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PHOTOELASTIC TECHNIQUES FOR THE COMPLETE DETERMINATION OF STRESSES IN COMPOSITE STRUCTURES

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ABSTRACT

Transmission photoelastic analysis of composite models has attracted increasing attention in recent years. The interpretation of the photoelastic response in terms of the average (macroscopic) composite stresses is more involved than for isotropic models. Methods of determining the individual principal stresses (or strains) which have been suggested so far are not satisfactory. In this paper, three new methods are examined. The first method is an extension of the oblique incidence technique in which the model (or the light beam) is rotated about one of the material symmetry axes. In the second method, transmission and reflection photoelastic responses are combined. The third method requires the drilling of small holes and the determination of the fringe orders at selected points on the hole boundary. The three methods are applied to an orthotropic circular disk under diametral compression. Results are compared with strain gage data.

INTRODUCTION

A complete stress analysis and reliable failure criteria are essential for optimum utilisation of the unique properties of composite materials in structural applications. The case for micromechanics analysis of composites is very strong because the materials are heterogenous and exhibit several modes of failure. However, micromechanics analysis is complicated and the results from such analysis cannot directly be applied to design. An engineering or macromechanics analysis of composites is, therefore, needed and the results from such analysis have been found to agree well with experimental results. For the design and analysis of composite structures on a macroscopic scale, for instance in the failure theories such as the tensor polynomial theory, the individual average composite stresses are required.

When polarized light is passed through a transparent birefringent composite, the phenomenon on a microscopic scale is very complicated. But over-all fringe patterns are observed. Considerable progress has been achieved in the application of transmission photoelastic techniques to composite orthotropic models in recent years. The developments in the subject have been reviewed by the author.[1] The isochromatic fringe order is a complex function of the principal stresses (or strains), their orientations, etc. The isoclinic parameter gives the directions of the principal birefringence components according to a Mohr circle of birefringence.

For the transmission photoelastic analysis of an orthotropic birefringent model to yield useful information, methods must be developed to determine the individual values of the principal stresses or strains. Several methods have already been proposed, such as shear difference, numerical solution of the compatibility equation and holography. These methods have been reviewed by the author.[2] Some of these proposed techniques suffer from the disadvantage that they use the photoelastic response partially and rely on analytical procedures which either give rise to error or are involved. The holographic method of combining isochromatics and isopachics is not feasible for composites because of the complex nature of both families of fringes.[3] There is consequently a need for a simple and completely experimental method of determining the individual values of principal stresses or strains. Three such methods are proposed and examined in this paper.

DRILLING SMALL HOLES

In order to determine the magnitudes and directions of the principal stresses at a given point in the interior of an isotropic photoelastic model, Tesar[4] suggested making a very small circular hole at the point. Referring to Fig. 1, the stresses at the points A and C on the boundary of the hole of radius a are determined from the isochromatic fringe orders at these points. It can be shown that the principal stress magnitudes, corresponding to the center of the hole and in the absence of the hole, are given by

$$\sigma_1 = \frac{\sigma_A + 3\,\sigma_C}{8} \tag{1}$$

$$\sigma_2 = \frac{\sigma_C + 3\,\sigma_A}{8} \tag{2}$$

This procedure has the disadvantage of depending on the precise determination of the boundary stresses at the edge of a small hole. Durelli and Murray[5] have overcome this disadvantage by determining the principal stresses corresponding to the hole center from the principal stress differences measured at interior points. If these interior points on a circle of radius 2a beyond A and C are designated as E and F, respectively, then it can be shown that

$$\sigma_1 = \frac{7\sigma_E + 15\sigma_F}{11} \tag{3}$$

$$\sigma_2 = \frac{7\sigma_F + 15\sigma_E}{11} \tag{4}$$

where σ_E and σ_F are the principal stress differences. Compared to Tesar's method, the improved procedure represented by equations (3) and (4) has the disadvantage of larger errors due to stress gradients.

The author[6] has suggested the extension of Tesar's method to birefringent composites. The state of stress around a circular hole in a composite plate subjected to a biaxial loading is quite complex in the general case. Simplifications can be made if the composite plate is considered to be subjected to stresses which act along the material symmetry axes, as shown in Fig. 1.

