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PROBLEM OF GAS ACCRETION ON A GRAVITATIONAL CENTER

V. A. Ladygin

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16. Abstract

The study deals with a method of the approximated solution of the problem of accretion on a rapidly moving gravitational center. This solution is obtained in the vicinity of the axis of symmetry in the region of the potential flow.

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PROBLEM OF GAS ACCRETION ON A GRAVITATIONAL CENTER

V. A. Ladygin

Examined in the study is a method of approximated solution of the problem of gas accretion on a rapidly moving gravitational center. The solution is obtained in some vicinity of the axis of symmetry in the region of potential flow. Calculations on a computer showed the effectiveness of the given method. /2*

Introduction /3

The solution of the problem of stationary gas accretion on a moving gravitational center simulates the movement of a substance in interstellar space in the vicinity of a black hole. A detailed picture of gas accretion on a black hole is of interest in connection with the problem of observation of black holes.

The qualitative study of such accretion, as well as two-dimensional numerical calculations of this problem, are available in studies [1] - [4]. The study of self-modeling solutions, which may represent asymptotics of the flow of gas near a gravitational center, was carried out in study [5].

In the present study, the system of equations of two-dimensional gas dynamics, which describes gas accretion, in contrast to studies [2] - [4], is solved in an approximate manner, by means of expansion into a linear series, according to one of the independent variables (angle θ), and by means of "abridging" of the obtained infinite system of common differential equations.

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*Numbers in the margin indicate pagination in the foreign text.

Although only the region of potential flow of the gas is studied in the present study, this method may be used also for a nonpotential flow behind a shock wave.

1. Formulation of the Problem and Basic Equations /4

Studied herein is the steady-state axisymmetrical flow of an ideal polytropic gas, devoid of viscosity and thermal conductivity in a gravitational field of a material point of mass M.

At infinity, the approach stream is assumed to be homogeneous and supersonic.

$$\begin{gathered} \rho \to \rho_\infty = \text{const}, \quad P \to P_\infty = \text{const}, \\ V_r \to -V_\infty \cos\theta, \quad V_\theta \to -V_\infty \sin\theta, \\ V_\infty = \text{const} > 0, \qquad r \to \infty \end{gathered} \qquad /2.1/$$

With these boundary conditions, the flow prior to the shock wave is potential and isentropic, and, in a spherical system of coordinates, it is described by the system of equations:

$$\frac{V_r^2 + V_\theta^2}{2} - \frac{GM}{r} + \gamma K \rho^{\gamma-1} = \Psi \qquad /2.2/$$

-energy integral,

$$\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial \theta} = 0 \qquad /2.3/$$

-condition of potentiality,

$$\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho V_\theta \sin\theta) = 0 \qquad /2.4/$$

-equation of continuity,

$$P = (\gamma - 1) K \rho^{\gamma} \qquad /2.5/$$

-equation of state,

where V_r, V_θ, ρ, and P are the radial and angular components of velocity, the density, and the pressure of the gas,

G is the gravitational constant, γ is the indicator of the polytropic curve.

The constants K and Ψ are determined with the boundary conditions (2.1).

$$K = \frac{P_\infty}{(\gamma-1)\rho_\infty^\gamma}, \quad \Psi = \frac{V_\infty^2}{2} + \frac{\gamma P_\infty}{(\gamma-1)\rho_\infty}. \qquad /2.6/$$

Through transformation of the analog

$$r = \frac{GM}{C_\infty^2} r_1, \quad \rho = \rho_\infty \rho_1, \quad V_r = C_\infty V_{r_1}, \quad V_\theta = C_\infty V_{\theta_1}, \quad P = \gamma P_\infty P_1,$$

where $C_\infty = (\gamma \frac{P_\infty}{\rho_\infty})^{1/2}$ is the speed of sound at infinity, the five-parameter problem (2.1)-(2.6) (parameters: γ, $G \cdot M$, P_∞, ρ_∞, V_∞,) is reduced to a two-parameter problem (parameters: γ and $M_\infty = \frac{V_\infty}{C_\infty}$ is the Mach speed at infinity). /5

