A MODEL DESCRIBING THE MICROWAVE EMISSION FROM A MULTI-LAYER SNOWPACK AT 37 GHz

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In this study, a multi-layer emission model is described and applied to the measured emission from snow in an attempt to relate the absorption and scattering parameters of the snow medium to its density and wetness.

The measured emission and ground-truth data used in this analysis were those collected during a 24-hour diurnal experiment conducted by Stiles and Ulaby (1980) on 2/17-2/18/77 in Steamboat Springs, Colorado. The emission measurements were acquired at 37 GHz and H-polarization using a microwave radiometer mounted atop a truck-mounted boom.

The Tinga et al. (1973) mixing formula is used to calculate the dielectric constant of snow at 37 GHz. These values of the dielectric constant of snow along with the multi-layer emission model are used to estimate the absorption and scattering coefficients of snow at 37 GHz and their dependence on wetness. It is found that the scattering coefficient is comparable in value to the absorption coefficient for dry snow. However, the absorption coefficient increases linearly with increasing snow wetness while the scattering (contd.)
Abstract (contd.)

The estimated values of the scattering and absorption coefficients are then used to calculate the emission from each layer of the snowpack throughout the diurnal experiment. It is shown that for dry snow, the ground underneath the snowpack contributes about 45% of all measured emission while the rest is due to emission from all the layers within the snowpack. However, when the wetness of the top 5 cm layer of the snowpack increases to 2% by volume, this top 5 cm snowlayer contributes more than 90% of all the measured emission.
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NOMENCLATURE

$c$  Velocity of Propagation, m/sec

d  Snow Depth, cm

$F$  Form Number

$f$  Frequency, HZ

$f_0$  Relaxation frequency, HZ

$K_0$  Free Space Permittivity = $8.85 \times 10^{-12}$ farad/m

$K_S$  Relative Dielectric Constant of Sr

$K'_S$  Real Part of the Dielectric Constant of Snow

$K''_S$  Imaginary Part of the Dielectric Constant of Snow

$K_W$  Relative Dielectric Constant of Water

$K'_W$  Real Part of the Dielectric Constant of Water

$K''_W$  Imaginary Part of the Dielectric Constant of Water

$K_i$  Relative Dielectric Constant of Ice

$K'_i$  Real Part of the Dielectric Constant of Ice

$K''_i$  Imaginary Part of the Dielectric Constant of Ice

$K_{ds}$  Dielectric Constant of Dry Snow

$K_o$  Optical Limit of the Dielectric Constant

$K_{dc}$  Static Limit of the Dielectric Constant

$L$  Loss, dB

$M_v$  Snow Wetness, Percent Liquid Water by Volume

$M_w$  Weight Percentage of Liquid Water Contained in Snow

$R_i$  Radius of Ice Particle, mm.

$R_w$  Radius of Water Covered Ice Particle, mm

$R_a$  Radius of the Air Space, mm.
T, T₀
  Snow Physical Temperature, \(^0\)K

Tₜₚ
  Brightness Temperature, \(^0\)K

Tₜₚ
  Apparent Radiometric Temperature of a Target, \(^0\)K

Tₛₙₖ
  Scattered Temperature, \(^0\)K, Representing the Downward Emitted Sky Radiation Scattered by the Target in the Direction of the Antenna

Tₛₚₖₚ
  Downward Emitted Sky Radiation, \(^0\)K

Tₖₚ
  Ground Contribution to Tₜₚ, \(^0\)K

Tₙₙ
  Ground Physical Temperature, \(^0\)K

Tₛ
  Snow Self-Emission Contribution to Tₜₚ, \(^0\)K

Tₛₑₚ
  Snow Self-Emission of a Thin Layer, \(^0\)K

\tan\(\delta\)
  Loss Tangent

W
  Snow Water Equivalent, cm

\(\alpha_a\)
  Attenuation Constant (Field), nepers/cm

\(\kappa_1\)
  Damping Coefficient, dB/m

\(\kappa_2\)
  Damping Coefficient, dB/m

\(\varepsilon\)
  Emissivity

\(\theta\)
  Angle of Incidence, Relative to Nadir, Degrees

\(\theta'\)
  Angle of Propagation, Relative to Nadir, Degrees

\(\kappa_e\)
  Snow Extinction Coefficient (Power), nepers/cm

\(\kappa_a\)
  Snow Absorption Coefficient (Power), nepers/cm

\(\kappa'_a\)
  Snow Mass Absorption Coefficient (Power), nepers g/cm²

\(\kappa'_s\)
  Snow Mass Scattering Coefficient (Power), nepers g/cm²

\(\kappa'_e\)
  Snow Mass Extinction Coefficient (Power), nepers g/cm²

\(\lambda\)
  Wave Length, cm

\(\rho\)
  Snow Density, g/cm³

\(\tau_{sa}\)
  Snow-Air Power Transmission Coefficient

\(\tau_{gs}\)
  Ground-Snow Power Transmission Coefficient
\[ \tau_{na} \] Power Transmission Coefficient from Layer \( N \), the Uppermost Layer of the Snowpack to Air

\[ \tau_{i(i+1)} \] Power Transmission Coefficient Across the Boundary Between the \( i \)th Snow Layer and the Snow Layer Just Above It

\[ \tau_{gl} \] Power Transmission Coefficient from the Ground to the Lowermost Snow Layer #1

\( \tau \) Relaxation Time of a Dipole, Seconds

\( r \) Linear Correlation Coefficient

\( \Delta h \) Thin Snow Thickness, cm

\( \sigma \) Ionic Conductivity, mho/m

\( \Delta \) Absolute Fluctuation in Tap Measurement
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ABSTRACT

In this study a multi-layer emission model is described and applied to the measured emission from snow in an attempt to relate the absorption and scattering parameters of the snow medium to its density and wetness.

The measured emission and ground truth data used in this analysis were those collected during a 24-hour diurnal experiment conducted by Stiles and Ulaby (1980) on 2/17 - 2/18/77 in Steamboat Springs, Colorado. The emission measurements were acquired at 37 GHz and H-polarization using a microwave radiometer mounted atop a truck-mounted boom.

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The estimated values of the scattering and absorption coefficients are then used to calculate the emission from each layer of the snowpack throughout the diurnal experiment. It is shown that for dry snow, the ground underneath the snowpack contributes about 45% of all measured emission while the rest is due to emission from all the layers within the snowpack. However, when the wetness of the top 5 cm layer of the snowpack increases to 2% by volume, this top 5 cm snowlayer contributes more than 90% of all the measured emission.
1.0 INTRODUCTION

During the Winter of 1977, an experimental program was conducted at a test site near Steamboat Springs, Colorado involving microwave measurements of snowpacks (Stiles and Ulaby, 1980). Among the investigations conducted was an experiment consisting of a 37 GHz passive microwave measurement over a 24 hour diurnal period. In conjunction with the microwave measurements, several snow parameters were recorded which include snow wetness, snow thermometric temperature and snow density at several depths in the snowpack.

The purpose of this study is to apply a multi-layer emission model to the measured emission from snow in an attempt to relate the absorption and scattering parameters of the snow medium to density and wetness.

Chapter 2 covers a brief description of the diurnal experiment reported by Stiles and Ulaby (1980) and reviews relevant results obtained in other parts of the same investigation.

The development and applications of the microwave emission model are given in Chapter 4. Results estimated by the different models for the scattering and absorption coefficients of snow at 37 GHz are shown.

In Chapter 5, an evaluation of the different models which were developed is shown. A comparison between scattering and absorption coefficients estimated in this study and those reported in the literature is also provided.

Finally Chapter 6 summarizes the findings of this study and the errors involved in the development and application of the model.
2.0 DIELECTRIC PROPERTIES OF SNOW

Snow is a mixture of ice, water and air. The absorption, emission and scattering of an electromagnetic wave by snow are governed by the geometrical and electrical (dielectric) properties of the snow medium. The relative dielectric constant of snow, \( K_s \), is in general a function of:

(a) Microwave frequency;
(b) Percentage volume of ice;
(c) Percentage volume of air;
(d) Percentage volume of free water;
(e) Snow thermometric temperature;
(f) Crystal size and structure of the snow medium; and
(g) Presence of impurities.

As will be seen in the next sections, the percentage of free water in a volume of snow has the strongest effect of all of the above parameters on the value of \( K_s \). The dielectric properties of water and ice which constitute, along with air, the snow medium will be examined first, followed by an examination of the dielectric and attenuation properties of dry and wet snow at 37 GHz.

2.1 Dielectric Properties of Water

The dielectric constant of water in the microwave region is governed by the Debye equation:

\[
K_w = K_\infty + \frac{K_{dc} - K_\infty}{1 - j2\pi f} + \frac{\sigma}{2\pi K_0 f} \tag{2-1}
\]
where

\[ K_\omega = \text{Complex relative dielectric constant of water, } K_W = K'_W - jK''_W \]

\[ K'_W = \text{Optical limit of dielectric constant} \]

\[ K''_W = \text{Static limit of dielectric constant} \]

\[ \tau = \text{Relaxation time of the water molecule dipole} \]

\[ K_0 = \text{Permittivity of free space} \]

\[ \sigma = \text{Ionic conductivity} \]

\[ f = \text{frequency in Hertz} \]

The real and imaginary parts resulting from the previous equation are

\[ K'_W = K_\omega + \frac{K_{dc} - K_\omega}{1 + (2\pi f)^2} \tag{2-2a} \]

\[ K''_W = \frac{\omega}{1 + (2\pi f)^2} + \frac{\sigma}{2\pi K_0 f} \tag{2-2b} \]

The Debye equation shows a single resonance phenomena at the relaxation frequency \( f_0 \) given by:

\[ f_0 = \frac{1}{2\pi \tau} \tag{2-3} \]

The relaxation frequency is temperature dependent as shown in Table 2-1 and is located in the microwave region. The limiting values on the equations for dielectric constant are
TABLE 2-1

Relaxation Frequency of Water $f_0$ at Different Temperatures (Royer, 1973)

<table>
<thead>
<tr>
<th>$T$(°C)</th>
<th>$f_0$(GHz)</th>
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<tbody>
<tr>
<td>0</td>
<td>8.5</td>
</tr>
<tr>
<td>10</td>
<td>11.7</td>
</tr>
<tr>
<td>20</td>
<td>15.8</td>
</tr>
<tr>
<td>30</td>
<td>21.2</td>
</tr>
<tr>
<td>40</td>
<td>27.0</td>
</tr>
<tr>
<td>50</td>
<td>33.9</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
K'_{w} &= K_{dc} \quad \text{for } f << f_{o} \quad (2-4a) \\
K''_{w} &= (K_{dc} - K_{\infty}) (2\pi f) \quad \text{for } f << f_{o} \quad (2-4b) \\
K'_{w} &= K_{\infty} \quad \text{for } f >> f_{o} \quad (2-5a) \\
K''_{w} &= \frac{(K_{dc} - K_{\infty})}{(2\pi f)} \quad \text{for } f >> f_{o} \quad (2-5b)
\end{align*}
\]

Figures 2.1 and 2.2 show the real and imaginary parts of the dielectric constant of water as a function of frequency and for different water temperatures. These curves were calculated by Royer (1973) using regression fits to obtain the values of \(K'_{w}\) and \(K''_{w}\) for several experimenters' data. The rate of attenuation in water is shown in Figure 2-3. It is seen that water is a very lossy medium throughout the microwave region. This loss is due to the fact that the relaxation frequency \(f_{o}\) is located in the microwave region. The attenuation of water at 37 GHz is between \(1.5 \times 10^{4}\) dB/meter and \(2.0 \times 10^{4}\) dB/meter depending on the temperature of the water. Because the dielectric constant of water is much larger than those of ice and air, small amounts of free water appearing in a snow medium can drastically alter the real and imaginary parts of the dielectric constant of snow and therefore change its emission, scatter and attenuation behavior.