When only the stress parallel to the reinforcement, σ_1, is acting, the tangential stress on the hole boundary is given by

$$\sigma_\theta = \sigma_1 \frac{E_\theta}{E_L}\left[-k\cos^2\theta + (n+1)\sin^2\theta\right] \tag{5}$$

where

$$k = \sqrt{\frac{E_L}{E_T}} \tag{6}$$

$$n = \sqrt{2\left(\frac{E_L}{E_T} - \nu_{LT}\right) + \frac{E_L}{G_{LT}}} \tag{7}$$

In the above equations E is the Young's modulus, G the shear modulus, ν the Poisson's ratio, L and T the material symmetry axes and θ the angle measured from the σ_1-direction. At the points A and B ($\theta = 0, \pi$)

$$\sigma_{A,B} = -\frac{\sigma_1}{k}. \tag{8}$$

and at the points C and D ($\theta = \frac{\pi}{2}, 3\frac{\pi}{2}$)

$$\sigma_{C,D} = \sigma_1 \; (1+n) \tag{9}$$

When the stress perpendicular to the reinforcement, σ_2, is acting alone, the tangential stress on the hole boundary is given by

$$\sigma_\theta = \sigma_2 \; \frac{E_\theta}{E_L} \left[(k+n) \; k \cos^2\theta - k \sin^2\theta \right] \tag{10}$$

At the points A and B

$$\sigma_{A,B} = \sigma_2 \; \frac{k + n}{k} \tag{11}$$

and at the points C and D

$$\sigma_{C,D} = -k \, \sigma_2 \tag{12}$$

Superposing the stresses σ_1 and σ_2 and solving for them,

$$\sigma_1 = \frac{k^2 \sigma_A + (k+n) \; \sigma_D}{n(n+k+1)} \tag{13}$$

$$\sigma_2 = \frac{k(1+n) \; \sigma_A + \sigma_D}{n(n+k+1)} \tag{14}$$

While the measurement of the isochromatic fringe order is difficult on the hole boundary and it would be preferable to make the measurement at interior points, the analytical expressions for stresses at interior points in an orthotropic plate with a circular hole are not available in a closed form. Experiments to substantiate this proposed method are described in a later section.

OBLIQUE INCIDENCE

In orthotropic birefringent models, the isochromatic fringe order under normal incidence of light is related to the principal stresses by the equation

$$N_n = h\left\{\left[\frac{1}{f_L} (\sigma_1 \cos^2\alpha + \sigma_2 \sin^2\alpha) - \frac{1}{f_T} (\sigma_1 \sin^2\alpha + \sigma_2 \cos^2\alpha)\right]^2 + \left[\frac{1}{f_{LT}} \; (\sigma_1 - \sigma_2) \sin 2\alpha \right]^2\right\}^{\frac{1}{2}} \tag{15}$$

where σ_1, σ_2 are the principal stresses, α the angle between σ_1 and L directions and f_L, f_T, f_{LT} are the principal stress-fringe values. As the author[7] has pointed out, rotation of the model about either of the principal stress directions is not possible because the principal stress angle, α, is not given by the optical isoclinic. It is possible to rotate the model about either of the principal strain directions, if it is assumed the the isoclinic parameter approximately gives the principal strain directions. The equations resulting from this approach are involved and the procedure is very complex. It is also necessary, in this approach, to directly or indirectly determine some out-of-plane elastic constants.

Instead of trying to determine the principal stresses or strains directly, the oblique incidence technique can be adapted to composite models by seeking the stress components σ_L, σ_T and τ_{LT}, referred to the material symmetry axes. One of the three equations required for this purpose is equation (15), which can be rewritten as

$$N_n = h\left\{\left(\frac{\sigma_L}{f_L} - \frac{\sigma_T}{f_T}\right)^2 + \left(\frac{2\,\tau_{LT}}{f_{LT}}\right)^2\right\}^{\frac{1}{2}} \tag{16}$$

According to the Mohr circle of birefringence, the optical isoclinic parameter, ϕ, is related to the stress components by

$$\tan 2\phi = \frac{2\,\tau_{LT}/f_{LT}}{\left(\sigma_L/f_L - \sigma_T/f_T\right)} \tag{17}$$

The third equation required can be obtained by rotating the model about the L-axis by θ, as shown in Fig. 2. The oblique incidence fringe order is given by

$$N_\theta = \frac{h}{\cos\theta}\left\{\left(\frac{\sigma_{L^1}}{f_{L^1}} - \frac{\sigma_{T^1}}{f_{T^1}}\right)^2 + \left(\frac{2\,\tau_{L^1T^1}}{f_{L^1T^1}}\right)^2\right\}^{\frac{1}{2}} \tag{18}$$

where $\sigma_{L'}$, $\sigma_{T'}$, $\tau_{L'T'}$ are the transformed stress components and $f_{L'}$, $f_{T'}$, $f_{L'T'}$ are the transformed stress-fringe values. While the transformed stress components are given by

$$\begin{aligned} \sigma_{L'} &= \sigma_L \\ \sigma_{T'} &= \sigma_T \cos^2\theta \\ \tau_{L'T'} &= \tau_{LT}\cos\theta \end{aligned} \tag{19}$$

the transformed stress-fringe values, due to transverse isotropy, are given by

$$\begin{aligned} f_{L'} &= f_L \\ f_{T'} &= f_T \\ f_{L'T'} &= f_{LT} \end{aligned} \tag{20}$$

