BUCKLING AND VIBRATION OF PERIODIC LATTICE STRUCTURES

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INTRODUCTION

Proposed concepts for large space structures are typified in the two photos of figure 1. Lattice booms and platforms composed of flexible members or large diameter rings which may be stiffened by cables in order to support membrane-like antennas or reflector surfaces are the main components of these large space structures. Analysis of these structures with a complete finite element model may be prohibitively expensive or impractical. However, the nature of these structures (repetitive geometry with few different members) makes possible relatively simple solutions for buckling and vibration of a certain class of these structures. This theory along with typical results will be discussed in this paper.

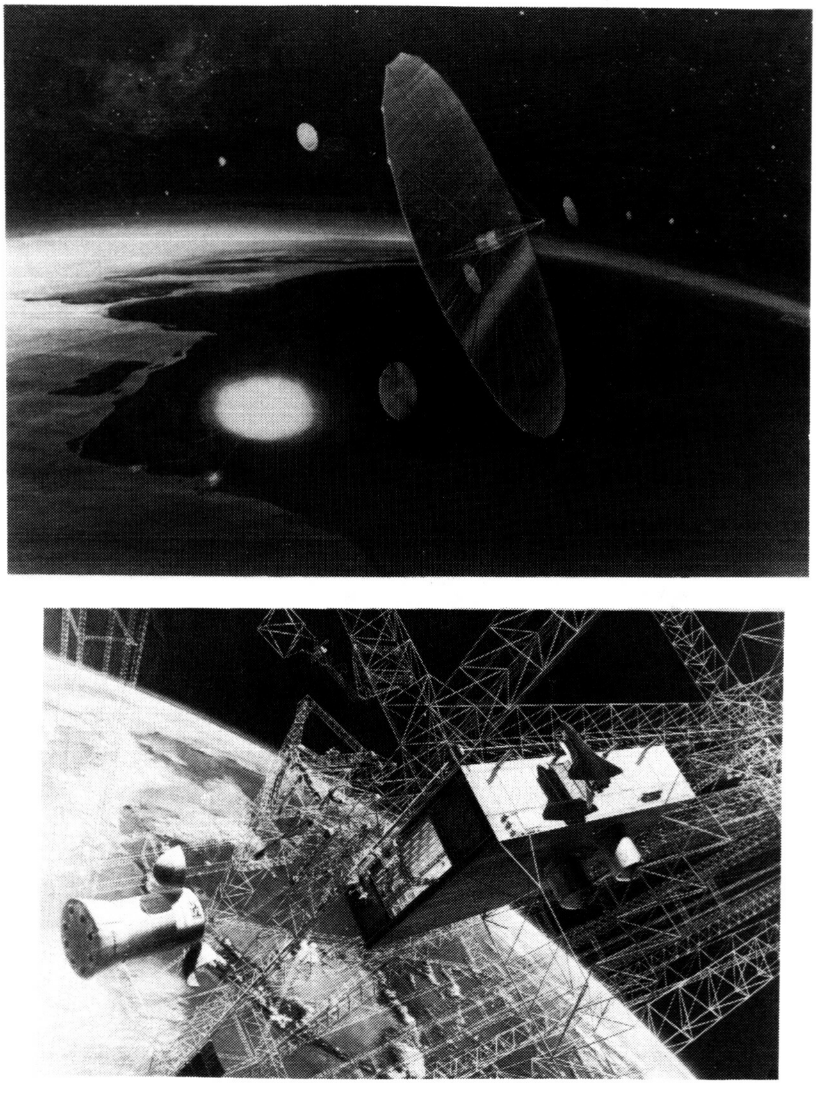

Figure 1

PERIODIC LATTICE CONFIGURATIONS ANALYZED

Exact buckling and vibration solutions have been obtained for the configurations shown in figure 2. The theory for buckling was developed in reference 1 and a simple extension yields vibration results as well. The column configurations are loaded with uniform axial load and are simply supported at the ends. If the ring is cable stiffened, the attachment point at the central mast must be assumed rigid. The flexibility of the mast may be included at the expense of increasing the problem size.

