BELLOWS FLOW-INDUCED VIBRATIONS

by

J.E. Johnson,
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W. J. Astleford,
C. R. Gerlach

FINAL REPORT
Contract No. NAS8-31994
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George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812
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Approved:

H. Norman Abramson, Vice President
Engineering Sciences
ABSTRACT

Results of theoretical and experimental investigations of bellows typical of those found in Space Shuttle external tanks are presented. New correlation parameters are identified which generalize the alternating stress calculations presented in an earlier SwRI study titled "Bellows Flow-Induced Vibrations and Pressure Loss." Alternating stress amplitudes and mean stress levels form the basis of a fatigue analysis incorporating seven-ordinate charts for 347 S.S., Alloy 21-6-9, and Inco 718. A crack propagation model is also presented. Computer programs for computing bellows fatigue life and Two Phase flow and material hardness topics are contained in the report.
ACKNOWLEDGEMENTS

Dr. C. Richard Gerlach, who is now the Chief Executive Officer of Gerlach Products, Inc. (San Antonio, Texas), served as a consultant to Southwest Research Institute. He contributed significantly to the early studies of bellows and likewise to the current study.

The authors of this report wish to express their sincere gratitude to Mrs. Adeline Raeke who cheerfully typed the text and to Mr. V. J. He andez for his skillful work on the figures.

We express a special word of thanks to Mr. Clinton Wood, Staff Technician, for his unique talents and ideas which were utilized in the design and fabrication of components required for the experimental apparatus. Mr. Wood also conducted many of the experimental tests and aided materially in the data reduction.
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I. INTRODUCTION

I.1 Overview

This report describes all work performed by Southwest Research Institute under Contract NAS8-31994, "Research Study of Flow Induced Vibrations." This study was performed for George C. Marshall Space Flight Center of the National Aeronautics and Space Administration, and it was administered by the Structures and Propulsion Laboratory, with Mr. R. H. Veitch serving as Technical Manager.

The general objective of this study was to evaluate bellows related theoretical assumptions either by analytical and/or experimental investigations. Emphasis was placed on obtaining a better understanding of the fluid-elastic excitation mechanism and upon developing a refined fatigue prediction methodology. The foundation of the current study is found in earlier research work performed by the Institute which is reported in a document titled "Bellows Flow-Induced Vibrations and Pressure Loss," by C. R. Gerlach, et al. (1)

Summary of Results

A number of significant findings have been made throughout this report; these are summarized below.

(a) Definition of $C_F^*$ Parameter - A stress correlation parameter has been defined which generalizes the existing bellows data contained in Reference 1. Previous data were characterized by a number of parameters such as the specific spring rate, fluid state, geometric factors and a vortex force coefficient. All of these factors are accounted for in the $C_F^*$ correlation and its usage.

(b) Damping Model - As an alternate method of predicting stress amplitudes, an empirical damping model was developed.

(c) Fatigue Prediction - A stress analysis has been coupled with the flow-induced vibration analysis in order to determine, with reasonable accuracy, the bellows fatigue life under varying environmental factors.

(d) Computer Program - A computer program has been developed to allow quick computation of the bellows mode frequencies, lock-in ranges, stress indicator, and stress level.
Acoustic Resonance - The acoustic resonances as identified by analysis have been verified by limited experimental investigation.

Special Problems - During the course of the contract, several urgent and special bellows related problems were addressed at NASA's request. The solution of these problems are included in this report.

I.2 Organization of Study

The bellows study has been broken into two separate methods of approach as indicated in the block diagram shown in Figure 1. The end objective of both methods is to predict the fatigue life of U-shaped bellows made of an arbitrary material, and in both cases, the alternating stress component is generated by flow induced vibrations. Method I incorporates the stress indicator concept, while Method II incorporates actual stress predictions which may be incorporated with 7-ordinate fatigue curves to predict bellows life. Method I suffers from the lack of a fatigue data base which must be generated by failing numerous bellows while influenced by flow induced vibrations. Method II suffers from underdevelopment of a realistic stress-deflection model where the convolute deflections can be predicted given an arbitrary geometry and flow conditions. Method I has been streamlined and somewhat generalized with the development of an envelope parameter designated as \( C_f^* \) which is then used to determine the stress indicator. Method II efforts were directed toward the development of a flow induced stress model.

I.3 General Discussion of Study

The main propulsion system of the Space Shuttle is configured with three engines, a complex array of liquid and gas flow lines, and two large external tanks (ET). An elementary schematic of the main propulsion system is shown in Figure 2. Bellows are contained throughout the flow network; however, the bellows of primary interest are contained in the feed lines (LO₂ and LH₂) and in the small recirculation lines.

Earlier studies have shown that unshrouded shuttle application bellows (see Figure 3 for bellows nomenclature) will vibrate violently when the contained fluid is moving at a specific critical velocity. The oscillation is shown to occur at a reduced velocity \((U/f_0)\) of approximately 4.5. Vortex shedding from the individual convolutes was found to be the flow induced vibration mechanism.
METHODS OF APPROACH

METHOD I
STRESS INDICATOR DEVELOPMENT

EXISTING DATA BASE
C_F, STRESS INDICATOR

DEVELOP C_F*

STRESS INDICATOR

FATIGUE TEST

FATIGUE LIFE PREDICTION

METHOD II
STRESS DEFLECTION DEVELOPMENT

DEVELOP BELLOWS FLOW
INDUCED VIBRATION
STRESS MODEL

7-ORDINATE
FATIGUE CURVES

FATIGUE LIFE
PREDICTION

FIGURE 1. METHODS OF APPROACH INCORPORATED IN BELLows STUDY
\( N_c \) - NUMBER OF CONVOLUTIONS COUNTED FROM THE OUTSIDE

\( N_p \) - NUMBER OF PLYS

\( D_m \) - MEAN BELLOWS DIAMETER

\( t \) - WALL THICKNESS (THICKNESS PER PLY IF MULTI-PLY)

\( \lambda \) - CONVOLUTE PITCH

\( \sigma \) - CONVOLUTE WIDTH

\( a \) - MEAN FORMING RADIUS

\( h \) - MEAN DISC HEIGHT

FIGURE 3. BELLows NOMENCLATURE
Experimental data, obtained from the earlier studies, were parametrically correlated in terms of (1) the Strouhal number (convolute width is the characteristic dimension), (2) the bellows modal frequencies which included added fluid mass terms, and (3) a stress indicator which is proportional to the maximum dynamic stress.

It has been shown that the stress indicator is a function of a vortex force coefficient, $C_F$, and a forced response dynamic amplification factor, $Q$. These experimentally derived factors are shown in Figures 4 and 5 and Table I.

Finally, the observed fatigue life was related to the stress indicator as shown in Figure 6. The fatigue data were obtained for 321 S.S. only; although the general presentation could be expanded to include other materials if appropriate material factors could be included. Bellows for Space Shuttle applications are constructed of Inco 718 and steel alloy 21-6-9 materials.
FIGURE 4. SUMMARY OF BELLows VORTEX FORCE COEFFICIENT EXPERIMENTAL DATA

Use this curve for higher longitudinal free bellows modes, flex hose longitudinal modes, and convolute bending mode.

Use this curve for first few free bellows longitudinal modes.
FIGURE 5. DYNAMIC AMPLIFICATION FACTORS FOR VARIOUS BELLOWS APPLICATIONS
### TABLE I.
APPLICATIONS INFORMATION FOR USE WITH Q VALUE DATA IN FIGURE 5

<table>
<thead>
<tr>
<th>Specific Spring Rate (see Note 1)</th>
<th>Number Plies</th>
<th>Internal Media (see Note 2)</th>
<th>Curve No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>all ranges</td>
<td>1</td>
<td>low pressure gases</td>
<td>1</td>
</tr>
<tr>
<td>over 2000 lb/in²</td>
<td>1</td>
<td>high pressure gases, light liquids</td>
<td>1</td>
</tr>
<tr>
<td>over 2000</td>
<td>1</td>
<td>water, dense liquids</td>
<td>2</td>
</tr>
<tr>
<td>under 2000</td>
<td>1</td>
<td>high pressure gases, light liquids</td>
<td>2</td>
</tr>
<tr>
<td>under 2000</td>
<td>1</td>
<td>water, dense liquids</td>
<td>3</td>
</tr>
<tr>
<td>over 3000</td>
<td>2</td>
<td>all pressure gases</td>
<td>3</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>2</td>
<td>all pressure gases</td>
<td>4</td>
</tr>
<tr>
<td>under 2000</td>
<td>2</td>
<td>all pressure gases</td>
<td>5</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>2</td>
<td>all liquids</td>
<td>5</td>
</tr>
<tr>
<td>under 2000</td>
<td>2</td>
<td>all liquids</td>
<td>6</td>
</tr>
<tr>
<td>over 3000</td>
<td>3</td>
<td>all pressure gases</td>
<td>5</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>3</td>
<td>all</td>
<td>6</td>
</tr>
<tr>
<td>under 2000</td>
<td>3</td>
<td>all pressure gases</td>
<td>6</td>
</tr>
<tr>
<td>under 2000</td>
<td>3</td>
<td>all liquids</td>
<td>6</td>
</tr>
</tbody>
</table>

**Use of Table** - To use table, first calculate bellows specific spring rate, then look up application curve number corresponding to this specific spring rate, number of plies, and internal media.

**Note 1:** The specific spring rate is here defined as

\[
S.S.R. = \frac{K_A N_C}{D_m N_p}
\]

or is the spring rate per convolute, per ply, per unit of diameter.

**Note 2:** Low pressure gases will be defined here as being those gases below 150 psia. Light liquids will be defined as having a density, relative to water, of less than 0.2.
FIGURE 6. PRELIMINARY BELLOWS FATIGUE LIFE DATA

Notes:
- Obtain $C_F$ from Figure 3
- Obtain $Q$ from Figure 4
- Use $C_E = 2.0$ for severe upstream elbow
- Use $C_E = 1.0$ for no upstream elbow
- This data is unconservative where acoustic resonance occurs
I.4 **Review of Relevant Literature**

The following list of reviewed sources of bellows information is included to help direct the interested reader build a background knowledge which is needed for detailed evaluation of bellows related topics.

1. **Kleppe, S. R., "High Pressure Expansion Joint Studies"**


   This paper presents a theoretical solution together with an experimental study of axial compression of certain bellows mainly of the corrugated-pipe type, used in the pressureless state. The total strain energy is written in terms of the circumferential stress and the axial loading moment. A Rayleigh-Ritz method is used to solve for a minimum strain energy condition. Ultimately the surface stresses are analytically determined. The paper contains a short literature review covering the period from 1916 to 1953.


   In this paper a formula has been derived to show the total stress induced in the material as a result of the combined effects of pressure and movement. The validity of the approximations used in the formula have been verified by laboratory strain measurements. The paper deals primarily with flat disc type bellows.
This paper presents the results of testing 19 bellows of various types to their endurance limit. The types include (1) welded roots, (2) hydraulically formed, and (3) welded disk. The bellows material consisted of stainless steel types 304, 321, and 347. A typical stress-distribution diagram for a 12-inch diameter hydraulically formed bellows is presented (case of axial extension and compression, and internal pressure). Strain measurements were taken with SR-4 strain gages. The maximum stress range for both radial and circumferential stresses occurs near the root of the corrugation.

Bellows may become unstable when loaded by internal pressure. The critical value of this pressure is governed by the rigidity of the bellows with respect to bending. Critical pressures have been analytically determined for rectangularly shaped corrugations and these critical pressures may be considered to agree with those obtained experimentally for U-shaped bellows when considering the approximations introduced and the variation of wall thickness.

An elastic analysis of U-shaped expansion joints under axial loads and internal or external pressure is presented. The analysis employs the energy method for the toroidal sections, and the theory of symmetrical bending of circular plates augmented by thick walled cylinder analysis for the annular plate connecting the two toroidal sections.

Flexline response frequencies are modeled as a lumped parameter system where the characteristic frequency is determined by
knowledge of the convolute effective mass and the effective fluid compressibility. Bellows longitudinal natural frequencies are modeled as a spring-mass analog where a dimensionless frequency parameter is utilized for evaluating all the longitudinal modal frequencies. An attempt has been made to define the maximum alternating stress.


An analytical and experimental study of the acoustic behavior of seven and nine corrugation expansion joints (bellows) used in a nuclear reactor is presented. Resonant frequencies obtained from a computer program using a matrix method are given. Experimental test results on seven corrugation expansion joints are in good agreement with the computations. It is concluded that the calculation of acoustic frequencies of expansion joints with internal sleeves can be utilized to avoid the coincidence of these frequencies with those of a mechanical or flow-induced noise nature and thus reduce the loads on expansion joint corrugations.


Life cycle testing was performed on 10" diameter bellows with nominal 3/8-inch high convolutions (.008-inch thick, Inconel 718). Testing was similar to that conducted for Boeing Company by Strazar. Metallurgical and fatigue properties were evaluated. This report does present a source of fatigue data as a function of bending stress (bellows), and percent of tensile ultimate strength (specimens only).


The report presents the results of a brief test program aimed at generating data on bending life of notched CRES sheet specimens. Emphasis of the study is directed toward the quantitative evaluation of bellows' defects, particularly those resulting from accidental damage. An empirically derived procedure for evaluating bellows' surface irregularities and determining service life is presented.
II. GENERALIZED CORRELATION PARAMETER: $C_F^*$

II.1 Introduction

Through the efforts of Gerlach, et al. and Sack is has been well established that a series of lumped spring-mass elements can represent a free bellows and the modal frequencies can be computed with a high degree of accuracy. The work of Gerlach went on to show that the flow excitation mechanism is a vortex shedding phenomena that occurs in the entrance region of a convoluted bellows. When the vortex shedding frequency is near a bellows longitudinal structural frequency, the vortex shedding frequency will "lock-on" and the structure will vibrate at an amplitude dependent upon the amount of fluid and structural damping present.

Ultimately, the most fundamental question is how to determine the amplitude of convolute displacement and hence the resultant maximum alternating stress amplitude. Two stress prediction models will be addressed in this section.

II.2 $C_F^*$ Correlation Parameter

Reference 1 contains the derivation and application of a stress indicator concept. It must be emphasized that the original form of the stress indicator was merely a benchmark showing relative stress intensities as a function of fluid and geometric parameters. Its purpose was to guide a designer when obtaining fatigue predictions. The stress indicator concept is a valid method for predicting fatigue life so long as a substantial data base is developed; unfortunately, a large data base does not exist.

Before describing the $C_F^*$ ("C sub F Star") model, the original stress indicator model is reviewed. It has been shown that the maximum convolute stress due to flow induced vibration is

$$\sigma_{alt} = K \frac{C_F Q}{N_p} \left(\frac{h}{t}\right)^2 \frac{1}{2} \rho_f v_{crit}^2$$  \hspace{1cm} (II.1)

The $K$ term contains factors of proportionality relating to geometric constraints and this factor was extracted from Equation (II.1) to produce a single simple expression for stress which contains only readily known bellows dimensional data, parameters, and flow variables. Therefore, the indicator is given as
S.I. = \frac{C_F Q}{N_p} \left(\frac{h}{t}\right)^2 \frac{1}{2} \rho \xi \frac{V_{crit}}{\xi}^2 \tag{II.2}

Table II compares the calculated stress indicator and measured stress on the crown of the second convolute (see Appendix B for a description of the experimental techniques). Several items are worth noting in this table. The K factor ranges from 0.585 to 3.61 for the limited test conducted and there is a downward trend in the K factor as the mode number increases. This shows that the stress indicator may or may not be a conservative estimator of stress levels, and the K factor is not constant as assumed in Reference 1.

**TABLE II. MEASURED CONVOLUTE RADIAL STRESS AND CALCULATED STRESS INDICATOR COMPARISON**

<table>
<thead>
<tr>
<th>Bellows* Ident.</th>
<th>Mode No.</th>
<th>Measured Radial Stress KSI (peak)</th>
<th>Stress Indicator KSI</th>
<th>Measured Calculated</th>
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<tr>
<td>4</td>
<td>1</td>
<td>2.03</td>
<td>2.21</td>
<td>1.325</td>
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<tr>
<td>4</td>
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<td>8.02</td>
<td>8.24</td>
<td>.97</td>
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<tr>
<td>4</td>
<td>3</td>
<td>8.94</td>
<td>11.48</td>
<td>.775</td>
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<td>6</td>
<td>1</td>
<td>.765</td>
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<td>.82</td>
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<td>.78</td>
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<td>E</td>
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<td>4.57</td>
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<tr>
<td>E</td>
<td>2</td>
<td>8.41</td>
<td>7.91</td>
<td>1.05</td>
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</tbody>
</table>

*Dimensional Data is contained in Appendix C.

The stress indicator contains two terms, \(C_F\) and \(Q\), that are dependent upon factors of damping, internal pressure, convolute geometry, and the flow media. Values for \(C_F\) are obtained from Figure 4 while values for \(Q\) are obtained from Figure 5 and Table I. The data contained in these sources have been correlated in the form of one universal stress function curve as discussed below.
All data contained in Reference 1 has been evaluated in terms of a correlation parameter defined as

\[ C_F^* = C_fQ(N/N_C) \]

Figures 7 through 9 show plots of the force coefficient parameter for representative samplings of the total data base. The effect of changes in \( \lambda \) on the force coefficient parameter \( C_F^* \) is illustrated in Figure 7. Here, a single bellows was tested at various pitch values, \( \lambda \), and the peak response of the first longitudinal mode (\( N=1 \)) was noted. It is noted that spring rate is affected somewhat by changes in \( \lambda \).

The reduced data shown in Figure 8 clearly illustrates the effect of vortex reinforcement and vortex retardation on the flow induced response of the bellows.