2.2 Dielectric Properties of Ice

The dielectric constant of ice, like water, is governed by the Debye Equation 2-1. However the water molecules in ice are bound, and therefore not as mobile as those of free water. This results in a relaxation frequency for ice that is much lower than that of water.
Figure 2-1  Relative Permittivity of Water at $T = 0^\circ$ C and $T = 20^\circ$ C Using Debye Equation (Royer, 1973).
Figure 2-2 Relative Permittivity of Water at $T = 10^\circ$ C and $T = 30^\circ$ C Using Debye Equation (Royer, 1973).
Figure 2-3 Rate of Attenuation in Water (Royer, 1973).
and is of the order of 10 KHz (Evans, 1965). The optical limit is therefore easily satisfied for the Debye equation and the real and imaginary parts of the dielectric constant of ice are given by equations (2-5a) and (2-5b).

Measurements up to 183 GHz indicate that the real part of the dielectric constant of ice is approximately 3.17 in the microwave region and is independent of frequency and temperature as shown in Figure 2-4 (Evans, 1965). The imaginary part was found to be temperature dependent at a frequency of 10 GHz showing a sharp minimum near 0°C (Lamb and Turney, 1949). Measurements of the dielectric constant of ice are shown in Figure 2-4. Above 24 GHz, the dielectric constant measurements for ice are limited to the results of Perry and Straiton (1972). Table 2-2 shows their results at 35.3 GHz. These data are suspected to be in error (Gough, 1972) since they are anomalously different from both lower and higher frequency data. However Hallikainen (1977) used the results shown for the loss tangent in Figure 2-4 to calculate an average value for the loss factor of ice as a function of frequency. His results are shown in Figure 2-5.

The rate of attenuation of tap-water ice in dB for different ice thickness at 35.3 GHz was also reported by Perry and Straiton (1972) and is shown in Figure 2-6.

2.3 Dielectric Mixing Formulas

Since snow is a mixture of ice, water and air, the varying volumetric properties of these components in snow make the dielectric properties of snow difficult to examine. In addition, the structure of snow is a significant factor in determining its dielectric properties.
Relative permittivity of ice (ordinates) versus logarithm of radio-frequency (abscissae).

Loss tangent of ice versus radio frequency. The quantity plotted vertically is $\log_{10} (f \tan \delta)$ where $f$ is the frequency in mc/sec. On the high-frequency tail of a relaxation spectrum this quantity is constant. It has the further useful property that the attenuation of a radio wave (measured in dB/m) passing through the medium is directly proportional to $f \tan \delta$. Temperatures are marked in °C.

- L: Lamb (1946) and Lamb and Turner (1949) -5°C at low frequencies, 0°C to -190°C at high frequencies: distilled water.
- C: Cumming (1952) Distilled water, tap water, and melted snow (no observable difference).
- Y: Hashimoto (1961) Antarctic ice core sampler, not annealed, density 0.91 g/cm³.
- W: Westphal (private communication) Greenland ice, annealed, density 0.95 g/cm³.
- B: Blue (1979)

Figure 2-4 Dielectric Properties of Ice (Evans, 1965).
TABLE 2-2

Relative Dielectric Constants of Ice at 35.3 GHz
as Reported by Perry and Straiton, (1972)

<table>
<thead>
<tr>
<th>$k'_i$</th>
<th>$k''_i$</th>
<th>tan $\delta$</th>
<th>Ice Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.91 ± 0.03</td>
<td>&lt;4 × 10^{-3}</td>
<td>&lt;2.1 × 10^{-3}</td>
<td>Deionized H$_2$O</td>
</tr>
<tr>
<td>1.89 ± 0.03</td>
<td>1.14 × 10^{-1}</td>
<td>6.03 × 10^{-2}</td>
<td>T$_{ap}$ H$_2$O</td>
</tr>
</tbody>
</table>
Figure 2-5  Loss Factor of Fresh Water Ice as a Function of Frequency (Hallikainen, 1977).
Figure 2-6  attenuation Curve (Tap-Water Ice) at 35.3 GHz (Perry and Straiton, 1972).
Table 2-3 shows several mixing formulas that have been proposed to calculate the dielectric constant of a mixture consisting of two components (Poe, 1971; Sweeny and Colbeck, 1974; Tinga et al., 1973). The form number $F$, used in the first five formulas accounts for structure in the dielectric medium. The numerical values of the form number can vary from $F = 0$ (vertical particles), to $F = 2$ (spherical particles), to infinity (for elongated horizontal particles). Values of $F$ for freshly fallen snow are normally between 2 and 10 (Evans, 1965). Edgerton et al. (1971) used a form number of $F = 37$ to get the best fit for their data for wet snow using the Weiner mixing formula.

The mixing formulas generally show no dependence on frequency; however, as the wavelength of interest approaches the order of the snow crystal size, scattering effects may become significant and the form number may require different values as a function of frequency. The wavelength at the desired frequency of 37 GHz is 0.81 cm which is of the order of snow crystal size; scattering, therefore, may become significant.

The first five mixing formulas were used to fit dielectric constant data for snow below 10 GHz (Evans, 1965) and were reported to have worked satisfactorily. However, since few measurements of snow dielectric constant were conducted above 10 GHz, the mixing formulas were never used to fit any data for wet or dry snow at or near the desired frequency of 37 GHz.

The Tinga et al. mixing formula was used to determine the dielectric constant of snow from the dielectric properties of ice and water at 5 GHz and 37 GHz (Tiuri and Schultz, 1980). The calculated values
TABLE 2-3
Mixing Formulas

Weiner (Poe, 1971)

\[
\frac{k_{ds} - 1}{k_{ds} + F} = \rho_i \frac{k_i - 1}{k_i + F} + (1 - \rho_i) \frac{k_o - 1}{k_o + F}
\]

where

\( \rho_i \) = Volumetric Fraction of Ice
\( k_o \) = Dielectric constant of air
\( k_i \) = Dielectric constant of ice
\( k_{ds} \) = Dielectric constant of dry snow
\( F \) = Form number

Bottcher (Poe, 1971)

\[
\frac{k_{ds} - k_o}{3 k_{ds}} = \rho_i \frac{k_i - k_o}{k_i + 2 k_{ds}}
\]

Wet Snow (Weiner) (Poe, 1971)

\[
k_{ws} = \frac{k_w \rho_w U + k_{ds} (1 - \rho_w)}{\rho_w U + (1 - \rho_w)}
\]

where

\( U = \frac{k_{ds} + F}{k_{ws} + F} \)

\( k_{ws} \) = Dielectric constant of wet snow
\( k_w \) = Dielectric constant of bulk water
\( \rho_w \) = Volumetric fraction of water
Pierce (Poe, 1971)

\[
k_{ds} = k_i + \frac{(1 - \rho_i)(1 - F)}{1 - (1 - \rho_i)F} (k_o - k_i)
\]

deLoor (Sweeny and Colbeck, 1974)

\[
k_{ws} = k_{ds} + \frac{\rho_w}{3} (k_w - k_{ds}) \sum_{j=1}^{3} \frac{1}{1 + \left(\frac{k_w}{k_n}\right) - 1} A_j
\]

where

\[k_n = \text{Dielectric constant of bound water}\]

\[A_j = \text{Particle Depolarization factor for each of the three major axes (Sweeney and Colbeck, 1974)}\]

Tinga et al. (1973)

\[
k_s = \frac{3\left[\left(\frac{R_w}{R_a}\right)^3 (k_w-1)(2k_w+k_i)\right]}{(2+k_w)(2k_w+k_i) - 2\left(\frac{R_i}{R_w}\right)(k_w-1)(k_w-k_i) - \left(\frac{R_w}{R_a}\right)^3}
\]

\[-\left(\frac{R_i}{R_a}\right)^3 (k_w-k_i)(2k_w+1)\]

\[
\frac{1}{(k_w-k_i)(2k_w+k_i) + \left(\frac{R_i}{R_a}\right)^3 (k_w-k_i)(2k_w+1)}
\]

but

\[
R_w = R_i \left(1 + \frac{0.92 M_w^{1/3}}{1 - M_w}\right)
\]

\[
R_a = R_i \left(\frac{0.92 \rho_s^{1/3}}{1 - M_w}\right) \left(1 + \frac{M_w^{1/3}}{1 - M_w}\right)
\]
where

\[ k_w = \text{Dielectric constant of water} \]
\[ k_i = \text{Dielectric constant of ice} \]
\[ R_i = \text{Radius of ice particle} \]
\[ R_{W} = \text{Radius of water covered ice particle} \]
\[ M_w = \text{Weight percentage of liquid water contained in snow} \]
\[ \rho_s = \text{Density of snow} \]
of the dielectric constant of snow were used to calculate the brightness
temperature of snow using a radiative transfer model and assuming the
snow to consist of spherical ice particles in air for dry snow. Wet
snow was assumed to consist of the same ice spheres with a thin water
shell surrounding them. Figure 2-7 shows the calculated dielectric
constants of snow for the different snow wetnesses. The density and
temperature of the snow were measured but the wetness and particle
size have been selected for best calculated fit. It can be seen that
a relatively good fit has been achieved for all cases.

2.4 **Dielectric Constant of Dry Snow**

Dry snow is a mixture of ice and air. Therefore, its response
to temperature and frequency is similar to that of ice. However if
the wavelength is of the order of the snow crystal size, scattering
may cause a frequency dependence in snow. Due to the difficulty associ-
ated with measuring the dielectric constant of snow while maintaining
the environment constant during the measurement, only one set of dielec-
tric constant measurements of snow above 10 GHz is reported. Table
2-4 shows the results of the experiments conducted by Edgerton et
al. (1971) at 37 GHz for dry snow. These measurements were made using
an ellipsometer to measure the reflection coefficient.

2.5 **Dielectric Constant of Wet Snow**

Due to the high values of the dielectric constant of water, small
amounts of water can significantly change the dielectric constant of
snow. The lack of a good method to measure the wetness of snow along
with the difficulty in keeping the environment constant during measurements
Figure 2-7 Measured and Calculated Brightness Temperatures of a 38 cm Thick Snow Field in the Morning and in the Afternoon. The Density and Temperature of Snow Were Known. The Wetness and Particle Size have been Selected for Best Fit (Tiuri and Schultz, 1980).
TABLE 2-4

Dielectric Constant of Dry Snow at 37 GHz as Reported by Edgerton et al. (1971)

<table>
<thead>
<tr>
<th>$k'_s$</th>
<th>$k''_s$</th>
<th>Temperature $^\circ$C</th>
<th>Density Units</th>
<th>Grain Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.77*</td>
<td>&lt; 0.03</td>
<td>-2</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>1.9</td>
<td>&lt; 0.03</td>
<td>-10</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

*The high values were explained as resulting from difficulty in sample preparation.
make dielectric constant measurements of wet snow difficult to obtain. Measurements of dielectric constant of wet snow above 10 GHz are nonexis-
tent. Therefore, there is no data for the dielectric constant of snow of any wetness at or near the desired frequency of 37 GHz.

2.6 Attenuation Through Snow

The absorption coefficient for a lossy medium is given by:

\[
\alpha_a = \frac{2\pi}{\lambda} \left[ \frac{\kappa'_S}{2} \left[ \left( 1 + \left( \frac{\kappa''_S}{\kappa'_S} \right)^2 \right)^{1/2} \right] - 1 \right] \right]^{1/2} \tag{2-6}
\]

However, experimental evidence indicates that for wet and dry snow:

\[
K''_S \ll K'_S \tag{2-7}
\]

This reduces equation (2-6) to:

\[
\alpha_a = \frac{\pi K''_S}{\lambda \sqrt{K'_S}} \tag{2-8}
\]

where

\[
\alpha_a = \text{Field absorption coefficient, nep/m}
\]

\[
\lambda = \text{Wavelength (meters)}
\]

\[
K'_S = K'_S - j K''_S = \text{Complex dielectric constant of snow}
\]

Attenuation rates in dB/m are given by:

\[
L = 20 \log_{10} \left( e^{-\alpha_a} \right) = -8.68 \alpha_a \tag{2-9}
\]
Power loss measurements made through a layer of snow are composed of two parts, mismatch loss and attenuation loss. If the power loss measurements are conducted with coherent transmission, the two types of loss cannot be separated because of the interference effects caused by multiple reflections.

