

* A5(15,22), A6(15,22)

COMMON/ICASE/B(4)*NM, NC, NG, NL

DIMENSION HDR(15)

NAMELIST/CASE/NM, NC, NAER, B, NG, NL

READ 100,(HDR(I), I=1,15)

PRINT 110,(HDR(I), I=1,15)

READ(5,CASE)

WRITE(6,CASE)

NMNC=NM+NC

IF (NAER.EQ.3) CALL FIT

IF (NAER.EQ.0) GO TO 10

DO 1 II=1,NM

DO 1 JJ=1, NMNC

*A3(11, JJ), A5(11, JJ)
1 CONTINUE
10 CONTINUE
CALL CLOTCA
STOP
100 FORMAT(15A4)
101 FORMAT(1H, 15A4)
200 FORMAT(6X, E10.4)
END

SUBROUTINE CONTRL(NP, P, QQ, QQ, NC, X, NTE, X)

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DETERMINATION OF THE NUMERATOR POLYNOMIAL FOR EACH CONTROL SURFACE

```fortran
C C DEFENITION OF THE NUMERATOR POLYNOMIAL FOR EACH CONTROL SURFACE
C
C CN(J)=X(NJ-5)
CALL PROPOL(CD2,J,CN,J,TEMP1,N)
C CN(J)=X(NJ-2)
CALL PROPOL(CD2,J,CN,J,TEMP2,N)
DO 4 K=1,N
P(1,2*N-1,K)=EH*(TEMP1(K)+TEMP2(K))
P(1,2*N,K)=EA*(TEMP1(K)+TEMP2(K))+C21*NTF(I)*QD(K,1)
4 CONTINUE
NP(1,2*N-1)=N
NP(1,2*N)=N
2 CONTINUE
RETURN
END
SUBROUTINE CDNRFL(NP,P,ND,QU,NC,BK,NTF,X)

U=1.0, T=0.0, CONTROL LAW FOR ANY NUMBER OF CONTROL SURFACES, CAN BE USED FOR BOTH FLUTTER AND GUST PROGRAMS, THE BASIC GAINS USED HEREIN ARE APPROPRIATE FOR 20 PERCENT L.E. AND 20 PERCENT T.E. CONTROL SYSTEMS WITH THE PURE SENSE LOCATED AT THE 50 PERCENT CHORD LOCATION - DIMENSIONS ARE LIMITED TO 8 CONTROLS.

IMPLICIT REAL(A-H,O-Z)
DIMENSION NP(6,1),P(12,1),QU(1,1),E(2,2),NTF(I),X(36),ND(6)
E(1,1)=4.00
E(1,2)=4.00
E(2,1)=4.00
E(2,2)=2.00
C21=-1.86
A=1000.00
NL2=2*NC
DO 1 J=1,NC
DO 1 K=1,NC
NP(1,J,K)=0.00
1 CONTINUE
DO 2 J=1,NC
IF(NTF(I) .EQ. 1) GO TO 3
2 CONTINUE
CASE OF T.E. CONTROL
EH=E(P,1)
EA=E(2,2)
IF(NTF(I) .EQ. 1) GO TO 3
CASE OF L.E. CONTROL
EH=E(1,1)
EA=E(1,2)
3 CONTINUE
DETERMINATION OF THE NUMERATOR POLYNOMIAL FOR EACH CONTROL SURFACE
```
C
C DETERMINATION OF THE DENOMINATOR POLYNOMIAL FOR EACH CONTROL SURF.
C
C QD(1,1)=A
QD(2,1)=B
QD(1,2)=A
QD(2,2)=B

2 CONTINUE
RETURN
END

SUBROUTINE FLUT
( IMPLICIT REAL*8(A-H,O-Z) )

THE EQUATIONS OF MOTION ARE BROUGHT IN THIS SUBROUTINE TO A
CONVENTION THAT WOULD FORM DYNAMIC-VIVI AND SOLVE FOR A GIVEN
VELOCITY AND MACH NUMBER AS A FUNCTION OF THE DYNAMIC PRESSURE Q
(WICH IS VARIED) WITHIN A PRESCRIBED RANGE. UP TO SIX ACTIVE
CONTROLS CAN BE USED IN THIS SUBROUTINE. RESULTS ARE SUITABLE
FOR BOTH LOCUS PLOTS.

NAMELIST/FLUT

MSS - MASS MATRIX (L X L X MAX)

HMANG - NATURAL FREQUENCIES, ARRAY(14 N) - (15 MAX) - NOTE-
STIFFNESS IS COMPUTED FROM MASS AND HMANG AND IS THEREFORE
CORRECT FOR DIAGONAL MASS MATRIX ONLY.

BEGIN - INITIAL VALUE OF DYNAMIC PRESSURE Q

END - FINAL VALUE OF DYNAMIC PRESSURE Q

NO - NUMBER OF EQUAL INTERVALS DIVIDING THE Q RANGE (NUMBER OF
VALUES OF Q=NO+1).

VEL - FLIGHT VELOCITY

BTRAN - ARRAY OF SEMI-CHORD LENGTHS OF WING(TAIL) SECTIONS
WHERE THE DIFFERENT CONTROLS ARE LOCATED (AT MIDDLE CONTROL SPAN
SECTIONS) - (6 MAX)

CTRAN - ARRAY OF DISTANCES BETWEEN THE TWO TRANSDUCERS AT EACH
CONTROL SURFACE MIDDLE SECTION(USED TO COMPUTE THE ANGLE OF
DEFORMATION) - (* MAX)

CREF - REFERENCE CHORD LENGTH (NORMALLY WING ROOT SEMI
CHORD) - SHOULD BE CONSISTENT WITH THE REFERENCE LENGTH USED IN
COMPUTING THE FREQUENCY K.

JMN - MATRIX WHERE JMN(I,J) INQUIRES THE DISPLACEMENT(POSITIVE
DOWN) OF THE I-TH TRANSDUCER DUE TO THE J-TH WAVE, FOR EACH
SECTION THERE ARE TWO TRANSDUCERS - THE FORE TRANSDUCER SHOUL
HE LOCATED AHEAD OF THE AFT TRANSDUCER(23 PERCENT CHORD FROM
L.E). THESE SETS OF TRANSDUCERS SHOULD BE ARRANGED IN THE SAME
ORDER AS THE CONTROLS - (12 X 15 MAX).

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C IREF - VALUES LIKE ZW OF REFERENCE TRANSDUCERS USED TO DETECT
THE RIGID BODY MOTION OF THE AIRCRAFT - (2 X 15 MAX).
C
C IPLOT - 1 --- ROOT LOCUS PLOT WILL BE MADE
C 0 --- NO PLOT WILL BE MADE
C
C CLR - ARRAY OF X DISTANCES (POSITIVE AFT) BETWEEN THE FORE
REFERENCE TRANSDUCER AND THE FORE CONTROL TRANSDUCER - (6MAX)
C
C CTR - DISTANCE BETWEEN THE TWO TRANSDUCERS AT THE REFERENCE
SECTION.
C
C NCACT - NUMBER OF ACTIVE CONTROLS FOLLOWING THE ORDER OF THE
CONTROLS.
C
C NAMELIST/PLUTPA
C XZ - LEFT HAND LIMIT OF REAL PART OF ROOT LOCUS
C
C YZ=0.
C
C XSCALE - ALPHISSA SCALE(VALUE PER INCH)
C
C YSCALE - ORDIATL SCALE (VALUE PER INCH)
C
C XL - LENGTH OF ALPHISSA IN INCHES
C
C YL - LENGTH OF ORDIATL IN INCHES
C
C ISYM - INTEGER DEFINING SYMBOL DURING ROOT LOCUS PLOT (=3 IS
RECOMMENDED).
C
C ENTRY - 1
C
C NAMELIST/NXSIZE
C MAXC - MAXIMUM NUMBER OF CONTROLS (=6 IN THIS PROGRAM)
C
C MAXNM - MAXIMUM NUMBER OF MODES (=15 IN THIS PROGRAM)
C
C MAXK - MAXIMUM NUMBER OF POLYNOMIAL TERMS PER ELEMENT IN THE
TRANSFER FUNCTION NUMERATOR AND DENOMINATOR MATRICES (=10 HEREIN)
C
C MAXT - MAXIMUM ORDER OF FINAL MATRIX AT=FINAL MATRIX AT=133
IN THIS PROGRAM)
C
C NAMELIST/CONC
C NOTE THAT IT IS NECESSARY TO DELETE ONE OF THE TWO SUBROUTINES
NAMED CONTU, ACCORDING TO THE DESIRED CONTROL LAW.
C
C WR - REFERENCE FREQUENCY (RAD/SEC), USED ONLY FOR THE D*T*T,F
CONTROL LAWS - VALUE CHOSEN IS NORMALLY AROUND THE FLUTTER
FREQUENCY VALUE.
C
C NTE - INTEGER ARRAY FOLLOWING THE ORDER OF THE CONTROLS AND
IDENTIFYING BETWEEN LE= AND TL= CONTROLS.
C
C IT IS IMPORTANT TO NOTE THAT WHENEVER A CONTROL IS NOT ACTIVE
PUT NTE=3.

X(1)*S**2/(5**2+2*X(2))*X(3)*S+X(2)**2 + X(4)*S**2/(5**2+2*X(5))*X(6)*S+X(5)**2

READ*4 XZ, YZ, XSCALE, YSCALE, XL, YL, BUF(100)
COMMON/AERF/AU(15,22),A1(15,22),A2(15,22),A3(15,22),A4(15,22),
*AU(15,22),A6(15,22),
COMMON/ICASE/8(4),NM,NC,NO,NL
DIMENSION OMEGAN(15),R(12,15),AO(15,6),AI(15,6),A2C(15,6),
*A3C(15,6),A4C(15,6),A5C(15,6),A6C(15,6),ZM(12,15),ZREF(2,16),
*BRTRAN(6),CTRAN(5),ACL(15,6,4),AL(15,15,4)*FCT(5,5),D(3),
*D1(15),CM(15,15,5),CA(15,15,5),CAC(15,5,7),NC(6,12),NPI(MX(6),
*NO(6),OO(10,6),XM(21,15,10),OM(21,6,10),PM(6,12,10),PH(6,15,13),
*C5(5),HMN(21,21),FS(189),L1(189),PV(189),CLR(6),NPC(6),
*VTE(6),X(36),
*T(100,100)
REAL*4 MAS5(15,15),KJAR(15,15)
NAMELIST/FLUT/MASS,UMEGAN,QREGEN,JEND,NQ,VEL,BTRAN,CTRAN,CREF,ZREF,
*PLUT,CLR,CTR,NC
NAMELIST/PLUTPA/XZ, YZ, XSCALE, YSCALE, XL, YL, I, J, XN, N, IENTRY
NAMELIST/MXSIZE/MAXX,MAXM,MAXK,MAXT
NAMELIST/CUNC/NC, NTE, X
READ(5,FLUT)
WRITE(6,FLUT)

PRINT 500
DO*100(D=NO-JUF(IN))/NJ
NOT=JU+1
PI=1.4159265AD0
CALL PLUTS(BUF,100,0,13,0)
CALL PLUT1(11111111)
IF(PLUT.EQ.1) READ(1,PLOTPA)
IF(PLUT.EQ.1) WRITE(6,PLOTPA)
READ(5,MXSIZE)
WRITE(6,4XSIZE)
IF(NC,N.EQ.0) READ(9,CUNC)
IF(NC,N.EQ.0) WRITE(6,CUNC)
NC=NC
MAX=MAXX+MAXM

COMPUTATION OF THE STIFFNESS MATRIX KBAR

DO 1 I=1,NM
UMEGAN(I)=2*D*PI1*UMEGAN(I)
DO 1 J=1,NM
KBAR(I,J)=MASS(I,J)*UMEGAN(I)**2
1 CONTINUE

FORMATION OF THE VARIOUS ALPH MATRICES

VEL1=CREF/VEL
VEL2=VEL1*VEL1
IF(NC*A.EQ.0) GO TO 60
DO 2 I=1,NM
DO 2 J = 1, NC
AOC(I, J) = AO(I, NM+J)
AIC(I, J) = AI(I, NM+J)
A2C(I, J) = A2(I, NM+J)
ACL(I, J, 1) = A3(I, NM+J)
IF(NL.LT.2) GO TO 2
ACL(I, J, 2) = A4(I, NM+J)
IF(NL.LT.3) GO TO 2
ACL(I, J, 3) = AB(I, NM+J)
IF(NL.LT.4) GO TO 2
ACL(I, J, 4) = A6(I, NM+J)
2 CONTINUE
DO 3 I = 1, NM
DO 3 J = 1, NM
AL(I, J, 1) = A3(I, J)
IF(NL.LT.2) GO TO 3
AL(I, J, 2) = A4(I, J)
IF(NL.LT.3) GO TO 3
AL(I, J, 3) = AB(I, J)
IF(NL.LT.4) GO TO 3
AL(I, J, 4) = A6(I, J)
3 CONTINUE
DO 4 I = 1, NL
B(I) = U(I)/VEI1
4 CONTINUE

C
C REDUCTION OF THE EQUATIONS OF MOTION TO A COMMON DENOMINATOR -
C IN THE FOLLOWING TWO STAGES: -
C (1) THE NM STRUCTURAL EQUATIONS WITHOUT THE CONTROL CONTRIBUTION
C
CALL FACTR(FCT, NL, LP, LF)
LSMX = LF + 2
DO 6 K = 1, LF
C(K) = FCT(K, LF)
6 CONTINUE
DO 5 I = 1, NM
DO 5 J = 1, NM
D(1) = KHAR(I, J)
D(2) = 0.0
D(3) = MASS(I, J)
CALL PROPMUL(C, LF, D, 1, SS)
DO 7 K = 1, LS
CM(I, J, K) = D1(K)
7 CONTINUE
D(1) = AO(I, J)
D(2) = AL(I, J)*VE11
D(3) = AL(I, J)*VE12
CALL PROPMUL(C, LF, D, 1, LS)
DO 8 K = 1, LS
CA(I, J, K) = D1(K)
8 CONTINUE
DO 9 K = 1, NL
D(1) = 0.0
D(2) = AL(I, J, K)
CALL PROPMUL(FCT(I, K), LP, D, 2, 1, LS)
DO 10 K = 1, LS
CA(I, J, KK) = CA(I, J, KK) + D1(KK)
10 CONTINUE
CONTINUE
CONTINUE
NR=LSMX
NRD=LSMX
NM=NM+NC
NMD=NM+1
NC2=2*NC
MAXC2=2*MAXC
IF (NC=EQ.0) GO TO 48

(2) ADJUSTMENTS TO THE NM STRUCTURAL EQUATIONS DUE TO THE NC
CONTROL SURFACES

DO 11 I=1,NM
DO 11 J=1,NC
D(1)=AIC(I,J)
D(2)=ACL(I,J)*VEL!
D(3)=ACL(I,J)*VEL?
CALL PROPUL(C,LF*U,J,DL*LS)
DO 12 K=1,LS
CAL(I,J,K)=DI(K)
12 CONTINUE
DO 13 K=1,NI
D(1)=0.00
D(2)=ACL(I,J,K)
CALL PNPUL(FCT(I,K),LP*D2+1*LS)
DO 14 K=1,LS
CAL(I,J,K)=CAL(I,J,K)+DI(K)
14 CONTINUE
13 CONTINUE
11 CONTINUE
DO 33 I=1,NC
DO 33 J=1,NC
DO 33 K=1,MAXK
P(I,J,K)=0.00
33 CONTINUE

FORMATION OF THE NP CONTROL SURFACE EQUATIONS THROUGH THE USE
OF THE CONTROL LAM

CALL CONTOL(NP,P,N),N,N,LP,NL,E,X)