Therefore, without bounding the generality, one can assume

$$\begin{array}{l} G \cdot M = 1, \quad \Psi = \frac{M_\infty^2}{2} + \frac{1}{\gamma-1}, \quad K = \frac{1}{\gamma(\gamma-1)}, \\ P_\infty = \frac{1}{\gamma}, \quad \rho_\infty = 1, \quad C_\infty = 1, \quad V_\infty = M_\infty > 1 \end{array} \qquad /2.7/$$

On the strength of (2.4), the expression

$$-r\rho \sin\theta V_\theta \, dr + r^2 \rho \sin\theta V_r \, d\theta$$

is a complete differential of some function $S(r, \theta)$ of the current lines, and, consequently,

$$\begin{array}{lll} & d\frac{\partial S}{\partial r} = -r \sin\theta V_\theta & /2.8/ \\ & d\frac{\partial S}{\partial \theta} = r^2 \sin\theta V_r & /2.9/ \\ /2.2/ \longrightarrow & \frac{V_r^2 + V_\theta^2}{2} - \frac{1}{r} + \frac{1}{\gamma-1} d^{1-\gamma} = \Psi & /2.10/ \\ /2.9/ \longrightarrow & \frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial \theta} = 0 & /2.11/ \end{array}$$

where $d = \frac{1}{\rho}$ is the specific volume.

2. Description of the Method of Solution of the Problem

We will derive the formulas for the approximated solution of the system (2.8)-(2.11). Since the flow is axisymmetrical, then

$$d(\tau,\theta)=d(\tau,-\theta),\quad V_\tau(\tau,\theta)=V_\tau(\tau,-\theta),$$
$$V_\theta(\tau,\theta)=-V_\theta(\tau,-\theta),\quad S(\tau,\theta)=S(\tau,-\theta) \qquad /3.1/$$

We will assume that the functions d, V_τ, V_θ, S are analytic according to θ in some vicinity of $\theta=0$. Then, d, V_τ, S are ex[illegible]ded into an exponential series according to even powers of θ, and V_θ, according to uneven powers of θ, <u>i.e.</u>,

$$d(\tau,\theta)=\sum_{k=0}^{\infty} d_{2k}(\tau)\,\theta^{2k},\quad V_\tau(\tau,\theta)=\sum_{k=0}^{\infty} q_{2k}(\tau)\,\theta^{2k},$$
$$S(\tau,\theta)=\sum_{k=0}^{\infty} S_{2k}(\tau)\,\theta^{2k},\quad V_\theta(\tau,\theta)=\sum_{k=1}^{\infty} p_{2k-1}(\tau)\,\theta^{2k-1} \qquad /3.2/$$

One can think that $S_0(\tau)=0$, since the function $S(\tau,\theta)$ is determined with an accuracy up to the additive constant, and is constant along the trajectory, while the axis of symmetry $\theta=0$ is the trajectory of the particles. /6

We will substitute the expansion (3.2) into the system (2.8)-(2.11), and group the terms with identical powers of θ. We will obtain the system of relationships:

$$/2.8/ \longrightarrow \sum_{i+j=2k} d_i \frac{dS_j}{d\tau} = -\tau \sum_{i+j=2k} p_i a_j, \qquad /3.3/$$
$$/2.9/ \longrightarrow \sum_{i+j=2k} i S_i d_j = \tau^2 \sum_{i+j=2k-1} q_i a_j, \qquad /3.4/$$
$$/2.10/ \longrightarrow \frac{V_0}{2} + \dots + \frac{h_0}{\gamma-1} = \Psi, \qquad /3.5/$$
$$\frac{V_{2k}}{2} + \frac{h_{2k}}{\gamma-1} = 0 \qquad /3.6/$$
$$/2.11/ \longrightarrow \frac{d}{d\tau}(\tau p_{2k-1}) = 2k\, q_{2k} \qquad /3.7/$$

$/k=1,2,\dots/$,

where a_i, V_i, h_i are the coefficients of the expansion into an exponential series, respectively, of the functions $\sin\theta$,

