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NASA Contract NAS5-25957 (MAGSAT)

Investigation of Geomagnetic Field Forecasting and Fluid Dynamics of the Core

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1. **Problems**

No new problems have arisen. Moreover, it has been learned that what was thought to be a problem (the fact that the domestic travel contract budget of $500 was exceeded in making the trip for the December 1980 MAGSAT Investigations Meeting at GSFC, see Progress Report #4) is not such a problem after all because of the government policy allowing automatic reallocation to cover such an excess up to a limit of $500. Therefore, official permission to exceed the contract budget is not being requested of NASA.

2. **Approach**

New approaches are being developed in two areas of interest, (a) use of MAGSAT data alone to evaluate theoretical upper limits to the present dipole and quadrupole magnetic moments of the earth, and (b) use of MAGSAT data to provide constraints on future geomagnetic field models.

a. **Upper Limits to the Present Dipole and Quadrupole Moments of the Earth.** Several possible global measures of the strength of the geomagnet exist (such as magnetic dipole moment, mean square surface field intensity, magnetic energy outside the core, etc.), but most of these can change quite rapidly (in a matter of decades to centuries). However, the "total pole-strength" of the earth (Bondi & Gold, 1950) defined as

\[
P(r_c, t) \equiv \int_0^{2\pi} \int_0^{\pi} |B_r(r_c, \theta, \phi, t)| r_c^2 \sin \theta d\theta d\phi
\]

where \( r_c \) is the core radius, should, theoretically, only vary on the several-thousand-year time scale for ohmic dissipation in the core.

We intend to make a careful evaluation of \( P \) from MAGSAT data above, but will also introduce a new way of doing so. Use of Stokes' theorem and the absence of magnetic monopoles permits (1)
to be re-expressed as a contour integral of a magnetic vector potential, \( \mathbf{A} \), around null-flux curves, \( C_0 \), on the core-mantle boundary:

\[
P(r_c, t) = 2 \oint_{C_0} \mathbf{A}(r_c, \theta, \phi, t) \cdot d\mathbf{s}
\]  

(2)

This form is more convenient, numerically, than (1) and is also less sensitive to truncation level, since spherical harmonic series for \( \mathbf{A} \) converge more rapidly than those for \( \mathbf{B} \). Recently, Hide and Malin calculated \( P \) from (1) at 1965 and obtained

\[
P(r_c, 1965) = 3.60 \cdot 10^{10} \text{ Tm}^2.
\]

By repeating this at 1980, using (2), we can (i) test the above new methodology, (ii) do away with a possible dependence of \( P \) on secular variation models (since Magsat data are essentially instantaneous in time), and (iii) compare \( P \) at 1965 with \( P \) at 1980.

We will also re-evaluate \( P \) at earlier epochs (up to a century or so ago) from existing geomagnetic models to verify that \( P \) is a slowly varying function of time (results of Hide and Malin suggest that \( P \) can perhaps be estimated to within less than 1% with geomagnetic field models truncated beyond degree \( N = 7 \)).

Assume, for the moment, that \( P \) is constant in the near term. Then we can obtain an upper limit to the dipole moment of the earth, as follows. Rearrange the field lines crossing \( r = r_c \) into an axial, purely dipolar configuration (without omitting any field lines or introducing new field lines). Calculate the dipole moment of the resulting field, calling its Gauss coefficient \( \mathbf{G}_1^0 \). Stated another way, the upper limit dipole moment \( (\mathbf{G}_1^0 r_e^3) \) is achieved by that pure dipolar magnetic field with the same total pole strength as the real earth. A useful "dipolarity index" for the earth is, then, the ratio

\[
\left[ (q_1^0)^2 + (q_1^1)^2 + (h_1^1)^2 \right]^{1/2} / \mathbf{G}_1^0.
\]
For 1965 this ratio was about 80%, i.e., the actual dipole strength was about 80% of its theoretical upper limit. The idea extends simply to higher order moments as well (e.g., at 1965, the quadrupole index was about 48%).

b. **Field Modelling Constraints from MAGSAT.** If the total pole strength of the earth is, indeed, nearly constant in time, then field models should be improved by incorporating this fact as a constraint. The accurate determination of \( P \) from MAGSAT data will provide the correct value of \( P \) to utilize.

We have also decided how to search for other constraints. As defined above, \( P \) is an integral on the core-mantle boundary and therefore is contributed to, significantly, by all of the retained spherical harmonics (although the work of Hide and Malin does indeed reveal encouraging tendency to convergence). It is suspected (though not yet proven) that the total pole-strength as observed from earth's surface, \( r = r_\text{e} \),

\[
P(r_\text{e},t) = \int_0^{2\pi} \int_0^{\pi} |B_r(r_\text{e},\theta,\phi,t)| r_\text{e}^2 \sin\theta \, d\theta \, d\phi,
\]

although not constrained in theory to vary as slowly as does \( P(r_\text{c},t) \), will nonetheless either be nearly constant or change smoothly in time over the near term. Then, its constancy or evaluated time change (using both pre-MAGSAT and MAGSAT models) can be used to constrain \( P(r_\text{e},t) \) at some future epoch. Since truncated series for \( B_r \) converge so much more satisfactorily at \( r = r_\text{e} \) than at \( r = r_\text{c} \), this constraint will be felt much more by the lower order harmonics.

Procedures similar in spirit have also been thought out in terms of magnetic energy (of internal origin) stored outside earth's core and outside the earth. This provides two further constraints of a global, integral character.
3. **Accomplishments**

The accomplishments during this reporting period fall into several categories.

a) Conceptualization of the new approaches to important geomagnetic issues outlined in 2 above.

b) Completion of the revision to a paper which will be published in *Geophysical and Astrophysical Fluid Dynamics* this summer under the title: "Inviscid, Frozen-Flux Velocity Components at the Top of Earth's Core from Magnetic Observations at Earth's Surface: A New Methodology." This constitutes a major theoretical step forward for this project. Part 2, dealing with an application of data to the method, is to be a focus of work during the remainder of this project.

c) Preparation of the first half of a paper proposed in the last progress report: "Rapid Diffusion of the Poloidal Geomagnetic Field Through the Weakly-Conducting Mantle: A Perturbation Solution." The second half of this paper requires numerical work that will be addressed during the next six months.

d) The truncation level study described in the last report has been formalized, somewhat, and advanced. It is now intended to examine five geomagnetic properties in a four-way collaboration between Langel, Estes, Muth, and Benton. The work is expected to be complete in time to present at the MAGSAT and the IAGA meetings in Edinburgh.

e) Three abstracts have been prepared and submitted for the IAGA meetings:

   i) "Total Pole-Strength of the Earth from MAGSAT and Upper Limits to the Low Order Geomagnetic Moments" by E. R. Benton, M. C. Coulter, and C. V. Voorhies.


   iii) "A Test for the Presence of Significant Vertical Motion in the Upper-most Layer of Earth's Core" by E. R. Benton.
4. **Significant Results**

The discovery of simple, theoretically-sound upper limits for geomagnetic moments (dipole, quadrupole, etc.) provides a significant new use for MAGSAT data, establishes useful constraints for future magnetic models, and bears strongly on the probable time required before the next polarity reversal can occur.

5. **Publication**

One sent by editor to the printer; three abstracts submitted.

6. **Recommendations**

None.

8. **Data Utility**

The field models of the MAGSAT data are of prime use in this investigation and are highly suitable as supplied to date.