THEORETICAL LINEAR APPROACH TO THE COMBINED MAN-MANIPULATOR SYSTEM IN MANUAL CONTROL OF AN AIRCRAFT

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SUMMARY

A new approach to the calculation of the dynamic characteristics of the combined man-manipulator-system in manual aircraft control has been derived from a model of the neuromuscular system similar to that described by McRuer and Magdaleno. (Ref. 1) This model combines the neuromuscular properties of man with the physical properties of the manipulator system which is introduced as pilot-manipulator model into the manual aircraft control. The assumption of man as a quasilinear and time-invariant control operator adapted to operating states - depending on the flight phases - of the control system gives rise to interesting solutions of the frequency domain transfer functions of both the man-manipulator system and the closed loop pilot-aircraft control system. It can be shown that it is necessary to introduce the complete precision pilot-manipulator model into the closed loop pilot-aircraft transfer function in order to understand the well known handling quality criteria of MIL-F-8785B/C, and to derive these criteria directly from human operator properties.

INTRODUCTION

The pioneer work on the precision pilot model presented by D.T. McRuer and his co-workers (Refs. 1...3) has become the most important step towards our understanding of the role of man in manual control. It is the combination of neurophysiology, physical dynamics, and control theory which gives the fascinating aspects of how the several complicated problems of manual control should be solved. But, there is a little gap between the theory and practical solutions, because the precision theory always turns out to be too complicated if engaged to solve such problems as the question for the best manipulator characteristics in a high performance aircraft.

In fig. 1 the precision pilot model is shown as it was presented by McRuer and Magdaleno (Ref. 2), which enlightens schematically what is running in the neuromotor system when the pilot puts his hand (or legs) on the manipulator. For practical use however, the human engineer wants to revise this model to also parametrically based but simpler facts.
Fig. 1: Precision pilot model, after McRuer and Magdaleno (ref. 1)

If you ask the pilot about manual control qualities, he explains his wishes on

- stick forces
- aircraft response (as it is)
- predictability of response (as he expects it is)
- lead elements (after inquiry),

but he is not accustomed to consider his stick force and displacement feedback. If the pilot must consider these, the handling qualities of the system may be bad.

So for practical use we tried to reorganize the pilot model. Fig. 2 shows another pilot model less sophisticated as that of fig. 1 but more practical in use. Force and displacement feedback now feeds into the spinal chord without becoming conscious to the operator who is engaged in the compensation of system lags by lead. This model too can be fully identified by physical parameters. (Fig. 3) We now can divide the pilot model again into two parts: part A identifies the mental parameters (lead) while part B represents the neuromuscular-physical parameters of the combined man-manipulator system (lag).

Now the intention of the following analysis is the formulation of the frequency domain transfer function of this quasilinear pilot model and the calculation of amplitude and phase shift of the man-manipulator combination; while further important investigations include the pilot-aircraft closed loop characteristics and their influence on handling qualities. (Ref. 5)
Fig. 2: Precision pilot model after P. Bubb (ref. 4), used in this report

Fig. 3: Physical properties of the precision pilot model after P. Bubb (ref. 4)
ANALYSIS

Formal arrangement of the model.

The model (fig. 2) can be arranged formally alike

(1) \[ F(s) = \frac{F_A(s)F_V(s)F_E(s)F_L(s)}{1+F_K(s)F_L(s)F_W(s)} , \]

where

(1.1) \[ F_A(s) = \frac{K_A}{1+T_A} e^{-\tau_A s} \] (information input term)

(1.2) \[ F_V(s) = \frac{K_V}{1+T_V} e^{-\tau_V s} \] (information processing term)

(1.3) \[ F_E(s) = \frac{K_E}{1+T_E} e^{-\tau_E s} \] (second order lag of manipulator)

(1.4) \[ F_L(s) = e^{-\tau_L s} \] (neuronal impulse delay)

(1.5) \[ F_K(s) = \frac{K_K}{1+T_K} e^{-\tau_K s} \] (delayed force-feedback)

(1.6) \[ F_W(s) = \frac{K_W}{1+T_W} e^{-\tau_W s} \] (delayed displacement feedback)

