A PRELIMINARY STUDY OF A VERY LARGE SPACE RADIOMETRIC ANTENNA

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A preliminary study to compute the size of a special radiometric reflector antenna is presented. Operating at 1 GHz, this reflector is required to produce 200 simultaneous contiguous beams, each with a 3 dB footprint of 1 km from an assumed satellite height of 650 km. The overall beam efficiency for each beam is required to be more than 90%.
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1. **Summary**

A preliminary study to compute the size of a special radiometric reflector antenna is presented. Operating at 1 GHz, this reflector is required to produce 200 simultaneous contiguous beams, each with a 3 dB footprint of 1 km from an assumed satellite height of 650 km. The overall beam efficiency for each beam is required to be more than 90%.
2. Introduction

The purpose of this report is to present a preliminary study for the design of a large radiometric reflector antenna system. When orbiting at a height of 650 km, this antenna system is required to produce simultaneously 200 contiguous 3 dB circular footprints on the ground, each having a diameter of 1 km. The lowest frequency of operation is 1 GHz. The footprints are required to be as identical to each other as possible. The single most important requirement on the system is that the overall beam efficiency for the copolarized component in each of the 200 beams be better than 90% within the two and a half 3 dB beamwidths. This means that among other things, the cross polarization be minimum too (<25 dB).

When in orbit, the reflector may undergo considerable thermal distortions and its performance may change. A method is therefore needed to predict the performance of even a distorted reflector. Such a technique is discussed in Section 6 of this report.
3. Spherical Reflector Approach

A solution that meets the requirements set in Section 2 is schematically shown in Figure 1. The 200 beams are simultaneously obtained by stacking 200 identical feed antennas along a concentric circular arc in front of a spherical reflector such that each feed is pointing radially towards the spherical reflector surface. Each feed thus creates its own independent footprint. And since each feed antenna essentially sees an identical segment of the spherical reflector, the resulting 200 footprints are also practically identical. Observe that the angular separation between any two consecutive feed antennas (called $\theta$) is the same as the angular separation between the two adjacent footprints. This, for an altitude of 650 km and a footprint size of 1 km, turns out to be $0.088^\circ$ and the 200 feeds stacked along the feed arc thus subtend a total of $16.6^\circ$ angle at the center of the spherical reflector (Figure 2).

Notice that the angular separation between any two consecutive feeds depends only upon the altitude and the footprint size; the physical separation, however, is the product of the angular separation ($\theta = 0.088^\circ$) and the feed arc radius, and therefore, depends upon the radius of the feed arc also. The radius of the feed arc, therefore, should be large enough to provide enough physical room for each of the 200 feed antennas. It is assumed
Figure 1 -- Multibeam Spherical Reflector Concept
Figure 2 -- Geometry of Spherical Reflector Antenna
at this stage that each feed antenna should have, on an
average, a room of at least 88 cm, which leads to a feed
arc radius of 577 m. And since for spherical reflectors,
the feed arc is generally located about halfway between
the reflector and its center of curvature (nearer to the
reflector), the radius of curvature of the spherical
reflector is chosen to be 1175 m. Note that there is no
specific reason to pick 88 cm for feed spacing except that
the estimations of practical feed horn sizes suggest that
a room of about a meter be available for each feed antenna.
And of course, whether a feed horn limited in size to 88 cm
at 1 GHz feeding a reflector with dimensions chosen above
can give a satisfactory secondary far field pattern or not,
remains to be checked.

Let us now consider an individual footprint which is
caused by an individual feed antenna located at the feed
arc. Each feed antenna illuminates a portion of the spherical
reflector and it is the far field of this illuminated reflec-
tor aperture which must have (a) a 3 dB beamwidth of 0.088°,
and (b) a beam efficiency of better than 90% within the two
and a half 3 dB beamwidths. The later requires that the
highest side lobe of the reflected pattern be less than -32
dB with wide angle side lobes below -80 dB. A study of
various aperture distributions [1] indicates that for an
operating frequency of 1 GHz, an aperture diameter of about
300 m (say, 305 m) with a rotationally symmetric cosine
squared field distribution produces both a 3 dB beamwidth of 0.088° and a side lobe at -32 dB, the side lobe fall off being -18 dB/Octave.

