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SUBPROGRAMS FOR INTEGRATING THE EQUATIONS OF MOTION OF SATELLITES.

FORTRAN FOUR.

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### Abstract

In this work is contained a description of the subprograms intended for the formation of the right members of the equations of motion of artificial earth satellites (ISZ), integration of systems of differential equations by Adams' method, and the calculation of the values of various functions from the AES parameters of motion.

These subprograms are written in the Fortran IV language and constitute an essential part of the package of applied programs for the calculation of navigational parameters AES.
PREFACE

The present preprint is a continuation of the description, begun in (5), of the description of the library of subprograms arising in the process of creation of a packet of applied programs for the calculation of navigational parameters of artificial Earth satellites of the Earth (AES), wherein are contained the subprograms having indexes from F to I. In the subprograms for the calculation of the right members of the AES equations of motion, presented in Chapter I, the subprograms described in (5) are utilized. Here there operate also the principles of scaling the dimensional quantities, determined in (1). The constants and scale factors for the subprograms of chapters 1, 3, and 4 are routed to the domain COMMON by conversion to the subprogram CONST. The system of coordinates determined in (5) is employed.

The subprogram for integration of the system of differential equations by the method of Adams, described in Chapter 2, on the other hand, is sufficiently autonomous and may be used for integration of any system of ordinary differential equations.

In Chapter 3 are presented subprograms ensuring the fixation of the attainment in the process of integration of the assigned values of different functions from the solution of systems of differential equations; the minimum and maximum of an arbitrary continuous function from the solution as a function of an independent variable; the exit of the AES at the ascending node of the orbit; the minimal and maximal altitudes of the AES above the surface of the terrestrial ellipsoid.

Chapter 4 contains subprograms for the computation of the values
of various functions from the parameters of motion of the AES.

The subprogram ROOT4(Ill), used for the computation of the moments of entry of the AES into the umbra of the earth, is intended for the calculation of the roots of the algebraic equations of the fourth, third and second degree, has a significantly independent character and may be used for the solution of other problems.

The subprogram FA GRAV (F 03) was written by E. E. Ryazanova. For the computation of the geomagnetic parameters B, L the subprograms BL, INVAR, LINES. STAR, CARMEL, INTEG, NEWMAG, ASIN (I 04) are utilized, submitted through the courtesy of Yu. N. Gal'perin and V. M. Sinitsyn. The author of the remaining subprograms is the author of the preprint.

The author wishes to express his thanks to E. A. Chistyakova for assistance in the editing of the texts of the subprograms for publication, to L. V. Zaytseva and V. F. Smirnova for help in the preparation of the manuscript.
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Chapter I. Right members of the system of equations of motion of the AES (index F)

I.1. Equation of motion of the AES. Various models.

We will write the differential equations of motion of the AES in an absolute system of coordinates in the general case in the following form:

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \Delta_N v_x + \Delta_A v_x + \Delta_0 v_x + \Delta_1 v_x + \Delta_L v_x, \\
\dot{v}_y &= \Delta_N v_y + \Delta_A v_y + \Delta_0 v_y + \Delta_1 v_y + \Delta_L v_y, \\
\dot{v}_z &= \Delta_N v_z + \Delta_A v_z + \Delta_0 v_z + \Delta_1 v_z + \Delta_L v_z.
\end{align*}
\]

In the Greenwich relative system of coordinates these equations have the form

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \omega_3^2 v_x + 2 \omega_3 v_y + \Delta_N \dot{v}_x + \Delta_A \dot{v}_x + \Delta_0 \dot{v}_x + \Delta_1 \dot{v}_x + \Delta_L \dot{v}_x, \\
\dot{v}_y &= \omega_3^2 v_y + 2 \omega_3 v_z + \Delta_N \dot{v}_y + \Delta_A \dot{v}_y + \Delta_0 \dot{v}_y + \Delta_1 \dot{v}_y + \Delta_L \dot{v}_y, \\
\dot{v}_z &= \Delta_N \dot{v}_z + \Delta_A \dot{v}_z + \Delta_0 \dot{v}_z + \Delta_1 \dot{v}_z + \Delta_L \dot{v}_z.
\end{align*}
\]

where the projections of acceleration, determined by the influence of the corresponding forces, are:

*Numbers in the margin indicate pagination in the foreign text.*
\[ \Delta N \hat{u}_x, \Delta N \hat{v}_y, \Delta N \hat{w}_z \] -- normal gravitational field of the earth
\[ \Delta A \hat{u}_x, \Delta A \hat{v}_y, \Delta A \hat{w}_z \] -- resistance of the earth's atmosphere
\[ \Delta_o \hat{u}_x, \Delta_o \hat{v}_y, \Delta_o \hat{w}_z \] -- gravitational anomalies
\[ \Delta_g \hat{u}_x, \Delta_g \hat{v}_y, \Delta_g \hat{w}_z \] -- gravitational perturbations of the moon and sun
\[ \Delta_l \hat{u}_x, \Delta_l \hat{v}_y, \Delta_l \hat{w}_z \] -- pressure of light
\[ \omega^3 x, \omega^3 y \] -- centrifugal force
\[ 2 \omega^3 v_y, 2 \omega^3 v_x \] -- force of Coriolis
\[ \omega^3 \] -- angular velocity of the earth's rotation

For the calculation of the right members of the system of equations of motion of AES one has the collection of subprograms: FNGRAV, FAGRAV, FATM, FGR.:., FLIGHT, each of which allows for its component in the right members of the equations of motion of the AES.

These subprograms have a subsidiary nature; from them it is possible to construct the subprogram for calculation of the right-hand sides for a system of equations, with this or that degree of completion for the described motion of the AES. In (4) were introduced the indexes KC, KG, KA, KS, KL and table 2.1, permitting the regulation of the variants of the system of forces, acting on the AES (and of the system of coordinates, in which the motion of the AES is analyzed).

Table 1.1 is a repetition of table 2.1 from work (4).

We recall also the designation of indexes.

The index KS characterizes the system of coordinates (possible values 1, 2).

To each of the forces acting is attached its index:

- KG--force of the earth's attraction (possible values 0,1, 2, 3, 4);
- KA--atmospheric resistance (possible values 0,1, 2, 3, 4);
- KS--gravitational perturbation by the moon and sun (possible values 0,1, 2, 3);
- KL--pressure of light (possible values 0,1, 2).

Giving to each of the above enumerated indexes the value determined, we give the determined model of forces, characterized by the five-valued index, consisting of the values of the indexes.
There is a subprogram FORCE, realizing all possible variants of the models of force, specified by table 1.1. However, for concrete models it is possible to recommend to the user the creation of "truncated" subprograms, which may be obtained from the subprogram FORCE by means of discarding conversions to those subprograms, which are not utilized in the given model.

Table 1.1 System of forces, considered in the equations of motion of the AES. Indexes KC, KG, KA, KS, KL

<table>
<thead>
<tr>
<th>Systems of Coord.</th>
<th>Grav. Field</th>
<th>Atmosphere</th>
<th>Grav. Perturbation</th>
<th>Pressure of Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>KC</td>
<td>KO</td>
<td>KA</td>
<td>KS</td>
<td>KL</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>0 oscilating e.lts.</td>
<td>central</td>
<td>not considered</td>
<td>not considered</td>
<td>not considered</td>
</tr>
<tr>
<td>1 Greenwich</td>
<td>normal</td>
<td>without calc.var.</td>
<td>from moon</td>
<td>without calculation of the earth's umbra</td>
</tr>
<tr>
<td>2 absolute</td>
<td>monal</td>
<td>with calc.of long period var.</td>
<td>from sun</td>
<td>with calculation of the earth's umbra</td>
</tr>
<tr>
<td>3 full anomalies</td>
<td>with calc. of long and short period var.</td>
<td>from moon and sun</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 harmonics</td>
<td>CIRA-72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22,30,40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The subprogram FC1100(FO6) may serve as an example of such a subprogram; in this subprogram is realized the model of the motion of the AES, corresponding to the following values of the indexes: KC equals 1 or 2, KG equals 0 or 1, KA equals 1, KS equals 0, KL equals 0.

In order that the designations of concrete subprograms of the right-hand sides, created by the user, reflect the model, realized in them, of the motion of the AES, it is suggested that one construct identifiers of these subprograms according to this sort of principle: the first letter — F, after it a five-valued index, corresponding to the chosen variant of model from table 1.1 (as this was done in the case of FC1100).
1.2 Calculation of the normal gravitational field of the earth and (for the case of the Greenwich system of coordinates) translational and coriolis accelerations (FON-FNGRAV).

1. Designation. The terms \( \Delta N \dot{V}_x, \Delta N \dot{V}_y, \Delta N \dot{V}_z \) are determined; they are dependent on the influence of the normal gravitational field of the earth in the right members of the equations of motion of the AEs (1.1) in the absolute system of coordinates. If the motion of the AEs is calculated in Greenwich system of coordinates (system of equations 1.2), then these sums are determined:

\[
\Delta N \dot{V}_x + \omega_x^3 \dot{x} + 2 \omega_x \dot{y}, \Delta N \dot{V}_y + \omega_y^3 \dot{y} - 2 \omega_y \dot{x}, \Delta N \dot{V}_z
\]

2. Structure. The subprogram FNGRAV. General units: /CAO/_, /CR/=1, /CAE/=2, /CAEL/=1, /COMZP/.

3. Conversion: CALL FNGRAV (KC, KG, Y, HC, RC, F)

4. Initial data: KC—index, characterizing the system of coordinates (KC = 1 for Greenwich system of coordinates, KC=2 for absolute system of coordinates);

KC—index, characterizing the gravitational field of the earth (for KG equal to zero there is considered only the central field)

\( V_0 \)—main part, containing \( x, y, z, V_x, V_y, V_z \) or \( x, y, z \), \( V_x, V_y, V_z \).

5. Results:

HC—elevation of AEs above the surface of the earth;
RC—modulus of the radius-vector of the AEs;
F—main part of the right members of the equation of motion of the AEs, containing \( x, i, z, V_x, V_y, V_z \) or \( i, j, z, \dot{x}, \dot{y}, \dot{z} \).

6. Utilization of the groups COMMON: The constants of the groups are utilized: /CAO/_, /CR/=1, /CAE/=2, /CAEL/=1, /COMZP/ (see in 4(3)) Nos. 2, 10, 11, 12, 15 table 2.1)

7. Algorithm:

a) absolute system of coordinates;
\[ \dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \]
\[ \dot{v}_x = \Delta_N \dot{v}_x + \omega_3^2 x + 2 \omega_3 v_y, \]
\[ \dot{v}_y = \Delta_N \dot{v}_y + \omega_3^2 y - 2 \omega_3 v_x, \]
\[ \dot{v}_z = \Delta_N \dot{v}_z, \]

where \( C = \frac{3}{2} \alpha_0^2 \frac{(R/r)^2}{(R/r)^2} \), \( D = \frac{3}{2} \frac{Z^2}{r^2} \), \( r = (x^2 + y^2 + z^2)^{1/2} \), \( R \) -- average radius of the earth, \( \alpha_0, \alpha_2 \) -- parameters of the normal gravitational field of the earth; for \( \alpha_2 = 0 \) there is considered only the influence of the central gravitational field of the earth.

b) Greenwich system of coordinates:

\[ \dot{x} = u_x, \quad \dot{y} = u_y, \quad \dot{z} = u_z, \]
\[ \dot{u}_x = \Delta_N \dot{u}_x + \omega_3^2 x + 2 \omega_3 u_y, \]
\[ \dot{u}_y = \Delta_N \dot{u}_y + \omega_3^2 y - 2 \omega_3 u_x, \]
\[ \dot{u}_z = \Delta_N \dot{u}_z, \]

... 

c) elevation \( h_e \) of the AES above the earth's surface ellipsoid is computed according to the formula \( h_e = r - (a_e - \alpha_1 z^2 / r^2) \), where \( a_e, \alpha_1 \) -- semimajor axis and constriction of the general terrestrial ellipsoid.

3. Text
1.3 Calculation of the influence of atmospheric resistance (FO2-FATM).

1. Designation. The terms \( \Delta_A \dot{V}_x, \Delta_A \dot{V}_y, \Delta_A \dot{V}_z \) are determined, depending on the influence of the atmospheric resistance in the equations of (1.1) or (1.2).

2. Structure. Subprogram FATM.

General groups: /BSB/, /COMI/.


4. Initial data: KC--index, characterizing the system of co-ordinates; \( V_0 \)--main part, containing \( X, Y, \dot{V}, V_X, V_Y, V_Z \); \( X, Y, Z, V_X, V_Y, V_Z \); \( \rho \)--density of the atmosphere; \( F_0 \)--main part of right members of equations of motion of \( X, Y, \dot{V}, V_X, V_Y, V_Z \) or \( X, Y, Z, V_X, V_Y, V_Z \) with the computation of the influence of atmospheric resistance.

6. Use of the domain COMMON. Before conversion to the subprogram FATM in the group COMMON/ESB/ESB it is necessary to address the value of the ballistic coefficient in the system of units, obtained from the calculations, for this it is sufficient that the value of the ballistic coefficient, given originally in the system of units \( kg, m, s \), be divided by the scale factor from the group COMMCN/BSB/ESB (see in (5) No. 12 table 2.3).
From the group COMMON COMMON COMMON COMMON: there is utilized the constant
(see in (5) No. 14 table 7.1)

7. Algorithm

\[
\begin{align*}
\Delta \dot{V}_x &= -3B \cdot P \cdot V_{\text{comp}} \cdot V_x, \\
\Delta \dot{V}_y &= -3B \cdot P \cdot V_{\text{comp}} \cdot V_y, \\
\Delta \dot{V}_z &= -3B \cdot P \cdot V_{\text{comp}} \cdot V_z,
\end{align*}
\]

\[
\dot{V}_x = F_x + \Delta \dot{V}_x, \quad \dot{V}_y = F_y + \Delta \dot{V}_y, \quad \dot{V}_z = F_z + \Delta \dot{V}_z,
\]

where \(3B\) -- ballistic coefficient \(3B = C_x \cdot F_w / 2m\)

\(C_x\) -- dimensionless coefficient of air resistance

\(F_w\) -- square of the mid section, \(m\) -- mass of AEs,

\(I\) -- air density

\(v_{\text{comp}}^x, v_{\text{comp}}^y, v_{\text{comp}}^z\) -- components of the vector of flight speed relative to the air

In the Greenwich system of coordinates

\[
v_{\text{comp}}^x = v_x, v_{\text{comp}}^y = v_y, v_{\text{comp}}^z = v_z.
\]

In the absolute system of coordinates

\[
v_{\text{comp}}^x = v_{x0} + \omega_3 Y, v_{\text{comp}}^y = v_{y0} - \omega_3 X, v_{\text{comp}}^z = v_z.
\]

\(F_x, F_y, F_z\) -- other terms in the right members of equations (1.1) or (1.2).