It is therefore possible to rewrite equation (18) as

$$N_\theta = \frac{h}{\cos\theta}\left\{\left(\frac{\sigma_L}{f_L} - \frac{\sigma_T\cos^2\theta}{f_T}\right)^2 + \left(\frac{2\,\tau_{LT}\cos\theta}{f_{LT}}\right)^2\right\}^{\frac{1}{2}} \tag{21}$$

The three stress components can be obtained by solving equations (16), (17) and (21). Experiments verifying the proposed method are described in a later section.

COMBINED TRANSMISSION AND REFLECTION

The author[8] had proposed, without applications, combining the tranmission and reflection photoelastic methods in order to determine the principal stresses or strains. If the symmetry axes for the composite model coincide with the material symmetry axes and if the loads are applied along these directions, then the principal stress and strain directions are the same. Assuming faithful strain transmission from the composite to the coating, the reflected isochromatic fringe order can be expressed as

$$N_r = \frac{2h^c}{f_\varepsilon^{\,c}} \left[\frac{\sigma_L^{\,s}}{E_L} (1 + \nu_{LT}) - \frac{\sigma_T^{\,s}}{E_T} (1 + \nu_{TL}) \right] \tag{22}$$

where the superscripts c and s refer to the coating and the composite specimen, respectively, and $f_\varepsilon^{\,c}$ is the strain-sensitivity of the coating. The transmitted isochromatic fringe order, N_t, given by equation (16), simplifies to

$$N_t = \pm h^s \left(\frac{\sigma_L^{\,s}}{f_L} - \frac{\sigma_T^{\,s}}{f_T} \right) \tag{23}$$

where the positive or negative sign is chosen appropriately to keep the fringe order positive. The principal stresses $\sigma_L^{\,s}$ and $\sigma_T^{\,s}$ can be obtained from equations (22) and (23) as

$$\sigma_L^{\,s} = \frac{2 N_r h^c \left\{ \frac{f_\varepsilon^{\,c} (1+\nu_c)}{E^c} \frac{f_L}{f_T} \right\} - N_t h^s \left\{ \frac{f_L^{\,2} (1+\nu_{LT})}{f_T E_L} - f_L \right\}}{\frac{1+\nu_{LT}}{E_L} \frac{f_L}{f_T} - \frac{1+\nu_{TL}}{E_T}} \tag{24}$$

$$\sigma_T^{\,s} = \frac{2 N_r h^c \frac{f_\varepsilon^{\,c} (1+\nu_c)}{E^c} - N_t h^s \frac{f_L (1+\nu_{LT})}{E_L}}{\frac{1+\nu_{LT}}{E_L} \frac{f_L}{f_T} - \frac{1+\nu_{TL}}{E_T}} \tag{25}$$

Experiments verifying the proposed method are described in the next section.

TESTS

To verify the proposed experimental methods, three circular disks of 7.6 cm. diameter were tested in diametral compression. The disks were machined from a unidirectionally reinforced E-glass-polyester laminate. The elastic and photoelastic constants for the material were determined by standard calibration procedures and are given in Table 1.

On one of the disks, circular holes of 0.32 cm. diameter were drilled on radial lines parallel and perpendicular to the reinforcement, at locations 1.27 cm. and 2.54 cm. from the center. Electrical resistance strain gages were mounted at similar points diametrically across from the holes. The disk was loaded parallel and perpendicular to the reinforcement and strain gage readings as well as fringe patterns were recorded. Typical isochromatic fringe patterns are shown in Fig. 3. The values of principal stresses given by equations (13) and (14) were found to differ from the strain gage results by a maximum of 10 per cent.

On a second disk, a circular photoelastic coating of slightly smaller diameter was bonded. The disk was again loaded under diametral compression, parallel and perpendicular to the reinforcement. The isochromatic fringe patterns for the coating are shown in Fig. 4. The fringe patterns for the composite disk in transmitted light were also recorded as a reflected pattern from the back of the photoelastic coating. Fringe patterns obtained in this manner are shown in Fig. 5. As birefringent composites incorporating glass fibres as reinforcement are usually photoelastically insensitive, this procedure doubles the maximum fringe order. For comparison, isochromatic fringe patterns for a third composite disk in transmitted light are shown in Fig. 6. All the fringe patterns shown in Figs. 3,4,5 and 6 correspond to a diametral compressive load of 1780 N. The values of principal stresses given by equations (24) and (25) were found to differ from the strain gage results by a maximum of 5 per cent.