Returning to Figure 2, an illuminated aperture with a diameter of 305 m on the reflector corresponds to a cone of 15° half angle emanating from each feed antenna. Therefore, each feed antenna whose nominal diameter has been fixed to 88 cm at 1 GHz has to be able to produce a rotationally symmetric cosine squared far field pattern over ± 15°. The overall diameter of the spherical reflector dish to produce 200 beams then turns out to be 660 m as shown in Figure 2.
4. Feed Considerations

In the previous section, it was assumed that after reflection, each feed pattern gave rise to a rotationally symmetric aperture field which varied as cosine squared in the radial direction. For the reflector dimensions under consideration, the portion of the reflector illuminated by a feed is such a small fraction of the full sphere, that it is practically flat and therefore a rotationally symmetric cosine squared aperture distribution is easily achieved by a feed which too has a rotationally symmetric cosine squared pattern.

A rotationally symmetric cosine squared feed pattern can be generated by any one of the several types of horns. In the present study, however, a circular corrugated horn [2] is considered. The feed pattern of the corrugated horn used for the computations presented in the following sections is shown in Figure 3. Observe that the horn diameter at the mouth is 2 m which is larger than 88 cm, the space designated for each feed at the feed arc. Therefore, the feed horns will have to be staggered around the feed arc so that they are still on an average 88 cm or 0.088° apart. The beam efficiency of the feed pattern within 15°, which corresponds to the edge of the reflector is 98.3%. This number is important because the overall beam efficiency of the antenna system is the product of this efficiency and the beam efficiency of the secondary pattern.
Figure 3 -- Feed Pattern of a Circular Corrugated Horn
5. **Secondary Pattern**

For the reflector geometry shown in Figure 2 and using the feed pattern presented in Section 4, the computed secondary radiation pattern is shown in Figure 4. It has a 3 dB beamwidth of 0.080° and a maximum cross polarization level of less than -200 dB. The beam efficiency of the secondary pattern at two and a half 3 dB beamwidths is 93.4%, the overall beam efficiency, therefore, being better than 91%.

One of the concerns in spherical reflector applications is the resulting spherical aberration. It is of interest to note that in the present case, such a small segment of the sphere is being used as reflector that the maximum spherical aberration near the edge of the illuminated aperture (where the field strength is -26.7 dB, Figure 3) is equivalent to a phase error of only about 18°. Such a small phase error causes a negligible degradation in the antenna gain.
Figure 4 -- Computed Secondary Pattern. Feed Pattern was Used Over $\pm 16^\circ$ with 1° Increment.
6. Thermal Distortion Considerations

The performance of a reflector antenna in space is sometimes not the same as predicted by the initial design because the reflector undergoes severe distortions due to thermal variations. If the distorted shape of the reflector is quite a bit different from the original spherical shape, the reflector performance may change significantly and may even become unsatisfactory. Therefore, it will be desirable to be able to predict the performance of even the distorted reflector. If the distorted reflector surface could be known analytically, then the reflector performance of course could be accurately predicted. It is not generally possible to know an analytic expression for the entire distorted reflector surface at all times. Alternatively, a sampling scheme can be implemented such that the coordinates of many discrete target points located along a rectangular grid on the reflector surface are known. Then, a smooth tight cubic surface can be fitted through the four corners of each of the rectangular grid patches such that the whole composite reflector surface is continuous and has continuous partial derivatives. Using this piecewise analytic expression for the reflector surface, the reflector properties can be computed. Needless to say, the target points on the reflector surface must be dense enough to sample the distortions and such that the surface between the measured points could be assumed tightly stretched.
To demonstrate that the far field radiation pattern can indeed be accurately computed even when the reflector surface is known only at certain discrete points, the following example is presented.

Computations presented in Figure 4 are made again, this time, instead of using a single analytic expression for the entire spherical reflector surface, though, the x-, y-, z- coordinates of 45 equispaced points on the reflector surface are used. These surface points are located on the reflector surface along a rectangular grid as shown in Figure 5, the points being 40 m or 133.3 wavelengths apart. For computational purposes, the reflector surface over any rectangular patch is expressed as a bi-spline under tension [4]. In Figure 6, the far field radiation pattern computed by using a single analytic expression for the entire spherical reflector surface (as in Figure 4) is shown by solid lines. On the same figure, the far field radiation pattern computed by using the piecewise analytic composite surface through 45 target points on the reflector is plotted with solid dots. The field values not shown by solid dots are too close to the solid line curve values to distinguish. The modifications needed in the computer program in Reference 3 to make the present surface fitting computations are shown in the Appendix.

