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Research Developing Closed Loop Roll Control for Magnetic Balance Systems

NASA, Langley Research Center
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Professor Eugene E. Covert
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Principal Investigators
Work during the period

During the past six months effort was directed to the following areas:

1. Procurement and installation of the line printer, A-D converter boards, clock and extra memory for the MINC 11-23 minicomputer, and the logic analyzer.

2. Interconnection of the computer inputs to magnetic balance outputs to provide computer position control and data acquisition. This necessitated construction of four junction boxes and two 27 pair cables 70 feet long to provide connections from balance to computer and computer to balance.

3. Completion of the investigation of the use of parameter identification of a means of determining dynamic characteristics. This study indicated that the technique was successful at providing preliminary measurements of $C_{m_a} + C_{m_q}$ for the ogive cylinder model. Sufficient measurements were not made to provide a comparison with other methods; i.e., forced oscillation motion. The measurements made indicated that results could be obtained with random noise inputs of .1, .2 and .4 degrees in pitch. This is reported in Gautham Ramohalli's M.S. thesis*, an abbreviated version of which constitutes the bulk of this report.

4. Repair of Thyratron and motor generator power supplies for the pitch and yaw degrees of freedom. Vacuum tubes and defective wiring in the Thyratron power supply used in pitch were

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replaced to permit use of the supply during extensive repair of the motor generators. Time since overhaul on these supplies was about 1100 hours, which is 100 hours longer than the expected life of the generator bearings.

Because of the increased effort required for computer connection and system repairs, grant funds have been expended as of this reporting period.

Personnel

During the past six months the following professional personnel have contributed to the effort under this program:

Professor Eugene E. Covert
Dr. Charles W. Haldeman
Mr. Gautham Ramohalli

Choice of a method for handling dynamic system data

The first method considered was the EEIC method of Merhav and Gabay (1). This has the attractive feature of being able to estimate not only poles but zeroes of a continuous system directly. Further, the method drives the pole zero excess coefficients (if a higher order model is guessed) automatically to zero and any excess pole zero pairs are driven to each other.

This method was tried on a simple first order model on a simulation on the VAX 11/780. The method showed good convergence to reasonably accurate values. However, it needed about 50 seconds of data before the values settled down. As the order of the model is increased, the complexity of the
process increases. For each order increase an additional filter is required of the form \( \frac{s}{(s+a)^k} \). Further, the pole zero pairs need to be monitored and factorization used to cancel any poles and zeroes that converge to each other (but do not converge to a steady value!). This method was not used.

If the memory and computational requirements are to be minimized, analog filters are indicated. However, since evaluating digital data processing for the application is an important goal, the data was processed using the MINC.

Since the MINC was used to acquire many data points in a short time, it was located near the wind tunnel. To permit its use for data reduction as well as acquisition, a reduction method was selected that could be easily implemented on the MINC rather than having to transfer the data over to the larger central VAX system. This led to choice of the second method.

The second method considered was that of the Fourier spectrum analysis using Fast Fourier Transforms algorithms. Using the Institute of Electrical and Electronics Engineers digital signal processing program (2), FFT subroutines were implemented on the MINC 11/23 described below. However, spectrum analysis methods have the shortcoming that their resolution in frequency is limited by the length of data 'T' as

\[ \Delta f \geq \frac{1}{T} \quad \text{(uncertainty principle)} \]
This is similar to a problem encountered in processing speech signals where the length of the data stream is limited due to the time varying nature of the vocal signals. In fact, it parallels the problem closely if we consider large amplitude motion where the dynamics are essentially non-linear and a time varying small increment linear model for the airplane and wind is needed.

The method used there is to model the signal using covariance/correlation methods digitally. This is based on the least squares norm minimization and is referred to as MEM or maximum entropy method.

The method relies on statistical covariance/correlation approach to estimating the poles of a digital filter using short streams of data and has the desirable property that it eliminates the restriction imposed by the uncertainty principle by effectively "making more data" using an analytical model of the system. Of course, this method has the obvious drawback that if the generated model is erroneous, then the data generated using it is meaningless. However, the method has been successfully used in the speech case and is presently being applied to speech recognition.

Further, the computational and memory requirements are quite modest and an estimate of the accuracy of the model is...
based on the residual energy. It can be used to decide when the model order is high enough if a suitable criteria on the size of the residual is available. This was the method chosen for use.

Application to the magnetic balance

The procedure followed was to adjust the model suspension to be very stiff in all degrees of freedom except pitch. The pitch position response was then stored during a run and analyzed using the MINC. For this case all displacements except pitch are negligible and the equation of motion for a second order system with a natural frequency $\omega_n$ and a damping ratio $\zeta$ is

$$\frac{d^2\theta}{dt^2} + 2\zeta\omega_n \frac{d\theta}{dt} + \omega_n^2 \theta = M \ldots$$

where $M$ is the forcing moment and $\theta$ is the angle of the body axis from the tunnel axis. See Figure 1.
Equation 1 in a more transparent form is

\[ J \frac{d^2 \theta}{dt^2} + D_{MP} \frac{d \theta}{dt} + K_{MP} \theta = P(t) + M_A(t) \]  

\[ M_A = \text{Aerodynamic moment function} \]

\[ D_{MP} = \text{Magnetic damping} \]

\[ J = \text{Moment of inertia about centre of rotation} \]

\[ K_{MP} = \text{Stiffness due to magnetic suspension system} \]

\[ P(t) = \text{Pitch input driving function} \]

The aerodynamic moment can be linearized as follows. An \( M_\theta \) term that is due to the lift force due to an angle of attack. An \( M_\theta \) term that is due to rotation of any point along the body and an \( M_\theta \) term that comes about due to motion of points along the body (except at centre of rotation) in the \( z \) direction. The total damping \( M \) is a combination of \( \dot{\theta} \) and \( \ddot{\theta} \) terms. When the centre of mass is fixed as it is in a wind tunnel, \( \dot{\theta} = \dot{\alpha}, \ddot{z} = 0 \). Hence 1a becomes

\[ J \frac{d^2 \theta}{dt^2} + (D_{MP} - M) \frac{d \theta}{dt} + (K_{MP} - M_\theta) \theta = P(t) \]

