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RECOGNITION OF BINARY X-RAY SYSTEMS UTILIZING THE DOPPLER EFFECT

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Studied herein is the possibility of the recognition of the duality of a single class of X-ray systems utilizing the Doppler effect. The procedure is based on the presence of a period which coincides with the orbital period at the intensity of the radiation in a fixed energy interval of the X-ray component of a binary system.
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Studied herein is the possibility of the recognition of the duality of a single class of X-ray systems utilizing the Doppler effect. The procedure is based on the presence of a period which coincides with the orbital period at the intensity of the radiation in a fixed energy interval of the X-ray component of a binary system.

1. We will use \( \psi(t, E) \) to designate the density of the flow of radiation from the X-ray source, that is, the average number of photons striking a site of unit area, perpendicular to the flow, per unit of time in the vicinity of the current time \( t \) in a unit energy interval in the vicinity of the energy \( E \). One of the following forms of the functional dependence of \( \psi \) on \( E \) is usually adopted [1]:

\[
\begin{align*}
\psi(t, E) &= A(t)E^{-\gamma(t)}, \\
\psi(t, E) &= B(t)E^2e^{-\kappa(t)E}, \\
\psi(t, E) &= B(t)E^{-1}e^{-\kappa(t)E}.
\end{align*}
\]

Let \( \phi(t, E) \) and \( L(t) \), respectively, be the energy and time spectra, that is, the average number of photons recorded by the radiation detector:

- in a unit energy interval per unit of time for the energy spectrum,
- in a unit of time for the time spectrum.

The energy spectrum is associated with the density of the flow by the relationship

*Numbers in the margin indicate pagination in the foreign text.*
where $\epsilon$ is the effectiveness of registration, and $S$ is the area of the detector. We will assume that the detector does not rotate in absolute space and

$$\epsilon = \begin{cases} 
\epsilon_0 = \text{const} \neq 0 & \text{with } E_H < E < E_K \\
0 & \text{with } E < E_H \text{ or } E > E_K.
\end{cases} \quad (4)$$

where $E_H$ and $E_K$ are some energy values which are characteristic for the given detector. It follows from (4) that $\phi(t,E)$ differs from zero only with $E_H < E < E_K$.

It is clear that

$$L(t) = \int \phi(t,E) dE = \int_{E_H}^{E_K} \phi(t,E) dE. \quad (5)$$

We will use $n = n(t_i, t_i + \Delta t, E_i, E_i')$ to designate the calculation of the pulses with energies from $E_i$ to $E_i'$ in the detector within the time interval of a magnitude $\Delta t$ from $t_i$ to $t_i + \Delta t$, where $t_i = t_0 + (i-1)\Delta t$, and $t_0$ is the beginning of the observation process. Then, the process of registration of the radiation may be described in the following manner: the sequence of measurements $\{h_i\}$ is generated by the Poisson process with variable intensity

$$\lambda(t) = \int_{E_i'}^{E_i} \int_{E_i'}^{E_i} G(E',E) \phi(t,E) dE dE'. \quad (6)$$

Here, $G(E',E)$ is the function of response of the detector, that is, the conditional density of distribution of the pulse energy, occurring during registration of a photon of a certain energy by the detector.
2. If the X-ray source is part of a binary system, then its flow density $\psi(t,E)$, and, consequently, its energy spectrum $\phi(t,E)$, changes with time for two reasons:

1) the orbital motion of the source, which evokes the Doppler effect,
2) the variability of the radiation process itself.

We will first examine the form of the energy spectrum $\psi_g(t,E)$, the variability of which is associated with the Doppler effect.

Let $\phi_o(E)$ be the "unperturbed" spectrum, that is, the spectrum which would have a source if it were not part of a binary system. It is not difficult to show that

$$\psi_g(t,E) = \left(1 + \frac{v(t)}{c}\right) \phi_o\left(\left(1 + \frac{v(t)}{c}\right)E\right). \quad (7)$$

Here, $v(t)$ is the projection onto the line of sight of the velocity of the X-ray component, relative to the center of mass of the binary system, and $c$ is the magnitude of the speed of light.