In conclusion, for actual distorted reflector conditions, where the whole distorted reflector surface is not known
Figure 5 -- Location of Surface Target Points used for Surface Fitting Calculations
Figure 6 -- Computed Secondary Pattern
analytically, accurate far field computations can be made by using our computer program which will accept for the reflector surface geometry, a set of discrete reflector surface points. The basic underlying assumption is that the surface is smooth between the target sample points.
7. References


Appendix

The computer program documented in Reference 3 has been modified such that now it can be used for making also the surface fitting reflector calculations similar to the ones presented in Section 6. The purpose of this appendix is to document the corresponding modifications, changes, and additions.

Two new features have been added to the computer program REFLCTR (documented in Reference 3). The first feature, which is not of direct concern to the subject matter of this report is that in addition to parabolic, spherical, and ellipsoidal reflectors, the program REFLCTR can now also handle planar reflectors. A planar reflector is specified by (a) three cartesian coordinates of a point on the surface of the planar reflector - PLNPNT(1), PLNPNT(2), and PLNPNT(3), and (b) the three cartesian components of a unit vector normal to the reflector surface - PLNORM(1), PLNORM(2), and PLNORM(3). The value of the integer variable SURFACE must be set to 4 for a planar reflector.

The second new feature is that in addition to specifying a reflector surface as being parabolic, spherical, ellipsoidal, or planar by setting SURFACE = 1, 2, 3 or 4 respectively, one can now also choose to specify any of the above reflector surfaces in terms of only a finite number of discrete target points located along a rectangular grid (assumed square grid here for simplicity) on the reflector surface. This is done by setting the integer variable NEEDFIT to a nonzero positive value. SIGMA is the tension factor (defined later) used for fitting the surface through the
grid points, the grid spacing being DISFAC for both the y- and the z- directions. All these newly defined variables along with the ones already defined in Reference 3 are read, as before, in the subroutine NPUT. A listing of modified NPUT is given in Figure A-1.

As a result of implementing the above two features, the subroutine APERTUR also changes. A listing of the new APERTUR is given in Figure A-2. Notice that in this subroutine, before statement number 100, the coordinates of surface target points are first stored in dimensioned arrays called EXTRAX, EXTRAY, and EXTRA2 and then the subroutine SURF1 is called to compute the parameters necessary to compute an interpolatory surface passing through the surface grid points. Later on in the subroutine APERTUR (before statement number 130), subroutine SURFD2 is called to interpolate the reflector surface at the given coordinate pair and to compute the components of a normal vector at the interpolated point.

