THE COOLING AND CONDENSATION OF FLARE CORONAL PLASMA

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ABSTRACT

We investigate a model for the decay of flare heated coronal loops in which rapid radiative cooling at the loop base creates strong pressure gradients which, in turn, generate large (supersonic) downward flows. Hence, the coronal material cools and "condenses" onto the flare chromosphere. The important features of this model which distinguish it from previous models of flare cooling are: (1) Most of the thermal energy of the coronal plasma may be lost by mass motion rather than by conduction or coronal radiation. (2) Flare loops are not isobaric during their decay phase, and large downward velocities are present near the footpoints. (3) The differential emission measure $q$ has a strong temperature dependence, $q \propto T^{3.5}$.

These results can account for recent observations of compact flare loops that are not consistent with the previous cooling models.
I. INTRODUCTION

Recent observations (Dere et al. 1977, Underwood et al. 1978, Dere and Cook 1979, and Widing and Spicer 1980) indicate that, during the decay phase of at least some compact flares, the differential emission measure \( q \) has a strong dependence on temperature for the hot coronal material. These authors found that \( q(T) \propto T^\delta \) with \( \delta > 3.0 \) for \( 10^5.5 \leq T \leq 10^7 \) K, where \( q \) is defined as:

\[
q = A n^2 \frac{1}{T} \frac{dT}{ds}^{-1},
\]

(1)

where \( A \) is the area of the emitting region and \( n \) is the electron density. Underwood et al. (1978) point out that this result is not compatible with models of a flare loop in which conduction to the chromosphere is the dominant energy loss mechanism of the flare plasma and suggest that radiation may dominate the cooling, particularly in view of the observation that the coronal density tends to be high in compact flares, (Moore et al. 1979). However, Antiochos (1980) found that models in which radiation dominates were also incompatible with the observations since such models also predict a weak dependence of \( q \) on \( T \), specifically, \( \delta \leq 1.5 \) over most of the temperature range.

Widing and Spicer (1980) have recently suggested that the observed large value for \( \delta \) may be due to a collection of many loops with different temperatures. This proposal is essentially identical to that of Dere and Cook (1979). However, as pointed out by Antiochos (1980), this interpretation is a tenable one if, for at least one of the loops, \( \delta > 1.5 \). Widing and Spicer (1980) do not justify such an assumption since
they do not discuss the form of the differential emission measure of each loop. It is, in fact, possible for the existing cooling models of flare loops to yield a large value for $6$ (Antiochos 1980), but only over a very small temperature range and only if there is a large variation in the cross-sectional area of the loops from their tops to their footpoints (Antiochos 1980). Although a carefully selected combination of such loops can reproduce the observed form for the differential emission measure, there are other observations that are in conflict with the existing models.

A feature common to both the conduction and the radiation dominated models is that all velocities are assumed to be small compared to the sound speed, so that the plasma in any particular flare loop is approximately isobaric. Several observations appear to be at odds with this assumption. Underwood et al. (1978) observed supersonic downward velocities in the cooler regions of the flare plasma ($T \lesssim 10^5$ K). Dere and Cook (1979) and Widing and Spicer (1980) inferred large pressure differences between the hot material ($T \sim 10^7$ K) and the cooler ($T \sim 10^5$ K), the hot plasma having a much larger pressure (by more than an order of magnitude). In addition, Cook and Dere (1979) have observed large downward velocities in the decay phase of a compact flare and have inferred that the enthalpy flux due to these velocities is an important energy loss mechanism for the coronal flare plasma. Such large velocities cannot be reconciled with the existing models even if a combination of many loops is assumed.

The results discussed above have led us to consider a model of a flare loop in which the main energy loss is due to mass motions, specifically that coronal material cools and condenses at high (supersonic)
velocity onto the flare chromosphere. In the next section, we investigate this model in detail and determine the resulting form for the differential emission measure.