In the Laplace domain, neglecting initial conditions,

\[ \Theta(s) \{ Js^2 + (D_{MP} - M)s + (K_{MP} - M_\theta) \} = P(s) \]

\[ \frac{P(s)}{\Theta(s)} = Js^2 + (D_{MP} - M)s + (K_{MP} - M_\theta) \]

2
If \( P(t) \) is white noise (or an impulse), \( P(\omega) \) is flat.

Hence equation 2 can be written more simply in the frequency plane as

\[
\frac{\text{constant}}{\Theta(s)} = J\omega_P^2 + (D_M - M)j\omega_P + (K_M - M\theta)
\]

The bandwidth at half power point of the corresponding power spectrum can be shown to be (3)

\[
\Delta \omega_P \approx 2 \omega_n \tau
\]

where \( \omega_P \) = frequency in pitch. Using this we have

\[
\Delta \omega_P = \frac{(D_M - M)}{J}
\]

M=0 with the wind off and we can measure \( \frac{D_M}{J} \). \( J \) is gotten either from the geometry of the model and its mass or using a step input and measuring the natural frequency (see 4 for details).

\[
M = J(\Delta \omega_P^{\text{off}} - \Delta \omega_P^{\text{on}})
\]
Hence \( C_{Ma} + C_{M\dot{a}} = \frac{M}{\frac{1}{2} \rho u_{\infty}^2 SD} \)

Now for pure plunging motion in the z direction we have

\[
m \frac{d^2 z}{dt^2} + D_M \frac{dz}{dt} + K_M z = L(t) + L_A(t)
\]

Again using the same argument as above, if \( L(t) \) is white noise and \( L_A(t) = L_A \alpha + L_A \dot{\alpha} \), with \( \alpha = \text{angle of attack} = \tan^{-1} \left( \frac{\ddot{z}}{u_{\infty}} \right) \).

Nose up to the wind \( \dot{A} \) positive angle of attack dictating the sign to be +.

Figure 2  Origin of lift from \( \alpha \).

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There is also a lift force due to $a$. Now for small

$$\frac{\ddot{z}}{u_\infty}, \quad \tan^{-1} \left( \frac{\dot{z}}{u_\infty} \right) = \frac{\dot{z}}{u_\infty}$$

7

and $\hat{a} = \frac{\ddot{z}}{u_\infty}$

8

Hence, equation 6 becomes

$$m \frac{d^2 z}{dt^2} + D_{M_L} \frac{dz}{dt} + K_{M_L} z = L(t) + L_a \left( \frac{\dot{z}}{u_\infty} \right) + L_{a} \left( \frac{\ddot{z}}{u_\infty} \right)$$

9

and again if $L(t)$ is white, then we can experimentally determine

$$(m - \frac{L_{a}}{u_\infty}) \text{ and } (D_{M_L} - \frac{L_{a}}{u_\infty})$$

by measuring the bandwidth at 1/2 power points as

$$\Delta \omega_L = \frac{(D_{M_L} - \frac{L_{a}}{u_\infty})}{(m - \frac{L_{a}}{u_\infty})}$$

10

and $\omega^2_{nL} = \frac{K_{M_L}}{(m - \frac{L_{a}}{u_\infty})}$

11
Having gotten $L_a$ and $L_a'$, we can relate them to $C_{L_a}$, $C_{L_a}'$ as

$$C_{L_a} = \frac{L_a}{\frac{1}{2} \rho u^2 S}; \quad C_{L_a}' = \frac{L_a'}{\frac{1}{2} \rho u^2 S}$$

Hence, $C_{L_a}$, $C_{L_a}'$, $(C_{M_a} + C_{M_a}')$ are determined.

In principle $C_{M_a}$ and $C_{L_a}$ can be determined from a plunging oscillation by measuring the currents and forces directly. However, two effects make this more difficult than using natural resonances. First, because of the large inertial loads, forces must be determined from differences in large numbers which have both phase and amplitude. Second, a small offset between the center of mass and the center of magnetization produces a moment as a result of vertical acceleration which must also be subtracted out. Further experience is needed to determine if these methods can be used
effectively and if these techniques can be used in the side slip planes.

The first step in incorporating Digital control of the magnetic balance system was to provide force and position inputs to the computer and position set signals to the balance. Figure 3 shows the block diagram for this interface.
A Digital Equipment Corporation minicomputer, the MINC 11/23, was used for the estimation of the stability derivatives. The E.P.S. signals from the balance provide six positions which, together with the six magnet currents, were sampled at 385 Hz simultaneously and recorded on an 8-inch floppy disk.

Two Data Translation analog-to-digital boards were used--a 16 channel single ended board that was used 'pseudo differentially' (the common side of the instrumentation amp was referenced to the magnetic balance ground) and an eight channel differential input board.

The signal levels being very low (typically < ± 1V) the boards were set up to operate with a full-scale range of 1V. The 14 bit boards hence had a resolution of \( \frac{1V}{8192} \) or approximately 0.1 mV.

The cables were shielded, twisted pair and there did not seem to be any significant degradation of the signals as seen at the computer.

An extensive library of support programs was written for
1. Input/output of data
2. Display of data
3. Plots of data, and
4. Processing of data.

Appendix A lists some of the programs that were felt to be useful in the understanding of the processing technique.
Most of the actual processing programs are interactive and require very little effort on the part of the experimenter. A concise procedure for the estimation is as follows:

I With the model hanging at the desired position, the Program 'Getdta' - gets in the positions and currents and saves them on a user specified file.

This process is repeated for different wind speeds. At each operating point four data files are recorded:

a. A wind-off noise off data
b. A wind-on noise off data
c. A wind-on noise on data
d. A wind-off noise on data

'noise' is the white noise source that is input to the desired degree of freedom.

II The Program 'Zerome' - (Zero means) is next run to take out the dc characteristics, and the operating point dc offsets for each file.

III The Program 'DISPLA' - displays any given data file and plots a graph of any given parameter. It also has the capability of doing a Fourier transform of the data and displaying the FFT.

This program also plots the graph on a line printer if desired. This is basically a program to evaluate the data and make quick 'by sight' decisions as to the validity of the results.
IV Program 'FILTER' filters out the dynamics of the magnetic balance.