The subroutines SURF1 and SURFD2 are from "A Spline Under Tension Package for Curve and Surface Fitting" by A. K. Cline, Department of Computer Science, University of Texas, Austin. This package of subroutines is an extension of Cline's work reported in Reference 4. A listing of subroutines SURF1 and SURFD2 which also includes definition of parameters used in the subroutines is presented in Figure A-3.
Figure A-1
* MINOR AXIS OF THE ELLIPTICAL REFLECTOR ..................*E7.21
590  FORMAT(1X,11X,2*)SPHERICAL REFLECTOR FOR FIELD RADIATION */
* PATTERN CALCULATION ..................*RA10/* RA1
  0/ WAVELENGTH OF ELECTRIC FIELD ..................*E9.4
  9 LOCATION OF ORIGIN WRT EEFN (X,Y,Z) ............*F7.2
  9 EEFN ROTATION ANGLE (ALPHA,PHI,THETA,GAMMA) ......*F7.2
  9 RADIUS OF THE REFLECTOR SPHERE ..................*E7.21
591  FORMAT(1X,11X,2*)PARABOLIC REFLECTOR FOR FIELD RADIATION */
* PATTERN CALCULATION ..................*RA10/* RA1
  0/ WAVELENGTH OF ELECTRIC FIELD ..................*E9.4
  9 LOCATION OF ORIGIN WRT EEFN (X,Y,Z) ............*F7.2
  9 EEFN ROTATION ANGLES (ALPHA,PHI,THETA,GAMMA) ......*F7.2
  9 FOCAL LENGTH OF THE REFLECTOR ..................*F7.2
592  FORMAT(* APERTURE PLANE LOCATION (X,C) ..............*F8.2/
  9 COORDINATES OF THE APERTURE PLANE CENTER ......*F7.2
  9 HALF MAJOR AXIS OF APERTURE PLANE (ALONG Y) .......*F7.2
  9 HALF MINOR AXIS OF APERTURE PLANE (ALONG Z) ......*F7.2/
  9 GRID SIZE USED FOR NUMERICAL INTEGRATION ..........*E9.4
  9 EEFN SHADOW CENTER COORDINATES IN APERTURE PLANE *F7.2
  9 HALF MAJOR AXIS OF EEFN SHADOW ..................*F7.2/
  9 HALF MINOR AXIS OF EEFN SHADOW ..................*F7.2/
  9 THETA RANGE FOR EEFN PATTERN (L+H,0 - DEGRFFS) ....*F7.2
  9 PHI RANGE FOR EEFN PATTERN (L+H) - DEGRFFS ..........*F7.2
)
593  FORMAT(* SIGMA USED FOR SURFACE FITTING .................*F7.2/
  9 SPACING BETWEEN SURFACE POINTS ...................*F7.21
590  FORMAT(* ---- INSUFFICIENT WORK STORAGE, NEED /*15 AVAILABLE IS
  9 ONLY *15 ---- *1)
PT=ACOST(-1.0)
PI2=PI+PI
PI2=PI+PI
TRNS=PI/180.
TRNS=180./PI
NP=(PHI(1)-PHI(1))/THETA(1)/THETA(1)+1.5
NP=(PHI(1)-PHI(1))/THETA(1)/THETA(1)+1.5
NPRINTF=NP
IF(NPRINTF.GT.MAXPTS) GO TO 600
CALL FILL(P,NT,NP)
RETURN
600  PRINT 590, NPRINTF, MAXPTS
STOP
END

Figure A-1 (Continued)
Figure A-2 (Continued)
IF($NEEDFIT.LE.0) GO TO 130
X=_SURF2D(YO,ZO,YNORM,ZNORM,IFIT,IFIT,IFIT,IFIT,IFIT,IFIT,EQTY,IFIT)
R=SORT((X0**2)**2+(YO**2)+(Z0**2)+(70+R(3,2))**2)
XMAG=SORT((1.0+YNORM**2+ZNORM**2)
NHAT(1)=1.0/XMAG
NHAT(2)=-SIGN(YNORM,YO)/XMAG
NHAT(3)=-SIGN(ZNORM,ZO)/XMAG
GO TO 130
130 GO TO (131,132,134,136), SURFACE
131 NHAT(1)=PLNORM(1)
NHAT(2)=PLNORM(2)
NHAT(3)=PLNORM(3)
GO TO 13A
132 NHAT(1)=X0/ANORRF
NHAT(2)=Y0/ANORRF
NHAT(3)=-Z0/ANORRF
GO TO 13A
134 NHAT(1)=X0/ANORRF
NHAT(2)=Y0/ANORRF
NHAT(3)=-Z0/ANORRF
GO TO 13A
136 NHAT(1)=2.0/ANORRF/SORT(4.0+ANORRF**2+Y0**2+Z0**2)
NHAT(2)= -Y0/SORT(4.0+ANORRF**2+Y0**2+Z0**2)
NHAT(3)=-Z0/SORT(4.0+ANORRF**2+Y0**2+Z0**2)
139 SCALAR=2.0*(A(1,1)+NHAT(1)+R(2,1)*NHAT(2)+R(3,1)*NHAT(3))
DO 1500 I=1,3
1500 SR(1)=R(1,1)-SCALAR+NHAT(1)
FTI=P(1,1,IP)/R
FPI=P(2,1,IP)/R
C(1)=CST*COS*FTI-SIN*FPI
C(2)=CST*SIN*FTI+COS*FPI
C(3)=-SINT*FTI
DO 2000 I=1,3
FI(I)=0.0
DO 2000 J=1,3
2000 FI(I)=FI(I)+A(I,J)*C(J)
SCALAR=2.0*(E(I,1)+NHAT(1)+F(2,1)+NHAT(2)+F(3,1)+NHAT(3))
DO 2500 I=1,3
2500 FR(I)=SCALAR+NHAT(1)-F(I,1)
Y=YO+(X-XO)*SR(1)/SR(1)
Z=ZO+(X-XO)*SR(3)/SR(1)
D=SORT((X-XO)**2+(Y-YO)**2+(Z-ZO)**2)
PHASE=PI2*(3.141)/XLAM+P(3,IP,IP)
PNFW(1)=PNFW(1)+Y
PNFW(2)=PNFW(2)+Z