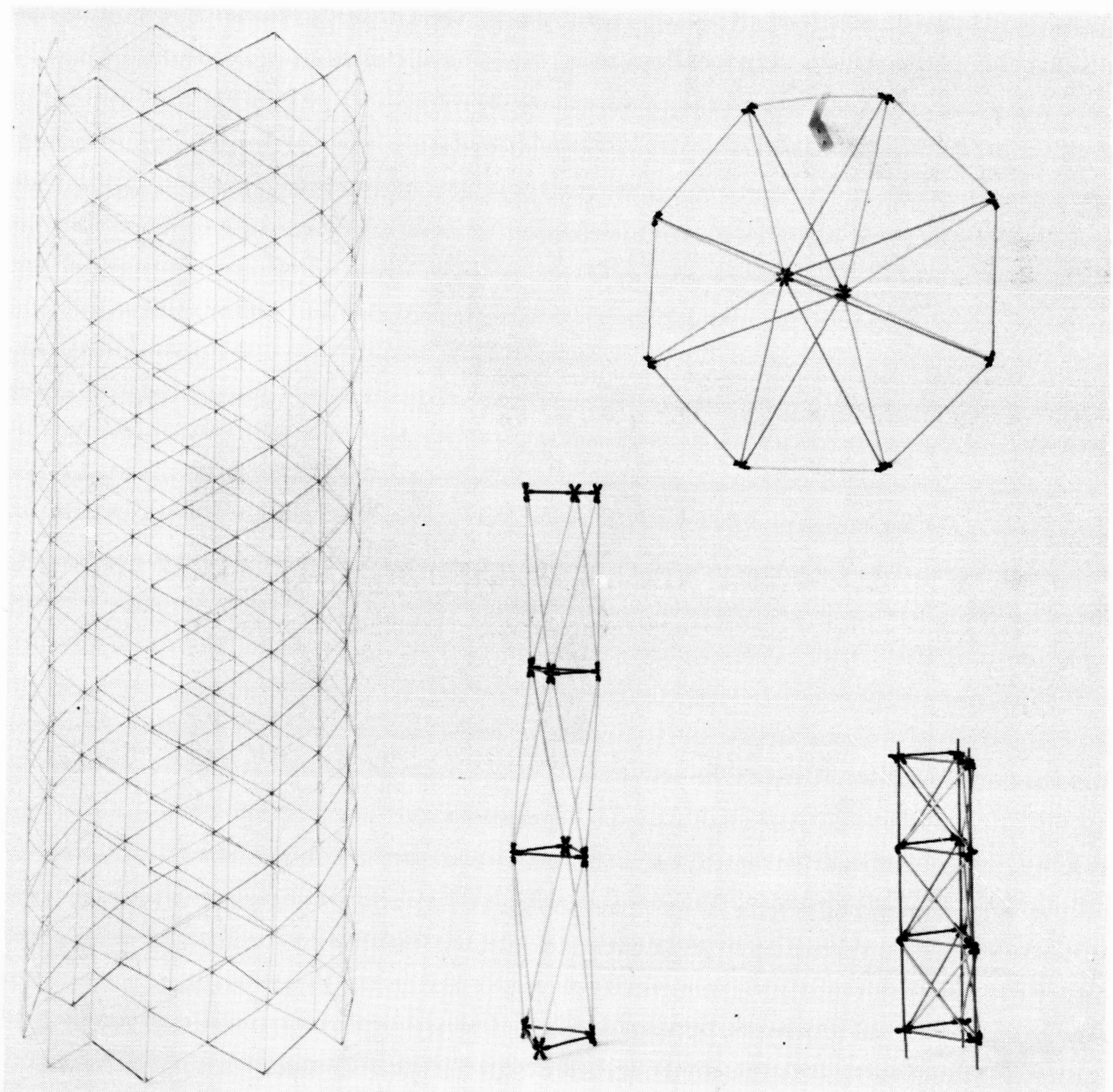

Figure 2

BASIS FOR THEORY

The theory in its simplest form is applicable to any configuration where the relative geometric relationship between nodes is identical for all nodes. This relation is true for all the configurations of figure 2. (The nodes at the ends of the mast of the ring configuration are assumed to have zero displacement, so they do not enter in the analysis.) Each member is represented by a stiffness matrix derived from the exact solution of the beam column equation. This transendental matrix gives the current member stiffness at any end load or frequency. It is not necessary to have intermediate nodes to insure accuracy. Using conventional finite element techniques, equilibrium equations can be written involving displacements and rotations of a typical node and its neighbors. The assumptions of a simple trigonometric mode shape is found to satisfy these equations exactly; thus the entire problem is governed by just one 6 x 6 matrix equation involving the amplitude of the displacement and rotation mode shapes. The boundary conditions implied by this solution are simple supported ends for the column type configurations.