Rearranging the equations (1.1)...(1.6) into eq. (1) results in

(2) \[ F(s) = \frac{A(s)B(s)}{1+R_E s^2 + M_E s^4} \]

and rewritten by the following introductions

\[ K_p, K_A, K_V, K_E, K_W \] (effective pilot gain)

\[ \tau_A, \tau_V, \tau_E \] (effective pilot delay time)

\[ \tau_K, \tau_L, \tau_W \] (feedback delay time, since \( \tau_K \approx \tau_L \))

yields

(3) \[ F(s) = \frac{A(s)B(s)}{1+R_E s^2 + M_E s^4} \]
To achieve the roots of this system, we first have to eliminate the delay terms as much as possible, which is done by multiplication of both the numerator and denominator of eq. (3) by $e^{+\bar{\tau}s}$. This results in the replacement of $e^{-\bar{\tau}es}$ by $e^{-\bar{\tau}es} = e^{-(C_{e}-\bar{\tau})s}$ in the numerator, and another form of the denominator $D_p(s)$ of eq. (3) like

$$D_p(s) = (1+R_E s+M_E s^2)[e^{s\bar{\tau}s} + K_e (1+T_E s)] + K_p K_e W (1+T_W s)$$

While the numerator has already its final root arrangement, the denominator has not. For better handling, the term $e^{+\bar{\tau}s}$ should be approximated by a Taylor series

$$e^{+\bar{\tau}s} = 1 + \bar{\tau}s + \frac{\bar{\tau}^2 s^2}{2} + R(s)$$

**5.1** $R(s) = \frac{\bar{\tau}^n s^n}{n}$, $n = 2, 3, ... 6$

the frequency range of which is $0 < (s=j) < 70$ rad/sec. with an amplitude and phase error below 20 %, for $n = 2$:

$$F_p(s) = A(s) B(s) =$$

$$\frac{K_p (1+T_E s)(1+T_W s) e^{-\bar{\tau}es}}{(1+R_E s+M_E s^2)[1+s\frac{\bar{\tau}^2 s^2}{2} + \frac{\bar{\tau}^n s^n}{n} + K_E (1+T_E s)] + K_p K_e W (1+T_W s)}$$

The roots of the denominator polynomial are solved by the calculation of the denominator polynomial form

$$D_p = a_0 + a_1 s^1 + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5$$

and by comparison of the coefficients $a_i$ and the coefficients $b_i$ of another, well-known polynomial the roots of which meet the 5th order arrangement

$$(1+b_1 s)(1+b_2 s+b_3 s')(1+b_4 s+b_5 s')$$

The necessary arithmetics are described and discussed in ref.5. It has to be mentioned, that an exact solution of a 5th order linear arithmetic equation does not exist.

**SOLUTION**

The best solution of eq. (6) is the following

$$F_p(s) = A(s) B(s) =$$

$$\frac{K_p (1+T_E s)(1+T_W s) e^{-\bar{\tau}es}}{K (1+T_W s)(1+\frac{1+KK}{K} R_E s + \frac{1+KK}{K} M_E s^2)(1+\frac{1}{K} R_E s + \frac{1}{K} M_E s^2)}$$

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which describes the required roots of a 5th order lag system \( B(s) \), if

\[
A(s) = K \left( 1 + T_A s \right) \left( 1 + T_V s \right) e^{-\delta s}.
\]

The conditions for the solution of \( B(s) \) within eq. (9) are:

1. \( K = 1 + K + K_K \) (combined feedback gain)
2. \( T_W = \frac{R}{E} = \frac{\beta}{E} \) (neuromuscular lag time constant)
3. \( \frac{1 + K_K}{K} R_E = \frac{25}{\omega E} \) (2nd order lag time constant)
4. \( \frac{1 + K_M}{K} E = \frac{1}{\omega E^2} \) (man-manipulator resonant frequency)
5. \( T_E = \frac{R + R_s}{2\sqrt{R_mR_s}} \geq 1.0 \) (Jamping of the 2nd order lag term)
6. \( M_E = \frac{M_m + M_s}{C_m + C_s} \) (Inertial resistance of the manipulator)
7. \( R_E = \frac{R_m + R_s}{C_m + C_s} \) (viscous resistance of the manipulator)
8. \( \delta = \sqrt{n + 2 \left( \frac{(R_m + R_s)^2}{2k\delta R_m R_s} \right)} - \frac{(3R_m + R_s)^2}{2k\delta R_m R_s} \)

\( \delta \) is a "high frequency weighting factor", in which \( \beta, k, \) and \( n \) are defined as follows

\[
\beta = \frac{1 + (K_K/4)}{1 + K_K/4}, \quad k = \frac{1 + K}{K}, \quad n = 2 \quad (\text{see eq. 5.1.).}
\]

The 5th order solution (9) degrades to a 3rd order solution, if \( \delta > \beta \). A limit of \( \delta \) is given by \( \delta < \beta \); if \( \delta < \beta \), set \( S = 0 \), while \( \epsilon \) is defined by

\[
\epsilon = \frac{1}{70 \sqrt{K_K E}}, \quad \text{i.e. if}
\]

\[
\omega^2 = \frac{1}{\sqrt{k\beta M_E}} \leq 70 \text{ rad/sec which is the limit in } \omega \text{ defined by the approximation in eq. (5.1)}.
\]

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Numerical evaluation.