$V_r^2 + V_\theta^2$, $d^{1-\gamma}$

$$\sin\theta = \sum_{k=1}^{\infty} a_{2k-1}\theta^{2k-1}, \quad a_{2k-1} = \frac{(-1)^{k+1}}{(2k-1)!}, \qquad /3.8/$$

$$V_r^2 + V_\theta^2 = \sum_{k=0}^{\infty} V_{2k}\theta^{2k}, \quad V_{2k} = \sum_{i+j=2k} q_i q_j + \sum_{i+j=2k} p_i p_j \qquad /3.9/$$

/K=0,1,.../,

$$d^{1-\gamma} = h = \sum_{k=0}^{\infty} h_{2k}\theta^{2k}$$

We will expand the equality

$$dh_\theta' = (1-\gamma) h d_\theta$$

into an exponential series according to θ.

We will obtain a system of relationships between the coefficients $\{h_i\}$ and $\{d_i\}$.

$$h_0 = d_0^{1-\gamma},$$

$$\sum_{i+j=2k} i h_i d_j = (1-\gamma)\sum_{i+j=2k} i d_i h_j \qquad /3.10/$$

/K=1,2,.../

We will transform the system (3.3)-(3.10). In place of the coefficients S_{2K}, we will examine the functions

/3.11/

$$F_{2K} = \frac{S_{2K}}{\tau^2} \qquad /K=1,2,.../,$$

which have finite limits with $\tau \to +\infty$. Using (3.5) and (3.6), we preclude the coefficients h_i in (3.10). We will introduce the new independent variable

$$u = \ln\tau \qquad /3.12/$$ /7

and also the functions

$$\mu_{2K} = d_{2K}/d_0$$

/K=1,2,.../.

We will obtain an infinite system of common differential equations relative to the functions p_{2K-1} , F_{2K}

/K=1,2,.../:

$$\begin{cases} \dfrac{dp_{2K-1}}{du} = 2K \cdot q_{2K} - p_{2K-1} \\ \dfrac{dF_{2K}}{du} = T_{2K} - F_{2K}\left(2 - \dfrac{2q_2(p_1+q_0) - e^{-u}}{q_0^2 - c_0^2}\right) \end{cases} \qquad /3.13/$$

The functions $\{T_{2K}\}_{K=1}^{\infty}$, $\{q_{2K}\}_{K=0}^{\infty}$, C_0^2 (square of the speed of sound on the axis of symmetry $\theta=0$), which are part of the right-hand portion of system (3.13), as well as the functions $\{V_{2K}\}_{K=0}^{\infty}$, $\{\mu_{2K}\}_{K=0}^{\infty}$, are sequentially determined through the relationships:

$$C_0^2=(\gamma-1)(\Psi-\varrho^{-M}-2F_1^2), \qquad /3.14/$$

$$q_0=2F_1, \quad V_0=4F_1^2, \quad T_2=-p_1 \qquad /3.15/$$

Then, if the coefficients μ_{2i}, V_{2i}, q_{2i} $(i<K)$ and T_{2j} $(j<K+1)$ have already been calculated, then μ_{2K}, V_{2K}, q_{2K} are determined from the system of three linear equations:

$$\begin{cases} V_{2K}-2q_0 q_{2K}=\sum\limits_{i+j=2K,\ i,j\neq 0} q_i q_j+\sum\limits_{i+j=2K} p_i p_j, & /3.16/ \\ 2K(V_{2K}-2C_0^2\mu_{2K})=(1-\gamma)\sum\limits_{i+j=2K,\ i\neq 2K} i\mu_i V_j-\sum\limits_{i+j=2K,\ i\neq 2K} i V_i \mu_j, & /3.17/ \\ 2F_1\mu_{2K}-q_{2K}=\sum\limits_{i+j=2K+1,\ j\neq 1} q_i a_j-\sum\limits_{i+j=2K+2,\ j\neq 2K} iF_i\mu_j-2(K+1)F_{2(K+1)} & /3.18/ \end{cases}$$