We would note that

$$\int \psi_g(t,E) dE = \int \left(1 + \frac{v(t)}{c}\right) \phi_o\left(\left(1 + \frac{v(t)}{c}\right)E\right) dE =$$

$$= \int \phi_o(U) dU \quad (8)$$

and does not depend on $t$.

We have indicated three forms of the functional dependence of the density of the radiation flow on the energy. Three forms of the spectra $\psi_g(t,E)$ correspond to these forms of the dependence.
If the density of the radiation flow is described by expression (1), then

\[ \Phi_g(t, E) = \left(1 + \frac{v(t)}{c}\right) A \varepsilon S \left((1 + \frac{v(t)}{c})E\right)^{-\gamma} = \]

\[ = A \varepsilon S \left(1 + \frac{v(t)}{c}\right)^{1-\gamma} E^{-1} \]

and with \( v(t) \ll c \)

\[ \Phi_g(t, E) = A \varepsilon S \left(1 + \frac{(1-\gamma)}{c} v(t)\right) E^{-\gamma}. \] (3)

If the density of the flow is described by (2), then

\[ \Phi_g(t, E) = \left(1 + \frac{v(t)}{c}\right) B \varepsilon S \left((1 + \frac{v(t)}{c})E\right)^2 e^{\kappa \left(1 + \frac{v(t)}{c}\right) E} = \]

\[ = B \varepsilon S \left(1 + \frac{v(t)}{c}\right)^2 E^2 e^{\kappa \left(1 + \frac{v(t)}{c}\right) E} \]

and with \( v(t) \ll c \)

\[ \Phi_g(t, E) = B \varepsilon S \left(1 + \frac{v(t)}{c}\right) \left(3 + \kappa E\right) E^2 e^{\gamma E}. \] (4)

If the density of the flow is described by (3), then

\[ \Phi_g(t, E) = \left(1 + \frac{v(t)}{c}\right) B \varepsilon S \left((1 + \frac{v(t)}{c})E\right)^{-1} e^{\kappa \left(1 + \frac{v(t)}{c}\right) E} = \]

\[ = B \varepsilon S E^{-1} e^{\kappa \left(1 + \frac{v(t)}{c}\right) E} \]
and with $v(t) \ll c$

$$\phi_p(t, E) = B e S \left( E^{-1} + \frac{v(t)}{c} \right) e^{KE}.$$ \hspace{1cm} (II)

We will now switch to the energy spectrum which describes the variability of the radiation process. We will designate this spectrum using $\phi_c(t, E)$. According to [2-5], there exists a class of X-ray sources in which the variability of the radiation process has a random nature. It is precisely this class with which we will be occupied subsequently. Judging from those same studies [2-5], the time spectra of the sources of this class are described well by a model of the random pulse process (shot noise), that is

$$L(t) = \sum_i l(t-t_i),$$ \hspace{1cm} (I2)

where $l(t)$ are pulses of a given form, and the times $t_i$ are random magnitudes, distributed according to Poisson's law with some parameter $\mu$.

If the time spectrum is described by a model of the random pulse process, then, keeping relationship (5) in mind, it seems natural to utilize a similar model for the description of the energy spectrum, as well. We will assume that

$$\phi_c(t, E) = \sum_i \phi(t-t_i, E),$$ \hspace{1cm} (I3)

where $\phi(t-t_i, E)$, similar to (12), are pulses of a given form, and they have the very same meaning as in (12). Following study [5] in the assumption of the nondependence of the times of appearance of the pulse $\phi(\cdot)$ on the energy, we will assume that $\phi(\cdot)$ can be
represented in the form

$$\varphi(t-t_i, E) = \varphi_1(t-t_i) \varphi_2(E).$$ \hspace{1cm} (14)

We will now construct a general model of the energy spectrum $$\varphi(t, E)$$. We will assume that it consists of two additive components, associated with the above-indicated reasons for the variability. Thus, in the case when the source is part of a binary system,

$$\varphi(t, E) = \varphi_g(t, E) + \varphi_c(t, E) - \delta \varphi_c(t, E).$$ \hspace{1cm} (15)

and in the opposite case

$$\varphi(t, E) = \varphi_0(E) + \varphi_c(t, E) - \delta \varphi_c(t, E).$$ \hspace{1cm} (16)

where $$\delta$$ is the sign of the mathematical expectancy.