8. Text

```
SUBROUTINE FATHM(KC,V,P,F)
COMMON /BBS/BB
COMMON /OMZ/OMZ
DIMENSION V(4),F(4)
DO 1 J=1,3
1 V(J)=V(J)+3)
GOTO(2,3),KC
3 V(1)=V(1)+OMZ+V(2)
```

```
V(2)=V(2)-OMZ*V(1)
2 W=0
DO 4 J=1,3
4 W=W+V(J)*V(J)
W=BB*P*SQR(T(W)
DO 9 J=1,3
9 F(J+3)=F(J+3)-W*V(J)
RETURN
END
```

1.4. Calculation of anomalies of gravitational field of the earth
(PO3-PO4-30V)

1. Designation. The terms \(\Delta G V_x\), \(\Delta G V_y\), \(\Delta G V_z\) in
the right members of the system of equations (1.1) or (1.2), derived
from the influence of anomalies of the earth's gravitational field.
2. Structure. Subprogram FACRAV.

There are utilized the exterior subprograms: DEG2, DEG3, DEG4 (E01) general group: /RAD/.


4. Initial data.

The main part \( Y_6 \), containing \( X, Y, Z, V_x, V_y, V_z \) or \( x, y, z \), \( V_x, V_y, V_z \); the main part \( XG_3 \), containing \( x, y, z \);

\( F_6 \)--main part of right members of equations (1.1) or (1.2), containing the terms, determined by the influence of other factors;

\( KG \)--index, determining the nature of the considered anomalies of the gravitational field of the earth (possible values: \( 2, 3, 4 \));

For \( KG=2 \) one considers only the zonal harmonics in the breakdown of the gravitational field of the earth; for \( KG=3 \) there are considered the zonal, tesseral, and sectorial harmonics, for \( KG=4 \) one considers only the harmonics \( 2^2, 30, 40 \);

\( NM \)--number of considered harmonics (\( NM \leq 2^2 \)).

Note 1. For \( KG=2 \) or 3 it is necessary first to turn to the subprogram CONGR(A02) for the addresses in the group COMMON/BCONGR/.

coefficients of the gravitational field of the earth. For \( KG=4 \) one uses the coefficients from the block COMMON/CA2F/4 (see (5) No. 4 table 2.1), value \( NM \) is not used here.

Note 2. The main part \( XG \) is used only for \( KG=3 \) or 4.

5. Results: Main part \( F_6 \) containing new values \( X, Y, Z, V_x, V_y, V_z \); or \( x, y, z, v_x, v_y, v_z \) with calculation of the influence of the anomalies of the earth's gravitational field.

6. Use of the domain COMMON. In the block COMMON/RAD/RC, RL one addresses the values \( RC=r \) and \( RL=r_1 \).

7. Algorithm:

\[
\begin{align*}
\Delta_0 \dot{V_x} &= -\Delta g_r X/r - \Delta g_m Z X/r_1 - \Delta g_c Y/r_1 , \\
\Delta_0 \dot{V_y} &= -\Delta g_r Y/r - \Delta g_m Z Y/r_1 + \Delta g_c X/r_1 , \\
\Delta_0 \dot{V_z} &= -\Delta g_r Z/r + \Delta g_m r_1 /r .
\end{align*}
\]
\[ V_X = F_X + \Delta g \cdot V_X, \quad V_Y = F_Y + \Delta g \cdot V_Y, \quad V_z = F_z + \Delta g \cdot V_z, \quad \text{where } r = (X^2 + Y^2 + Z^2)^{1/2}, \]

\[ r_1 = (X^2 + Y^2)^{1/2}, \quad F_X, \quad F_Y, \quad F_z \quad \text{-- components of other terms in the right members of equations (1.1) or (1.7),} \]

\[ \Delta g_r \quad \text{-- radial component of vector } \Delta g, \quad \text{acceleration due to the influence of anomalies of the earth's gravitational field,} \]

\[ \Delta g_m \quad \text{-- meridional component of the vector } \Delta g, \]

\[ \Delta g^*_1 \quad \text{-- projection of the vector } \Delta g \text{ on the normal to the plane of the meridian.} \]

**Note.** The components of the vector \( \Delta g \), for some or some other recommended anomalies of the earth's gravitational field, respectively, are calculated with the subprograms with index 501: \( \text{DG 2}, \text{DG 3} \) or \( \text{DG 4} \) see in (5) p. 6.1 (there is introduced also the algorithm for the computation of the components of vector \( \Delta g \)).

### Subroutine

```fortran
SUBROUTINE +GRAV(X, F, Y, V, N)
DIMENSION X(6), F(6), Y(6)
DIMENSION XG(3)
COMMON/RAD/R1, R
X2=X(1)*X(1)
Y2=Y(1)*Y(1)
Z2=X(2)*X(2)
R12=X2+Y2+Z2
R1=SQRT(R12).
R2=R12-Z2
R=SQRT(R2).
NV=J-1
W=R+1
GO TO (;2,3,4,N
1 CALL DEG2(X, DG, N)
GO TO 6
2 CALL DEG3(XG, DG, N)
GO TO 4
3 CALL DEG4(XG, DG)
4 F(1) = F(4) - (DG(1)*X(1)) + (DG(2)*X(2)) + (DG(3)*X(3))/R1
      F(2) = F(5) - (DG(1)*X(2)) + (DG(2)*X(2))*X(3))/R1
      F(3) = F(6) - (DG(1)*X(3)) + (DG(2)*X(3))/R1
RETURN
END
```
1. Calculation of the gravitational perturbations connected with the influence of the moon or sun (F04-FGRSS).

1. Designation. The terms $\delta_S \dot{V}_X, \delta_S \dot{V}_Y, \delta_S \dot{V}_Z$ in the right members of the equations of motion (1.1) or (1.2), dependent on the gravitational perturbation of the moon or sun.

2. Structure. Subprogram FGRSS.


4. Initial data: $GDR3 = \mu_s/r_s^2$, where $\mu_s$ -- product of the gravitational constant and the mass of the moon (or sun), $r_s$ -- modulus of the radius vector of the moon (or sun).

The main part $Y_0$, containing the values $X, Y, Z, V_X, V_Y, V_Z$ or $x, y, z, v_x, v_y, v_z$; main part $X_0$, containing the values $X_0, Y_0, Z_0$ or $x_0, y_0, z_0$ -- directed cosines of the radius vector of the moon (or sun); main part $F_0$, containing the values $X, Y, Z, V_X, V_Y, V_Z$ or $x, y, z, v_x, v_y, v_z$, considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In the main part $F_0$ the new values of the right members of the equations of motion of the AES are addressed with the computation of the influence of gravitational perturbations, produced by the moon (or sun).

6. Algorithm:

$$\Delta_S \dot{V}_X = \mu_s/r_s^2 \left( (x_s - X/r_s) r_s^3 / |r_3 - r|^{3/2} - X_s^3 \right),$$
$$\Delta_S \dot{V}_Y = \mu_s/r_s^2 \left( (y_s - Y/r_s) r_s^3 / |r_3 - r|^{3/2} - Y_s^3 \right),$$
$$\Delta_S \dot{V}_Z = \mu_s/r_s^2 \left( (z_s - Z/r_s) r_s^3 / |r_3 - r|^{3/2} - Z_s^3 \right),$$
$$\ddot{V}_X = F_X + \Delta_S \dot{V}_X, \quad \ddot{V}_Y = F_Y + \Delta_S \dot{V}_Y, \quad \ddot{V}_Z = F_Z + \Delta_S \dot{V}_Z,$$
$$|r_3 - r|/r_s = \left( (x_s^2 - X/r_s)^2 + (y_s^2 - Y/r_s)^2 + (z_s^2 - Z/r_s)^2 \right)^{1/2},$$
1.6 Computation of the influence of the pressure of light (FLIGHT)

1. Designation. The terms $\Delta_L V_X$, $\Delta_L V_Y$, $\Delta_L V_Z$ are determined, considering the influence of light pressure on the right members of equations (1.1) or (1.2).

2. Structure. Subprogram FLIGHT.

General blocks: /EB/1, /CRZ/1.


4. Initial data: KL--index, governing the computation of the umbra of the earth (for KL=1 is produced the calculation of the influence of light pressure independently of the shadow, for KL=2 the shadow is considered).

The main part $F_6$, containing the values $X, Y, Z, V_X, V_Y, V_Z$ or $x, y, z, v_x, v_y, v_z$; main part $XS_3$, containing the directed cosines of the radius-vector of the sun: $X^0, Y^0, Z^0$ or $x^0, y^0, z^0$; considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In main part $F_6$ are addressed the new values of right members of equations of motion of the AEs with calculation of the influence of the pressure of light.

6. Use of the domain COMMON. Before conversion to the

SUBROUTINE FORSS (GDR3, V, XP, RP, F)
DIMENSION VTL, XP(3), F(6), XPR(3), W(15)!

DO 1 J=1,3
XPRI(J)=V(J)/RP
W(J)=XP(J)-XPR(J)
1 R=W(J)*W(J)+R
R=1./RSQRT(R)
DO 2 J=1,3
F(J+3)=F(J+3)+GDR3*(W(J)+R-XP(J))
RETURN
END
subprogram FLIGHT in the block COMMON/BB/B it is necessary to address the value of the coefficient B (see par. 7). Starting from this, that in the kg m, a system of units this coefficient has the dimensions m/s², it is necessary to make it agree with the system of units used in the computations, dividing by the scale factor from the block COMMON/CB/LB (see (5), table 2.3, No. 13).

From the block COMMON/CP1, one uses the constant (No. 10 table 2.1 (5)).

7. Algorithm:

\[ \begin{align*}
\dot{v}_x &= -B\left( X_0 - X/r_0 \right) r_0^3/|r_0 - r|^3 + F_x, \\
\dot{v}_y &= -B\left( Y_0 - Y/r_0 \right) r_0^3/|r_0 - r|^3 + F_y, \\
\dot{v}_z &= -B\left( Z_0 - Z/r_0 \right) r_0^3/|r_0 - r|^3 + F_z,
\end{align*} \]

where \( B = \frac{F_M}{M} \) kq₀

where \( F_M \) --square of the mid section, \( M \) --mass of the satellite, 
q₀ = 4.5 \( 10^{-7} \) kg/m² --pressure of light in the region of the earth's orbit, 
k = 1 for full optical reflection, k = 1.44 for full diffused reflection.

In the case of calculation of the influence of light pressure with calculation of the shadow of the earth the entry condition of the \( \text{ABE} \) into the earth's shadow is verified:

\[ \cos \gamma < 0 \text{ и } r \sin \gamma < R, \]

where \( \cos \gamma = \frac{(XX_0^C + YY_0^C + ZZ_0^C)}{r} \), \( r = \left( X^2 + Y^2 + Z^2 \right)^{1/2} \),

R --average radius of the earth.
1.7 Right members of the system of equations of motion of the AES, considering the normal gravitational field of the earth and standard five-layered atmosphere (F06-FC1100)

1. Designation. The subprogram FC1100 is intended for use in the capacity of a subprogram for the right members for integration of a system of differential equations of motion of the AES models, characterized by the indexes 11100, 21100, 10100, 20100 (see table 1.1).

2. Structure. Subprogram FC1100
External subprograms: FNGRAV (F01), RO (D1), FAT: (F02).
General blocks: /BHC/, /BK/5, /BSB/1.


4. Initial data: T -- independent variable (time), Y -- main part, containing X, Y, Z, Vx, Vy, Vz or x, y, z, vx, vy, vz.

5. Results: F -- main part, containing X, Y, Z, Vx, Vy, Vz or x, y, z, vx, vy, vz.

6. Use of the domain CCMCN. In the block CCMCN /BSE/SEB is necessary to give the value of the ballistic coefficient, relative to the scale factor from the block CCMCN/CESB/ESB (see (5), table 2.3, No. 12).

In the block CCMCN/BK/KC, KG, KA, KS, KL there must be given the values of the indexes of the model of forces (see par. 1.1). In the given case for the index KC the possible values are 1, 2; for KG -- 0, 1; the values of the remaining indexes are not used.

In the block CCMCN/BK/KC, as a result of the work of the subprogram FC1100 is addressed the value HC -- elevation of the AES above the surface of the terrestrial ellipsoid.
1.3. Universal subprogram for calculation of right members of equations of motion of AES with the use of various models of forces (FCF, FORCE, RODEN, LOGMOD).

1. Designation. Subprogram FORCE is intended for use as a sub-program of right-hand sides for integration of the system of equations of motion of the AES for any models of forces, described in Table 1.1. The subprogram LOGMOD for the given model of forces determines the value of the logical parameters, governing the computation of sidereal time, the positions of the moon and sun.

2. Structure. The package of subprograms.

Inputs: for the users: FORCE, LOGMOD.
Internal inputs RODEN.
Utilized external subprograms:

\[ \text{FNORAV(F01), FATM(F02), FAORAV(F03), FORSS(F04), FLIGHT(F05), AG10AC(B06), RO(D01), DENS(D02), SELENA(C01), STT(B05), GCLTLN(B10), SUN(C02), ADEN, AMBAR, GRAV, TLOCAL(D03), DEG2, DEG3, DEQ4(E01).} \]

General blocks

\[ \text{O0MNL, O0KN, BK/5, BLOO/4, BSB/1, BB/4, BNLW/1, BRO/6, BDT/1, BSO/5, BDDYEAR/1, BCONGR/30A, OXOEF/60, SYEAR/59, CAED/1, CAEB/2, CA22/4, CRE/16, CRZ/1, CAE/2, CGRL/1, OGRS/2, CDSJS/1, COMZ/4, COMZP/2, CAEL/2.} \]
3. Conversion C.I. FORCE (T, Y, F)

4. Initial data: 
   - T -- independent variable (time), 
   - Y -- main part of the functions sought: X, Y, Z, Vx, Vy, Vz, or x, y, z, vx, vy, vz

5. Results: 
   - F -- main part of functions derived from those sought: X, Y, Z, Vx, Vy, Vz or x, y, z, vx, vy, vz -- right members of the system of differential equations (1.1) or (1.2).

6. Use of the domain COMMON. Before conversion to the subprogram FORCE it is necessary to ensure in the domain COMMON the values of the parameters utilized. In the block /BK/KC, KA, KS, KL it is necessary to address the values of the indexes of the chosen model of forces (see table 1.1). In the block COMMON/BLOG/L3UN, LSUN, LSER, LST by means of conversion to the subprogram LCCMID (described below, in par. 3), it is necessary to address the values of the logical variables, used for regulation of the computation of sidereal time (for LST-TRUE), positions of the moon (for LSER-TRUE), and sun (for LSUN-TRUE).

   The remaining blocks COMMON are used only for the determined values of the indexes of the model of forces, for which in each case there is an indication. For KA greater than or equal to 1 in the block COMMON/TSR/3R it is necessary to address the value of the ballistic coefficient, relative to the scale factor from the block COMMON/SSR/1 (see (5), table 2.3, No. 17).