A vortex reinforcement occurs when the vortex shedding from an upstream convolute arrives at the adjacent downstream convolute at the right moment to aid in the formation of the vortex forming at that adjacent convolute. Vortex retardation has the opposite effect. The vortex shed from an upstream convolute arrives at the adjacent downstream convolute at the right moment to detract from the formation at that location. As we will soon discuss, it is our present concept that vortex reinforcement is most prevalent and effective in the higher longitudinal modes. (Figure 6 from the final report "Bellows Flow-Induced Vibrations and Pressure Loss" clearly shows a visualization of vortex reinforcement for a higher longitudinal mode.) In the first two or three modes of a bellows, vortex reinforcement and vortex cancellation both come into play, as illustrated by Figure 8. However, for the intermediate modes, the vortex retardation phenomena is prevalent.

Figure 8 presents a plot of \( C_F^* \) versus the mode number \( N \) for four test bellows that have constant values of the parameter \( (h/t) \) but have \( h \) values ranging from 0.2 to 0.5. Since spring rate is proportional to \( (h/t) \), this family had similar modal frequencies, so that the effect of convolute height \( h \) should be revealed. Also, however, each of the four bellows was tested for three or four values of \( \lambda \) achieved by stretching. Note that there is a spread of the combined \( C_F^* \) values for these four bellows for each mode number of \( N \) value. This spread is caused by a
FIGURE 7. VORTEX FORCE COEFFICIENT $c_F^*$ VS. PITCH FOR THE FIRST MODE OF BELLOWS 105
FIGURE 8. VORTEX FORCE COEFFICIENT $C_F^*$ AS A FUNCTION OF MODE NUMBER FOR BELLOWS WITH CONSTANT (h/t)
combination of two factors. First, it represents the influence of the effect of changing $\lambda$ as illustrated previously in Figure 7, and, secondly, it reflects the normal variation expected in flow-induced vibration experiments of bellows where slight changes in alignment, clamping of the ducting, etc., cause changes in the peak response point.

From Figure 8, we have concluded the following:

(a) Other than for the No. 1 specimen, which had $h = 0.2$ or a very short convolute, the effect of $h$ was not apparent between the bellows. Specimen No. 1 had lower $C_F^*$ values than the other bellows, probably because short convolutes do not couple so well as taller convolutes. After all, the limiting case is $h = 0$ which represents a straight pipe which has no response of the type under consideration.

(b) The vertical spread of $C_F^*$ for each mode is primarily caused by vortex reinforcement or vortex cancellation.

(c) The pronounced minimum of $C_F^*$ is a result of an optimum vortex cancellation effect for this mode number range.

(d) The rapid rise of $C_F^*$ for the higher longitudinal modes is a result of a predominance of vortex reinforcement for these modes.

(e) Many of the higher modes simply never appear because other modes close to them predominate and prevent their occurrence.

Figure 9 presents $C_F^*$ as a function of mode number $N$ for three bellows having similar convolute geometry but different numbers of convolutes. The bellows No. 19 illustrates yet another phenomena. Note that the $C_F^*$ values for this bellows are quite low for the first two longitudinal modes. Also note the strong presence of the first cocking mode plotted for $N = 1.5$. For this bellows the cocking mode was stronger than normal so it suppressed the first and second longitudinal modes causing their $C_F^*$ values to be abnormally low.
FIGURE 9. VORTEX FORCE COEFFICIENT $c_F^*$ AS A FUNCTION OF MODE NUMBER FOR BELLows WITH DIFFERENT $N_c$. 

$\bullet$ No.7 - $N_c = 7$
$\circ$ No.5 - $N_c = 13$, $h = .3$, $t = .008$
$\square$ No.6 - $N_c = 19$
The primary intent of the $C_F^*$ relation is to mathematically collapse all of the experimentally generated Q surfaces into one relationship that applies to all ranges of the bellows operational parameters; hence, the stress indicator is computed

$$
S.I. = \frac{C_F^* N_C}{N N_p} (h/t)^2 \left(1/2 \rho \frac{V_{crit}}{V^2}\right) \quad (II.3)
$$

The parameter $C_F^*$ is obtained from Figure 10 which is a somewhat conservative curve that envelops all previously generated experimental bellows data. This curve contains all inherent information relating to $C_F$ and Q.

**Summary of Design Analysis Procedure**

The procedure for analyzing a given bellows design to assure freedom from flow-induced vibration failure consists of several distinct steps which are listed below.

- **Step 1.** Calculate the natural frequencies for all modes of the bellows.
- **Step 2.** Determine the lock-in or critical velocity range for each possible mode of vibration.
- **Step 3.** Calculate the Stress-Indicator for each mode at the critical velocity.
- **Step 4.** Determine the potential for failure of the bellows using the Stress-Indicator versus Cycles-to-Failure curve.

Pages 23 and 24 present a detailed step-by-step procedure that may be used for hand calculations. A more sophisticated calculation procedure is contained in a computer program (see Appendix A).
TABLE III.
SUMMARY OF FREQUENCY AND STRESS LEVEL CALCULATIONS

<table>
<thead>
<tr>
<th>STEP</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Consider the bellows structure representable by a lumped mass-spring mechanical model.</td>
</tr>
<tr>
<td>B</td>
<td>Calculate the elemental spring rate value $K$ from the expression $K = 2N_C K_A$, where $K_A$ is the overall spring rate determined from a force-deflection test or from the following expression: $K_A = D_m E \frac{N_p}{N_C} (t/h)^3$.</td>
</tr>
<tr>
<td>C</td>
<td>Calculate the elemental metal mass $M_m$: $M_m = \pi \rho_m t N_p D_m [\pi a + (h-2a)]$.</td>
</tr>
<tr>
<td>D</td>
<td>Calculate the fluid added mass $M_f$, for the first few longitudinal modes and for the higher longitudinal modes as $M_f = \pi/2 \rho_f D_m h (2a-tN_p)$ (First few $N$ values) and $M_f = \frac{\pi D_m \rho_f h^3}{3\delta}$ (Higher $N$ values).</td>
</tr>
<tr>
<td>E</td>
<td>Calculate the reference frequency $f_0$ from the expression $f_0 = 1/2\pi \sqrt{k/m}$, where $m = m_m + m_f$.</td>
</tr>
</tbody>
</table>
TABLE III (CONTD)

SUMMARY OF FREQUENCY AND STRESS LEVEL CALCULATIONS

STEP F Calculate the dimensionless frequencies and then multiply the
dimensionless frequencies by the reference frequencies to
obtain the true mode frequencies

\[ B_i = \sqrt{2 \left[ 1 + \cos \left( \frac{(2N_c-1)}{2N_c} \right) \right]} \]

Dimensionless frequency for the i-th mode

\[ i = 1, 2, 3, \ldots 2N_c-1 \]

\[ f_i = B_i f_o \]

True frequency for the i-th mode

Alternately, the dimensionless frequency factors may be
obtained from Table I, Appendix A.

STEP G Calculate the first convolute bending mode from the
expression

\[ f_b = \frac{1}{2\pi} \sqrt{8k/m} \]

where \( m = m_m + m_f \)

and \( m_f = \pi D m \rho_f h^3/3\delta \)

STEP H Calculate stress indicator from the following expression:

\[ S.I. = C_F^* \left( \frac{N_c}{N_p} \right) (h/t)^2 \left( 1/2 \rho V_{crit}^2 \right) \]

The parameter \( C_F^* \) is obtained from the curve presented in
Figure 10.

STEP I Calculate bellows expected life from the data presented in
Figure 6, which is a plot of stress indicator versus cycles
to failure. If the fatigue life is greater than \( 10^5 \) cycles,
then the data are conservative for materials classified as
Inco 718 and alloy 21-6-9.

If the calculated number of cycles is less than \( 10^5 \), then
the expected life of alloy 21-6-9 will be less than that
indicated for SS-321 or its equivalent SS-347.
III. STRESS LEVELS

III.1 Introduction

While section II presented a method for calculating vibration frequencies and stress-like quantities that may be used with the appropriate analysis to predict fatigue life, this section will explore various properties of actual stress levels experienced during the flow induced vibration process. As of this writing, an exact method has not been developed to calculate actual stresses; however, several important aspects of the problem are presented along with a reasonable stress calculation procedure.

III.2 Stress Envelope

Test data, shown in Table IV, has been reduced in terms of non-dimensional stress and velocity ratios for each longitudinal mode of vibration. The velocity ratio is formed by dividing the critical velocity of a particular mode by the first mode velocity and the stress ratio is formed in a similar fashion. The correlation in Figure 11 shows that similar families of curves are developed. The data may be further collapsed by referencing the curves to a particular damping ratio. For the present case, an average damping ratio of 0.00635 served as the reference damping value. Figure 12 shows the results of the damping normalization. From the limited data presented, the second and third mode stress may be calculated by the following empirical equation,

\[ \sigma_{alt, N} = \sigma_{alt, 1} \left( \frac{0.00635}{\zeta} \right) F_N \]  \hspace{1cm} (III.1)

\[ F_2 = 2.75 \]
\[ F_3 = 3.05 \]

Equation III.1 was developed from data obtained from a series of 3″, 321 S.S. bellows with a constant convolute height. The material thickness, number of plies and number of convolutes were allowed to vary and the measured spring rates were significantly different. The alternating stress component referred to is the convolute radial stress. Radial stresses were calculated from biaxial strain data (radial and circumferential) as described below.
TABLE IV. THREE-INCH BELLOWS STRESS RESULTS

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Mode No.</th>
<th>V_p, fps</th>
<th>( \sigma_{alt, ksi} )</th>
<th>Velocity Ratio ( \frac{V_p}{V_1} )</th>
<th>Stress Ratio ( \frac{\sigma_{alt @ N}}{\sigma_{alt @ N=1}} )</th>
<th>Average Damping Ratio, ( \zeta )</th>
<th>( \zeta / \zeta_{ref} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>5.40</td>
<td>2.93</td>
<td>1.0</td>
<td>1.0</td>
<td>.007</td>
<td>1.102</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.80</td>
<td>8.02</td>
<td>2.0</td>
<td>2.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15.89</td>
<td>8.94</td>
<td>2.99</td>
<td>3.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4.18</td>
<td>0.765</td>
<td>1.0</td>
<td>1.0</td>
<td>.0027</td>
<td>.425</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.35</td>
<td>3.67</td>
<td>1.99</td>
<td>4.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12.53</td>
<td>4.59</td>
<td>2.99</td>
<td>6.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>7.26</td>
<td>2.82</td>
<td>1.0</td>
<td>1.0</td>
<td>.0064</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>14.52</td>
<td>7.84</td>
<td>2.0</td>
<td>2.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>21.79</td>
<td>8.51</td>
<td>3.0</td>
<td>3.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 11. Velocity Ratio vs. Stress Ratio

- Bellows No. 15
- Bellows No. 14
- Bellows No. 16

Stress Ratio, $\frac{\sigma_{\text{alt}}(\theta)}{\sigma_{\text{alt}}(\theta_{N=1})}$

Velocity Ratio, $\frac{V_N}{V_1}$

$\xi = 0.0027$

$\xi = 0.007$

$\xi = 0.0064$
FIGURE 12. NORMALIZED STRESS RATIO
Each bellows was strain gaged (see Figure 3, Appendix B) on the second and middle convolute in the radial and circumferential directions which are the assumed principal directions. Principal stresses are calculated from the measured principle strains,

\[
\sigma_R = \frac{E}{1 - \mu} (\epsilon_R + \mu \epsilon_C) \quad \text{(III.2)}
\]

\[
\sigma_C = \frac{E}{1 - \mu} (\epsilon_C + \mu \epsilon_R) \quad \text{(III.3)}
\]

where

- \( \sigma_R \) = radial stress, psi
- \( \sigma_C \) = circumferential stress, psi
- \( E \) = modulus of elasticity, psi
- \( \mu \) = Poisson ratio
- \( \epsilon_R \) = radial strain, microinches
- \( \epsilon_C \) = circumferential strain, microinches

### III.3 Two-Ply Bellows

Multi-ply bellows flow-induced strain characteristics are significantly different than those of single-ply bellows. Figure 13 shows the flow-induced strain for a 3" single-ply bellows and a 3" two-ply bellows. In each case, the first mode has been flow excited. Note that the alternating strain level for the single-ply bellows is independent of internal pressure, while the strain magnitude and lock-in range for the two-ply bellows is strongly dependent upon internal pressure. For the particular bellows exhibited, it was found that the alternating strain component varies inversely and as a linear function of pressure (see Figure 14).

The most plausible explanation of this phenomena is that Coulomb friction damping is experienced between the plies of the bellows. The Coulomb friction force is directly proportional to the normal force acting in a manner to compress the plies together. To bear out this fact, a two-ply bellows was impulsed into vibration and then allowed to decay. The decay traces are shown in Figure 15 where it is obvious that the damping is a function of the internal pressure which is the mechanism generating the normal force on the convolute sidewalls.
FIGURE 13. FLOW INDUCED STRAIN FOR SINGLE AND DOUBLE PLY BELLOWS AS A FUNCTION OF PRESSURE

(a) 3" Bellows No. 6 - 1 Ply

(b) 3" Bellows No. 15 - 2 Ply
FIGURE 14. CONVOLUTE STRAIN (ALTERNATING COMPONENT) AS A FUNCTION OF INTERNAL PRESSURE
Figure 15. Damping ratio as a function of internal pressure for a two ply bellows.
The results of these pressure tests suggest that multi-ply bellows vibrate with a lesser magnitude when they are internally pressurized; thus, when single-ply stress calculations are performed on a multi-ply bellows exhibiting the same damping ratio at zero gage internal pressure, the calculated alternating stress component will be conservative (higher stress) for the internal pressurization case. These tests suggest that it may be practical to include damping material between plys as an alternative to including flow liners.

III.4 Convolute Mean Stress

Typically, alternating stresses which are generated by flow induced vibrations are superimposed upon a mean stress which results from internal static pressure and/or bellows axial extension or compression preload forces. By observing a typical seven-ordinate fatigue chart (for example, see Figure 23), it is noted that fatigue life is decreased with increasing mean stress magnitude. For example, a bellows that is operated at high static pressures would fail sooner than one operated at lower pressure even if the alternating stress component were equal for both cases. The derivation and use of the seven ordinate curves will be discussed in Section IV; however, the important issue is that the seven ordinate charts allow for mean stress contribution which is not present in cycle-to-failure (S-N) curves.

III.4.1 Internal Pressure Stress

Figure 16 presents the strain data obtained on Bellows No. SwRI-E during an internal pressurization test (ends of the bellows were clamped). The maximum principal stress was calculated for Convolute No. 2 and No. 7 and these stress values compared to the following equation taken from Reference 4.

\[ \sigma_p = \frac{P}{2} \left(\frac{h}{t}\right)^2 \]  

Table V summarizes the results which are evaluated at a pressure of 30 psig. Note, \( \sigma_p \) is a compressive stress on the convolute crown. The table compares the compressive stress, \( \sigma_p \), to the measured radial stress, \( \sigma_{max} \).
FIGURE 16. STRAIN DATA FOR INTERNAL PRESSURE LOADS – 6" BELLOWS NO. E.

(a) Radial Strain - Convolute No. 2

Pressure, psi

Bridge Volts

\[ \mu = 30.96P + 124.8 \]

(b) Radial Strain - Convolute No. 7

Pressure, psi

Bridge Volts

\[ \mu = 1.67P + 15.05 \]

(c) Circumferential Strain - Convolute No. 2

Pressure, psi

Bridge Volts

\[ \mu = 22.6P + 120.7 \]

(d) Circumferential Strain - Convolute No. 7

Pressure, psi

Bridge Volts

\[ \mu = 1.87P + 5.5 \]
TABLE V. INTERNAL PRESSURE STRESS AT 30 FSI

<table>
<thead>
<tr>
<th>Convolute No.</th>
<th>( \sigma_{\text{max}} ) (KSI)</th>
<th>( \sigma_p ) (KSI)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-24.9</td>
<td>-21.1</td>
<td>-18</td>
</tr>
<tr>
<td>7</td>
<td>-33.0</td>
<td>-21.1</td>
<td>-56</td>
</tr>
</tbody>
</table>

It is noted that Equation (111.4) under-predicts the radial stress (maximum principal) by as much as 56%. It is also noted that the radial stress in the center region of the bellows is higher. Most likely this higher center stress is caused by a "ballooning" effect in the mid-span of the bellows. The conservative approach when considering multi-ply bellows is to assume that the plies are not in complete contact; thus, the effective thickness is less than \( N_p \cdot t \). Due to the limited data obtained with respect to ply-coupling effects, it is recommended that the calculated single ply stress be used when multiple plies are incorporated in a design.

III.4.2 Compression Preload Stress

The same 6" bellows that was used for pressure tests was subjected to compression loading test. This is accomplished by placing calibrated weights on the open end edge of a free bellows which is placed in an upright position on a hard surface. This procedure is used to obtain the bellows spring constant \( K_A \); however, in this test the strain gage readings are also recorded. Figure 17 shows the strain data obtained versus compression loads. By noting that

\[
\frac{d \mu e}{d \ell} = (K_A) \left( \frac{d \mu e}{d F_c} \right)
\]

where \( \frac{d \mu e}{d \ell} \) = change in microstrain per unit change in live length \((\mu e/\text{in})\)

\( K_A \) = bellows spring constant \((\text{lb/in})\) (slope of deflection-load curve)

\( \frac{d \mu e}{d F_c} \) = change in microstrain per unit change of load \((\mu e/\text{lb})\) (slope of strain-load curve),

where
FIGURE 17. STRAIN DATA FOR AXIAL COMPRESSION LOAD - 6" BELLOWS NO. E

(b) Radial Strain - Convolute No. 7

\[ m = 36.66 \, \mu \varepsilon/\text{lb} \]

Load, lb:
Bridge Volts

(a) Radial Strain - Convolute No. 2

\[ m = 36.95 \, \mu \varepsilon/\text{lb} \]

Load, lb:
Bridge Volts

(c) Circumferential Strain - Convolute No. 2

\[ m = 4.44 \, \mu \varepsilon/\text{lb} \]

Load, lb:
Bridge Volts

(d) Circumferential Strain - Convolute No. 7

\[ m = 3.56 \, \mu \varepsilon/\text{lb} \]

Load, lb:
Bridge Volts
it is possible to determine the convolute strain-load characteristic. The deflection-load curve is presented below (Figure 18) from which the bellows spring rate, $K_A$, can be determined.