NCMX=0
NPMX=0
DO 45 I=1,NC
NPMX(I)=NP(I,1)
IF (NP(I,1).EQ.NCX) NCMX=NCX(I)
45 CONTINUE
IF (NPX.0=LSMX) NPK=NPMX
IF (NCMX.GT.LSMX) NRD=NCMX

46 CONTINUE
   DO 46 I=1,NMT
   DO 46 J=1,NN
   DO 46 K=1,MAXK
   RM(I,J,K)=0.0
46 CONTINUE
   IF (NC.EQ.0) GO TO 51
   DO 47 I=1,NMT
   DO 47 J=1,NC
   DO 47 K=1,MAXK
   DM(I,J,K)=0.0
47 CONTINUE
   INC=0
   DO 31 I=1,NC+2
   INC=INC+1
   DO 31 J=1,NN
   DO 31 K=1,NN+2
   H(I,J,K)=Z(I,J)+Z(I,J,K+1)-Z(I,J,K)+ZK(IF(I,J)-NN)
   H(I+1,J)=(Z(I+1,J)-Z(I,J))/CTR(K+1)-ZK(IF(I,J)-NN)
   CONTINUE
   NNPMX=0
   DO 52 I=1,NN+1
   IF (NPIMX(I).LT.NNPMX) NNPMX=NPIMX(I)
52 CONTINUE
   IF (NNPMX.GT.NN) NN=NNPMX

C FORMATION OF THE EXPANDED FIRST ORDER DIFFERENTIAL EQUATIONS
C OF VARIATION (SUITABLE FOR EIGENVALUE SOLUTION OF THE T MATRIX) -
C REPEATED IN A LOOP FOR THE VARIOUS VALUES OF DYNAMIC PRESSURE Q
C
   DO 10 J=1,NN+1
   DO 32 I=1,NT
DO 32 J=1,NT
T(I,J)=0.00
32 CONTINUE
Q=QBEGIN+(ICASE-1)*DU
C
PRINT 100,Q
DO 15 I=1,NM
DO 15 J=1,NM
DO 15 K=1,LSMX
HM(I,J,NRR-K+1)=CM(I,J,LSMX-K+1)+Q*CA(I,J,LSMX-K+1)
15 CONTINUE
IF(NC.EQ.0) GO TO 49
DO 16 I=1,NC
DO 16 J=1,NC
DO 16 K=1,LNSMX
DM(I,J,NRR-K+1)=Q*CAC(I,J,LSMX-K+1)
16 CONTINUE
49 CONTINUE
DO 19 I=1,NT
DO 19 J=1,NT
RDMN(I,J)=HM(I,J,NRR)
19 CONTINUE
IF(NC.EQ.0) GO TO 19
DO 21 J=1,NC
RDMN(I,J)=DM(I,J,NRR)
21 CONTINUE
CALL MXINVR(NMT,LSMX,RDMN)
DO 22 K=1,NRM
NJ=NMT*(K-1)+1
KK=NRR-K
CALL MXPRJU(RDMN,RJ(I,J,I),NJ,NMT,LSMX,MAX,MAX)
22 CONTINUE
IF(NC.EQ.0) GO TO 23
DO 23 K=1,NC
KK=NRM-K
IF(K.LE.NRM1) NJ=NMT*K-NC+1
IF(K.GT.NRM1) NJ=NMT*KNRM+NC*(K-NRM1-1)+1
CALL MXPRJU(RDMN,RJ(I,J,I),NJ,NMT,LSMX,MAX,MAX)
23 CONTINUE
50 CONTINUE
DO 24 I=1,NTS
T(NMT+I,1)=0.00
24 CONTINUE
IF(NTSS.EQ.0) GO TO 26
DO 25 I=1,NTS
T(NIS+I,NIS-NC+1)=1.00
25 CONTINUE
26 CONTINUE
500 EIGENVALUE SOLUTION OF THE FINAL T MATRIX
CALL EHALAF(T,NT,MAXT,PV,INK,INL)
CALL EHESSF(T,INK,INL,NT,MAXT,PV)
CALL EQRH3F(T,NT,MAXT,INK,INL,FR,EL,ZZ,0,IER)
PRINT 600,IER
DO 82 I=1,NT
PRINT 200,FR(I),EL(I)
82 CONTINUE
IF(IPLUT*EQ.1) CALL PLTDAT(INTEGER,E1,XZ,YZ,XSCALE,YSCALE,XYL,YL)
+ISYM,ENTRY)
400 CONTINUE
    CALL PLOT(0,0,994)
100 FORMAT(/10X* DYNAMIC PRESSURE ='',F10.3,/) 00000581
203 FORMAT(10X,E15.4,*1.,E15.4) 00000582
500 FORMAT(/10X* ROOT LOCUS — CLOSED LOOP — REAL ACTUATORS *//) 00000583
600 FORMAT(10X* EENK = *14) 00000584
900 FORMAT(7E13.6) 00000585
901 FORMAT(5I5) 00000586
902 FORMAT(9E13.5) 00000587
903 FORMAT(4X,E14.6) 00000588
RETURN 00000589

SUBROUTINE FACTR(FCT,6,NL,LP,LF) 00000590

C THIS SUBROUTINE COMPUTES THE VARIOUS FACTORS WHICH ARE NECESSARY
C SO AS TO BRING THE AERO PADE APPROXIMANTS TO A COMMON DENOMINATOR
C FCT(I,J) IS THE FACTOR WHICH MULTIPLIES THE J-TH LAG AERO TERM —
C IN TERMS OF POLYNOMIAL FCT(1,J)+FCT(2,J)*S+FCT(3,J)*S**2+........
C
REAL*8 FCT(5,5),H(4)

IF(NL.NE.1) GO TO 1
1 LP=1 00000600
LF=2 00000601
FCT(1,1)=1.D0 00000602
FCT(1,2)=0(1) 00000603
FCT(2,2)=1.D0 00000604
RETURN 00000605

IF(NL.NE.2) GO TO 2
2 LP=2 00000606
LF=3 00000607
FCT(1,1)=B(2) 00000608
FCT(2,2)=1.D0 00000609
FCT(2,3)=B(3) 00000610
FCT(3,2)=1.D0 00000611
RETURN 00000612

IF(NL.NE.3) GO TO 3
3 LP=3 00000613
LF=4 00000614
FCT(1,1)=B(2)*B(3) 00000615
FCT(2,1)=U(2)+H(3) 00000616
FCT(3,1)=1.D0 00000617
FCT(1,2)=B(1)*B(3) 00000618
FCT(2,2)=B(1)+B(3) 00000619
FCT(3,2)=1.D0 00000620
FCT(1,3)=B(1)*B(2) 00000621
FCT(2,3)=H(1)+H(2) 00000622
FCT(3,3)=1.D0 00000623
FCT(1,4)=H(1)*B(2)*H(3) 00000624
FCT(2,4)=B(1)*B(2)+H(1)*B(3)+H(2)*B(3) 00000625
FCT(3,4)=B(1)+B(2)*H(3) 00000626
FCT(4,4)=1.D0 00000627
RETURN 00000628
3 IF(NL .NE. 4) GO TO 4
LP=4
LF=5
FCT(1,1)=B(2)*H(3)*B(4)
FCT(2,1)=B(2)*B(3)+B(2)*H(3)*B(4)
FCT(3,1)=B(2)*B(3)*H(4)
FCT(4,1)=1.0D0
FCT(1,2)=B(1)*B(3)*B(4)
FCT(2,2)=B(1)*B(3)+B(1)*B(4)+B(3)*B(4)
FCT(3,2)=B(1)*B(3)*H(4)
FCT(4,2)=1.0D0
FCT(1,3)=B(1)*B(2)*B(4)
FCT(2,3)=B(1)*B(2)+B(1)*B(4)+B(2)*B(4)
FCT(3,3)=B(1)*B(2)*B(4)
FCT(4,3)=1.0D0
FCT(1,4)=B(1)*B(2)*B(3)
FCT(2,4)=B(1)*B(2)+B(1)*B(3)+B(2)*B(3)
FCT(3,4)=B(1)*B(2)+B(1)*B(3)*B(4)
FCT(4,4)=1.0D0
FCT(1,5)=B(1)*B(2)*B(1)*B(3)*B(4)
FCT(2,5)=B(1)*B(2)+B(1)*B(3)+B(2)*B(3)+B(1)*B(4)*B(2)*B(3)
B(4)
FCT(3,5)=B(1)*B(2)+B(1)*B(3)+B(2)*B(3)+B(1)*B(4)+B(2)*B(3)+B(1)*B(4)
B(4)
FCT(4,5)=B(1)*B(2)+B(1)*B(3)+B(2)*B(3)+B(1)*B(4)+B(2)*B(3)+B(1)*B(4)
B(4)
FCT(5,3)=1.0D0
RETURN
4 PRINT 100
100 FORMAT(5X*, NUMER OF AERODYNAMIC LAG TERMS EXCEEDS THE MAXIMUM OF
*FOUR TERMS /)
STOP
END
SUBROUTINE FIT
IMPLICIT REAL*B(A-H,0-2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC00000670
CC00000671
C FITS THE AERO COEFFICIENTS IN TERMS OF PADE APPROXIMANTS USING
C LEAST SQUARE TECHNIQUE
C NAMELIST/FIT
NVS - NUMBER OF REDUCED FREQUENCIES K USED FOR INTERPOLATION
C AK - ARRAY CONTAINING THE K VALUES (20 Max) (FIRST REDUCED K
C MUST BE EQUAL TO ZERO)
C MAXNK - MAX VALUE OF NK (MAX NK=20 IN PRESENT PROGRAM)
C NPRINT - 0 NO PRINTED OUTPUT FROM SUBROUTINE FIT
C - I PRINTED OUTPUT IS AVAILABLE (FOR DEBUGGING PURPOSES)
C NPUNCH - 0 NO PUNCHED OUTPUT FROM SUBROUTINE FIT
C - I PUNCHED OUTPUT (INTERPOLATION COEFFICIENTS)
C IRIGID, J = I = CURVE FITTED (WITH NO LEAST SQUARES TECHNIQUE)
C OF THE FIRST IRIGID ROWS AND J = I = COLUMNS OF AERO MATRIX
C ASSUMED TO CONTAIN RIGID BODY AERO TO IMPROVE RESULTS
C READ(I,J,K) AERO(I,J,K) FORMAT(2E15,5)
C00000672
C00000673
C00000674
C00000675
C00000676
C00000677
C00000678
C00000679
C00000680
C00000681
C00000682
C00000683
C00000684
C00000685
C00000686
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C00000688
C00000689
C00000690
C00000691
C00000692
C00000693
C00000694
COMMON/AERU/A0(15,22),A1(15,22),A2(15,22),A3(15,22),A4(15,22),
*AS(15,22),A6(15,22)
COMMON/ICASC/(4,NM,NC,NG,NL)
COMPLEX*16 AERU(15,22,20),AERO
DIMENSION AK(20),AK2(20),X(40,6),XT(6,40),Y(40),XTX(6,6),
XTY(6,6),S(6),CRLU(4,20),CRLI(4,20)
NAMES/FT/NK,AK,MAMNK,PRINT,NPUNCH,IRIGID,JRIGID
READ(5,FT)
WRITE(6,FT)
MAXNK=2*MAXNK
NMNC=NM+NC+NG
DO 1 K=1,NK
AK2(K)=AK(K)*AK(K)
DO 1 J=1,NMNC
DO 1 I=1,NM
HEAD(2,201) AERU(I,J,K)
1 CONTINUE
DO 5 I=1,NL
S2=H(I)*S(I)
DO 5 K=1,NK
CLR(I,K)=AK2(K)/(1+2*AK2(K))
CLI(I,K)=H(I)*AK(K)/(1+2*AK2(K))
5 CONTINUE
IF(AK(I)*NE.*2) PRINT 130
IF(AK(I)*NE.*0) STOP
C
C DETERMINATION OF THE INTERPOLATION LEAST SQUARE MATRIX XTX AND
C THE KNOWN AERO VECTOR XTY
C
DO 2 I=1,NM
DO 2 J=1,NMNC
IF((I+1)*NMNC/(J+1)*NMNC) SD TO 7
NK=NK
NK=(3+NL)/2+1
7 CONTINUE
DO 3 K=1,NK
X(2*K-3,1)=0.00
X(2*K-2,1)=AK(K)
X(2*K-3,2)=-AK(K)
X(2*K-2,2)=0.00
Y(2*K-3)=DREAL(AERU(I,J,K)-AERO(I,J,1))
Y(2*K-2)=IMAUG(AERO(I,J,K))
DO 3 L=1,NL
X(2*K-3*L)=CLR(L,K)
X(2*K-2*L)=CLI(L,K)
3 CONTINUE
NROWS=2*(NK-1)
NCOLS=2+NL
IF(NROWS.LT.NCULS) PRINT 119
IF(NROWS.LT.NCULS) STOP
DO 4 IR=1,NROWS
DO 4 JR=1,NCOLS
XT(IR,JR)=X(IR,JR)
4 CONTINUE
CALL MPNUM(XT,XT,XTX,NCOLS,NROWS,NCOLS,6,MAXNKK,6)
CALL MPNUM(XT,Y,XTY,NCOLS,NROWS,1,6,MAXNKK,6)
C
C SOLUTION FOR THE UNKNOWN INTERPOLATION COEFFICIENTS
C
CALL MGINVR(NGL5,0,N,XTX)
CALL MXPRED(XTX,XTY,5,NGLS,NGLS,1,6,6)
A0(I,J)=AERO(I,J,1)
A1(I,J)=S(1)
A2(I,J)=S(2)
A3(I,J)=S(3)
IF(NL.LT.2) GO TO 10
A4(I,J)=S(4)
IF(NL.LT.3) GO TO 13
A5(I,J)=S(5)
IF(NL.LT.4) GO TO 10
A6(I,J)=S(6)
10 CONTINUE
IF(I.EQ.1.RIGID.AND.J.LE.JRIGID) NK=NKT
C
C PRINTED AND OR PUNCHED OUTPUTS
C
IF(NPRINT.NE.1.AND.NPUNCH.NF.1) GC TO 2
IF(NPUNCH.NE.1) .GO TO ,
PUNCH 600,1,J,A0(I,J),(S(JJ),JJ=1,NGLS)
C CONTINUE
IF(NPRINT.NE.1) .GO TO 13
PRINT 700,1,J
PRINT 200,AJ(1,J),(S(JJ),JJ=1,NGLS)
DO 8 IK=1,NK
COEF=CMPLX(0,0,0,0)
DO 6 11=1,NL
CUEF=CMPLX(CLH(II,IK),CLL(II,IK))
6 CONTINUE
CUEF=CMPLX(AK(1,J,IK))
PRINT 210*,IJ,IK,A0(1,J,IK),COEF
END
C NUM = 4*NUMBER OF MODES
C XZ = LEFT HAND LIMIT OF REAL PART
C 1
C YZ = 0.
C CROSS = ABCISSA SCALE
C YSCALE = ORDINATE SCALE
C XL = LENGTH OF PLOT IN INCHES OF PAPER
C YL = HEIGHT OF PLOT IN INCHES OF PAPER
C
Dim Ex(150), El(150), Ey(150)
M = 0
DO 1 I = 1, N
EX(I) = CLK(I)
EY(I) = CLY(I)
RHTLM = XZ * XSCALE * XL
UPMLT = YZ + YL * YSCALE
IF (EY(I) > TLT, YZ) GO TO 1
IF (EY(I) < LTL, YZ) GO TO 1
IF (EY(I) > RHTLM) GO TO 1
IF (EY(I) < UPMLT) GO TO 1
IF (ABS(EX(I) + LTL, Y) AND ABS(EY(I) + LTL, Y) = 0) GO TO 1
M = M + 1
EX(M) = EX(I)
EY(M) = EY(I)
1 CONTINUE
EX(M + 1) = XZ
EY(M + 1) = YZ
EX(M + 2) = XSCALE
EY(M + 2) = YSCALE
GO TO (.+1), ENTRY
2 CONTINUE
YAXIS = ABS(1.)/(XSCALE)
CALL AXI3(0, 0, 0, 'REAL PART', -9, XZ, XZ, XSCALE)
CALL AXI3(YAXIS, 0, 0, 'REAL PART', -9, YL, YZ, YSCALE)
ENTRY = 2
3 CONTINUE
CALL PLT(J, 5, D, 1)
CALL LINL(FX, YZ, M, 1, 1, SYM)
RETURN
END
FUNCTION DREAL(Z)
C THIS SUBROUTINE CAN BE USED WITH EITHER THE SLOW OR FAST IBM.
C DOUBLE PRECISION, COMPRESSIBLE ALKDYNA Rt) COEFFICIENTS PROGRAM.
IMPLICIT REAL*8(D)
RETURN
DREAL = Z(2)
RETN
ENTRY DAIMAG(Z)
DAIMAG = Z(2)
RETURN
FND
SUBROUTINE DCOOL(A, N, 3, M, C, L)
IMPLICIT REAL*8(A, N, L, Z)
RETURN
C A ROUTINE FOR MULTIPLYING POLYNOMIALS COEFF WHERE A, B, C ARE
C POLYNOMIALS OF THE FORM 
C A(1)*X+ A(2)*X**2+ A(3)*X**3+ ......... A(N)*X**(N-1) 
C B(1)*X+ B(2)*X**2+ B(3)*X**3+ ......... B(M)*X**(M-1) 
C C(1)*X+C(2)*X**2+C(3)*X**3+ ......... C(N+M-1)*X**(N+M-1) 
C AND WHERE 
C L=N+M-1 
C 
C DIMENSION C(1), A(1), B(1), C(1) 
C N=M=N-1 
C DO 1 I = 1, N+M 
C 1 C(I) = 0.0 
C DO 2 I = 1, N 
C DO 2 J = 1, M 
C 2 C(I+J-1) = A(I)*B(J)+C(I+J-1) 
C L = NM 
C RETURN 
C END 
C 
C SUBROUTINE MXINV - DOUBLE PRECISION 
C SUBROUTINE MXINV(N, M, MAX, A) 
C 
C REAL MATRIX INVERSION WITH SOLUTION OF LINEAR EQUATIONS 
C 
C CAVM = DABS(A(MAX)) , CAVA = DABS(A(1,1)) 
C CADM = DABS(UFTL(N)) , CAHV = DABS(PIVUT) 
C 
C IMPLICIT REAL*A-H, O-Z) 
C DIMENSION A(MAX+1, B(15+1), IPIV(N+1), INDEX(15+1)) 
C IF (M.NF.0) GO TO 1 
C 
C DO 2 I = 1, N 
C 2 A(I, I) = 0.0 
C GO TO 10 
C 1 IF (IPIV(I) .EQ. 1) 10 
C 
C 10 CONTINUE 
C 
C C CAVM = DABS(A(J, K)) 
C IF (CAVM .GE. CAVA) 100 
C 
C 100 CONTINUE 
C 
C RETURN 
C END 
C 
C CONSTANTS, INITIALIZATION 
C 
C C = J +J 
C C = J +J 
C DEF = CI 
C CADM = J +J 
C DO 20 J = 1, N 
C 20 IPIV(J) = 0 
C DO 50 I = 1, N 
C 50 CONTINUE 
C 
C SLASH FOR PIVOT ELEMENT 
C 
C CAVM = DABS(A(J, K)) 
C IF (CAVM .GE. CAVA) 100 
C 
C 100 CONTINUE 
C 
C RETURN 
C END 
C
CAVM = CAVA
100 CONTINUE
105 CONTINUE
IF (CAVM.EQ.0.000) GO TO 120
IPIVICOL) = IPIVICOL + 1