   For KL greater than C (computation of the pressure of light) in the block /C3/B it is necessary to replace the value of the coefficient B (see par. 1.5) relative to the scale multiplier from block /CBEB/1 (see (1)), table 2.3, No. 13). For KG=2 or 3 it is necessary in the block COMMON/BNM/1 to address the value of the number of harmonics considered in the gravitational field of the earth, in the block COMMON/BCUNGR/46 -- value of the coefficients of the breakdown of the gravitational field of the earth (by conversion to the subprogram CCUNGR/302/1). For KG=4 in the block COMMON/CA00/2 it is necessary to transfer the values of the corresponding variables from the block /CA00/2A/2 (see (5), table 2.1, Nos. 2, 3). /20

   For KA greater than 1 or KS greater than 0 or KL greater than 0 in the block COMMON/BDT/1 it is necessary to address the value of the date of tying the time in the mode RUD (relative to the Julian date). For KA=1, if KA is greater than 1 or KS is greater than 0
or KL greater than 0, and for KG=3, if KG is greater than 0, in the block COMMON/RSC/SC, Ts, N3 it is necessary to address: the value SC--sidereal time at midnight Greenwich of the date DT, and TS and NS to set equal to zero.

For KA=2 or 3 it is necessary in the blocks COMMON/KOEP/50 and SYEAR/50, to address: the values of the coefficients of the model of the atmosphere and set of numerical corrections for the semi-annual effect, which is accomplished by conversion to the subprogram VMKA(102); in the block BYEAR-1 one addresses the date and time in the form of the number of 4-hr. periods from the beginning of the year; in the block COMMON/BRO/K107, FO, AI, D, I one addresses the values F10.7 of the intensity of solar radio-radiation F10.7 with computation of the time of retardation (for KA=1 one sets F10.7 less than 0.5), D--of the average level of solar radio-radiation (possible values: 75, 100, 125, 150), A--trihourly index of geomagnetic perturbation with computation of the time lag (for KA=2 set AI less than 0.5), D--parameter, regulating the calculation of the semi-annual effect (for 3 less than 0 the semiannual effect is not considered), 1--parameter, regulating the calculation of the diurnal effect (CE) (for KG CE is considered without the term with the coefficient C, for I<0 CE is not considered, for I greater than 0 CE is considered fully) For KA=4 (atmosphere CIRA-72) in the block COMMON/K/BRO/F107, FO, AKI, D, I it is sufficient to address: the values F10.7--index F10.7(time of retardation 1.17 days), FC--values F10.7, averaged for 4-hr. solar rotations, AKI--geomagnetic index Kp, considering $Kp=2.66, 3=3.00, 3=3.33$ and so on (time of retardation $\sim$0.79 days), values of D and I without importance.

In the block COMMON/BHC/HC/RAD/RC, RL/B1/CL/CE, SE/BFT/TM, ST, RJ, AS, BJ, NL, XG(3), XL(3), GS, GL in the process of operation of the subprogram FORCE one addresses the values HC--elevation of the AER above the surface of the terrestrial ellipsoid, RC, RL--radius-vector of AER and projections of the radius vector of the AER on the plane of the earth's equator (in the subprogram AGRAV); CE, SE--cosine and sine of the angle of inclination of the plane of the earth's equator to the plane of the ecliptic (in the subprogram SUN); TM=T--current time (Moscow); ST--sidereal time; AS, BS--right ascension and declination of the sun; RS, XG(3), RL, XL(3)--are, respectively, for the modules and directed cosines (in the system of coordinates, determined by the index KG) radius vector of the sun and moon; GS, GL--mu/RS and mu/RL. In the remaining blocks COMMON, enumerated in para. 2, it is necessary to address: the values of the constants and scale factors in conjunction with the tables P.1-2.4 (!) by means of conversion to the subprogram CONST(A01).

The subsidiary subprogram RODENS, intended for switching into the subprogram FORCE of the subprograms of computation of the density of the earth's atmosphere according to various models. For KG=2 or 3 one uses the model, realized in the subprogram DENS(D02), for KG=4 one uses the model CIRA-72, subprogram ADEN (D03).

Structure. Subprogram RODENS.
External subprograms AGIGAC(BCO), GCLTLN(BLO), DENS(KE2), AMBAR, GRAV, TECAL(DCO).

General blocks: /BK/1, /RTO/1, /KCOEF/0, /SYEAR/8,
/BDYCAR/1, /ECGM/1, /CGT3/1, /CS1AY/1, /CE3/1, /BFM/2, /CERO/1.

Conversion: CALL KOBEC (T, Y, HC, RC, ST, AS, BS, 1).

Initial data: T--time; Y--X, Y, Z, V_X, V_Y, V_Z (R x, y, z),
V_x, V_y, v_z; RC, HC--modulus of radius vector and elevation of ABS
above the surface of the terrestrial ellipsoid; ST--sidereal time;
AS, BS--right ascension and declination of the sun.

Result: F--density of the atmosphere (in the system of units
used for the computation). Concerning the use of blocks COMMON
see para. 5.

8. Subprogram LOGMHD, starting from the values of the indexes of the
model of forces, given in the block COMMON/BK/KC, KG, KA, KL,
addresses in the domain COMMON/ALCG/LSUN, LSUNS, LSTL, LST the values /22
of the logical parameters, which may be used for the regulation of
the calculation of sidereal time (LST) and positions of the moon
(LM3) and sun (LSUN). Below are introduced the conditions, under
which each of these parameters takes on the value TRUE.

LSUN="TRUE" for KG greater than 1 or KL greater than 0 or KA
greater than 1;

LSUNS="TRUE" for KG greater than 1 or KL greater than 0;

LSPL="TRUE" for KS=1 or K3=6;

LST="TRUE" for KG, greater than 1 or KL greater than 0 or KL
greater than 0, if KG=1 and for KG greater than 7, if KG=2.

Conversion: CALL LOGMHD.
SUBROUTINE FORCE(X, V, F)

DIMENSION V(6), F(6), VG(3)
LOGICAL LSUN, LSUNS, LSEL, LST
COMMON/BFT/T, ST, RS, AS, BS, RL, XS(3), XL(3), GS, GL
COMMON/BLOG/LSUN, LSUNS, LSEL, LST
COMMON/BK/KC, KG, KA, KS, KL
COMMON/CORS, ORS, ORS
COMMON/BHC/HC
COMMON/BNM/NMO
COMMON/CORL/ORL
COMMON /BDT/DT
COMMON/CNNO/NMO
COMMON/BK/KC
COMMON/GN0/NO
COMMON/BNC/NC
COMMON /NNO/NMO
COMMON/OI/R0, OR, R1, RT
CALL SUN(DT, X, RS, AS, BS, XS)
GOTO(20, 10), KC
20 IF(LSUNS)
   CALL AGIGAC(ST, XS, 1, KS)
GOTO 6
9 IF(.NOT., LSEL)
   GOTO 6
21 CALL RRSSTC(L, XL, AL, V)
22 CALL RRSSTC(L, RL, AS, BS, P)
GOTO 3
11 CALL RD(HC, P)
13 CALL FAM(KC, V, P, F)
102 IF(KL) 103, 105, 13
13 CALL LIIIGHT(KL, V, RS, XS, RS, F)
103 IF(KS-1) 104, 21, 22
22 CALL FRSS(GS, V, XS, RS, F)
GOTO(21, 104, 21), KS
21 CALL FRSS(GL, V, XL, RL, F)
104 CONTINUE
RETURN
END

GO TO 6
CALL SELENA(DT, X, XL, RL)
GL=GL/RL/RL
GOTO(5, 6), KC
9 CALL AGIGAC(ST, XL, 1, KL)
6 CONTINUE
6 CALL FNGRAV(KC, KG, V, HC, R, F)
101 IF(KG-2) 101, 24, 1
1 DO 23 J=1, 3
23 VG(J)=V(J)
GOTO(24, 25), KC
25 CALL AGIGAC(ST, VG, 1, VG)
24 CALL FNGRAV(V, VG, F, KG, NMO)
101 IF(KA-1) 102, 11, 12
12 CALL RODENS(X, V, HC, R, ST, AS, BS, P)
GOTO 3
11 CALL RD(HC, P)
3 CALL FAM(KC, V, P, F)
102 IF(KL) 103, 105, 13
13 CALL LIIIGHT(KL, V, RS, XS, RS, F)
103 IF(KS-1) 104, 21, 22
22 CALL FRSS(GS, V, XS, RS, F)
GOTO(21, 104, 21), KS
21 CALL FRSS(GL, V, XL, RL, F)
104 CONTINUE
RETURN
END
SUBROUTINE RODENS(T, V, HC, R, ST, AS, TS, P)
DIMENSION V(6), RO(3), SU(2), SAT(3), X(I), TEMP(2), ALINN(6)
COMMON/RK/KC, KG, KA, KS, KL
COMMON /BRO/GEQ(3), D, I
COMMON /DDY/DT
COMMON /BDE/AR/DTE
COMMON /GHM/EM
COMMON /CT3/T3
COMMON /CDAY/SDAY
COMMON /CE3/ES
COMMON /REM/EM, ESEC
COMMON /LEQ/ERO
DO 1 J = 1, 5
1 X(J) = V(J)/R
GOTO (3, 2), KC
CALL AG16AC(ST, X, 2, X)
SU(1) = AS
SU(2) = BS
SAT(3) = HC/ES = EM
IF(KA.EQ.4)
   GOTO 7
6 IF(D) 4, 5, 9
5 D = DTNG + T/SDAY
4 CALL DENS(SAT(3), X(1), X(2), X(3), SU, GEQ(3), GEQ(1), D, 1, RO)
   P = RO(4) = ERO
GOTO 10
7 CALL GC2LHN(X, SAT(2), SAT(1))
   AMJD = DT + (T-13)/SDAY + 15019.9
   CALL ADEN(AMJD, SU, SAT, GEQ, TEMP, ALNN, AMMN, P)
   P = P/M = ERO
10 CONTINUE
RETURN
END

SUBROUTINE LOG00
COMMON/ALOG/LSUN, LSOU, LSL, LST
LOGICAL LSUN, LSOU, LSL, LST
COMMON/RK/KC, KG, KA, KS, KL
LOGICAL LSOU, LSL, LKL, LK2
COMMON/BLF/T, ST(13)
T = 100
LSOU = KA.GT.1
LSUN = KS.GT.1, OR, KL.GT.U
LSEL = KS.EQ.1, OR, KS.EQ.3
LG = KG.GT.2
LSUN = LSUN .OR. LSUNS
LSL = LSUN .OR. LSEL
LKG = KG .EQ. 1
LST = LS .AND. LKG .OR. LG .AND. .NOT. LKG
RETURN
END
Chapter 2. Integration of systems of differential equations
(index G).

2.1. Integration of a system of differential equations by the method of Adams (GCl-ADMSF)

1. Designation: In the subprogram the method of Adams is realized with the use of 8 differences, with automatic choice of step for integration of a system of differential equations:

\[ Y = F(t, Y) \]

with the initial conditions \( Y(t_0) = Y_0 \), where \( Y, \dot{Y}, Y_0 \), \( F \)-n-dimensional vectors, \( t \)-independent variable. The start is produced by the Runge-Kutta method of the fourth order.

2. Structure. Subprogram ADMSF.

One uses the external subprograms: TRP and BKK—composed by the user.


4. Initial data: KII—variable, ensuring the possibility of integrating with constant pace or with automatic choice of pace (in the first case it is necessary to take KII=1, in the second KII=2); H—initial step of integration; T—first value of the independent variable;

\[ Y \]—unidimensional block of dimension \( N \), containing the initial values of the desired functions;

IRI—name of the subprogram, computing the right members of equations, set up by the user;

BKK—name of the subprogram, prepared by the user, intended for control of the end of integration. In this subprogram there occurs conversion from the subprogram ADMSF after each step of the integration;

MKY, MKY—number of the first and last function in the block \( Y \), respectively, according to which proceeds the control of the accuracy of the integration with automatic choice of step;

EL, EP—blocks of dimension \( N \), containing lower and upper bounds of the permissible errors of the controlled functions;

These values are used: \( EL(NKY_0), EP(NKY), EL(NKY+1), EP(NKY+1) \ldots \)

5. Subprogram IRI.

NKY, MKY—number of the first and last function in the block \( Y \), respectively, according to which proceeds the control of the accuracy of the integration with automatic choice of step;

NKY, MKY—number of the first and last function in the block \( Y \), respectively, according to which proceeds the control of the accuracy of the integration with automatic choice of step;

EL, EP—blocks of dimension \( N \), containing lower and upper bounds of the permissible errors of the controlled functions;

These values are used: \( EL(NKY_0), EP(NKY), EL(NKY+1), EP(NKY+1) \ldots \)

EL(MKY), EP(MKY);

N—dimension of the system of equations

AF, Y1, Y2, Y3—auxiliary blocks of dimension \( N \);

F6—two-dimensional block of dimension \( (N, 3) \);

F4—two-dimensional block of dimension \( (N, 4) \).

Note 1. The designations of subprograms TRP and BKK must be written in EXTERNAL subprogram, used for subprogram ADMSF.

Subprogram IRI must have the form SUBRUTING IRI(T, Y, F), where \( T \)-independent variable

\[ Y \]—block of current values of the desired functions

F—block of right members, length of blocks \( Y \) and \( F \) equal to \( N \).

Subprogram BKK: SUBRUTING BKK(T, Y, F6, IBKK, H), where \( T \)—current value of independent variable

\( Y \)—current value of desired functions, block of dimension \( N \), \( H \)—step of integration;

F6—block of dimension \( (N, 5) \), containing values of the right members.
in current points of integration, IBKK--integral variable, which, within BKK, must be ascribed the value \( \alpha \) for fulfillment of the conditions, according to which is determined the moment of exit from the integration (before the start of integration the subprogram \( \text{AdAMS} \) ascribes to IBKK the value 1).

In subprograms BRI and BKK the values \( r, Y, F, H \) are established by subprogram \( \text{AdAMS} \) in the process of integration.

Note 2. In some times in the starting portion there arises a need to break down the step, not connected with ensuring the prescribed accuracy of integration. Ascribing to index IBKK the value \( \beta \) leads to interruption of the process of starting that was begun, disconnecting the influence of KII on the choice of step, return to the starting point and to the start with a halved step.

Giving the index KBKK the value \( \gamma \) leads to a new interruption of the process of integration, establishment of the initial step, reestablishment of the influence of KII on the choice of step, return to the initial point and to the transfer to the standard process of integration.

The subprogram \( \text{AdAMS} \) may serve as an illustration of the possibility of using this method of integration. See part 7...

5. Results: TI and the block \( Y \)--values of the independent variable and the desired functions at the final point of the integration.