Linlor (1980) measured the rate of attenuation in dB/cm as a function of snow wetness and at selected frequencies between 4 and 12 GHz using phase shift and transmission loss measurements through two identical containers. One of the containers was empty and used as the reference unit and the other was filled with the snow sample. Figures 2-8 and 2-9 show the results of his experiment. These attenuation curves were plotted using an empirical relation derived by Linlor (1980) and given as:

\[
\text{Attenuation (dB/cm)} = M_v \left[ 0.045(f-4) + 0.066 \right]
\]

(2-10)

where

\[ M_v = \text{Percentage wetness by volume} \]
\[ f = \text{frequency in GHz} \]

Notice the increase in attenuation with increasing snow wetness and increasing frequency. Figure 2-8 shows that the attenuation through dry snow (zero wetness) is zero. This, according to the author, is not the case but the attenuation was less than 1 dB and so could not be measured using the available measuring instruments.

The Tinga et al. (1973) mixing formula along with Equation (2-8) are used to calculate the attenuation through snow as a function of its wetness at 37 GHz, 18 GHz and 12 GHz, for snow with a density of 0.21 g/cm³.
Figure 2-8  Variation of Attenuation with Snow Wetness at Selected Frequencies (Linlor, 1980).

Figure 2-9  Variation of Attenuation with Frequency at Selected Snow Wetness (Linlor, 1980).
Figure 2-10 shows the results of this calculation. As seen by the figure, there is an increase in attenuation with increasing wetness of the snow and with increasing frequency. However, at 12 GHz, the attenuation starts to saturate at a percentage wetness, $M_v$, of 3%. In Figure 2-11, the attenuation in Nep/cm is plotted as a function of $M_v$ at 12 GHz using both Equation (2-10) of Linlor (1980) and the Tinga et al. (1973) mixing formula. It is seen that the attenuation values obtained by the mixing formula are higher in value than those obtained by Linlor's empirical formula for percentage wetness by volume of snow that are less than $M_v = 2.75$.

Battles and Crane (1965, 1965) used interferometry techniques to measure loss from artificially created snow. Figure 2-12 shows their results in dB/ft as a function of snow temperature at 35.2 GHz. The loss shows a slow increase with increasing temperature until the temperature nears the melting point. Near $32^\circ$ F, the melting point of ice, a sharp increase in loss through snow is seen which is due to the appearance of moisture in the snow. Table 2-5 shows the results of another experiment reported by Battles and Crane (1966), in which loss from different types of snow is reported at 35.26 GHz. The higher losses for locally packed and ice crystal cases were explained to be the result of scattering of the wave by the snow crystals.

Currie et al. (1977) made measurements using a pulsed radar operating at 35 GHz. Their loss measurements were calculated by comparing the return from a corner reflector placed below the snow layer with the return measured from the same corner reflector placed above the snow layer. Table 2-6 shows their results. The loss measure-
Figure 2-10  The Rate of Attenuation in Nep/cm as a Function of Snow Wetness and Frequency Calculated Using the Tinga et al. (1973) Mixing Formula for a Snow Density of 0.21 g/cm³.
Figure 2-11 The Attenuation Through Snow as a Function of its Wetness as Calculated by Linlor's (1980) Empirical Formula and by the Tinga et al. (1973) Mixing Formula at 12 GHz and Snow Density of 0.21 g/cm$^3$. 

Figure 2-11 The Attenuation Through Snow as a Function of its Wetness as Calculated by Linlor's (1980) Empirical Formula and by the Tinga et al. (1973) Mixing Formula at 12 GHz and Snow Density of 0.21 g/cm$^3$. 

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Figure 2-12  Absorption of Radiation by Snow as a Function of Temperature (Battles and Crane, 1965).
TABLE 2-5

Loss Caused by Snow and Ice at 35.26 GHz
(Battles and Crane, 1966)

<table>
<thead>
<tr>
<th>Type</th>
<th>Thickness (cm)</th>
<th>Density g/cm³</th>
<th>Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine, loose snow</td>
<td>14.0</td>
<td>0.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Locally packed loose snow</td>
<td>14.0</td>
<td>0.33</td>
<td>7.2</td>
</tr>
<tr>
<td>Large ice crystals</td>
<td>14.0</td>
<td>0.39</td>
<td>9.2</td>
</tr>
<tr>
<td>Packed snow</td>
<td>14.0</td>
<td>0.47</td>
<td>1.4</td>
</tr>
<tr>
<td>Ice</td>
<td>5.1</td>
<td>0.92</td>
<td>1.2</td>
</tr>
</tbody>
</table>
TABLE 2-6
Loss Measurements at 35 GHz
(Currie et al., 1977)

<table>
<thead>
<tr>
<th>Snow Condition</th>
<th>Layer Thickness (cm)</th>
<th>Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet Snow Crust</td>
<td>3.5</td>
<td>11</td>
</tr>
<tr>
<td>Dry Snow Crust</td>
<td>4.6</td>
<td>21</td>
</tr>
<tr>
<td>Dry Snow Crust</td>
<td>5.0</td>
<td>5</td>
</tr>
<tr>
<td>Dry Snow Crust</td>
<td>3.5</td>
<td>3</td>
</tr>
</tbody>
</table>
ments are due to both mismatch and attenuation losses and, therefore, an attenuation coefficient cannot be calculated.

Stiles and Ulaby (1980) measured path loss at 35 GHz as a function of depth for three snow conditions using transmission techniques. Figure 2-13 shows their results in dB as a function of snow depth.

Stiles and Ulaby (1980) also calculated the emissivity of snow at 37 GHz as a function of the angle of incidence of the radiometer relative to nadir. The emissivity \( \varepsilon(\theta) \) was calculated using the equation below

\[
\varepsilon(\theta) = \frac{T_{ap}(\theta) - T_{sky}(\theta)}{T_0 - T_{sky}(\theta)} \quad (2-11)
\]

where

\[
T_{ap} = \text{Apparent temperature of snow measured by radiometer}; \quad {^0K}
\]

\[
T_{sky} = \text{Sky brightness temperature}; \quad {^0K}
\]

\[
T_0 = \text{Physical temperature of snow}; \quad {^0K}
\]

Stiles and Ulaby (1980) developed a simple emission model for a homogeneous layer of snow and the ground underneath it, and were able to fit their emissivity data calculated using the previous equation to the following equation developed by their model for emissivity as a function of the angle of incidence \( \theta \):

\[
\varepsilon(\theta) = \left( \tau_{sa} \frac{\kappa_a}{\kappa_e} \right) + \left( \tau_{sa} \tau_{gs} - \tau_{sa} \frac{\kappa_a}{\kappa_e} \right) \exp \left( -\kappa_e' \sec \theta \right)
\]

\[
= A + B \exp \left( -\kappa_e' \sec \theta \right) \quad (2-12)
\]
Figure 2-13 Measured Path Loss as a Function of Snow Thickness for Three Snow Conditions (Stiles and Ulaby, 1980).
where \( A \) and \( B \) represent the constant terms inside the parenthesis and

\[
\begin{align*}
\tau_{sa} &= \text{Snow-air power transmission coefficient} \\
\tau_{gs} &= \text{Ground-snow power transmission coefficient} \\
\kappa_a &= \text{Snow absorption coefficient (nep/cm)} \\
\kappa_e &= \text{Snow extinction coefficient (nep/cm)} \\
&= (\kappa_a + \kappa_s) \\
\kappa'_e &= \text{Mass extinction coefficient of snow (nep g^{-1} cm^2)} \\
\kappa_s &= \text{Snow scattering coefficient (nep/cm)} \\
W &= \text{Snow water equivalent, cm} \\
\theta &= \text{Angle of incidence, relative to nadir (degrees)}.
\end{align*}
\]

Figure 2-14 shows a plot of measured radiometric emissivity for dry snow at 37 GHz and for \( \theta = 27^0 \) and \( 57^0 \) as a function of the water equivalent \( W \) of snow. The best fit curves obtained using Equation (2-12) to fit the emissivity data are also shown. The values of \( A \) and \( B \) estimated by regression analysis to produce the best fits are shown in Figure 2-14 and Table 2-7. From these values it is seen that at \( \theta = 27^0 \):

\[
\begin{align*}
\tau_{sa} \frac{\kappa_a}{\kappa_e} &= 0.517 \quad (2-13a) \\
\tau_{sa} \tau_{gs} - \tau_{sa} \frac{\kappa_a}{\kappa_e} &= 0.481 \quad (2-13b) \\
\kappa'_e \sec 27^0 &= 0.0235 \quad (2-13c)
\end{align*}
\]

Assuming \( \tau_{sa} = 0.99 \), for the snow density measured during the experiment (0.41 g/cm\(^3\)), the above three equations are used to calculate \( \kappa_a \),
Figure 2-14 Measured Radiometric Emissivity Response to Dry Snow Water Equivalent at 37 GHz (Stiles and Ulaby, 1980).
TABLE 2-7
Values of $A$, $B$, $\kappa_e$, $\kappa_a$ and $\kappa_S$ Generated by Fitting the Experimental Values of $\varepsilon$ at 37 GHz to Equation 2-11: (Stiles and Ulaby, 1980)

<table>
<thead>
<tr>
<th>$\theta^\circ$</th>
<th>$A$</th>
<th>$B$</th>
<th>$\kappa_e$</th>
<th>$\kappa_a$</th>
<th>$\kappa_S$</th>
<th>Maximum Error*</th>
</tr>
</thead>
<tbody>
<tr>
<td>27$^\circ$</td>
<td>0.517</td>
<td>0.481</td>
<td>$8.60 \times 10^{-3}$</td>
<td>$4.50 \times 10^{-3}$</td>
<td>$4.10 \times 10^{-3}$</td>
<td>0.02</td>
</tr>
<tr>
<td>57$^\circ$</td>
<td>0.586</td>
<td>0.273</td>
<td>$1.38 \times 10^{-2}$</td>
<td>$8.17 \times 10^{-3}$</td>
<td>$5.63 \times 10^{-3}$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*Maximum Error = $|\varepsilon_c - \varepsilon_M|$;  
$\varepsilon_c$ = Calculated from best fit eq. 2-12.  
$\varepsilon_M$ = Measured (equation 2-11).
\( \kappa_s \) and \( \kappa_e \) for snow at \( \theta = 27^\circ \). The results of the calculations are shown in Table 2-7. Similar calculations are carried out for \( \theta = 57^\circ \) and the values obtained are also shown in Table 2-7.

Attenuation rates for snow were calculated using the temperature measurements of a snow layer of known thickness over a metal plate (Hofer and Matzler, 1980). The attenuation rates were calculated using two simple models proposed by Hofer and Matzler (1980). The first model included the effects of multiple reflections but neglected interference effects, whereas the second model was a radiative transfer model which included volume scattering \( \kappa_s \) and absorption \( \kappa_a \). Figure 2-15 and 2-16 show the results. \( \kappa_2 \) is the power damping coefficient which combines the effects of absorption and scattering and is given by (Hofer and Matzler, 1980):

\[
\kappa_2 = \sqrt{\kappa_a^2 + 2\kappa_a\kappa_s}
\]  

(2-14)

\( \kappa_1 \) is the power attenuation coefficient calculated using the first model. Hofer and Matzler (1980) reported moist snow with wetness of 1 to 3 percent by volume exhibited power damping coefficients greater than 30 dB/meter at 36 GHz.
Figure 2-15 Scattering, Absorption and Damping Coefficients for Dry Winter Snow (Hofer and Matzler, 1980).
Figure 2-16 Scattering, Absorption and Damping Coefficients for Dry Spring (Metamorphosed) Snow (Hofer and Matzler, 1980).
3.0 EXPERIMENT DESCRIPTION AND RESULTS

This chapter provides a description of the microwave diurnal experiment conducted by Stiles and Ulaby (1980) on 2/17 - 2/18/77 and on which this study is based. The first section gives a brief description of the sensor used, test site and the different microwave and ground truth experiments conducted. Later sections of the chapter illustrate the results of the experiment.