II. MODEL

From physical considerations, and in order to simplify the calculations, we divide a cooling flare loop into three distinct physical regions characterized by their temperatures. The bulk of the loop volume and mass is essentially at the maximum coronal temperature \( T > 10^7 \) K. [It is well known that flare loops appear to be almost isothermal all the way down to their footpoints (Dere and Cook 1979), and the form of the differential emission measure (the large values of \( \delta \) implies that most of the flare mass and volume must be at the highest temperature.] At the base of the loop is cool plasma \( T \leq 10^5 \) K which we consider to be part of the flare chromosphere. Material at intermediate temperatures \( 10^5 \leq T \leq 10^7 \) K forms the flare transition region.

We expect that physical conditions are quite different in each of the three regions. The relevant time scales for the evolution of the plasma (the conductive and radiative cooling time scales) have strong temperature dependences (see, for example, Antiochos 1980):

\[
\tau_c \propto P H^2 T^{-7/2}, \tag{2a}
\]

and

\[
\tau_r \propto P^{-1} T^{5/2}, \tag{2b}
\]

where \( P \) is the pressure, \( H \) is the temperature scale height, and we note that the radiative loss coefficient for optically thin coronal
plasma (Cox and Tucker 1969 and Raymond et al. 1976) varies approximately as $T^{-1/2}$. It is evident from relations (2) that the low temperature material has the shortest cooling time since it loses energy very rapidly by radiation. We expect this to be the case for the cool base plasma. On the other hand, we expect that the $10^7$ K plasma has a relatively long cooling time because its radiative time scale is much longer [from (2b), approximately five orders of magnitude larger than for the $10^5$ K material], and its conductive time scale is known to be of the same order as its radiative time scale (Moore et al. 1979). If the base region of a flare loop does cool more rapidly than the hot region, we expect that strong pressure gradients will be established and large downward velocities will develop in the intermediate transition region.

This situation is, indeed, found to occur in numerical simulation of the decay of flare loops (Antiochos and Krall 1979 and Antiochos 1980a). In these calculations, it was found that a cool region formed at the loop base and that large downward velocities were generated near the base. However, due to numerical difficulties associated with the small size scale of the transition region (Antiochos and Krall 1979), it was not possible by numerical simulation to determine the detailed temperature and density profiles in the transition region and hence obtain the differential emission measure.

Therefore, we make the key simplification in this model that the transition region is assumed to be in a steady state. Of course, a flare loop is never in a true steady state during its decay phase since the plasma is continuously cooling. However, we expect that the time scale for the evolution of the loop as a whole is the cooling time of the hot region, which is large compared to the time scale either for
cooling or for mass propagation in the transition region. The physical model that we propose is that the hot coronal plasma, which comprises the bulk of the loop, acts as a reservoir of mass and heat for a steady-state flow through the transition region onto the cool base, which acts as both a mass sink and a heat sink because it is such an efficient radiator. Since the time scale for the evolution of the hot reservoir is long compared to the mass propagation time scale in the transition region, an approximately steady-state flow may be established. We check subsequently that this condition is, indeed, satisfied by our model.

III. EQUATIONS

Under the assumption that the flow is in a steady state, the relevant equations for the transition region are

\[ \rho(s) v(s) = \rho_0 v_0 , \quad (3) \]

\[ \rho v^2 + P = \rho_0 v_0^2 + P_0 , \quad (4) \]

and

\[ \frac{d}{ds} \left( \frac{1}{2} \rho v^3 + \frac{5}{2} \rho v^2 \frac{dT}{ds} \right) = -n^2 \Lambda(T) , \quad (5) \]

together with the equation of state,

\[ P = 2k nT . \quad (6) \]

In equations (3) through (5), \( \rho \) is the mass density, \( v \) is the velocity along the loop (i.e., parallel to the magnetic field), \( \kappa \) is the coefficient of conduction given by Spitzer (1962) as
\[ \kappa = 10^{-6} T^{5/2}, \quad (7) \]

\( A(T) \) is the radiative loss coefficient which we assume has the simple form

\[ A(T) = 10^{-19.2} T^{-1/2} \quad (8) \]

in the temperature range \( 10^5 \leq T \leq 10^7 \), and the subscript "o" indicates that the variable is to be evaluated at the top of the transition region which is defined to be at \( s = 0 \) (\( s \) increases downwards).