- EACH NODE HAVING IDENTICAL GEOMETRICAL RELATIONSHIP WITH ITS NEIGHBORS

- STIFFNESS OF EACH MEMBER REPRESENTED BY "EXACT" FINITE ELEMENT MODEL THAT ACCOUNTS FOR FREQUENCY AND BEAM COLUMN EFFECT

- PERIODIC MODE SHAPE

- EIGENVALUES OF 6 x 6 DETERMINANT FOR BUCKLING OR VIBRATION

Figure 3

BUCKLING OF LATTICE ISOGRID CYLINDER

A simple approach to analysis of lattice structures containing a large number of members is to develop equivalent beam, plate, or shell stiffness so that they may be analyzed with a continuum theory. The present discrete lattice theory can be used to evaluate the accuracy of such an approach. In the example given in figure 4, the buckling load of an isogrid cylinder similar to that shown in figure 2 is given in terms of the slenderness ratio of an individual member. The load is normalized by the load obtained from continuum theory; thus, a value of 1 means agreement between the two approaches. Two orientations of the isogrid are shown, one with one set of members aligned axially and the other orientation has one set aligned circumferentially. For each orientation two curves are given, one for all members straight, the other applicable to configurations having helical members curved to lie in the surface of the circular cylinder containing the vertices. The results of the figure show the following: (1) as member slenderness ratio increases, the discretness effect compared to continuum theory is much more significant; (2) the highest load capability is for straight members having a principal direction in the circumferential direction. This orientation results in lower individual member loads for a given end load and thus greater resistance to buckling; (3) buckling loads are further reduced if helical members are curved. If there is a compressive force in the curved member which occurs for the $\pm 30^{\circ}$, 90° configurations the reduction in buckling load is the greatest.