3.1 Description of the Microwave Radiometer and Ground Truth Data

A microwave radiometer was mounted atop a truck-mounted boom and used to acquire apparent temperature \( T_a \) data at a test site near Steamboat Springs, Colorado. The radiometer used during the experiment was a dual-polarized Dicke type manufactured by Aerojet General Corporation. It operated at 37 GHz, had a bandwidth of 300 MHz, a sensitivity of 0.5°K, an accuracy of 1°K and operated over a temperature range of 0-50°C. The radiometer had an approximate gain of 0.010 volts/K and used an automatic gain control mechanism. Calibration of the radiometer was conducted at the University of Kansas after completion of the experiment (Stiles and Ulaby, 1980).

The test site chosen for the experiment was a 40-acre hayfield near Steamboat Springs. The surface of the field was very flat and was covered with close-cut hay, approximately 6 cm in height. A snow-pack approximately 30 cm in depth lay on the surface of the field during the 2/17 - 2/18/77 diurnal experiment.

The diurnal experiment consisted of continuous data acquisition over a 28-hour period commencing at 6:00 AM on 2/17/77. \( T_a \) data
was acquired at 37 GHz at both the horizontal and vertical polarizations and at 0°, 20° and 50° angles of incidence. Ground truth data were gathered in a snow pit on the test site and included measurements of snow density profiles, snow wetness profiles, snow temperature profiles along with soil moisture samples and soil temperature in the top 5 cm of the soil. These measurements were obtained at hourly intervals except for the snow wetness measurements which required a slightly longer duration. Snow density profiles were obtained by taking horizontal cores from each layer in the snowpack. Snow wetness was measured with a freezing calorimeter at 5 cm intervals vertically throughout the snowpack. Snow temperature was measured using a digital thermometer and was monitored at 2 cm intervals vertically throughout the snowpack. Since water equivalent is the total amount of water contained in the form of snow per unit area, it is calculated as the product of density and depth of each snow layer. Results of $T_{ap}$ and ground truth measurements are discussed below (Stiles and Ulaby, 1980).

3.2 Diurnal Variation of $T_{ap}$ at 37 GHz

Since the model developed later in this paper uses only the $T_{ap}$ data obtained at 37 GHz in the horizontal polarization mode, only the horizontal polarization data are illustrated here. Figure 3-1 shows the apparent temperature $T_{ap}$ response at $\theta = 0°, 20°$ and $50°$ throughout the diurnal experiment. Notice the increase in $T_{ap}$ starting at 1000 hours and ending at 2200 hours. This increase in $T_{ap}$ is due to the appearance of water in the snowpack as a result of warmer snow.
Date: 2/17 - 2/18/77
Polarization: H
Frequency: 37 GHz
Snow Depth: 30 cm
Water Equivalent: 6.3 cm

Figure 3-1  Apparent Temperature as a Function of Time at 37 GHz, H Polarization and for $\theta = 0^\circ, 20^\circ$ and $50^\circ$ (Stiles and Ulaby, 1980).
temperatures. Notice also the little difference between the $T_{ap}$ response at $\theta = 0^0$ and $20^0$. Figure 3.2 shows the snow wetness in individual 5 cm layers in the top 15 cm of snow as a function of time. By comparing Figures 3-1 and 3-2 one notices that $T_{ap}$ is shown to lag behind $M_v(0-5 \text{ cm})$, the percentage wetness by volume of the snow in the top 5 cm of the snowpack, by about two hours. $T_{ap}$ exhibits sharp increases and decreases as the snow starts to melt and refreeze respectively.

3.3 *Ground Truth Diurnal Variations*

As mentioned in the section above, Figure 3-2 shows the wetness $M_v$ of the snowpack at the depths of 0-5 cm, 5-10 cm and 10-15 cm from the snow surface. The snowpack at depths lower than 15 cm was frozen throughout the experiment and showed no wetness. $M_v$ is defined as the volume percentage of liquid water contained in a unit volume of snow.

Snow thermometric temperatures measured every 2 cm from the top of the snowpack to a depth of 5 cm below the soil surface were used to generate the average values shown in Table 3-1. In this table, the snow thermometric temperatures are averaged to give an average temperature for the snow every 5 cm from the snow surface. Table 3-1 shows 21 different data sets representing snow thermometric temperatures throughout the diurnal experiment. Figures 3-3, 3-4, 3-5 and 3-6 show the snow thermometric temperature along with the snow wetness at intervals of 5 cm of snow and at the hours of 5:30, 10:00, 18:00 and 6:45 respectively. Notice that as the snow starts getting warm and starts melting, it warms up first at the surface layer of snow and then the lower layers of the snow begin to warm up. Later in
Figure 3-2  Snow Wetness $M_v$ as a Function of Time in the Top 15 cm of Snow (Stiles and Ulaby, 1980).
TABLE 3-1

Snow Thermometric Temperature in $^0K$ for Different Depths Within the Snowpack on 2/17/77 - 2/18/77.
(Miles and Ulaby, 1980)

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<th>10-15 cm</th>
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Figure 3-3  Snow Thermometric Temperature and Wetness at 5:30 AM on 2/17/77.

\[ T_{ap} \text{ at } 50^\circ = \begin{cases} 165.096 \\ \Delta = 9.031 \end{cases} \]
Figure 3-4  Snow Thermometric Temperature and Wetness at 10:00 AM on 2/17/77.
Figure 3-5  Snow Thermometric Temperature and Wetness at 1800 Hours on 2/17/77.
Figure 3-6  Snow Thermometric Temperature and Wetness at 6:45 AM on 2/18/77.
the day as the surface layer becomes very wet, the lower layers warm up to almost the same temperature as the top 5 cm snow layer as shown in Figure 3-5. Similarly at the refreezing stage after the sun goes down and the snow starts drying up in moisture, the top 5 cm layer starts decreasing in temperature first followed by the lower snow layers. Therefore, the refreezing process starts, like the melting process, from the snow surface down to the lower layers of snow.

Snow density was measured for each physical layer from the top of the snowpack down to the soil surface. The average density was found to be 0.21 g/cm$^3$ and did not change significantly at different depths in the snowpack or with time throughout the diurnal experiment. Therefore, the water equivalent of the total snowpack did not change and was measured to be 6.3 cm throughout the entire experiment. Also, the snow depth remained constant at 30 cm.

3.4 Wetness Response

Figures 3-7, 3-8 and 3-9 illustrate the response of $T_{ap}$ to snow wetness of the top 5 cm layer at $\theta = 0^0$, $20^0$ and $50^0$ respectively. $T_{ap}$ is shown to exhibit a hysteresis-like pattern. This hysteresis-like pattern is due to the fact that as the snow gets wet, the microwave radiometer operating at 37 GHz becomes sensitive to a snow depth that is smaller than the top 5 cm layer of the snow from which the snow wetness measurement was obtained. The snow depth to which the sensor is sensitive when the snow is wet depends on the penetration depth of the wave in the wet snow layer. In other words, the snow wetness actually measured is the average wetness of the top 5 cm of snow. The wetness values for
Figure 3-7 Tap Response to Snow Wetness $M_v$, Showing the Hysteresis Effect at $\theta = 0^\circ$. 
Figure 3-8  Tap Response to Snow Wetness Mv, Showing the Hysteresis Effect at $\alpha = 20^\circ$. 
Figure 3-9  $T_{ap}$ Response to Snow Wetness $M_v$, Showing the Hysteresis Effect at $\theta = 50^\circ$.  

$M_v$ (0-5 cm) (%)

$T_{ap}$ in °K
the top 5 cm snow layer therefore gives a value of wetness that is smaller than the actual wetness of the snow depth to which the sensor is sensitive at the melting stage and a larger value of wetness at the refreezing stage. This causes the $T_{ap}$ response to lead $M_v$ in the melting stage and to lag behind it in the refreezing stage which is shown in Figure 3-1 and is the cause of the hysteresis effect.

3.5 Depth Response

Stiles and Ulaby (1980) conducted an experiment in which they measured $T_{ap}$ at 37 GHz for different snow depths and different angles of incidence. The depth of snow was varied by piling dry snow to a depth of 170 cm. Apparent temperature $T_{ap}$ shows an exponential-like decrease with increasing water equivalence, as shown in Figure 3-10.

Data collected from an adjacent undisturbed snowpack shows values of $T_{ap}$ that are considerably lower than those from the snow pile. This is due to the presence of ice layers in the natural snow and to differences in layering and in the distribution of crystal sizes which contribute to a larger scattering loss at 37 GHz and, therefore, lower emission.
Frequency: 37 GHz
Polarization: H
d = Snow Depth
- Experiment 1 $c = 27^\circ$
- Reference [4] $\theta = 45^\circ$
- Experiment 2 $\theta = 57^\circ$
- Undisturbed Snowpack

Figure 3-10 Radiometric Apparent Temperature Response to Snow Water Equivalent at 37 GHz (Stiles and others, 1980).
4.0 MULTILAYER EMISSION MODEL DEVELOPMENT AND TESTING

The relationship between the measured microwave data ($T_{ap}$) from a snowpack and the physical and dielectric parameters of the snowpack can be examined through the development of models to explain this relationship. In the first section of this chapter, an emission model for snow is developed in a more general form. Later sections of this chapter show the use of a mixing formula to calculate the complex dielectric constant of snow at 37 GHz for different snow wetness. The calculated complex dielectric constant of snow is used along with the different forms of the emission model developed to estimate the absorption and scattering coefficients of snow at 37 GHz and their dependence on snow wetness. These coefficients are then used along with the snow model to estimate the values of $T_{ap}$ measured throughout the diurnal experiment.

4.1 Development of the Multilayer Emission Model for Snow

Consider a snowpack of height $d$ above the ground and consisting of $N$ different snow layers. It is assumed that:

(a) Each snow layer in the snowpack has a uniform thickness $d_i$ cm, where $i$ stands for the snow layer number from the soil surface up to the snow surface. Dielectric and thermal properties of each snow layer are assumed to be constant within the layer, at a given time.

(b) The surface of the snow and the soil underneath the snowpack are smooth.

(c) Multiple reflections at layer boundaries are ignored.

(d) The radiation received by the radiometer is incoherent.
(e) Since the radiometer used in the experiment was a ground based system, there is neither attenuation nor emission between the surface of the snowpack and the radiometer.

(f) The snowpack is a layered scattering volume with $\kappa_{ai}$ and $\kappa_{si}$ taken to be the absorption and scattering coefficients of the $i$th snow layer respectively. $\kappa_{ei}$ is the sum of $\kappa_{ai}$ and $\kappa_{si}$ and is the extinction coefficient of the $i$th snow layer. Explicitly, diffuse scatter is ignored; however it is understood that the above coefficients are effective values that at least partially compensate for this approximation.

The apparent temperature measured by the radiometer at different angles of incidence $\theta$ is given by:

$$T_{ap}(\theta) = T_B(\theta) + T_{sc}(\theta)$$  \hspace{1cm} (4-1)

where $T_B$ is the brightness temperature due to emission from the snowpack and the ground underneath it, and $T_{sc}$ is the component of the downward emitted sky radiation scattered by the scene in the direction of the radiometer. The brightness temperature of the scene measured at an angle of incidence $\theta$ is given by:

$$T_B(N^+) = \tau_{na} T_B(N^-)$$  \hspace{1cm} (4-2)

where

$T_B(N^+)$ = The brightness temperature measured directly above the snow-air boundary, $^0K$

$\tau_{na} = The power transmission coefficient from layer $N$, the uppermost layer of the snowpack to air

$T_B(N^-)$ = The brightness temperature measured directly below the snow-air boundary
$T_B(N^-)$ is the sum of the components due to emission by all the layers within the snowpack $T_S(SUM)$, and the component due to the emission by the ground $T_G$, therefore

$$T_B(N^-) = T_S(SUM) + T_G$$

(4-3)

Self-emission by a thin layer of snow of thickness $\Delta h$ within the $i^{th}$ layer of snow is given by:

$$T_{sei} = T_i \left[ 1 - \exp \left( -\kappa_{a_i}' \rho_i' \sec \theta_i' \Delta h \right) \right] \approx T_i \kappa_{a_i}' \rho_i' \sec \theta_i'$$

(4-4)

The letter $i$ above is an indication of the position of the snow layer considered within the snowpack, starting with $i = 1$ for the snow layer on the bottom of the snowpack directly above the ground surface, and $i = N$ for the snow layer at the top of the snowpack directly below the snow-air boundary. $T_i$ is the physical temperature of the $i^{th}$ layer of snow, $\rho_i$ is its density, $\kappa_{a_i}$ is its absorption coefficient and $\theta_i$ is the angle of propagation in the $i^{th}$ snow layer relative to nadir.