Since the size scale for the transition region is small compared to both the gravitational scale height and the size scale for variations of the magnetic field, we neglect the effects of gravity and a variable cross-sectional area for the loop in equations (3) through (5).

Much insight into the possible forms of the plasma flow can be obtained by examining only the equations of continuity and momentum, (3) and (4). Combining these, the temperature can be expressed in terms of the velocity

\[ T = \frac{m}{2k} v(v_b - v), \quad (9) \]

where

\[ v_b = v_o \left( 1 + \frac{3}{5M_o^2} \right) \quad (10) \]

and \( M_o \) is the initial Mach number,

\[ M_o^2 = \frac{v_o^2}{\frac{5}{3} \rho_o} \] \quad (11)

Equation (9) describes a simple parabola. There are two types of solutions corresponding to the two branches of the parabola: as \( T \) decreases to zero, either \( v \) decreases to zero or \( v \) increases to \( v_b \).
The former case corresponds to subsonic flow and leads back to the conductive and radiative models discussed previously (Antiochos 1980); we are now interested in the latter case.

For this case, the smallest possible initial velocity is \( v_0 = \frac{v_b}{2} \), since this corresponds to the maximum point of the parabola. This selection for \( v_0 \) implies, by equation (11), that we are choosing the upper boundary of the steady state region to occur where the flow begins to be supersonic; the initial Mach number is \( M_0^2 = 3/5 \). It is now evident that the velocity does not vary greatly throughout the transition region: the velocity at the loop base must be less than a factor of two greater than at the top. Since the density is inversely proportional to the velocity, by equation (3), the density also must vary little in the steady state region and actually decreases slightly as the temperature decreases. Clearly, the loop plasma is far from being isobaric.

In order to obtain the differential emission measure and, hence, \( \delta \), the heat equation (5) must be solved for the temperature gradients. It is convenient to do so using dimensionless variables. We define

\[
\theta = \frac{T}{T_0} \tag{12}
\]

and

\[
x = s \left( \frac{2kT_0}{\rho p_0 v_0} \right)^{-1} \quad . \tag{13}
\]

Expressed in terms of these variables and using (9), equation (5) becomes

\[
\frac{d}{dx} \left( 2\theta + \sqrt{1 - \theta} - \frac{d\theta^{7/2}}{dx} \right) = -\frac{\alpha}{\sqrt{\theta(2 + 2\sqrt{1 - \theta} - \theta)}} \quad , \tag{14}
\]
where $\alpha$ is a numerical constant,

$$\alpha = 10^{-2.7}.$$  \hfill (15)

Two boundary conditions must now be specified. One is simply that

$$\Theta(0) = 1.$$ \hfill (16)

The other condition is obtained by specifying the initial temperature gradient or, equivalently, $f_0$, where

$$f = -\frac{d\Theta^{7/2}}{dx}.$$ \hfill (17)

Changing back to physical variables, we note that $f_0$ corresponds to the initial ratio of the conductive heat flux to the mechanical energy flux,

$$f_0 = \frac{\kappa(T_0)}{p v_o} \left(\frac{dT}{ds}\right)_0.$$ \hfill (18)

Since this quantity is not known a priori, we use it as a free parameter and investigate the forms of the solutions for various $f_0$. Although equation (14) is highly nonlinear, it has the basic form appropriate for an initial value problem and can be readily solved numerically using a standard ordinary differential equation routine.

**IV. RESULTS**

In Figure 1, the variation of $f$ with $\Theta$ along the loop is shown for various values of $f_0$. Since $f$ is directly proportional to the
heat flux, Figure 1 corresponds to a plot of heat flux versus temperature. As is indicated by the figure, there are two classes of solutions separated by the critical initial flux, \( f_{oc} \approx 0.148 \). For \( f_0 < f_{oc} \), the heat flux vanishes at a finite temperature. These cases are unphysical because they imply an upward heat flux at the base of the transition region. For example, for the case \( f_0 = 0.14 \), the flux changes sign at \( \theta = 0.95 \), so that the temperature profile has a minimum at this value.

