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DETERMINATION OF MAGNETIC HELICITY IN THE SOLAR WIND AND IMPLICATIONS FOR COSMIC RAY PROPAGATION

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ABSTRACT

The mean value of the correlation between local magnetic field and vector potential, known as the magnetic helicity, is a measure of the lack of mirror reflection symmetry of magnetic covariances in a turbulent medium. We present a method for extraction of helicity spectra from magnetometer data, and apply it to an evaluation of the magnetic helicity of interplanetary magnetic fluctuations. Circumstances in which the effects of helicity on the pitch angle scattering of cosmic rays is strongly influenced by the helicity spectrum are discussed in the following paper.

1. Introduction

Magnetic helicity, the mean value of the correlation between a turbulent magnetic field $\underline{B}$ and the magnetic vector potential $\underline{A}$ may be defined as

$$H_m = \int d^3x \; \underline{A}\cdot\underline{B} \tag{1}$$

where $\underline{B} = \nabla\times\underline{A}$ and $\nabla\cdot\underline{A} = 0$. The pseudoscalar H_m measures the departure of $\underline{B}$ from mirror symmetry. Non-zero H_m may be thought of as signifying a net topological linkage of magnetic flux tubes, or a net handedness of the magnetic spectrum (Moffat, 1978). If the fields are decomposed into Fourier modes $\underline{A}(\underline{k})$ and $\underline{B}(\underline{k})$, H_m can be written $H_m = \Sigma\, H_m(\underline{k})$, where the summation is over $\underline{k}$. $H_m(\underline{k}) = \underline{A}^*(\underline{k})\cdot\underline{B}(\underline{k})$ takes on its maximum absolute value when $\underline{A}(\underline{k})$ corresponds to a pure left of right handed circularly polarized wave.

Another characterization of the helicity spectrum, which enters the expressions for both the pitch angle and spatial diffusion coefficients for cosmic ray propagation, is

$$\sigma(\underline{k}) = \frac{kH_m(\underline{k})}{E(\underline{k})} \tag{2}$$

where $E(\underline{k}) = |\underline{B}(\underline{k})|^2$ and $-1 \leq \sigma(\underline{k}) \leq 1$.

Magnetic helicity has received a considerable amount of attention since Woltjer (1958) showed that H_m is an integral invariant of three dimensional magnetohydrodynamics (MHD). In the non-steady dissipative selective decay theories (Matthaeus and Montgomery, 1981), the conjecture is made that while magnetic energy is generally transferred to small scales where it is dissipated, the magnetic helicity is preferentially transferred to large scales and is nearly conserved in the limit of large Reynolds number.

2. Theory

In the past, tests of MHD turbulence theories have been almost completely limited to computer simulations of analogous two-dimensional conjectures, probably because it has not been clear that either H_m, or $\underline{A}$ are unambiguously measurable. We present here a summary of a technique for determination of H_m and its "reduced" spectrum from the two point magnetic correlation matrix,

$$R_{ij}(\underline{r}) = \langle B_i(\underline{x}) B_j(\underline{x}+\underline{r})\rangle \tag{3}$$

where only homogeneity is assumed $[R_{ij}(\underline{r}) = R_{ji}(-\underline{r})]$.

It can be shown (Matthaeus et al., 1981) that the antisymmetric part of $R_{ij}(\underline{r})$ always takes the form

$$\frac{1}{2}\,[R_{ij}(\underline{r}) - R_{ji}(\underline{r})] = \epsilon_{ijl}\,\frac{\partial}{\partial r_l}\,\Phi(\underline{r}) \tag{4}$$

where $\Phi(\underline{r}) = \Phi(-\underline{r})$ is a scalar function.