$$/K=1,2,\ldots/,$$

and the function $T_{2(K+1)}$ is calculated according to the formula

$$T_{2(K+1)}=-\sum_{i+j=2K+2} T_i\mu_j \ \sum_{i+j=2K+2} p_i q_j \qquad /3.19/$$

From (2.1), (2.9), and (3.2), we will obtain the boundary conditions for system (3.13)

$$p_{2K} \longrightarrow \frac{(-1)^{K+1}K!}{(2K+1)!}$$

with $u\to+\infty$

$$F_{2K}\to \qquad /3.20/$$

$$/K=1,2,\ldots/$$

If the system (3.13)-(3.20) is solved, then the coefficients d_{2K} and S_{2K} are determined by the formulas /8

$$d_0=C_0^{\frac{2}{\gamma-1}}, \quad d_{2K}=d_0\cdot\mu_{2K}, \qquad /3.21/$$

$$S_{2K}=\varrho^{2u}\cdot F_{2K}/d_0$$

$$/K=1,2,\ldots/.$$

The calculations according to formulas (3.13)-(3.19), with small values of τ, requires large outlays of computer time, since the right-hand portions of the system (3.13) are unbounded with $\tau \to 0$. Therefore, in the region of $u<0$, it is advisable to make a substitution of the variables:

$$p^{*}_{1k} = e^{ku} p_{1k}, \qquad F^{*}_{1k} = e^{ku} F_{1k}, $$
$$q^{*}_{1k} = e^{ku} q_{1k} \qquad T^{*}_{1k} = e^{ku} T_{1k}, \qquad /3.22/$$
$$C^{*}_{k} = e^{ku} C_{k} \qquad V^{*}_{1k} = e^{k} V_{1k}$$

In this case, only equations (3.13) and (3.14) change, taking on the form:

$$\begin{cases} \frac{dp^{*}_{1k-1}}{du} = 2kq^{*}_{1k} \cdot \frac{p^{*}_{1k-1}}{2} \\ \frac{dF^{*}_{1k}}{du} = T^{*}_{1k} - F^{*}_{1k}(1.5 \cdot \frac{2q^{*}_{1}(p^{*}_{1} + q^{*}_{1}) - 1}{q^{*}_{1} - C^{*}_{1}}), \end{cases} \qquad /3.23/$$

$$C^{**}_{1} = (\gamma - 1)(\gamma e^{u} + 1 \cdot 2F^{**}_{1}) \qquad /3.24/$$

For approximated calculation of the first N coefficients of expansion (3.2), we will examine system (3.13) for k=1,2,...N. Formulas (3.14)-(3.19) make it possible, in the right-hand portion of (3.13), to express q_{2i}, μ_{2i}, V_{2i} (i <N) and T_{2j} (j <N+1) in the form of functions from u, p_1, ..., p_{2N-1}, F_2, ..., F_{2N} For a similar recording of the coefficients q_{2N} and V_{2N}, we will make use of formulas (3.16) and (3.17) for k=N, with $\mu_{2N}=0$ being assumed in (3.17). This additional assumption does not contradict the boundary conditions

$$\mu_{1k} \to 0 \text{ with } u \to +\infty$$
$$k=1,2,...$$

3. Discussion of Results /9

The obtained closed system 2N of common differential equations relative to the functions p_1, ..., p_{2N-1}, F_2, ..., F_{2N}

was integrated numerically on a computer by the Runge-Kutt method in the interval $-4\ln 10<u<10\ln 10$ (i.e., $10^{-4}<\tau<10^{10}$) for the indicator of the polytropic curve $\gamma=5/3$ and the Mach speed $M_\infty=2.4$. In this case, the boundary conditions with $\tau\to+\infty$ were transposed to the point $\tau=10^{10}$ Bondi radii.

The calculation was carried out for N=1,2,...,10,15,20.

For N≥2, within the limits of accuracy of integration $\varepsilon=10^{-5}$ of the system (3.13), the values of the density $\rho(\tau)$, Mach speed $M(\tau)$, and modulus of speed $V(\tau)$ on the axis of symmetry $\theta=0$ practically do not depend on the selection of N, and are represented in the table. The density, speed, and Mach speed approach infinity monotonously with $\tau\to 0$.