6. Method. We consider the system (7.1). According to the known values \( Y_k \) in the Kth point and the approximate values \( Y_{k+1} \) produced at the Kth and 7 preceding points, one determines by the extrapolation formula of Adams

\[
Y_{k+1} = Y_k + h_t \sum_{i=0}^{\gamma} \alpha_i Y_i, \quad i = 0, 1, 2, \ldots, \gamma
\]

where \( h_t \)--step of integration.

Interpolation formula of Adams

\[
Y_{k+1} = Y_k + h_t \sum_{i=0}^{\gamma} \beta_i F_i, \quad i = 0, 1, 2, \ldots, \gamma
\]

allows the obtained values of \( Y_{k+1} \) to be made more precise.

The coefficients of the interpolation and extrapolation formulas of Adams have the following values:

\[
\begin{align*}
\alpha_0 &= 3.5895535, \\
\alpha_1 &= 9.5250668, \\
\alpha_2 &= 18.0545337, \\
\alpha_3 &= -22.027793, \\
\alpha_4 &= 77.976544, \\
\alpha_5 &= 8.6121797, \\
\alpha_6 &= 2.44516369, \\
\end{align*}
\]

\[
\begin{align*}
\beta_0 &= 0.304234537, \\
\beta_1 &= 1.15615906, \\
\beta_2 &= -1.00691934, \\
\beta_3 &= 1.01796461, \\
\beta_4 &= 0.732035384, \\
\beta_5 &= 0.34900257, \\
\beta_6 &= 0.0935405392, \\
\end{align*}
\]
For automatic choice of step of integration one computes the differences $\Delta y_{k+1} = y_{k+1} - y_k$.

We give the allowable bounds of error for integration: 0 less than or equal to $\epsilon_{1,j}$ less than or equal to $\epsilon_{2,j}$. If there are fulfilled the conditions $\epsilon_{1,j}$ less than or equal to absolute value of $\Delta y_{j,k+1}$ less than or equal to $\epsilon_{2,j}$, $j = N_1, N_1 + 1, \ldots, N_2$, where $N_1, N_2$--numbers of the first and last controlled functions, then the step of integration $h_t$ does not change.

If the absolute value of $\Delta y_{j,k+1}$ is less than $\epsilon_{1,j}$ for all $j = N_1, \ldots, N_2$ then there results a doubling of the step.

For this integration continues with step $h_t$ until the accumulation of the necessary number of points, for which after an interval of time, the multiple $2h_t$ the known values of the function $F$, after which the step is doubled.

If the absolute value of $\Delta y_{j,k+1}$ is greater than $\epsilon_{2,j}$ even if for one value of $j$: $N_1$ less than or equal to $j$ less than or equal to $N_2$, then a breakdown of the step is produced. For this it is necessary to have the values of $F$ in the seven preceding points with the interval $h_t/2$. It is possible to obtain them by means of the interpolation according to the formula of Lagrange for known values of $F$ with step $h_t$.

We introduce the variable $x_t = (t - t)/h_t$, then the interpolation formula of Lagrange for the determination of $F(t)$ is written in the form:

$$F(t) = F_{-7}(x_t^7 - 21 x_t^6 + 175 x_t^5 - 735 x_t^4 + 1624 x_t^3 - 1764 x_t^2 + 720 x_t)/5040 -$$
$$- F_{-6}(x_t^7 - 22 x_t^6 + 190 x_t^5 - 820 x_t^4 + 1849 x_t^3 - 2038 x_t^2 + 840 x_t)/720 +$$
$$+ F_{-5}(x_t^7 - 23 x_t^6 + 270 x_t^5 - 925 x_t^4 + 2144 x_t^3 - 2412 x_t^2 + 1008 x_t)/240 -$$
$$- F_{-4}(x_t^7 - 24 x_t^6 + 226 x_t^5 - 1056 x_t^4 + 2549 x_t^3 - 2952 x_t^2 + 1260 x_t)/144 +$$
$$+ F_{-3}(x_t^7 - 25 x_t^6 + 287 x_t^5 - 1219 x_t^4 + 3312 x_t^3 - 3796 x_t^2 + 1680 x_t)/144 -$$
$$- F_{-2}(x_t^7 - 26 x_t^6 + 270 x_t^5 - 1420 x_t^4 + 3929 x_t^3 - 5274 x_t^2 + 2520 x_t)/240 +$$
$$+ F_{-1}(x_t^7 - 27 x_t^6 + 295 x_t^5 - 1665 x_t^4 + 5104 x_t^3 - 8028 x_t^2 + 5040 x_t)/720 -$$
$$- F_{1}(x_t^7 - 28 x_t^6 + 322 x_t^5 - 1960 x_t^4 + 6769 x_t^3 - 13132 x_t^2 + 13068 x_t - 5040)/5040.$$
To obtain the values of $F$ in the first 7 points one uses the Runge-Kutta method of the fourth order.

$$Y_{h+1} = Y_k + \frac{h}{6} \left( \ell_1 + 2 \ell_2 + 2 \ell_3 + \ell_4 \right),$$

$$\ell_1 = h \cdot F(t_k, Y_k),$$
$$\ell_2 = h \cdot F(t_k + \frac{1}{2} h, Y_k + \frac{1}{2} \ell_1),$$
$$\ell_3 = h \cdot F(t_k + \frac{1}{2} h, Y_k + \frac{1}{2} \ell_2),$$
$$\ell = h \cdot F(t_k + h, Y_k + \ell_3).$$

**Literature:** (C), p. 3:3.

7. Text.

```plaintext
SUBROUTINE ADAMS (KP, H, X, Y, PRP, BKX, XK, MA, EL, E2,
                  HAF, F1, X1, V1, U2, U3)

DIMENSION V(N), A(N), F(N-1), F(N-1), EL(N), E2(N),
        V1(N), V2(N), V3(N), A(0), F(0), R(0)

H = H
100  IA = KP1
    H = HN
1004  IBKK = 1
    DO 1001 J = 1, N
    DO 1001 I = 1, N
1001  F(I, J) = 0
C RUNGE-KUTTA
101  Z = X + 6.7 * H
100  DO 2 J = 1, N
    V1(J) = V(J)
2   X1 = X
    A(1) = 5 * H
    A(2) = A(1)
    A(3) = H
    A(4) = H
    A(5) = A(1)
3   X2 = X1
    DO 4 J = 1, N
    V2(J) = V1(J)
4   V3(J) = V1(J)
    DO 19 I = 1, 4
    CALL PRP(X1, V2, AF)
    IF (I-1) = 6 * 7
6   GOTO (10, 7, 7) 1A
10   DO 12 J = 1, N
12   F(J, 0) = AF(J)
    IF (IA-1) = 6 * 8
5   CALL BKX(X1, V1, F, IBKK, H)
    GOTO (8, 1000, 1003, 1002) 18%4
1003  IA = 1
    H = H * 5
    GOTO 1004
8   DO 9 K = 1, 7
9   DO 9 J = 1, N
```

(Original page is not clear and may contain errors or unclear text.)
C ADAMS

103 A(1) = -.304225376H
   A(2) = 2.449613669H
   A(3) = -0.612127975H
   A(4) = 17.3796544H
   A(5) = -22.027755H
   A(6) = 16.0545387H
   A(7) = -9.52920666H
   A(8) = 3.989955335H
   B(1) = .01136739424H
   B(2) = .09364009320H
   B(3) = .3430003370H
   B(4) = .732835364H
   B(5) = 1.017816610H
   B(6) = -1.0082964H
   B(7) = 1.19619986H
   B(8) = 3.042245370H

104 CALL PRP(X1,V1,AF)

DO 23 J=1,N
   V3(J) = V1(J)
   V2(J) = V1(J)

23 F(J, 0) = AF(J)

CALL BBK(X1,V1,F,JBK,H)

GOTO(105,100,1003,1002,1000)

105 IF(IA-7) = 118, 54, 118

106 H = H + 2.

DO 39 K=1, N
   V3(J) = V1(J)
   V2(J) = V1(J)

39 F(J, L) = F(J, M)

DO 36 K=1,4

DO 38 J=1,N

36 F(J, K) = F(J, K)

DO 50 J = 1, 8
   A(J) = A(J) = 2.

50 B(J) = B(J) = 2.

119 IA = 0

118 X1 = X1 + H

DO 24 K = 1, 8
100 M=M/2

104 W1=W1-M
263,5
DO 37 K=1,N
   F(J,K)=0
   DO 30 L=1,N-1
30 F(J,K)=F(J,K)+F(L,J)+R(J)*F(L,J)
37 Z=Z-1.
   DO 39 K=5,7
      L=K-8.
   DO 39 J=1,N
39 F(J,J)=F(J,J)+R(J)*F(J,J)
   DO 40 K=1,4
      L=Z+K-1.
40 F(J,J)=F(J,J)+R(J)*F(J,J)
   DO 41 J=1,N
41 V2(J)=Y2(J)
   DO 44 I=1,N
44 V1(J)=Y1(J)
   DO 51 J=1,N
51 B(J)=B(J)+H*W(J)
   DO 51 J=1,N
   M=M*2
   F(J,J)=F(J,J)+R(J)*F(J,J)
   GOTO 106
290 CONTINUE
GOTO 106
106 CONTINUE
2.2. Interpolation according to Adams (G02-ADINT).

1. Designation. The subprogram allows in the process of integration by the method of Adams of the system of differential equations \( Y = F(t, Y) \), where \( Y, Y, F \) are \( N \)-dimensional vectors, the computation of the values of the desired functions at the moments of time \( t_{nu} \), not multiples for the step of integration.

2. Structure. Subprogram ADINT.


4. Initial data:
   - Value of the variable \( xi = (t_k - t_{nu})/h_t \), where \( t_{nu} \) -- given moment of time, \( h_t \) -- step of integration, \( t_k \) -- current value of the variable of integration, corresponding to the condition: \( t_k = h_t \) less than \( t_{nu} \) less than \( t_k \);
   - \( F8 \) -- block of dimension \( (N, 8) \), containing those produced from the desired functions in the eight last points: \( t_k - 7, t_k - 6, \ldots, t_k \);
   - \( Y \) -- block of dimension \( N \), containing the values of the desired functions at the point \( t_k \);
   - \( M \) -- number of interpolated functions (\( M \) less than or \( = N \));
   - \( H \) -- step of integration (\( h_t \));
   - \( N \) -- order of the system of equations.

5. Results: \( YR \) -- block of dimension \( N \), containing \( y_j(t_{nu}) \), \( j = 1, \ldots, M \).

6. Algorithm: We introduce the variable \( xi = (t_k - t_{nu})/h_t \), \( t_{nu} \), \( h_t \) and \( t_k \) determined above).

The value of the desired functions at the point \( t_{nu} \) is computed by the formula

\[
y_j(t_{nu}) = y_j(t_k) - h_t \sum_{i=k-7}^{k} f_{i,j} \psi_{i}/120960,
\]

where \( j = 1, \ldots, M \) less than or \( = N \); \( f_{i,j} \) -- values of the right members on the points \( t_i \) (\( i = k-7, \ldots, k \)); \( y_j(t_k) \) -- values of the desired functions at the point \( t_k \);
\[
\begin{align*}
\Psi_{k-7} &= (3^{6} - 72^{7} + 700^{8} - 3528^{9} + 9744^{10} - 14112^{11} + 8640^{12}), \\
\Psi_{k-6} &= -(21^{6} - 52^{7} + 5320^{8} - 27552^{9} + 77658^{10} - 114128^{11} + 70560^{12}, \\
\Psi_{k-5} &= (63^{6} - 1656^{7} + 17388^{8} - 93240^{9} + 270144^{10} - 405216^{11} + 254016^{12}, \\
\Psi_{k-4} &= -(105^{6} - 2880^{7} + 31640^{8} - 177408^{9} + 534450^{10} - 826560^{11} + 529200^{12}), \\
\Psi_{k-3} &= (105^{6} - 3000^{7} + 34580^{8} - 204792^{9} + 653520^{10} - 1062880^{11} + 705600^{12}), \\
\Psi_{k-2} &= -(63^{6} - 1872^{7} + 22680^{8} - 143136^{9} + 495054^{10} - 886032^{11} + 635040^{12}), \\
\Psi_{k-1} &= (21^{6} - 648^{7} + 6260^{8} - 55944^{9} + 214368^{10} - 449568^{11} + 423360^{12}), \\
\Psi_{k} &= -(3^{6} - 96^{7} + 128^{8} - 9408^{9} + 40614^{10} - 105056^{11} + 156816^{12} - 120960^{13}).
\end{align*}
\]

Дополнительно: \([2]\).

SUBROUTINE ADINT(Z,F,V,VR,N,M,N)
DIMENSION F(N,6),V(N),VR(N),R(8)
R(1)=((((3,-72,),=2+700.),=2-3528.)=2
  -9744.,=2-14112.,=2-70560.,=2
R(2)=(((1-21.,=2-526.,=2-3320.),=2+27552.),=2
  -77658.,=2-17388.,=2-93240.,=2
R(3)=((((163.,=2-1656.,=2-17388.),=2-93240.),=2
  +270144.,=2-405216.,=2-254016.,=2
R(4)=(((1-109.,=2-2860.,=2-31608.),=2-17388.,=2
  -53450.,=2-82860.,=2-529200.,=2
R(5)=(((1-105.,=2-3000.,=2-34580.),=2-204792.,=2
  +55320.,=2-1062880.,=2-705600.,=2
R(6)=(((1-63.,=2-1872.,=2-22680.),=2-143136.,=2
  -495054.,=2-886032.,=2-635040.,=2
R(7)=(((121.,=2-648.,=2-9260.),=2-55944.),=2
  +214368.,=2-449568.,=2-423360.,=2
R(8)=(((1-3.,=2-96.,=2-1288.,=2-79408.),=2-40614.),=2
  -105056.,=2-156816.,=2-120960.)
DO 1 J=1,M
  VR(J)=V(J)
DO 2 K=1,B
  R(K)=R(K)*H/Z/120960.
DO 2 J=1,M
  VR(J)=VR(J)-F(J,K)*R(K)
RETURN
END
Chapter 3. SUBPROGRAMS, ENSURING IN THE PROCESS OF INTEGRATION OF THE SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS THE ATTAINING OF THE PRESCRIBED VALUES OF THE FUNCTIONS FROM THE SOLUTION (INDEX H)

3.1. Arriving at the prescribed value of an arbitrary function from the solution (HOI-REACH) in the process of integration.

1. Designation. In the process of integration of the system of differential equations (2.1) the subprogram REACH allows one to determine the value of the independent variable TR, corresponding to reaching the given value FRI of an arbitrary continuous function 
\[ \phi(T)=\text{FREACH}(T,Y), \quad Y--N\text{-dimensional vector}. \]

2. Structure. Subprogram REACH. Utilized external subprograms: ADINT(G02), subprogram--function, FREACHI (composed by the user).