Oblique incidence measurements were conducted on the third composite disk for which the angle of oblique incidence with the chopped prism arrangement was found to be about 30°. The oblique incidence fringe order was combined with the normal incidence fringe order and the isoclinic para-

-meter by equations (16), (17) and (21). The results obtained in this manner differed from the strain gage results by a maximum of 7 per cent.

CONCLUSIONS

Three completely experimental procedures have been proposed and compared. In one of the methods, small circular holes are drilled at the points of interest and the isochromatic fringe order at selected points on the hole boundaries are measured. In the second method a photoelastic coating is bonded to the birefringent composite model and the transmitted and reflected fringe orders are combined. In the third method, the oblique incidence fringe order, obtained by rotating the model or the light beam about the material symmetry axis, is combined with the normal incidence fringe order. The three methods have been applied to a circular disk under diametral compression, with the load parallel or transverse to the direction of reinforcement. Comparison with strain gage results indicate that the method of drilling holes is the least accurate, due to the difficulty in determining the fringe order on the boundary of a hole and the stress gradient from the hole center to the hole boundary; this method also requires extension to the more general biaxial loading where the material symmetry axes are not the principal stress directions.

The method of combining transmitted and reflected isochromatic fringe orders has the added advantage of doubling the transmitted photoelastic response if it is obtained by reflection from the back of the coating. However, the method requires several corrections due to the coating.

The oblique incidence method is easily applicable to general biaxial loading but the method depends on the isoclinic parameter and also yields the stress components referred to the material symmetry axes. Use of the chopped prism restricts the angle of oblique incidence to just one value.