For the evaluations which we will carry out subsequently, it is desirable to know the connection between the characteristics of the time and energy spectra. In this connection, we will give our model in detail. We would note that, in virtue of (5), (12), (14) and (15), and (16),

$$\sum_i \ell(t-t_i) = \sum_i \varphi_1(t-t_i) \int \varphi_2(E) dE$$

and it is natural to assume that $$\ell(t) = \phi_1(t)$$, that is,

$$\ell(t) = \int_{E_n}^{E} \varphi(t-t_i, E) dE.$$
We will assume that the distribution of the pulse \( I(t) \), according to the energies, takes place in accordance with the energy spectrum \( \phi_o(E) \), that is,

\[
\varphi_2(E) = \frac{\phi_0(E)}{\int_{E_u} E_{\text{th}} \phi_0(E) dE}
\]

or

\[
\varphi(t-t_i,E) = \frac{I(t-t_i)\phi_0(E)}{\int_{E_u} E_{\text{th}} \phi_0(E) dE}
\]  \hspace{1cm} (17)

It is evident that the model we constructed generalizes the model in [2] in a natural manner. In principle, one can complicate the model by taking into account the dependence of the distribution of the pulse \( I(t) \) according to the energies on the true spectrum \( \phi(t,E) \), and not on \( \phi_0(E) \).

3. We will examine the probability properties of the introduced magnitudes. We would note that, since relationships (15) and (16) differ only in their determined additive terms, the probability characteristics of the spread for these cases coincide. Thus, if \( \varphi(t,E) \) and \( \varphi(t,U) \) are functions determined by expressions (15) or (16), with various values of the energy \( E \) and \( U \), then, making use of the results in [2], we obtain an expression for the mutual spectral density \( S(\nu,E,U) \) of the random processes \( \varphi(t,E) \), \( \varphi(t,U) \)

\[
S(\nu,E,U) = m|F(\nu)|^2 \frac{\phi_o(E)\phi_o(U)}{\left( \int_{E_u} E_{\text{th}} \phi_0(E) dE \right)^2}
\]  \hspace{1cm} (18)
Here, $F(v)$ is the transform of the Fourier function $f(t)$. The mutual covariation function of the processes $\phi(t,E)$ and $\phi(t,U)$ is stationary, and is determined by the expression

$$K(t,E,U) = \int_{-\infty}^{\infty} S(v,E,U) e^{j 2 \pi v t} dv = \frac{\Re\{F(v)\phi_0(U)\}}{\int_{E_0}^{E} \phi_0(E) dE}$$  \hspace{1cm} (19)$$

The average values of the magnitudes determined from relationships (15) and (16) will be distinguished using the indexes $g$ and $o$, respectively. Thus, the mathematical expectancy of the random process $\phi(t,E)$ is equal to $\phi_{g,o}(t,E)$.

From the obtained results and expression (6), we conclude that the intensity $\lambda(t)$ is a random process with a mathematical expectancy

$$M_{g,o}(t) = \int_{E'}^{E_1} \int_{E''}^{E''_0} G(E',E) \phi_{g,o}(t,E) dE dE'$$  \hspace{1cm} (20)$$

and a stationary covariation function

$$B(t) = \frac{\int_{E_0}^{E_1} |F(v)|^2 e^{j 2 \pi v t} dv}{\left( \int_{E_0}^{E_1} \phi_0(E) dE \right)^2} \left( \int_{E_0}^{E_1} \int_{E_0}^{E_1} G(E',E) \phi_0(E) dE' dE \right)^2.$$  \hspace{1cm} (21)$$

We will now examine the probability properties of the sequence of measurements $\{n_i\}$. It is evident that

$$\mathbb{E} n_i = \int_{t_i}^{t_i+\Delta t} M_{g,o}(t) dt.$$  \hspace{1cm} (22)$$

We will obtain expressions for the covariation of the magnitudes
\( n_i \) and \( n_{i+j} \). Initially, let \( j > 0 \), that is, the time intervals in the course of which the calculation is recorded do not intersect. It is not difficult to see that, for this case,

\[
\text{cov}(n_i, n_{i+j}) = \int_{t_i}^{t_i+\Delta t} \int_{t_{i+j}}^{t_{i+j}+\Delta t} B(t_2-t_1) dt_2 dt_1.
\]

Here, \( \text{cov} \) is the sign of covariation.