3. Conversion:
\[ \text{CALL REACH}(T, Y, F, H, E, FR, FR1, FRK, TR, FREACHI, N, YR). \]

4. Initial data: 
- T--independent variable;
- \( Y \) \(_N\)--block of current values of the vector Y;
- \( F \)--two-dimensional block \((N, 8)\)--values of right members in 8 points with step \( H \);
- \( H \)--value of the step of integration;
- FREACHI--designation of the subprogram--function, computed
\[ \phi(T) \]
- FR--current value of the function FREACHI;

*For the subprograms of this chapter the conversion must take place in the process of integration of the system of differential equations, i.e., from the subprograms BKK, to which is transferred the regulation from the subprogram ADAMSF (see par. 2.1) after each step of integration.*
FRK—value of FREACH at the previous step of integration (at the point T=1);
FR1—given value of the function FREACH;
N—order of system of equations;
I—allowable relative error, ensuring the choice of value of the independent variable TR, satisfying the relationship

\[
\frac{|FR1 - FREACH(TR, YR)|}{\Delta F} \leq E
\]

where \( \Delta F \)—increment in the function FREACH from the step of integration.

Note 1. Subprogram—function FREACH, composed by the user, must have the form: FUNCTION FREACH(T, Y), where T—dependent variable;
Y_n—block of current values of the vector Y.

Note 2. Function FREACH must be described in the external subprogram, using the subprogram FREACH.

Note 3. Before use of subprogram FREACH it is necessary to find that moment of time in the process of integration, for which is satisfied one of the relationships FRK less than or equal to FR1 less than or equal to FR or FR1 greater than or equal to FR.

5. Results: TR—value of independent variable, corresponding to reaching the function FREACH of the value FR1;
YR_n—block of values of the desired functions Y at the point TR.

6. Algorithm. Let us consider, in the process of integration of the system of equations (2.1) a continuous function \( \phi(T) = FREACH(T, Y) \), where Y—N-dimensional vector, taking on the values FR at the current moment of time T, FRK—at the moment of time T_k=T-n, where N—step of integration.

It is necessary to find the moment of time, for which the function \( \phi(T) \) takes on the value FR1.

Under the condition, that there is fulfilled one of the inequalities FRK less than or equal to FR1 less than or equal to FR or FRK greater than or equal to FR1 greater than or equal to FR, we determine F=FR--FRK, \( x_1 = (FR - FR1) / \Delta F \).

We set some epsilon greater than 0. If the absolute value of \( x_1 \) is less than or equal to epsilon, that moment of time TR=T-x_1, H gives the solution of the problem proposed. In the opposite case, using the subprogram of interpolation by Adams' method (par. 2.2), we determine the value YR at the moment of time TR and the value P=\( \phi(TR) = FREACH(TR, YR) \).
For the new iteration we determine \( x_{i_2}^+ = (P - FR1)/\Delta F \), we verify the condition absolute value of \( x_{i_2} \) less than or equal to \( \epsilon \). If this condition is satisfied, then \( TR = T - x_{i_1} - x_{i_2} H \) gives the solution to the proposed problem, otherwise we go on to a new iteration and so on.

### Text.

```fortran
SUBROUTINE REACH1(V,F,H,E,FR,FR1,PK,TH,FREACH,N,VR)
DIMENSION V(N),F(N+1),VR(N)

TR=E
IF(ABS(F(1,1)+F(2,1)+F(3,1)-T)=1.E-15)10,1,4
1 DP=FR-FR1
2 DO=FR-FRK
4 W=0,
3 W1=DP/OZ
4 W=W+H1
5 TR=T-W0H
6 IF(ABS(W1)-E)=1,2
1 CALL ADINT(W,F,V,VR,N,N)
2 P=FREACH(TR,VR)
3 DP=FR-TR1
GOTO 3
1 RETURN
END
```
3.2 Determination, in the process of integration, of the minimum and maximum of an arbitrary continuous function from the solution as functions of an independent variable (H02-EXTREM, PARAB)

1. Designation. Subprogram EXTRM determines in the process of integration of the system of equations (2.1) the minimum and maximum of an arbitrary continuous function phi(T) = FREACH(T, Y), where Y—N-dimensional vector, and the value of the independent variable, corresponding to the minimal and maximal values of this function.

2. Structure. Subprogram EXTRM.
   - **Internal inputs:** T, R, B, A, B.
   - **Utilized external subprograms:** ADINT(T0, T), FREACH (composed by the user).
   - **General blocks:** /BMIN/2, /BFC/1, /B3M/1.

3. Conversion:
   CALL EXTRM (T, Y, F, H, I1, I2, FMIN, TMIN, FMAX, TMAX, FREACH, N, YR)

4. Initial data: T—-independent variable; YN—block of current values of vector Y; H—step of integration; F—two-dimensional block (N, I), containing the values of the right-hand sides at 8 points with step H; F1, I2—permissible relative errors, ensuring the choice of extremum, satisfying the relationships

\[
\left| \frac{FIE - \phi(TE)}{\Delta F} \right| < P1 \quad \text{OR} \quad \left| \frac{TIE - TE}{H} \right| < P2,
\]

where FIE, TIE—value of the function and of the argument corresponding to the actual extremum; phi(TE, TE)—approximate values of the extremum of the function and in the corresponding argument, deltaF—increment in the function phi(T) for the step of integration; H—order of the system of equations; FREACH—designation of the subprogram—function, computed phi(T).

This subprogram is composed by the user and has the following construction:

\[ \text{FUNCTN: FREACH (T, Y)} \]

where T—-independent variable; YN—block of values of the vector Y.

Note. The name of the subprogram—function FREACH must be described in EXTRM subprogram, used for subprogram EXTRM.

5. Results: FMIN—last local minimum of function FREACH for the interval of time from the beginning of integration to the current moment; TMIN—moment of time, corresponding to the value FREACH = FMIN; FMAX—last local maximum of function FREACH in the same time interval, TMAX—moment of time, corresponding to the value FREACH = FMAX; YN—block of values of vector Y at the point of the extremum.

6. Use of the domain COMMON: After the beginning of integration
in the block COMMON/BAIN/H1, H2 one must substitute these values: H1=0, H2=0.

In the block COMMON/BMC/1 at each step of integration it is necessary to address the current value of the function FREACH.

In the block COMMON/BSM/ the subprogram EXTRM addresses the value of the step of integration.

7. Algorithm.

Let HC, H2, H1 be the values of the function FREACH at three consecutive moments of time t_i: t_1=t, t_2=t-h, t_3=t-h-h where h is the step of integration, h_1=h for integration with constant step (for integration with automatic choice of step h_1 may be different from h).

If the inequality (H2-H1)(HC-H2) less than 0 is satisfied, then the extremum (for H2-H1 greater than 0---maximum, for H2-H1 less than 0---minimum) is found in the time interval (t_3, t_1).

We will term such a situation extremal. It remains to determine more precisely the moment of time, corresponding to the extremum, and the value of the extremum.

We proceed from the moments of time t_1 to the moments tau_1 = (t-t_1)/h; then tau_1=0, tau_2=1, tau_3=1+h_1/h correspond to the values of the function HC, H2, H1.

For convenience in further discussions, we shall rename the moments of time and the corresponding values of the function, starting from the following considerations.

\[
\begin{align*}
& \text{IF } |HC - H2| < |H2 - H1| \text{ THEN }, \quad B_1 = H2, \quad x_1 = \tau_2 = 1; \\
& \quad B_2 = HC, \quad x_1 = \tau_1 = 0; \quad B_3 = H1, \quad x_3 = 1 + h_1/h. \\
& \text{IF } |HC - H2| > |H2 - H1| \text{ THEN }, \quad B_1 = H2, \quad x_1 = \tau_2 = 1; \\
& \quad B_2 = H1, \quad x_2 = \tau_3 = 1 + h_1/h; \quad B_3 = HC, \quad x_3 = \tau_1 = 0.
\end{align*}
\]

Thus, we have 3 values of the function B_1 for three values of the arguments x_1. We construct a parabola, passing through these points, and find the value of W, the argument of the extremum guaranteed by this parabola.

Turning to the subprogram of interpolation following Adams (par. 3.2) with the obtained value of W, we determine the value YR of the vector Y at the moment in time TR=t-Wh, then the value HR of the function FREACH at this moment in time.

If the absolute value of (HR-B_1)/delta F is less than epsilon_1, or the absolute value of W-x_1 is less than epsilon_2, where epsilon_1, epsilon_2---given values of the permissible errors, deltaF =HC-H2, then the problem is solved, otherwise we go on over to a new iteration, using the values B_3=B_2, B_2=B_1, B_1=HR at the moments in time x_3=x_2, x_2=x_1, x_1=W and so forth.
SUBROUTINE ExtTa(T, Y, F, N, P1, P2, FMIN, TMIN, FMAX, TMAX, FREACH, VR, NVR)

DIMENSION V(N), F(N, 8), VR(N)
DIMENSION X(3), B(3)
COMMON /BSM/S?
COMMON /SHIN/H1, H2
COMMON /BFC/HC
IF(H1) 1, 1, 3
1 IF(H1) 3, 3
3 H1 = H2 = H
W2 = HC = H
IF(H1 = H2) 4, 5, 9
4 IF(IF(1, 1) + F(2, 1) + F(3, 1) > 0.3, 3)
30 K = SIGN(1, W1)
B(1) = M
X(1) = 1.
X(3) = 1. - S2/H
IF((HC = H1) = K) 19, 8, 8
8 B(2) = HC
X(2) = 0.
B(3) = M
GOTO 10
9 B(2) = H1
X(2) = X(3)
B(3) = HC
X(3) = 0
10 CALL PARAB(X, B, XM)
K = X
CALL ADINT(XM, F, V, VR, N, H, N)
TR = T = M
HR = FREACH(TR, VR)
IF((HR - B(1)) * K) 29, 21, 20
20 IF(ABS((HR - B(1))/K2 - P1) > 21, 21, 23
23 IF(ABS(XM - X(1)) - P2) 21, 21, 24
24 DO 26 J = 1, 2
L = 4 - J
X(L) = X(L - 1)
26 B(L) = B(L - 1)
B(1) = HR
X(1) = X;
GOTO 10
25 HR = B(1)
21 IF(K) 32, 33, 33
32 FMIN = HR
TMIN = TR
GOTO 5
33 FMAX = HR
TMAX = TR
5 H1 = H2
2 H2 = HC
S2 = H
RETURN
END
9. The subsidiary subprogram FARAB, intended for the determination of the extremum of the parabola, passing through three points: \( \phi(x_1), \phi(x_0), \phi(x_2) \) of some function \( \phi(x) \).

Conversion: CALL FARAB (X, B, XM).

Initial data: \( X \) -- block of values of the argument, \( B \) -- block of values of the function.

Results: XM -- value of the argument, giving the extremum of the parabola, passing through the given 3 points.

Algorithm. Let the values \( B \) of some function \( \phi(x) \) be known for three values of the argument \( x_1 \): \( B = \phi(x_1) \).

It is necessary to determine XM -- the value of the argument, giving the extremum of the parabola, passing through the indicated values \( B \).

\[
XM = \frac{1}{2} \left( \frac{(B_2 - B_1)(x_2^2 - x_1^2) - (B_3 - B_1)(x_3^2 - x_1^2)}{(B_2 - B_1)(x_2 - x_1) - (B_3 - B_1)(x_3 - x_1)} \right).
\]

TEXT.

```
SUBROUTINE PARAB(X,B,XM)
DIMENSION X(3),B(3),W(2),T(2),Z(2)
V=1(1)X(1)
DO 1 J=2,3
W(J-1)=B(J)-B(1)
T(J-1)=X(J)-X(1)
1 T2(J-1)=X(J)*X(J)-V
XM=90*(W(1)*Z2(2)-W(2)*Z2(1))/4(W(1)*T(2)-W(2)*T(1))
RETURN
END
```
3.3. Exit of the ABS at the ascending node of the orbit (NO3-K60;)

1. Designation. The subprogram permits one to determine in the process of integration of the systems of equations (1.1) or (1.2) the moment of time, corresponding to the ABS crossing the ascending node, and also the coordinates and components of the velocity vector of the ABS at this moment in time.


General domain: /B-Y/ E, /B1IN/ 1.

3. Conversion:

4. Initial data: T-time; Y, block of current values X, Y, Vx, Vy, Vz;

H-step of integration;

F=two-dimensional block of dimension (e, 0), containing the values of X, Y, Vx, Vy, Vz produced at 8 points: T-2H, ..., T-H, T;

NB--loop number;

5. Results: If TV is greater than 0, then TV, Vy, Vy, ...--moment of time and block of values Xv, Yv, Vx, Vy, Vz corresponding to the ABS crossing the ascending node.

Note. If the condition of exit of ABS at the ascending node is fulfilled at the take-off stage by the method of Runge-Kutta (i.e., initial conditions are given in the vicinity of the ascending node), then the procedure of determining more precisely the moment of exit at the node is disconnected, and the loop number (NB) is increased by 1.

6. Use of the domain COMMON.

Before the beginning of integration in the block C="/3 K/"K, it is necessary to address the value ,t, to the place ,K and ,K, in the block COMMON/BIN/INOD one, sets IP<1.

7. Algorithm.

In the process of integration of equations of motion of the ABS one determines the moment of time t, for which is fulfilled the condition: S(t-h) <(t) less than 0, '(t) greater than 0.

Starting from the value of the parameter x, (t), delta t, where delta t=S(t) -S(t-h), using the subprogram of integration according to Adams (par. 2.2), we determine the values Xv, Yv, Vx, Vx, Vy, Vy, Vz, Vz, at the moment of time tnu =t-xih, where h--step of integration.

If the absolute value of Z/delta 3 is less than epsilon as given, then tv, Xv, Yv, Zv, Vx, Vy, Vz, Vz, give the solution to the proposed problem. If the condition is not fulfilled, then a new iteration is carried out, starting from the value xi=x1+
Subprogram APSIS.

External subprograms employed: HEIGHT(B10), ADINT(G02), PARAB(H02).

3.4. Determination in the process of integration of the minimal and maximal elevation of the \( \text{ABS} \) above the surface of the terrestrial ellipsoid (H04-APSIS).

1. Purpose. The subprogram APSIS allows the determination of the minimal and maximal value of the elevation of the \( \text{ABS} \) above the surface of the terrestrial ellipsoid in the process of integration of the equations of motion of the \( \text{ABS} \) (1.1) or (1.2).

2. Structure. Subprogram APSIS.

External subprograms employed: HEIGHT(B10), ADINT(G02), PARAB(H02).


4. Initial data: T--time;
   Y--block of current values of X, Y, Z, V_X, V_Y, V_Z;
   H--step of integration;
   F8--two-dimensional block (i, j), containing the values of
   X, Y, Z, V_X, V_Y, V_Z derived at the moments of time: T=TH, T=2H, ...
   3KK--variable, controlling the exit from the integration by the
   subprogram 'DRAW' (par. 3.4)

5. Results:
   HMIN, HMAX--minimal and maximal values of the elevation of the
   AES above the surface of the earth's ellipsoid in the time interval
   from the beginning of integration (or from the start of the loop)
   to the current moment of time.