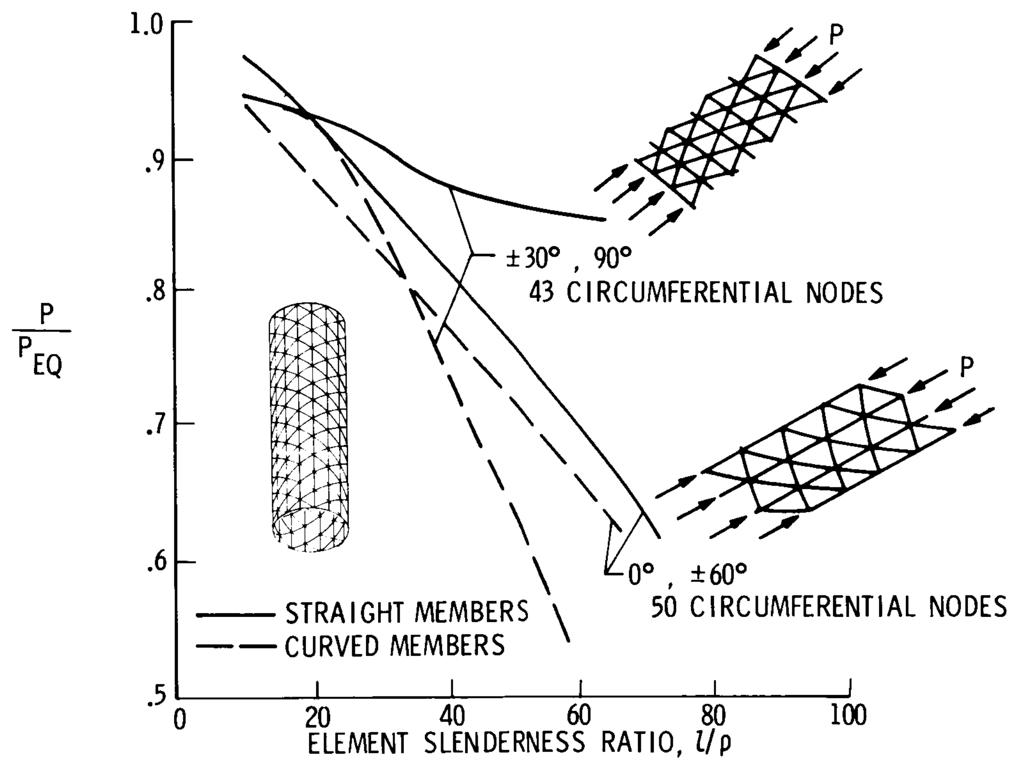

Figure 4

BUCKLING OF TRIELEMENT BEAM

A beam configuration that has been proposed for many applications because of its structural efficiency for carrying axial load consists of three longerons with cross-braced diagonals. The buckling characteristics for a simply supported three element beam loaded in axial compression are shown in figure 5. The buckling load is plotted as a function of buckle length. Three different modes are possible. Buckling of an individual member as an Euler column supported at the joint locations is illustrtated on the left. The load for all buckle lengths has been normalized with respect to the local buckling load corresponding to a buckle length equals bay length ℓ. If the column is sufficiently long, in this case somewhat greater than 17 bays, it will buckle in an overall column mode. The classical Euler load based on continuum stiffnesses is shown dashed and indicates some effect of the shear flexibility of the diagonals. A third mode is possible when the diagonals are small with respect to the longerons. In this case, a moderate length buckle involving movement of the diagonals can result in lower loads than assuming the diagonal force nodes. The discrete lattice buckling theory gives all these results from the same analysis as a function of the wavelength parameters.