In terms of the energy spectrum tensor

$$R_{ij}(\underline{k}) = \left(\frac{1}{2\pi}\right)^3 \int d\underline{x}\, e^{-i\underline{k}\cdot\underline{r}}\, R_{ij}(\underline{r})$$

equation (4) is equivalent to

$$\frac{1}{2}\,[R_{ij}(\underline{k}) - R_{ji}(\underline{k})] = \epsilon_{ijl}\underline{k}_l G(\underline{k}) \tag{5}$$

where the even function $G(\underline{k})$ can be shown to equal $iH_m(\underline{k})/2$. Thus the total helicity is $2\Phi(\underline{r}=0) = H_m$.

3. Observations

To apply the above formalism to solar wind magnetic fluctuations, for which only single point measurements are available, we employ the usual "frozen in approximation". If $\hat{r}_1$ is the direction of solar wind flow (in the radial direction), the known values of the correlation (3) are just $R_{ij}(r_1, 0, 0)$. Then $\Phi(r_1, 0, 0)$ and the reduced spectral tensor $S_{ij}(k_1) = \int dk_2 \int dk_3 R_{ij}(k_1, k_2, k_3)$ can be determined.

The reduced helicity spectrum is

$$H_m(k_1) = 2 \text{ Im } S_{23}(k_1)/k_1$$

while the total helicity becomes

$$H_m = \int dk_1 H_m(k_1) = 2\Phi(r_1=0, 0, 0).$$

We have applied this approach to an analysis of several periods of Voyager 2 interplanetary magnetometer data near 2.8 AU. During this period the correlation length, or energy containing length, was found to be $L_C \sim 3 \times 10^{11}$ cm. The helicity containing length $L_H \sim 2 \times 10^{12}$ cm. The energy spectra during this period have the usual power law form. The helicity spectrum peaks strongly near the largest scales sampled and oscillates about zero thereafter, with an envelope closely resembling the power spectrum. The relative helicity, $\sigma(k_1)$, oscillates rapidly between values of ±0.85 over lengths ranging from the L_H to r_g, the proton gyroradius. The implications of this result for charged transport are dicussed in the following paper.

Some rather tentative conclusions concerning solar wind field structure may be drawn from this analysis. The magnetic field appears to be characterized by a well defined sense of twist in each particular region. The length scale of the twist is somewhat larger than that of the typical energy containing magnetic fluctuation structures, but much less than the characteristic length of the variation of the mean field, the Parker spiral. This picture is reminiscent of one proposed by Lee and Fisk (1981) in their discussion of the role of large scale drifts in cosmic ray modulation. The helicity of the fluctuations may well be appearing at these large scales due to selective decay processes occuring in transit from the sun, a possiblity we are presently investigating.

4. Acknowledgments

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THE ROLE OF MAGNETIC HELICITY IN COSMIC RAY TRANSPORT THEORY

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ABSTRACT

The pitch angle scattering of cosmic rays can be modified greatly if the medium through which they propagate contains significant amounts of magnetic helicity. When helicity is present in the medium, the often used expressions for the pitch angle diffusion coefficient derived from quasilinear theory must be modified. We illustrate these modifications for "slab" and "isotropic" symmetries. Using measurements of helicity in the solar wind determined from single point measurements of the correlation tensor of magnetic fluctuations obtained from Voyager 2, the effects of the observed helicity on both the pitch angle and spatial diffusion coefficients can be estimated.

1. Introduction

For some years it has been apparent that the computed values of the mean free path for pitch angle scattering of low rigidity cosmic rays computed from weak turbulence theory are systematically larger than the values inferred from analyses of solar particle events. An extensive review of this problem can be found in Fisk (1979).

Goldstein (1980) has argued that the discrepancy may arise because the observed fluctuations in the interplanetary magnetic fields tend to be constrained in ways not usually included in the weak turbulence formalism. In particular, the magnitude of the field is known to be nearly constant, with a root mean square fluctuation, η, that is typically less than 5% (Chang and Nishida, 1973). Inclusion of this constraint could, at least in principle, lessen the discrepancy between theory and observation.