V The program 'Model' finds the performance function of the system using a least squares error criteria. This is again an interactive program that uses a guess of the model order. The residual error energy is monitored and the model order chosen that has a reasonably small residual energy.

'Model' also performs a Fourier transform of the performance function which can be plotted.

The half power bandwidth of this resulting spectrum is measured and the appropriate parameters calculated. To calculate the initial balance parameters and the moment of inertia:

The program 'IGTDTA' - (Impulse and get data) may be used. This pulses the model with a small (1°) impulse and records the resulting damped oscillations. Using methods described in (3), the J and the natural frequency are evaluated.

Figure 4 Size of blocks correspond approximately to computation time.
Wind tunnel tests and results including error analysis

Wind Tunnel Tests - The model was aligned with the tunnel axis in yaw and slip and lift and centered in drag. It was then set up at a 2-degree pitch angle.

White noise at 2 intensities was used as an input + 0.125V maximum and ± 1V maximum which corresponded approximately to about 0.05 degrees and 0.2 degrees. Later, repeat runs were made at 0.05, 0.1, 0.2 and 0.4 degrees. Data was taken for four conditions:

1. Wind off noise off
2. Wind off noise on
3. Wind on noise on, and
4. Wind on noise off.

These are shown in Figures 5 - 8.

Except in one region, the spectral power did not change when the wind or the white noise was turned on, as can be seen from the figures. The magnetic suspension feedback system poles are distinctly visible with the one varying peak corresponding to the model.

Since the amplitudes of the various other peaks are comparable to the one due to the model, the analytical model formed will not be a very accurate measure of the airplane model. The data was first filtered using a digital filter of length 64 using the Ramez exchange program (2).
Do you want ffts? y=1; 
ret to read new dta; 2 to print plot; (-1 to quit)

Figure 5  FFT of wind off noise off data.
Do you want fits: y=1, ret to read new dta; 2 to print plot; (-1 to quit)

Figure 6  FFT of wind off noise on data.
Figure 7  FFT of wind on noise off data. Note the reduction of model unsteadiness! (around 11 Hz)
Figure 8: FFT of wind on noise on data.
A pass band was used extending to 18 Hz and a stop band extending from 22 Hz. The pass band ripple was designed to about 0.65 db and stop band ripple was -20 db.

The choice of this filter was made arbitrarily. If better accuracy in the parameters is desired, the length can be increased at the cost of computation time. The effect of the filter was dramatic, as can be seen from Figures 8 and 9.

It is worth pointing out that the remaining points (past 22 Hz) cannot be just chopped off because of the ringing that results between samples (Gibbs phenomenon).

It is also worth pointing out that the modified signal is still only a small signal length (original sample time) and hence, still governed by the 'uncertainty principle'.

The signal was modelled using an autocovariance method and the stationarity of the residual energy when model order five was reached taken to be a sign of good fit. The Fourier transform of the model position is shown in Figures 10 and 11 for the wind off and on respectively. The excellent resolution in frequency is noteworthy.
Figure 9  FFT of wind-on noise on data after filtering. The little peaks at 60 and 120 are due to the non-ideal nature of the filter.
Figure 10  FFT of analytical model of the wind off data for 0.2° pitch noise.
Sample Calculations

The model used had the following properties:

\[ l = 12.5 \text{ cms.} = 0.125\text{m} \]
\[ \rho = 1.19 \frac{\text{kg}}{\text{meter}^3} \text{ at 0 altitude, } T = 20^\circ\text{C} \]
\[ m = 345 \text{ gms.} \]
\[ \frac{1}{2} \rho u_\infty^2 = 4.92 \times 10^3 \text{ Newtons/meter}^2 \]
\[ \frac{1}{2} \rho u_\infty^2 SD = 0.06 \text{ Newton meters} \]
\[ \frac{1}{2} \rho u_\infty^2 S = 2.41 \text{ Newtons} \]
\[ u_\infty = 300 \text{ ft/sec} = 90.9\text{m/sec} \]
\[ D = 0.0254 \text{ meters (1 inch)} \]

Figure 14 is the white noise, as seen on a scope and Figure 15 is a pitch position response, as seen on the computer.
Figure 14

Typical white noise input.
Figure 15

Typical model response in pitch.
Results

Results of the measurements are summarized below in Table 1. Runs A and B were made first. The extensive repairs were made on the power supplies, the controls were readjusted, and a week later Runs C, D, E and F were made. From examining Table 1 it is apparent that resolution is good but scatter from Run to Run is large, particularly with respect to the natural frequency.
### Table 1

Preliminary Pitch Damping Data for Ogive Cylinder Model at 300 ft/sec, $\alpha=2^\circ$, $Re_D=0.5\times10^6$

<table>
<thead>
<tr>
<th>Run</th>
<th>Noise Amplitude Degrees</th>
<th>$\omega_n$ radians</th>
<th>$\Delta\omega$</th>
<th>Damping Ratio</th>
<th>$(C_{Ma}^* + C_{Mq}^<em>)^</em>$</th>
<th>$u_\infty$ ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.05</td>
<td>86.8</td>
<td>20.7</td>
<td>0.11</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>0.05</td>
<td>79.7</td>
<td>14.2</td>
<td>0.09</td>
<td>+ 154.6</td>
<td>300</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>80.3</td>
<td>1.8</td>
<td>0.011</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>73.8</td>
<td>4.7</td>
<td>0.032</td>
<td>- 71.6</td>
<td>300</td>
</tr>
</tbody>
</table>

Repeated 1 week later after balance repair and tuning:

<table>
<thead>
<tr>
<th>Run</th>
<th>Noise Amplitude Degrees</th>
<th>$\omega_n$ radians</th>
<th>$\Delta\omega$</th>
<th>Damping Ratio</th>
<th>$(C_{Ma}^* + C_{Mq}^<em>)^</em></th>
<th>$u_\infty$ ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.4</td>
<td>87.4</td>
<td>12.4</td>
<td>0.071</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0.4</td>
<td>76.2</td>
<td>18.3</td>
<td>0.12</td>
<td>- 143.1</td>
<td>300</td>
</tr>
<tr>
<td>D</td>
<td>0.2</td>
<td>93.9</td>
<td>12.4</td>
<td>0.066</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>0.2</td>
<td>92.1</td>
<td>20</td>
<td>0.109</td>
<td>- 186.1</td>
<td>300</td>
</tr>
<tr>
<td>E</td>
<td>0.1</td>
<td>111</td>
<td>42.5</td>
<td>0.19</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>0.1</td>
<td>86.8</td>
<td>47.2</td>
<td>0.272</td>
<td>- 114.5</td>
<td>300</td>
</tr>
<tr>
<td>F</td>
<td>0.05</td>
<td>119.8</td>
<td>46.1</td>
<td>0.192</td>
<td>+ 150.3</td>
<td>300</td>
</tr>
<tr>
<td>F</td>
<td>0.05</td>
<td>115.1</td>
<td>40.2</td>
<td>0.175</td>
<td>+ 150.3</td>
<td>300</td>
</tr>
</tbody>
</table>

This is a conventional notation with $C_{Ma}^* = \frac{\partial C_M}{\partial (\frac{\partial \alpha}{2U_\infty})}$ and $C_{Mq}^* = \frac{\partial C_M}{\partial (\frac{\partial \phi}{2U_\infty})}$. It is obtained from the values of Reference 4 by dividing by $\frac{\partial C_M}{\partial (\frac{\partial \phi}{2U_\infty})}$.  

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This indicates that the control system should have been adjusted to produce the same wind off natural frequency for each data point. Also, a larger data sequence should be tried to reduce scatter.

The negative damping (positive coefficient) observed at .05° amplitude is probably a result of system noise. Since noise tends to broaden the response curve, it is surmised that at very low excitation the decrease in noise resulting from damping on the wind offsets the broadening caused by aerodynamic damping. At higher excitations where the only power points are well above the noise level, the effect is not important. This suggests that excitation should be .2 to .4 degrees.

An estimate of the relative error can be made as follows:

\[
e = \sqrt{\left(\frac{\delta\rho}{\rho}\right)^2 + \left(2\frac{\delta u}{u}\right)^2 + \left(2\frac{\delta D}{D}\right)^2 + \left(\frac{\delta f}{f}\right)^2}
\]

\[
+ \left(\frac{\delta m}{M}\right)^2 + \left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta A}{D}\right)^2
\]

\[
e = \sqrt{\left(\frac{0.005}{1.19}\right)^2 + \left(\frac{2 \times 0.5}{300}\right)^2 + \left(\frac{2 \times 0.05}{2.5}\right)^2 + \left(\frac{0.05}{12.5}\right)^2}
\]

\[
+ \left(\frac{0.5}{345}\right)^2 + \left(\frac{0.005}{18}\right)^2 + \left(\frac{0.1}{200}\right)^2
\]

\[
e = 0.041; \text{ i.e., } -4\% \text{ error.}
\]

*These are typical values.
This holds for repeated experiments at the same amplitudes.

The scatter in the three experimental values of runs D,E,F (dynamic stability derivatives -143,-186,114) may be related to the shift in stiffness in runs. The change in value with amplitude may be due to an additional error made in the fit due to the S/N being different for the three cases but further experiments should clarify these issues.

As with any new technique of magnetic balance experimentation, a sizeable base of experience is needed to establish the best way to obtain a given type of data.

Summary and conclusions

A technique using random noise excitation for estimating dynamic stability derivatives using the magnetic suspension system has been implemented. The method is not sensitive to drift and noise in the system. It also has the advantage of being simple and computationally not demanding. A 64K Byte MINC was adequate. Typical turnaround times from the start of the experiment to the evaluation of dynamic stability derivatives is of the order of an hour.

Theoretical calculations based on linear theory and assuming an \( x_{ref} = 2.75 \) inches from the nose, give a value of \(-286\) for \( C_{Mq} + C_{Ma} \) for this body.
Theoretical calculation of $C_{Ma}$ gives a value of 1.477.

The experimental values obtained by measuring natural frequency are 3.32, 0.9 and 5.94 corresponding to 0.1°, 0.2° and 0.4° noise.

Since $C_{Ma}$ is computed directly from the change in stiffness wind on, the scatter is probably due to the change in wind off stiffness from run to run. If this can be eliminated, data scatter should be greatly reduced.

More work is needed to determine the cause of this scatter and whether static derivative estimation using this method can be competitive to the method of measuring currents and inferring the forces and moments acting on the body.

Some interesting facts about the balance can be seen from Figures 5 to 9 with the wind on, the balance is very quiet. Compare this power (Figure 5) to the spectrum with the wind on (Figure 7). This says that tunnel unsteadiness alone is not enough to excite the frequencies in the model for reasonable resolution.

It is reassuring to note that the system model derived by the white noise method ($s^2+12.44s+87.6^2$) is very close to that obtained by Peter Luh using forced oscillation techniques ($s^2+12s+85.21^2$) (5).

Two hundred and fifty-six data samples were taken at 385 Hz, which corresponds to approximately 1 second of data. This sampling rate for twelve channels is more than proportionately lower than the 15 KHz possible for only one channel.
It may be possible to increase this to nearer 1 KHz with software changes. However, increasing the sample rate well above the natural frequencies of the signal has adverse effects. It tends to cluster the poles right on the unit circle and the resolution of poles close together is reduced. 385 Hz is more than adequate since the highest frequency in the system seems to be around 30 Hz.

The simultaneous acquisition of data from all channels has the added advantage of being very useful for the evaluation of cross couplings between different degrees of freedom. Essentially the same procedure may be used to estimate the effect of an input in one degree of freedom on another.

Static stability derivatives may be measured simultaneously to the dynamic ones by computation of the mean value of the signal with the wind on compared to the mean value with the wind off. However, this method has not been compared with direct force measurement for accuracy.

The parameter estimation programs can be incorporated as a subroutine in another program that analytically computes the half power bandwidth from the system performance function and hence, directly outputs the dynamic stability derivatives. However, a direct involvement in the mechanics of the procedure was retained in order to make sure that the numbers made sense.

The 64K Byte memory limitation on the MINC restricted taking more than 256 samples at a time. An additional 64K which is on order will eliminate this limitation. Ideally,
the number of samples should be approximately 100 times model order for a good estimate of the performance function.