The total emission by the $i^{th}$ snow layer thickness $d_i$ is (Stiles and Ulaby, 1980):

$$T_{S_i} = T_i \int_0^{d_i} \rho_i' \sec \theta_i' \exp \left[ -\left( \kappa_{a_i}' + \kappa_{s_i}' \right) \rho_i (d_i - h) \sec \theta_i' \right] \text{d}h$$

(4-5)

where $\kappa_{s_i}$ is the scattering coefficient of the $i^{th}$ layer of snow.

After integration, Equation (4-5) reduces to:
The radiation emitted by the $i^{th}$ snow layer travels through the snow layers above it before it is seen directly below the air-snow boundary. The radiation emitted by the $i^{th}$ layer of snow is, therefore, attenuated by the factor

$$T_{si} = T_i \left( \frac{\kappa_{ai}}{\kappa_{ai} + \kappa_{si}} \right) \left[ 1 - \operatorname{Exp} \left\{ -\left( \kappa_{ai} + \kappa_{si} \right) \sigma_i d_i \sec \beta_i \right\} \right]$$

(4-6)

The above factor is the loss due to the propagation of the radiation emitted by the $i^{th}$ snow layer through the $(N-i)$ snow layers above it. Therefore

$$\operatorname{Exp} \left\{ \sum_{j=i+1}^{N} \left[ -\left( \kappa_{aj} + \kappa_{sj} \right) \sigma_j d_j \sec \beta_j \right] \right\}$$

(4-7)

The above factor is the loss due to the propagation of the radiation emitted by the $i^{th}$ snow layer through the $(N-i)$ snow layers above it. Therefore

$$T_s^{(SUM)} = \sum_{i=1}^{N} \left( \prod_{i \neq i(i+1)} \right) \cdot T_{si} \operatorname{Exp} \left\{ \sum_{j=i+1}^{N} \left[ -\left( \kappa_{aj} + \kappa_{sj} \right) \sigma_j d_j \sec \beta_j \right] \right\}$$

(4-8)

$\tau_{i(i+1)}$ is the power transmission coefficient across the boundary between the $i^{th}$ snow layer and the snow layer just above it. Similarly if $T_g$ is the ground thermometric temperature underneath the snowpack and $q_1$ is the power transmission coefficient at the ground-snow interface, the ground contribution measured directly below the snow-air boundary is:
\[ T_G = \tau_{ql} \cdot T_g \cdot \prod_{i=1}^{N-1} \tau_i(i+1) \cdot \exp \left\{ \sum_{i=1}^{N} \left[ -(\kappa_{a_i} + \kappa_{s_i}) \varphi_i d_i \right. \right. \]
\[ \left. \left. \left. \left. \sec \varphi_i \right] \right) \right) \right) \right) \right) \]  
(4-9)

\( \tau_{ql} \) is the power transmission coefficient at the boundary between the ground and the lowermost snow layer (layer =1).

The value of \( T_{si} \) given by Equation (4-6) is substituted into Equation (4-8), and this latter equation giving an expression for \( T_s(SUM) \) along with Equation (4-9) giving an expression for \( T_G \), are substituted into Equation (4-3) producing a complete expression for \( T_B(N^-) \). The final expression for \( T_B(N^-) \) is substituted into equation (4-2) to give:

\[ T_B(N^+) = \tau_{na} \sum_{i=1}^{N} \left( \prod_{i=1}^{N-l} \tau_i(i+1) \right) T_i \left( \frac{\kappa_{al}}{\kappa_{al} + \kappa_{al}} \right) - \exp \left\{ \sum_{i=1}^{N-l} \left[ -(\kappa_{al} + \kappa_{al}) \varphi_i d_i \sec \varphi_i \right) \right) \right) \right) \right) \]
\[ + \tau_{na} \tau_{ql} \sum_{i=1}^{N} \left( \prod_{i=1}^{N-l} \tau_i(i+1) \right) \exp \left\{ \sum_{i=1}^{N-l} \left[ -(\kappa_{al} + \kappa_{al}) \varphi_i d_i \sec \varphi_i \right) \right) \right) \right) \right) \]  
(4-10)

The above equation describes the emission from a snowpack of \( N \) different layers above the ground and the emission from the ground underneath the snowpack. This equation when substituted into equation (4-1) gives the general expression of the model developed.

\( T_{sc} \), the component of the downward emitted sky radiation scattered by the scene in the direction of the radiometer, shown in equation (4-1) is given as:

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\[ T_{sc}(\lambda) = (1 - \tau_{na}) T_{sky} \]  \hspace{1cm} (4-11)

\( T_{sky} \) is the downward emitted sky radiation. Equation (4-11) when added to Equation (4-10) produces an expression for the apparent temperature, \( T_{ap} \), measured by the radiometer.

### 4.2 Calculations of the Dielectric Constant and the Power Absorption Coefficient of Snow

In order to apply the emission model developed in the previous section and described by Equation (4-1), calculations of the power transmission coefficients at every boundary connecting two different snow layers and at the snow-air interface and the ground-snow interface are needed. In addition, values of the propagation angle in every snow layer and the absorption and scattering coefficients of every snow layer need to be calculated. To obtain all these values, the complex dielectric constant of snow at 37 GHz is needed as a function of snow wetness and snow density. Since measurements of the dielectric constant of snow at 37 GHz do not exist, one of the mixing formulas shown in Table 2-3 will be used. The Tinga et al. (1973) mixing formula is used to obtain the dielectric constant of snow as a function of its wetness and density at 37 GHz. The Tinga et al., mixing formula is used here because it was shown to yield good results by Tiuri and Schultz (1980) when used at 37 GHz. The radius of the ice particle \( r_i \) is taken to be 0.5 mm. \( \rho_s \), the density of snow, was measured during the ground truth experiment, and previously reported in Section 3-3, equal to 0.21 g/cm\(^3\). The value of \( k_w \), the dielectric constant of water
at 37 GHz, is obtained from Figure 2-1 and is \( k_w = 9.55 - j 19.10 \).
The value of \( k_i \), the dielectric constant of ice at 37 GHz, is obtained
from Figure 2-5 an’ is equal to \( k_i = 3.15 - j 0.003 \). The values of the
dielectric constant of snow, obtained from the mixing formula, as a
function of the percentage wetness by volume of the snow is shown in
Table 4-1 and Figures 4.1 and 4.2.

The imaginary part of the dielectric constant of snow, \( k_s'' \), shows
a very small value when the snow is of zero wetness. However, this
value increases substantially as the snow wetness increases by a very
small amount, as shown in Table 4-1 for a percentage of snow wetness
by volume of 0.1%. The difference in wetness between completely dry
snow and that of 0.1% wetness by volume cannot be detected by any avail-
able experimental methods. This leads us to suspect that the value of
the loss factor calculated for completely dry snow is unrealistic and,
therefore, should be replaced by a value comparable to that calculated
for a snow wetness of 0.1% by volume. If a linear fit is used to fit
the values of \( k_s'' \) at percentages of snow wetness by volume of 0.1%
and 0.2% and then the fit is extended to predict the value of \( k_s'' \)
for snow of zero wetness, the value of \( k_s'' \) predicted is 0.0015 as
shown in Figure 4-3. This value is very close to that reported by
Sweeny and Colbeck (1974) who experimentally measured the loss factor
for dry snow at a frequency of 6 GHz and found it to be 0.003. Hence,
the value \( k_s'' = 0.003 \) is the one chosen in the investigation for \( M_v = 0 \).

The values of \( k_s' \) and \( k_s'' \) obtained by the mixing formula of Tinga
et al. (1973) and modified at one value of \( k_s'' \) for dry snow are sub-
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</table>
Figure 4-1 The real Part of the Dielectric Constant of Snow $k_r$, as a Function of $M_v$, the Percentage Wetness by Volume of Snow as Calculated by Tinga et al. (1973) Mixing Formula.
Figure 4-2  The Imaginary Part of the Dielectric Constant of Snow $k_s''$, as a Function of $M_v$, the Percentage Wetness by Volume of Snow as Calculated by Tinga et al. (1973) Mixing Formula.
Figure 4-3 A Linear Fit for the Calculated Loss Factor Using the Tinga et al. (1973) Mixing Formula at 0.1% and 0.2% Wetness by Volume. The Fit is Extended to Predict a Value of $L_s$ for Dry Snow.
stituted in equation (2-8) to give the values of $\alpha_a$, the field absorption coefficient of snow as a function of the snow wetness. The corresponding values of $\kappa_a$,

$$\kappa_a = 2\alpha_a$$

(4-12)

where $\kappa_a$ is the power absorption coefficient in Nep/cm, are shown in Table 4-2. Figure 4-4 shows a plot of $\kappa_a$ in Nep/cm versus the percentage wetness by volume of snow, $M_v$. This figure shows that the dependence of $\kappa_a$ on wetness is linear and its linear best fit is given by:

$$\kappa_a = 0.0203 + 0.2741 M_v$$

(4-13)

The equation above is used throughout the rest of this investigation to describe the power absorption coefficient of snow at 37 GHz.

4.3 Emission Model for a Single Homogeneous Snow Layer:

During the diurnal experiment conducted on 2/17 - 2/18/77 by Stiles and Ulaby (1980) the total depth of the snowpack above the ground was 30 cm. The snowpack was divided into four layers. The thermometric temperatures of each snow layer were measured throughout the experiment and are shown in Table 3-1. The percentage wetness by volume, $M_v$, of each snow layer were measured throughout the experiment and are shown in Figure 3-2. $T_{ap}$ measurements for $\theta = 0^0, 20^0$ and $50^0$, as measured by the radiometer are reported in Figure 3-1. Combining the results shown in Table 3-1 and Figure 3-2, produces 21 complete
TABLE 4-2

Values of $\kappa_a$ in Nep/cm as a Function of Snow Wetness

<table>
<thead>
<tr>
<th>$M_v$ (%)</th>
<th>$\kappa_a$ in Nep/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>$0.2030 \times 10^{-1}$</td>
</tr>
<tr>
<td>0.11</td>
<td>$0.3949 \times 10^{-1}$</td>
</tr>
<tr>
<td>0.21</td>
<td>$0.7649 \times 10^{-1}$</td>
</tr>
<tr>
<td>0.32</td>
<td>0.1129</td>
</tr>
<tr>
<td>0.42</td>
<td>0.1487</td>
</tr>
<tr>
<td>0.53</td>
<td>0.1839</td>
</tr>
<tr>
<td>0.63</td>
<td>0.2183</td>
</tr>
<tr>
<td>0.74</td>
<td>0.2523</td>
</tr>
<tr>
<td>0.84</td>
<td>0.2855</td>
</tr>
<tr>
<td>0.95</td>
<td>0.3181</td>
</tr>
<tr>
<td>1.05</td>
<td>0.3500</td>
</tr>
<tr>
<td>1.15</td>
<td>0.3812</td>
</tr>
<tr>
<td>1.26</td>
<td>0.4117</td>
</tr>
<tr>
<td>1.36</td>
<td>0.4416</td>
</tr>
<tr>
<td>1.47</td>
<td>0.4707</td>
</tr>
<tr>
<td>1.58</td>
<td>0.4992</td>
</tr>
<tr>
<td>1.68</td>
<td>0.5269</td>
</tr>
<tr>
<td>1.78</td>
<td>0.5539</td>
</tr>
<tr>
<td>1.89</td>
<td>0.5803</td>
</tr>
<tr>
<td>1.99</td>
<td>0.6059</td>
</tr>
<tr>
<td>2.10</td>
<td>0.6309</td>
</tr>
<tr>
<td>2.20</td>
<td>0.6551</td>
</tr>
<tr>
<td>2.31</td>
<td>0.6787</td>
</tr>
<tr>
<td>2.41</td>
<td>0.7016</td>
</tr>
<tr>
<td>2.52</td>
<td>0.7237</td>
</tr>
<tr>
<td>2.62</td>
<td>0.7453</td>
</tr>
</tbody>
</table>
Figure 4-4 $K_a$ in Nep/cm as a function of $M_V$, the Percentage Wetness by Volume of Snow.
ground truth and microwave measurements at different times in the
diurnal experiment. For eight out of the 21 different measurements,
the snow was totally dry in all its layers within the snowpack.