C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
IF (IHOM.EQ. ICCL) GO TO 230
DET = -JLT
DU 200 I=1,N
SWAP = A(IHOM+L)
A(IHOM+L) = A(IICOL+L)
A(IICOL+L) = SWAP
200 CONTINUE
IF (M .LE. 0) GO TO 21
DU 220 L=1,M
SWAP = A(IICOL+L)
H(IICOL+L) = H(IICCL+L)
H(IICCL+L) = SWAP
220 CONTINUE
230 C - I N T I N D
INUX(i,i) = IHOM
INUX(i,i) = ICUL
PIV = A(IICOL+ICCL)
(CAPV) = A(PIV)
IF (CAPV .LE. 0.00) GO TO 710

C DIVIDE PIVOT FROM PIVOT ELEMENT
C
APV = 1.0/PIV
IX ?) = 1,N
350 A(IICCL+L) = A(IICCL+L)*PIV
IF (M .LE. 0) GO TO 390
DU 370 L=1,M
370 H(IICCL+L) = H(IICCL+L)*PIV

C REDUCE NON-PIVOT ROWS
C
450 CONTINUE
DU 500 L=1,N
IF (L .EQ. UC) GO TO 530
SWAP = A(L,ICUL)
AL(I,ICUL) = CO
DU 400 L=1,N
400 A(L,1) = A(L,1) - A(L,ICUL)*SWAP
IF (M .LE. 0) GO TO 700
DU 450 L=1,M
450 H(L,1) = H(L,1) - H(L,ICUL)*SWAP
500 CONTINUE

C INTERCHANGE COLUMNS
C
DU 700 I=1,N
L = N+1-1
IF (INUX(L,L+1) .EQ. 0) GO TO 700
IHOM = INUX(L,L+1)
ICUL = INUX(L,L+1)
DU NGO K=1,N
SWAP = A(K,1,FUL)
A(K,1,1,FUL) = A(K,1,ICOL)
A(K,1,ICOL) = SWAP
690 CONTINUE
700 CONTINUE
90 SCALE = J
700 RETURN
END
SUBROUTINE MAXCAL(A,H0,A,N,I,NJ,J,M,S,T,MAXA,MAXH,MAXS)
REAL A,*
DIMENSION A(M,N,J)
DO 100 J=1,NJ
A(J)=-9999
100 continuation
DO T=1,NJ
A(J)=A(J)+A(J-1)
100 CONTINUE
END
SUBROUTINE MAXJCL(A,H0,A,N,I,NJ,J,M,S,T,MAXA,MAXH,MAXS)
REAL A,*
DIMENSION A(M,N,J)
DO 100 J=1,NJ
A(J)=A(J)+A(J-1)
100 CONTINUE
END
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**Note:** The table contains floating-point numbers with a high degree of precision, indicating a scientific or technical context. The values are likely used in calculations or simulations, given the context of the text in the image.
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APPENDIX F

SOURCE LISTING AND INPUT/OUTPUT EXAMPLE

FOR GUST OPTIMIZATION PROGRAM

The first part of the Appendix consists of the source listing of the program which is used for both gust optimization and gust sensitivity purposes. The operating instructions indicate which subroutines and which cards need be deleted or replaced.

The example chosen relates to the same DAST configuration (M=0.9) chosen for the flutter example with one active T.E. control surface, (using L.D.T.T.F.). Therefore, the aerodynamic data AERO (I,J,K) which resides on file 2) is not listed again in this Appendix. All the data required by the program appears in the output. The two PSD plots for the control surface deflection and for the control surface rate of deflection are supplemented by a tabulation of these plots. These appear in four tables as follows:

The first table shows XF(I)(=ω rad/sec), DEFLN(I)(=δ_i,PSD) and PSD(I) (= the Von Karman gust spectrum).

The second table is similar to the first but shows DEFNR(I)(= ε_i,PSD).

The third table shows DEFLN2(I)(=δ_i^2,PSD)

The fourth table similarly shows DEFLNR2(I)(= δ_i^2,PSD).

Note that all the control deflections are given in degrees per unit gust velocity.

The last table summarizes the optimization iterations and is very important in studying the progress of the minimization process. The notation used is as follows:
ITERNS = Iteration number.

FOPT. = Value of the target function FUNCTN during the present iteration.

GMAX = The absolute value of the maximum gradient component during the present iteration.

IGMAX = The active control law variable number to which GMAX relates.

DELMAX = The maximum absolute value of the optimum direction component during the present iteration.

IDMAX = The active control law variable number to which DELMAX relates.

E(LOWEST) = The step size to the minimum along the optimum direction.

The output also includes the initial values of the gradients G(I) (with respect to the control variables) and the final values of the gradients G(I) (after completing the minimization process) together with the optimum values for the control variables X(I) and the minimum value of the target function FUNCTN.

Note also that when a control variable resides on a constraint and its gradient leads to the violation of that constraint, the gradient is artificially changed to assume zero value.

Note that the plotted output shows labels which appear to be displaced. These displacements reflect transient difficulties encountered while using a new plotter and they do not originate from the programs used.
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GUST RESPONSE PACKAGE WHICH PERMITS THE COMPUTATION OF THE
SPECTRAL RESPONSE OF AIRCRAFT DUE TO CONTINUOUS GUST ENVIRONMENT.
THE EFFECTS OF ACTIVE CONTROLS ON THE RESPONSE CAN BE ACCOUNTED
FURTHERMORE, THE BASIC GUST PROGRAM IS COUPLED IN THE
PRESENT PACKAGE WITH AN OPTIMIZATION ROUTINE WHICH ENABLES THE
DETERMINATION OF THE VARIOUS CONTROL GAINS SO AS TO MINIMIZE THE
AIRCRAFT RESPONSE TO GUST. SENSITIVITY STUDIES AROUND THE
GIVEN (OR OPTIMAL) CONTROL GAINS CAN ALSO BE MADE. THE
FOllOWING DATA IS REQUIRED:--

- HEADER (FORMAT 15A4)
- NAMELIST/CASE
- NM - NUMBER OF MODES (15 MAX)
- NL - NUMBER OF CONTROLS (15 MAX)
- NAER - 1 IF AERU IN TERMS OF INTERPOLATION COEFFICIENTS OF K
- 0 IF AERU FOR DIFFERENT VALUES OF K - INTERPOLATION
- COEFFICIENTS TO BE COMPUTED IN SUBROUTINE FIT.
- B - ARRAY OF LAG TERMS USED DURING INTERPOLATION (4 MAX)
- NG - 1 IF JUST AERU IS SUPPLIED
- 0 IF JUST AERU IS NOT SUPPLIED
- NL - NUMBER OF LAG TERMS TO BE USED DURING INTERPOLATION.
- IF NAER=1 THEN AERO COEFFICIENTS ARE READ (FORMAT 6X,7E10.4)
NAMELIST/GST

MASS - MASS MATRIX(15X15 MAX)

OMEGAN - NATURAL FREQUENCIES ARRAY(IN HZ) - (15 MAX) - NOTE-
STIFFNESS IS COMPUTED FROM MASS AND OMEGAN AND IS THEREFORE
CORRECT FOR DIAGONAL MASS MATRIX ONLY.

VEL - FLIGHT VELOCITY

UTHAN - ARRAY OF SEMICHORD LENGTHS OF WING(OR TAIL) SECTIONS
WHERE THE DIFFERENT CONTROLS ARE LOCATED AT MID CONTROL SPAN
SECTIONS) - (6 MAX)

CTRAN - ARRAY OF DISTANCES BETWEEN THE TWO TRANSDUCERS AT EACH
CONTROL SURFACE MID SECTION(USED TO COMPUTE THE ANGLE OF
DEFORMATION) - (6 MAX).

CHEF - REFERENCE SEMI CHORD LENGTH (NORMALLY WING ROOT SEMI
CHORD) - SHOULD BE CONSISTENT WITH THE REFERENCE LENGTH USED IN
COMPUTING THE REDUCED FREQUENCY K.

ZM - MATRIX WHERE ZM(I,J) INDICATES THE DISPLACEMENT(POSITIVE
DOWN) OF THE I-TH TRANSDUCER DUE TO THE J-TH MODE FOR EACH
SECTION THERE ARE TWO TRANSDUCERS -- THE PURE TRANSDUCER SHOULD
BE LOCATED AHEAD OF THE AFT TRANSDUCER(AT 30 PERCENT CHORD FROM
L.E.). THESE SETS OF TRANSDUCERS SHOULD BE ARRANGED IN THE SAME
ORDER AS THE CONTROLS - (12 X 15 MAX).

ZREF - VALUES LIKE 7% OF REFERENCE TRANSDUCERS USED TO DETECT
THE RIGID BODY MOTION OF THE AIRCRAFT - (2 X 15 MAX).

Q - FLIGHT DYNAMIC PRESSURE

CLR - ARRAY OF X DISTANCES (POSITIVE AFT) BETWEEN THE FORE
REFERENCE TRANSDUCER AND THE FORE CONTROL TRANSDUCER - (6 MAX).

CTR - DISTANCE BETWEEN THE TWO TRANSDUCERS AT THE REFERENCE
SECTION.

WR - REFERENCE FREQUENCY(RAD/SFC). USED ONLY FOR THE D.T.T.F.
CONTROL LAWS -- VALUE CHOSEN IS NORMALLY AROUND THE FLUTTER
FREQUENCY VALUE.

NTE - INTEGER ARRAY FOLLOWING THE ORDER OF THE CONTROLS AND
IDENTIFYING BETWEEN L.E. AND AFT CONTROLS.

=1, AFT CONTROL
=0, L.E. CONTROL

IT IS IMPORTANT TO NOTE THAT WHENEVER A CONTROL IS NOT ACTIVE
PUT NTE=0

NACT - NUMBER OF ACTIVE CONTROLS

IF NACT=0 NEXT INPUTS ARE READ IN SUBROUTINE FIT.

ETA1,PHI - ETA1 - ACCURACY OF COMPUTER RELATIVE TO 1 (ON 1.8X)
DOUBLE PRECISION=5-E13, ABSOLUTE ACCURACY==ETA1.