![Graph of Load-Deflection Curve - Bellows No. E]

**FIGURE 18. LOAD-DEFLECTION CURVE - BELLows NO. E**

Axial compression stresses as obtained experimentally have been compared to the following equation:

$$
\sigma_c = \frac{E \Delta}{h^2 N_c}
$$

(III.6)

where

- $\sigma_c$ = stress due to compression or extension load ($\sigma_c > 0$ for compression load, $\sigma_c < 0$ for extension load), psi
- $\Delta$ = deflection of live length, inch.

Table VI has been prepared to compare experimental results with Equation (III.6) for a preload of 20 lbs.
TABLE VI. PRELOAD STRESS AT 20 POUNDS

<table>
<thead>
<tr>
<th>Convolute No.</th>
<th>$\sigma_{\text{max}}$ (KSI)</th>
<th>$\sigma_c$ (KSI)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22.96</td>
<td>22.09</td>
<td>-3.9</td>
</tr>
<tr>
<td>7</td>
<td>24.21</td>
<td>22.09</td>
<td>-9.6</td>
</tr>
</tbody>
</table>

It is observed that Equation (III.6) gives reasonable accuracy and it provides a means for relating relative convolute motion to convolute radial stress level. Equation (III.6) can be used in a dynamic situation; however, it must be emphasized that the deflection value used is relative to adjacent convolutes.

Equation (III.6) is easily modified to incorporate preload rather than deflection if the bellows spring constant is known.

$$\Delta = \frac{F_c}{K_A}$$

thus,

$$\sigma_c = \frac{Et F_c}{h^2 N_c K_A}$$  \hspace{1cm} (III.7)

III.4.3 Compression Preload With Internal Pressure

A schematic illustrating the nature of the radial fiber strains in the region of the bellows root and crown is shown in Figure 19. The strains are the result of bending moments generated in root and crown. For analytical considerations, the bellows is envisioned to be restrained by the external piping for the case of intercal pressurization. It is immediately obvious from Figure 19 that while it may be possible to reduce the crown radius stress state by simultaneous compression loading and internal pressurization, the root stresses are intensified by the combination loading. Therefore, the root stress may be estimated as follows:

$$\sigma_{\text{cp}} = \sigma_p + \sigma_c$$  \hspace{1cm} (III.8)

where $\sigma_{\text{cp}}$ = combined stress due to pressure and compression load. By substitution of Equation (III.7) and (III.4) into (III.8).
FIGURE 19. NATURE OF FIBER STRAINS

(a) Unstressed State

(b) Compression Preload

T = Tension (+)  C = Compression (-)

(c) Internal Pressure
(Ends Restricted)
III.5 **Convolute Alternating Stress and Displacement**

A series of three-inch diameter bellows were flow tested to validate several assumptions made in earlier studies (Reference 1). The vibratory peak stress in the bellows convolute was assumed to be given by

\[
\sigma_p = \frac{F_c}{K_A} \left( \frac{E_t}{h^2 N_c} \right)
\]

where \( F_c \) is a geometric stress factor and the other terms are as defined earlier. The Reference 1 work utilized a single point strain gage to infer displacement and stress which is difficult under the best of test conditions. In the present study, stress was measured via a biaxial gage arrangement and convoluted displacement was obtained independently via a displacement probe (see Appendix B for details).

By assuming a mode shape over the first quarter wavelength of the form

\[
x = \frac{x_0}{2} \left[ \frac{(N/\lambda)y + \sin (N\pi y/\lambda)}{N} \right]
\]

where \( x \) denotes the axial absolute displacement of a given point along the bellows defined by the axial position coordinate \( y \), we may determine the relative displacement by differentiating Equation (III.11) with respect to \( y \). Thus,

\[
\Delta \delta = \frac{x_0}{2} \left[ \frac{(N/\lambda) + N\pi y/\lambda \cos (N\pi y/\lambda)}{N} \right]
\]

The above method was used to convert absolute displacement, \( \delta \), data into equivalent relative displacement, \( \Delta \delta \).

A summary of the deflection and stress results are shown in Table VII for each test specimen at the first, second, and third modes and a summary of the damping characteristics is shown in Table VIII.

Calculated alternating stress levels as determined by Equation (III.10), have been correlated with actual measurements. Results shown in Figure 20 indicate that \( C_s \) may be considered to equal unity.
### TABLE VII. THREE-INCH BELLOWS DEFLECTION AND STRESS RESULTS

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Mode No.</th>
<th>$V_F$, fps</th>
<th>2nd Convolute</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\delta$, mills</td>
<td>$\Delta \delta$, mills</td>
<td>$\sigma_{Alt}$, ksi</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>5.40</td>
<td>4.0</td>
<td>2.33</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.80</td>
<td>15.0</td>
<td>3.21</td>
<td>8.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15.89</td>
<td>22.6</td>
<td>3.58</td>
<td>8.94</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4.18</td>
<td>0.8</td>
<td>0.40</td>
<td>0.765</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.35</td>
<td>4.8</td>
<td>1.65</td>
<td>3.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12.53</td>
<td>7.8</td>
<td>1.93</td>
<td>4.59</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>7.26</td>
<td>4.0</td>
<td>1.96</td>
<td>2.82</td>
<td></td>
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<tr>
<td></td>
<td>2</td>
<td>14.52</td>
<td>14.0</td>
<td>4.71</td>
<td>7.84</td>
<td></td>
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<tr>
<td></td>
<td>3</td>
<td>21.79</td>
<td>21.0</td>
<td>3.34</td>
<td>8.51</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE VIII. THREE-INCH BELLOWS DAMPING CHARACTERISTICS

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Mode No.</th>
<th>$f_r$, Hz</th>
<th>$\Delta f_{0.707}$, Hz</th>
<th>$Q$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>120</td>
<td>2.0</td>
<td>60</td>
<td>.0083</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>234</td>
<td>3.0</td>
<td>78</td>
<td>.0064</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>133</td>
<td>.55</td>
<td>242</td>
<td>.0021</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>255</td>
<td>1.7</td>
<td>150</td>
<td>.0033</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>147</td>
<td>1.8</td>
<td>82</td>
<td>.0061</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>288</td>
<td>3.8</td>
<td>76</td>
<td>.0066</td>
</tr>
</tbody>
</table>
FIGURE 20. ALTERNATING STRESS VERSUS DEFLECTIONS

\[
\frac{E_t \Delta \delta}{h^2}, \text{ ksi}
\]

\[\sigma_{Alt}, \text{ ksi}\]

FIGURE 20. ALTERNATING STRESS VERSUS DEFLECTIONS
For all single ply bellows tested, the alternating stress component was observed to be insensitive to internal pressure variation. Tests were conducted at pressures of 0, 10, 20 psig over the first three modes of excitation.

III.6 Recommended Stress Prediction Equation

III.6.1 Alternating Stress

Two methods are available for calculating the alternating stress component. The Stress Indicator may be used as a predictor of actual stress for single ply bellows by incorporating a factor 2 into the S.I. equation over the first three modes of vibration, or

\[ \sigma_{alt} = 2 \frac{C_F \cdot N_c}{N \cdot N_p} (h/t)^2 \left( \frac{1}{2} \rho \sqrt{V_{crit}^2} \right) \] (III.13)

The second method is merely a refinement of the above method. The stress envelope factor may be applied to the value of the S.I. calculated for mode 1, or

\[ \sigma_{alt,N} = S.E.1 \left( \frac{0.00635}{\xi} \right) F_N \] (III.14)

where \( F_2 = 2.75 \)
\( F_3 = 3.05 \)

The second method requires more detailed knowledge of the bellows; however, it provides a means to infer the effects of combined fluid and structure damping.

III.6.2 Convoluted Mean Stress

Significant errors have been observed in the measured and calculated stress values that relate to internal pressure while axial extension of compression preloads may be more accurately modeled. Therefore, the recommended mean stress model is

\[ \sigma_{mean} = P \left( \frac{h/t}{2} \right)^2 + \left( \frac{E_t}{h^2 N_c} \right) |\Delta| \] (III.15)

where \( \Delta \) is the total live length extension or compression displacement.

For the multi-ply case, it has been assumed that the plies do not fully couple; hence, the calculations for the single ply are applied to multi-ply designs. Due to the inaccuracy of the simple pressure-stress model, the .5 factor has been deleted.
III.6.3 Combined Stress

The maximum stress developed is composed of the two additive components, or

\[ \sigma_{\text{max}} = \sigma_{\text{alt}} + \sigma_{\text{mean}} \]  \hspace{1cm} (III.16)

and by substitution of Equations (III.13) and (III.15), the proposed combined elastic stress model is

\[ \sigma_{\text{max}} = \left( \frac{E t}{N_c h^2} \right) |\Delta| \div (h/t)^2 \left[ P + \frac{2C_p^* N_c}{N N_p} P_d \right] \]  \hspace{1cm} (III.17)

III.7 Material Hardness Properties

A 3" bellows with 13 convolutes was sectioned and prepared for micro-hardness testing. This is accomplished by cutting axial strips of approximately 1/2" wide that contain several convolutes. Subsequently, these sections are imbedded in an epoxy molding compound, then the compound and bellows specimens are ground until their surfaces exhibit a highly polished finish. The bellows specimen is placed into a Diamond Pyramid Hardness (DPH) testing apparatus where a specific sized diamond needed is allowed to penetrate the bellows surface. The driving weight used is 10 kg. By an appropriate measuring technique, the dimensions of the penetration, rhomboid shaped, are measured and then converted into a DPH number.

Figure 21 shows a general bellows section. Three measurements were taken at the approximate locations shown in the figure; therefore, 24 data points per convolute were obtained. The results are tabulated in Table IX.

Upon careful review of the data, several observations are apparent which include the following:

1. Global averaging of the convolute center region produced a lower average hardness than found in global averaged outer edges.

2. Zonal averaged hardness numbers in the "inside diameter" region exhibits hardness close to slightly below the global average.

3. The "outer diameter" region exhibits hardness numbers significantly larger than the global averages.

4. The "straight wall" region exhibits hardness numbers significantly lower than the global averages.
FIGURE 21. CROSS SECTION OF BELLOWS SHOWING LOCATIONS OF HARDNESS MEASUREMENTS

$n =$ convolute number

Fluid Side

Outer Surface

Middle Surface

Inner Surface
# TABLE IX. HARDNESS READINGS

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>DIAMOND PYRAMID HARDNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CONVOLUTIONS 1, 2, &amp; 3</td>
</tr>
<tr>
<td></td>
<td>OUTER SURFACE</td>
</tr>
<tr>
<td>h₁</td>
<td>224</td>
</tr>
<tr>
<td>a₂ (inside diameter)</td>
<td>241</td>
</tr>
<tr>
<td>a₂</td>
<td>232</td>
</tr>
<tr>
<td>c₂ (straight wall)</td>
<td>239</td>
</tr>
<tr>
<td>d₂</td>
<td>268</td>
</tr>
<tr>
<td>e₂ (outer diameter)</td>
<td>270</td>
</tr>
<tr>
<td>f₂</td>
<td>266</td>
</tr>
<tr>
<td>g₂ (straight wall)</td>
<td>239</td>
</tr>
<tr>
<td>h₂</td>
<td>241</td>
</tr>
<tr>
<td>aₐ (inside diameter)</td>
<td>235</td>
</tr>
</tbody>
</table>

mean (μ)  245.5  240.1  245.8  
Std. Dev. (σ)  16.35  21.5  18.06

|          | CONVOLUTIONS 6, 7, & 8    |
| h₆       | 236           | 263    | 283           |
| a₇ (inside diameter) | 236           | 253    | 252           |
| b₇       | 257           | 266    | 247           |
| c₇ (straight wall)  | 235           | 250    | 250           |
| d₇       | 281           | 279    | 285           |
| e₇ (outer diameter) | 273           | 265    | 297           |
| f₇       | 285           | 279    | 273           |
| g₇ (straight wall)  | 236           | 232    | 236           |
| h₇       | 281           | 253    | 261           |
| a₈ (inside diameter) | 265           | 257    | 250           |
| b₈       | 268           | 250    | 273           |

mean (μ)  259.4  259  264.3  
Std. Dev. (σ)  20.3  13.78  19.15
### TABLE IX. HARDNESS READINGS (Cont'd)

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>DIAMOND PYRAMID HARDNESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CONVOLUTIONS 11, 12, &amp; 13</td>
</tr>
<tr>
<td></td>
<td>OUTER SURFACE</td>
</tr>
<tr>
<td>h₁₁</td>
<td>236</td>
</tr>
<tr>
<td>a₁₂ (inside diameter)</td>
<td>221</td>
</tr>
<tr>
<td>b₁₂</td>
<td>233</td>
</tr>
<tr>
<td>c₁₂ (straight wall)</td>
<td>247</td>
</tr>
<tr>
<td>d₁₂</td>
<td>275</td>
</tr>
<tr>
<td>e₁₂ (outer diameter)</td>
<td>313</td>
</tr>
<tr>
<td>f₁₂</td>
<td>300</td>
</tr>
<tr>
<td>g₁₂ (straight wall)</td>
<td>243</td>
</tr>
<tr>
<td>h₁₂</td>
<td>267</td>
</tr>
<tr>
<td>a₁₃ (inside diameter)</td>
<td>261</td>
</tr>
<tr>
<td>b₁₃</td>
<td>275</td>
</tr>
<tr>
<td>mean (μ)</td>
<td>261</td>
</tr>
<tr>
<td>Std. Dev. (σ)</td>
<td>286</td>
</tr>
</tbody>
</table>
5. Outer diameter zonal averaged DPH numbers ranged from 274 to 293 which corresponds to a Rockwell Hardness range of 26C to 29C.

6. Inner diameter zonal averaged DPH numbers ranged from 229 to 260 which corresponds to Rockwell readings in the range of 96B to 24C.

III.8 Conclusions

1. The outer diameter region exhibits a yield stress of approximately 132,000 psi whereas the inner diameter region exhibits a yield stress of approximately 100,000 psi. These yield values are somewhat lower (\textit{30%}) than those reported in Reference (13); however, it is speculated that the hydroforming process work hardens to a lesser extent than the rolling process.

2. Failures most often occur in the root or crown region; therefore, in view of the hardness data, it can be concluded that failures are not the result of material weakness in the failure region.
IV. FATIGUE LIFE

IV.1 Crack Propagation Model

A bellows fatigue life model was developed based on the assumption that crack propagation in the convolute wall is the failure mechanism. It was further assumed that the crack was initiated by a pre-existing surface or material flaw. The state of stress in the bellows wall was taken to be the sum of the mean stress due to internal fluid pressure plus a cyclic bending stress that is associated with convolute deflection in any given mode of excitation. The stress model used here is different than that used in Section III; however, the features of the crack modeling and general results are valid.

The mean internal pressure stress is, \( \sigma_p = p/2 \left( \frac{h}{t} \right)^2 \) \hspace{1cm} (IV.1)

where \( p \) = internal pressure,
\( h \) = root-to-crown height, and
\( t \) = bellows wall thickness \((N_{ply} \times t_{ply})\)

Superimposed onto this steady state stress is a cyclic, deflection-related bending stress that is caused by the flow induced vibration of the bellows convolutes at given excitation mode. The peak-to-peak amplitude of this cyclic stress component is given by, \( \Delta \sigma = \frac{2(1.5) E \ell t}{h^{1.2} \lambda^{3.2}} \left( \frac{\Delta}{2N_c} \right) \) \hspace{1cm} (IV.2)

where \( E \) = Young's modulus
\( \lambda \) = convolute pitch
\( \Delta/N_c \) = flow-induced convolute deflection

The deflection per convolute is calculated from Reference 1, \( \frac{\Delta}{N_c} = \frac{C_m \rho V^2 A_p}{g^2 K_A} (C_F Q) \) \hspace{1cm} (IV.3)

where \( \rho \) = fluid density
\( V \) = critical flow velocity as a function of mode number
\( A_p \) = \( \pi/2 \ h (D_i + D_o) \)
\( D_i, D_o \) = bellows inside and outside diameter, respectively
\( K_A \) = bellows spring rate
\( C_F Q \) = force amplification factor from Figures 4 and 5
\( C_m \) = bellows mode factor.
The bellows mode factor, $C_m$, is of the form

$$C_m = \frac{1}{8N} \left[ \frac{N}{N_C} + \sin \left( \frac{\pi}{2} \frac{N}{N_C} \right) \right] \quad (IV.4)$$

where $N =$ mode number
$N_C =$ number of bellows convolutes.

Thus, Equations (IV.1) through (IV.4) define the mode-dependent state of stress in the bellows wall. This state of stress can be illustrated schematically as shown in Figure 22. In this figure, tensile stresses are positive. Depending on the mode number and the magnitude of the mean stress, the minimum stress can be compressive, in which case the sign of the stress is negative.