In this first application of the model described by equations
(4-10) and (4-11), the snowpack is assumed to be made up of one homo-
geneous snow layer that is 30 cm thick. The thermometric temperature
of this homogeneous snow layer, its density and its wetness are assumed
similar to those measured for the top 5 cm snow layer of the snowpack.
In other words, an attempt will be made in this section to see whether
the temperature, wetness and density of the top 5 cm layer of snow
can be used to represent the whole snowpack.

When Equation (4-10) is used to describe the emission from one
layer of snow above the ground it reduces to:

\[
T_B = \tau_{sa} \left( \frac{\tau_{al}}{\tau_{al} + \tau_{s1}} \right) \left[ 1 - \exp \left( -\left( \frac{1}{\tau_{al}} + \frac{1}{\tau_{s1}} \right) \vartheta_1 \right) \right]
\]

\[
d_1 \sec \theta_1 \right] + \tau_{gs} \tau_{sa} T_g \exp \left( -\left( \frac{1}{\tau_{al}} + \frac{1}{\tau_{s1}} \right) \right)
\]

\[
\vartheta_1 d_1 \sec \vartheta_1 \right] \right)
\]

where

\[
\tau_{sa} = \text{Power transmission coefficient at the snow-air boundary},
\tau_{gs} = \text{Power transmission coefficient at the ground-snow boundary},
T_g = \text{The snow thermometric temperature in } \text{°K}.
\]

The equation above is solved in the following way:

68
(a) $T_1$, the snow thermometric temperature values were shown in Table 3-1. $T_1$ values used are those reported for the top 5 cm snowlayer. These temperatures are assumed indicative of the whole snowpack.

(b) $\rho_1$, the density of snow was measured during the ground truth experiment and reported equal to 0.21 g/cm$^3$. This value did not change throughout the diurnal experiment.

(c) $d_1$, the total depth of the snowpack was measured throughout the experiment and was equal to 30 cm. The depth of the snowpack did not change throughout the diurnal experiment.

(d) $T_g$, the thermometric temperature of the ground underneath the snow was measured for the top few centimeters of the ground and was equal to 273 K. That is the ground was frozen throughout the diurnal experiment.

(e) The relative dielectric constant of frozen ground $k_g$ is assumed equal to 3.

(f) The power transmission coefficient $\tau_{sa}$ at the snow-air boundary is calculated for each of the 21 measurements considered using:

$$
\tau_{sa} = 1.0 - \left| \frac{\cos \theta - \sqrt{k_s' - \sin^2 \theta}}{\cos \theta + \sqrt{k_s'' + \sin^2 \theta}} \right|^2
$$

(4-15)

where

$\theta$ = The look angle of the radiometer, $0^\circ$, $20^\circ$ and $50^\circ$.

$k_s' - jk_s''$ = The complex dielectric constant of snow.
\( \tau_{sa} \) is a function of the radiometer's look angle and the snow dielectric constant. The snow dielectric constant is calculated using the Tinga et al. (1973) mixing formula shown in Table 2-3, for each of the 21 different measurements considered. \( \tau_{sa} \) is calculated as shown above for the different \( k_s \) values and are shown as a function of the percentage wetness by volume of snow in Figures 4-5, 4-6 and 4-7 for \( \varphi = 0^\circ, 20^\circ \) and \( 50^\circ \) respectively.

(g) The propagation angles in the snow layer are calculated using Snell's law given below as:

\[
\sec \theta' = \frac{\sqrt{k_s}}{\sqrt{k_s - \sin^2 \varphi}}
\]

The propagation angle, \( \theta'_s \), is a function of the snow dielectric constant and, therefore, is a function of the snow wetness.

(h) The power transmission coefficient \( \tau_{gs} \) at the ground-snow boundary is calculated using a similar expression to that shown by Equation (4-15), that is:

\[
\tau_{gs} = 1.0 - \left| \frac{\cos \theta'_s - \sqrt{\frac{k_g}{k_s} - \sin^2 \theta'_s}}{\cos \theta'_s + \sqrt{\frac{k_g}{k_s} - \sin^2 \theta'_s}} \right|^2
\]

\( k_g \) is the dielectric constant of ground. \( \tau_{gs} \) is calculated for each of the 21 different measurement considered and for the different propagation angles in the snow layer.

(i) The value of \( \kappa_a \) is taken equal to that previously shown by equation (4-13).
(j) The scattering coefficient \( \kappa_s \), is assumed to have the form:

\[
\kappa_s = A + BM_v
\]

(4-18)

That is \( \kappa_s \) is assumed to have a linear dependence on the snow wetness \( M_v \), similar to \( \kappa_a \). A and B are constants to be determined.

(k) \( T_{sc} \), the downward emitted sky radiation scattered by the snowpack in the direction of the radiometer and previously shown by Equation (4-11), is assumed to be small in value and is neglected in this analysis. That is \( T_B \), the brightness temperature of the snow and the ground underneath as described by Equation (4-10) is taken equal to \( T_{rad} \), the apparent temperature measured by the radiometer at \( \theta = 0^\circ \), \( 20^\circ \) and \( 50^\circ \). This is because as shown in Figures 4-5, 4-6 and 4-7 for \( \tau_{sa} \) as a function of \( M_v \) at \( \theta = 0^\circ \), \( 20^\circ \) and \( 50^\circ \) respectively, the minimum value of \( \tau_{sa} \) calculated is 0.9715 for \( \theta = 50^\circ \) and \( M_v(%) = 2.53 \). This value when substituted into Equation (4-11) and \( T_{sky} \) is taken equal to \( 300^\circ K \), the maximum possible value for \( T_{sky} \), \( T_{sc} \) is only equal to \( 8.55^\circ K \). This value is small enough to be neglected. \( T_{rad} \) values measured are shown in Figure 3-1.

The values calculated by steps (a) - (k) above are substituted into Equation (4-14). This produces 21 equations, each describing the emission from the homogeneous layer of snow at a particular time throughout the diurnal experiment. These equations are solved using non-linear regression analysis (BMDP, UCLA: 1977) to estimate the values of the constants \( A \) and \( B \). This is done by solving Equation (4-14) for the eight measurements in which snow was totally dry in all its layers. In this case Equations (4-13) and (4-18) reduce to:
Figure 4-5  $T_{sa}$, the Power Transmission Coefficient at the Snow-Air Boundary as a Function of $M_v$, the Percentage Wetness by Volume of Snow at $\theta = 0^\circ$. 
Figure 4-6 $\tau_{sa}$, the Power Transmission Coefficient at the Snow-Air Boundary as a Function of $M_v$, the Percentage Wetness by Volume of Snow at $\vartheta = 20^\circ$. 

\begin{align*}
M_v(\%) & \quad 0.0 \quad 0.7 \quad 1.4 \quad 2.1 \quad 2.8 \quad 3.5 \quad 4.2 \quad 4.9 \quad 5.6 \\
\tau_{sa} & \quad 0.995 \quad 0.994 \quad 0.993 \quad 0.992 \quad 0.991 \quad 0.990 \quad 0.989 \quad 0.988 \quad 0.987 
\end{align*}
Figure 4. \( r_{sa} \), the Power Transmission Coefficient at the Snow-Air Boundary as a Function of \( M_v \), the Percentage Wetness by Volume of Snow at \( \theta = 50^\circ \).
\[ \kappa_a = 0.0203 \text{, Nep/cm} \]  \hspace{1cm} (4-19)

and

\[ \kappa_s = A \text{, Nep/cm} \]  \hspace{1cm} (4-20a)

Once the value of \( A \) has been estimated, it is substituted into Equation (4-18) and the expression for \( \kappa_s \) with only constant \( B \) unknown is substituted into Equation (4-14). Non-linear regression analyses are carried out for both the dry and wet snow conditions to estimate the value of the Constant \( B \). The results of these analyses show that:

\[ A = 0.0171 \text{, Nep/cm} \]  \hspace{1cm} (4-20b)

\[ B = 0 \]

The above equation gives a value for the scattering coefficient of snow at 37 GHz. \( \kappa_s \) predicted shows no dependence on snow wetness. Figure 4-8 shows a plot of \( \kappa_a \), \( \kappa_s \) and \( \kappa_e \) as predicted by this model as a function of \( M_v \), the percentage wetness by volume of the snow layer.

The values of \( \kappa_a \) and \( \kappa_s \) shown by Equations (4-13) and (4-20b) respectively are now substituted into Equation (4-10) and this equation is solved for the 21 different measurements to produce estimates of \( T_B \) at different times throughout the diurnal experiment. Figures 4-9, 4-10 and 4-11 show the observed \( T_{ap} \) measurement along with the calculated \( T_B \) values as a function of time along the diurnal for \( \theta = 0^\circ, 20^\circ \) and \( 50^\circ \) respectively. For all the three angles considered, the predicted
Figure 4-8  \( \kappa_a, \kappa_s, \) and \( \kappa_e \) as a Function of Snow Wetness as Predicted by the Single Homogeneous Snowlayer Model.
Figure 4-9 Observed $T_a$ and $T_b$ Predicted by the Single Homogeneous Snowlayer Model as a Function of Time at $\theta = 0^\circ$. 
Figure 4-10  Observed $T_{ap}$ and $T_B$ Predicted by the Single Homogeneous Snowlayer Model as a Function of Time at $\theta = 200^\circ$. 
Figure 4-11: Observed $T_{\alpha p}$ and $T_B$ predicted by the Single Homogeneous Snowlayer Model as a Function of Time at $c = 50^\circ$. 
$T_b$ response leads that of the observed $T_{ap}$ response when the snow is melting and lags behind it when the snow is refreezing. This is due to the fact that the response of the predicted $T_b$ is a function of the snow wetness of the top 5 cm layer of snow. The wetness of the top 5 cm layer of snow is assumed here to be descriptive of the whole snowpack. However, the wetness of the top 5 cm layer of snow experimentally shows higher values of wetness than the rest of the sub-snow layers when the snow is melting and in turn shows lower wetness values than the rest of the sub-snow layers when the snow is refreezing.

4.4 Emission Model for a Multilayered Snowpack

As was previously mentioned in the last section, the snowpack during the 2/17 - 2/18/77 diurnal experiment had a total depth of 30 cm and modeled to consist of four distinct snow layers. Figure 4-12 shows the thickness of each of the snow layers within the snowpack. In this analysis, Equation (4-10) describing the emission from a multilayered snowpack is used to estimate the value of $\kappa_S$ and its dependence on snow wetness. The snowpack in this case is treated as consisting of four distinct snow layers.