   Remark 1. In order that HMIN, HMAX be determined as minimal and
   maximal values of the elevations not on the whole interval of
   integration, but on each loop separately, it is necessary to address
   the values HC--of the current altitude of AES--at the moments of
   exit of the AES at the point of time for HMIN, HMAX.

   Remark 2. If the extremal situation arises in the starting
   portion, then the variable 13KK receives the value 3.

   In this case the program of integration ADAMS automatically
   reduces the step in half, switches out the influence of KPI on the
   choice of step and begins the start from the initial point. After
   HMIN or HMAX is found, 13KK receives the value 4, the initial step
   of integration and the influence of KPI on choice of step are
   automatically renewed, and a new start will begin from the initial
   point and the integration enters the standard regime.

6. Use of the domain COMMON. Before the beginning of inte-
   gration in the blocks COMMON/3XK, 3XN/BAI 3/1HM, H1, H2 it is
   necessary to give the value 3(t_0) to the variables 3X, 3XN, to set
   HM=1, H1=0, H2=0.

   In the block COMMON/3XK/HC at each step of integration it is
   necessary to address the value HC, current elevations of AES above
   the surface of the earth.

   In the block COMMON/3X2/1, the subprogram A.FJ13 addresses the
   value of the step of integration. In the block COMMON/3TAH/S/
   TMIN/TMAX are addressed the moments of time, corresponding to
   HMIN, HMAX.

7. The Algorithm is completely analogous to the algorithm, used
   in the subprogram EXTRM(H02), only in place of the function FREACCH
   one uses the altitude of AES above the surface of the earth's ellip-
   soid.

   For the permissible errors are chosen the values epsilon_1 =
   0.0001, epsilon_2 = 0.01 (variables P1, P2, values are given to these
   by the operator DATA).
SUBROUTINE APS1S(T, V, F, HMIN, HMAX, IBKK)

DIMENSION X(3), B(3), VR(6), P(6, 9)
DIMENSION V(6)

DATA P1, P2, 0.01, 0.01

COMMON/BS2/S2
COMMON/SAWS/INI, 1, 1
COMMON/BBZK/2ZK, 2K
COMMON/BBHC/HC
COMMON/BAPS/TMIN, TMAX

IF (H2) 1, 11: 1

11 HMIN=HC
HMAX=HC
GOTO 2
1 IF (H1) 3, 9, 3
3 H1=H2-H1
H2=HC-H2
IF (H1=0) 4, 9, 9
4 IF (F(1, 1)+F(2, 1)+F(3, 1)) = 0, 3, 30
31 GOTO (91, 91: 91, 1) 1HM
51 1HM=2

IBKK=3

6 H1=0
H2=0
ZK=2K
RETURN

30 X=SIGN(1, H1)
B(1)=H2
X(1)=1.
X(3)=1. 32/1
IF ((H1=H1)*K) 3, 20
8 B(2)=HC
X(2)=0, 22
B(3)=H1
GOTO 10
9 B(2)=H1
X(2)=X(3)
B(3)=HC
X(3)=0

10 CALL PARAB(X, B, XH)
CALL APRINT(X, F, V, VR, 6, H, 6)
CALL HEIGHT(V, HR)
IF ((1>R-1(1)*K) 23, 21, 20
20 IF (ABS(HF-H(1))/M2-P1) 21, 21, 23
25 IF (ABS(XH-X(1)) 21, 24
24 DO 26 J=1, 2
L=L-J
X(L)=X(L-1)
26 B(L)=B(L-1)
B(1)=HF
X(1)=XH
GOTO 10
25 HR=B(1)
21 IF (K) 27, 28, 28
27 IF (HF-HMIN) 32, 29, 29
32 HMIN=HR
TMIN=T-XH*H
GOTO 29
28 IF (HR-HMAX) 29, 29, 33
33 HMAX=HR
TMAX=T-XH*H
29 GOTO (5, 53, 5), 1HM
33 1HM=3
IBKK=4
GOTO 16
9 H1=H2
2 H2=HC
S2=H
RETURN
END
4.1. Right ascension of the \textit{ABS}, local time and zenith distance of the sun (IO1--RIGHTA, TLCC, ZDSUNL, ZDSUNC).

1. Purpose. The subprogram TLCC for known geographical longitude (lambda) of \textit{ABS} and sidereal time (ST) determines the right ascension of the \textit{ABS}.

The subprogram TLOC for known geographical longitude of the point (lambda), sidereal time (ST), and right ascension of the sun (alpha) determines local time.

The subprogram ZDSUNL for known geographical latitude (phi) and longitude (lambda) of a point and for the directed cosine of the radius vector of the Sun \((x_0^0, y_0^0, z_0^0)\) in the Greenwich system of coordinates determines the zenith distance of the sun.

The subprogram ZDSUNC determines the zenith distance of the sun for known directed cosines of the normal to the surface of the earth's ellipsoid \((x_N^0, y_N^0, z_N^0)\) and for directed cosines of the radius vector of the sun \((x_0^0, y_0^0, z_0^0)\) in the Greenwich system of coordinates.


Common domain: /CHI/3, /CD2GR/, /CHRAD/.

Remark 1. Subprogram--function ZDSUNC refers to the subprogram-function ZDSUNL.


4. Initial data: ST--sidereal time in radians; ALT, ALN--geographical latitude and longitude of the point in radians; AS--right ascension of the sun in radians.

Block XNG--directed cosines of the normal to the surface of the earth's ellipsoid at the point underneath the satellite.

Block X3G--directed cosines of the radius vector of the sun.

5. Results: A--right ascension of the \textit{ABS} in degrees; TL--local time in hours; AS--zenith distance of the sun in degrees.

6. Use of the domain COMMON. The constants from these blocks are used: COMMON/CHI/3, /CD2GR/, /CHRAD/ (see 90, No. 1, 2, 3 table 2,3).

7. Algorithm. Right ascension of the \textit{ABS}--lambda, local time --t, and the zenith distance of the sun --\textit{ZS} are determined according to the following formulas:
\[ \lambda^o = \lambda + ST, \]
\[ t^o = \lambda^o - \alpha^o + \pi, \]
\[ \cos ZS^o = x^o_n x^o_0 + y^o_n y^o_0 + z^o_n z^o_0. \]

### 3. Texts

FUNCTION RIGHTA(ST, ALN)
COMMON /DEGR/DEGR
COMMON /PI/PI, PI2, PI12
T=ST+ALN
K=K+PI12
T=T-K+PI12
RIGHTA=T*DEGR
RETURN
END

FUNCTION TLOC(ST, ALN, AS)
COMMON /PI/PI, PI2, PI12
COMMON /HRAD/HRAD
T=ST+ALN-AS+PI
6 IF(T>4,5,9
4 T=PI12
GOTO 6
9 K=K+PI12
T=T-K+PI12
TLOC=T*HRAD
RETURN
END

FUNCTION ZDSUNC(XE, XSG)
DIMENSION XE(3), XSG(3)
COMMON /PI/PI, PI2, PI12
COMMON /DEGR/DEGR
C=0
DO 3 J=1,3
3 C=XE(J)*XSG(J)+C
S=SQR(1-C*C)
IF(C<0) 7, 8, 7
6 ZS=PI12
GOTO 11
7 ZS=ATAN(S/C)
10 ZS=ZS+PI
11 ZDSUNC=ZS*DEGR
RETURN
END

FUNCTION ZDSUNL(ALT, ALN, XSG)
DIMENSION XSG(3), XR(3)
CB=COS(ALT)
XR(1)=COS(ALN)*CB
XR(2)=SIN(ALN)*CB
XR(3)=SIN(ALT)
ZDSUNL=ZDSUNC(XR, XSG)
RETURN
END

### 4. Geomagnetic coordinates of the AFS, geomagnetic local time

(I02-GHLL, GEOM),
1. Purpose. For the point, given by directed cosines \((x_N, y_N, z_N)\) in the Greenwich system of coordinates for the normal to the earth's ellipsoid, one determines the geomagnetic latitude and longitude (subprogram GMILLO) and geomagnetic time (subprogram TGEOM).

2. Structure. Subprogram GMILLO and subprogram-function TGEOM.

   a. General domain: /CII/3, /CDGRA/1, /CLLL/2, /BG7/2.
   b. Conversion: C.Li. GMILLO \((X_NG, GLT, GLLN)\),
      TM=TGEOM \((X_NG, XSG)\).
   c. Initial data: Block XSG--directed cosines of the normal to the surface of the earth's ellipsoid in the Greenwich system of coordinates; the block XSG--directed cosines of the radius vector of the sun in the Greenwich system of coordinates.
   d. Results: GLT, GLLN--geomagnetic latitude and longitude in degrees;
      TM--local geomagnetic time in hours.
   e. Use of the domain COMMON. From the blocks COMMON/CII/3, /CDGRA/1, the constants are used (see \(\delta\), Nos. 1, 2, table 2.3)

   In the block COMMON/BGLL/GLT, GLN the subprogram TGEOM gives the value of the geomagnetic latitude and longitude of the point, in the block COMMON/BGLL/GLT, GLN--the values of the geomagnetic latitude and longitude of the sun (in degrees).

3. Algorithm. The geomagnetic latitude \(\phi_m\) and longitude \(\lambda_m\) are computed by the formulas:

   \[
   \sin \varphi_m = x_0^0 y_0^0 + y_0^0 z_0^0 + z_0^0 x_0^0,
   \sin \lambda_m = \frac{y_0^0 x_0^0 - x_0^0 y_0^0}{z_0^0 \sin \varphi_m - z_0^0},
   \]

   \(x_0^0 = 0.71447078, \ y_0^0 = -0.186126001, \ z_0^0 = 0.979924705 - \)

directed cosines (in the Greenwich system of coordinates) of the radius vector of the geomagnetic pole (\(\phi_G=73.5^\circ, \lambda_G=-69^\circ\)).

Geomagnetic time TM=\(\lambda_m\)--\(\lambda_m\), where \(\lambda_m\)--geomagnetic longitude of the sun, calculated by the same formulas, as \(\lambda_m\), if instead of \(x_0^0, y_0^0, z_0^0\) one substitute the values \(x_0^0, y_0^0, z_0^0\), directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

4. Texts.
SUBROUTINE GM1L(XE, GLT, GLN)
DIMENSION XE(3)
COMMON/COEGR/DEGR
COMMON/CP1/P1:PID2,P2
SF=XE(1)*0.071447079+16612601*XE(2)*0.07924369*XE(3)
CF=SQRT(1-SF*SF)
IF(CF)<1.21 2
GLN=SIGN(PID2, SF) *DEGR
GOTO 3
1
GLT=ATAN(SF/CF)*DEGR
W=SF*.07924369-XE(3)
CF=XE(2)*.071447079+16612601*X(1)
IF(W)<13.43 4
GLN=SIGN(PID2, CF)
GOTO 9
3
GLN=ATAN(CF/W)
IF(N)<7.88 7
GLN=GLN*PI
GOTO 6
6
IF(GLN)<6.6 6
9
GLN=GLN*PI2
6
GLN=GLN*DEGR
RETURN
END

FUNCTION TVGEO(XE, XSG)
COMMON/BGS/LLT, SLN
COMMON/BGL/LLT, GLN
DIMENSION XE(3), XSG(3)
CALL GM1L(XE, LLT, GLN)
CALL GM1L(XSG, SLT, SLN)
W=(GLN-SLN)/15.+12.
I='(W)'/1.2.2
1
W=24.+W
2
TVGEO=W
RETURN
END
4.3. Auroral longitude, auroral time
(103---AULONG, TAUR)

1. Purpose. For the point, given by directed cosines of the
normal to the surface of the earth's ellipsoid \((x_N^0, y_N^0, z_N^0)\)
in the Greenwich system of coordinates, the auroral longitude
(subprogram AULONG) and auroral time (subprogram TAUR) are determined.

2. Structure. Subprogram--functions AULONG, TAUR.
Common blocks: /CHI/3, /CHRAD/1, /BAU/1.

3. Conversion: EU=AULONG (XNG),
TA=TAUR (XNG, XSG).

Remark 1. Subprogram--TAUR refers to the subprogram AULONG.

4. Initial data: The block XNG_--directed cosines of the nor-
mal to the surface of the earth's ellipsoid in the Greenwich system of
coordinates; block XSG_--directed cosines of the radius vector of
the sun in the Greenwich system of coordinates.

5. Result: AU=--auroral longitude in hours; TA=--auroral time in
hours.

6. Use of the domain COMMON. From the blocks COMMON/CHI/3,
/CHRAD/1 one uses the constants (see (5), Nos. 1, 3 table 2,3).
In the block COMMON/BAU/AU the subprogram TAUR addresses the
value of the auroral longitude.

7. Algorithm. The auroral longitude \(\lambda_{\alpha}\) according to
(6) is computed by the formulas:

\[
\lambda_{\alpha} = \text{arctg} (A/B) - 56^\circ, 61,
\]

\[
\begin{align*}
A_\alpha &= 0.939414 x_\alpha - 0.331032 y_\alpha - 0.157062 z_\alpha, \\
A_\alpha &= -0.353629 x_\alpha - 0.923585 y_\alpha - 0.147963 z_\alpha.
\end{align*}
\]

\[
\begin{align*}
\alpha &= x_N^0 + 0.062724, \\
\alpha &= y_N^0 - 0.02319, \\
\alpha &= z_N^0 - 0.010589.
\end{align*}
\]

Auroral time \(T_{\alpha}\) =\(\lambda_{\alpha} - \lambda_{\alpha}^0\), \(\lambda_{\alpha}^0\) computed by the same formulas as \(\lambda_{\alpha}\), if instead of
\((x_N^0, y_N^0, z_N^0)\) one substitutes \((x^0, y^0, z^0)\), directed cosines of the
radius vector of the sun in the Greenwich system of coordinates.

8. Texts.

1. Purpose. Subprograms intended for the calculation of B, L, the parameters of the magnetic field of the earth.


3. Conversion: CALL BL (XC, YC, ZC, NG1, TE1, B3, FL).

4. Initial data: XC, YC, ZC--Greenwich coordinates of the point in meters, NG1--number of harmonics considered (NG1 less than or = 9), TE1--period, for which are computed the coefficients of the expansion for the magnetic field of the earth, minus 1960 (for example, if the period 1965 is chosen, then TE1=5).

5. Results B3--block, containing four actual magnitudes: Bphi, Btheta, Bz, B, coordinates of the vector B in the geographical
system of coordinates and the modulus of the vector B in degrees, FL--value of L in radius of the earth.
6. Use of domain COMMON.
   In the block COMMON/COH/ there are assigned HC, phi, lambda -
   geographic coordinates of the magnetic conjugate point (HC in km, phi, lambda---in degrees).
7. There is a more detailed description in (6).