Figure A-2 (Continued)
Fig. A-2 (Continued)
2705 FORMAT(1X,**,23X,2F10.4,2F10.7,F10.4,10X,*INTERPOLATED, ALOCRN*)
GO TO 2715
2710 CALL MOVEPM(PINT,P(1,5),PFIN,2712) PINT
2712 FORMAT(1X,**,23X,2F10.4,2F10.7,F10.4,10X,*INTERPOLATED*)
2715 CONTINUE
2800 CALL MOVEPM(PNEW,P(1,5))
TFST=TEST
TFST=(DIGTHEI-90.0)**2+(DIGPHI-180.0)**2
IF (TFST-BMTEST) 2900,3000,3000
2900 BMTEST=TEST
BM=T=DIGTHEI
BM=DI(GPHI)
HMSRX=SR(1)
HMSRY=SR(2)
HMSRZ=SR(3)
3000 CONTINUE
4000 CONTINUE
5000 CONTINUE
5025 FORMAT(* NUMBER OF INTERNAL POINTS:*******************JS
* NUMBER OF EDGE POINTS:*******************JS
COSMMP=HMSRZ/ROOT(HMSRX**2+HMSRY**2+HMSRZ**2)
SINMMP=HMSRY/ROOT(HMSRX**2+HMSRY**2)
MPP=DEG=ACOS(COSMMP)
MPP=DEG=ASIN(SINMMP)
PRINT 5050, MTT, MPP, RMSX, RMSY, RMSZ, AMP, AMP
5050 FORMAT(* THE FFFD MAX IS:*******************JS
* THE REFLECTED RAY VECTOR (X,Y,Z):*******************JS
* THE REFLECTED RAY MAX IS:*******************JS
RETURN
END

Figure A-2 (Continued)
SUBROUTINE SIRE1 (M,N,X,Y,Z,I7,ZX1,ZZX,ZZY,ZYN,ZZY1) *
    7XYM1,7XY1H,ZZYM1,ISLB5W,7P,TMP, *
SIGMA,IER1)
C THIS SUBROUTINE DETERMINES THE PARAMETERS NECESSARY TO
C COMPUTE AN INTERPOLATORY SURFACE PASSING THROUGH A RECT-
C ANGULAR GRID OF FUNCTIONAL VALUES. THE SURFACE DETERMINED
C CAN BE REPRESENTED AS THE TENSOR PRODUCT OF SPLINES UNDER
C TENSION, THE X- AND Y-PARTIAL DERIVATIVES AROUND THE
C BOUNDARY AND THE X-Y-PARTIAL DERIVATIVES AT THE FOUR
C CORNERS MAY BE SPECIFIED OR OMITTED FOR ACTUAL MAPPING
C OF POINTS INTO THE SURFACE IT IS NECESSARY TO CALL THE
C FUNCTION SIRD2.
C
C IN INPUT--
C
M IS THE NUMBER OF GRID LINES IN THE X-DIRECTION, I. E.
LINES PARALLEL TO THE Y-AXIS (M .GE. 2).

N IS THE NUMBER OF GRID LINES IN THE Y-DIRECTION, I. E.
LINES PARALLEL TO THE X-AXIS (M .GE. 2).

X IS AN ARRAY OF THE M X-COORDINATES OF THE GRID LINES
IN THE X-DIRECTION. THESE SHOULD BE STRICTLY INCREASING.

Y IS AN ARRAY OF THE M Y-COORDINATES OF THE GRID LINES
IN THE Y-DIRECTION. THESE SHOULD BE STRICTLY INCREASING.

Z IS AN ARRAY OF THE M X X FUNCTIONAL VALUES AT THE GRID
POINTS, I. E. Z(I,J) CONTAINS THE FUNCTIONAL VALUE AT
(X(I),Y(J)) FOR I = 1,...,M AND J = 1,...,N.