The solution of system (3.13) may be continued into the randomly small vicinity of the point $\tau=0$, which indicates the absence of a departed shock wave in front of the gravitational center for a gas with $\gamma=5/3$. The density $\rho(\tau,\theta)$, modulus of speed $V(\tau,\theta)$, and function of the current lines $S(\tau,\theta)$ were calculated approximately according to the formulas

$$\rho=(d_0+d_2\theta^2+\ldots+d_{2N-2}\theta^{2N-2})^{-1}, \qquad /4.1/$$

$$V=(V_0+V_1\theta^2+\ldots+V_{2N}\theta^{2N})^{1/2}, \qquad /4.2/$$

$$S=S_1\theta^2+\ldots+S_{2N}\theta^{2N}. \qquad /4.3/$$

For N=10, the level lines ρ, V, and S are given in figures 1, 2, and 3, respectively. Here, the x-axis is the axis of symmetry. The gravitational center is located at the origin of the coordinates. Plotted along the axes of the coordinates are the distances in Bondi radii ($R_B=G\cdot M/C_\infty^2$). The gas flows into the center from the right.

The calculation of the level lines of ρ, V, S for N=5, 10, 15, and 20 shows that, in the range $0<\theta<\pi/2$, the values of ρ, V, and S practically do not depend on N, i.e., there occurs /10 convergence of the approximated solution to the potential flow.

For $\pi/2<\theta<\pi$, and especially close to the axis of symmetry $\theta=\pi$, the approximated values of ρ, V, and σ depend strongly on N. In this region, there is no convergence to the point solution, since the flow is nonpotential.

τ	$\rho(\tau)$	$M(\tau)$	$V(\tau)$
10.	1.[illegible]01	2.4412	2.4413
6.	1.0002	2.4[illegible]2	2.4684
4.	1.0005	2.5014	2.5018
2.	1.0018	2.5978	2.5903
1.	1.0061	2.7779	2.7835
0.5	1.0188	3.0987	3.1181
0.1	1.1[illegible]28	4.8124	5.0460
0.01	[illegible].0644	11.214	14.279
0.001	5.[illegible]078	25.635	44.717
0.0001	15.733	56.424	141.39