We will make a substitution in the integral of the variables \( \tau_1 = t_1; \tau = t_2 - t_1 \). Then,

\[
\int_{t_i}^{t_i+\Delta t} \int_{t_{i+j}}^{t_{i+j}+\Delta t} B(t_2-t_1) dt_2 dt_1 = \Delta t \int_{(j-\Delta t)}^{(j+\Delta t)} B(\tau) d\tau + \int_{(j-\Delta t)}^{(j+\Delta t)} H(j, \tau) d\tau,
\]

where

\[
H(j, \tau) = \begin{cases} 
(\tau - j \Delta t) & \text{with } \tau < j \Delta t \\
(j \Delta t - \tau) & \text{with } \tau > j \Delta t.
\end{cases}
\]

It is evident from expression (23) that the covariation is stationary, that is, it depends only on the magnitude of \( j \). We will use \( c_j \) to designate \( \text{cov}(n_i, n_{i+j}) \).

Now, let \( j = 0 \). Utilizing the results given in [6], we obtain the following expression for the dispersion of the magnitude \( n_k \):

\[
\mathbb{D} n_k = c_0 \omega = \text{cov}(n_k, n_k) = \int_{t_k}^{t_k+\Delta t} M_g d(t) dt + \int_{0}^{\Delta t} (\Delta t - \tau) B(\tau) d\tau. \tag{24}
\]
The spectral density of the sequence of measurements \( (n_i) \)
is expressed via the covariation and dispersion according to the formula

\[
\Gamma_{g,0}(\nu) = \sum_{j=-\infty}^{\infty} c_j e^{-2\pi j \nu t},
\]

where \( c_{-j} = c_j \). Specifically,

\[
\Gamma_{g,0}^2(\nu) = \sum_{j=-\infty}^{\infty} c_j^2 = \int M_{g,0}(t) dt + \Delta t \int B(t) dt,
\]

with

\[
\Gamma(\nu) \leq \Gamma(0).
\]

4. We will now switch to the verification of the duality of the source. In the current study, for simplicity, we will examine only circular orbits. In this case, \( v(t) \), which is part of relationship (7), is a harmonic function, and \( M_{g}(t) \) for a binary source, as follows from (20) and (9) or (10), or (11), may be represented in the form

\[
a_0 + a_1 \cos(\omega_0 t + \phi),
\]

where \( \omega_0 \) is the circular frequency of rotation along the orbit, and \( a_0, a_1, \phi \) are some constants.

We will examine the sample spectrum of the sequence of measurements, that is [7],
Here, \( N \) is the total number of measurements, taking part in the processing, \( \bar{n} = \frac{1}{N} \sum_{j} n_j \), and \( W(K) \) is the correlation window. In our analysis, we will limit ourselves to the simplest case, when \( W(K) = 1 \) with \( K = 1, 2, \ldots, N \), that is, to the study of the unleveled spectrum \( C(v) \) [7].