Equation (4-10) is solved in a similar manner to that previously discussed in Section 4-3. In this case, however, the power transmission coefficients are calculated at every boundary between the snow layers, the boundary between the uppermost snow layer and the air, and the boundary between the ground and the lowermost snow layer. Similarly, the propagation angles are calculated in each snow layer for the three different look angles. Values of the thermometric temperatures in each layer and the wetness $M_v$ of each layer for the 21 different times
Figure 4-12  A Geometrical Presentation of the Snowpack Cross Section During the 2/17/77 - 2/12/77 Diurnal Experiment.
considered throughout the diurnal experiment are shown in Table 3-1 and Figure 3-2 respectively. \( \rho \), the density of the snow was reported equal to 0.21 g/cm\(^3\) for all the snow layers throughout the diurnal experiment. The results of this analysis gives

\[
\rho_s = 0.0175 - 0.0046 \rho \quad \text{deg/cm}
\]

That is, \( \rho_s \), the scattering coefficient, is smaller in value than the absorption coefficient, \( \rho_a \), for dry snow. Also, \( \rho_a \) increases linearly with increasing wetness but, \( \rho_s \) decreases linearly with increasing wetness as shown in Figure 4-13. The rate of decrease of \( \rho_s \) with wetness is very small as shown by Equation (4-21).

Figures 4-14, 4-15 and 4-16 show the \( T_d \) observed during the diurnal experiment along with the \( T_B \) predicted by the multilayer model at \( \alpha = 0^\circ, 20^\circ \) and \( 50^\circ \) respectively. The values of \( T_B \) are calculated by substituting the values of \( \rho_a \) and \( \rho_s \) of every layer into Equation (4-10) along with the other parameters to give the calculated values of \( T_B \) throughout the diurnal experiment. It would be seen from Figures 4-14, 4-15 and 4-16 that \( T_B \) calculated still leads \( T_d \) observed by the radiometer when the snow is melting and lags behind it when the snow is refreezing. However, the difference in values between \( T_d \) observed and \( T_B \) predicted is smaller than those previously shown for the single homogeneous layer model. The fits shown by Figures 4-14, 4-15 and 4-16 are, therefore, better fits for they produce estimates of \( T_B \) that are closer in value to the \( T_d \) measured by the radiometer.
Figure 4-13 $\kappa_a$, $\kappa_S$ and $\kappa_e$ for Snow as a Function of Snow Wetness at 37 GHz Estimated by the Multilayered Emission Model.
Figure 4-14  Observed $T_{ap}$ and $T_B$ Predicted by the Multilayered Emission Model as a Function of Time at $z = 0^\circ$. 
Figure 4-15  Observed $T_{ap}$ and $T_{g}$ Predicted by the Multilayered Emission Model as a Function of Time at $\theta = 20^\circ$. 
Figure 4-16  Observed $T_a$ and $T_b$ Predicted by the Multilayered Emission Model as a Function of Time at $e = 50^\circ$. 
4.5 Multilayered Model for Snow with Split Top Layer

In order to modify the models applied in the previous two sections to give better estimates of the observed $T_{ap}$, an understanding of the snow melting and refreezing process is necessary. Figures 3-3 to 3-6 along with Table 3-1 indicate that as the air temperature above the snowpack increases sufficiently above freezing combined with the radiation from the sun shining on the snowpack, the snow begins to melt. The process of melting starts on the top of the snowpack with the top 1 cm of snow or less becoming much more wet and so having a higher $M_v$ than the rest of the snowpack below it. If the air temperature remains warm enough, the snow wetness $M_v$ increases with time in the top few centimeters of snow, becoming wettest on the top of the snowpack and decreasing in wetness with increasing snow depth. At this time the radiometer observing the snowpack at 37 GHz is sensitive to a small snow depth on the top of the snowpack. Due to the decrease in penetration depth, this very small snow depth is responsible for the majority of the emission of microwaves from the snowpack. This top and very wet snow layer could be smaller in thickness than the top 5 cm layer of snow considered in the previous model and whose average wetness was measured by Stiles and Ulaby (1980) and is shown in Figure 3-2.

As the air temperature starts decreasing later in the afternoon to a temperature below that of freezing, the snowpack starts to refreeze. The refreezing process, similar to the melting process, starts from the top of the snowpack. A small depth of snow on the top of the snowpack, which could be 1 cm in depth or smaller, starts to refreeze and the $M_v$
of that depth decreases greatly. With time, the wetness, $M_v$, of this small depth drops to values that are less than the values of $M_v$ of the lower depths of the snowpack. The snow in the top of the snowpack then becomes dry and its $M_v$ drops to zero in the top few centimeters of the snowpack, while the snow in the lower layers of the snowpack remains higher in wetness. With time and continuing decrease in air temperature, the frozen layer on the top of the snowpack increases in thickness gradually down the snowpack until the whole snowpack is frozen.

The radiometer observing the snowpack at the start of the refreezing process will be sensitive at times to a snow depth that is larger than the 5 cm top layer of snow due to the increased penetration depth of the wave with the disappearance of free water from the top of the snowpack. The snow wetness measurements collected from the top 5 cm layer of snow are average values of wetness throughout the snow thickness considered and do not give the exact wetness profile in the snow thickness. This average value of wetness will, therefore, lag behind the $T_{ap}$ observed when the snow is melting and lead the $T_{ap}$ observed when the snow is refreezing as shown in Figure 3-1. This causes the hysteresis pattern shown in Figures 3-7, 3-8 and 3-9.

The model developed as described by Equation (4-10) needs to be modified to account for the physical behavior of snow at its melting and refreezing process. This is done by splitting the uppermost snow layer, shown in Figure 4-12 as Layer 4, into two sublayers each 2.5 cm in thickness. The thermometric temperature of these two sublayers is taken to be the same as the original 5 cm top layer. The percentage wetness by volume $M_v$ of the top 2.5 cm layer, shown in Figure 4-17
Figure 4-17  A Geometrical Presentation of the Revised Snowpack Cross Section During the 2/17/77 - 2/18/77 Diurnal Experiment. The Top 5 cm Layer is Shown Split into Two 2.5 cm Sublayers.
as Layer 5, is made seven times as wet as the original 5 cm top layer in the melting stage and then allowed to saturate at a value of $M_v = 2.53$ which was the maximum value of $M_v$ reached by the original 5 cm top layer. The percentage wetness $M_v$ of the lower 2.5 cm layer, shown in Figure 4-17 as Layer 4, is calculated during the melting process such that the average value of wetness $M_v$ of both 2.5 cm snow layers is equal to that of the original 5 cm uppermost snow layer. The wetness $M_v$ of the second 2.5 cm layer is made seven times as wet as the original 5 cm uppermost snow layer during the refreezing stage allowing it to saturate at the same value of $M_v = 2.53$. The wetness of the top 2.5 cm layer during the refreezing stage is then calculated such that the average wetness $M_v$ of both 2.5 cm snow layers is equal to that of the original 5 cm uppermost snow layer. Figure 4-17 shows the snow layers within the snowpack and their various thicknesses. Figure 4-18 shows the calculated wetness curves of the two 2.5 cm layers that make up the original 5 cm uppermost snow layer, along with the wetness curve of the original 5 cm top layer of snow.

Equation (4-10) describing the emission from a multilayered snowpack is solved in a manner similar to that shown in the last two sections, taking into account the modifications introduced in this section. The scattering coefficient estimated by this model as modified above is the same as that estimated in the previous section and shown by Equation (4-21). Figures 4-19, 4-20 and 4-21 show the $T_{ap}$ observed during the diurnal experiment along with the $T_B$ predicted by the multilayered model with the split top layer at $\theta = 0^\circ, 20^\circ$ and $50^\circ$ respectively. The difference between the fits shown here by Figures 4-19, 4-20 and 4-21 and those
Figure 4-18  Percentage Wetness by Volume $M_v$ Curves for the Top 2.5 cm Snow Layers Along With the Original 5 cm Snow Layer Wetness Curve of the 2/17/77 - 2/18/77 Diurnal Experiment. The Top 2.6 cm Layer is Seven Times as Wet as the Original 5 cm Layer in the Melting Stage While the Lower 2.5 cm Layer is Seven Times as Wet as the Original 5 cm Layer in the Refreezing Stage.
Figure 4-19  The Observed $T_{ap}$ and the Predicted $T_B$ as Calculated by the Multilayered Emission Model With Split Top Layer at $e = 0^\circ$. The uppermost sublayer is 2.5 cm thick and seven times as wet as the original 5 cm top snow layer.
Figure 4-20 The Observed $T_{ap}$ and the Predicted $T_B$ as Calculated by the Multilayered Emission Model with Split Top Layer at $\theta = 20^\circ$. The Uppermost Sublayer is 2.5 cm Thick and Seven Times as Wet as the Original 5 cm Top Snowlayer.
Figure 4-21 The Observed $T_{\text{ap}}$ and the Predicted $T_B$ as Calculated by the Multilayered Emission Model with Split Top Layer at $\beta = 50^\circ$. The Uppermost Sublayer is 2.5 cm Thick and Seven Times as Wet as the Original 5 cm To, Snowlayer.
calculated previously and shown by Figures 4-14, 4-15 and 4-16, for 
$\theta = 0^0, 20^0$ and $50^0$ respectively, for $T_B$ are very small.

The uppermost 2.5 cm snow layer was made eight, 12 and 15 times 
as wet as the original 5 cm uppermost layer of snow. The original 
top 5 cm layer of snow was split into two sublayers. The thickness 
of the top layer was made 1 cm and that of the second layer 4 cm in 
thickness and the analysis repeated again for the different wetnesses 
considered. However, the fits produced by the different thickness 
of the snow sub-layers or the different wetnesses considered were not 
much different from those shown by Figures 4-19, 4-20 and 4-21 and are, 
therefore, not shown here.
5.0 EVALUATION OF THE EMISSION MODELS AND THEIR RESULTS

In this chapter, all of the emission models developed and applied in Chapter 4 to describe the emission from a snowpack are evaluated. The model that predicts the best estimates of the $T_{ap}$ observed by the 37 GHz radiometer throughout the diurnal experiment is considered the superior model. The values of $\kappa_a$, the absorption coefficient and $\kappa_s$, the scattering coefficient estimated by the superior model chosen are compared to similar values reported in the literature and are used to calculate the percentage contribution from each layer of the snowpack throughout the diurnal experiment.

5.1 Evaluation of the Models

Two methods are used to evaluate the models developed in Chapter 4. The first is a residual sum of squares analysis in which the sum of the squares of residuals occurring between $T_B(N^+)$ predicted by the model and $T_{ap}$ observed is calculated for each model. The model that shows the least residual sum of squares at all of the radiometer's look angles is therefore considered a better model for it gives the best estimates of $T_{ap}$ observed. Table 5-1 shows the results of such an analysis conducted on all the models developed. The model describing a multilayered snowpack with split uppermost layer of snow and whose 2.5 cm top layer is seven times as wet as the original 5 cm uppermost layer of snow shows the least residual sum of squares for all $N$.