We conclude that, if the initial heat flux is too small, it is not possible to obtain a temperature profile in which the temperature decreases monotonically down to chromospheric values. This result has also been found for static models of the transition region (e.g., Moore and Fung 1972). In the static models, the lower limit to the initial heat flux is due basically to the requirement that the conductive flux be sufficiently large to power the radiative losses. Although this requirement also applies to our steady-state model, we find that the effect of the mass motion is to place an even more stringent requirement on the initial heat flux.

Using (17) to rewrite the energy equation (14) in terms of \( \theta \) as the independent variable, we obtain

\[
\left(2 - \frac{1}{2\sqrt{1 - \theta}}\right) f + f \frac{df}{d\theta} = \frac{7}{2} \alpha (1 - \sqrt{1 - \theta})^2.
\]

The first term represents the effects of mass motion. For \( \theta > 15/16 \), the mass motion acts as a heat sink and the heat flux must decrease from its initial value. For \( \theta < 15/16 \), on the other hand, mass motion acts as a heat source and the heat flux may increase depending on the relative
importance of the radiative loss term. If, for the moment, we neglect the radiative losses, equation (19) may be integrated directly to yield

\[ f = f_0 + 2(1 - \theta) - \sqrt{1 - \theta} \quad . \quad (20) \]

Since the radiative losses can act only to decrease \( f \), equation (20) may be regarded as setting an upper limit to the heat flux at any temperature in the transition region.

It is evident from (20) that, if \( f_0 < 1/8 \), the heat flux will vanish at some value of \( \theta > 15/16 \). Even if \( f_0 > 1/8 \), the heat flux may still vanish at a finite \( \theta \) due to the effects of radiative losses; as noted previously, this will occur if \( f_0 < f_{oc} = 0.148 \). However, if \( f_0 \) is sufficiently near the critical value \( f_{oc} \), then even though \( f \) may vanish at a finite value of \( \theta \), this value may be so small as to be well below the temperature at the base of our model, \( \theta_{min} > 10^{-2} \), so that these cases will be physically acceptable because they imply a monotonically decreasing temperature throughout the transition region.

It turns out, however, that these cases do not lead to a steep dependence of the differential emission measure on temperature. For example, consider the case \( f = f_{oc} \). The heat flux vanishes at \( \theta = 0 \) where it has a minimum. For \( \theta \ll 1 \), equation (19) becomes

\[ \frac{3}{2} f + f \frac{df}{d\theta} = \frac{7}{8} \alpha \theta^2 \quad . \quad (21) \]

Since \( f \) has a minimum at \( \theta = 0 \), \( df/d\theta \) vanishes there and thus we obtain that

\[ f = \frac{7}{12} \alpha \theta^2 \quad . \quad (22) \]
near $\theta = 0$. However, from equation (1),

$$q \propto n^2 \frac{\theta^{7/2}}{|f|}.$$  \hspace{1cm} (23)

Since, as previously noted, $n$ is approximately constant in the steady state region, we find that

$$q \propto \theta^{3/2}.$$  \hspace{1cm} (24)

This result is confirmed by exact solution of (19). In Figure 2, $q$ is plotted on a function of $\theta$ for $f_0 = f_{OC}$. It is evident that, over most of the temperature range, $q \propto \theta^{3/2}$ so that $\delta$ is no larger than that characteristic of the static models.