4.5. Invariant geomagnetic latitude.
   (INP---AINLAT, OINLAT)

1. Purpose. The subprogram AINLAT determines the invariant geomagnetic latitude for known values B and L, of the geomagnetic parameters. The subprogram OINLAT determines the invariant latitude for the basis of the line of force.

7. Structure. Subprograms---functions AINLAT, OINLAT.
   Common block /CDEGR/.
3. Conversions: ALI=AINLAT (B, FL),
   ALO=OINLAT (FL).
4. Initial data: B, FL--B, L---parameters of the magnetic field of the earth.
5. Results: ALI---invariant geomagnetic latitude in degrees,
   ALO---invariant latitude of the basis of the line of force in degrees.

6. Use of the domain COMMON: One uses the constant from the block /CDEGR/ (see (5) No. 2 table 2.3)

7. Algorithm. The invariant geomagnetic longitude Lambda, according to (6), is computed according to the formula:

\[ \Lambda = \arctg(\sqrt{1/B_5} - 1), \]

где \( B_5 = \sum_{n=0}^{\infty} b_n a^n \),
\[ a = \left( 0.311653/(B_5) \right)^{1/3} \]

Инвариантная широта у основания силовой линии,

\[ \Lambda_0 = \arctg(\sqrt{L-1}), \]

где \( B, L \)---геомагнитные параметры.
1. **Purpose.** From the known Greenwich coordinates of the AES (x, y, z) and the directed cosines of the radius vector of the sun (x_0, y_0, z_0) one determines the geographical coordinates of T (GEOG). 

2. **Structure:** Subprogram PTERM. One uses the external sub-program GEOGRC (B10).

3. **Conversion:** CALL PTERM (XG, XSG, HT, TLT, TLN).

4. **Initial data:** Block XG--Greenwich coordinates of AES; block XSG--directed cosines of radius vector of the sun in Greenwich system of coordinates.

5. **Results:** HT, TLT, TLN--elevation above the surface of the
earth, latitude and longitude of T-point (angles in radians).

6. Algorithm: \( x_T, y_T, z_T \) -- Greenwich coordinates of the T-point, points of intersection with the plane of the terminator of the line, connecting the AES with the sun (or continuation of this line), are computed with the formulas:

\[
\begin{align*}
x_T &= x - a_s x^o, \\
y_T &= y - a_s y^o, \\
z_T &= z - a_s z^o,
\end{align*}
\]

\( a_s = x x^o + y y^o + z z^o \)

The geographical coordinates of T-point a (or the algorithm par. 3.10 (2)).

7. Text.

```
SUBROUTINE PT5 (XG,XSG,LT,C,EX,ET,LTN,XT)

DIMENSION XG(3),XSG(3),XT(3)

C=0
DO 3 J=1,3
3 C=C+XG(J)*XSG(J)
DO 4 J=1,3
4 XT(J)=XG(J)*C*XSG(J)
CALL GEOGR(XT,HT,TLT,TLN,XT)
RETURN
END
```

4.7. Determination of the Greenwich coordinates of the point, given geographical latitude, longitude and elevation above the surface of the earth's ellipsoid, and matrices of the transformation from the Greenwich to the point system of coordinates, connected with this point (I/O7-I/I5).”

1. Purpose. The Greenwich coordinates \((x_G, y_G, z_G)\) are determined for the point \(I\), prescribed geographical latitude, \((\phi_1)\) longitude \((\lambda_1)\) and elevation \((h_1)\) above the surface of the earth's ellipsoid.

The point topocentric system of coordinates, connected with the point \(I\), is determined in the following manner.

The origin of the coordinates coincides with the point \(I\), the axis \(ix_1\) is directed to the north pole of the earth by the tangent to the meridian of the point \(I\), the axis \(iy_1\) -- the external normal to the surface of the earth's ellipsoid, the axis \(iz_1\) completes the system up to the right side.

The subprogram GEINT determines the matrix of the transformation from the Greenwich system of coordinates to the point system.

2. Structure. Subprogram GEINT.

Common domain: /CAE/.
3. Conversion: CALL POINT (it, ILT, ILM, XG, GI).

4. Initial data: M, ILT, ILM--elevation, geographical latitude, longitude of the point i.

5. Results: Block XING--Greenwich coordinates of the point i;

GP--two-dimensional 3 x 3 matrix of the transformation from Greenwich system to coordinate in the point system.

6. Use of the domain COMMON: one uses the constants from the block /GA5/ (see (.), No. 11, Table 2.1).

7. Algorithm: Coordinates of point i in the Greenwich system of coordinates are determined with the formulas:

\[
\begin{align*}
  x_{p0} &= (a_e/A + h_p) \cos \varphi_p \cos \lambda_p, \\
  y_{p0} &= (a_e/A + h_p) \cos \varphi_p \sin \lambda_p, \\
  z_{p0} &= (a_e(1-\alpha)^2/A + h_p) \sin \varphi_p, \\
  A &= (\cos^2 \varphi_p + (1-\alpha)^2 \sin^2 \varphi_p)^{1/2},
\end{align*}
\]

\[ \alpha_e, \alpha_h = \text{semimajor axis and coefficient of compression of the earth's ellipsoid.} \]

Matrix of the transformation (G_{ij}) from the Greenwich system of coordinates to the point system (determined in par. 1), which has the following form:

\[
\begin{pmatrix}
  [G_{Pij}] = & \\
  & \\
  & \\
  & \\
\end{pmatrix}
\]

48
SUBROUTINE POINT(HP, ALTP, ALNP, XP, GP)

DIMENSION XP(3), GP(3,3)
COMMON/CAS/CE/AC, AL
CL=0.9(ALTP)
SL=0.9(ALNP)
CLN=0.9(ALNP)
SLN=0.9(ALNP)
ALK=1, AL
ALK=ALK+ALK
AN=SQR(CL+CL+ALK+SL+SL)
AE/AO
GP(1,1)=SL+CL
GP(1,2)=SL+SLN
GP(1,3)=CL
GP(2,1)=CL+CL
GP(2,2)=CL+SLN

GP(3,3)=SL
GP(3,1)=SLN
GP(3,2)=SLN
GP(3,3)=0.
XP(1)=(4*ALK+HP)*SL
AA=HP
XP(1)=A*GP(2,1)
XP(2)=A*GP(2,2)
RETURN
END

4.5. Change from the Greenwich system of coordinates to the point

topo-centric form, determination of ranging, angle of location,

azimuth (103=GRI X, TURN).

1. Purpose. Subprogram GRIX for known Greenwich coordinates

of the AER (x, y, z) and Greenwich coordinates of the point i

(x1, y1, z1) determines the coordinates of AER (x1, y1, z1) in

topo-centric system of coordinates; AVL, inclined range

distance AES to point i; AL, angle of location---angle between

radius vector of AES and the plane, perpendicular to the axis \( y_i \)

of the point topo-centric system of coordinates; AL azimuth---angle

between direction for the sun (angle \( x_i \)) and the projection of

the radius vector of the AES on the plane perpendicular to the axis

\( y_i \) (opposite the hand of the clock). The components of the vector

of velocity in the point system of coordinates may be determined,

using the subprogram TURN, intended for the multiplication of the

matrix of dimension (N, N) by the N-dimensional vector.

2. Structure. The subprograms GRIX, TURN.

External subprograms utilized: GCLTLN (R10)

3. Conversion to GRIX:

CALL GRIX(XS, XG, GI, XI, DAL, WI, WI).

Initial data: Block XS----Greenwich coordinates of AER; block

XG----Greenwich coordinates of point i; GI---twodimensional block

(z,3)----matrix of the transformation from the Greenwich to the point

system of coordinates.
Results: block $X_{2}$—coordinates of $\text{AES}$ in topocentric point system of coordinates, $D\lambda L$, $A\mu M$, $A\zeta$, determined above.

4. Conversion to subprogram TURN.

CALL TURN $(X, A, Y, N)$.

Initial data: $X$—block of dimension $N$; $A$—block of dimension $(N, N)$; $N$—dimension.

Results: $Y$—block of dimension $N$ ($Y = AX$).

6. Algorithm

\[
\begin{bmatrix}
  z_p \\
  y_p \\
  x_p
\end{bmatrix}
= \begin{bmatrix}
  x - x_{PG} \\
  y - y_{PG} \\
  z - z_{PG}
\end{bmatrix} \begin{bmatrix}
  v_{x_p} \\
  v_{y_p} \\
  v_{z_p}
\end{bmatrix} = \begin{bmatrix}
  v_x \\
  v_y \\
  v_z
\end{bmatrix}
\]

\[
D\lambda L = \left( (x - x_{PG})^2 + (y - y_{PG})^2 + (z - z_{PG})^2 \right)^{1/2};
\]

\[
A\mu M = \arctan \left( \frac{z_p}{x_p^2 + y_p^2} \right);
\]

\[
A\zeta = \arctan \left( \frac{y_p}{x_p} \right);
\]

\[
(G_{ij}) \text{—matrix of transformation from Greenwich system of coordinates to the point form;}
\]

7. Text:

```fortran
SUBROUTINE GRPX(XG, XGP, GP, XP, CAL, AM, AZ)
  DIMENSION XG(3), XGP(3), GP(3, 3), XP(3, 3)
  DAL = 0.
  DO 2 J = 1, 3
    X(J) = XG(J) - XGP(J)
    2 DAL = DAL + X(J) * X(J)
  D = SQRT(DAL)
  CALL TURN(X, GP, XP, 3)
  X(1) = XP(1)
  X(2) = XP(2)
  X(3) = XP(3)
  CALL GCLTLN(X, AM, AZ)
RETURN
END
```

```fortran
SUBROUTINE TURN(XG, GP, XP, H)
  DIMENSION XG(N), GP(N, N), XP(N, N)
  DO 1 I = 1, N
    XP(I) = 0.
    DO 1 J = 1, N
      1 XP(I) = XP(I) + GP(I, J) * XG(J)
RETURN
END
```
1. Purpose. For known coordinates of $AES (X, Y, Z)$ in the absolute system of coordinates one determines the coordinates of the $AES$ in the solar-ecliptic system of coordinates, determined by the following means: the center of the system $O$ coincides with the center of the earth, axes $CX_{ce}, CY_{ce}$ lie in the plane of the ecliptic, the axis $CX_{ce}$ directed towards the sun, axis $OX_{ce}$ perpendicular to the plane of the ecliptic and forming an acute angle with the axis of rotation of the earth directed toward the north.

   Subprograms utilized: `GCLTLN` (B10). Common block `/BIECL/`.

3. Conversion: CALL `ABSECL` ($X_A, X_A3, X_SE, CLTJ, CLNS$).

4. Initial data: Block $X_A$--absolute coordinates of $AES$;
   Block $X_A3$--directed cosines of radius vector of the sun in the absolute system of coordinates.

5. Results: Block $X_SE$--coordinates of $AES$; $CLTJ, CLNS$--geocentric latitude and longitude of $AES$ in solar-ecliptic system of coordinates.

6. Use of the domain COMMON. From the block COMMON `/BIECL/` one uses $\cos \epsilon$ and $\sin \epsilon$, computed in the subprogram `SUN(Re)`, where $\epsilon$--angle of inclination of the plane of the earth's equator to the plane of the ecliptic.

7. Algorithm:

$$
\begin{bmatrix}
X_{SE} \\
Y_{SE} \\
Z_{SE}
\end{bmatrix} = 
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} 
+ 
\begin{bmatrix}
X_0^2 Y_0^2 \sin Y_0 \\
X_0 Z_0 \sin Y_0 \\
Y_0 \sin Y_0
\end{bmatrix} 
\begin{bmatrix}
r^1 \sin Y_0 \\
r^1 \sin Y_0 \\
r^1 \sin Y_0
\end{bmatrix} 
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix},
$$

where

$$
r^1 = (Y_0^2 + Z_0^2)^{1/2}.
$$

8. Text:
4.10. Moments of time of the entry of AES into the earth's shadow and exit from the shadow (IIU--III'Z'W')
1. Argument. For known elements of the orbit: a, e, i, 
Omega, omega, for position in the orbit (u) at a given moment in 
time t one determines the moments of time of entry of the AES into 
the shadow of the earth and exit from the shadow.
2. Structure: Subprogram SHADOW.
One uses external subprograms ROOT 4 (IIU), SUN (COX).
Common blocks: /CHR/1, /CRE/1, /CHR/1
3. Conversion: CALL SHADOW (DT, T, A, T1, T2).
4. Initial data: Block Ai, containing elements of the orbit and 
the argument of latitude: a, e, i, Omega, omega, u; DT--data 
in the form RJD, T--Moscow time, corresponding to the argument of 
latitude u.
5. Results: T1, T2--are, respectively, the moments of entry of 
the AES into the shadow of the earth and exit from the shadow (in units 
given by the scale factor EJEC). If AES does not get into the shadow, 
then T1=T2.0.
6. Use of the domain COMMON: One uses the constants from the 
blocks COMMON: /CHR/1, /CRE/1, /CHR/1 (see (5), Nos. 5, 9, 
table 2.1, No. 1 table 2.3).
7. Algorithm: Moments of time (T1) of entry of AES into the 
shadow of the earth and (T2) of exit from the shadow are determined 
without computation of the effect of the penumbra; with the assump-
tion that the earth has no compression and is not displaced orbit-
ally. (7). These moments of time correspond to actual anomalies of 
the AES, satisfying the following relationship:

\[ \cos \psi = \frac{R_\theta}{r} = -\frac{(r^2-a_e^2)^{1/2}}{r} \]  

(1)

where \( R_\theta \)--radius vector of the sun, \( r \)--radius vector of the 
AES, \( a_e \)--equatorial radius of the earth, \( \psi \)--angle between 
radius-vector of AES and radius vector of the sun. The relation-
ship (1) leads to the equation of the fourth degree with respect to
the cosines of the actual anomaly:

\[ S' = A_0 \cos^4 \nu + A_1 \cos^3 \nu + A_2 \cos^2 \nu + A_3 \cos \nu + A_4, \quad (2) \]

where

\[ A_0 = (\frac{\alpha e}{\beta})^4 e^4 - 2(\frac{\alpha e}{\beta})^2 (\xi^2 - \beta^2) e^2 + (\beta^2 + \xi^2)^2, \]
\[ A_1 = 4(\frac{\alpha e}{\beta})^4 e^3 - 4(\frac{\alpha e}{\beta})^2 (\xi^2 - \beta^2) e, \]
\[ A_2 = 6(\frac{\alpha e}{\beta})^4 e^2 - 2(\frac{\alpha e}{\beta})^2 (\xi^2 - \beta^2) - 2(\frac{\alpha e}{\beta})^2 (1 - \xi^2) e^2 + 2(\xi^2 - \beta^2) (1 - \xi^2) - 4\beta^2 \xi^2, \]
\[ A_3 = 4(\frac{\alpha e}{\beta})^4 e - 4(\frac{\alpha e}{\beta})^2 (1 - \xi^2) e, \]
\[ A_4 = (\frac{\alpha e}{\beta})^4 - 2(\frac{\alpha e}{\beta})^2 (1 - \xi^2) + (1 - \xi^2)^2; \]

\[ \rho = \text{parameter of orbit} = \alpha (1 - e^2), \]
\[ \beta = X_o \rho_x + Y_o \rho_y + Z_o \rho_z, \]
\[ \xi = X_o Q_x + Y_o Q_y + Z_o Q_z, \]

\[ X_o, Y_o, Z_o \] - направляющие косинусы радиуса-вектора Солнца, \( R_o \),
directed cosines of radius vector of the sun

\[ \rho_x = \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i, \]
\[ \rho_y = \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i, \]
\[ \rho_z = \sin \omega \sin i, \]
\[ Q_x = -\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i, \]
\[ Q_y = -\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i, \]
\[ Q_z = \cos \omega \sin i. \]

\[ a \]
The solutions to equations (2), satisfying the conditions:

\[ \beta \cos \nu + x_i \sin \nu \] less than 0,
\[ S = (1 + \cos \nu)^2 + (p/a_e)^2 \]
\[ (\beta \cos \nu + x_i \sin \nu)^2 - (p/a_e)^2 = 0. \]

are assumed for further consideration.