PHI - RELATIVE SIZE OF SUCTION ZONE WITHIN WHICH
THE OPTIMIZED PARAMETER IS SUCKED TO THE CONSTRAINT
IN ORDER TO AVOID FALSE CONVERGENCE, ABSOLUTE SIZE OF ZONE=X(I)PHI OR XE(I)PHI DEPENDING WHETHER NEAR LOWER OR UPPER CONSTRAINTS (FORMAT 4E10.0) C00000093

C NV, NMR, NDK - NV - AN INPUT INTEGER (36 MAX) CONTAINING THE
NUMBER OF INDEPENDENT CONTROL GAINS IN THE SYSTEM C00000097
- NMR - )
- NDK - 3 OPTIMIZATION BASED ON MINIMIZATION OF RMS
RESPONSE OF CONTROLS.
- 1 OPTIMIZATION BASED ON MINIMIZATION OF RMS
RESPONSE RATES OF CONTROLS. (FORMAT B19) C00000101
C
C NONACT - NUMBER OF NON ACTIVE OPTIMIZATION PARAMETERS (FORMAT B15) C00000105
C
C NA - INTEGER INPUT ARRAY CONTAINING THE LOCATION OF THE NON
ACTIVE PARAMETERS IN THE X ARRAY (SEE BELOW) (FORMAT B15).
C
C IF NONACT=0, A BLANK CARD SHOULD BE PLACED HERE.
C
C W, WT - THE WEIGHTS FOR EMPHASIZING THE RMS CONTROL RESPONSE
C (OR WT) OF ANY DESIRED SPECIFIC CONTROL SURFACE, THIS IS USED
C IN CONJUNCTION WITH THE DEFINITION OF THE TARGET FUNCTION
C 'FUNCTN' (FORMAT 4E10.0).
C
C X(I), X(I), X(I), EPS(I) - THERE ARE NV SUCH CARDS (FORMAT 4E10.0)
X(I) - DENOTES THE LOWEST BOUND OF THE I - TH CONTROL
GAIN PARAMETER.
X(I) - DENOTES THE INITIAL VALUE OF THE I - TH CONTROL
GAIN PARAMETER.
X(I) - DENOTES THE UPPER BOUND OF THE I - TH CONTROL
GAIN PARAMETER.
EPS(I) - THE DESIRED ABSOLUTE ACCURACY OF THE OPTIMAL
FINAL X(I) VALUE (IN CASE MINIMIZATION IS MADE), IN
CASE OF CONTROL GAIN SENSITIVITY STUDY EPS(I) DENOTES THE
INCREMENTAL VARIATION OF X(I) WITHIN THE REGION X(I) -- X(I).
MAX. NUMBER OF INCREMENTS=34. MAX. SIZE OF ARRAYS=36.
C
C FMINT, ETA - FMINT - INPUT PARAMETER CONTAINING AN APPROXIMATION
OF THE MINIMUM RMS RESPONSE VALUE. IF UNKNOWN
USE FMINT=0.
C
C ETA - INPUT PARAMETER CONTAINING AN ESTIMATE OF THE
RELATIVE ACCURACY OF THE RMS RESPONSE EVALUATIONS WHICH ARE USED TO DETERMINE THE TYPE OF
DIFFERENCE APPROXIMATION TO THE GRADIENT
(FORMAT 4E10.0).
C
C ITMAX, Iw - ITMAX - AN INPUT/OUTPUT INTEGER. ON INPUT, ITMAX
CONTAINS THE MAXIMUM ALLOWABLE NUMBER OF
OPTIMIZATION ITERATIONS. ON OUTPUT, ITMAX
CONTAINS THE NUMBER OF ITERATIONS USED.
C
C Iw - AN INPUT INTEGER CODE FOR PRINTING DURING
COMPUTATION:
- 3 NO PRINTING
- 1 PRINT GRADIENT VECTOR DIRECTION OF EACH LINEAR C00000145
MINIMIZATION, AND FUNCTION VALUE BEFORE AND AFTER
EACH LINEAR MINIMIZATION.
- 2 IN ADDITION TO THE ABOVE PRINT FUNCTION VALUES C00000146
CALCULATED DURING THE COURSE OF LINEAR MINIMIZATION C00000147
- 3 IN ADDITION TO THE ABOVE, PRINT FUNCTION VALUES C00000150
CALCULATED IN EVALUATING THE GRADIENT (FORMAT B15) C00000151
NF,FBEGIN,FEND - NUMBER OF FREQUENCY INTERVALS USED IN COMPUTING
THE SPECTRAL RESPONSE AND ESTIMATE OF OUTPUT POWER
(IN W), RESPECTIVELY (FORMAT 110,2E10.0). TOTAL
NUMBER OF FREQUENCIES USED NFT=NF+1

LENGTH - GUST SCALE LENGTH USED TO DETERMINE THE VON
KARMAN GUST SPECTRUM (FORMAT 4E10.0)

NOTE THAT THE MAIN PROGRAM WRITES AND READS FROM A TEMPORARY
FILE 15 (SHOULD BE DEFINED AS NEW. PASS). FILE 4 IS USED TO OUTPUT
OPTIMIZATION RESULTS AND SHOULD BE DEFINED AS EQUAL TO OUTPUT.

NOTE ALSO THAT PLOTS SHOWING THE SENSITIVITY OF ANY CONTROL
LAW WITH RESPECT TO THE VARIOUS X(I) (PARAMETERS CAN BE MADE
USING THIS PACKAGE TO ACCOMPLISH THIS, THE CALL TO SUBROUTINE
GUSPLT SHOULD BE REPLACED WITH A CALL TO SUBROUTINE GUSPLT* WHEN
SENSITIVITY PLOTS ARE NOT REQUIRED IT IS POSSIBLE TO DELETE THE
SUBROUTINE GUSPLT AND PLT (WHICH IS CALLED BY GUSPLT). THE
SENSITIVITY RANGE IS MOUNT BETWEEN X1(I) AND X2(I) IN STEPS OF
EPS(1), XA KTING WITH X(1).

NOTE ALSO THAT TWO GENERALIZED CONTROL LAWS ARE INCLUDED IN
MAKE SURE TO DELETE THE SUPER-FLUVIOUS CONTROL LAW.

INPUT/OUTPUT OF DATA

READ 132, (HOL(1), I=1,15)
READ 132, (HOL(1), I=1,15)
READ (5, CASE)
WRITE (6, CASE)
NMNC=NM+NC+Nu
IF (NAERelN=0) GO TO 17
DO 1 II=1,NM
DO 1 JJ=1,NMNC
READ 132,AO(11, JJ),A1(11, JJ),A2(11, JJ),A3(11, JJ),A4(11, JJ)
A5(11, JJ),A6(11, JJ)
1 CONTINUE
10 CONTINUE
READ(6,ST)
WRITE(6,ST)
IF(NAEI.EQ.0) CALL PIT
NMAC=NMAC+NS
10 IF(NPC.EQ.0) CALL PIT
MC=MC+NC
READ 100,ETAI PHI
IU=6

PI=3.1415926535
READ 103,NV,P,NK
READ 103,NM,NACT
READ 103,(NA(I),I=1,NM,NACT)
NVACT=NV-NM,NACT
READ 103,WT
WRITE(IU,113)
WRITE(IU,102)
IU=1
MV=20
READ 100,X1(I),X2(I),EPS(I)
WHITE(IU,102)
DO 200 1=1,NV
MV=20
READ 100,X1(I),X2(I),EPS(I)
WRITE(IU,102)
DO 200 1=1,NV
200 CONTINUE
READ 100,MIN,F
READ 103,ITMAX,IB
READ 123,NF,FBEGIN,FEND
READ 100,LNGTH
PRINT 101,LNGTH
CF=FFNO-FBEGIN/F
PRINT 105,CF+FBEGIN,FEND
NF=NF+1
READ 100,CF
PRINT 135,CM
PRINT 103,CM
220 CONTINUE
READ(13,140) (LABELX(I,J),J=1,NJ)
PRINT 121
PRINT 103.(NA(I),I=1,NM,NACT)
C

COMPUTATION OF THE TRANSFORMATION MATRIX \( H \) WHICH EXPRESS ES
THE DEFORMATION AND TORSION OF THE 50 PERCENT CHORD LINE FOR THE
VARIOUS MID SPAN SECTIONS OF THE CONTROLS, IN TERMS OF THE
GENERALIZED COORDINATES.

C

DO 240 J=1,NM
INC=INC+1
D0 240 J=1,NM
M(I,J)=(2*(I,J)+(CLH(INC)/CTH-1,00)*2KFF(I,J)-CLH(INC)/CTH)*
*2KFF(I,J)/UTRAN(INC)
M(I+1,J)=(2*(I+1,J)-2*(I,J))/UTRAN(INC)-(2KFF(I,J)-2KFF(I,J))/CTH
240 CONTINUE

C

FORMATION OF NEW ARRAYS WHERE ALL THEIR ELEMENTS RELATE TO
ACTIVE PARAMETERS ONLY.

C

CALL X2XX(NV, NVACT, X, XX, NA)
CALL X2XX(NV, NVACT, EPSACT, NA)
CALL X2XX(NV, NVACT, IN, NVACT, NA)
CALL X2XX(NV, NVACT, X1, XI, ACT, NA)
CALL X2XX(NV, NVACT, X2, X2ACT, NA)
IF (NVACT.EQ.0) GO TO 241
IF (FINAL. EQ. 0) GO TO 241

C

THE FOLLOWING OPTIMIZATION SUBRoutines CALL SHOULD BE REPLACED
BY A CALL TO DOPRINT WHEN SENSITIVITY PLOTS ARE REQUIRED.

C

CALL 500(NVACT, XIACT, XX, X2ACT, MIN, EPSACT, X1, X2ACT, NA)
IMAX=1, FMNXT, SGLST, IFNF(VN, NA)

C

CALL X2XX(NV, NVACT, X, XX, NA)
PRINT 114, ITimer+1
PRINT 111, FUNKTN

C

PRINTOUT OF OPTIMAL CONTROLL GAINS.

C

PRINT 112
PRINT 102, (X(J), J=1,NV)
PRINT 115
241 CONTINUE

C

PRINTOUT OF OPTIMAL UST RESPONSE OF CONTROLS.

C

IF (FINAL. EQ. 1)
CALL SGLST(XX, FUNKTN)
STOP

100 FORMAT (4F10.0)
101 FORMAT (1P4, LEN=4+13.6)
102 FORMAT (1P4, LEN=14.6)
103 FORMAT (15S)
104 FORMAT (4*NV=12* MPW=12* NDR=12*)
105 FORMAT (1P4, WL=13.6, WT=13.6)

C CALCULATE() IN THIS SUBROUTIN USING THE VON KÁRMÁN GUST SPECTRUM
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C 107 FORMAT(1. INITIAL (INPUT) VECTOR DRY(1),/)
108 FORMAT(10. X(1), X(1), EPS(1),/100000320
109 FORMAT(10., PHIN=1.E13., ETA=1.E13.,/)
110 FORMAT(10., ITHMAX=113., IWT=112./)
111 FORMAT(10., FUNCTN=1.E13.,/)
112 FORMAT(10., OPTIMUM VECTOR X(1),/)
113 FORMAT(10., IERK=10., ITERATIONS PERFORMED=1./)
114 FORMAT(/)
115 FORMAT(/)
116 FORMAT(10., ETA=1.E13., PHI=1.E13.,/)
117 FORMAT(10., NONACT=12., NVACT=12.,/)
120 FORMAT(10., THE NON ACTIVE PARAMETERS NA(I),/)
123 FORMAT(10., 2E10.0)
130 FORMAT(154A4)
131 FORMAT(154A4)
132 FORMAT(6E7E10.0)
133 FORMAT(8F4.2, IQ=10, FB=1, 'CONTROL NUM',12,E HZ')
135 FORMAT(1H1, 'M=1.', F4.2, '')</n140 FORMAT(154A4)

FND
SUBROUTINE WULGST(X, FUNCTN)
IMPLICIT REAL*8(A-H, O-Z)
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C THE VON KÁRMÁN GUST SPECTRUM
C
C CALCULATED IN THIS SUBROUTIN USING THE VON KÁRMÁN GUST SPECTRUM
C
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C COMMON/CUSTF/X(30),P(131),CHL/FX(131),UVEL,H(12,15)
C *RMASS(15,15),WFLGAM(15,M),AL+TR(103),Y(103),AK,PI,CH(6),VRMS(6)
C COMMON/CUSTF/X(30, TVACT, FIN, NCH, NAL(36), NV, LAMAX(150), NMNC,
C *NCACT, NTC(6)
C REAL*8 NH, Y, FPN
C
C COMMON/ALFDF/NA(15,22),A1(15,22),A2(15,22),A3(15,22),A4(15,22)
C *A5(15,22),A6(15,22)
C COMMON/CAS/ICASE/H(4),NM+=C+NUM, NL
C DIMENSION X(1),U(10),U(15,12),U(10,10), HFP(15,10),
C *DF2(10,6), DF2(10,6),UP(10)
C *W(15,2),PIVUT(15),NP(12), N(36)
C COMPLEX*16 FST, E(6), M(6,12),L(10,15),F(15,15), F(I(15,15)
C *H(15), AK, DETERM, ZHUM
C *AKB1,AKB2,AKB3,AKB4,FNC(15,6)
C PI=3.141592653589
C *ZD=180.00/PI
C
C CALL XX2(INV, TVACT, X, XX, NA)
C NC2=2*NC
C
C CALL CONT( NA, PI, BD, NC, W, H, X, PI)
C
C THE EQUATIONS OF MOTION ARE CONSTRUCTED AND SOLVED FOR NFT
C VALUES OF FREQUENCIES.
C
C DO 200 JF=1,NFT
C AK=DCMPLX(0,0,XF(JF)*CHLIF/VEL1)
C FD=DCMPLX(0, DO,XF(JF))
C ZER=DCMPLX(0,0,0,0,0)
C FZ=FZ
C AKZ=AKAK
C AKB1=AK/(AK+B1))
C AKB2=AK/(AK+B2))

C
THE TRANSFORMATION MATRIX WHEN GENERALIZED COORDINATES AND
CONTROL ROTATIONS IS CONSTRUCTED NEXT.

DO 300 I=1,NC
NN=ND(I)
BD(I)=ZERD
DO 250 K=1,NN
250 BD(I)=BD(I)+UD(K, I)*F**(K-1)
DO 300 J=1,NC
BN(I, J)=ZERD
NJ=NP(I, J)
DJ=320, K=1,NN
IF(P(I, J,K)+QO(0,0,0)) GO TO 320
BN(I, J)=BN(I, J)+P(I, J,K)*F**(K-1)
320 CONTINUE
300 BN(I, J)=BN(I, J)/HD(I)
DO 340 I=1,NC
DU 340 J=1,NN
CM(I, J)=ZERD
DO 340 K=1,NC2
CM(I, J)=CM(I, J)+HN(I, K)*F**(K-1)
340 CONTINUE

CONSTRUCTION AND SOLUTION OF THE EQUATIONS OF MOTION

DO 360 I=1,NC
DO 360 J=1,NN
NMJ=NM+J
FNC(I,J)=(AO(I, NMJ)+AI(I, NMJ)*AK+AZ(I, NMJ)*AK2+A3(I, NMJ)*AK3*
*AKB1+AKB2+AKB3+AKB4+AKB5+AKB6+AKB7+AKB8+AKB9+AKB10)*Q
360 CONTINUE

DO 380 I=1,NC
DO 380 J=1,NN
FG(I)=Q(I, VFL)*(AO(I, NMNC)+AI(I, NMNC)*AK+AZ(I, NMNC)*AK2+
-A3(I, NMNC)*AKB1+AKB2+AKB3+AKB4+AKB5+AKB6+AKB7+AKB8+AKB9)+
FK(I, J)+H(I, J)*F2+Q0(AO(I, J)+AI(I, J)*AK+AZ(I, J)*AK2+
-A3(I, J)*AKB1+AKB2+AKB3+AKB4+AKB5+AKB6)+*H(I, J)*FG(I, J)
380 CONTINUE

CONTINUE
DO 400 I=1,NN
DO 400 J=1,NN
T(I, J)=RMA55(I, J)*F2+Q0(AO(I, J)+AI(I, J)*AK+AZ(I, J)*AK2+
-A3(I, J)*AKB1+AKB2+AKB3+AKB4+AKB5+AKB6)+*AKB4+FG(I, J)
395 CONTINUE

T(I, J)=T(I, J)+MASS(I, I)*UM(AN(I)+UMEGAN(I)*4.00*P1*PI
+*DCMPLX(1.0, 0.0, 0.15, 0.0))
400 CONTINUE
CALL CXINVM(T, NM, FG, 1, DETERM, IPVOT, INX, 15, ISCALE)
DO 420 I=1,NC
K(I)=ZERD
DO 420 J=1,NC
420 K(I)=K(I)+CM(I, J)*FG(I)
IF(NDR.EQ.0.0 AND I.FINAL.EQ.0) GO TO 430
DO 421 I=1,NC
\[
\text{DEF}(JF, J) = \text{CDABS}(R(J)) \\
421 \quad \text{DEF}(JF, J) = \text{DEF}(JF, J) \times \text{DEF}(JF, J) \\
430 \quad \text{IF}((NDR, EQ, 0.0) \text{AND} (IFINAL, EQ, 0)) \text{GO TO 200} \\
\text{DO} 422 \quad I = 1, NC \\
\text{DEFR}(JF, J) \equiv \text{CDABS}(R(J) \times F(J)) \\
422 \quad \text{DEFR}(JF, J) = \text{DEFR}(JF, J) \times \text{DEFR}(JF, 1) \\
200 \quad \text{CONTINUE} \\
\text{IF}((NDR, EQ, 1.0) \text{AND} (IFINAL, EQ, 0)) \text{GO TO 440} \\
\]