The length 64 FIR filter has a stop band ripple of -20 dB and pass band ripple of 0.65 dB. This was arbitrarily chosen and the accuracy of the derivatives may be improved by increasing the length of the filter. This will increase computation time, however.

An important thing to keep in mind is that different model dynamics require a change in the filter characteristics (see Appendix D).

More experiments need to be done with models whose characteristics are either known or analytically determined and the results evaluated. Once this is done the system is ready for use as an easy to use and extremely versatile balance that can perform experiments until now not possible.
Listed in the order referenced:


This is the channel table for the magnetic wind tunnel computer interface.

<table>
<thead>
<tr>
<th>data file</th>
<th>Single Ended</th>
<th>data file</th>
<th>Differential Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>x array</td>
<td>ch clm</td>
<td>ch clm</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Drag Position</td>
<td></td>
<td>Lift Current</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Slip Position</td>
<td></td>
<td>Yaw Current</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Yaw Position</td>
<td></td>
<td>Pitch Current</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Lift Position</td>
<td></td>
<td>Drag Current</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Pitch Position</td>
<td></td>
<td>Side force Current</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Magnetizing Current</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Input Signal</td>
</tr>
</tbody>
</table>

Data files and their interpretation:

- **WNDFFn.DAT**: Raw data with wind off noise off
- **WNDFFn.DAT**: Raw data with wind off noise on
- **WNDNFn.DAT**: Raw data with wind on noise off
- **WNDNNn.DAT**: Raw data with wind on noise on
- **ZWDFFn.DAT**: Zero mean wind off noise off data
- **ZWDFFn.DAT**: Zero mean wind off noise on data
- **ZWDNFn.DAT**: Zero mean wind on noise off data
- **ZWDNNn.DAT**: Zero mean wind on noise on data
- **FWDFFn.DAT**: Filtered wind off noise off data
- **FWDFFn.DAT**: Filtered wind off noise on data
- **FWDNFn.DAT**: Filtered wind on noise off data
- **FWDNFnn.DAT**: Filtered wind on noise on data

(The n denotes the run number)

Coutham Ramohalli
M.I.T., Cambridge
8-May-81

37
C THIS PROGRAM GETS IN THE 6 CURRENTS, 5 POSITIONS AND THE INPUT
FROM THE MAGNETIC BALANCE AND SAVES THEM ON DISK ON A FILE
THAT YOU ASK IT TO

C

Gautham Ranohalli
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PROGRAM GETDATA

LOGICAL*! LIN, BLANK
DIMENSION LIN(10), BLANK(10)
INTEGER I, J, K
REAL X(256,5), Y(256,7), SUM, MEAN
INTEGER IX(256)
N=256
DATA X, Y/128000., 179280./
WRITE(5,500)
500 FORMAT(' ENTER 10 BLANKS')
READ(5,501) BLANK
501 FORMAT(10A1)
1 DO 555 I=1, 10
LIN(I)=BLANK(I)
555 CONTINUE
CALL A2DM(0.0, N, X, 1.)
CALL A2DM(1.0, N, Y, 1.)
CALL SCAL(Y(1,7), IX, N)
SUM=0.
DO 200 I=1, N
SUM=SUM+Y(I,7)
200 CONTINUE
MEAN=SUM/FLOAT(N)
CALL CLEAR
CALL GRAPN(N, IX)
WRITE(5,999) MEAN
999 FORMAT(' Mean Value=' , F8.3, ' The data displayed is the white noise
X input')
2 WRITE(5,998)
998 FORMAT(' Do you want this data written to a file? y=1, Read no
Xream=0, oui=1, -2')
READ(5,997) KR
IF(KR.EQ.-2) GO TO 1000
IF(KR.EQ.1) GO TO 1
CALL CLEAR
WRITE(5,996)
997 FORMAT(12)
996 FORMAT(' Enter the name of the file to be saved as')
READ(5,557) LIN
557 FORMAT(10A1)
OPEN(UNIT=1, NAME=LIN, FORM=' UNFORMATTED', TYPE=' NEW')
DO 1200 I=1, N
WRITE(1) (X(I,J), J=1, 5)
WRITE(1) (Y(I,J), J=1, 7)
1200 CONTINUE
CLOSE(UNIT=1)
GO TO 2
1000 CALL CLEAR
This program takes an input file and filters a given channel and writes the file back as a filtered file. The program uses a 64 length finite impulse response filter with a pass band that extends to 18Hz and stop band that starts at 27Hz. This effectively filters out the magnetic balance system feedback effects from the signal and what remains is a pure signal due to the airplane dynamics.

Gautham Ramachalli
8-may-81
M.I.T., Cambridge

Program filter

dimension LIN(10), LOUT(10), BLANK(10)
LOGICAL1 LIN, LOUT, BLANK, i,p
Real x(256:5), u(256:7), h(64), rdum(7)
complex xx(1024), hh(1024)
Integer i, j, k, ichnl, f2a9