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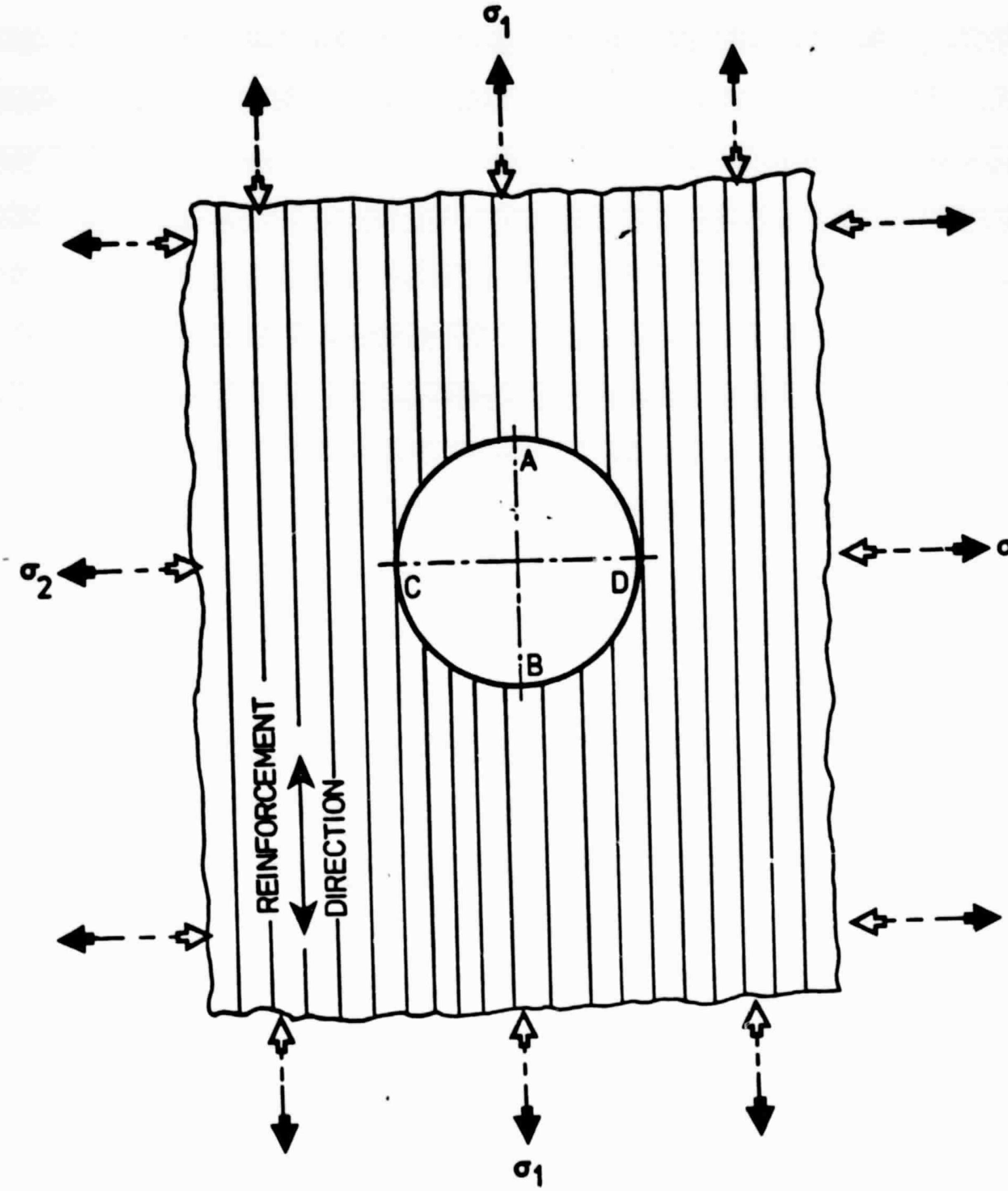
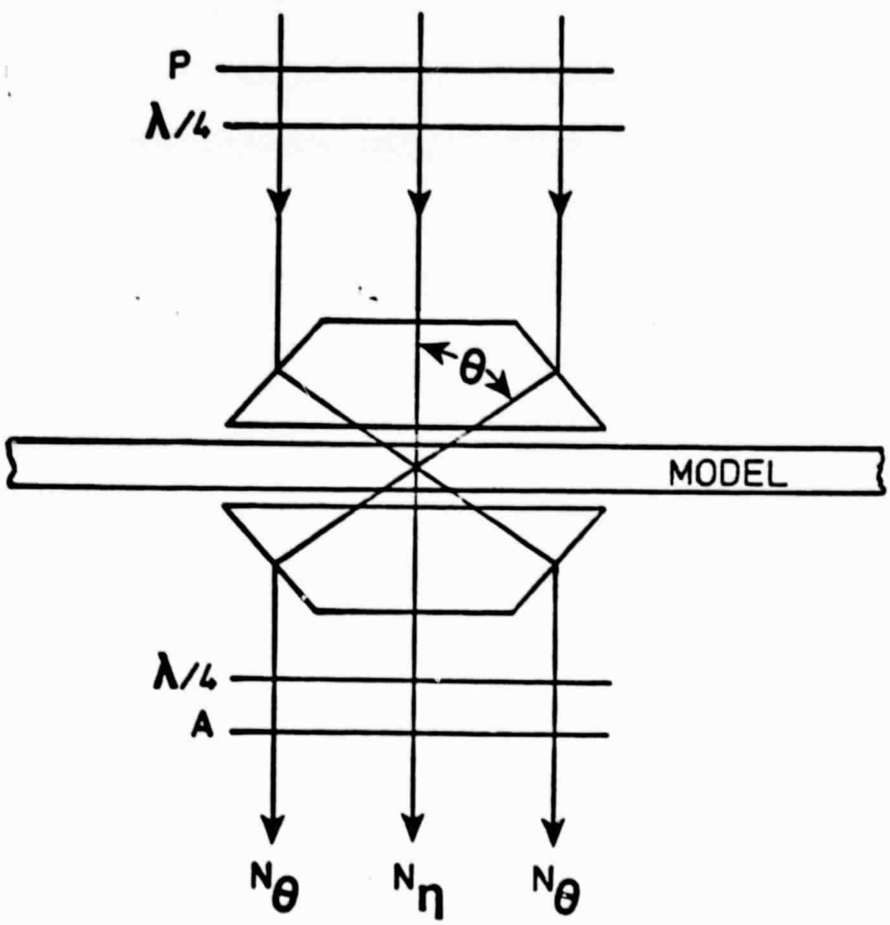
LIST OF FIGURES

Table 1 ELASTIC AND PHOTOELASTIC PROPERTIES OF BIREFRINGENT COMPOSITE MODEL

Property	Value
E_L	28.8 GPa
E_T	9.4 GPa
G_{LT}	3.2 GPa
ν_{LT}	0.3
f_L	156 KPa - m/fringe
f_T	78 KPa - m/fringe
f_{LT}	69 KPa - m/fringe



σ1
A
C
D
B
σ2
σ2
REINFORCEMENT
DIRECTION
σ1


P
λ/4
θ
MODEL
λ/4
A
Nθ
Nη
Nθ