L IS THE ROW DIMENSION OF THE MATRIX Z USED IN THE
CALLING PROGRAM (L >= M).

7X1 AND 7YM ARE ARRAYS OF THE M X-PARTIAL DERIVATIVES
OF THE FUNCTION ALONG THE X(I) AND X(M) GRID LINES.
(REPECTIVELY. THIS 7X1(I) AND 7YM(I) CONTAIN THE X-PART-
IAL DERIVATIVES AT THE POINTS (X(I),Y(J)) AND
(X(M),Y(J)), RESPECTIVELY, FOR J = 1,...,N. EITHER OF
THREE PARAMETERS WILL BE IGNORED (AND APPROXIMATIONS
SUPPLIED INTERNALLY) IF ISPBW = 0 INDICATES.

7Z1 AND 7YN ARE ARRAYS OF THE M Y-PARTIAL DERIVATIVES
OF THE FUNCTION ALONG THE Y(I) AND Y(M) GRID LINES.
(REPECTIVELY, THIS 7Z1(I) AND 7YN(I) CONTAIN THE Y-PART-

Figure A - 3

27
Figure A - 3 (Continued)
Figure A - 3 (Continued)
Figure A-3 (Continued)
TEMP(1) = DTAG1*SN1AG1
IF (M .LE. 2) GO TO 34

DO 33 J = 2, NM
   JM1 = J-1
   JP1 = J+1
   NDMP,J = NDMP,J+1
   DFL,J = Y(JP1)-Y(J)
   IF (DFL,J .LE. 0.) GO TO 47
   DFL,J = 1./DFL,J
   DO 31 I = 1, N
   31 ZP(I,JP1,J) = DFL,J*(Z(I,JP1,J)-Z(I,JM1,J))
   ZP(I,JP1,J) = DFL,J*(ZP(I,JP1,J)-ZP(I,JM1,J))
   7P(I,JP1,J) = DFL,J*(7P(I,JP1,J)-7P(I,JM1,J))
   CALL TERMS (DTAG2,SN1TAG2,SIGNAN,DELP)
   DTAGN = 1./(DTAG1+DTAG2-SN1TAG2*TEMP(JM1,J))
   DO 32 I = 1, N
   32 ZP(I,JP1,J) = DTAGN*(ZP(I,JP1,J)-ZP(I,JM1,J))
   ZP(I,JP1,J) = DTAGN*(ZP(I,JP1,J)-ZP(I,JM1,J))
   7P(I,JP1,J) = DTAGN*(7P(I,JP1,J)-7P(I,JM1,J))
   7P(I,JP1,J) = DTAGN*(7P(I,JP1,J)-7P(I,JM1,J))
   TEMP(J) = DTAGN*SN1AG2
   DTAG1 = DTAG2
   SN1AG1 = SN1AG2
   14 DTAGM = 1./(DTAG1-SN1AG1)*EXP(NM1)
   DO 35 I = 1, N
      NM1 = NM1 *
   35 ZP(I,NM1) = DTAGM*(7P(I,NM1)-7P(I,NM1+1))
   ZP(I,NM1) = DTAGM*(7P(I,NM1)-7P(I,NM1+1))
   7P(I,NM1) = DTAGM*(XYNS-7P(I,NM1))
   7P(I,NM1) = DTAGM*(XYNS-7P(I,NM1))
   TEMP(N) = DTAGNS*(XYNS-7P(NM1,N))
   TEMP(N) = DTAGNS*(XYNS-7P(NM1,N))
   DO 37 J = 2, NM
      JHAK = M1-J
      JHAKP1 = JHAK+1
      T = TEMP(JHAK)
      DO 46 I = 1, N
   46 ZP(I,JHAKP1,I) = ZP(I,JHAK,I)-T*ZP(I,JHAKP1,I)
   ZP(I,JHAKP1,I) = ZP(I,JHAK,P1,I)-T*ZP(I,JHAKP1,I)
   7P(I,JHAKP1,I) = ZP(I,JHAK,P1,I)-T*7P(I,JHAKP1,I)
   7P(I,JHAKP1,I) = ZP(I,JHAK,P1,I)-T*7P(I,JHAKP1,I)
   TEMP(JHAK) = ZP(I,JHAK,P1,I)-T*TEMP(JHAKP1)
   DFL,J = X(J)-X(J1)
   IF (DFL,J .LE. 0.) GO TO 47
   DFL,J = 1./DFL,J
   DO 32 J = 1, N

Figure A - 3 (Continued)
Figure A-3 (Continued)
This function interpolates a surface at a given coordinate pair using a set of spline functions under tension. The subroutine SURF should not be called earlier in the program for the given necessary parameters.