If a crack is initiated on the bellows surface, the rate at which it will propagate into the wall thickness is governed by

$$\frac{da}{dn} = C \left( \frac{Y \Delta \sigma \sqrt{a}}{1-R} \right)^m \quad (IV.5)$$

where $a =$ crack length
$n =$ number of imposed stress cycles
$\Delta \sigma =$ cyclic stress range, Equation (2)
$C, m =$ curve fit coefficients that describe the experimental crack growth rate as a function of stress intensity factor, which is the expression within the brackets in Equation (IV.5). These parameters are dependent upon the bellows material.

The factor, $R$, accounts for the mean stress effect, and it is defined as

$$R = \frac{\sigma_p - \Delta \sigma / 2}{\sigma_p + \Delta \sigma / 2} \quad (IV.6)$$

It is worth noting that when $\sigma_p$ is equal to zero (no pressure stress), the value of $R$ is -1.0, which describes a fully reversed state of stress. The quantity $Y$, which is an explicit function of the crack length, is a geometric correction factor that accounts for the decrease in load bearing area during crack propagation. As such, $Y$ can be satisfactorily approximated by a second order polynomial...
Mean Stress, $\sigma$, Period of Oscillation, $T$, and $\Delta \sigma = f$ (mode number)

FIGURE 22. DEFINITION OF BELLOWS STRESSES
where \( a, \epsilon \) = curve fit coefficients.

Combining Equations (IV.5) and (IV.7), and separating variables, yields

\[
y = \alpha(a/t)^2 + \epsilon
\]  
(IV.7)

The failure model, Equation (IV.8), is valid only for values of \( a_c < t/2 \).

Evaluation of the Model

The fatigue life integral and its supporting equations were programmed for solution on the CDC6600/Cyber 74 system. A trapezoidal integration scheme was used to evaluate the definite integral in Equation (IV.8). A listing of the computer program, FATLIF, is contained in Appendix E.

Since prediction of fatigue life is currently accomplished by a stand-alone program, it was necessary to first exercise program "Bellow" to generate critical flow excitation velocities for a given bellows configuration. The essential input-output data for program Bellow is summarized in Table X. The reader will be able to identify the bellows and fluid input parameters that are common to the FATLIF program. Excitation velocities are shown for the first four modes. It should be noted that the fatigue life program accepts fluid pressure in psia rather than psig.

To complete the input data for program FATLIF, it was necessary to specify numerical values for \( C, m, \alpha, \epsilon, a_i, \) and \( a_c \). The constants, \( \alpha \) and \( \epsilon \), were obtained by curve-fitting the correction factors for a single edge-notched strip that are presented in Table 4 of Reference 14. The results of the manipulation yielded

\[ \alpha = 6.79 \]
\[ \epsilon = 1.12 \]
### TABLE X

**INPUT/OUTPUT DATA FOR PROGRAM BELLOW**

**BELLOW PARAMETERS (INPUT)**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIGRA (CONVOLUTE WIDTH, IN)</td>
<td>0.158</td>
</tr>
<tr>
<td>LAMBDA (CONVOLUTE PITCH, IN)</td>
<td>0.269</td>
</tr>
<tr>
<td>H (MEAN DISC HEIGHT, IN)</td>
<td>0.390</td>
</tr>
<tr>
<td>Y (CONVOLUTE THICKNESS/PLY, IN)</td>
<td>0.018</td>
</tr>
<tr>
<td>DI (INSIDE DIAMETER, IN)</td>
<td>4.000</td>
</tr>
<tr>
<td>DO (OUTSIDE DIAMETER, IN)</td>
<td>4.828</td>
</tr>
<tr>
<td>NC (NUMBER OF CONVOLUTES)</td>
<td>9,000</td>
</tr>
<tr>
<td>NPLY (NUMBER OF PLIES)</td>
<td>3,000</td>
</tr>
<tr>
<td>E (YOUNG'S MODULUS, LBS/SQ. IN)</td>
<td>2.9400E+07</td>
</tr>
<tr>
<td>KA (OVERALL SPRING RATE, LBS/IN)</td>
<td>373.366</td>
</tr>
<tr>
<td>RHOM (MATERIAL DENSITY, LBS/CU. IN)</td>
<td>0.282</td>
</tr>
</tbody>
</table>

**FLUID PARAMETERS (INPUT)**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (PRESSURE, PSIG)</td>
<td>100.000</td>
</tr>
<tr>
<td>TEMP (TEMPERATURE, DEG F)</td>
<td>70.000</td>
</tr>
<tr>
<td>RHOF (FLUID DENSITY, LBS/CU. IN)</td>
<td>3.611E-02</td>
</tr>
<tr>
<td>NFLUID (1=GAS, 2=LIQUID)</td>
<td>2</td>
</tr>
</tbody>
</table>

**THEORETICAL PERFORMANCE (OUTPUT)**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hz</th>
<th>Lower</th>
<th>Critical</th>
<th>Upper</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>191</td>
<td>7.0</td>
<td>9.8</td>
<td>16.2</td>
</tr>
<tr>
<td>2</td>
<td>365</td>
<td>13.4</td>
<td>18.7</td>
<td>30.9</td>
</tr>
<tr>
<td>3</td>
<td>523</td>
<td>19.2</td>
<td>26.8</td>
<td>44.3</td>
</tr>
<tr>
<td>4</td>
<td>666</td>
<td>24.4</td>
<td>34.1</td>
<td>56.4</td>
</tr>
</tbody>
</table>

*ORIGINAL PAGE IS OF POOR QUALITY*
Specification of the values of $C$ and $m$ was impeded by the lack of basic crack propagation data in the open literature for the bellows materials of interest, i.e., Inco 718, 21-6-9 and 321. As an alternative, for evaluation purposes only, the following values of $C$ and $m$ were obtained from Reference 15 for Type 316 stainless steel.

$$C = 7 \times 10^{-16}$$

$$m = 6.5$$

The number of fatigue cycles needed to effect failure is strongly affected by the magnitude of $a_1$ and $a_c$. In evaluating the model, the initial flaw size, $a_1$, was chosen to be 0.001 inch. This value is believed to be representative of a typical surface flaw. The crack length at failure, $a_c$, was taken to be $t/2$, the validity limit of the model.

Based upon the above input data, fatigue life predictions were made for the specific bellows geometry, fluid properties, and critical excitation velocities in Table X. The results are summarized in Table XI. For this example problem, the following observations can be made on the validity of the model.

1. The cyclic stress range, $\Delta \sigma$, increases with mode number because the product of flow excitation velocity and dynamic amplification factor is an increasing function of mode number.

2. The maximum bending stresses is tensile at all mode numbers. The minimum stress is tensile initially but becomes compressive as mode number increases. In the presence of internal pressure, a fully-reversed stress field is not achieved.

3. For this example, in which Type 316 stainless steel was employed, the maximum tensile stress in the first three modes did not exceed the material yield point of 42 ksi. In the fourth mode, the maximum tensile stress exceeded the material yield point.

4. In this example, the model predicts high cycle fatigue when $a_1$ and $a_c$ are 0.001 and 0.012 inch, respectively.
TABLE XI

PREDICTED FATIGUE LIFE FOR A TYPE 321 STAINLESS STEEL BELLOWS

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>$\sigma_p$ (psi)</th>
<th>$\Delta \sigma$ (psi)</th>
<th>Frequency (Hz)</th>
<th>Fatigue Life (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15,144</td>
<td>23,808</td>
<td>191</td>
<td>$1.76 \times 10^{12}$</td>
</tr>
<tr>
<td>2</td>
<td>15,144</td>
<td>33,024</td>
<td>365</td>
<td>$6.34 \times 10^{11}$</td>
</tr>
<tr>
<td>3</td>
<td>15,144</td>
<td>45,684</td>
<td>523</td>
<td>$1.94 \times 10^{11}$</td>
</tr>
<tr>
<td>4</td>
<td>15,144</td>
<td>98,783</td>
<td>666</td>
<td>$6.19 \times 10^9$</td>
</tr>
</tbody>
</table>
At this point, realization of the full utility of this approach to fatigue modeling is impeded by:

1. The lack of basic crack propagation data for the three materials of interest. Currently, the fatigue data that are available from the materials manufacturers were obtained using fully-reversed stress fields at room temperature. What is needed are crack propagation tests which yield crack growth rates as a function of stress intensity factor and mean stress over the range of temperatures of interest.

2. A correlation between the fatigue life as predicted by the crack propagation model and experimental fatigue life of actual bellows in a common temperature and stress environment.

3. The lack of an experimental definition of the flaw size, \( a_i \), that is needed to initiate and propagate a crack and \( a_c \), the crack length at which failure occurs.

VI.2 Fatigue Curves

Due to the limitations posed by the crack growth model, an alternate approach was developed to predict bellows life. Seven ordinate charts were developed (Figures 23, 24, and 25) from data listed in References 17 through 27. The materials studied included 347SS (a close substitution for 321SS), Alloy 21-6-9, and Inco 718. Data reviewed were mainly in the form of "cycles to failure" or S-N curves for various R values and for temperatures of 70°F and -423°F.

Seven-Ordinate charts relate stress and stress ratios to cycle life. Most of the seven-ordinate data is based upon data banks maintained by the Department of Defense and the Federal Aviation Agency if its source is contained in the MIL-HDBK-5B.

Seven-ordinate charts are convenient to use and they relate fatigue life in terms of mean stress which could be an important factor when predicting bellows life. Design or analysis parameters can be specified as stress amplitude, mean stress, maximum or minimum stress, cycle life, R-values, and A-values. (The A value is defined as the stress amplitude divided by the
STAINLESS ALLOY 21-6-9

Minimum Stress (ksi)

Maximum Stress (ksi)

R = -1.0 - .8 - .6 - .4 - .2 0 .2 .4 .6 .8 1.0

10^4 Cycles

10^5

10^6

160

120

80

40

Alternating Stress (ksi)

Mean Stress (ksi)

-200 -160 -120 -80 -40 0 40 80 120 160 200

Minimum Stress (ksi)

-423°F

70°F

R = Minimum Stress

R = Maximum Stress

FIGURE 23. SEVEN-ORDINATE CHART FOR ALLOY 21-6-9
FIGURE 24. SEVEN-ORDINATE CHART FOR INCONEL 718
347 STAINLESS STEEL

FIGURE 25. SEVEN-ORDINATE CHART FOR 347 SS
mean stress.) To determine the fatigue life, only three parameters are required. These parameters are usually determined from the bellows stress analysis.

The seven-ordinate fatigue data is built into the computer program listed in Appendix A. Each constant life cycle curve is modeled as a power law, or

\[ C_j = B \sigma_{alt}^m \]

where \( C_j \) is a constant life value for \( j \) mean stress. Curves are developed for mean stress levels of 0, 20, 40, 60, and 80 ksi. The alternating stress component, \( \sigma_{alt} \), is in the units of ksi. Simple linear interpolation may be used for intermediate values.

The seven-ordinate curves are applicable for fatigue life predictions once the stress levels have been determined; however, stress indicator values may be used directly as a calculated alternating stress value with reasonable accuracy even though the stress indicator’s intended use was to predict fatigue life with the aid of data presented in Figure 6.
REFERENCES


REFERENCES


APPENDIX A

BELLOWS FLOW-INDUCED VIBRATION COMPUTER PROGRAM
A.1 Governing Equations

The performance equations, which will be presented in this section, are based upon the derivations given in Reference 1. Therefore, detailed algebraic manipulations and derivations have been eliminated for clarity.

Figure A-1 illustrates a longitudinal cross-section of a typical bellows together with pertinent notation. The overall bellows spring rate is

\[ K_A = D_m E \frac{N_p}{N_c} (t/h)^3 \]  \hspace{1cm} (A-1)

where \( E \) is the Young's modulus for bellows material and \( D_m \) is the mean bellows diameter which is defined as

\[ D_m = (D_t + D_o)/2 \]  \hspace{1cm} (A-2)

The elemental spring rate, \( K \), is given by

\[ K = 2 N_c K_A \]  \hspace{1cm} (A-3)

The corresponding elemental metal mass of the bellows is

\[ m_m = \pi \rho_m t N_p D_m \left[ \pi a + (h - 2a) \right] \]  \hspace{1cm} (A-4)

where \( \rho_m \) is the metal density and the mean crown or convolute forming radius is

\[ a = (\sigma - t N_p)/2 \]  \hspace{1cm} (A-5)

As the bellows vibrates in any one of its \( 2N_c - 1 \) longitudinal modes, fluid is accelerated within the convolutes. The process of moving the fluid is manifested as an apparent of added mass which must be taken into account in calculating the frequencies at which a fluid-elastic instability is likely to occur. This added mass is a function of the longitudinal mode number, \( N \). That is

\[ m_f = m_{f1} \left( \frac{2 N_c - 1 - N}{2 N_c - 2} \right) + m_{f2} \left( \frac{N - 1}{2 N_c - 2} \right) \]  \hspace{1cm} (A-6)

where

\[ m_{f1} = \frac{\pi \rho_f D_m h (2a - t N_p)}{2 g} \]  \hspace{1cm} (A-7)

and

\[ m_{f2} = \frac{\pi D_m \rho_f h^3}{3 g \delta} \]  \hspace{1cm} (A-8)
$N_c$ = NUMBER OF CONVOLUTIONS COUNTED FROM THE OUTSIDE

$N_p$ = NUMBER OF PLYS

$D_m$ = MEAN BELLows DIAMETER

$t$ = WALL THICKNESS (THICKNESS PER PLY IF MULTI-PLY)

$\lambda$ = CONVOLUTE PITCH

$\sigma$ = CONVOLUTE WIDTH

$a$ = MEAN FORMING RADIUS

$h$ = MEAN DISC HEIGHT

FIGURE A-1. BELLows NOMENCLATURE
In these expressions $\rho_f$ is the fluid density, $g$ is the gravitational acceleration constant, and

$$\delta = \sigma - 2t N_p \quad (A-9)$$

The mode number, $N$, ranges between 1 and $2N_c-1$. A reference frequency for a particular mode number can be defined as

$$f_o (N) = \frac{1}{2\pi} \sqrt{\frac{K}{m_m + m_f}} \quad (A-10)$$

The true modal frequency, $f_N$, is then obtained by multiplying the reference value by the dimensionless frequency corresponding to the desired mode number and system degree of freedom. Dimensionless frequencies can be calculated as

$$B_i = \left(2 \left[1 + \cos \left(\frac{\pi (2N_c-i)}{2N_c}\right)\right]\right)^{\frac{1}{2}}; \quad i = 1, 2, 3, \ldots 2N_c-1 \quad (A-11)$$

Alternately, for purposes of hand calculations, the dimensionless frequency factors may be determined from Table A-I.

It has been observed that flow excitation of a particular mode can occur over a broad range of fluid velocities, which is termed the "lock-in-range." In fact, if the modal frequencies are sufficiently close together, the lock-in ranges may overlap, thus producing nearly continuous excitation of the bellows. These lock-in ranges are estimated as follows. Extensive experimental studies have revealed that the Strouhal number provides an excellent means of correlating the vibration frequency, fluid velocity and bellows geometry as shown in Figure A-2. The Strouhal number is based on convolute pitch, $\sigma$. For a bellows having a convolute pitch-to-convolute tip width ratio of $\lambda/\sigma$, three values of the Strouhal number are indicated. Peak bellows excitation corresponds to the curve marked $\sigma_{crit}$ from which the critical flow velocity may be calculated, i.e.

$$v (N)_{crit} = \frac{f_N\sigma}{\sigma_{crit}} \quad (A-12)$$

Similarly, the upper and lower values of velocity, which define the lock-in-range are obtained from

$$v (N)_{upper} = \frac{f_N\sigma}{\sigma_2} \quad (A-13)$$

and

$$v (N)_{lower} = \frac{f_N\sigma}{\sigma_0} \quad (A-14)$$
| Degrees of Freedom, 2Nc+1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 6                        | 0.195 | 0.362 | 0.547 | 1.122 | 1.602 | 2.082 | 2.650 | 3.460 | 1.082 | 1.595 | 1.925 | 2.359 | 3.624 | 4.876 | 6.462 | 9.000 | 11.195 | 13.202 | 15.000 | 16.989 | 19.000 | 21.000 | 23.000 | 25.000 |
| 7                        | 0.174 | 0.347 | 0.518 | 0.848 | 1.328 | 1.772 | 2.354 | 3.208 | 1.082 | 1.595 | 1.925 | 2.359 | 3.624 | 4.876 | 6.462 | 9.000 | 11.195 | 13.202 | 15.000 | 16.989 | 19.000 | 21.000 | 23.000 | 25.000 |
| 8                        | 0.165 | 0.336 | 0.509 | 0.499 | 1.123 | 1.602 | 2.082 | 2.650 | 1.082 | 1.595 | 1.925 | 2.359 | 3.624 | 4.876 | 6.462 | 9.000 | 11.195 | 13.202 | 15.000 | 16.989 | 19.000 | 21.000 | 23.000 | 25.000 |
| 9                        | 0.157 | 0.323 | 0.457 | 0.618 | 1.208 | 1.691 | 2.285 | 3.208 | 1.082 | 1.595 | 1.925 | 2.359 | 3.624 | 4.876 | 6.462 | 9.000 | 11.195 | 13.202 | 15.000 | 16.989 | 19.000 | 21.000 | 23.000 | 25.000 |
| 10                       | 0.149 | 0.300 | 0.298 | 0.540 | 0.851 | 1.351 | 1.968 | 2.650 | 1.082 | 1.595 | 1.925 | 2.359 | 3.624 | 4.876 | 6.462 | 9.000 | 11.195 | 13.202 | 15.000 | 16.989 | 19.000 | 21.000 | 23.000 | 25.000 |

TABLE A-1 - DIMENSIONLESS FREQUENCIES FOR BELLOWS MECHANICAL MODEL
Figure A-2. Composite of all Strouhal Number Correlation Data
The stress indicator is a relative measure of the stress intensity. Two methods of calculation are allowed in the computer program. The first method, and the more exacting one, involves a greater number of calculations and a substantial amount of input data. The second method incorporates a "Universal CfQ Function" and, due to its data compression requirement, it is by nature a more conservative calculation, i.e., the SI values will be high. These calculation methods are given as:

**Method I: Conventional Stress Indicator**

\[ SI = \left[ \frac{C_f \cdot C_e \cdot P_d}{N_p} \right] \cdot \left( \frac{h}{t} \right)^2 \cdot Q \]  \hspace{1cm} (A-15)

where

- \( C_f \) = vortex force coefficient which is a function of \( \lambda / \sigma \) and is obtained from Figure A-3.
- \( C_e \) = elbow factor to account for above average forces exerted on bellows convolutes if an elbow located immediately upstream of the bellows.
- \( P_d \) = fluid dynamic pressure.
- \( Q \) = dynamic amplification factor.