The second evaluation method conducted, is a linear correlation analysis between the values of the $T_B(N^+)$ predicted by each model and the $T_{ap}$ observed by the radiometer. Figures 5-1 to 5-9 show the results
TABLE 5-1

The Residual Sum of Squares Analysis Conducted for all the Models Developed and at $\theta = 0^\circ$, $20^\circ$ and $50^\circ$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$0^\circ$</th>
<th>$20^\circ$</th>
<th>$50^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single homogenous layer Snowpack</td>
<td>2044.23</td>
<td>3732.51</td>
<td>9022.11</td>
</tr>
<tr>
<td>Multilayered Snowpack</td>
<td>1690.29</td>
<td>2077.31</td>
<td>3220.84</td>
</tr>
<tr>
<td>Multilayered Snowpack with Split Top Layer</td>
<td>1380.06</td>
<td>1598.42</td>
<td>3148.98</td>
</tr>
</tbody>
</table>
$T_B (\text{Pred}) = 0.940 \ T_a (\text{Obs}) + 16.150$

![Graph showing linear correlation coefficient between predicted $T_B$ and observed $T_a$.](image)

Figure 5-1  Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Single Homogeneous Layer Snowpack Model at $\phi = 0.0$ and the $T_a$ Observed.
\[
T_B (\text{Pred}) = 0.936 \ T_{\text{ap}} (\text{Obs}) + 19.325
\]

Figure 5-2  Linear Correlation Coefficient Between the Predicted \( T_B \) as Calculated by the Single Homogeneous Layer Snowpack Model at \( \theta = 20^\circ \) and the \( T_{\text{ap}} \) Observed.
Figure 5-3 Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Single Homogeneous Layer Snowpack Model at $\phi = 50^\circ$ and the $T_{ap}$ Observed.
Figure 5-4 Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Multilayered Snowpack Model at $\theta = 0^\circ$ and the $T_{ap}$ Observed.
\[ T_B (Pred) = 0.979 \, T_{ap} (Obs) + 3.004 \]

Figure 5-5  Linear Correlation Coefficient Between the Predicted \( T_B \) as Calculated by the Multilayered Snowpack Model at \( \theta = 20^\circ \) and the \( T_{ap} \) Observed.
$I_B (Preo) = 0.987 \ T_{ap} (Obs) + 11.640$

Figure 5-6  Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Multilayered Snowpack Model at $\theta = 50^\circ$ and the $T_{ap}$ Observed.
Figure 5-7  Linear Correlation Coefficient Between the Predicted $T_b$ as Calculated by the Multilayered Snowpack with the Split Top Layer and the $T_o$ Observed at $\theta = 0^\circ$. New Uppermost Layer is 2.5 cm in Thickness and Seven Times as Wet as the Original 5 cm Top Snow Layer.
Figure 5-8  Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Multilayered Snowpack with the Split Top Layer and the $T_{ap}$ Observed at $z = 200$. New Uppermost Layer is 2.5 cm in Thickness and Seven Times as Wet as the Original 5 cm Top Snow Layer.
$T_B\ (Pred) = 0.966\ T_{ap\ (Obs)} + 16.494$

Figure 5-9 Linear Correlation Coefficient Between the Predicted $T_B$ as Calculated by the Multilayered Snowpack with the Split Top Layer and the $T_{ap}$ Observed at $\theta = 50^\circ$. New Uppermost Layer is 2.5 cm in Thickness and Seven Times as Wet as the Original 5 cm Top Snow Layer.
of such an analysis carried out on all the results of the models developed at $\theta = 0^o, 20^o$ and $50^o$. Figures 5-1, 5-2 and 5-3 show the linear correlation between the $T_B$ predicted by the single homogeneous layered snowpack and the $T_{ap}$ observed at $\theta = 0^o, 20^o$ and $50^o$ respectively. Figures 5-4, 5-5 and 5-6 show the linear correlation between the $T_B$ predicted by the multilayered snowpack and the $T_{ap}$ observed at $\theta = 0^o, 20^o$ and $50^o$ respectively. Figures 5-7, 5-8 and 5-9 show the linear correlation between the $T_B$, predicted by the multilayered snowpack model with the original uppermost snow layer split into two sublayers, each 2.5 cm in thickness, and the $T_{ap}$ observed at $\theta = 0^o, 20^o$ and $50^o$ respectively. Both multilayered emission models show superior estimates of $T_B$ than the single homogeneous layer model. However, there is little difference in the correlation coefficient between the values of $T_B$ predicted by both multilayered models and the $T_{ap}$ observed. On this basis the multilayered snowpack model, with the uppermost layer split into two sublayers each 2.5 cm in thickness and the top sublayer is made seven times as wet as the original 5 cm uppermost snowlayer is considered the best model of the three developed.

5.2 Evaluation of the Values of $\kappa_a$ and $\kappa_s$ Estimated by the Multilayered Snowpack Model

Equations 4-13 and 4-21 show the absorption coefficient calculated by the Tinga et al. (1973) mixing formula and the scattering coefficient estimated by the multilayered emission model of snow as a function of snow wetness. The values of $\kappa_a$, $\kappa_s$ and $\kappa_e$ calculated in this study for dry snow are compared to those previously reported in the literature.
as shown in Table 5-2. The values of $\kappa_a$, $\kappa_s$ and $\kappa_e$ calculated for dry snow in this study are greater in value than those reported by both Stiles and Ulaby (1980) and Hofer and Matzler (1980). This is due to the following reasons:

(a) The difference in the snow depth, snow density, snow temperature, and snow crystal structure in all the experiments conducted. There were also differences in air temperature and other environmental conditions which could have caused the differences seen in the values of $\kappa_a$, $\kappa_s$ and $\kappa_e$.

(b) The values of $\kappa_a$, $\kappa_s$ and $\kappa_e$ estimated by Stiles and Ulaby (1980) are obtained from the results of the snow pile experiment shown in Figure 3-10. In their experiment, the snow was piled up to a depth of 170 cm and the apparent temperature of the snow pile at 37 GHz was measured for different depths. At the same time, $T_{ap}$ was measured from the undisturbed snowpack around the snow pile to see the difference in the $T_{ap}$ measurement for the same depth. Figure 3-10 shows that for a snow depth, above the ground, of 30 cm or for a 6.3 cm water equivalent at $\theta = 27^0$, the $T_{ap}$ measured from the snow pile was $260^0K$ whereas the $T_{ap}$ measured from the undisturbed snow was $200^0K$. Similarly for $\theta = 57^0$ and a water equivalent of 15 cm, $T_{ap}$ measured from the snow pile was $200^0K$ whereas $T_{ap}$ measured from the undisturbed snow was $175^0K$. That is the values of $T_{ap}$ measured from the snow pile are higher than those measured from the undisturbed snow. This difference is due to the alteration in the crystal structure of the snow as it was piled causing the scattering coefficient to become smaller in the snow pile than it is for the undisturbed snow and, giving higher


TABLE 5-2

Values of $\kappa_a$, $\kappa_s$, $\kappa_e$ for Dry Snow as a Function of $e$ Calculated Using Stiles and Ulaby (1980) Model, the Multilayered Snowpack Emission Model and the Hofer and Matzler (1980) Model.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\theta$ In Degrees</th>
<th>$\kappa_a$ dB/cm</th>
<th>$\kappa_s$ dB/cm</th>
<th>$\kappa_a/\kappa_e$</th>
<th>$\kappa_e$ dB/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiles and Ulaby (1980)</td>
<td>27°</td>
<td>0.04</td>
<td>0.04</td>
<td>0.52</td>
<td>0.08</td>
</tr>
<tr>
<td>Stiles and Ulaby (1980)</td>
<td>57°</td>
<td>0.07</td>
<td>0.05</td>
<td>0.59</td>
<td>0.12</td>
</tr>
<tr>
<td>Multilayered Emission Model</td>
<td>$0^\circ$, $20^\circ$ &amp; $50^\circ$</td>
<td>0.18</td>
<td>0.15</td>
<td>0.55</td>
<td>0.33</td>
</tr>
<tr>
<td>Hofer &amp; Matzler (1980) (High Winter)</td>
<td>$0^\circ$-$60^\circ$</td>
<td>0.02</td>
<td>0.03</td>
<td>0.40</td>
<td>0.05</td>
</tr>
<tr>
<td>Hofer &amp; Matzler (1980) (Spring)</td>
<td>$0^\circ$-$60^\circ$</td>
<td>0.02</td>
<td>0.04</td>
<td>0.33</td>
<td>0.06</td>
</tr>
</tbody>
</table>
values of $T_a$ from the snow pile than undisturbed snow. Since the values of $\kappa_a$, $\kappa_s$, and $\kappa_e$ calculated in this study are those of undisturbed snow while those calculated by Stiles and Ulaby (1980) are those for the snow pile, the larger values of $\kappa_a$, $\kappa_s$, and $\kappa_e$ calculated here are, therefore, explainable. Figures 5-10 and 5-11 show the brightness temperature calculated using Equation (4-14) for a snowpack made up of one homogeneous layer of snow as a function of water equivalent at $\theta = 27^\circ$ and $57^\circ$ respectively. These figures are calculated by substituting the values of $\kappa_a$, $\kappa_s$, and $\kappa_e$ obtained in this study for undisturbed snow into Equation (4-14). The figures also show the measurements of Stiles and Ulaby (1980) for undisturbed snow at $\theta = 27^\circ$ and $57^\circ$ of Figure 3-10. Notice the small difference in $T_B$ calculated here and that measured by Stiles and Ulaby (1980) at the two angles considered.

(c) The ratio of $\kappa_a/\kappa_e$ is very similar in all the measurements shown in Table 5-2, however the value of $\kappa_e$ tends to be much different. The dependence of $T_B$ on $\kappa_e$ is shown in Figure 5-12. In this figure, $T_B$ is plotted versus $\kappa_e$ for a snowpack made up of one homogeneous layer and 30 cm in depth and a snow temperature of 262.33$^\circ$K. $\kappa_e$ calculated in the study is comparable in value to the total loss measured by Stiles and Ulaby (1980) for dry snow as shown in Figure 2-10. This adds to the credibility of the value of $\kappa_e$ calculated here.

5.3 Total Emission from Each Snow Layer

The emission from each snow layer within the snowpack, shown in Figure 4-9 is calculated using the values of $\kappa_a$ and $\kappa_s$ given by
Figure 5-10  $T_B$ Calculated From an Undisturbed Snowpack as a Function of $W$ for One Layered Snowpack at $\theta = 27^\circ$ Using the $k_a, k_s$ and $k_e$ Calculated in This Study.
Figure 5-11  $T_R$ Calculated From an Undisturbed Snowpack as a Function of $W$ for a Single Layer Snowpack at $\theta = 57^\circ$ Using the $\kappa_a$, $\kappa_s$ and $\kappa_e$ Calculated in the Study.
Figure 5-12  $T_B$ as a Function of $\kappa_e$ for a Snowpack of One Homogeneous Layer at Nadiř. The Snow Depth is 30 cm and Snow Temperature is 262.330K.
Equations 4-13 and 4-21 respectively, and Equation (4-10). Figure 5-13 shows the percentage contribution of each layer within the snowpack as a function of time throughout the diurnal experiment. It is seen that for dry snow, the ground contributes the most to the total microwave emission measured by the radiometer. The contribution from the ground underneath a totally dry snowpack is above 45% of the total emission measured. As the topmost snow layer of the snowpack increases in wetness, its percentage contribution to the total emission increases as well. When the wetness of the top 5 cm layer is 1% by volume it contributes almost 80% of the total emission. When the snow wetness of the top 5 cm layer of snow is 2% by volume, it contributes 90% of the total emission. For dry snow, therefore, the emission from the snowpack and the ground underneath it is a volume emission stemming from each layer within the snowpack and the ground underneath it. However, this volume emission changes rapidly with the appearance of free water in the uppermost layer of the snowpack to become a surface emission stemming almost totally from the surface layer of the snowpack.
Figure 5-13 The Percentage Contribution of Each Layer Within the Snowpack as a Function of Time.
6.0 CONCLUDING REMARKS

In this paper a model describing the microwave emission from snow at 37 GHz was developed as a function of the physical and dielectric parameters of the snowpack. These parameters included the consideration of a snowpack consisting of different layers. The model provided information on the contribution of each snow layer and the ground to the total emission, for both wet and dry snow conditions. The model also provided solutions of the scattering, absorption and extinction coefficients for both wet and dry snow. The errors that exist in this model are due to the following:

(a) The model developed predicted the brightness temperature of the snow and not the apparent temperature actually measured. It, therefore, did not take into account the contribution of the downward emitted sky radiation scattered by the scene in the direction of the radiometer antenna.

(b) The lack of information of the dielectric constant of wet snow at 37 GHz, and of the dielectric constant of each snow layer considered throughout the experiment.

(c) The lack of wetness profiles that provide the wetness of the snowpack accurately and for every 1 cm thickness of the snowpack.

(d) Lack of information on the exact contribution of the ground to the apparent temperature measured by radiometer, since no $T_{ap}$ measurements were conducted for the frozen ground of the scene during the experiment.
The model is the first to calculate values of $\kappa_a$, $\kappa_s$ and $\kappa_e$ as a function of snow wetness. The model is also the first to show the contribution of each snow layer to the total emission from snow and ground as a function of snow wetness.
REFERENCES