The condition:

\[ S' = \rho(\beta \cos \nu + \xi \sin \nu)(\xi \cos \nu - \beta \sin \nu) - a_e^2 e(1 + \cos \nu) \sin \nu > 0 \]
determines the value of the actual anomaly \( \nu \), in the entry of the ISZ into the shadow of the earth, \( S' \) less than zero corresponds to the value \( \nu_2 \), determining the exit of the ISZ from the shadow.

53
With the value of \( t \), time, for corresponding given position of AES in orbit, we determine \( \tau \), the time of passage of the AES through perigee:

\[
\tau = L - (E - \varepsilon \sin E) / \lambda ,
\]

\[
\text{where } \lambda = \sqrt{\mu / a^3} , \quad t g \, E / 2 = [(1 - \varepsilon) / (1 + \varepsilon)]^{2} t g \, v / 2 , \quad v = u - \omega ,
\]

mu-produse of gravitational constant times the mass of the earth. Moments of time \( T_i \) \((i=1, 2)\) are determined by the known values \( t \) and \( v_i \):

\[
T_i = \tau + (E_i - \varepsilon \sin E_i) / \lambda ,
\]

\[
t g \, E_i / 2 = [(1 - \varepsilon) / (1 + \varepsilon)]^{2} t g \, v_i / 2 .
\]
CALL SUM (T1, T2, W1, W2, W3, X1)

CONTINUE
AP = P(1) + P(8)
AQ = P(1) + P(7)
P(5) = P(2) + P(8)
P(6) = P(4)
P(9) = P(2) + P(7)
DO 2 J = 1, 2
P(J) = P(7) + P(J + 3) - AQ + P(J + 4)
P(6) = P(9)
B = 0
S = 0
DO 3 J = 1, 3
B = X5(J) + P(J) * R
S = X5(J) * P(J)
1 W1 = A(2) + A(3)
PK = A(1) * (A - 1)/AE
Q(3) = 2.0 * W3 * W4 - 4.0 * AP * AQ
Q(4) = 0
Q(5) = W6 - W4
DO 5 J = 1, 3
Q(J) = Q(J) + W1 * P(J) + W2 * P(J + 3)
Q(J + 2) = Q(J + 2) + W1 * P(J + 3)
CALL ROOT4 (Q, V, JR)
N = 4
K = 1
IF (F(1)) 8, 6
6 K = 3
8 IF (F(2)) 19, 11
11 N = 2
9 IF (K = N) 12, 12, 7
12 DO 17 J = K, N
10 IF (AES(V(J)) = 1) 18, 18, 17
18 W1 = B + V(J)
W3 = 0 + A(2) * V(J)
W6 = W5 + W3 * PK
DO 30 I = 1, 2
W4 = S + W3
W4 = W1 + W4
V = W4 + W3 * W6 / W4
IF (AES(V)) = 1, E = 3) 27, 30, 30
27 IF (W6) 19, 30, 30
19 V = T1
W2 = 1.0 * V(J)
IF (W2, E = 19)
- V = ATAN(W3 * EK / W2) * 2.
IF (V, LT, 0.1)
- V = V + 12
V = T0 * (V = A(2) + S1P(V) = TP) * AL
23 W2 = PK * W4 + (S1P(V) = B0) * W3
IF (W2) 24, 25, 25
25 T3 = V
GOTO '30
24 T2 = V
30 W3 = W3
17 CONTINUE
9 RETURN
END
4.11. Calculation of the roots of algebraic equations of the
fourth, third and second degree (ILL-RC4T4)

1. Purpose. Determination of the roots of the algebraic
equation
\[ A_0 x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0, \]
of the fourth \( A_0 \neq 0 \), third \( (A_0 = 0) \) and second \( (A_0 = 0, A_1 = 0) \)
degree with real coefficients.


4. Initial data: Block A--coefficients of equation (1) in
order: \( A_0, A_1, A_2, A_3, A_4 \).

5. Results: Block \( Y \) --roots of equation (1);
Block \( IR \) --characteristics of the roots:

\[
\begin{align*}
IR (1) = & \ 0, \text{ if } Y(1), Y(2) \text{--real roots} \\
& \ 4, \text{ if } Y(1), Y(2) \text{ real and imaginary part of a} \\
& \ \text{complex root (for complex-conjugate root } Y(3) \text{)} \\
& \ 1, \text{ if } A_0 = A_1 = A_2 = 0 \text{ (equation of first degree)} \\
& \ 0, \text{ if } Y(3), Y(4) \text{--real roots} \\
& \ 4, \text{ if } Y(3), Y(4) \text{--real and imaginary part of} \\
& \ \text{complex root (for complex conjugate root } Y(5) \text{ taken} \\
& \ \text{with opposite sign)} \\
& \ 3, \text{ if } A_0 = 0 \text{ (equation of third degree)} \\
& \ 2, \text{ if } A_0 - A_1 = 0 \text{ (equation of second degree)} \\
& \ 1, \text{ if } A_0 = A_1 = A_2 = 0 \text{ (equation of first degree).}
\end{align*}
\]

Remark. For \( IR (1) = 1 \) \( Y(1) \)--root of equation \( 1 \), of the first
degree. For \( IR (1) = 3 \) \( Y(3) \)--real root of an equation of the third
degree.

6. Algorithm. For the solution of equation (1) one uses the
algorithm, described in [7] (p. 445):

a) Equation of 4 degree is solved by the method of Descartes:
Dividing equation (1) by \( A_0 \) (if \( A_0 = 0 \), then we proceed to part delta),
we obtain
\[ y^4 + B_1 y^3 + B_2 y^2 + B_3 y + B_4 = 0. \]
We transform this equation, excluding from it the term of the third
degree:

\[
\begin{align*}
& x^4 + px^2 + qx + r = 0, \\
p &= 6h^2 + 3B_1 h + B_2, \\
q &= 4h^3 + 3B_1 h^2 + 2B_2 h + B_3, \\
r &= h^4 + B_1 h^3 + B_2 h^2 + B_3 h + B_4, \\
h &= -B_1/4, \quad y = x + h.
\end{align*}
\]
For $c = 0$ we obtain the biquadratic equation, for the solution of which we use the formulas of part V. For $c \neq 0$ we consider the cubic resolvent

$$
t^3 + c_2 t^2 + c_3 t + c_4 = 0,
$$

with

$$
c_2 = 2P, \quad c_3 = P^2 - 4R, \quad c_4 = -Q^2.
$$

We transform equation (3) into the form:

$$
x^3 + ax + b = 0,
$$

where

$$
\alpha = c_3/3 - s^2, \quad \beta = b/2 = +s^2 - c_3 a/2 + c_4 / 2,
$$

and

$$
s = c_3/3, \quad x = t + s,
$$

$$
\Delta = \frac{a^3}{27} + \frac{b^2}{4}.
$$

If $\Delta$ is greater than 0, then the solution of equation (4) is found by the formula of Kaplan:

$$
z_3 = (-\beta/2 + (\Delta)^{1/2})^{1/3} + (\Delta/2 - (\Delta)^{1/2})^{1/3}
$$

If $\Delta = 0$, then the roots of equation (4) are determined by the formula:

$$
z_2 = \frac{-\beta}{2}, \quad z_1 = \frac{-\beta}{2} + \frac{\Delta^{1/2}}{2}
$$

If $\Delta$ is less than 0, then we use the representation of the roots as resolvents in trigonometric form,

$$
z_1 = E_0 \cos \varphi/3, \quad z_2 = E_0 \cos(\varphi/3 + 120^\circ), \quad z_3 = E_0 \cos(\varphi/3 + 240^\circ),
$$

where $E_0 = 2 (-\alpha/3)^{1/2}$, $\cos \varphi = -\frac{\beta}{2}/(-\frac{\alpha}{3})^{1/2}$, $0 < \varphi < \pi$.

We select a critical root of equation (3):

$$
R' = \max (z_1 - s, z_2 - s, z_3 - s),
$$

where $z_1 - s$ -- real root of equation (3).

Knowing $R'$, it is possible to break down equation (2) with the multipliers:

$$
(x^2 + x\sqrt{R'} + \xi)(x^2 - x\sqrt{R'} + \beta) = 0,
$$

where

$$
\xi = 1/2(P + R' - Q/\sqrt{R}), \quad \beta = 1/2(P + R' + Q/\sqrt{R}).
$$
Using the relationships of division V, we obtain the solutions \( x_i \) \((i=1,4)\) of two quadratic equations \( (1) \), and consequently—the roots of equation \( (1) \): \( y_i = x_i + h \).

6. Cubic equation we write in the form

\[
A_1 y^3 + A_2 y^2 + A_3 y + A_4 = 0.
\]

Dividing the left side by \( A_1 \) (if \( A_1 = 0 \), we go on to division b), we obtain

\[
y^3 + c_2 y^2 + c_3 y + c_4 = 0.
\]

This equation coincides with equation \( (3) \), the solution of which is described in division a.

Remark For \( \Delta \) greater than zero, the complex roots \( y_1, y_2 \) of equation \( (4) \) are obtained by the solution of a quadratic equation

\[
y^2 + (c_3 + z_3 - s)y + c_3 + (c_2 + z_3 - s)(z_3 - s),
\]

where \( z_3 \)—real root of equation \( (4) \), \( s = c_2/3 \).

b) Dividing the left side of the quadratic equation

\[
A_1 y^2 + A_3 y + A_4 = 0
\]

by \( A_1 \) (if \( A_1 = 0 \), then we go on to division c), we obtain:

\[
y^2 + P_3 y + P_4 = 0.
\]

The roots of this equation

\[
y_{1,2} = \frac{-B_3 \pm \sqrt{B_3^2 - 4B_4}}{2}
\]

c) Equation of the first degree \( A_3 y + A_4 = 0 \).

Solution \( y = -A_4/A_3 \).
SUBROUTINE ROOT4(A,V,IR)
DIMENSION A(9),V(4),IR(2),IN(3),R14
DATA P1,P02,P12/3.14159265,1.57079632,6.28318531/
DATA IN/1,0,2/
K=2
IR(1)=0
IR(2)=1
IF(A(1))=1.100.1
1 N=1
2 DO 4 J=1,4
3 V(J)=0.
2 D(J)=A(J+1)/A(1)
4 M=-8(1)/4.
V(2)=1.
3 DO 3 J=1,4
3 V(2)=V(2)+M*B(J)
4 Q=4.
W=3.
5 DO 4 J=1,3
5 Q=Q+M*B(J)*W
4 N=W-1.
P=(2.)*H+B(1))3.)*H*B(2)
5 V(1)=P
6 IF(ABS(Q)-1.E-16)103,103,100
103 CONTINUE
GOTO 7
1 IF(IN(J))
2 D=V(J)+V(J)-4.0V(J+1)
3 V(J)=-V(J)/2.
4 IF(0)5,6,6
5 D=SQR(T(0))/2.
6 V(J+1)=V(J)-D
7 V(J)=V(J)+D
8 IR(1)=0
9 GOTO 7
10 V(J+1)=SQR(T(-D))/2.
11 IR(1)=6
7 CONTINUE
GOTO(104,105,107),K
104 IF(IR(1))10,11,10
11 V(4)=-V(1)
12 V(2)=-V(2)
13 V(3)=0.
14 V(1)=0.
N=3
K=3
GOTO 103
10 W1=V(1)+V(1)
W=W1+V(2)+V(2)
R1=SQR(T(W)
W=SQR(T(W+1)
R1=SQR(T(R1)*2.
V(1)=V(2)/R1
V(2)=W/R1
V(3)=W-V(1)
V(1)=V(1)+W
V(4)=V(2)
IR(2)=4
100 IF(A(2))15=16X3
105 DO 16 J=2,4
106 B(1)=A(J-1)/A(2)
107 IR(2)=3
108 GOTO 109
109 S=B(2)/3.
110 W=S
111 AR=B(3)/3,-W
112 BR=S-B(3)/2,-5*W-B(4)/2.
113 DEP=AR+BR+BR
114 IF(D)17,18,19
115 M=2,-SQR(T-AR)
116 GOTO 20
20 W1=P1D2
21 GOTO 23
22 W1=SQR(T-D)
23 IF (BR)51,50,51
24 W1=P1D2
25 GOTO 23
26 W1=ATAN(W1/BR)
27 IF(BR)22,23,23
28 W1=P1W1
29 W=SQRT(D)
30 V(1)=BR+W
31 V(2)=BR-W
32 WH=ABS(V(J))
33 W1=ALOG(W)
34 W1=EXP(W1/3,)
35 V(J)=SIGN(W1,V(J))
36 V(3)=V(1)+V(2)-5
37 V(1)=0.
38 V(2)=0.
39 K=IR(2)
40 W1=V(3)
41 GOTO (111,205,205),K
42 N=ABS(BR)
43 W1=ALOG(N)
44 W1=EXP(W1/3,)
45 V(1)=SIGN(W1, BR)
46 V(2)=-V(1)
47 V(1)=2*V(1)
48 V(3)=V(2)
49 N=1
50 CONTINUE
51 V(J)=V(J)-H
52 GOTO 20
53 N=1
54 K=2
55 GOTO 103
56 V(1)=B(2)+V(3)
57 V(2)=V(1)+V(3)+B(3)
58 N=1
59 CONTINUE
60 GOTO 103
61 IF(A(3))36,102,36
62 V(1)=A(4)/A(3)
63 V(2)=A(5)/A(3)
64 IR(2)=2
65 N=1
66 GOTO 103
67 V(1)=A(5)/A(4)
68 GOTO 103
69 END
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