The bracketed term in Equation (A-15) is termed the "bellows operational parameter". This parameter is used in conjunction with the bellows specific spring rate and Table A-II to determine the dynamic amplification factor (Figure A-4), where specific spring rate is defined as

\[ SSR = \frac{K_A \cdot N_c}{D_m \cdot N_p} \]  \hspace{1cm} (A-16)

The computer program currently calculates the stress indicator corresponding to the critical flow velocity defined by Equation (A-12).

If the internal medium is a gas, a radial acoustic resonance condition is likely to occur, wherein the acoustic pressure fluctuations couple with the vortex shedding process to produce a force amplification that is significantly larger than would be predicted by the value of \( Q \) obtained from Figure A-4. Physically, these pressure fluctuations are attenuated at approximately a constant rate for all vortex shedding frequencies less than the radial acoustic resonance or cutoff frequency. In the vicinity of the cutoff frequency, the increased amplification must be taken into account since it results in much higher bellows stress levels. To this end, the first mode radial acoustic resonant frequency is obtained from Figure 5 for a particular bellows geometry. This cutoff frequency is then compared with the predicted longitudinal modal frequencies. The predicted \( Q \) value from Figure A-4 is modified by a suitable constant for all longitudinal frequencies that exceed the cutoff frequency. In other words, this adjustment of \( Q \) states that the radial acoustic resonance is capable of coupling with higher longitudinal modes not just at the condition where the frequencies coincide. Figure A-5 is valid for convolute pitch-to-tip width ratios of 1.4 to 2.0. These values
Use this curve for higher longitudinal free bellows modes, flex hose longitudinal modes, and convolute bending mode.

**FIGURE A-3. SUMMARY OF BELLOWS VORTEX FORCE COEFFICIENT EXPERIMENTAL DATA**
TABLE A-II. APPLICATIONS INFORMATION FOR USE WITH Q VALUES DATA IN FIGURE A-4

<table>
<thead>
<tr>
<th>Specific Spring Rate</th>
<th>Number Plies</th>
<th>Internal Media (see Note 1)</th>
<th>Curve No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Ranges</td>
<td>1</td>
<td>low pressure gases</td>
<td>1</td>
</tr>
<tr>
<td>over 2000 lb/in²</td>
<td>1</td>
<td>high pressure gases, light liquids</td>
<td>1</td>
</tr>
<tr>
<td>over 2000</td>
<td>1</td>
<td>water, dense liquids</td>
<td>2</td>
</tr>
<tr>
<td>under 2000</td>
<td>1</td>
<td>high pressure gases, light liquids</td>
<td>2</td>
</tr>
<tr>
<td>under 2000</td>
<td>1</td>
<td>water, dense liquids</td>
<td>3</td>
</tr>
<tr>
<td>over 3000</td>
<td>2</td>
<td>all pressure gases</td>
<td>3</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>2</td>
<td>all pressure gases</td>
<td>4</td>
</tr>
<tr>
<td>under 2000</td>
<td>2</td>
<td>all liquids</td>
<td>5</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>2</td>
<td>all pressure gases</td>
<td>5</td>
</tr>
<tr>
<td>under 2000</td>
<td>2</td>
<td>all liquids</td>
<td>6</td>
</tr>
<tr>
<td>over 3000</td>
<td>3</td>
<td>All</td>
<td>4</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>3</td>
<td>All</td>
<td>5</td>
</tr>
<tr>
<td>under 2000</td>
<td>3</td>
<td>all pressure gases</td>
<td>5</td>
</tr>
<tr>
<td>under 2000</td>
<td>3</td>
<td>all liquids</td>
<td>6</td>
</tr>
</tbody>
</table>

NOTE 1: Low pressure gases will be defined here as being those gases below 150 psia. Light liquids will be defined as having a specific gravity of less than 0.2.
FIGURE A-4. DYNAMIC AMPLIFICATION FACTORS FOR VARIOUS BELLOWS APPLICATIONS
FIGURE A-5. BELLOWS CUT-OFF FREQUENCY FOR
FIRST MODE RADIAL ACOUSTIC RESONANCE
correspond to total convolute thickness of 0.3σ and 0.0σ (theoretical zero wall thickness). In addition, Figure A-5 is valid for fluid damping numbers, \( D_N \), of the order of \( 10^{-6} \) where \( D_N = \nu/r_1 c_0 \), \( \nu \) = fluid kinematic viscosity and \( c_0 \) = isentropic speed of sound.

**Method II: Calculation of SI with \( C_F^* \) Function**

Calculation of the stress indicator may be greatly streamlined if the universal \( C_F^* \) function shown in Figure A-6 is incorporated as follows:

\[
\text{SI} = C_F^* \left( \frac{N_C}{N_N} \right) C_e \left( \frac{h}{t} \right)^2 P_d
\]

Note that the calculation requires the use of only one curve, and hence, this method is favored for hand calculations; however, if \( JCFQ \) is set to 0, the calculation is performed by the computer code. Input cards 9 through 15 may be blank cards.

Calculation of fatigue life is accomplished in a subroutine called XLIFE where the input parameters of material type, alternating stress, and mean stress are manipulated in conjunction with a "Seven-Ordinate" fatigue chart to determine the bellows expected life.

The current version of the program assumes a mean stress of 0 psi; however, several simple program statements could be included to account for internal pressure and slight angulation. Room temperature conditions are assumed, but these conditions predict shorter life expectancies than cryogenic conditions.

The room temperature conditions compensate somewhat for unknown work hardening effects. From the limited amount of data available (AFRPL-TR-68-22), it is generally shown that hyd-gformed bellows life expectancy is shorter by one order of magnitude than that of a coupon made of the same material. Therefore, it is not advisable to expect longer bellows life due to low temperature operation.

A typical Seven Ordinate Chart is shown in Figure A-7. The alternating and mean stress ordinates are used exclusively in the bellows code. Each constant life curve is represented by a simple power law of the form

\[
\text{Cycles} = B \sigma_{alt}^m
\]

\( B \) and \( m \) values are obtained from data cards 16 through 21. For example, the cycles to failure for INCONEL 718 operating with a mean stress of 0 psi at room temperature is

\[
\text{Cycles} = 2.1410 \times 10^5 \sigma_{alt}^{5.1097}
\]

Similar curves are generated for mean stress levels of 20, 40, 60, and 80 KSI. A linear interpolation process is used to compute cycle values between successive 20 KSI mean stress levels.
Figure A-6. Envelope Curve for $C_F^*$ Correlation
INCONEL 718

FIGURE A-7. SEVEN-ORDINATE CHART FOR INCONEL 718
A.2 **Equivalence of Theoretical and Computer Program Variables**

This section is intended to establish the correspondence between the analysis variables presented in the previous section and the computer coded variables. Internally generated variables as well as curve fit coefficients will be discussed in subsequent sections.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Computer</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_c )</td>
<td>NC</td>
<td>Number of bellows convolutes</td>
</tr>
<tr>
<td>( N_p )</td>
<td>NPLY</td>
<td>Number of plies</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>SIGMA</td>
<td>Convolute width</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>LAMBDAYA</td>
<td>Distance between adjacent convolute crowns</td>
</tr>
<tr>
<td>( h )</td>
<td>H</td>
<td>Mean convolute disc height</td>
</tr>
<tr>
<td>( t )</td>
<td>T</td>
<td>Thickness per convolute ply</td>
</tr>
<tr>
<td>( D_i )</td>
<td>DI</td>
<td>Bellows inside diameter</td>
</tr>
<tr>
<td>( D_o )</td>
<td>DO</td>
<td>Bellows outside diameter</td>
</tr>
<tr>
<td>( E )</td>
<td>E</td>
<td>Young's modulus of bellows material</td>
</tr>
<tr>
<td>( \rho_\theta )</td>
<td>RHOM</td>
<td>Bellows material density</td>
</tr>
<tr>
<td>( r_a )</td>
<td>KA</td>
<td>Overall bellows spring rate</td>
</tr>
<tr>
<td>( C_e )</td>
<td>CE</td>
<td>Elbow loss factor</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>RHOF</td>
<td>Fluid density</td>
</tr>
<tr>
<td>( D_m )</td>
<td>DMEAN</td>
<td>Mean bellows diameter</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>K</td>
<td>Elemental spring rate</td>
</tr>
<tr>
<td>( a )</td>
<td>A</td>
<td>Mean convolute forming radius</td>
</tr>
<tr>
<td>( m_m )</td>
<td>MME(\gamma)</td>
<td>Elemental metal mass</td>
</tr>
<tr>
<td>( m_f1 )</td>
<td>MFLUID1</td>
<td>Apparent fluid mass at low mode numbers</td>
</tr>
<tr>
<td>( m_f2 )</td>
<td>MFLUID2</td>
<td>Apparent fluid mass at higher mode numbers</td>
</tr>
<tr>
<td>( m_f )</td>
<td>MFLUID</td>
<td>Apparent fluid mass</td>
</tr>
<tr>
<td>( \delta )</td>
<td>DELTA</td>
<td>Internal convolute width</td>
</tr>
<tr>
<td>Analysis</td>
<td>Computer</td>
<td>Comment</td>
</tr>
<tr>
<td>---------------</td>
<td>----------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td>$S_{\sigma_k}$</td>
<td>STLO</td>
<td>Strouhal number defining the lower and upper bounds on lock-in-range</td>
</tr>
<tr>
<td>$S_{\sigma_u}$</td>
<td>STUP</td>
<td>Strouhal number for severe excitation</td>
</tr>
<tr>
<td>$S_{\sigma_{crit}}$</td>
<td>STCRIT</td>
<td>Lower velocity bound on lock-in-range</td>
</tr>
<tr>
<td>$V(N)_{lower}$</td>
<td>V(MODE,1)</td>
<td>Flow velocity for maximum excitation</td>
</tr>
<tr>
<td>$V(N)_{crit}$</td>
<td>V(MODE,2)</td>
<td>Upper velocity bound on lock-in-range</td>
</tr>
<tr>
<td>$V(N)_{upper}$</td>
<td>V(MODE,3)</td>
<td>Vortex force coefficient</td>
</tr>
<tr>
<td>$C_f$</td>
<td>CF</td>
<td>Envelope stress coefficient</td>
</tr>
<tr>
<td>$C_F^*$</td>
<td>CFSTAR</td>
<td>Specific spring rate</td>
</tr>
<tr>
<td>$Q$</td>
<td>Q</td>
<td>Dynamic amplification factor</td>
</tr>
<tr>
<td>$S_l$</td>
<td>SI</td>
<td>Stress indicator</td>
</tr>
<tr>
<td>$\sigma_{alt}$</td>
<td>ALTSTR</td>
<td>Alternating stress</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>MEANSTR</td>
<td>Mean stress</td>
</tr>
<tr>
<td>$\omega_{CO} r_i / C_O$</td>
<td>FNCO</td>
<td>Frequency number for first mode radial acoustic resonance</td>
</tr>
<tr>
<td>$\omega_{CO}$</td>
<td>FREQCO</td>
<td>Angular cutoff frequency for first mode radial acoustic resonance</td>
</tr>
<tr>
<td>$C_O$</td>
<td>CO</td>
<td>Isentropic speed of sound</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>GAMMA</td>
<td>Ratio of gas specific heats</td>
</tr>
</tbody>
</table>

A.3 Curve Fit Requirements

When predicting the performance of complex systems, it is frequently necessary to describe experimentally observed relationships between two or more variables through the use of empirical expressions, i.e., curve fits. In predicting bellows flow-induced vibrations, it was necessary to curve fit the data shown in Figures A-2, A-3, and A-4. To this end, all data in these figures were fitted to a hyperbolic equation of the form
where $k$, $a$, $b$, and $d$ are the coefficients to be determined. Coordinate pairs are input to the fitting routine, and the resulting equations are solved simultaneously for the unknown coefficients. A listing of the curve fit routine is included in the next section. Note that there is an option for either a four- or eight-point fit. It was necessary to use an eight-point fit only for the curves labeled 1, 2, and 3 in Figure A-4 (Q-surface). Curve fit coefficients are supplied on input cards 6 through 15.

A.4 Computer Program Structure and Listing

The computer programs listed in this section were written in FORTRAN IV language. In the form presented here, the programs must be compiled each time they are submitted to the computer; however, multiple runs can be accomplished at each submittal. The user of this program may find it more convenient to compile and store the program on tape, thus necessitating minor program modifications.

Four program listings are contained in this section:

(1) MAIN (PROGRAM BELLOW)

(2) Curve generating routine (CURVE)

(3) First mode acoustic response frequency (ACOURES)

(4) Fatigue life routine (XLIFE)

The source deck for the performance program consists of a main program in which a majority of the calculations are performed and three subroutines: CURVE, which is called from the main program, and it contains the logic for selecting the appropriate curve on the Q-surface (Figure A-4); ACOURES, which evaluates the first mode cutoff or acoustic resonance frequency as a function of bellows geometry, and XLIFE, which calculates the bellows life expectancy based upon seven ordinate fatigue data.

The execution structure of the program consists of the following items in the order presented.

(1) Program control cards - number and type of these cards varies with the user facility.

(2) Main program designated Program Bellow.

(3) Subroutine CURVE

(4) Subroutine ACOURES

(5) Subroutine XLIFE

(6) End of record (EOR) card; multi-punch 7-8-9 in column 1.

(7) Data package containing one or more runs.

(8) End of file (EOF) card; multi-punch 6-7-8-9 in column 1.
PROGRAM BELLON(INPUT, OUTPUT, TAPE6=INPUT)
C THIS PROGRAM GENERATES A THEORETICAL PREDICTION OF THE NATURAL
C VELOCITIES WHICH PRODUCE FLOW-INDUCED VIBRATIONS (EXCITATION) OF
C FREQUENCIES FOR A GIVEN BELLONS INCLUDING THE FLUID FLOW
C READS THE BELLONS NATURAL LONGITUDINAL M ODES.
C
C INPUT
C IFLAG = 1(CALCULATE KA), 2(USE EXPERIMENTALLY DETERMINED KA), KA
C IS THE OVERALL BELLONS SPRING RATE, LB/IN
C NFLUID = 1(GAS), 2(LIQUID)
C NDEG = NUMBER OF BELLONS LONGITUDINAL DEGREES OF FREEDOM, 2+N/2-1
C JMAX = NUMBER OF CURVES NEEDED TO DESCRIBE A SURFACE
C NC = NUMBER OF BELLONS CONVOLUTE
C NPLT = NUMBER OF PLYS IN THE BELLONS CONVOLUTE
C SIGMA = CONVOLUTE WIDTH, IN.
C LAMBD = DISTANCE BETWEEN ADJACENT CONVOLUTE CROWNS, IN.
C M = PLANCHEIM HEIGHT, IN.
C T = THICKNESS PER CONVOLUTE PLY, IN.
C DI = BELLONS INSIDE DIAMETER, IN.
C DO = BELLONS OUTSIDE DIAMETER, IN.
C E = YOUNG'S MODULUS OF THE BELLONS MATERIAL, LBS/IN2
C RHO = HUGO DIAMETRAL MATERIAL, LB/CU IN.
C CE = DIMENSIONLESS ELASTIC NUMBER
C IF NFLUID = 1(GAS), THE PERFECT GAS EQUATION OF STATE IS USED FOR
C CALCULATING GAS DENSITY AT THE STATE DEFINED BY P AND TEMP.
C IT IS ASSUMED THAT THE GAS PROPERTIES ARE KNOWN AT A REFERENCE
C STATE DEFINED BY RHOFE, PREFERENCE, AND TREF.
C P = GAS PRESSURE, PSIG
C TEMP = GAS TEMPERATURE, DEG. F.
C PREFERENCE AND TREF = REFERENCE GAS STATE, PSIG AND DEG. F.
C RHOFE = GAS DENSITY AT REFERENCE STATE, LB/CU FT
C GAMMA = RATIO OF SPECIFIC HEATS FOR GAS
C IF NFLUID = 2(LIQUID), THE LIQUID DENSITY MUST BE KNOWN A PRIORI AT
C THE LIQUID STATE(P AND TEMP)
C P = LIQUID PRESSURE, PSIG
C TEMP = LIQUID TEMPERATURE, DEG. F.
C RHO = LIQUID DENSITY AT P AND TEMP, LB/CU FT
C MLMATERIAL Indicator(ELNCU 719) BELLONY 1=4,322155
C SUMA, SUMA, SUMA, STOR, STOR = CURVE FIT COEFFICIENTS FOR UPPER BOUND
C ON STRAUDUS NUMBER VS. LAMBDAT/SIGMA
C BELLONY, STROG, STROG, STM = SAME AS ABOVE EXCEPT LOWER BOUND
C STRAUDUS NUMBER VS SIGMA IS THE SAME AS ABOVE EXCEPT FOR OPTIMUM
C OF CRITICAL STRAUDUS NUMBER FOR BELLONS EXCITATION
C CFP, CFA, CFP, CFP = CURVE FIT COEFFICIENTS FOR VORTEX FORCE
C COEFFICIENT
C QK(J), QA(J), QG(J), WJ(J) = CURVE FIT COEFFICIENTS FOR THE DYNAMIC
C AMPLIFICATION FACTOR(C) SURFACE
C N = DIMENSIONLESS NATURAL FREQUENCY AS A FUNCTION
C OF HUGO NUMBER FOR NDEG BELLONS LONGITUDINAL
C DEGREES OF FREEDOM.
C
C X(I,ML) = TWO DIMENSIONAL MATRIX CONTAINING VALUES OF M IN
C CALCULATING LIFE CYCLES, ML IS MATERIAL INDICATOR.
C E(I,ML) = TWO DIMENSIONAL MATRIX CONTAINING VALUES OF S IN
C CALCULATING LIFE CYCLES, ML IS MATERIAL INDICATOR.
C SUBROUTINE LIFE CALCULATES THE NUMBER OF PREDICTED LIFE CYCLES
C GIVEN ALTERNATING STRESS(PSI), MEAN STRESS(PSI), M, AND B VALUES
C
C ORIGIAL PAGE 15
OF POOR QUALITY
C ***** DIMENSION FREQ(25), V(25, 3), SI(25), TFAIL(25)
000003 DIMENSION G1(10), GA(15), GD(10), GD(10)
000003 DIMENSION XNL(5, 5), XCS(5, 5)
000003 DIMENSION XL(5, 5), XR(5, 5)
000003 REAL MELOG(3), MECH(3), MEANL(3)
000003 REAL NC, NPL, AMIND, NPLI, NPLD, NPLD, NPLD, NMET, N, M, N
000003 REAL MEANST
000003 COMMON NPLY, SRR, NPLD, RHOF, JCRVE, SP, Q, CYCLE, RHE
000003 DATA NPL, NPLC, RRR, RRR, RRR, RRR
000003 DATA MECH(1), XCS(1), XCS(1)
000003 DATA MEANL(1), MEANL(1), MEANL(1)