**Computation of RMS Response of Control Surfaces**

\[
\text{DO} 423 \quad I = 1, NC \\
\text{CALL INTH1}(NFT, XF, DLF2(J), AREA, PSD) \\
423 \quad \text{DRMS}(1) = \text{DSQRT}(\text{AREA}) \\
\text{FUNCTN} = \text{DRMS}(1) \\
\text{IF}((FINAL, EQ, 0)) \text{GO TO 480} \\
440 \quad \text{CONTINUE} \\
\text{DU} 424 \quad I = 1, NC \\
\text{CALL INHLS}(NFT, XF, DEFR(J), AREA, PSD) \\
424 \quad \text{DRRMS}(1) = \text{DSQRT}(\text{AREA}) \\
\text{FUNCTN} = \text{DRRMS}(1) \\
\text{IF}((FINAL, EQ, 0)) \text{GO TO 480} \\
\]

**Print and Plot Outputs**

\[
\text{PRINT} 100 \\
\text{DO} 425 \quad I = 1, NFT \\
\text{PRINT} 110, (XF(I), (DEF(I, J), J = 1, NC), PSD(I)) \\
425 \quad \text{CONTINUE} \\
\text{PRINT} 120, (DRMS(I), I = 1, NC) \\
\text{PRINT} 125 \\
\text{PRINT} 127 \\
\text{DU} 470 \quad I = 1, NFT \\
\text{DO} 471 \quad J = 1, NC \\
\text{DEF2}(I, J) = \text{DEF2}(I, J) \times \text{PSD}(I) \\
471 \quad \text{DEFR2}(I, J) = \text{DEFR2}(I, J) \times \text{PSD}(I) \\
\text{PRINT} 110, (XF(I), (DEF2(I, J), J = 1, NC), PSD(I)) \\
470 \quad \text{CONTINUE} \\
\text{PRINT} 120, (DRMS(I), I = 1, NC) \\
\text{PRINT} 127 \\
\text{PRINT} 126 \\
\text{DU} 427 \quad I = 1, NFT \\
\text{PRINT} 110, (XF(I), (DEF2(I, J), J = 1, NC), PSD(I)) \\
427 \quad \text{CONTINUE} \\
\text{PRINT} 121, (DRMS(I), I = 1, NC) \\
\text{CALL PLTS}(BUF, 10, 6, 10) \\
\text{CALL SCALE(XR, 5, NFT, 1) \\
\text{CALL PLOT}(10, 12.5, J) \\
\text{DU} 900 \quad IP = 1, NC \\
\text{DU} 900 \quad IR = 1, 2 \\
\text{IF}((IR, EQ, 1) \text{AND} (DRMS(IP, EQ, 0.0)) \text{GO TO 900} \\
\text{IF}((IR, EQ, 0.0) \text{AND} (DRMS(IP, EQ, 0.0)) \text{GO TO 900} \\
\text{IF}((IR, EQ, 2)) \text{GO TO 910} \\
\]

\[\text{END}\]
DO 901 11P=1,NFT
901 Y(1P)=DEFR2(11P,IP)
FPN=DRMS(IP)
GO TO 950
910 CONTINUE
DO 911 11P=1,NFT
911 Y(1P)=DEFR2(11P,IP)
FPN=DRMS(IP)
950 CONTINUE
CALL AXIS(0,0,LABELX(I,IP),-56.7,0,XR(NFT+1),XR(NFT+2))
CALL SCALE(Y,5,NFT+1)
IF(IR.EQ.1) CALL AXIS(0,0,DEFLN PSD*10,5,90,Y(NFT+1))
+Y(NFT+2))
IF(IR.EQ.2) CALL AXIS(0,0,DEFLN RATE PSD*16,5,90,Y(NFT+1))
+Y(NFT+2))
CALL LINE(KP,Y,NFT,15,1)
CALL SYMUL(5,5,4.75,15,1,1,-1)
CALL NUMBER(5,75,4,675,0,15,FPN,0,0)
CALL PLUT(15,0,0,-1)
900 CONTINUE
CALL PLOT(1,0,0,4,4,1)
480 RETURN
100 FORMAT(' XF(I) DEFLN(I)...... PSD(I)\',/)
101 FORMAT(' XF(I) DEFLN(I)...... PSD(I)\',/)
110 FORMAT(' XF(IP,DEFLN(I)...... PSD(I)\',/)
120 FORMAT(' XF(IP,DEFLN(I)...... PSD(I)\',/)
125 FORMAT(' XF(IP,DEFLN(I)...... PSD(I)\',/)
126 FORMAT(' XF(IP,DEFLN2(I)...... PSD(I)\',/)
127 FORMAT(')
END
C THIS SUBROUTINE IS NECESSARY ONLY WHEN SENSITIVITY PLOTS ARE
C REQUIRED AROUND GIVEN X(1) CONTROL GAINS AND VARIATION BETWEEN
C X1(I) AND X2(I) IN STEPS OF EPS(I). THIS SUBROUTINE REPLACES THE
C CALL TO SJFP SUBROUTINE. THE FOLLOWING SUBROUTINE CALLS
C SUBROUTINE PLT. BOTH OF THESE SUBROUTINES HAVE NC CARD
C DATA INPUTS. THE PLOT OUTPUT IS ALSO PRINTED IN SUBROUTINE PLT.
C ONLY ACTIVE X(I) GAINS ARE Plotted. MAXIMUM IF 3A INTERVAL
C VARIATIONS OF X(I) ARE ALChED FOR.
C
COMMON/CUSTN/X(JL-)9P'(101),XP(I),UE(101),11P,FI,HH(12.15)
*RMS(15,15),UIMEGA(15),F(1),RE(101),T(101),X(I),R
*VRMS(1),VRMS(1)
COMMON/CUSTF(NFT,15,15), SMEGA(NFT,15,15), UIMEGA(NFT,15,6),NMNC
*NCACT,NC(6)
DIMENSION XX(1),XX1(I),XX2(I),JELX(I),D(36,6),DX(30,6),XP(36)
REAL*4 CJ,UX,XP,XR,Y
ISTART=0
NC=NCACT
DG=255,1,1,19,1
ISTART=ISTART+1
NP=0
XX=XX(I)
XXM=XX(I)
DO 150 K=1,34
  IF(XM*LT.XXI(I)) GO TO 150
  XM=XM-DELX(I)
150 CONTINUE
XX(I)=XM
DO 250 K=1,34
  XX(I)=XX(I)+DELX(I)
  IF(XX(I).GT.XX(1(I)+UR.XX(I)+T.IX(I(I)))) GO TO 250
NP=NP+1
  XP(NP)=XX(I)
  CALL SLG5T(XX,FUNCT)
DO 300 IC=1,NC
  D(NP,IC)=DRMS(1C)
300 CONTINUE
DX(NP,IC)=DRMS(1C)
250 CONTINUE
XX(I)=XT
DO 251 IC=1,NC
  CALL PLT(NV,NVACT,NA,XP(1,IC),NP,1,1,Q,EM,IC,ISTART,NC)
  IF(XX(I).LT.XX(I(I))..IX(I(I))=XX(I(I))=XX(I(I))
  IF(XX(I).LT.XX(I(I))..IX(I(I))=XX(I(I))=XX(I(I))
  NP=NP+1
  CALL PLT(NV,NVACT,NA,XP(1,IC),NP,1,1,Q,LM,IC,ISTART,NC)
251 CONTINUE
260 CONTINUE
  CALL PLT(10.0,0.,0.99)
RETURN
END

SUBROUTINE PLT(NV,NVACT,NA,XP,IC,PLT,K,EMDP,IC,ISTART,NC)
REAL*8 UUP,

DIMENSION NA(1),X(1),Y(1),LABELX(15),LABELY(3),BUF(100)

IF(ISTART.EQ.1) CALL PlTS(HUF,10,J,6,10))

Q=QDP
F=EMDP
YMAX=Y(1)
DO 15 I=2,NP
15 IF(Y(I).LT.YMX) YMX=Y(I)
RETURN

NX=12
NY=36
IF(NV.EQ.NVACT) IND=X=IPLT
IF(NV.EQ.NVACT) GO TO 3
II=1
INONAC=0
DO 10 1=1,NV
10 IF(I.NE.NA(1)) GO TO 2
INONAC=INONAC+1
IF(INONAC.LT.(NV-NVACT)) II=II+1
3 IACTIV=I-INONAC
IF(IACTIV.EQ.IPLT) IND=X=I
IF(IACTIV.EQ.IPLT) GO TO 3
10 CONTINUE
3 CONTINUE
REWIND 13
WRITE(13,100) INDEX,EM,U
108

IF(K.EQ.1) WRITE(13,121) IC
IF(K.EQ.2) WRITE(13,123) IC
REWIND 13
READ(13,120) (LABELX(J),J=1,9)
READ(13,121) LABELY
NX=-NX
IF(1STRT.EQ.1) CALL PLOT(10,2,5,-3)
CALL SCALE(X,5,NP,1)
CALL SCALE(Y,5,NP,1)
CALL AXIS(0,0,0,0,LABELX,NX,7,0,0,X(NP+1),X(NP+2))
PRINT 120,LABELX
PRINT 102,(X(J),J=1,NP)
PRINT 120,LABELY
PRINT 102,(Y(J),J=1,NP)
CALL AXIS(0,0,0,0,LABELY,NY,5,0,0,Y(NP+1),Y(NP+2))
CALL LINE(X,Y,NP,1,1)
CALL PLOT(15,0,0,-3)
RETURN

100 FORMAT('VAR.X(*,12,1),DASI M=*',F4.2,'DYN.PRESS=',F5.1)
C 121 FORMAT('DRMS(*,11,1) PSD')
123 FORMAT('DRRMS(*,11,1) PSD')
102 FORMAT(4E14.6)
120 FORMAT(15A4)
END

SUBROUTINE CONTKLTHP

C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
DETERMINATION OF THE DENOMINATOR POLYNOMIAL FOR EACH CONTROL SURF.

CD1(1) = (6*1-4)*X(6*1-3)
CD1(2) = (6*1-3)*X(6*1-2)
CD2(1) = (6*1-1)*X(6*1-2)
CD2(2) = (6*1-3)*X(6*1-2)
CD2(3) = (6*1-3)*X(6*1-3)

CALL PRUPOL(CU1,3,CD1,3,ND(1))

DETERMINATION OF THE NUMERATOR POLYNOMIAL FOR EACH CONTROL SURFACE.

CN(J) = X(6*1-5)
CALL PRUPOL(CDK,3,CNJ,TEMP2,ND)
CN(J) = X(6*1-1)
CALL PRUPOL(CDK,3,CNJ,TEMP2,ND)
DO 4 K=1,ND
P(1,2*1-1,K) = EH*(TEMP1(K) + TEMP2(K))
P(1,2*1-K) = EH*(TEMP1(K) + TEMP2(K)) + C21*NTF(1)*ND(K,1)
4 CONTINUE
NP(1,2*1-1)=N
NP(1,2*1-1)=N
2 CONTINUE
RETURN

END

IMPLICIT RFAL*8(A-H.O-L)
DIMENSION NP(E..L),P(091291),(1UILJ91),E(;e...NTE(X)

C D.T.T.F. CONTROL LAW FOR ANY NUMBER OF CONTROL SURFACES. CAN
C BE USED FOR BOTH FLUTTER AND GUST PROGRAMS. THE BASIC GAINS
C USED HEREIN ARE APPROPRIATE FOR 20 PERCENT L.E. AND 20 PERCENT
C T.F. CONTROL SYSTEMS WITH THE FINE SENSOR LOCATED AT THE 30
C PERCENT CHORD LOCATION - DIMENSIONS ARE LIMITED TO 6 CONTROLS.
C
ILMPLICIT RFAL*8(A-H.O-L)
DIMENSION NP(E..L),P(091291),(1UILJ91),E(;e...NTE(X)

E(1,1) = 4.0D0
E(1,2) = 4.0D0
E(2,1) = 4.0D0
F(2,2) = 3.0D0
(A=1000.J0)
NC2=2*NC
DO 1 I=1,NC
DO 1 J=1,NC2
NP(I,J)=1
DO 1 K=1,2
P(I,J,K)=0.0D0
1 CONTINUE

P(1,2*1-1,1)=0.0D0
P(1,201,1) = A1 + C1 + H(1)

CASE OF T.E. CONTROL

E1 = E(2,1)
E2 = E(2,2)
IF(NE1(1),EQ,1) GO TO 3

CASE OF L.E. CONTROL

E1 = E(1,1)
E2 = E(1,2)
J CONTINUE

DETERMINATION OF THE NUMERATOR POLYNOMIAL FOR EACH CONTROL SURFACE

P(1,201-1,2) = A1 + C1 + H(1)
P(1,201,2) = A1 + C1 + H(1)

DETERMINATION OF THE DENOMINATOR POLYNOMIAL FOR EACH CONTROL SURFACE

QD(1,1) = A
QD(2,1) = 1.0
NP(1,201-1,2) = 2
NP(1,201,2) = 2
ND(1) = 2
Z CONTINUE
RETURN
END

SUBROUTINE FIT

IMPLICIT REAL*3(A-H,L-P)

BLAU (21, A) = C1 + (1,J,K) . FORMAT (2L15.5)

FREE THE AERO COEFFICIENTS IN TERMS OF PAD APPROXIMANTS USING LEAST SQUARE TECHNIQUE.

NAMELIST /FIT

NK - NUMBER OF REDUCED FREQUENCIES K USED FOR INTERPOLATION
AK - ARRAY CONTAINING THE K VALUES (20 MAX) - FIRST REDUCED K MUST BE EQUAL TO ZERO
MAXNK - MAX VALUE OF NK (MAX NK = 20 IN PRESENT PROGRAM)
NPRINT - 0 NO PRINTED OUTPUT FROM SUBROUTINE FIT
- 1 PRINTED OUTPUT IS AVAILABLE (FOR DEBUGGING PURPOSES)
NPUNCH - 0 NO PUNCHED OUTPUT FROM SUBROUTINE FIT
- 1 PUNCHED OUTPUT (INTERPOLATION COEFFICIENTS)

IH10=HIH10 - CURVE FITTING WITH NO LEAST SQUARES TECHNIQUE
OF THE FIRST RIGID RUNS AND JUST COLUMNS OF AERO MATRIX - ASSUMED TO CONTAIN RIGID BODY AERO - TO IMPROVE RESULTS.