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Density ρ

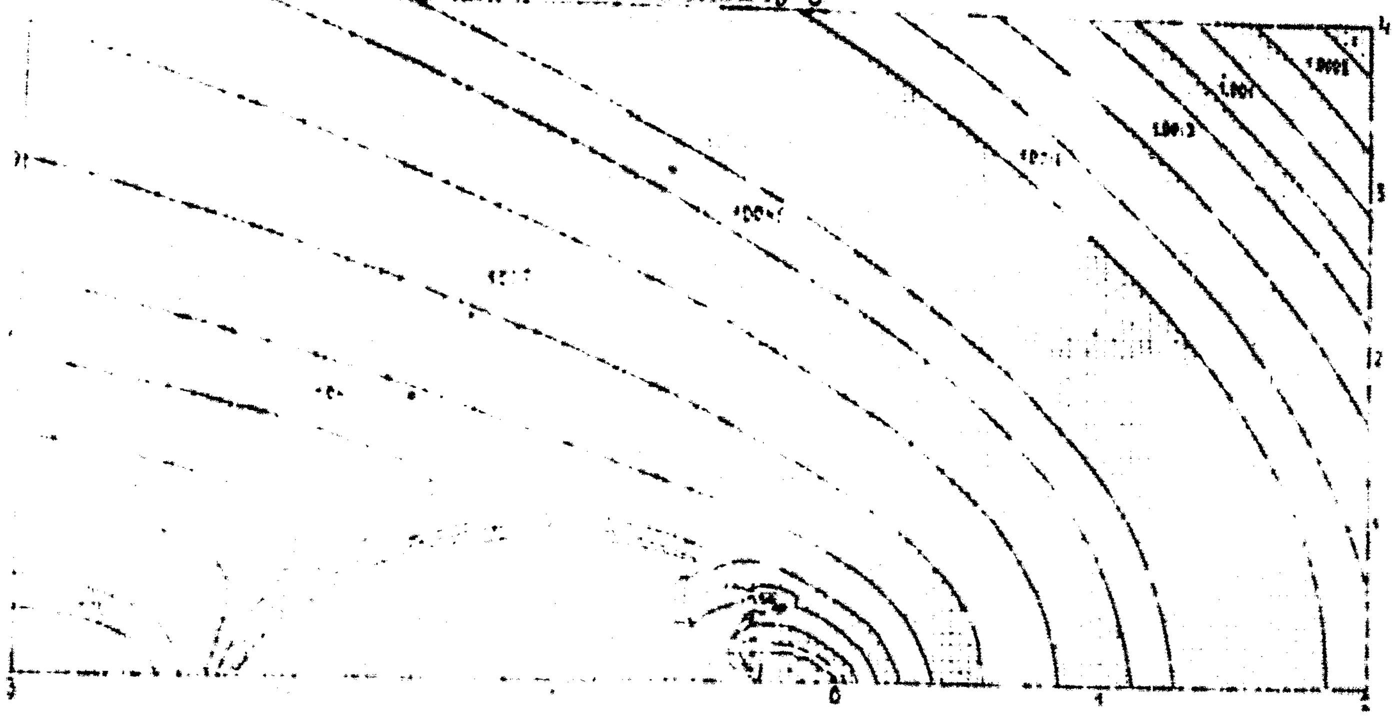

Figure 1

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