000003 C *****
000003 1 READ 1000, TITLE
000003 IF (LEF, NO) 5, 10
000003 5 STOP
000003 10 READ 1000, JFL, FPL, NPLD, RRR, JMAX, JCFD, JLL
000003 20 READ 1001, NC, NPL, SIGMA, LAMB, M
000003 25 READ 1010, DI, DQ, E, RHOM, RACE
000003 30 GO TO (11, 12), NPLD
000003 11 READ 1010, P, TEM, NREF, TREF, RHE0, NRM
000003 12 GO TO 13
000003 13 READ 1010, P, TEM, RHE0
000003 14 READ 1000, JFL, FPL, NPLD, RRR, JMAX
000003 15 READ 1001, (XN(1, 1), XN(1, 1), XN(1, 1)
000003 20 READ 1002, (XN(1, 2), XN(1, 2), XN(1, 2)
000003 25 READ 1003, (XN(1, 3), XN(1, 3), XN(1, 3)
000003 30 READ 1004, (XN(1, 4), XN(1, 4), XN(1, 4)
000003 35 READ 1005, (XN(1, 5), XN(1, 5), XN(1, 5)

C ***** CALCULATION OF NATURAL FREQUENCIES AND EXCITATION VELOCITIES *****
000003 25 PI03, 1415927
000003 30 PI03, 1634736
000003 35 PI03, 1634736
000003 40 PI03, 1634736
000003 45 PI03, 1634736
000003 50 PI03, 1634736
000003 55 PI03, 1634736
000003 60 PI03, 1634736
000003 65 PI03, 1634736
000003 70 PI03, 1634736
000003 75 PI03, 1634736
000003 80 PI03, 1634736
000003 85 PI03, 1634736
000003 90 PI03, 1634736
000003 95 PI03, 1634736
000003 100 PI03, 1634736
000003 105 PI03, 1634736
000003 110 PI03, 1634736
000003 115 PI03, 1634736
000003 120 PI03, 1634736
000003 125 PI03, 1634736
000003 130 PI03, 1634736
000003 135 PI03, 1634736
000003 140 PI03, 1634736
000003 145 PI03, 1634736
000003 150 PI03, 1634736
000003 155 PI03, 1634736
000003 160 PI03, 1634736
000003 165 PI03, 1634736
000003 170 PI03, 1634736
000003 175 PI03, 1634736
000003 180 PI03, 1634736
000003 185 PI03, 1634736
000003 190 PI03, 1634736
000003 195 PI03, 1634736
000003 200 PI03, 1634736
000003 205 PI03, 1634736
000003 210 PI03, 1634736
000003 215 PI03, 1634736
000003 220 PI03, 1634736
000003 225 PI03, 1634736
000003 230 PI03, 1634736
000003 235 PI03, 1634736
000003 240 PI03, 1634736
000003 245 PI03, 1634736
000003 250 PI03, 1634736
000003 255 PI03, 1634736
000003 260 PI03, 1634736
000003 265 PI03, 1634736
000003 270 PI03, 1634736
000003 275 PI03, 1634736
000003 280 PI03, 1634736
000003 285 PI03, 1634736
000003 290 PI03, 1634736
000003 295 PI03, 1634736
000003 300 PI03, 1634736
000003 305 PI03, 1634736
000003 310 PI03, 1634736
000003 315 PI03, 1634736
000003 320 PI03, 1634736
000003 325 PI03, 1634736
000003 330 PI03, 1634736
000003 335 PI03, 1634736
000003 340 PI03, 1634736
000003 345 PI03, 1634736
000003 350 PI03, 1634736
000003 355 PI03, 1634736
000003 360 PI03, 1634736
000003 365 PI03, 1634736

ORIGINAL PAGE 15
(?) POOR QUALITY
001017 175 IF(FREQ(MODE),GE,FREQCO) Q=Q+ADJUST
001024 180 SI(MODE)=80P+Q
001027 AMODE=A+MODE+1.
001031 20C CONTINUE
001033 285 PRINT 2800, TITLE
001041 PRINT 1040, SIGMA, LAMBDA, M, 1, 0.0, NC, MPLY, E, K, RHOM, P, TEMP, RHOF, NF
001104 PRINT 1050
001111 PRINT 1040
001115 MEANSTR=0.0
001116 DO 44 MODE=1, NDEG
001120 ALTSTR=SI(MODE)/1000.
001122 DO 101 I=1, 5
001124 RAM(I)=RM(1,MTL)
001127 BM(I)=B(1,MTL)
001132 CONTINUE
001133 CALL XLIKE(RAM,BM,ALTSTR,MEANSTR)
001136 GO TO (140,130,140,210), MRET
001146 210 PRINT 1270, MODE, SI(MODE), FREQ(MODE), V(MODE,1), V(MODE,2), V(MODE,3),
001156 8 CYCLE
001170 GO TO 44
001171 130P PRINT 1300, MODE, SI(MODE), FREQ(MODE), V(MODE,1), V(MODE,2), V(MODE,3),
001180 MSEL=
001193 GO TO 44
001214 140P PRINT 1400, MODE, SI(MODE), FREQ(MODE), V(MODE,1), V(MODE,2), V(MODE,3),
001227 8 MESH=
001234 GO TO 44
001237 1410 PRINT 1415, MODE, SI(MODE), FREQ(MODE), V(MODE,1), V(MODE,2), V(MODE,3),
001241 MEANLG
001246 CONTINUE
001259 IF(FK, FLUID, EQ, 1) 1216.1
001270 215 PRINT 1200, FRECU, ADJUST
001300 GO TO 1
001301 1003 FORMAT(810)
001301 1010 FORMAT(8E1.6)
001301 1020 FORMAT(1DF7.3)
001301 1030 FORMAT(813)
001301 10-0 FORMAT(19D, 18E, BELLOWS PARAMETERS/
001301 19D) 19D, 19D, SIGMA(CONVOLVE WIDTH, IN), 19X, F6.3,
001301 19D) 19D, LAMBDA(CONVOLVE PITCH, IN), 19X, F6.3,
001301 19D) 19D, MEAN DIS HEIGHT, IN), 19X, F6.3,
001301 19D) 19D, CONVOLVE THICKNESS/PLY, IN), 19X, F6.3,
001301 19D) 19D, INSIDE DIAMETER, IN), 19X, F6.3,
001301 19D) 19D, OUTSIDE DIAMETER, IN), 19X, F6.3,
001301 19D) 19D, NUMBER OF CONVOLUTES), 19X, F7.3,
001301 19D) 19D, NUMBER OF PLIES), 19X, F7.3,
001301 19D) 19D, MODULUS, 19X, E11, 8,
001301 19D) 19D, OVERALL SPRING RATE, 19X, F6.3, 8,
001301 19D) 19D, MATERIAL DENSITY, LB/CU. IN), 19X, F7.3, 8,
001301 19D) 19D, FLUID PARAMETERS/
001301 19D) 19D, P(1, TEMP, DEG F), 19X, F7.3, 8,
001301 19D) 19D, TEMP(FLUID DENSITY, LB/CU. IN), 19X, E11, 4,
001301 19D) 19D, FLUID DENSITY, LB/CU. IN), 19X, E11, 4,
001301 19D) 19D, ALLOY (ENCO 316, 4130, 2014, 2024, 3032), 19X, E11, 4,
001301 19D) 19D, ALLOY (ENCO 316, 4130, 2014, 2024, 3032), 19X, E11, 4,
001301 19D) 19D, ALLOY (ENCO 316, 4130, 2014, 2024, 3032), 19X, E11, 4,
001301 1050 FORMAT(19D, 18M, THEORETICAL BELLOWS PERFORMANCE
001301 19D) 19D, ADJUST, NO. STRESS INDICATOR NATURAL FREQUENCY F
001301 SLO- EXCITATION RANGE, FT/SEC LIFE CYCLES,
<table>
<thead>
<tr>
<th>Hello</th>
<th>World</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>456</td>
</tr>
</tbody>
</table>

**Table:**

<table>
<thead>
<tr>
<th>Format Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10100</td>
<td>1000</td>
</tr>
<tr>
<td>10200</td>
<td>1000</td>
</tr>
<tr>
<td>10300</td>
<td>1000</td>
</tr>
</tbody>
</table>

---

**Message:**

OF POOR QUALITY
SUBROUTINE CURVE
DIMENSION DIFKREU(25),FREQ(25),X(25,3),3I(25)
DIMENSION D(1),H(1),G(1),U(1)
COMMON NPLY,SR,RFLUID,SPHOF,JCURVE,DOM,F,CYCLE,METAL,KAR
REAL NC,NCYCLE,MDA,RFLUID1,RFLUID2,RFLUID3,METAL,KAR
IF(NPLY,og.1.,110.10)
IF(NPLY,og.0.,90.20)
20 IF(SRM,0.3000.,30.40)
30 JCURVE#4
40 IF(2ND0.0.,LE.SSR,AND.SSR.LE.3000.)50,60
50 JCURVE#5
RETURN
60 IF(SMR.LE.2000.,AND.,NFLUID,og.1.)70,80
70 JCURVE#5
RETURN
80 JCURVE#5
RETURN
90 IF(3SMR,0.3000.,100.110.
100 JCU#E#3
RETURN
110 IF(NFLUID,og.1.)120,140
120 IF(2000.,LE.SSR,AND.,SSR,LE.3000.)130,140
130 JCURVE#4
RETURN
140 JCURVE#6
RETURN
150 IF(2000.,LE.SSR,AND.,SSR,LE.3000.)160,170
160 JCURVE#5
RETURN
170 JCURVE#6
RETURN
180 IF(NFLUID,og.1.,AND.,PL.1.,150.)190,200
190 JCURVE#1
RETURN
200 IF(SRM,0.2000.,210,220
210 GO TO (220,230),NFLUID
220 JCURVE#1
RETURN
230 SPCRAREM0F/(62.9/1728.)
240 IF(SPMAR,LY,0.202.250
250 JCURVE#1
RETURN
260 RETURN
270 JCURVE#2
RETURN
280 GO TO (290,300),NFLUID
290 JCURVE#2
RETURN
300 RETURN
310 SPCRAVEM0F/(62.9/1728.)
320 IF(SPMAR,LY,0.202.300
330 JCURVE#2
RETURN
340 JCURVE#3
RETURN
350 RETURN
360 ENU
SUBROUTINE ACOURES(X,Y,Z,FRECO,GADJUST)

PI=3.1415927

M1=M1*X

C0=C0/Z

IF(M1.LE.0.5) GO TO 20

10 FRECO=FRECO-1.222*M1

GO TO 30

20 FRECO=FRECO+1.222*M1

30 FRECO=FRECO/(2.*PI*PI)

GADJUST=G

RETURN

END
SUBROUTINE XLIFE(XM,B,ALTSTR,MEANSTR)
DIMENSION XM(5),B(5)
COMMON NPLY,SSR,NFLUID,P,RHOF,JCURVE,SOPO,CYCLE,HRET

REAL MEANSTR

I=1
IF(MEANSTR.LE.20.)GO TO 200
I=I+1
IF(MEANSTR.LE.40.)GO TO 200
I=I+1
IF(MEANSTR.LE.60.)GO TO 200
I=I+1
IF(MEANSTR.LE.80.)GO TO 200
HRET=I
RETURN

200 CYCLE=CYCLE+ALTSTR**XM(I)
CYC2=B(I+1)*ALTSTR**X2(I+1)
FRAC=(MEANSTR-20.)*(I-1)/20.0
CYCLE=CYCLE+CYC2=CYCLE
FRAC
IF(CYCLE.LE.1000.)GO TO 200,300
HRET=2
RETURN

300 IF(CYCLE.GE.1000000.)GO TO 400,500
HRET=3
RETURN

400 RETURN

500 RETURN

END
A.5 Data Input Package

Instructions for preparation of a data input package are located at the beginning of the PROGRAM BELOW listing. An experienced programmer will have no difficulty in constructing the input, but for the inexperienced user the following supplementary remarks may be useful.

Input Card 1

This card is an identification card on which the user can place information that will aid in identifying and classifying the run. Any alpha-numeric characters can be placed in columns 2 through 80. Column 1 must either contain a 1 for printer carriage control or be left blank.

Input Card 2

<table>
<thead>
<tr>
<th>Word 1</th>
<th>Word 2</th>
<th>Word 3</th>
<th>Word 4</th>
<th>Word 5</th>
<th>Word 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>JFLAG</td>
<td>NFLUID</td>
<td>NDEG</td>
<td>JMAX</td>
<td>JCFQ = 1</td>
<td>MTL</td>
</tr>
</tbody>
</table>

- See Program Listing
- See Program Listing
- Is the number of individual curves necessary to describe the Q-surface (Figure A-4). As shown in that figure, JMAX = 6. If future data indicate that more than six curves are necessary, then the dimension statement pertaining to Q must be altered accordingly.

Word 5 JCFQ = 1 (Use Method I Stress Indicator Calculation)

Word 6 MTL - See Program Listing

Input Card 3

<table>
<thead>
<tr>
<th>Word 1</th>
<th>Word 2</th>
<th>Word 3</th>
<th>Word 4</th>
<th>Word 5</th>
<th>Word 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>NPLY</td>
<td>SIGMA</td>
<td>LAMBDA</td>
<td>H</td>
<td>T</td>
</tr>
</tbody>
</table>

- See Program Listing
- See Program Listing
- See Program Listing
- See Program Listing

Input Card 4

<table>
<thead>
<tr>
<th>Word 1</th>
<th>Word 2</th>
<th>Word 3</th>
<th>Word 4</th>
<th>Word 5</th>
<th>Word 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>DI</td>
<td>DO</td>
<td>E</td>
<td>RHOM</td>
<td>KA</td>
<td>CE = 1.0</td>
</tr>
</tbody>
</table>

- See Program Listing
- See Program Listing
- See Program Listing
- See Program Listing

May be left blank if JCFQ = 1

Input Card 5

<table>
<thead>
<tr>
<th>Word 1</th>
<th>Word 2</th>
<th>Word 3</th>
<th>Word 4</th>
<th>Word 5</th>
<th>Word 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>TEMP</td>
<td>PREF or RHOF</td>
<td>TREF</td>
<td>RHOFREF</td>
<td>GAMMA</td>
</tr>
</tbody>
</table>

- See Program Listing
- See Program Listing
- See Program Listing
- See Program Listing

- See Program Listing

- See Program Listing
Input Card 6

This card contains 4 curve fit coefficients for the upper bound of the Strouhal number vs. lambda/sigma function. They are as follows:

Word 1  STUPK  =  +.2535226+00
Word 2  STUFA  =  +.40487805+00
Word 3  STUPB  =  +.2229595+00
Word 5  STUPD  =  -.34329268-01

Input Card 7

This card contains 4 curve fit coefficients for the lower bound of the Strouhal number vs. lambda/sigma function. They are as follows:

Word 1  STLOK  =  +.11870422+00
Word 2  STLOA  =  +.46569343+00
Word 3  STLOB  =  +.73139166-01
Word 4  STLOD  =  -.79927007-02

Input Card 8

This card contains 4 curve fit coefficients for the critical curve of the Strouhal number vs. lambda/sigma function. They are as follows:

Word 1  STCRITK  =  +.43502697+00
Word 2  STCRITA  =  -.6870504-01
Word 3  STCRITB  =  +.37269292-02
Word 4  STCRITD  =  +.40647482-02

Input Card 9

This card contains 4 curve fit coefficients for the vortex force coefficient vs. lambda/sigma function. They are as follows:

Word 1  CFK  =  -.19458000+03
Word 2  CFA  =  +.25500000+02
Word 3  CFB  =  -.74460000+01
Word 4  CFD  =  -.39900000+00

Input Cards 10 through 15

The curve fit coefficients for the Q-surface are read in at a rate of four words per card, i.e., QK (1), QA (1), QB (1), QD (1) are present on Card 10; QK (2), QA (2), QB (2), QD (2) are on Card 11 of this. Reading continues per this format until JMAX sets of coefficients below have been read in.
| Card 10 | QK(1) | = 4.087388E+04 |
| QA(1) | = 1.405253E+02 |
| .1 | = 3.741973E+01 |
| QD(1) | = 2.257494E-03 |

| Card 11 | QK(2) | = 3.398047E+04 |
| QA(2) | = 1.749869E+02 |
| QB(2) | = 3.878355E+01 |
| QD(2) | = 2.703427E-03 |

| Card 12 | QK(3) | = 2.008199E+04 |
| QA(3) | = 1.491777E+02 |
| QB(3) | = 4.539384E+01 |
| QD(3) | = 4.868938E-03 |

| Card 13 | QK(4) | = 9.879988E+03 |
| QA(4) | = 1.248987E+02 |
| QB(4) | = 4.995059E+01 |
| QD(4) | = 6.800111E-03 |

| Card 14 | QK(5) | = 7.826471E+03 |
| QA(5) | = 2.068204E+02 |
| QB(5) | = 4.357605E+01 |
| QD(5) | = 4.061292E-03 |

| Card 15 | QK(6) | = 2.350669E+04 |
| QA(6) | = 8.443207E+02 |
| QB(6) | = 2.477333E+01 |
| QD(6) | = 1.481069E-03 |

**Input Card 16**

This card contains exponent values (M) for material 1

| Word 1 XM (1,1) | = 5.11 (mean stress = 0 KSI) |
| Word 2 XM (2,1) | = 5.479 ( " " = 20 " ) |
| Word 3 XM (3,1) | = 5.519 ( " " = 40 " ) |
| Word 4 XM (4,1) | = 5.645 ( " " = 60 " ) |
| Word 5 XM (5,1) | = 5.972 ( " " = 80 " ) |

**Input Card 17**

This card contains coefficient values (B) for material 1

| Word 1 B (1,1) | = +0.21410+16 (mean stress = 0 KSI) |
| Word 2 B (2,1) | = +0.72280+16 ( " " = 20 " ) |
| Word 3 B (3,1) | = +0.44440+16 ( " " = 40 " ) |
| Word 4 B (4,1) | = +0.24367+16 ( " " = 60 " ) |
| Word 5 B (5,1) | = +0.3200+16 ( " " = 80 " ) |
Input Card 18

This card contains exponent values (M) for material 2

Word 1 XM (1,2) = -13.003
Word 2 XM (2,2) = -13.170
Word 3 XM (3,2) = -16.008
Word 4 XM (4,2) = -14.168
Word 5 XM (5,2) = -5.345

Input Card 19

This card contains coefficient values (B) for material 2

Word 1 B (1,2) = +0.67770+27
Word 2 B (2,2) = +0.32560+28
Word 3 B (3,2) = +0.29480+32
Word 4 B (4,2) = +0.49910+27
Word 5 B (5,2) = +0.89980+11

Input Card 20

This card contains exponent values (M) for material 3

Word 1 XM (1,3) = -2.447
Word 2 XM (2,3) = -3.567
Word 3 XM (3,3) = -4.387
Word 4 XM (4,3) = -4.683
Word 5 XM (5,3) = -6.124

Input Card 21

This card contains coefficient values (B) for material 3

Word 1 B (1,3) = +0.14360+10
Word 2 B (2,3) = +0.13630+12
Word 3 B (3,3) = +0.22900+13
Word 4 B (4,3) = +0.12990+13
Word 5 B (5,3) = +0.12510+13

A.6 Example Problem

Listed below is an input data deck constructed in accordance with the instructions presented at the beginning of PROGRAM BELLOW. The notations that appear in columns 73 through 80 serve to identify the data group in each card. Following this listing is the corresponding computer output. The output is grouped into three sections. The first group summarizes the pertinent bellows input parameters. For this example, only the overall spring rate, \( KA \), was inserted as data. The next group summarizes the fluid parameters. The next group contains the predicted longitudinal bellows performance. Bellows lock-in-range for a particular mode of vibration is defined by the upper and lower flow velocities. Stress indicator was calculated based on
the critical flow velocity for each mode. Note, that for this particular bellows configuration, the lock-in-ranges for successive modes overlap, which indicates a more or less continuous spectrum of excitation velocities. Note also that all performance variables at the highest mode numbers are less than the corresponding quantities at previous mode numbers. Physically this behavior is accounted for in the fact that the apparent fluid mass is increasing at a faster rate than the dimensionless frequency numbers in Table A-I for this bellows.
### Stainless Steel Heliums, SWR No. 15 at 27F and 10 PSIG

**Helium Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma \text{(Convolute Width, IN)} )</td>
<td>.120</td>
</tr>
<tr>
<td>( \Lambda \text{(Convolute Pitch, IN)} )</td>
<td>.250</td>
</tr>
<tr>
<td>( H \text{(Mean Disc Height, IN)} )</td>
<td>.300</td>
</tr>
<tr>
<td>( t \text{(Convolute Thickness/PLY, IN)} )</td>
<td>.006</td>
</tr>
<tr>
<td>( D \text{(Inside Diameter, IN)} )</td>
<td>3.000</td>
</tr>
<tr>
<td>( Do \text{(Outside Diameter, IN)} )</td>
<td>3.600</td>
</tr>
<tr>
<td>( NC \text{(Number of Convoluting)} )</td>
<td>13.000</td>
</tr>
<tr>
<td>( NP \text{(Number of Plies)} )</td>
<td>2.000</td>
</tr>
<tr>
<td>( E \text{(Young's Modulus, Lb/In2)} )</td>
<td>2.800E+07</td>
</tr>
<tr>
<td>( K \text{(Overall Spring Rate, Lb/IN)} )</td>
<td>96.000</td>
</tr>
<tr>
<td>( R \text{(Material Density, Lb/Cu.In)} )</td>
<td>.280</td>
</tr>
</tbody>
</table>

**Fluid Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P \text{(Pressure, PSIG)} )</td>
<td>10.000</td>
</tr>
<tr>
<td>( T \text{(Temperature, Deg F)} )</td>
<td>70.000</td>
</tr>
<tr>
<td>( \rho \text{(Fluid Density, Lb/Cu.In)} )</td>
<td>3.4093E+02</td>
</tr>
<tr>
<td>( N \text{(Fluid) Liquid} )</td>
<td>2</td>
</tr>
<tr>
<td>( M \text{(1=Gas, 2=Alloy 21-6-4, 3=316 SS)} )</td>
<td>3</td>
</tr>
</tbody>
</table>

### Theoretical Heliums Performance

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>5.49E+02</td>
<td>170.191</td>
<td>4.931</td>
<td>6.581</td>
<td>10.877</td>
<td>ABOVE 10E+03</td>
</tr>
<tr>
<td>2</td>
<td>2.44E+01</td>
<td>270.365</td>
<td>8.511</td>
<td>11.925</td>
<td>14.740</td>
<td>ABOVE 10E+07</td>
</tr>
<tr>
<td>3</td>
<td>5.13E+01</td>
<td>905.912</td>
<td>12.511</td>
<td>17.529</td>
<td>24.099</td>
<td>ABOVE 10E+07</td>
</tr>
<tr>
<td>4</td>
<td>8.00E+01</td>
<td>524.335</td>
<td>16.337</td>
<td>22.089</td>
<td>37.931</td>
<td>7.71E+06</td>
</tr>
<tr>
<td>5</td>
<td>1.22E+02</td>
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<td>19.485</td>
<td>26.000</td>
<td>44.402</td>
<td>3.13E+10</td>
</tr>
<tr>
<td>6</td>
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<td>23.353</td>
<td>32.859</td>
<td>54.153</td>
<td>1.54E+10</td>
</tr>
<tr>
<td>7</td>
<td>2.03E+02</td>
<td>846.780</td>
<td>26.736</td>
<td>37.940</td>
<td>62.077</td>
<td>4.92E+05</td>
</tr>
<tr>
<td>8</td>
<td>2.47E+02</td>
<td>945.189</td>
<td>29.837</td>
<td>43.799</td>
<td>64.269</td>
<td>5.61E+05</td>
</tr>
<tr>
<td>9</td>
<td>2.94E+02</td>
<td>1061.156</td>
<td>32.791</td>
<td>45.873</td>
<td>76.014</td>
<td>3.47E+05</td>
</tr>
<tr>
<td>10</td>
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<td>98.054</td>
<td>1.27E+05</td>
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<td>44.136</td>
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<td>102.491</td>
<td>1.05E+05</td>
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<td>5.28E+02</td>
<td>1751.057</td>
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<td>107.002</td>
<td>8.45E+04</td>
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<td>1866.567</td>
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<td>110.597</td>
<td>7.79E+04</td>
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<td>48.038</td>
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<td>6.29E+04</td>
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<td>5.88E+04</td>
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<td>73.208</td>
<td>121.310</td>
<td>5.17E+04</td>
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<td>122.102</td>
<td>5.02E+04</td>
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<td>23</td>
<td>6.65E+02</td>
<td>2746.849</td>
<td>55.636</td>
<td>74.907</td>
<td>122.944</td>
<td>4.96E+04</td>
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<td>24</td>
<td>6.69E+02</td>
<td>2869.479</td>
<td>56.419</td>
<td>75.672</td>
<td>123.708</td>
<td>4.94E+04</td>
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<td>25</td>
<td>6.74E+02</td>
<td>3001.014</td>
<td>57.096</td>
<td>76.319</td>
<td>124.380</td>
<td>5.07E+04</td>
</tr>
</tbody>
</table>

**Table A-I**
APPENDIX B

EXPERIMENTAL FACILITY
B.1 Flow Loop

All liquid flow tests were conducted in a closed loop water flow tunnel shown schematically in Figure B-1. The bellows upstream piping was sized to the nominal bellows size, i.e., 3" PVC pipe was used during 3" bellows test and 6" PVC was used for 6" bellows. The flow loop can be pressurized to pressures in excess of 100 psig.

Flow rate was accurately measured by a 4" turbine meter (Flow Technology SN - 64033), and the loop's static pressure was determined by a calibrated bordon pressure gauge located one diameter upstream of the bellows.

Fluid motion is generated by a Goulds propeller pump rated at 40 ft. head at 6000 GPM. The prime mover is a 75 hp variable speed hydraulic motor which provides a means to vary the loop flow velocity. Piping components are fabricated of carbon steel or PVC. A large antisurge reservoir (air over water) has been incorporated into the basic tunnel design.

B.2 Bellows Instrumentation

During a typical bellows flow test, three time dependent variables are normally recorded. These include the (1) volumetric flow rate, (2) the strain time history at various bellows locations, and (3) the displacement time history of selected bellows convolutes.

The overall instrumentation setup is shown in Figure B-2 where it can be seen that three modes of recording data are possible. For quick look information polaroid pictures of the scope face may be obtained. As a second mode of operation, a high speed direct write galvonometer (CEC Model 5-124) is used to obtain high frequency hard copy bellows strain and displacement time histories; however, the majority of data (3rd mode of operation) was recorded in a form more useable for analysis, i.e., a dependent variable was plotted versus an independent variable on the x-y plotter while a test was in progress.

Typical data collected in the form of two dimensional plots are presented in Figure 13. The vertical scale is proportional to either peak to peak strain amplitude or peak to peak displacement amplitude. Special circuitry, to be described subsequently, converted convolute peak to peak displacement motions to an equivalent D.C. analog voltage which was input to the y-axis of a model x-y recorder. Peak to peak strain signals (radial and circumferential) were processed in a similar fashion. The horizontal axis is proportional to volumetric flow rate through the bellows. Since the primary flow measurement element was a turbine meter, its output frequency (directly proportional to the volume rate) was converted to a D.C. signal and then input to the recorder's x-axis.

A typical instrumented bellows is shown in Figure B-3. Four strain gages were attached to the convolute crowns each test bellows. Two gages were placed on convolute number two, one responded to radial strains and the other responded to circumferential strain. Convolute number two was chosen as a representative and convolute where peak strains occur (maximum relative displacement occurs in this region) but due to the end restraint. The middle convolute was gaged in the same manner as convolute number two. By observing the middle convolute
FIGURE B-1. WATER FLOW TUNNEL
FIGURE B-2. INSTRUMENTATION
<table>
<thead>
<tr>
<th>Gage No.</th>
<th>Gage Type</th>
<th>Strain Direction</th>
<th>Convolute No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EA-09-031ED-120</td>
<td>Radial</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>EA-09-031ED-120</td>
<td>Radial</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>EA-06-031DE-120</td>
<td>Circumferential</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>EA-06-031DE-120</td>
<td>Circumferential</td>
<td>2</td>
</tr>
</tbody>
</table>

FIGURE B-3. STRAIN GAGE AND TAB LOCATION FOR 3" BELLows
response simultaneously with the second convolute, the mode number is positively identified and insight is gained with respect to the mode shape.

All strain gages used were 1/32" long and each was selected to the base material of the bellows (321 stainless steel). Due to the small size of the gage and its associated installation difficulty, single arm active bridge circuits were employed. Figure B-4 shows a schematic of the signal conditioning circuit that converts gage resistance changes into a measurable voltage. The first stage amplifier (Analog Devices 610) is a high quality instrumentation amplifier operated in a differential voltage measurement mode. The second stage amplifier (Analog Devices 3140) provides offset voltage control and boosts the 610's output by a fixed gain of 10.

Consolute displacement was obtained by measuring the displacement of a small metal tab that was epoxied to the crown of a convolute. A Bently probe (Model 316) was attached to a fixed structure above the test bellows. The tab couples with the transducer to produce an analog signal directly proportional to the displacement of the tab with respect to the transducer face; hence, the convolute absolute displacement was recorded. A sufficient number of tests were performed to insure that the virtually massless attached tab did not influence the vibration process.
APPENDIX C

BELLOWS GEOMETRIC AND MECHANICAL PROPERTIES DATA
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bellows No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>$D_n$ (in)</td>
<td>3.3</td>
</tr>
<tr>
<td>$D_i$ (in)</td>
<td>3.0</td>
</tr>
<tr>
<td>$D_o$ (in)</td>
<td>3.6</td>
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<tr>
<td>$h$ (in)</td>
<td>.3</td>
</tr>
<tr>
<td>$t$ (in)</td>
<td>.006</td>
</tr>
<tr>
<td>$N_p$</td>
<td>1</td>
</tr>
<tr>
<td>$N_c$</td>
<td>13</td>
</tr>
<tr>
<td>$\lambda$ (in)</td>
<td>.228</td>
</tr>
<tr>
<td>$\sigma$ (in)</td>
<td>.12</td>
</tr>
<tr>
<td>$\lambda/\sigma$</td>
<td>1.9</td>
</tr>
<tr>
<td>$K_A$ (lb/in)</td>
<td>44.2</td>
</tr>
<tr>
<td>$du_e/dfc$ (min/lb)</td>
<td>94.96</td>
</tr>
<tr>
<td>$du_e/dl$ (min/in)</td>
<td>4197</td>
</tr>
</tbody>
</table>
APPENDIX D

FATIGUE LIFE COMPUTATION PROGRAM, FATLIF
PROGRAM FATLEF (INPUT, OUTPUT, TAP1, INPUT, TAP2, OUTPUT)

DIMENSION N(10), V(10)

REAL NPL, M, LAMDA, MODENO, KA, NC, NONC

N=10

WRITE(NW, 1)

READ(NW, 1000)

READ(NR, 1010) N, M, NPL, TPL, F, LAMDA

READ(NW, 1010) NC, MODENO, KA, GT, G

READ(NR, 1010) C, M, LAMDA, MODENX, AT, AC

READ(NR, 1015) MC, NC

READ(NR, 1011) V, (I), T(I), TMAX

READ(NR, 1012) (N(I), T(I), TMAX)

WRITE(NW, 1000)

WRITE(NW, 1020)

G=.32, 12

PI=3.1415927

NPL=ATPL

STEP=(H/T)*2/3

DO 1010 T=1, TMAX

MODENO=M

NONC=MODENO/NC

CMX=(NONC+TNS*(PI/NC), T)/(.8, MODENO)

IF (NONC LE 0.075) GO TO 2

IF (NONC LE 0.100) GO TO 9

IF (NONC LE 0.3) GO TO 6

IF (NONC GT 0.3) GO TO 10

2 CFATARA1.0

GO TO 12

4 CFATARA0.04754354*NONC=(0.139054)

GO TO 12

6 CFATARA0.0259256*NONC=(0.48502)

GO TO 12

8 CFATARA0.0253149*NONC=(0.22127)

GO TO 12

10 CFATARA0.0533572*NONC=(0.51917)

12 CFATARA0.0533572*NONC=0.51917

APRHA=(0.04754354/2)

DELONC=CH2900E6APCFO(V(T)+12.)*2/(2.012.0GA/KA)

DELSTG1=SAP+DELONC/SORT(LAMDA+HAX3)

SIGMAX=SIGP+DELSTG/2

SIGMIN=SIGP-DELSTG/2

RESIGN=SIGMAX

COEFF1((I), C)=(1., R)/(DELSTG=(E-3))**N

C FATIGUE LIFE DEFINITE INTEGRAL

A=AC=1/20

ALPHA

A=ALPHA

PZ1=((ALPHA+1.)*2*EPSILON)+ALPHA*(H/2.3)

25 PZ2=(((ALPHA+2.)*EPSILON)+ALPHA*(H/2.3))

APFARAP=APR2/(PZ1+2)

FL=FL

F=AFA

IF(AF=AC)=5.25, 0

GO FATLEF, COEFF1, AREAF
WRITE(6,1010)NONC,CM,CF,STAR,CFG,DELONC,COEFF
100 WRITE(6,1025)IN(1),Y(1),SIGP,DELSIG,SIGMAX,SIGNIN,AREA,H,FAILIFE
1000 FORMAT(AM)

1010 FORMAT(6E12.4)
1015 FORMAT(13I)
1020 FORMAT(14H,10M,MODE,NO,2X,LM(HFPS),2X,9HMSIG(Psi),2X,11HDELSIG)P

13I,3X,11HSIGMAX(Psi),3X,11HSIGNMIN(Psi),4X,SHAERA,11X,1M,2X,1HMPA

2TIGUE CYCLES)
1025 FORMAT(5X,13X,FB,1X,EX,2X,EB,0.6X,FX,0.6X,EK,0.6X,FK,0.6X,E10.4,5X)

STOP
END
APPENDIX E

TWO PHASE FLOW STUDY
E.1 Two Phase Flow

During the performance period of the bellows study, several special studies were conducted on an as needed basis. One particularly noteworthy study conducted was a simplified analysis of the Shuttle LH₂ chilldown or recirculation system. Four possible operating conditions of the chilldown system were assumed and the analysis of the chosen "worst case," indicates low probability of a bellows failure due to a two phase flow phenomena. Results are presented below.