There are, however, other properties of the solar wind magnetic fields not usually included in cosmic ray propagation theory in addition to the constancy of the field magnitude. In particular, the resonant scattering theory usually relates the pitch angle scattering coefficient, D_μ, only to the power spectrum of the magnetic fluctuations (μ is the cosine of the particle's pitch angle). However, other components of the spectral tensor of the magnetic field fluctuations can contribute to pitch angle scattering. The contributions of these additional terms can be estimated from the interplanetary magnetic field, and we show below that these additional contributions can be significant. The formalism we present includes the possibility that the field magnitude is constant, and thus among other things, provides a formal basis for the heuristic arguments made by Goldstein (1980).

2. Theory

In computing the pitch angle scattering coefficient from weak turbulence theory, one is required to assume something about the symmetry properties of the magnetic fluctuations which characterize the medium. Unfortunately, because present observations are limited to single point measurements in a supersonic flow, it is impossible to derive those properties from the data (Fredicks and Coroniti, 1976). Thus, from the onset, one must construct models which, one hopes, adequately reflect those properties that can be measured. Common to all models, and fundamental to any theory which treats the interplanetary medium as an example of turbulence, is the assumption that the medium is homogeneous. The two models generally used in propagation theory are the "slab" and "isotropic".

An additional assumption often employed, albeit sometimes tacitly; is that the turbulence is mirror reflection invariant. The role played by this assumption in the theory will become more apparent in the ensuing discussion.

The structure of correlation tensors in homogeneous anisotropic turbulence has recently been analyzed by Matthaeus and Smith (1981). The general form of such tensors is too lengthy to reproduce here, but their results are readily adapted to the simpler, and more familiar, cases of slab and isotropic turbulence. For brevity, details of our analysis will not be given here.

For the slab model, one can show that the Fourier transform of the correlation tensor $R_{ij}(\underline{r})$ can be written as

$$R_{ij}(\underline{k}) = \delta(\underline{k}_{\perp})[R(k_{\parallel})N_{ij} + iH(k_{\parallel})k_{\parallel}\Omega_{ij}] \tag{1}$$

where, $N_{ij} = \delta_{ij} - \beta_i\beta_j$, $\Omega_{ij} = \epsilon_{ijk}\beta_k$, and $\hat{\beta}$ is a unit vector in the direction of the mean magnetic field, $\underline{B}_o$. The function $R(k_{\parallel})$ is the usual reduced power spectrum defined so that

$$\mathrm{Tr}\ R_{ij}(\underline{r}=0) \equiv R_{ii}(\underline{r}=0) = \frac{2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk_{\parallel} R(k_{\parallel}) = \frac{\langle B'(x)B'(x)\rangle}{B'^2_{rms}} = 1 \tag{2}$$

The second term in (1) is a measure of the lack of mirror symmetry in the medium. Equivalently, it is related to the degree of circular polarization or helical twist present in each mode, $k_{\parallel}$. That $R_{ij}(\underline{k})$ can be written in the form (1) has been noted before (Hasselmann and Wibberenz, 1968). What has not been appreciated is the fact that $H(k_{\parallel})$ can be obtained from magnetic field data in as straightforward a manner as the power spectrum, $R(k_{\parallel})$.

To demonstrate how $H(k_{\parallel})$ is related to $R_{ij}(\underline{r})$, let us define the magnetic helicity, $H_m \equiv \langle \underline{A}\cdot\underline{B} \rangle$, where $\underline{A}$ is the vector potential chosen so that $\nabla\cdot\underline{A}=0$. Because $\underline{B} = \nabla\times\underline{A}$, the correlation $\langle \underline{AB} \rangle$ can be related to $R_{ij}(\underline{r})$.