READ (2, 1) AK(C1, J, K) . FORMAT (2L15.5)

COMMON ICASE/B(4),NN,NC,NL
COMMON/ICASE/B(4),NN,NC,NL
COMPLEX*16 AFKU(15,2,20),COFF
DIMENSION AK(20),AK2(2),X(40,6),XT(6,40),Y(40),XTX(6,6)
*XTY(6,6),CL1(4,2),CL2(4,2)
NAMELST/FT/NK,AK,MAXNK,NPRINT,NPUNCH,INRID,IRIGID
READ(5,FT)
WRITE(5,FT)
MAXNK2=2+MAXNK
NNMC=NN+NC+NL
DO 1 K=1,NK
AK2(K)=AK(K)+AK(K)
DO 1 J=1,NNMC
DO 1 I=1,NM
READ(2,POH)AEKH(I,J,K)
1 CONTINUE
DO 5 I=1,NN
DO 5 K=1,NK
CLH(I,K)=AK2(K)/(U2+AK2(K))
CL1(I,K)=AK(K)/(U2+AK2(K))
5 CONTINUE
IF(AK(1).NEs0)PRINT 100
IF(AK(1).NEs0)STOP
10 CONTINUE
IF(NROW.LT.NCULS) STOP
CALL MKPRUD(XT,XTX,XTY,NCOLM,NCOLM,MAXNK2,6)
CALL MKPRUD(XT,XTX,NCOLM,NCOLM,MAXNK2,6)
C
DETERMINATION OF THE INTERPOLATION LEAST SQUARE MATRIX XTX AND
THE KNOWN AKV2 VECTOR XTY
C
DO 2 I=1,NN
DO 2 J=1,NNMC
2 CONTINUE
DO 3 K=1,NK
X(2*K-1,1)=AK(K)
X(2*K-2,1)=AK(K)
X(2*K-3,2)=AK2(K)
X(2*K-2,2)=0.0
DO 3 J=1,NCULS
Y(2*K-3)=AJMAG(AEHG(I,J,K))—AEHG(I,J,K))
3 CONTINUE
DO 4 L=1,NL
X(2*K-3,2*L)=CLH(L,K)
X(2*K-2,2*L)=CL1(L,K)
4 CONTINUE
NRWS=2*(NK+1)
NCOLM=2*NL
IF(NK0W=1,NCOLS) PRINT 110
IF(NK0W=1,NCOLS) STOP
DL 4 IR=1,NROWS
DO 4 JH=1,NCOLS
XT(IH,IR)=X(1H,JH)
4 CONTINUE
CALL MXPROD(XT,XTX,NCOLS,1,6,MAXNK2,6)
CALL MXPROD(XT,XTZ,NCOLS,1,6,MAXNK2,6)
C
SOLUTION FOR THE UNKNOWN INTERPOLATION COEFFICIENTS
C
CALL MXINVK(1NCOLS,1,6,XT)
CALL MXPROD(XT,XTX,NCOLS,1,6,6)
SUBROUTINE X2KX(NV, NVACT, X, XX, NA)
IMPLICIT REAL*8 (A-H, O-Z)

1 CONTINUE
IF(NV .LT. NVACT) GO TO 10
IF(NV .GT. NVACT) GO TO 10
NVACT = NV - NVACT

2 CONTINUE
210 FORMAT(2X,315*$E12.4)
600 FORMAT(2X,312*$F12.4)
700 FORMAT(2X,15*$E12.4)
RETURN
100 FORMAT(' FIRST REDUCED FREQUENCY MUST BE EQUAL TO ZERO')
110 FORMAT(' THERE ARE LESS EQUATIONS THAN Unknowns')
201 FORMAT(2E15.5)
END

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C THIS SUBROUTINE REDUCES THE X(I) ARRAY INTO AN XX(I) ARRAY
C WHERE THE NON ACTIVE PARAMETERS (NV-NVACT) HAVE BEEN ELIMINATED.
C THE POSITION OF THESE NON ACTIVE PARAMETERS ALONG THE X ARRAY
C IS GIVEN BY THE NA(I) ARRAY.
C
C DIMENSION X(NV), XX(NVACT), NA(NVACT)
NUNACT = NV - NVACT
NCOUNT = 0
DO 100 I = 1, NV
IF (NONACT .EQ. NCOUNT) GO TO 115
DO 110 J = 1, NONACT
IF (I .NE. NA(J)) GO TO 110
NCOUNT = NCOUNT + 1
GO TO 100
110 CONTINUE
115 X(I) = X(NCOUNT)
100 CONTINUE
RETURN
END

SUBROUTINE XX(NV, NVACT, X, XX, NA)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION X(NV), XX(NV, NVACT, X, XX, NA)
IMPLICIT REAL*8 (A-H, O-Z)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC000000919
C
C THIS SUBROUTINE KEEPS THE X(I) ARRAY USING THE REDUCED XX(I)
C ARRAY AND THE INITIAL VALUES OF THOSE X(I)'S THAT ARE NOT
C ACTIVE. THIS IS THE INVERSE PROCESS OF SUBROUTINE XX.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC000000938
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC000000939
DIMENSION X(I), XX(I), NA(I)
NONACT = NV - NVACT
NCOUNT = 0
DO 100 I = 1, NV
IF (NONACT .EQ. NCOUNT) GO TO 115
DO 110 J = 1, NONACT
IF (I .NE. NA(J)) GO TO 110
NCOUNT = NCOUNT + 1
GO TO 100
110 CONTINUE
115 X(I) = XX(NCOUNT)
100 CONTINUE
RETURN
END

SUBROUTINE CMXP(A, B, NA, NJA, NJD)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*8 A(JA), B(JA)
COMPLEX*8 A(JA), B(JA)
DO 100 J = 1, JA
DO 100 K = 1, JA
100 CONTINUE
RETURN
END

SUBROUTINE CMXAD(A, B, K, J)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*8 A(K, J), B(K, J)
DO 100 K = 1, K
DO 100 J = 1, K
100 CONTINUE
RETURN
END

SUBROUTINE CMXAS(A, B, K, J)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*8 A(K, J), B(K, J)
DO 100 K = 1, K
DO 100 J = 1, J
100 CONTINUE
RETURN
END

SUBROUTINE CMXAM(A, B, C, D)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*8 A(K, J), B(K, J), C(K, J), D(K, J)
DO 100 K = 1, K
DO 100 J = 1, J
100 CONTINUE
RETURN
END

SUBROUTINE CMXAB(A, B, C, D)
IMPLICIT REAL*8 (A-H, O-Z)
COMPLEX*8 A(K, J), B(K, J), C(K, J), D(K, J)
DO 100 K = 1, K
DO 100 J = 1, J
100 CONTINUE
RETURN
END
100 C(I,J) = A(I,J) * B(I,J)
RETURN
END
SUBROUTINE INTGSL(MN,X,Y,A,B)
IMPLICIT REAL*8 (A-H,O-Z)

DIMENSION X(1), Y(1), W(1)
A = 0.0
N = NN-1
DO 1 I = 1, N
1 A = A + (Y(I) * W(I) * (Y(I+1) - Y(I)) / X(I+1) - X(I))
RETURN
END
SUBROUTINE SDFP(N,XI,XU,X2,FMIN, EPS, ETA, ETAI, PHI, DRV, ITMAX, IWO, FO)
IMPLICIT REAL*8 (A-H,O-Z)

MINIMIZATION SUBROUTINE BASED ON THE STEWART'S ADAPTATION OF
THE DAVIDON-FLETCHER-POWELL ALGORITHM. A VARIATION HAD BEEN INCORPORATED HEREIN TO PERMIT THE CONSTRAINT OF THE INDEPENDENT VARIABLES WITHIN A SPECIFIED LOWER AND UPPER BOUNDS.

N - NUMBER OF INDEPENDENT VARIABLES.
XI(I) - DENOTES THE LOWEST BOUND OF THE I-TH INDEPENDENT VARIABLE.
XU(I) - DENOTES THE INITIAL VALUE OF THE I-TH INDEPENDENT VARIABLE.
X2(I) - DENOTES THE UPPER BOUND OF THE I-TH INDEPENDENT VARIABLE.
FMIN - INPUT APPROXIMATION TO THE FUNCTION MINIMUM.
EPS(1) - INPUT ARRAY CONTAINING THE DESIRED ABSOLUTE ACCURACY OF THE INDEPENDENT VARIABLES.
EPA - INPUT PARAMETER CONTAINING AN ESTIMATE OF THE RELATIVE ACCURACY OF THE FUNCTION EVALUATIONS WHICH ARE USED TO DETERMINE THE TYPE OF DIFFERENCE APPROXIMATION TO THE GRADIENT (ABSOLUTE ACCURACY*FUNCTION*ETA).
ETA - RELATIVE ACCURACY OF COMPUTER UNIT IN B.M. DOUBLE PRECISION.
* 5.0E-13 = ABSOLUTE ACCURACY*ETAI.
PHI - RELATIVE SIZE OF 'SUCTION ZONE' WITHIN WHICH THE OPTIMIZED FREE PARAMETER IS SUNK TO THE CONSTRAINT TO AVOID FALSE CONVERGENCE. ABSOLUTE SIZE OF ZONE = XI(I)*PHI OR X2(I)*PHI DEPENDING ON WHETHER NEAR LOWER OR UPPER CONSTRAINTS.
DRV - A ONE DIMENSIONAL INPUT ARRAY OF AT LEAST LENGTH N CONTAINING INITIAL STEP SIZES FOR DIFFERENCE APPROXIMATIONS TO THE GRADIENT.
ITMAX - AN INPUT/OUTPUT INTEGER. ON INPUT, ITMAX CONTAINS THE
MAXIMUM ALLOWABLE NUMBER OF OPTIMIZATION ITERATIONS. ON OUTPUT, ITMAX CONTAINS THE NUMBER OF ITERATIONS USED.

IV - AN INPUT INTEGER CODE FOR PRINTING DURING COMPUTATION.
- 0 NO PRINTING EXCEPT FOR SELECTED RESULTS DURING EACH ITERATION.
- 1 PRINT GRADIENT VECTOR AND FUNCTION VALUE BEFORE AND AFTER EACH LINEAR MINIMIZATION.
- 2 IN ADDITION TO THE ABOVE, PRINT FUNCTION VALUES CALCULATED DURING THE COURSE OF LINEAR MINIMIZATION.
- 3 IN ADDITION TO THE ABOVE PRINT FUNCTION VALUES CALCULATED IN EVALUATING THE GRADIENT.

FC - FUNCTION MINIMUM ON OUTPUT.
EVAL - THE NAME OF A USER CCODF SUBROUTINE WHICH EVALUATES THE FUNCTION BEING MINIMIZED. THIS NAME MUST APPEAR IN AN EXTERNAL STATEMENT OF THE CALLING PROGRAM.

IERR - OUTPUT ERROR CODE.
- -1 DISTANCE TO THE MINIMUM IS OPPOSITE THE DIRECTION INDICATED BY THE GRADIENT OF THE FUNCTION. OPTIMUM HAS PROBABLY BEEN REACHED.
- 0 NORMAL CONVERGENCE.
- 1 DERIVATIVE OF FUNCTION ALONG THE DIRECTION OF LINEAR MINIMIZATION WAS NOT NEGATIVE. USER SHOULD TRY SMALLER VALUES IN THE DV ARRAY.
- 2 NO PROGRESS IN THE LINEAR MINIMIZATION. THE FUNCTION MINIMUM HAS PROBABLY BEEN REACHED. USER SHOULD TRY DIFFERENT INITIAL CONDITIONS FOR XO.
- 3 THE LINEAR MINIMIZATION FAILED TO CHANGE THE FUNCTION VALUE. THE FUNCTION MINIMUM HAS PROBABLY BEEN REACHED ON A FLAT SURFACE. USER SHOULD TRY DIFFERENT INITIAL CONDITIONS FOR XO AND SEE IF THE SAME MINIMUM IS REACHED.
- 4 FAILURE TO CONVERGE WITHIN ITMAX ITERATIONS.

NV - TOTAL NUMBER OF PARAMETERS NECESSARY FOR THE DETERMINATION OF THE FUNCTION IN SUBROUTINE EVAL. SOME OF THESE PARAMETERS CAN BE MADE INACTIVE DURING OPTIMIZATION AND THUS LEAD TO A VALUE OF N WHICH IS SMALLER THAN NV.
NA - INTEGER INPUT ARRAY CONTAINING THE LOCATIONS OF THE NONACTIVE PARAMETERS IN THE EXPANDED XO ARRAY.

NOTE - THE ABOVE TWO PARAMETERS ARE USED IN SUBROUTINE EVAL THROUGH THE USE OF SUBROUTINES X2XX AND XX2X.

DIMENSION XU(I), FPS(I), DRV(I), H(60,60), X(60), G(60), Y(60), DEL(60), IC(60), E(4), EE(4), F(4), 2,G(60), 00001085
3,X1(1),X2(1), TST(60), WEX(60), X1PHI(60), Y2PHI(60), XIETA(60), X2ETA(60), 00001086
DIMENSION NA(I)
LOGICAL IDENT