Let us assume that
\[
\Delta n_j = v_j - \bar{n} ; \bar{n} = \frac{1}{N} \sum_{j} n_j ; \bar{v} = \frac{1}{N} \sum_{j} v_j ,
\]
and
\[
C_1(v) = \frac{\Delta t}{N} \left( \sum_{j} (\bar{v}_j - \bar{v})^2 + 2 \sum_{k} W_k \cos 2\pi v_k \Delta t \right) \times \sum_{j} (\bar{v}_j - \bar{v})(\bar{v}_{j+k} - \bar{v}) ;
\]
\[
C_2(v) = \frac{\Delta t}{N} \left( \sum_{j} (\Delta n_j - \bar{n})^2 + 2 \sum_{k} W_k \cos 2\pi v_k \Delta t \right) \times \sum_{j} (\Delta n_j - \bar{n})(\Delta n_{j+k} - \bar{n}) ;
\]
\[
C_3(v) = \frac{\Delta t}{N} \left( \sum_{j} (\bar{v}_j - \bar{v}) (\Delta n_j - \bar{n}) + 2 \sum_{k} W_k \cos 2\pi v_k \Delta t \right) \times \sum_{j} (\bar{v}_j - \bar{v})(\Delta n_{j+k} - \bar{n}) .
\]

Then
We will occupy ourselves with the study of $C_1(v)$. We will assume that

$$\omega_0 \Delta t \ll 1$$

(30)

and, for convenience, we will examine the continuous analog $C_1(v)$, that is,

$$\frac{1}{T} \int_0^T \cos 2\pi \nu u \int_0^{T-\nu u} (\hat{e}_n(t) - \hat{e}_n)(\hat{e}_n(t+u) - \hat{e}_n) dt du.$$  (31)

Since relationship (22) is fulfilled, then

$$\bar{e}_{n_0}(t) = \int_{\text{t} \Delta t} M_{n_0}(\nu t) dt,$$  (32)

For evaluation, we will limit ourselves to the main loop of $C_1(v)$. Then, making use of relationships (22) and (30), and having carried out the operations determined by expressions (31), (32), and (33), we obtain, in the case when the source is binary:

$$\max C_1(v) = C_1 \left( \frac{\omega_0}{2\pi} \right) = 2a_1^2 \Delta t^2.$$  (34)

We will switch to the study of $C_2(v)$. We will evaluate the maximally possible magnitude of $C_2(v)$. Since $C_2(v)$ is a random function of the argument $v$, then we will make use of the traditional
probability evaluation $\mathcal{E} C_2(\nu) + 3 \sqrt{\mathcal{D}} C_2(\nu)$ for the maximally possible values of this function. Utilizing the results of [7], we obtain

$$\mathcal{E} C_2(\nu) = \Gamma(\nu); \quad \mathcal{D} C_2(\nu) = 2 \Gamma^2(\nu)$$

and

$$\mathcal{E} C_2(\nu) + 3 \sqrt{\mathcal{D}} C_2(\nu) = (1 + 3\sqrt{2}) \Gamma(\nu). \quad (35)$$

From (35) and (27), the following evaluation follows:

$$C_2(\nu) \leq (1 + 3\sqrt{2}) \Gamma(0). \quad (36)$$

On the other hand, according to the properties of the sample spectrum, $C_2(\nu) > 0$. Thus,

$$0 \leq C_2(\nu) \leq (1 + 3\sqrt{2}) \Gamma(0)$$

and, in that case when the X-ray source is part of a binary system with sufficiently large values of $T$ (greater than some $T^*$), the maximum value of the function $C(\nu)$ exceeds the threshold value, determined by the expression $(1 + 3\sqrt{2}) \Gamma_0(0)$. If the source is single, then the values of the sample spectrum are no greater than $(1 + 3\sqrt{2}) \Gamma_0(0)$.

5. We will evaluate the magnitude of $T^*$, having adopted the Gaussian form of the pulse of the time spectrum. Thus, let

$$l(t) = a e^{-\frac{t}{b^2}},$$

where $a$ and $b$ are some constants.
The transform of the Fourier function \( t(t) \) is then determined by the expression

\[
\tilde{f}(\nu) = a b \sqrt{\pi} e^{-\pi^2 a^2 \nu^2}.
\]

Let the function of response of the detector be ideal, that is,

\[
G(E', E) = \delta(E' - E),
\]

where \( \delta(\cdot) \) is Dirac's delta-function. Then, the covariation function of intensity \( \lambda(t) \), according to (21), is determined by the expression

\[
B(\tau) = \frac{\left( \int_{E_k}^{E_{k+1}} \alpha_0(E) dE \right)^2}{\left( \int_{E_k}^{E_{k+1}} \alpha_0(E) dE \right)^2} \sqrt{\frac{\pi}{2}} a^2 b e^{-\frac{\sigma^2}{2}}.
\]