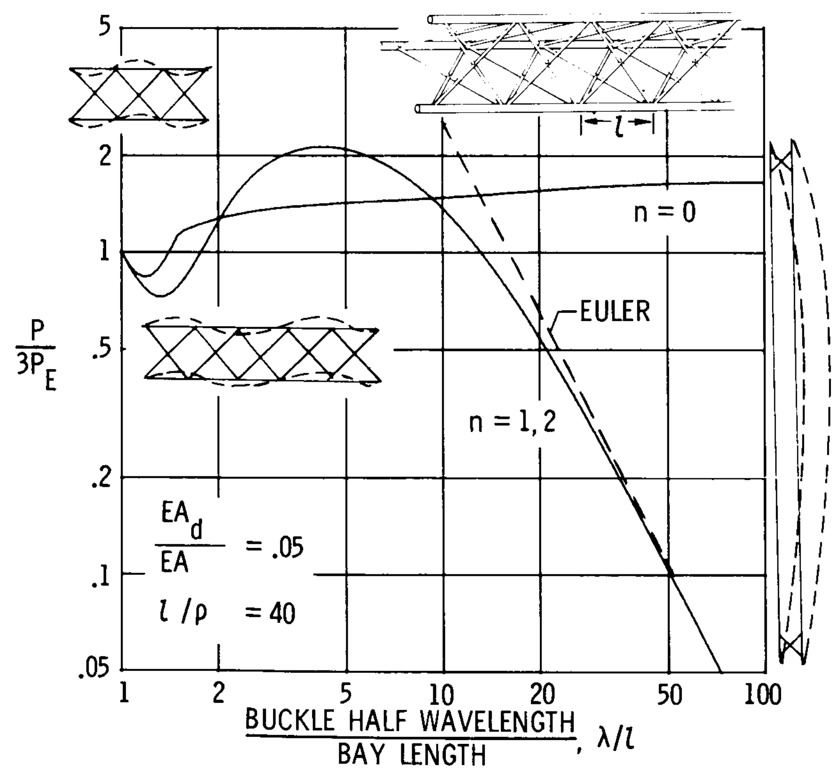

Figure 5

STRUCTURAL EFFICIENCY OF COLUMN CONFIGURATIONS

A study was made of the column mass required to sustain a given load for the three element column discussed in the previous slide. Another column configuration sometimes referred to as a geodesic beam is also shown in figure 2. The configuration is somewhat unusual in that there are no pure axial members. The load carrying members are inclined to the axis of the column and provide the transverse shear stiffness as well as contribute to the buckling stiffness. The battens are loaded in tension when axial compression is applied. A comparison of the mass of the two different configurations is shown in figure 6. The mass parameter is plotted against a compressive loading index. The results were obtained by systematically varying proportions in the buckling analysis and observing the minimum mass required for a given end load. The minimum mass conventional three element beam with diagonal bracing is significantly lighter than the geodesic beam. However, the proportions for minimum weight are much different. The area of the diagonal stiffeners is less than 1% of the longerons whereas the geodesic beam has all areas essentially equal. An optimum configuration with equal areas would be of advantage for small loads that lead to minimum size members. The upper curve is for the trielement beam with all members equal area. In this case, it is much heavier than the geodesic beam.

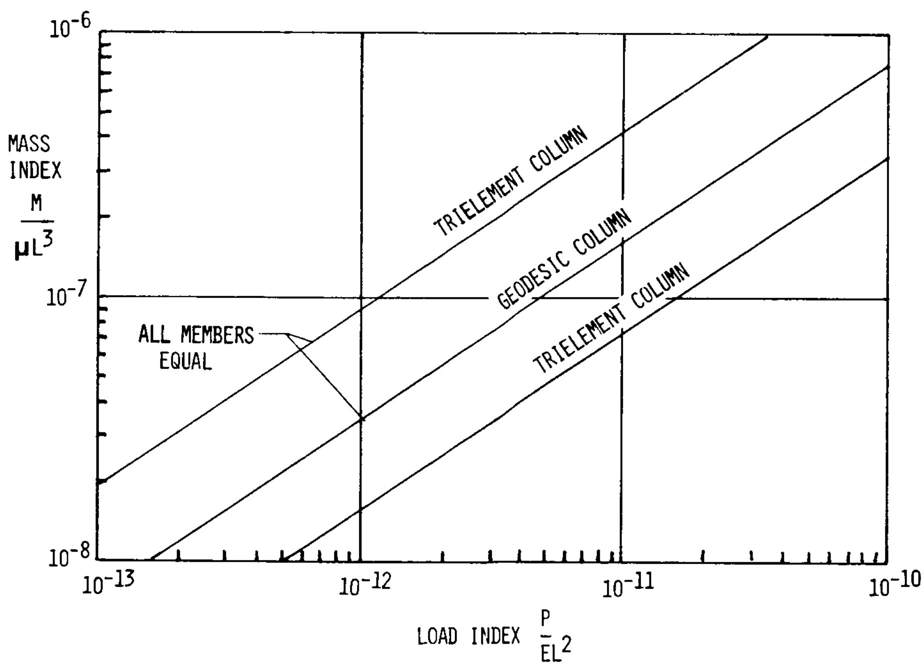

Figure 6

BUCKLING OF CABLE STIFFENED RING

The polygonal ring configuration with central mast used to attach cable stiffening has been proposed to support thin reflector or antenna surfaces The pretension of the cable puts the ring members in compression; additional compression is caused from the inward radial loads due to stretching a membrane like surface in the interior of the ring. For very large structures, the prevention of ring buckling can be a significant design requirement. The buckling problem for this configuration has been solved and typical results shown in figure 7. The radial buckling load Q which is applied at each vertex is plotted against a parameter proportional to the pretension in the cables. Also shown is the internal force in the ring, P. Both P and Q are expressed in terms of classical ring buckling parameters. The dashed line at the lower part of the figure indicates the load capability without cable stiffening. As pretension is increased, the buckling load increases. The single line on the left represents the point at which the cables go slack. The maximum load is reached when a general mode appears even while cables are still in tension. For this case the critical mode has 5 circumferential waves. Note that the n=2 mode, which is the lowest for an unstiffened ring, is higher than the n = 5 mode. Once the general buckling mode occurs, further increases in pretension cause a slow drop in the buckling load Q but the force in the ring remains essentially constant.