OPTIONAL OUTPUT FORMATS

2000 FORMAT(1), 'FUNCTION VALUE = ',E20.10, ' VARIABLES X(I) = ',E20.10/ 00001095
1(17x,E20.10))
2001 FORMAT(1HO,'COMPUTE GRADIENT')
2002 FORMAT(1HO,'GRADIENT = ',6x,E20.10/(17x,E20.10))
2003 FORMAT(1HO,'DIRECTION OF MINIMIZATION = ',4x,E20.10))
2004 FORMAT(1HO,'LINEAR MINIMIZATION - FUNCTION VALUE = ',E20.10)
2005 FORMAT(1HO,'MINIMUM FUNCTION EVALUATION = ',E20.10)
2006 FORMAT(HO,'END OF ITERATION ',13/) 
2007 FORMAT(1P,3X,13,2X,E14.6,1X,E14.6,1X,13,2X,E14.6)
2008 FORMAT(1) ITERMS FURT UMAX IGNX DELMAX
+IDNX 3 (LOWEST ) */
2009 FORMAT(1E14.6)
2010 FORMAT* INITIAL GRADIENTS VECTOR G(I)*;/
2011 FORMAT* FINAL GRADIENTS VECTOR G(I)*;/
2012 FORMAT* (+++++++)
4XTRIS=25
IU1=6
IU2=4
DO 2 I=1,N
XIETA(I)=DABS(X1(I)*ETA1)
IF(DABS(X1(I))LT.1.) XIETA(I)=-ETA1
X2ETA(I)=DABS(X2(I)*ETA1)
IF(DABS(X2(I))LT.1.) X2ETA(I)=ETA1
X1PHI(I)=DABS(X1(I)*PHI1)
IF(DABS(X1PHI(I))LE.DABS(EPS(1))) X1PHI(I)=DABS(1.1*EPS(1))
X2PHI(I)=DABS(X2(I)*PHI1)
IF(DABS(X2PHI(I))LE.DABS(EPS(1))) X2PHI(I)=DABS(1.1*EPS(1))
2 CONTINUE
C EM= .1D-13
FM=FMIN
ILIN = 1
LOWEST=1
CALL EVAL(XO,FO)
C COMPUTE GRADIENT
C 4 IF (IW.GT.2) WRITE(IU2,2001)
DO 10 I=1,N
X(I)=XO(I)
10 X(I)=XO(I)+DRV(I)
CALL EVAL(XO,FG)
IF (IW.GT.2) WRITE(IU2,2000) FG,(XO(J),J=1,N)
7 G(I)*(FG-F0)/ DRV(I)
10 X(I)=X(I)
CALL XX2X(NV,N,NV,G,NA)
PRINT 2010
PRINT 2009,(GEX(J),J=1,NV)
WRITE(IU1,2008)
20 IDENT=.TRUE.
DO 30 I=1,N
DO 25 J=1,N
25 M(I,J)=0.D0
M(I,I)=1.D0
30 C(I)=1.D0
IF (IW.GT.0) WRITE(IU2,2002) (G(I),I=1,N)
C DETERMINE DIRECTION AND DIRECTIONAL DERIVATIVE
C 50 D=0.00
IF CONSTRAINTS ARE VIOLATED: SET DEL(1)=0.00
IF(X(I)+EQ.X(2(I)) AND DEL(I)+GT+0.00) DEL(I)=0.00
IF(X(I)+EQ.X(1(I)) AND DEL(I)+LT+0.00) DEL(I)=0.00
IF(DEL(I)+EQ+0.00) GO TO 60
EP=DMINI(EP,DABS(EP(I)/DEL(I))
D=D+G(I)*DEL(I)
GO TO 60
CONTINUE
EP=FP+.0000
IF(DL+LT+.3*DO) GO TO 73
IF(.NOT.+IDENT) GO TO 20
ERR = 1
GO TO 500
IF(DL+00*FU) GO TO 71
71 CONTINUE
F(2)=DMINI(1.+2.+0.+2.+DO*FM-FL)/DO
IF (I+GT+1) WRITE(1,2,20) (DF(I),I=1,N)
IF(1+GT+0) WRITE(1,2,200) FU,(XU(I),I=1,N)
F(1)=FO
F(I)=0.00
NIT=0
CALL MX(N,GMAX,IMAX)
CALL MX(N,DL,DELMAX,IDMAX)
PROCEED WITH LINEAR MINIMIZATION
KKK=1
103 IF(DABS(F(2))+LE.+EP) E(2)=F(2)+1.1D0*EP
NTRIES=0
GO 105 1+1,N
105 X(I)=XO(I)+E(2)*DEL(I)
CALL EVAL(X,F(2))
IF (I+GT+1) WRITE(1,220) F(I)
IF(F(2)+NE+F(I)) GO TO 107
501 E(2)=2.00*D(I)
GO TO 103
CONTINUE
DENOM=D+F(I)+F(1)+F(2)
IF(DABS(DENOM)+LT+1.0-20) GO TO 501
ED+=.500*D+F(1)+2/DENOM
IF(ED+LE+.DO) ED+=2.00*E(2)
IF(F(2)+LT+F(I)) UC TC 120
IF(KKK+LT+5+AND+DABS(LD)+GT+EP) E(2)=E(I)
KKK=KKK+1
IF(KKK+LT+5+AND+DABS(LD)+GT+EP) GO TO 103
F(3)=F(2)
CALL EVAL(X+F(1))
IF (IF*F(I)+F(1)>F(1)) GOTO 150
F(1)=F(I)
F(1)=F(I)
CALL EVAL(X+F(1))
IF (IF*F(I)+F(1)>F(1)) GOTO 150
170 OKE 150
180 CONTINUE
190 CALL EVAL(X+F(1))
IF (IF*F(I)+F(1)>F(1)) GOTO 150
F(1)=F(I)
KKK=1
IF(F(EQ.4)) GO TO 220
IF(F(1).GT.F(4)) GO TO 280
CALL INTIPM(E,F,EE,A+O)
IF(E(2)+EE(2).LT.E(4).AND.A.GT.0.OO) GO TO 160
GO TO 210
200 KKK=2
CALL INTIPM(E,F,EE,A+1)
IF(E(3)+EE(2).GT.E(1).AND.A.GT.O.D0) GO TO 220
210 KKK=1
IF(F(2).LT.F(1).AND.F(2).LE.F(3).OR.F(2).LE.F(1).AND.F(2).LT.F(3)) GO TO 150
150 GO TO 160
220 DO 230 I=1,3
F(I)=E(1+I)
230 F(I)=F(I+1)
GO TO (153,160),KKK
250 IF(I=GT.0) WRITE(IU2,2005) F(LOWEST)
NIT=NIT+1
C
END OF MINIMIZATION ALONG DEL
C
IF THERE WAS NO MOTION RETURN
C
IF(E(LW)GT.0.0) GO TO 260
IF(.NOT.IDENT) WRITE(IU1,2037) LIN,FOPT,UMAX,UMAX,DELMAX,IMAX,E(LW)
+E(LW)
IF(.NOT.IDENT) GO TO 20
IERR=2
GO TO 500
260 IF(F(LOWEST).GT.E(2005)) GO TO 270
IERR=3
GO TO 500
C
CHANGE L(LOWEST) IF NECESSARY SO AS NOT TO VIOLATE CONSTRAINTS
C
270 IF(E(LOWEST).GE.0.0) GO TO 271
WRITE(IU1,2037) LIN,FOPT,UMAX,UMAX,DELMAX,IMAX,E(LOWEST) +E(LOWEST)
IF(.NOT.IDENT) GO TO 20
IERR=-1
IMAX=ILIN
GO TO 650
271 F0=F(LOWEST)
DU 262 I=1,N
XT=XO(I)+E(LOWEST)*DEL(I)
TST(I)=0.D0
IF(XT-X2(I).GT.X2ETA(I)) TST(I)=XT-X2(I)
IF(XT-X1(I).LT.X1ETA(I)) TST(I)=X1(I)-XT
262 CONTINUE
CALL MX(N,TST,TSTMX,IMX)
IF(TSTMX.EQ.O.D0) GO TO 272
DECLAM=TSTMX/DEL(IMX)
E(LOWEST)=E(LOWEST)-DABS(DECLAM)
272 CONTINUE
WRITE(IU1,2037) LIN,FOPT,UMAX,UMAX,DELMAX,IMAX,E(LOWEST)
C
CHECK FOR CONVERGENCE AND CREATE A SUCTION ZONE NEAR CONSTRAINTS
C
OF THICKNESS XI(I)*PHI OR X2(I)*PHI
C
IERN=0
**ETEST** = DABS(E(LOWEST))

DO 260 I = 1, N

IR1 = 10

IR2 = 10

IR3 = 10

IF (DABS(ETEST)*DEL(1)) LE DABS(EPS(I))) IR1 = 0

IF (X(1) - EQ. X(2) AND DEL(1) GT X(2PHI(1))) IR2 = J

IF (X(1) - EQ. X(1) AND DEL(1) LT X(1PHI(1))) IR2 = 0

IF (IR1 .EQ. 0 OR IR2 .EQ. 0) IR3 = 0

IF (IR3 .NE. 0) IEHR = 10

X(I) = XO(I) + E(LOWEST)*DEL(I)

IF (X(I) - X2(I) - GT X2PHI(1)) X(I) = X2(I)

IF (X(I) - X1(I) - LT X1PHI(1)) X(I) = X1(I)

260 CONTINUE

CALL EVAL(XU, FIST)

IF (FIST .LE. FOPT, UN, NLT.GT.2) GO TO 274

E(2) = E(LOWEST)

F(1) = 0.0

F0 = F0PT

F(1) = F0PT

KKK = 0

GO TO 100

274 CONTINUE

DO 276 I = 1, N

DEL(I) = X(I) - X0(I)

X0(I) = X(I)

276 G(I) = G(I)

F0 = F0PT

IF (IEHR .EQ. 0 AND IDENT) GO TO 212

C IF TOO MANY ITERATIONS RETURN

C IF (T. .GT. 0) WRITE (1U2, 200) ILIN

ILIN = ILIN + 1

IEHR = 4

IF (ILIN .GT. ITMAX) GO TO 500

C CALCULATE NEw GRADIENT

C

281 IF (T. .GT. 2) WRITE (1U2, 2001)

DO 300 I = 1, N

X(I) = XU(I)

IF (FU .EQ. 0.0) GO TO 285

IF (G(I) .EQ. 0.0) GO TO 285

IF (IDENT) GO TO 285

ETAM = DMAXI (ETAMXABS(FU)**G(I)**XU(I)/FU))

IF (G(I) .EQ. 2.0**G(I)**XU(I)/FU)) ETAM = TAN(I) TO 282

DRV(I) = 2.0**G(I)**XU(I)/FU) DAHS(U(I)) DAHS(U(I))**2) EXP(I) 33333330

DKV(I) = DRV(I) ** (1.0) - DAHS(U(I)) / (1.5)**(1.0) DRV(I) ** (2.0)** U(I)**2)

GO TO 283

282 DRV(I) = 2.0**U(I)**DAHTXU(I)**U(I)/FU)**(1.0)

DRV(I) = DRV(I) ** (1.0) U(I)**2) DRV(I) ** (1.0) U(I)**2) DRV(I) ** (1.0) U(I)**2)

GOTO 283

281 DRV(I) = 3.0**G(I)**XU(I)**2)

IF (3.0**DAHS(U(I))**G(I)**XU(I)**2) GO TO 285

285 XU(I) = X(I) + DHV(I)

CALL EVAL(XU, F)

IF (T .GT. 2) WRITE (1U2, 200) FU, (XU(J), J = 1, N)
G(I)=(FG-FQ)/DRV(I)
GO TO 300
295 DY=100.000*AUX*(FC*ETAN**G(I))
DRV(I)=ABLS(G(I))+SQRT(G(I)**2+20.00*AUX*AUX+DABS(FQ)*C(I)*ETAN)
DRV(I)=DRV(I)/C(I)
DRV(I)=MIN(DRV(I),DY)
XD(I)=X(I)*DRV(I)
CALL EVAL(XD,FP)
IF (INO.GT.2) WRITE(I12,2000) F2,(XJ(J),J=1,N)
XD(I)=X(I)-DRV(I)
CALL EVAL(XD,FM1)
IF (INO.GT.2) WRITE(I12,2000) FM1,(XJ(J),J=1,N)
G(I)=.5D0*(FP-DFM1)/DRV(I)
300 XD(I)=X(I)
C
IF ON CONSTRAINTS .SET G(I)=0.00
C
DO 105 I=1,N
IF (XJ(I).EQ.X2(I).AND.G(I).LT.0.00) G(I)=0.00
305 CONTINUE
C
IF MIN ALONG -DEL SET H= C(INV)
C
100 IF( (LUST).LT.0.00) GO TO 10
10 IF(IFNN.LT.0) GO TO 2
C
MODIFY H AND WRITEATL
C
IDENT=.FALSE.
A=0.00
DO 310 I=1,N
Y(I)=G(I)-G(I)
310 A=A+Y(I)**2
B=B-X(I)*Y(I)
DO 310 I=1,N
CALL EVAL(X2(I),F2)
DO 310 J=1,N
C(I)=C(I)+CL*Y(I)**2+CL*Y(I)*Y(J)
X(I)=X(I)+Y(J)
330 B=B-X(I)*Y(I)
DO 340 I=1,N
IF (C(I).LE.0.00) GO TO 20
340 H(I,J)=H(I,J)+DEL(I)*DEL(J)/A
340 H(J,I)=H(I,J)
GO TO 50
C
RETURN TO CALLING PROGRAM
C
500 IF (INO.GT.2) WRITE(I12,2000) F2,(XJ(J),J=1,N)
ITMAX = ILIN
IF (IERR.EQ.0) UN=1KX.EQ.4) GO TO 500
IF (IERR.EQ.0) ILIN=620,620
600 ITENS=ILIN
IF (IERH.EQ.0) IFRNS=ILIN+1
WRITE(IUI,2007) IFRNS,FU
GO TO 650
610 WRITE(IUI,2007) ILIN,FUPT,GMAX,IGMAX,DELMAX,IDMAX
GO TO 650
620 WRITE(IUI,2007) ILIN,FUPT,GMAX,IGMAX,DELMAX,IDMAX,F(LUWEST)
IFRNS=ILIN+1
WRITE(IUI,2007) IFRNS,F(LUWEST)
650 PRINT 2011
CALL XX2XNVN,GEX,GOA)
PRINT 2009,(GEX(J),J=1,NV)
RETURN
END
SUBROUTINE INTIPM(E,F,CE,A1)
IMPLICIT REAL*6 (A-H,O-Z)
DIMENSION E(I),F(I),FE(I),E(3),C(1+3)
EE(I)=E(1+1)-E(I+2)
DF1=LE(1)*(F(I+3)-F(I+2))
DF3=FE(3)*(F(I+1)-F(I+2))
T=0.500*(EE(I)*DF1—EE(3)*DF3)
ACURY=1.0D-09
ERM=5.0D-14
EHE3=(DABS(E(I+3))+DABS(E(I+2)))*ERM
ELE1=(DABS(E(I+1))+DABS(E(I+2)))*ERM
EHE23=(DABS(F(I+3))+DABS(F(I+2)))*ACURY
CERF12=(DABS(F(I+1))+DABS(F(I+2)))*ACURY
IF (EHE23.LT.2.00*ACURY) ERF3=2.00*ACURY
IF (EHE12.LT.2.00*ACURY) ERF3=2.00*ACURY
ENDF1=ERE1*EABS(F(I+1)-F(I+2))+ERF2*DAHS(EF(I))
ENDF3=ERE1*EABS(F(I+1)-F(I+2))+ERF2*DAHS(EE(I))
ENDF1=ERDF1+ERF1*ERF2
ENDF3=ERDF3+ERF1*ERF2
ET3=EABS(E(I+1)-EE(I))/DABS(OF(I))+EABS(LF(I))/DABS(EF(I))
ETF3=EABS(E(I+1)-EE(I))/DABS(OF(I))+EABS(LF(I))/DABS(EF(I))
E2F3=0.500*DABS(T)*E2F3
E2F1=ERDF1*ERF2
ERF1=ERF1*E2F3
ERF2=ERF2*E2F3
ERF3=ERF3*E2F3
TFK=FLX1*DAHS(DF(I)+DFJ)*EABS(EF(I)+DFJ)+DAHS(EF(I)+DFJ)*EABS(DJF3)*DABS(EE0001496+(3)))/0.500+DAHS(T)*E2F3
ERF1=ERF1+0.500*(ERF1*ERDF1*ERF3*ERF3)
ERF2=ERF2*ERDF1*ERF3
ERF3=EABS(EF(I)+DFJ)+DABS(EE(I)+DFJ)+EABS(EF(I)+DFJ)+EABS(EE(I)+DFJ)
T=0.500*DAHS(T)*ERF1
EE(E(I)+DFJ)=EE(I)+DFJ
DFJ=DFJ+DFJ
DA=EE(I+1)+EE(I+2)+EE(I+3)
IF (DDF3-DFJ).LE.0.0D0 GO TO 1
EE(2)=T/(DF3-DFJ)
A=(DF3-DFJ)/DA
RETURN
1 CONTINUE
EE(2)=0.00
IF(F(1+3)+LT:F(1+2)) KE(1)=EE(3)
A=1.00
RETURN
END

SUBROUTINE MX(NV,Y,YMAX,IMAX)
IMPLICIT REAL*B(A-H,U-Z)

C
DIMENSION Y(1)
YMAX=DAUS(Y(1))
IMAX=1
IF(NV.EQ.1) RETURN
DO 100 1=2,NV
IF(DAUS(Y(1))LE.YMAX) GO TO 110
YMAX=DAUS(Y(1))
IMAX=1
100 CONTINUE
RETURN
C

END OF QUADRATIC CONVERGENCE PACKAGE(WITHOUT PEnalty FUNCTIONS).

C

FUNCTION DREAL(Z)
IMPLICIT REAL*8(A-H,U-Z)

C
THIS SUBROUTINE CAN BE USED WITH EITHER THE SLOW OR FAST.IUM.