While the numerical values of $M_m$ (limb inertia), $M_s$ (manipulator inertia), $C_s$ (feel spring constant), and $R_s$ (manipulator viscous resistance) can be achieved by measurement or effectively are known - the values of $C_m$ (limb muscular tension at operating point) or $R_m$ (limb viscous resistance at operating point) have to be assumed in order to achieve reasonable results.

The same is done for the values of $K_k$ and $K_w$ which only can be defined statically as in the following way:

\[
K_k = \frac{F_s \text{ (oper.point)}}{F_s \text{ max}} \quad \text{(} F_s = \text{ control stick force)}
\]

\[
K_w = \frac{\delta_s \text{ (operat. point)}}{\delta_s \text{ max}} \quad \text{(} \delta_s = \text{ control stick displacement)}
\]

while $K_E$ might be defined as:

\[
K_E = \frac{F_s \text{ max}}{F_s / n} \quad \text{(} F_s / n = \text{ stick force gradient at operating point, eq. } n = n_z \text{ for pitch axis)}
\]

For force-displacement-manipulators, the product $K_k K_w$ is defined to be unity at every operating point.

RESULTS

Comparison with experimental data

Some calculations of $T_w$, $\gamma_E$, $\omega_E$, and $\delta$ are made based on experimental data published in ref. 2. Most interesting data were those of ref. 2 which have been evaluated from tracking experiments with all manipulator characteristics but only the simplest controlled element (pitch axis)

\[
F_c(s) = K_c,
\]

because these data are most characteristic for the manipulator system alone. If one assumes $K_A = K_V = 1$ and $T_A - T_V = 0$ for a controlled element without the necessity of lead compensation, which will be almost true for open loop control, the resulting transfer function is

\[
F(s) = B(s) K_C, \quad K_C = 1.
\]

Using the data of experimental runs no. 3, 8, and 23 of ref. 2 for $M_s$, $R_s$, $C_s$ and assuming:
Fig. 4: Comparison between experimental data (o) measured by Magdaleno and McRuer (ref. 2) and theoretical data (-) - pitch axis, open loop -
\[ M_m = 0.0.0199 \text{ [kgm}^2\text{]} = \text{hand-to elbow inertia for side stick} \]
\[ C_m = 1.5 \text{ [Nm}^2/\text{sec}^2\text{]} \]
\[ R_m = \frac{(M_m+M_s)(C_m+C_s)}{2 s \sqrt{M_sC_s}} \text{ (see ref. 5) for } \gamma_s = 0.7 \]

The calculated open loop Bode characteristics are plotted against the experimental data from ref. 2 in fig. 4. As one can recognize, only the amplitude differences are quite noticeable, because the calculation was based upon \( K_C = 1 \) and \( K_E = 0.666 \) for run 3 (force-displacement stick) \( 0.666 \ldots 8 \) (displacement stick) \( 0.00913 \ldots 23 \) (force stick).

Nevertheless, the calculated plots in fig. 4 clearly show the same characteristics as the experimentally evaluated ones.

Handling quality requirements

According to the Handling quality specification MIL-F-8785-C (ref. 6) the phase angle between \( F_S \) (stick force) and \( \delta_C \) (control area displacement) should not exceed 35 degrees, for level 1, related to the axis resonant frequency, \( \omega_n \). The minimum requirements for \( 1/T_W \) and \( \gamma_E/\omega_E \) are, while other phase shift between \( F_S \) and \( \delta_C \) be excluded,

\[ \frac{1}{T_W} \geq 3.5 \omega_n \]
\[ \omega_E = (3.5 \ldots 7) \omega_n \text{ for } \gamma_E \geq 1.0 \]

Assume, the pitch axis resonant frequency \( \omega_{\text{nsp}} = 3.5 \) rad/sec, these minimum requirements mean:

\[ \frac{1}{T_W} = (3.5 \ldots 3.5) = 10 \ldots 17.5 \text{ rad/sec (} T_W = 0.057 \ldots 0.1 \) \]
\[ \omega_E = (3.5 \ldots 3.5 \ldots 7) = 10 \ldots 35 \text{ rad/sec} \]

As an example, we should remember the high resonant frequency of the T33 manipulator system, used by CAL during the Neal-Smith experiments, which was \( \omega_C = 31 \text{ rad/sec (ref. 7).} \)