E.2 Case A - Pure Liquid Flow

For this case the entire recirculation system was assumed to be flowing pure liquid hydrogen with the pump curve shown in Figure E.1 defining the pressure head versus flow for each of the three pumps. Table E.I lists the assumed bellows geometry for this analysis. Table E.II lists the LH₂ properties and analysis results for the Case A pure liquid flow problem. As shown, because of the very low velocity and $1/2 \rho V^2$ valve, the stress indicator valve is quite low and no bellows flow-induced vibration problem is anticipated.

E.3 Case B - Pure Gas Flow

For this case we assume pure liquid flow through the pump followed by pure gaseous flow through the recirculation system. The reason for this assumption is to ensure the maximum possible driving head at the pump is available to "push" the gas through the lines. With gaseous flow through the pumps, a very low head would occur hence no means would exist to continue to introduce liquid into the system.

It is assumed that sufficient heat is transferred into the liquid to cause complete boiling hence a pure gaseous flow through the recirculation lines. This is definitely a possibility at the first stage of chilldown.
FIGURE E-1. RE Circulation Pump Pressure Flow Characteristic

Estimated System Characteristic
Liquid Flow in Pumps, Gaseous
Remainder

This Point Used
for Pure Liquid
Case A

Nominal

Total System Characteristic
Liquid Flow Only

Δp - psi

Flow Rate - lb/sec

1.0 1.5

1.0 1.5

0 0.5 1.0 1.5 2.0

0 0.5 1.0 1.5 2.0

0 0.5 1.0 1.5 2.0
TABLE E-I Summary Of Bellows Data For Case A

Bellows Geometry (Arrowhead Drawing 13619)

- Material - ARMCO 21-6-9
- O.D. = 5.0 inches
- I.D. = 4.0 inches
- \( N_C \) = 8 convolutes
- \( N_P \) = 2 plys
- \( t \) = 0.008 inches per ply
- \( \lambda_C \) = 2.0 inches convoluted length
- \( \lambda \) = 0.50 inches
- \( \sigma \) = 0.267
- \( \sigma \) = 0.134

Calculated Data

- \( K_A \) = 138.24 lb/inch overall spring rate
- \( M_m \) = \( 1.002 \times 10^{-4} \) lb-sec\(^2\)/in\(^4\)
- \( f_0 \) = 748 Hz, reference frequency
- \( f_1 \) = 148.9 Hz, first mode frequency
- \( V_1 \) = 7.56 fps, first mode critical velocity
- \( f_{15} \) = 1488 Hz, highest longitudinal mode frequency
- \( V_{15} \) = 75.5 fps, highest longitudinal mode critical velocity
TABLE E-II  Summary Of LH₂ Properties And Analysis Results
For Case A - Pure Liquid Flow

Liquid Hydrogen Properties And Conditions

1. LH₂ @ - 420°F, 16.1 psig
2. \( \rho_f = 0.002564 \text{ lbm/in}^3 = 4.431 \text{ lbm/ft}^3 \)
3. \( \dot{\omega} = 4.2 \text{ lb/sec total flow, 3 pumps} \)

Calculated Data

1. Volume flow = \( \frac{\dot{\omega}}{\rho_f} = 0.9479 \text{ ft}^3/\text{sec} \)
2. \( v = \frac{\text{Volume flow}}{\text{Area}} = \frac{0.9479}{0.08722} = 10.87 \text{ fps} \)
3. \( 1/2 \rho_f v^2 = 0.0565 \text{ psi} \)
4. \( C_f Q = 8 \text{ (first mode)} \)
5. \( \text{S.I.} = \left( \frac{C_f Q}{N \rho} \right) (\frac{\text{h}}{\text{f}})^2 (1/2 \rho v^2) = 882.8 \text{ psi} \)

Conclusions

1. The Stress Indicator is so low no significant bellows response is possible.
Pure gaseous flow at the 4.2 lb/sec rate achieved for the pure liquid case is not possible because the flow loss would far exceed the available head at the pumps. Therefore, a downward adjustment in flow occurs until the loss matches the available pump head. The total flow from three pumps which satisfies this requirement is about 0.823 lb/sec; see Figure E-I.

Based on this flow, the bellows of Table E-I has been analyzed and results are shown in Table E.III. As shown, the stress indicator is quite low and there is no possibility of acoustic resonance, hence the bellows is safe.

E.4 Case C - Liquid Flow for Part of Line, Gaseous Flow for Rest

For this case we assume pure liquid flow through the pumps and through a fraction of the total recirculation line length. The flow through the remainder of the recirculation line is assumed to be pure gaseous. The transition from liquid to gas is assumed to occur suddenly at a single point in the line.

As with Case B, the total pressure loss along the line is assumed equal to the head available from the pump. When the percentage of line with gaseous flow is large, we expect the mass flow to be smaller than the nominal 4.2 lb/sec value and the total head greater than the nominal 8.0 psi value. As a starting point we assume the presence of the gas will restrict the flow so that the pump is operating in the region of a 12 psi head value. Other assumptions are:

- The liquid density is always \( \rho_c = 4.431 \text{ lb/ft}^3 \)
- The gas density is always \( \rho_g = 0.0939 \text{ lb/ft}^3 \)
- The pump head of 12 psi produces an average \( \frac{1}{2} \rho V^2 \) of 0.0897 psi along the line
- \( X \) is the percentage of the total line over which the flow is pure liquid
- \( (1-X) \) is the percentage of the total line length over which the flow is pure gaseous
- There is sufficient heat transfer to convert the \( \text{LH}_2 \) to \( \text{GH}_2 \) at the point \( X \)
TABLE E-III. Summary Of \( \text{GH}_2 \) Properties And Analysis Results
For Case B - Pure Gaseous Flow

<table>
<thead>
<tr>
<th>Gaseous Hydrogen Properties And Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{GH}_2 @ - 422^\circ F, 19.0 \text{ psia} )</td>
</tr>
<tr>
<td>( \rho_g = 0.00005435 \text{ lbm/in}^3 = 0.09392 \text{ lbm/ft}^3 )</td>
</tr>
<tr>
<td>( \dot{\omega} = 0.823 \text{ lb/sec total flow, 3 pumps} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Volume flow} = \frac{\dot{\omega}}{\rho_g} = 8.763 \text{ ft}^3/\text{sec} )</td>
</tr>
<tr>
<td>( V = 100.5 \text{ fps (could excite highest mode)} )</td>
</tr>
<tr>
<td>( 1/2 \rho V^2 = 0.1023 \text{ psi} )</td>
</tr>
<tr>
<td>( C_f Q = 3.2 ) (highest mode)</td>
</tr>
<tr>
<td>( \text{S.I.} = \frac{C_f Q}{N_p} \left( \frac{h}{E} \right)^2 \left( 1/2 \rho V^2 \right) = 629.5 \text{ psi} )</td>
</tr>
<tr>
<td>( \text{Velocity required for acoustic resonance} = 394 \text{ fps} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Stress Indicator too low for problem. No acoustic resonance possible. Bellows safe.} )</td>
</tr>
</tbody>
</table>
We now have

\[ \left( \frac{1}{2} \rho_L v_L^2 \right) X + \left( \frac{1}{2} \rho_g v_g^2 \right) (1-X) = 0.0897 \text{ psi} \]

From fluid continuity we find that

\[ \left( \frac{1}{2} \rho_g v_g^2 \right) = 47.0 \left( \frac{1}{2} \rho_L v_L^2 \right) \]

thus

\[ \frac{\left(1/2 \rho_g v_g^2\right)}{47.0} X + \left( \frac{1}{2} \rho_g v_g^2 \right) (1-X) = 0.0897 \text{ psi} \]

or

\[ \left( \frac{1}{2} \rho_g v_g^2 \right) (1-0.0970 X) = 0.0897 \text{ psi} \]

From the above equation we find that a given value of X we have a unique value of \((1/2 \rho_g v_g^2)\) or \(v_g\). Figure E-2 shows a plot of \(v_g^2\) versus X from the above equation.

Note that as X increases toward a value of 1.0, the \(v_g\) value also increases. For example, if there is liquid flow over 90% of the recirculation line, with the final 10% being gaseous flow, we can expect to have \(v_g = 272 \text{ fps}\) from the gaseous flow over the final 10% of the line.

Figure E-2 also shows the stress indicator values for the bellows defined in Table E-1. Of course there must be a bellows located in the portion of the line over which the gaseous flow exists to experience this flow condition.

From this analysis we can see that if the flow conditions assumed were to really exist then a bellows placed very near the end of the recirculation line could be subject to rather high stresses. Also we are getting into gaseous velocity ranges where acoustic resonances might be possible. Table E-IV summarizes the results of this analysis.

The question remaining then is: Can such a flow condition occur? The answer to this question depends on the results of a heat transfer analysis to find out if sufficient heat can be introduced into the fluid to produce the required phase change from liquid to gas.
FIGURE E-2. RESULTS OF CASE C ANALYSIS
TABLE E-IV  Case C - Portion Of Line Pure Liquid Flow
And Portion Pure Gaseous Flow

Hydrogen Properties And Conditions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LH₂ @</td>
<td>-420°F, 16.1 psig</td>
<td></td>
</tr>
<tr>
<td>ρₗ</td>
<td>4.431 lbm/ft³</td>
<td></td>
</tr>
<tr>
<td>GH₂</td>
<td>-422°F, 19.0 psia</td>
<td></td>
</tr>
<tr>
<td>ρₔ</td>
<td>0.09392 lbm/ft³</td>
<td></td>
</tr>
<tr>
<td>Fluid mass flow variable</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analysis

- Pure liquid flow over X percent of line length
- Pure gaseous flow over (1-X) percent of length
- Pump head always 12 psi, average line 1/2 ρV² = 0.0897 psi.
- X and flow head related by

\[
(\frac{1}{2}\rho_g V_g^2)(1-0.979X) = 0.0897 \text{ psi}
\]
- Solution to above given in Figure 2
- For example, if X = .90 or 90%, then

\[
\frac{1}{2}\rho_g V_g^2 = 0.754 \text{ psi}
\]

\[V_g = 272.6 \text{ fps}\]

\[S.I. = 4713 \text{ psi (SAFE)}\]
- Acoustic resonance occurs @ \(V_g = 394 \text{ fps}\)

Conclusions

- Only a bellows located at end of line would be in possible damage if the postulated flow condition can actually occur.
This question will be answered in the next section; however, if no bellows exists over the final 10% of the line length then no problem exists.

E.5  Case D - Slug Flow:

For this case we assume that a pocket of gas has formed in the recirculation line and is growing because of further boiling of LH₂. This gas product growth pushes the LH₂ in front of it out of the line; hence, we need to determine if the liquid and/or gas velocities can become high enough to create a bellows problem.

Figure E-3 shows a schematic diagram of the physical problem for Case D. As shown, we assume a gas pocket of length Y which is growing because of boiling caused by heat transfer through the tube from the surroundings. The rear boundary of the gas pocket is assumed moving at a velocity \( V_1 \), while the front boundary is assumed moving at a velocity \( V_2 \). The difference in velocity of the two boundaries relates to the volume growth of the gas pocket because of boiling.

The first problem to be solved is a determination of the boiling volume growth of the gas pocket. Table E-V summarizes an analysis to solve this problem. We assume a gas pocket of length Y is being formed by boiling from heat transferred through the tube wall. It has been determined that the boiling transfer coefficient on the inside of the tube is so very high relative to the external heat transfer coefficient that the tube wall can be assumed at the same temperature as the LH₂. Therefore the boiling rate is limited or determined by the heat transfer from the ambient surroundings to the tube wall.

On this basis the analysis shows that the maximum weight rate of LH₂ boiled into GH₂ will be

\[ \dot{w}_{\text{max}} = 5.29 \text{ lbm/hr} \]

per foot of tube over which boiling is assumed to occur. From this rate of boiling the volume rate of growth of the gas pocket has been calculated to be

\[ Q_{\text{vol}} = 0.9199 \text{ ft}^3/\text{min per foot length}. \]
Y is length of gas pocket
Z is length of liquid filled line beyond gas pocket
Q is heat input through line from ambient surroundings
V₁ is the fluid-gas boundary velocity at the rear of the gas pocket
V₂ is the fluid-gas boundary velocity at the front of the gas pocket

FIGURE E-3. Schematic of Case D Problem
TABLE E-V  Summary Of Heat Transfer Through Recirculation Line Walls

\[ Q = \text{heat transfer through wall to induce boiling of LH}_2 \]

\[ Y = \text{length of line over which boiling assumed occurring} \]

\[ \text{LH}_2 \text{ assumed @ - 420°F, 16.1 psig} \]

Heat transfer limited by convection to tube on O.D. - tube wall assumed at temperature virtually equal to LH2

From above

\[ Q = h_o A (T_a - T_T) \]

\[ h_o = \text{convection heat transfer coefficient assumed equal to 2.0 Btu/hr-ft}^2 \text{ °F} \]

\[ A = \pi D_o Y = \text{area of tube O.D. for length Y} \]

Per foot of tube we have where \( T_a = 70°F \) and \( T_T = -420°F \)

\[ Q = 1026 \text{ Btu/hr per foot of tube} \]

If the heat of vaporization of LH2 is assumed at \( 194 \text{ Btu/lbm} \) then the weight rate of fluid boiled is

\[ \dot{\omega} = \frac{1026 \text{ Btu/hr}}{194 \text{ Btu/lbm}} = 1.29 \text{ lbm/hr per foot of tube} \]

From above the relative boundary velocities of the gas pocket has been calculated at

\[ V_2 - V_1 = 0.1758Y \text{ fps} \]
Finally this volume growth rate permits calculation of the differential gas pocket boundary velocities as

\[ V_2 - V_1 = 0.1758 \text{ Y-fps} \]

From the above it is clear that boiling over very long lengths of line would be required to cause significant increases in the advancing liquid-gas boundary. For example, we might make some probably impossible assumptions to show that there is no real problem from bellows flow excitation for the Case D situation. Let's assume:

- The liquid weight flow at the rear boundary is \( \dot{\omega}_2 = 4.2 \text{ lb/sec.} \)
- The rear boundary advances at a rate corresponding to the above or, \( V_1 = 10.87 \text{ fps} \) (see Table E-II).
- Boiling occurs over a 50 foot length of recirculation line. The line may or may not be this long.

Based on the above, we have:

\[ V_2 = 10.87 + 0.1758 \times 50 = 19.66 \text{ fps} \]

The liquid in front of the gas boundary is therefore being "pushed" along at a velocity of 19.66 fps. The stress indicator for this particular case would be (\( C_{fQ} = 2.67, \) 3rd mode)

\[ S.I. = 963.7 \text{ psi} \]

which is clearly too low to cause any problem.

**Discussion and Conclusions**

Figure E-4 shows a realistic but simplified schematic of the LH\(_2\) feed and recirculation systems. During chilldown the prevalves are closed, the recirculation pumps are operative and the recirculation valves are open. From our analysis so far we anticipate the following chain of events.

1. LH\(_2\) will start to flow into the feed system from the recirculation pumps at a rate greater than the nominal 4.2 lb/sec since the system is empty.

2. Massive boil off will initially occur as the feed lines and pumps begin to cool down.
FIGURE E-4. SIMPLIFIED SCHEMATIC OF FEED AND RECIRCULATION SYSTEMS
The initial boil off will raise the gas pressure in the feed line and pump areas, but as the pressure increases the LH₂ flow from the recirculation pumps will slow down or shut off as the maximum pump head pressure is achieved.

The initial flow through the recirculation line will be pure gaseous under conditions outlined in Case B.

As the feed system and pumps begin to cool down, LH₂ will enter the recirculation lines. We can expect a condition of slug flow where we have alternate pockets of gas and liquid. As shown in Case D, there is not sufficient heat transfer into the recirculation lines to create a high velocity condition from local boiling.

When the system is chilled down to the required extent, pure liquid flow will occur and Case A analysis covers this situation.

The Case C analysis is, we feel, unrealistic since, as shown in the Case D analysis, we cannot expect sufficient heat transfer through the recirculation lines to achieve boiling at a rate necessary to create a high velocity problem.

At this time we feel there is little chance of a bellows related problem in the feed and recirculation system because of two phase flow problems.