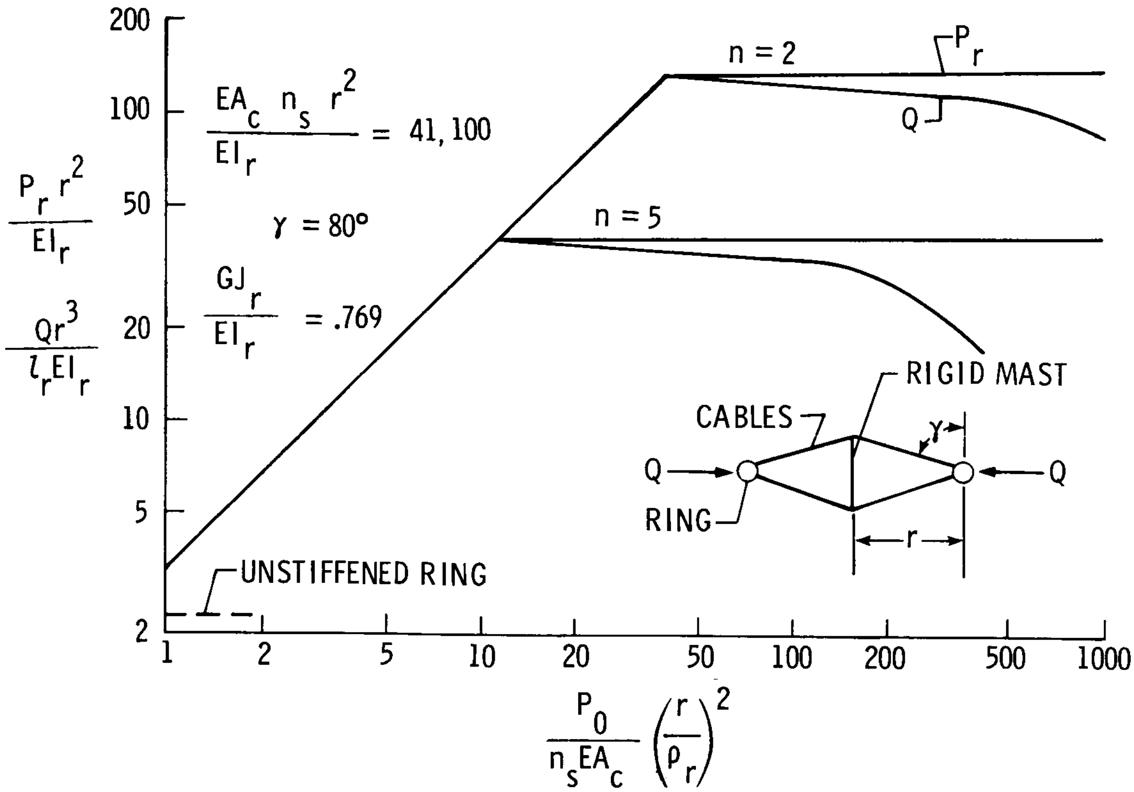

Figure 7

VIBRATION OF CABLE STIFFENED RING

The vibration characteristics of the same ring studied in figure 7 are shown in figure 8. Frequency is plotted against pretension for various values of the radial load Q. No frequency is shown at values of pretension lower than that required to prevent cables from going slack due to the load Q. The frequency then drops slowly until the buckling condition is approached. It drops very rapidly then to zero which represents the buckling solution of the previous figure. One effect that is missing from these calculations is the mass of the membane that is exerting the force Q. A method of including this effect is currently under development.