data b/1HP/
OPEN(UNIT=1, NAME='FILTER.DAT', TYPE='OLD')
READ(1,8)(h(i), i=1,32):
close(unit=1)
do 9081 i=1,32
  h(i+32)=h(33-i)
8081 continue
do 8082 i=1,64
  hh(i)=h(i)
8083 format(' h(''I2'') = ''E20.9)
8082 continue
do 8084 i=65,1024
  hh(i)=0.
8084 continue
call fourera(hh,1024,-1)
WRITE(5,950)
950 FORMAT(' ENTER 10 BLANKS PLEASE')
READ(5,501) BLANK
501 FORMAT(10A1)
1 DO 1000 i=1,10
  LIN(i)=BLANK(i)
  LOUT(i)=BLANK(i)
1000 continue
write(5,999)
999 format(' enter the name of the input file')
read(5,998) LIN
998 format(10A1)
write(5,950) LIN
write(5,2716)
2716 format(' enter the channel to be filtered! (ex: 5) ')
read(5,2717) ichnl
2717 format(A1,12)
write(5,2718) ichnl
2718 format(' filter channel ''A1,12''? ')
read(5,2719) kr
2719 format(12)
    if(kr.eq.-1) go to 1
958 format(' file name=' ,10A1)
1040 continue
    file=0
    if(r.ne.b) file=1
    open(unit=1, name=lin, form='unformatted', type='old')
    do 1100 i=1,256
        read(1) (x(i,j),j=1,5)
        read(1) (w(i,j),j=1,7)
    1100 continue
    close(unit=1)
    if(file.eq.1) go to 1200
    do 1150 i=1,256
        xx(i)=x(i,ichn1)
    1150 continue
70 to 1250
1200 do 1250 i=1,256
    xx(i)=v(i,ichn1)
1250 continue
    do 1270 i=1,256,1024
        xx(i)=0.0
    1270 continue
    call fousxx(xx,1024,-1)
    do 1280 i=1,1024
        xx(i)=xx(i)*hh(i)
    1280 continue
    call fousxx(xx,1024,1)
    if(file.eq.1) go to 1310
    do 1290 i=1,256
        xx(i)=real(xx(i))
    1290 continue
    go to 1320
1310 do 1320 i=1,256
    w(i,ichn1)=real(xx(i))
1320 continue
997 format(' enter the name of the output file')
957 format(' input file=' ,10A1, ' output=' ,10A1)
write(5,997)
997 read(5,998) lout
write(5,957) lin,lout
open(unit=2, name=lout, form='unformatted', type='new')
1500 do 1500 i=1,256
    write(2) (x(i,j),j=1,5)
    write(2) (w(i,j),j=1,7)
1500 continue
    close(unit=2)
    write(5,990)
990 format(' Do you have any more files to work on? y=')
rebd(5,989) kr
989 format(12)
    if(kr.eq.1) go to 1
stop
This program reads in an input file (magnetic balance data) and performs a covariance method of signal modelling. The program is interactive and asks for the file name and the length of data and model order. It looks back for a change in model order.

Program Model

logical LIN, blank, r, b
dimension LIN(10), blank(10)
double precision X(256), A(21), AC(21), wv(300), A2(21)
double precision ALPHA
real rda(7), v(512), xx(512), PI
integer nn, nmr, j, k, IX(512), jfr, ich1
complex z(1024), W

DATA b: w/1HP, 5120, ff=12
PI=ATAN(1.)/(4.)
W=CMPLX(0., PI)
write(5, 15) b
DO 17 I=1, 256
X(I)=0.
17 continue
DO 19 I=1, 21
RC(I)=0.
A(I)=0.
19 continue
ALPHA=0.
n=256
write(5, 500)
DO 500 I=1, 256
write(5, 501) blank
DO 510 I=1, 21
write(5, 501) blank
DO 501 I=10A1
500 format(' Enter a blank line please')
read(5, 501) blank
501 format(10A1)
500 format(' Enter the name of the file to be read')
1 do 5 I=1, 10
LIN(I)=blank(I)
5 continue
write(5, 999)
read(5, 998) LIN
998 format(10A1)
write(5, 997) LIN
997 format(' The file name is ', 10A1)
write(5, 996)
996 format(' To display a particular parameter type the appr no')
write(5, 995)
995 format(' x PI=drag position', 140, ' C1=lif current')
write(5, 994)
994 format(' x PI=drag position', 140, ' C2=wa current')
write(5, 993)
993 format(' x PI=drag position', 140, ' C3=pitch current')
41
write(5,992)
992 format(' P4=lift position',T40,'C4=drag current')
write(5,991)
991 format(' PS=pitch position',T40,'C5=slip current')
write(5,990)
990 format(T40,'C6=armature current')
write(5,989)
989 format(T40,'C7=torque signal')
read(5,988) r,ichnl
988 format(A1,I2) r,ichnl
987 do mat('1',' display A1,I2'+data'+(-1 to read file again)')
986 do mat('1') r,ichnl
985 if(kr,e<=1) go to 1
984 call clear
983 if(r,e<=1) go to 1190
982 OPEN(UNIT=1,NANE=LIN,FOM='UNFORMATTED',TYPE='OLD')
981 do 1100 i=1,nn
980 read(i) (rdum(j),j=1,5)
979 do 1100 i=1,nn
978 read(i) (rdum(j),j=1,7)
977 v(i)=rdum(ichnl)
976 X(i)=rdum(ichnl)
975 1100 continue
974 close(unit=1)
973 1190 OPEN(UNIT=1,NANE=LIN,FOM='UNFORMATTED',TYPE='OLD')
972 do 1250 i=1,nn
971 read(i) (rdum(j),j=1,5)
970 do 1250 i=1,nn
969 w(i)=rdum(ichnl)
968 X(i)=rdum(ichnl)
967 read(i) (rdum(j),j=1,7)
966 1250 continue
965 close(unit=1)
964 1280 call scal(v,IX,256)
963 call graph(256;IX)
962 write(5,985)
961 format(' ret to read new data? model signal?(-1 to quit)')
960 1275 read(5,986) kr
959 format(I2)
958 call clear
957 if(kr,e<=1) go to 3000
956 if(kr,e>2) go to 2500
955 so to 1
954 write(5,969)
953 format(' Make sure the line printer is set up right!')
952 write(5,968)
951 format(' The vertical 1r=6,forelen=66')
950 1567 read(5,967) kr
949 format(I2)
948 557 write(6,777) ff
947 write(6,970) LINr,ichnl
946 format(' ///// Data file=10Al,T30,Pos/Cur=A1,T50,Chnl=12')
945 777 format(A1)
WRITE (6, 979)
979 FORMAT (' -----------------------',//)
WRITE (5, 950)
950 FORMAT (' model order=??')
READ (5, 951) m
951 FORMAT (I2)
WRITE (6, 952) n
952 FORMAT (' model order=??')
WRITE (5, 953)
953 FORMAT (' length of signal=??')
READ (5, 954) n
954 FORMAT (I2)
call covar(n, x, m, alpha, rc)
955 FORMAT (I2)
call covar(n, x, m, alpha, rc)
956 FORMAT (/* model coefficients, T50, reflection coefficient
DO 350 i=1, m
   WRITE (6, 957) (1, a(i), RC(i), i=1, m)
350 CONTINUE
WRITE (6, 958) ALPHA
957 FORMAT (/* length of the signal=??')
WRITE (6, 959)
958 FORMAT (/* residual energy=??')
WRITE (6, 959)
350 CONTINUE
WRITE (6, 958) ALPHA
959 FORMAT (/* which model do you want ffts for covar=0, auton=1)
WRITE (5, 986) kr
960 CONTINUE
WRITE (6, 979)
961 CONTINUE
WRITE (5, 555)
555 FORMAT (/ return to change model order!!! to read new file!!
   X 2 to set ffts of models!!-1 to auto!!)
   READ (5, 986) kr
   IF (kr .eq. 2) GO TO 8092
   WRITE (5, 7070)
7070 FORMAT (/* which model do you want ffts for covar=0, auto=1)
   READ (5, 986) kr
   IF (kr .eq. 0) STOP
if(kr.eq.1) flag=1.
write(5,8089)
8089 format(’computing ffts’)
if(flag.eq.1) go to 8082
C
do 8082 i=1,mp
C
z(i)=A(i)
8082 continue
C
do 7072 J=2,mp
  z(i)=z(i)+A(J)*CEXP(-(W*FLOAT(i-1)/2048.)*FLOAT(J-1))
7072 continue
C
z(i)=1./z(i)
707 continue
C
do 8083 i=1,mp
  y(i)=cabs(z(i))
8083 continue
C
call fourera(z,1024,-1)
do 707 i=1,512
  z(i)=1.0
do 702 J=2,mp
  z(i)=z(i)+A(J)*CEXP(-(W*FLOAT(i-1)/2048.)*FLOAT(J-1))
702 continue
z(i)=1./z(i)
707 continue
do 8084 i=1,mp
  xx(i)=(float(i-1)/4096.)*385.
8084 continue
write(5,8085)
8085 format(’ret to loop? 2 to print plot -1 to quit’)
read(5,986) kr
if(kr.eq.2) go to 8092
  do 8086 i=1,mp
    xx(i)=(float(i-1)/4096.)*385.
8086 continue
write(5,8087)
8087 format(’enter no of points to be plotted’)
read(5,8088) npts
8088 format(’13)
write(6,777) ff
write(6,970) lin,richnl
write(6,979)
write(6,952) n
write(6,979)
write(5,8090)
8090 format(’make sure printer is set for 12 lpi and 132 lpi’)
pause
call plot(xx,y,npts)
call clear
so to 8091
8092 call clear
if(kr)3000,557,1275
3000 call clear
stop
Appendix C