Modulus of Speed V

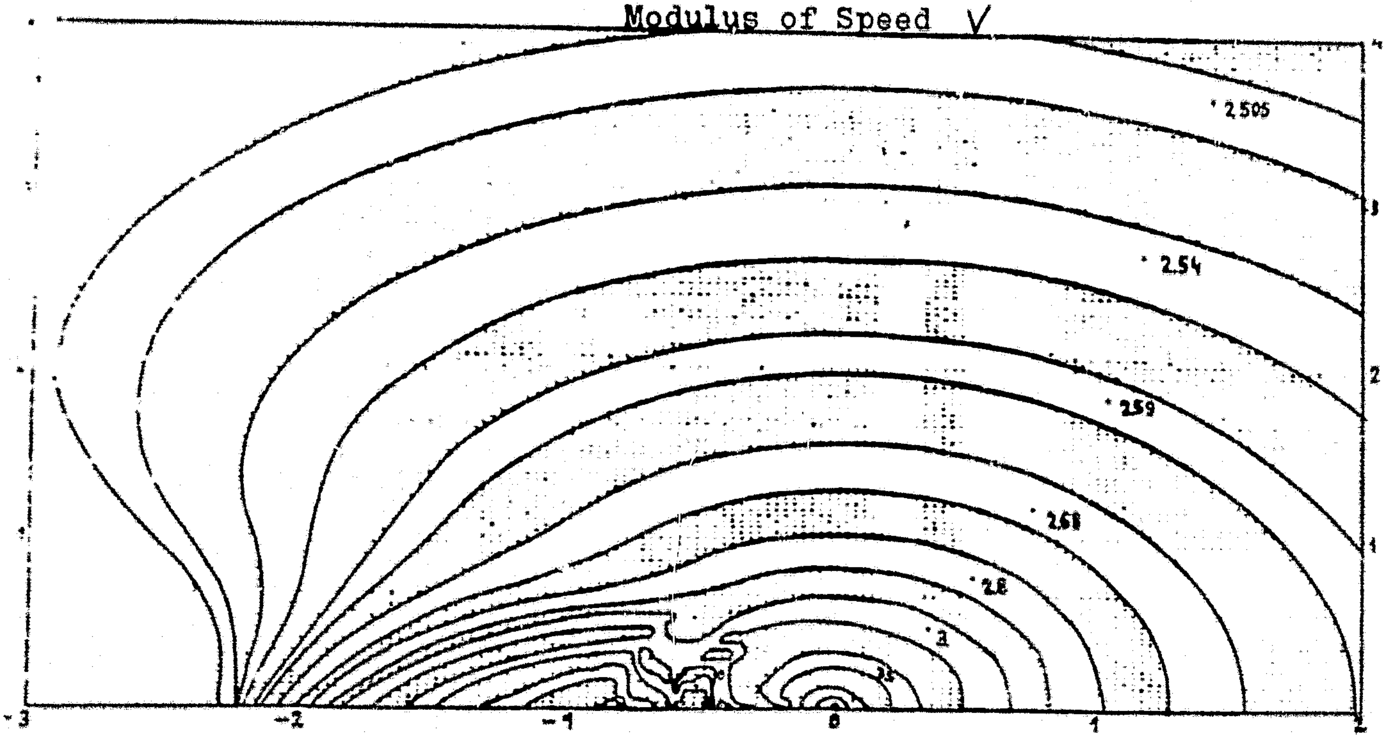

Figure 2

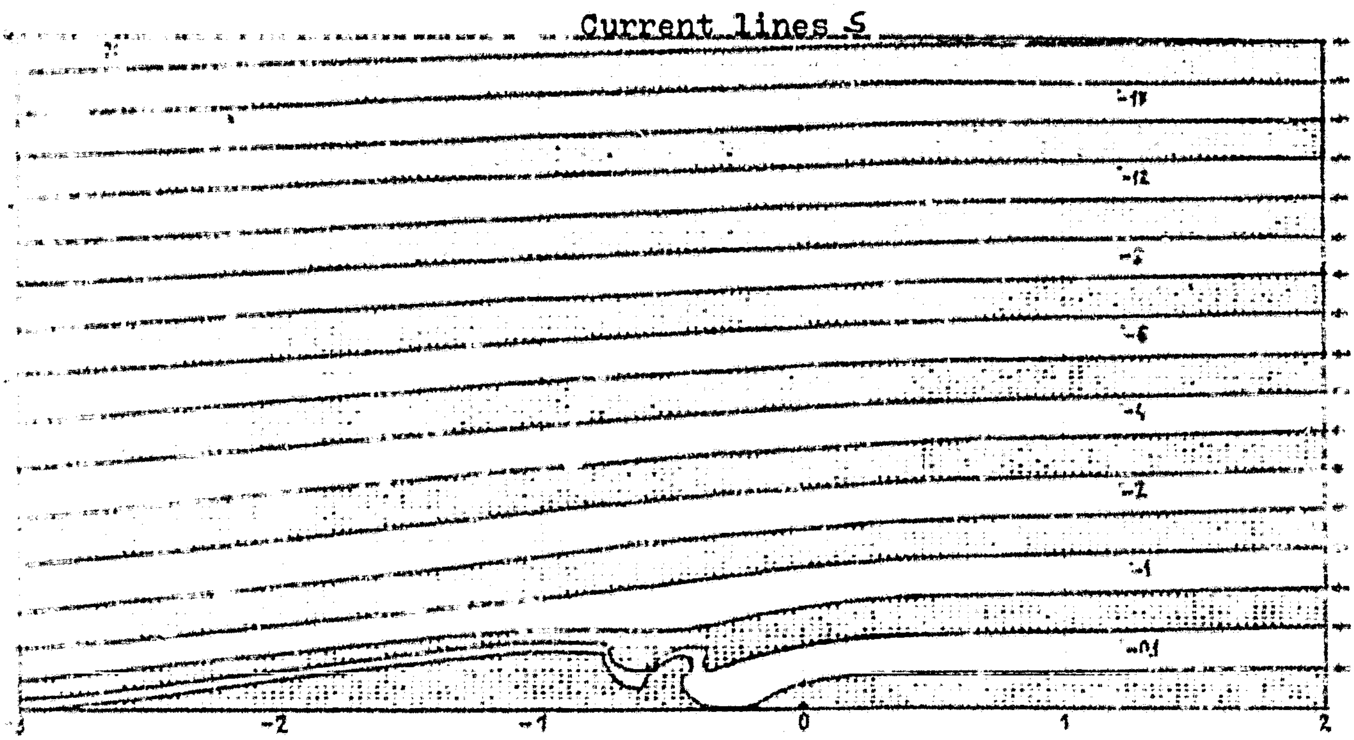

Figure 3