The low values of \( T_W \) as well as the high values of \( \omega_E \) from the above assessment are realized only by light, stiff manipulators (hand controls) used aboard fighter aircraft with high resonant frequencies (\( \omega_{\text{nsp}} = 3 \ldots 10 \text{ rad/sec} \)).
Handling quality criteria derivations

The derivation of well known handling quality criteria is possible by the introduction of the solution of eq. (9) into the closed-loop transfer function of the system pilot-aircraft. With the assumption that $\omega_E$ is high enough to satisfy the requirement

$$\omega_E \geq 3.5 \ldots 7 \cdot \omega_{nsp} \quad \text{pitch axis}$$

the solution of eq. (9) is simplified to

$$F_p(s) = \frac{K_p (1 + T_A s) (1 + T_V s) e^{T e s}}{K (1 + T_W s)}$$

If this pilot model is used in a closed loop together with the well known pitch axis aircraft transfer function

$$F_n(s) = \frac{1}{F(s) = \frac{K_n K_c (1 + T_0 s)}{(1 + T_s) F(s) = \frac{1}{s}}$$

the closed loop response is

$$F_G(s) = \frac{F_n(s)}{1 + F_n(s) F_p(s)} = \frac{1}{F(s)}$$

Again, $e^{-T e s}$ has to be substituted. In this case the 1st order Padé approximation

$$e^{-T e s} = 1 + \frac{T e}{2} s + \frac{T e^2}{2}$$

is the best substitution. The solution of the closed loop problem (13) can be achieved by using a method similar to that applied to eq. (6). We may suggest

$$F_G(s) = \frac{\Theta}{\Theta_c} = \frac{(1 + T_A s) (1 + T_V s) (1 + T_0 s) (1 - T e s)}{(1 + T_W s) (1 + T e s) (1 + T_0 s) (1 + T e s)}$$

with the conditions

$$T_{nsp} = \frac{T_0 - T_e}{T_{nsp}} \quad \text{for} \quad T_e > T_{nsp}$$

$$T_V = \frac{T_{nsp}}{\omega_{nsp}} = \frac{1}{2 \omega_{nsp} \omega_{nsp}}$$

$$T_A = \frac{T_W}{T_{nsp}}$$

$\overline{K}$ as defined above

$K_e = K_p K_c \Theta$

$T_e < T_e (PIO)$
These conditions show a very strong dependence between the aircraft parameters $T, \gamma_{nsp}, \omega_{nsp}$, the most critical parameter of the man-manipulator $T_w$, and the lead time constants generated by the pilot himself in order to stabilize the closed loop. If level 1 proven aircraft parameters are used, the closed loop response satisfies the Neal and Smith tracking criterion (ref. 7) for good handling, if the Bandwidth of the "band-pass filter" described by eq. (15) is restricted to

$$BW = 3.5 \text{ rad/sec}$$
$$\log |A|(BW) = -3 \text{dB}$$

The pilot conditions are then:

$$\bar{t_e} < 0.4 \text{ sec (PIO-criterion)}$$
$$\frac{K}{K_e} \sim \frac{F_s}{n_z}$$

$$T_v + T_A < 2.0 \text{ sec (ref. 8)}$$

This "band-pass filter criterion" is shown by fig. 5. While $T_v$ is the indicator for critical a/c parameters, $T_A$ is that for critical man-manipulator parameters. Both should be as small as possible for good handling qualities.

The addition of the 2nd order term $(1+k_{Re}s+k_{M_s}^2)$ disturbs this Bandwidth conditions only, if $\frac{1}{\sqrt{k_{Re}}} \sim \omega_{nsp}$. In this case, another lead time constant $T_{Al}$ has to be generated by the pilot in order to compensate the lag time constant $k_{Re}$, resulting in a higher order pilot lead element

$$(1+T_{As})(1+T_{Al}s)(1+T_v s),$$

the sum of which again must satisfy

$$T_{A} + T_{Al} + T_v < 2.0 \text{ (Arnold, ref. 8)}.$$

The higher order of the lead element surely will also degrade the pilot rating further.

Fig. 5: Handling quality criterion proposed by Neal and Smith (ref. 7). Bode diagram of closed loop response of system pilot-aircraft.
CONCLUSIONS

The formal arrangement of the full quasilinear pilot model first proposed by McRuer et al. into both open loop and closed loop transfer functions of the manual aircraft control is a powerful means to assess aircraft handling quality criteria. The derivation of known criteria from lead term and delay term limits of the pilot has been shown clearly.

REFERENCES


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