**LIST OF SYMBOLS**

- $D_{M_L}$: Magnetic damping in lift
- $K_{M_L}$: Magnetic stiffness in lift
- $D_{M_P}$: Magnetic damping in pitch
- $K_{M_P}$: Magnetic stiffness in pitch
- $\zeta$: Damping ratio
- $\omega$: Frequency in radians/sec
- $J$: Moment of inertia about the center of gravity
- $u_\infty$: Free stream velocity in the $-x$ direction
- $q$: $\frac{6D}{2u_\infty}$
- $\theta$: Pitch angle with respect to the $x$ axis
- $M$: Moment acting on the body
- $\alpha$: Angle of attack
- $\rho$: Density of air
- $m$: Mass of body
- $S$: Characteristic area
- $l$: Characteristic length
- $\#$: By definition is
L = aerodynamic lift force on the body.

\[ C_L = \frac{L}{\frac{1}{2} \rho u_\infty^2 S} \]

\[ C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} \]

\[ C_{L_{\dot{\alpha}}} = \frac{\partial C_L}{\partial \dot{\alpha}} \]

\[ C_M = \frac{M}{\frac{1}{2} \rho u_\infty^2 SD} \]

\[ C_{M_\theta} = \frac{\partial C_M}{\partial \theta} \]

\[ C_{M_{\dot{\alpha}}} = \frac{\partial C_M}{\partial \dot{\alpha}} \]

NOTE. THIS DEFINITION LEADS TO UNITS OF SECS. FOR \( C_{M_\alpha} + C_{M_{\dot{\alpha}}} \) AND \( C_{L_\alpha} \).
Appendix D

FILTER DESIGN GUIDE

The Remez exchange algorithm needs the following inputs:

Card 1  N, ITYPE, NBANDS, NGRID

N - order of filter. (This is an interactive procedure where you keep increasing N until the desired attenuation in the stop band and pass band ripple is achieved.)

N=1: Low pass high pass or band pass

NBANDS = Number of bands, eg., a low pass filter has two bands.

NGRID: Grid density always set to sixteen.

For example, a length fifteen low pass filter would have the following first card: 15, 1, 2, 16

Card 2  F1LOW, F1HIGH, F2LOW, F2HIGH

FNBANDS LOW, FNBANDS HIGH

This card specifies the edges of the bands. There are NBANDS as specified on Card 1. These band edges are entered as real numbers (not more than four per line) and normalized to (0.0, 0.05) interval corresponding to the interval (0-1/2T) where T is the sampling time. In this case 1/2T = 187 Hz.
Thus a low pass filter with pass band to 18 Hz, stop band from 22 Hz would have Card 2. 0.0, 0.0467, 0.057, 0.5

Card 3 AMPl,.. AMPNBANDS

This card specified the desired magnitude response in each band. For example, a low pass filter would have

Card 3 1.0, 0.0

Card 4 Wl, ... WNBANDS

Specifies the ripple weighting in each band. These are real numbers. The ripple ratio is given by δ₁/δ₄, where δ₁ is the maximum derivation allowed in the first band and δᵣ is the derivation allowed in the ith band.

Thus for a low pass filter with δ₁ = 0.1, δ₂ = 0.2, this card would be 1.0, 0.5

The result of running this program is a data file called FILTER.DAT that gives the impulse response. This file should then be transferred to the disk where the FILTER.SAV program resides (i.e., the same disk that you have your magnetic balance data). The FILTER.SAV program reads in the filter characteristics before it operates on the magnetic balance data file.