The cases that are of most interest to us are those with $f_0 > f_{OC}$ since these exhibit the largest value for $\delta$. From Figure 1, we note that, for $f_0 \geq 0.15$, the heat flux is approximately constant. In fact, if $f_0$ is sufficiently large, then we expect that the radiative losses are negligible and equation (20) is valid. But equation (2) indicates that $f$ actually increases as $\theta$ decreases (at least for $\theta < 15/16$) and hence we expect from (23) that $\delta > 7/2$. In Figure 2, $q(\theta)$ is plotted for the case $f_0 = 0.15$; we find, indeed, that $\delta \approx 3.5$ over most of the temperature range. We therefore conclude that the steady-state model can produce a sufficiently large value of $\delta$ to account for the flare observations, but only if the parameter $f_0$ (the ratio of conductive flux to mechanical flux at the top of the transition region) is not too small ($f_0 > 0.15$).
On the other hand, \( f_o \) cannot be too large because our use of equations (3) through (5) and, in particular, of the form of the conductivity (7) is only valid if the heat is "unsaturated." The exact magnitude of the conductive flux at which this assumption becomes invalid is not well known. Clearly, the flux cannot be larger than the product of the pressure and the electron sound speed so that \( f_o \leq 40 \); however, there are strong reasons to expect that it is actually limited to values \( (f_o \leq 6) \) substantially less than this (Manheimer 1977).

We also note that, even if the initial heat flux is well within these limits, the model indicates that the heat flux will saturate at some lower temperature because we found that both the flux and the density are approximately constant throughout the steady-state region and, of course, the sound speed varies as \( \theta^{1/2} \) so that

\[
\frac{f}{PC} \propto \theta^{-3/2}.
\]  

(25)

Since we require that \( f_o > 0.15 \) in order to account for the observation of a large \( \delta \), relation (25) implies that our model can be applicable for, at most, one and a half decades in temperature. However, the temperature interval over which \( \delta \) is observed to be large is only a decade or so (Underwood et al. 1978, Dere and Cook 1979, Widing and Spicer 1980), so that our model may, in fact, be applicable throughout the observed interval.

In order for this to occur, the ratio of the conductive flux to the mechanical flux at the top of the transition region must be very nearly equal to 0.15 for all observed flares. This value for \( f_o \) is not
inconsistent with the observed parameters of compact flares. Using (18) and the result that

\[ v_o = (3/5)^{1/2} C_o, \]  

where \( C_o \) is the initial sound speed, we obtain

\[ f_o = 10^{5.4} T_o^{-2} n_o^{-1} H_o^{-1}, \]

where \( H_o \) is the temperature scale height at the top of the steady-state region. Assuming typical flare coronal parameters of \( T_o \sim 10^7 \) K and \( n_o \sim 10^{11} \) cm\(^{-3}\) and that the scale height \( H_o \) is of the order of the loop length, \( H_o \sim 10^{9.3} \) cm, we obtain \( f_o \sim 10^{-0.9} \). Therefore, in view of the large uncertainties in \( T_o, n_o, \) and \( H_o \), for the flares observed so far, the flux \( f_o \) can easily take on values near 0.15.

Even if \( f_o \) is significantly larger than 0.15, so that the model used above is not valid, we believe it likely that the effect of mass motion would still be to yield a large value for \( \delta \). However, to prove this contention, a detailed calculation of the electron distribution function would be required. In addition, the interpretation of the observations would be much more difficult if the plasma is nonthermal (e.g., Shoub 1981).

V. DISCUSSION

The results of the previous section indicate that, in our model, energy loss by mass motion dominates that by conduction for the coronal plasma. The conductive flux is only \( \sim 10\% \) of the enthalpy flux. For the
parameters used in the previous section, we find that the downward velocity at the top of the transition region is $v_0 = 10^{7.6}$ cm sec$^{-1}$, the mass flux is $\rho_0 v_0 = 10^{5.2}$ gm sm$^{-2}$ sec$^{-1}$, and the mechanical energy flux is $F_m = 10^{10}$ ergs cm$^{-2}$ sec$^{-1}$. Although the magnitude of the enthalpy flux is quite large, the effective rate of energy loss is reduced because the cross-sectional area of the loop at the top of the transition region is expected to be somewhat less than that in the corona due to the divergence of the magnetic field lines (see Antiochos and Sturrock 1976). Assuming a compression factor (i.e., ratio of cross-sectional area at the loop top to that at the base) $\Gamma = 10$ implies a time scale for cooling by mass motions of

$$\tau_c = \frac{\Gamma L}{v_0} = 10^{2.7} \text{ sec}.$$ 

Of course, this is also the time scale for draining the coronal mass out of a flare loop.