Figure A-3 (Continued)

The parameters M, N, X, Y, Z, and SIGMA should be altered in the calling routine. The input locations and the output values corresponding to the grid lines in the X- and Y-directions, respectively, are contained in the array ZP. X and Y are arrays containing the X- and Y-coordinates of the point of the surface. M and N contain the number of grid lines in the X- and Y-directions, respectively, increasing in order.

SIGMA contains the tension factor (its sign is ignored).

2X is partial derivative with respect to X.

2Y is partial derivative with respect to Y.

SIGMA contains the tension factor (its sign is ignored).
Figure A-3 (Continued)
Figure A - 3 (Continued)
SUBROUTINE SNHCSH (SINHM, COSHM, X, ISM)

THIS SUBROUTINE RETURNS APPROXIMATIONS TO

\[
\begin{align*}
\sinh(x) &= \sinh(x) - x \\
\cosh(x) &= \cosh(x) - 1
\end{align*}
\]

AND

\[
\sinh(x) = \cosh(x) - 1 + x^2/2
\]

WITH RELATIVE ERRORS LESS THAN 3.42E-14

IN INPUT:

X CONTAINS THE VALUE OF THE INDEPENDENT VARIABLE.

ISW INDICATES THE FUNCTION DESIRED

\[
\begin{align*}
&= -1 \text{ IF ONLY SINHM IS DESIRED}, \\
&= 0 \text{ IF BOTH SINHM AND COSHM ARE DESIRED}, \\
&= 1 \text{ IF ONLY COSHM IS DESIRED}, \\
&= 2 \text{ IF ONLY COSHM IS DESIRED}, \\
&= 3 \text{ IF BOTH SINHM AND COSHM ARE DESIRED}.
\end{align*}
\]

IN OUTPUT:

SINHM CONTAINS THE VALUE OF SINH(X) IF ISW \leq 3 OR

ISW \geq 4. SINHM IS UNALTERED IF ISW = 0 OR ISW = 1.

COSHm CONTAINS THE VALUE OF COSH(X) IF ISW \geq 4 OR

ISW \leq 1 AND CONTAINS THE VALUE OF COSH(X) IF ISW

\geq 2. COSHM IS UNALTERED IF ISW = -1).

AND

X AND ISM ARE UNALTERED.

INTENT(ISM)

REAL SINHM, COSHM, X

DATA 5.4/4.54/2.75/6.33.1335-.1335E-1/6. 
3.13/1.952/2.84/4.1/5/3.1/0.86-.86E-1/6. 
3.72/5.72/6.7/6.79/7.16.2E-6-.16/6. 
3.84/3.84/5.44/3.17/1/10/10E-1/6.

DATA 0.24/1.73/0.356/7.9891/01-.01E-7/6. 
0.23/7.977/722/70/4.4E-15/6. 
0.29/2.13/1.960/2.82-.82/6. 
0.21/7.92/1.97/2.85E-12/6.

Figure A - 3 (Continued)
** Figure A - 3 (Continued)**
REAL DIAG, SDIAG, SIGMA, DEL
IF (SIGMA .NE. 0.) GO TO 1
DIAG = DEL/2.
SDIAG = DEL/6.
RETURN
1 SIGFEL = SIGMA*DEL
CALL SNHCSC (SINHH, COSH, SIGFEL, 0)
DENOM = DEL/((SINHH+SIGFEL)*SIGFEL*SIGFEL)
DIAG = DENOM*(SIGFEL*COSH-SINHH)
SDIAG = DENOM*STANH
RETURN
END

SUBROUTINE CEL2 (DEFL, DEL2, SIGMA, C1, C2, C3, N)