Hence, utilizing (26), we obtain

\[
\Gamma_{\nu}(\tau) = \int_{t_k}^{t_{k+1}} M_{\nu}(\tau) d\tau + \Delta t \int_{E_k}^{E_{k+1}} \alpha^2 dE \frac{\left( \int_{E_k}^{E_{k+1}} \alpha_0(E) dE \right)^2}{\left( \int_{E_k}^{E_{k+1}} \alpha_0(E) dE \right)^2}.
\]

We would note that, according to [2],

\[
\mathcal{E} \mathcal{L}(t) = \int_{-\infty}^{\infty} \mathcal{L}(t) dt = \int_{-\infty}^{\infty} a e^{-\frac{t^2}{2a^2}} dt = \sqrt{2\pi} a b \sqrt{\pi}.
\]

On the other hand,

\[
\mathcal{E} \mathcal{L}(t) = \mathcal{E} \int_{E_k}^{E_{k+1}} \mathcal{L}(t,E) dE = \int_{E_k}^{E_{k+1}} \mathcal{L}(t,E) dE.
\]

Therefore,
\[
\Gamma_g(0) = \int_{t_k}^{t_k + \Delta t} M_g(\tau) d\tau + \frac{\Delta t}{t_k} \left( \int_{E_1}^{E_f} \mathcal{M}_0(E) dE \right).
\]

For definiteness, let the dependence of \( \psi \) on \( E \) be described by relationship (1); then

\[
a_1 = \mathcal{A} \varepsilon S \frac{V_0}{c} \left( E_2^{1-\gamma} - E_1^{1-\gamma} \right).
\]

Here, \( V_0 \) is the maximum value of the projection of the velocity of the X-ray component, relative to the center of mass of the binary system, on the line of sight.

Hence,

\[
\max_{\psi} C_4(\psi) = 2a_1^2 \Delta t = \frac{2 \mathcal{A} \varepsilon^2 S V_0^2}{c^2} \left( E_2^{1-\gamma} - E_1^{1-\gamma} \right) \Delta t
\]

and \( T^* \), according to (34) and (36), is determined from the equality \( (14) \)

\[
(1 + 3V_2^2) \left( 1 + \frac{V_0}{c} \right)^{1-\gamma} \frac{\varepsilon S A(E_2^{1-\gamma} - E_1^{1-\gamma})}{1-\gamma} + \varepsilon^2 S^2 A^2 \left( E_2^{1-\gamma} - E_1^{1-\gamma} \right)
\]

\[
= 2 \frac{V_0^2}{c^2} \mathcal{A} \varepsilon^2 S^2 (E_2^{1-\gamma} - E_1^{1-\gamma})
\]

that is,

\[
T^* = \frac{(1 + 3V_2^2) \left( 1 + \frac{V_0}{c} \right)^{1-\gamma} \frac{1-\gamma}{\varepsilon S A(E_2^{1-\gamma} - E_1^{1-\gamma})} + \frac{1}{M}}{(1-\gamma)^2 \frac{V_0^2}{c^2}}
\]
We will utilize the obtained formula for the evaluation of $T^*$ during the registration of the radiation by the detector with the parameters of the SNEG-2MP instrument [8]:

$$\varepsilon S = 12 \text{cm}^2 \cdot \text{me}^2^' \cdot E_1^' = 20 \text{keV}; \ E_2^' = 280 \text{keV}.$$ 

We will take the parameters of the Sud X-1 as the characteristics of the X-ray source [5,9]:

$$\gamma = 1.93; \ \nu_0 = 581 \text{keV} \cdot \text{sec}^ {-1}; \ \nu = 20 \frac{\text{sec}}{\text{sec}}; \ \lambda = 3.89 \text{photons/cm}^2 \text{keV}.$$ 

Through the utilization of these data, the value of $T^*$ is obtained as equal to 4.4 days, which should evidently be recognized as acceptable.
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