In fact,

$$R_{ij}(\underline{k}) = i\epsilon_{jlm}k_l H_{im}(\underline{k}) \quad \text{or,} \quad H_{ij}(\underline{k}) = i\epsilon_{jkl}k_k R_{il}(\underline{k})/k^2 \tag{3}$$

Thus, with (1) for $R_{ij}(\underline{k})$, we find that $H(k_\parallel)$ is the reduced helicity spectrum, where

$$H_m = \frac{2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk_\parallel H(k_\parallel) = \frac{\langle A\cdot B' \rangle}{B'^2_{rms}} \tag{4}$$

When (1) is used to compute D_μ, one has (Goldstein et al., 1975)

$$D_\mu = \sqrt{(2\pi)}(\epsilon\eta)^2(1-\mu^2)R(\epsilon/|\mu|)[1-\sigma(\epsilon/|\mu|)]/|\mu| \tag{5}$$

where $\epsilon \equiv \lambda_c/r_g$, the ratio of the correlation length to the Larmor radius, and $|\sigma(k_\parallel)| \equiv |k_\parallel H(k_\parallel)/R(k_\parallel)| \le 1$. The spatial diffusion coefficient can then be written as (Hasselmann and Wibberenz, 1968)

$$\kappa_\parallel = \frac{\epsilon}{2}\int_0^1 d\mu \frac{(1-\mu^2)^2}{D_\mu[1-\sigma^2(\mu)]} \tag{6}$$

A similar calculation for the isotropic model can be performed. In this case $R_{ij}(\underline{k})$ is

$$R_{ij}(\underline{k}) = R(k)(\delta_{ij} - k_i k_j/k^2) + iH(k)\epsilon_{ijk}k_k \tag{7}$$

$\kappa_\parallel$ for the isotropic model differs from (6) in that $D_\mu \rightarrow D^e_\mu$, and σ^2 is replaced by D^o_μ/D^e_μ, with $D_\mu = D^e_\mu + D^o_\mu$ satisfying $D^e_\mu(-\mu) = D^e_\mu(\mu)$ and $D^o_\mu(-\mu) = -D^o_\mu(\mu)$. The even function D^e_μ arises from the first term in (7), while D^o_μ follows from the second term and is proportional to the helicity spectrum. Unlike the slab model, in the isotropic model $\kappa_\parallel$ is not infinite if $\sigma(k) = \pm 1$. Therefore, it is possible to estimate the maximal effect on $\kappa_\parallel$ in this limiting case. We have evaluated $\kappa_\parallel$ numerically (ignoring the effects of the Landau resonance which is not influenced by the existence of helicity) for several choices of the power spectral index. In all cases, $\kappa_\parallel(\sigma=\pm 1) \approx \kappa_\parallel(\sigma=0)$. That is, when the turbulence is isotropic, the presence or absence of helicity can have no substantial effect on particle propagation.

Therefore, the existence of helicity will influence pitch angle scattering only to the extent that the solar wind can be modeled as a slab.

3. Observations

Whether the slab is a valid representation of solar wind fluctuations will not concern us here, rather we ask the question: is the amount of helicity in the solar wind sufficient to influence particle propagation even if one assumes that a slab model is appropriate? The techniques for extracting σ are discussed in Matthaeus and Goldstein (1981) and Matthaeus et al. (1981). We have used Voyager 2 magnetometer data near 3 AU in this preliminary analysis. This 55 hour span of data may not be typical, however, we find that during this period the reduced spectrum of σ (reduced in the sense of being obtained from a single point measurement) oscillates rapidly as a function of k with maximal excursions reaching ± 0.85. Because the reduced spectrum represents a lower bound on σ, these preliminary data suggest that much of the solar wind at that time is comprised of fluctuations with nearly maximal helicity. Quantitative estimates of the effects of these large values of σ on $\kappa_{\parallel}$, and more detailed presentation of the data will be deferred to a more extensive paper.

4. Acknowledgments

We would like to thank the Voyager magnetometer team, N. F. Ness, Principal Investigator, for their help and cooperation in the data analysis. Dr. Matthaeus is a NRC/NAS Resident Posdoctoral Associate.

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