Detailed instructions for the use of this procedure will be found in a user manual that is being planned and should be available at the balance.
CALLING REMEZ AND RUNNING PROGRAM. DON'T INTERRUPT

DEVIATION = -0.000231585
DEVIATION = -0.039413721
DEVIATION = 0.132467500
DEVIATION = 0.150773944
DEVIATION = -0.155212869
DEVIATION = -0.156569904
DEVIATION = -0.156635265
DEVIATION = -0.156635449

FINITE IMPULSE RESPONSE (FIR)
LINEAR PHASE DIGITAL FILTER DESIGN
REMEZ EXCHANGE ALGORITHM

BANDPASS FILTER

FILTER LENGTH = 64

*** IMPULSE RESPONSE ***
H( 1) = -0.76954342E-01 = H( 64)  
H( 2) =  0.33773184E-02 = H( 63)  
H( 3) =  0.45766904E-02 = H( 62)  
H( 4) =  0.63917860E-02 = H( 61)  
H( 5) =  0.85576177E-02 = H( 60)  
H( 6) =  0.10764614E-01 = H( 59)  
H( 7) =  0.12635142E-01 = H( 58)  
H( 8) =  0.13839498E-01 = H( 57)  
H( 9) =  0.14070064E-01 = H( 56)  
H(10) =  0.13136059E-01 = H( 55)  
H(11) =  0.10920897E-01 = H( 54)  
H(12) =  0.74827075E-02 = H( 53)  
H(13) =  0.29894039E-02 = H( 52)  
H(14) =  0.21979585E-02 = H( 51)  
H(15) =  0.76648444E-02 = H( 50)  
H(16) =  0.12855738E-01 = H( 49)  
H(17) =  0.17256759E-01 = H( 48)  
H(18) =  0.19906990E-01 = H( 47)  
H(19) =  0.20732537E-01 = H( 46)  
H(20) =  0.19146100E-01 = H( 45)  
H(21) =  0.14848895E-01 = H( 44)  
H(22) =  0.77870712E-02 = H( 43)  
H(23) =  0.17256759E-01 = H( 42)  
H(24) =  0.19906990E-01 = H( 41)  
H(25) =  0.20732537E-01 = H( 40)  
H(26) =  0.19146100E-01 = H( 39)  
H(27) =  0.14848895E-01 = H( 38)  
H(28) =  0.77870712E-02 = H( 37)  
H(29) = -0.17256759E-01 = H( 36)  
H(30) = -0.19906990E-01 = H( 35)  
H(31) = -0.20732537E-01 = H( 34)  
H(32) = -0.19146100E-01 = H( 33)  
H(33) = -0.14848895E-01 = H( 32)  
H(34) = -0.77870712E-02 = H( 31)  
H(35) =  0.17256759E-01 = H( 30)  
H(36) =  0.19906990E-01 = H( 29)  
H(37) =  0.20732537E-01 = H( 28)  
H(38) =  0.19146100E-01 = H( 27)  
H(39) =  0.14848895E-01 = H( 26)  
H(40) =  0.77870712E-02 = H( 25)  
H(41) = -0.17256759E-01 = H( 24)  
H(42) = -0.19906990E-01 = H( 23)  
H(43) = -0.20732537E-01 = H( 22)  
H(44) = -0.19146100E-01 = H( 21)  
H(45) = -0.14848895E-01 = H( 20)  
H(46) = -0.77870712E-02 = H( 19)  
H(47) =  0.17256759E-01 = H( 18)  
H(48) =  0.19906990E-01 = H( 17)  
H(49) =  0.20732537E-01 = H( 16)  
H(50) =  0.19146100E-01 = H( 15)  
H(51) =  0.14848895E-01 = H( 14)  
H(52) =  0.77870712E-02 = H( 13)  
H(53) = -0.17256759E-01 = H( 12)  
H(54) = -0.19906990E-01 = H( 11)  
H(55) = -0.20732537E-01 = H( 10)  
H(56) = -0.19146100E-01 = H(  9)  
H(57) = -0.14848895E-01 = H(  8)  
H(58) = -0.77870712E-02 = H(  7)  
H(59) =  0.17256759E-01 = H(  6)  
H(60) =  0.19906990E-01 = H(  5)  
H(61) =  0.20732537E-01 = H(  4)  
H(62) =  0.19146100E-01 = H(  3)  
H(63) =  0.14848895E-01 = H(  2)  
H(64) =  0.77870712E-02 = H(  1)  

 console.log(impulseResponse);
| H(23)     | 0.19024462E-02 | H(42)     |
| H(24)     | 0.18868979E-01 | H(41)     |
| H(25)     | 0.27566157E-01 | H(40)     |
| H(26)     | 0.42291783E-01 | H(39)     |
| H(27)     | 0.57208348E-01 | H(38)     |
| H(28)     | 0.71434379E-01 | H(37)     |
| H(29)     | 0.84089835E-01 | H(36)     |
| H(30)     | 0.94392702E-01 | H(35)     |
| H(31)     | 0.10165238E+00 | H(34)     |
| H(32)     | 0.10535892E+00 | H(33)     |

| LOWER BAND EDGE | 0.0000000 | BAND 1 | 0.0571000 |
| UPPER BAND EDGE | 0.0468000 | BAND 2 | 0.5000000 |
| DESIRED VALUE   | 1.0000000 |        | 0.0000000 |
| WEIGHTING       | 2.0000000 |        | 1.0000000 |
| DEVIATION       | 0.0783177 |        | 0.1566354 |
| DEVIATION IN DB | 0.6549352 |        | -16.1021996 |

**EXTREMAL FREQUENCIES** -- MAXIMA OF THE ERROR CURVE

| 0.0117187 | 0.0273437 | 0.0400391 | 0.0468000 | 0.0571000 |
| 0.0649125 | 0.0795609 | 0.0951859 | 0.1108109 | 0.1264360 |
| 0.1430375 | 0.1586625 | 0.1742875 | 0.1899125 | 0.2065141 |
| 0.2221391 | 0.2377641 | 0.2543457 | 0.2699007 | 0.2856157 |
| 0.3012407 | 0.3178422 | 0.3334672 | 0.3490922 | 0.3647172 |
| 0.3813188 | 0.3969438 | 0.4125688 | 0.4281938 | 0.4447953 |
| 0.4604203 | 0.4760453 | 0.4916703 |        |        |

1 . 0 . 0 . 0 . 0