THIS SUBROUTINE DETERMINES THE COEFFICIENTS C1, C2, AND C3
USED TO DETERMINE EQUATION SLOPES, SPECIFICALLY, IF
FUNCTION VALUES Y1, Y2, AND Y3 ARE GIVEN AT POINTS X1, X2,
AND X3, RESPECTIVELY, THE QUANTITY C1*Y1 + C2*Y2 + C3*Y3
IS THE VALUE OF THE DERIVATIVE AT X1 OF A SPLINE UNDER
TENSION (WITH TENSION FACTOR SIGMA) PASSING THROUGH THE
THREE POINTS AND HAVING THIRD DERIVATIVE EQUAL TO ZERO AT
X1. OPTIONALLY, ONLY TWO VALUES, C1 AND C2 ARE DETERMINED.

ON INPUT--
DEFL IS X2-X1 (>0, 0).

Figure A - 3 (Continued)

40
DFL2 IS X3-X1 (C.F. N.). IF N EQ. 2, THIS PARAMETER IS IGNORED.

SIGMA IS THE TENSION FACTOR.

AND

N IS A SWITCH INDICATING THE NUMBER OF COEFFICIENTS TO BE RETURNED. IF N EQ. 2 ONLY TWO COEFFICIENTS ARE RETURNED, OTHERWISE ALL THREE ARE RETURNED.

ON OUTPUT--

C1, C2, AND C3 CONTAIN THE COEFFICIENTS.

NONE OF THE INPUT PARAMETERS ARE ALTERED.

THIS SUBRUTINE REFERENCES PACKAGE MODULE SMHC SH.

------------------------------------------------------------------------

REAL DFL, DFL2, SIGMA, C1, C2, C3
IF (N EQ. 2) GO TO 2
IF (SIGMA .NE. 0.) GO TO 1
DFL = DFL2/DFL
C1 = -(DFL + DFL2)/(DFL * DFL2)
C2 = DFL2/DFL
C3 = -DFL1/(DFL * DFL)
RETURN
1 CALL SMHC SH (HIMMY, COSHIM, SIGMA*DFL1, 1)
CALL SMHC SH (HIMMY, COSHIM, SIGMA*DFL2, 1)
DFLP = SIGMA*(DFL2 + DFL1)/2.
DFLM = SIGMA*(DFL2 - DFL1)/2.
CALL SMHC SH (SINHIM, HIMMY, DFLP - 1)
CALL SMHC SH (SINHIM, HIMMY, DFLM - 1)
DENOM = COSHIM*(DFL2 - DFL1)*2.*DFL1*(SINHIM + DFLP)*
   (SINHIM + DFLM)
C1 = 2.*((SINHIM + DFLP) + (SINHIM + DFLM))/DENOM
C2 = -COSHIM/DENOM
C3 = COSHIM/DENOM
RETURN
2 C1 = -1./DFL
C2 = -C1
RETURN
END

FUNCTION INTUNL (T, Y, N)

Figure A - 3 (Continued)
C THIS FUNCTION DETERMINES THE INDEX OF THE INTERVAL
C (DETERMINED BY A GIVEN INCREASING SEQUENCE) IN WHICH
C A GIVEN VALUE LIES.
C
C ON INPUT--
C
C T IS THE GIVEN VALUE.
C X IS A VECTOR OF STRICTLY INCREASING VALUES.
C AND
C N IS THE LENGTH OF X (N .GE. 2).
C
C ON OUTPUT--
C
C INTERVAL RETURNS AN INTEGER I SUCH THAT
C
C I = 1 IF T .LE. X(2) .
C T = N-1 IF X(N-1) .LE. T .
C OTHERWISE X(I) .LE. T .LE. X(I+1).
C
C NOTE THE INPUT PARAMETERS ARE ALTERED.
C

DEFINIEN N
<REAL> I,X(N)

IF = 1
IF (T .LE. X(N)) GO TO 2
IF (T .GE. X(N-1)) GO TO 6
I = 2
I = N-1
11 = I + INTX(FLAT((I-1)X(I))/X(I)-X(I-1)))
IF (T .LE. X(I)) GO TO 2
IF (T .GE. X(I+1)) GO TO 3
I = I + 1
GO TO 11
2 T = 1
GO TO 11
6 I = I + 1
GO TO 1
5 RETURN = 1
RETUR...

Figure A-3 (Continued)