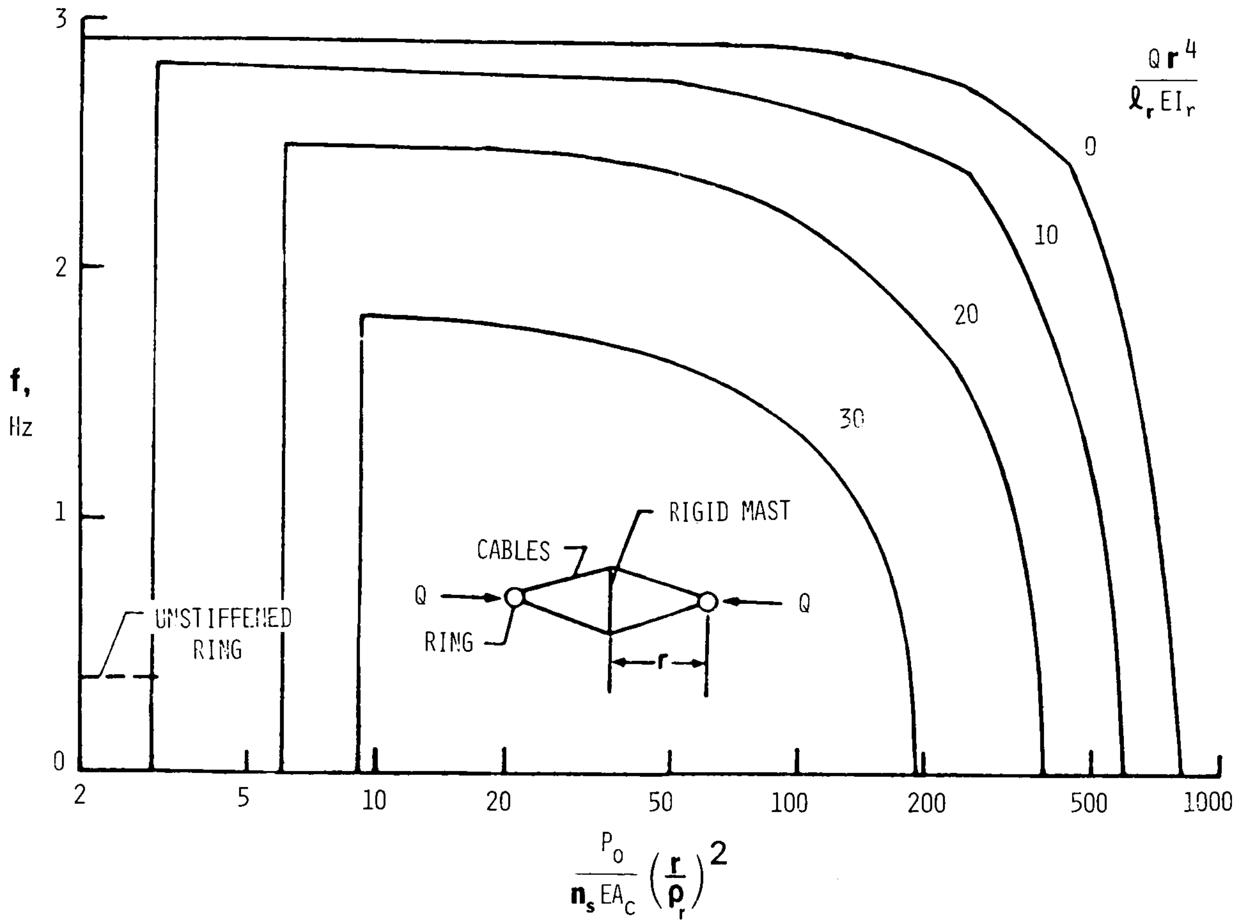

Figure 8

SUMMARY

An analysis for buckling and vibration of repetitive lattice structures has been developed. The results are essentially exact for ring configurations and for column configurations having simply supported ends. A wide variety of configurations are possible within the framework of one analysis. Results were given for typical configurations. In many areas, discrete effects not possible to determine with simple theory were identified. Yet the solution is no more complicated than the eigenvalues of a 6 x 6 matrix.

REFERENCE

1. Anderson, Melvin S.: Buckling of Periodic Lattice Structures. Presented at AIAA/ASME 21st Structures, Structural Dynamics, and Materials Conference Seattle, Washington. May 12-14, 1980. AIAA Paper No. 80-0681. To be published in AIAA Journal.

- ESSENTIALLY EXACT BUCKLING AND VIBRATION ANALYSIS DEVELOPED
- APPLICABLE TO A NUMBER OF LATTICE CONFIGURATIONS WITH REPETITIVE GEOMETRY
- COMPLEX BEHAVIOR ACCURATELY MODELED
- SOLUTION REQUIRES ONLY EIGENVALUES OF 6 X 6 MATRIX

Figure 9