C
DOUBLE PRECISION, COMPRESSIBLE AERODYNAMIC COEFFICIENTS PROGRAM
IMPLICIT REAL*8(D)

REAL*8 Z(D)
DREAL=Z(1)
RETURN
ENTRY DAIMG(Z)
DAIMG=Z(2)
RETURN
END

SUBROUTINE PRODUL(A,N,U,M,C,L)
IMPLICIT REAL*B(A-IU,C-Z)

C

A ROUTINE FOR MULTIPLYING POLYNOMIALS C=A*B WHERE A,B,C ARE

C
POLYNOMIALS OF THE FORM

C
A=A(1)+A(2)*X+A(3)*X**2+A(4)*X**3+...........A(N)*X**N-1
B=B(1)+B(2)*X+B(3)*X**2+B(4)*X**3+...........B(M)*X**M-1
C=C(1)+C(2)*X+C(3)*X**2+C(4)*X**3+...........C(N+M-1)*X**(N+M-2)
AND WHERE L=N+M-1
C
DIMENSION C(1),A(1),B(1)
N=N+M-1
DO 1 =1, NM
C(1)=0.00
1
DO 2 I=1,N
DO 2 J=1,N
2 C(I+J-1)=A(I)*B(J)+C(I+J-1)
L=NM
RETURN
END

SUBROUTINE CXINV(A,N,B,M,DET,IPIV,INDX,MAX,IBSCALE)

IMPLICIT REAL*8(A-H,O-Z)
COMPLEX*16 A(MAX,N),B(MAX,M),SWAP,DET,PIV,PIVI,CO,CI,PIVR
DIMENSION IPIV(N), INDEX(MAX,N)

C
C THIS SUBROUTINE IS IDENTICAL TO SUBROUTINE CXINV EXCEPT FOR
C MINOR MODIFICATIONS THAT CAUSE IT TO BE FASTER (FOR EXAMPLE,
C THE PIVOT IS DETERMINED BY AVOIDING THE USE OF CABS() BY USING
C DABS(REAL)+DABS(IMAG)). FOR DETAILS REGARDING USAGE SEE
C SUBROUTINE CXINV.
C
C
C IMPLICIT REAL*8(A-H,O-Z)
COMPLEX*16 A(MAX,N),B(MAX,M),SWAP,DET,PIV,PIVI,CO,CI,PIVR
DIMENSION IPIV(N), INDEX(MAX,N)

C
C CONSTANTS, INITIALIZATION
C
C CJ=DCMPLX(J,J,J,0.00)
CI=DCMPLX(1.000,0.000)
DET = CI
CAVM=0.000
DO 20 J=1,N
20 IPIV(J) = 0
DO 50 J=1,N
C
C SEARCH FOR PIVOT ELEMENT
C
C CAVM=5.000
DO 105 J=1,N
IF (IPIV(J) .EQ. 1) GO TO 105
DO 100 K=1,N
IF (IPIV(K) .EQ. 1) 50,100,750
50 CONTINUE
DI=DAIMAG(A(J,K))
C
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
C IF (CAVM .LE. 0.000) GO TO 720
IPIV(ICOL) = IPIV(ICOL) + 1
105 CONTINUE
IF (CAVM .LE. 0.000) GO TO 720
IPIV(ICOL) = IPIV(ICOL) + 1
C
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
C IF (IRUW .EQ. 1) GO TO 230
DET = -DET
DO 230 L=1,N
SWAP = A(IRUW*L)
A(IRUW*L) = A(ICOL*L)
A(ICOL*L) = SWAP
IF (IRUW .EQ. 1) 230,720,290
720 CONTINUE
C
C
C
200 CONTINUE
IF (M .LE. 0) GO TO 230
DO 220 L=1,N
SWAP = 0(IROW=L)
B(IROW,L) = M(ICOL,L)
B(ICOL,L) = SWAP
220 CONTINUE
230 CONTINUE
INDX(I+1) = IROW
INDX(I+2) = ICOL
PIV = A(ICOL,ICOL)
CAPV = CAPV(PIV)
IF (CAPV .GE. 0.3D0) GO TO 720
C                  DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
A(ICOL,ICOL) = 1
PIVR=1.0D0/PIV
DU 350 L=1,N
350 A(ICOL,L) = A(ICOL,L)*PIVR
IF (M .LT. 0) GO TO 330
DU 370 L=1,N
370 ICOL(L) = M(ICOL,L)*PIVR
C                  RESULT NON-PIVOT ROWS
C
380 CONTINUE
DU 400 L=1,N
IF (L .EQ. ICOL(L)) GO TO 530
SWAP = A(L,ICOL(L))
A(L,ICOL(L)) = 0
DU 400 L=1,N
400 A(L,L) = A(L,L) - A(L,ICOL(L))*SWAP
IF (M .LT. 0) GO TO 450
DU 450 L=1,N
450 B(L,L) = M(L,L) - M(L,ICOL(L))*SWAP
500 CONTINUE
C                  INTERCHANGE COLUMNS
C
DU 700 L=1,N
L = N+1-L
IF (INDX(L) .EQ. ICOL(L)) GO TO 700
IROW = INDX(L+1)
ICOL = INDX(L+2)
DU 690 K=1,N
SWAP = A(K,IROW)
A(K,IROW) = A(K,ICOL)
A(K,ICOL) = SWAP
690 CONTINUE
700 CONTINUE
GU TO 750
750 END
SUBROUTINE M81611V (N,M,X,P,A)
SUBROUTINE M81611V(N,M,X,P,A)
C
REAL MATRIX INVERSION WITH SOLUTION OF LINEAR EQUATIONS

CAVM = DABS(A(MAX)), CAVA = DABS(A(I,J))
CADM = DABS(DETERM), CAPV = DABS(IPIV

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(MAX,1), I(150,1), IPIV(150), INDX(150,2)

IF(M+N.E.0) GO TO 1
DO 7 I=1,N
2 B(I,1)=0.00
GO TO 10
1 PRINT 1000
1000 FORMAT(3, "NO SOLUTION OF LINEAR EQUATIONS IS ALLOWED FOR IN "+ THIS VERSION OF MKINVR")
STOP

10 CONTINUE

CLUTANTS, INITIALIZATION

CG=0.00
C1=1.00
DET = C1
CADM=1.00
DO 20 J=1,N
20 IPIV(J) = J
DO 500 I=1,N

SEARCH FOR PIVOT ELEMENT

CAVM=0.000
DO 105 J=1,N
IF (IPIV(J) .EQ. 1) GO TO 105
DO 100 K=1,N
IF (IPIV(K) .EQ. I) GO TO 100
50 CONTINUE
CAVM=DABS(A(J,K))
IF (CAVM .LT. CAVA) GO TO 100
IROW = J
ICUL = K
CAVM = CAVA
100 CONTINUE
105 CONTINUE
IF (CAVM .LT. U0*0.000) GO TO 700
IPIV(ICUL) = IPIV(IROW) + 1

INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

IF (IROW .EQ. ICUL) GO TO 700
DET = -DET
DO 200 L=1,N
SWAP = A(IROW,L)
A(IROW,L) = A(ICUL,L)
A(ICUL,L) = SWAP
200 CONTINUE
IF (M .LT. J) GO TO 230
DO 220 L=1,M
SWAP = B(IROW,L)
B(IROW,L) = B(ICUL,L)
B(ICUL,L) = SWAP
220 CONTINUE
126
220 CONTINUE
230 CONTINUE
   INDEX(1,1) = IROW
   INDEX(1,2) = ICUL
   PIV = A(ICUL,ICOL)
   CAPV=UASSPI(PIV)
   IF(CAPV.EQ.0.0D0) G0 TO 720
   
   C       DIVIDE PIVOT ROW BY PIVOT ELEMENT
   C
   A(ICUL,ICUL) = C1
   PIVH = 1.0D0/PIV
   DO 350 L = 1,N
   350 A(ICUL,L) = (A(ICUL,L)*PIV)
   IF (M .LT. 0) GO TO 180
   DO 370 L = 1.N
   370 B(ICUL,L) = B(ICUL,L)*PIV
   C       REDUCE NON-PIVOT ROWS
   C
   380 CONTINUE
   DO 500 L = 1,N
   IF (L .EQ. ICUL) GO TO 500
   SWAP = A(L,ICOL)
   A(L,ICOL) = C0
   DO 400 L = 1,N
   400 A(L,L) = A(L,L) - A(ICL,L)*SWAP
   IF (M .LT. 0) GO TO 530
   DO 450 L = 1,N
   450 B(L,L) = B(L,L) - B(ICL,L)*SWAP
   500 CONTINUE
   C       INTERCHANGE COLUMNS
   C
   510 CONTINUE
   DO 700 L = 1,N
   700 L = M+1-L
   IF (INDEX(L,1) .EQ. INDEX(L,1)) GO TO 730
   IROW = INDEX(L,1)
   ICOL = INDEX(L,2)
   DO 690 K = 1,N
   690 SWAP = A(K,ICOL)
   A(K,ICOL) = A(K,IROW)
   A(K,ICOL) = SWAP
   690 CONTINUE
   GO TO 750
   720 DET = C0
   ISCALF = J
   750 RETURN
END
DO 30 J = 1, NJM
300 C(I,J) = D(I,J)
100 CONTINUE
RETURN
END
SUBROUTINE MXADD(A,B,C, NIA, NJ, MAXA, MAXB, MAXC)
REAL A,B,C
DIMENSION A(MAXA,1),B(MAXB,1),C(MAXC,1)
DO 100 I = 1, NIA
DO 10 J = 1, NJ
100 C(I,J) = A(I,J) + B(I,J)
RETURN
END
SUBROUTINE MXSUB(A,B,C, NIA, NJ, MAXA, MAXB, MAXC)
REAL A,B,C
DIMENSION A(MAXA,1),B(MAXB,1),C(MAXC,1)
DO 100 I = 1, NIA
DO 10 J = 1, NJ
100 C(I,J) = A(I,J) - B(I,J)
RETURN
END
SUBROUTINE MXSCAL(A,B,C, NIA, NJ, MAXA, MAXC)
REAL A,B,C
DIMENSION A(MAXA,1), C(MAXC,1)
DO 100 I = 1, NIA
DO 10 J = 1, NJ
100 C(I,J) = A(I,J) * B(I,J)
RETURN
END
END OF JOH. CONDITION CODE: WAS 0
<table>
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<tr>
<th>Mass</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
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</table>
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{X1(I)} & \textbf{X(I)} & \textbf{X2(I)} & \textbf{EPS(I)} \\
\hline
0.0 & 4.50000000D+00 & 5.00000000D+00 & 1.00000000D-05 \\
6.00000000D+01 & 7.00000000D+01 & 1.50000000D+02 & 1.00000000D-05 \\
5.00000000D-01 & 9.00000000D-01 & 1.00000000D+00 & 1.00000000D-05 \\
0.0 & 9.10000000D-01 & 5.00000000D+00 & 1.00000000D-05 \\
6.00000000D+01 & 1.20000000D+02 & 1.50000000D+02 & 1.00000000D-05 \\
5.00000000D-01 & 6.00000000D-01 & 1.00000000D+00 & 1.00000000D-05 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
9.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 \\
\hline
\end{tabular}

\textbf{LENGTH} = 3.00000000D+04 \\
\textbf{NF} = 30 \textbf{FREGIN} = 0.50000000D+00 \textbf{FEND} = 0.40000000D+02 \\
\textbf{W} = 0.90 \\
\textbf{NV} = 12 \textbf{NPR} = 0 \textbf{NDR} = 1 \\
\textbf{ETA1} = 5.00000000D-13 \textbf{PHI} = 1.00000000D-04 \\
\textbf{NONACT} = 6 \textbf{NVACT} = 6 \\
\textbf{THE NON ACTIVE PARAMETERS NA(I)} \\
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APPENDIX G

INPUT/OUTPUT EXAMPLE FOR GUST SENSITIVITY PROGRAM

The source listing of the program is identical to that of the gust optimization program. The operating instructions given in Appendix B indicate which cards need be deleted or replaced together with the required changes in the data.

The example chosen relates to the same DAST configuration (M=0.9) chosen for the gust optimization example. All the data required (except for the aerodynamic coefficients which are identical to the ones used in the previous examples) appears in the output. The control law used is based on the L.D.T.T.F. and it employs only three control variables. The sensitivity of these 3 variables is tested herein. Note that the array NA(I) involves 9 control variables.

The variation of URMS(I) ($\delta_i,_{\text{rms}}$) and DRRMS(I) ($\delta_i,_{\text{rms}}$) with the control variables is printed in the output and is supplemented by plots illustrating this variation.

It is important to note the following points:

1) Reference to X(I) in the plotted output implies reference to the active X(I) array.

2) In studying the sensitivity of the response to the various control parameters, one should remember the constraints imposed on the control variables during optimization. This is important since a control variable lying on a constraint will not necessarily exhibit a minimum type variation during the sensitivity studies.

Note that all the control deflections are given in degrees per unit gust velocity. The plotted output shows labels which appear to be
displaced. These displacements reflect transient difficulties encountered using a new plotter and they do not originate from the programs used.
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<td>1.0000000D-00</td>
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LENGTH = 3.0000000D+04
NP = 30 PREGION = 0.5000000D+00 PRED = 0.4000000D+02

M = 0.50

AV = 12 AFF = 0 ACE = 1
ETA1 = 5.0000000D-03 PHI = 1.0000000D-04
NCACI = 9 AVACT = 3

THE NUM ACTIVE PARAMETERS NA(1)

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BL = 1.0000000D+00 WT = 1.0000000D+00

INITIAL (INPUT) VECTOR DRY(1)

| 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 |
| 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 |
| 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 | 1.0000000D-04 |

FMH = 5.4000000D+00 ETA = 1.0000000D-09

ITMAX = 8 IM= 0

V=0.X(1) CAST M=0.Y0UP运河PRESS= 5.0

C=0.3000000E+01 0.3500000E+01 C=0.4000000E+01 0.4500000E+01
C=0.5000000E+01 C=5000000E+01 C=6000000E+01

DRMS(1) FSD

0.610795E-01 | 0.649550E-01 | 0.691183E-01 | 0.93278JE-01 |
0.574055E-01 | 0.614696E+00 | 0.105451E+00 |

VAR.X(1) CAST M=0.Y0UP运河PRESS= 5.0

0.3000000E+01 | 0.3500000E+01 | 0.4000000E+01 | 0.4500000E+01 |
0.5000000E+01 | 0.5500000E+01 | 0.6000000E+01 |

DRMS(1) FSD

0.613637E+01 | 0.642680E+01 | 0.668460E+01 | 0.6368B6E+01 |
0.632082E+01 | 0.631326E+01 | 0.631236E+01 |

VAR.X(1) CAST M=0.Y0UP运河PRESS= 5.0

0.5000000E+02 | 0.5500000E+02 | 0.6000000E+02 | 0.650000E+02 |
0.7000000E+02 |

DRMS(1) FSD

0.102940E+00 | 0.578759E-01 | 0.974055E-01 | 0.556503E-01 |
C=0.445555E-01
VAR.X(2), CAST M=0.90CYN
PRES= 5.0
0.500000E+02 0.300000E+02 0.600000E+02 0.660000E+32
0.790000E+02
DRAMS(1) PSD
0.613022E+01 0.622221E+01 0.632062E+01 0.642662E+01
0.664043E+01
VAR.X(3), CAST M=0.90CYN
PRES= 5.0
0.750000E+00 0.800000E+00 0.850000E+00 0.900000E+00
0.950000E+01
DRAMS(1) PSD
0.964674E+00 0.101634E+00 0.994928E+01 0.974058E+01
0.953333E-01
0.953333E-01
0.953333E-01

VAR.X(3), CAST M=0.90CYN
PRES= 5.0
0.750000E+00 0.800000E+00 0.850000E+00 0.900000E+00
0.950000E+01
DRAMS(1) PSD
0.634040E+01 0.643055E+01 0.632348E+01 0.632062E+01
0.632154E+01
0.632154E+01
0.632154E+01

IERR= 0  ITERATIONS PERFORMED= 6

CUTIUM VECILH X(1)

2.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
1.200000E+02 0.000000E+00 0.000000E+00 0.000000E+00
0.0 0.0 0.0 0.0
AR.X (1), DIST M-0.90 DYN, PRESS=5.0