The time scale above is consistent with the observed lifetimes of compact flare loops. Note that, as claimed in Section II, the time scale is long compared to the mass propagation time across the transition region. For the cases $f_0 > 0.15$, the temperature scale height $H \propto \theta^{7/2}$

and, hence, the propagation time decreases very rapidly for lower temperatures. The situation is similar to that of active region loops in that the fact that the heat flux is approximately constant implies that the size scale of the transition region is very small compared to coronal length scales.
An important consideration is the nature of the base of the steady-state flow region; in particular, the effect on the flare chromosphere of the large mechanical energy flux yielded by our model. Since the flow is supersonic, a shock should occur where the condensing material impinges on the chromosphere. We defer for another article the calculation of the structure of the shocked material, which we expect to be complex and possibly not in a steady state. As discussed previously, we expect that radiation cooling dominates in the base region due to the high density and the strong line emission at $T \sim 10^5$ K (Cox and Tucker 1969); in fact, it is this rapid cooling rate that creates the pressure gradients which drive the flow in the first place. A key feature of the steady-state model is that the large mechanical energy flux acts as an efficient mechanism to transfer the thermal energy of the hot coronal plasma to the flare chromosphere where it can be dissipated as UV and EUV radiation. This distinguishes the present model from previous models for the cooling (Antiochos and Sturrock 1978, Antiochos 1980) in which soft x-ray emission is assumed to be the dominant mechanism for dissipating the thermal energy of the flare corona.

The main conclusion of this paper is that condensation cooling can result in a steep dependence of the differential emission measure on temperature $\delta \approx 3.5$, in a loop. Unfortunately, a flare does not consist of a single loop. If it did, then one would expect to observe a loop structure when viewing the highest temperature line, and two bright points representing the loop footpoints when viewing in cooler lines. Instead, one usually observes a loop structure in lines covering a wide temperature range, $10^5 \lesssim T \lesssim 10^7$, implying that the emission in cooler lines is dominated by emission from cool loops rather than from the base.
points of the hottest loops. Therefore, the fact that the value of $\delta$ is somewhat less than 3.5 and that the density is not approximately constant (as in our model) but tends to decrease with decreasing temperature (Widing and Spicer 1980), can be explained as due to the contribution of cooler loops to the observed emission. We emphasize again, however, that a multi-loop model cannot account for the observation that $\delta > 3$ unless at least some of the loops have this value (or larger values) for $\delta$, and this is not consistent with previous models for flare cooling.

Additionally, the observation of large downward velocities in the decay phase of flares strongly favors the model presented here, but these observations were made for lower temperature lines, below the temperature interval in which $\delta$ is large. In order to critically test the condensation cooling model, observations of high temperature material ($T > 10^6$ K) are required with good spatial resolution, so as to minimize the contamination by cool loops, and with good spectral resolution, so as to accurately measure doppler shifts. Experiments on the SMM satellite may be able to provide such data.

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CAPTIONS

1. The variation of the heat flux $f$ with temperature $\theta$ along the loop for three values of the parameter $f_0$, the flux at the top of the steady-state transition region.

2. The variation of the differential emission measure $q$ with temperature $\theta$ along the loop for two values of the parameter $f_0$. The broken curve refers to the model with $f_0 = f_{0c}$ and the solid curve to $f_0 = 0.15$. 

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FIGURE 1
We investigate a model for the decay of flare heated coronal loops in which rapid radiative cooling at the loop base creates strong pressure gradients which, in turn, generate large (supersonic) downward flows. Hence, the coronal material cools and "condenses" onto the flare chromosphere. The important features of this model which distinguish it from previous models of flare cooling are: (1) Most of the thermal energy of the coronal plasma may be lost by mass motion rather than by conduction or coronal radiation. (2) Flare loops are not isobaric during their decay phase, and large downward velocities
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