

THEORETICAL LINEAR APPROACH TO THE COMBINED
MAN-MANIPULATOR SYSTEM IN MANUAL CONTROL OF
AN AIRCRAFT

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SUMMARY

A new approach to the calculation of the dynamic characteristics of the combined man-manipulator-system in manual aircraft control has been derived from a model of the neuromuscular system similar to that described by McRuer and Magdaleno. (Ref. 1) This model combines the neuromuscular properties of man with the physical properties of the manipulator system which is introduced as pilot-manipulator model into the manual aircraft control. The assumption of man as a quasilinear and time-invariant control operator adapted to operating states - depending on the flight phases - of the control system gives rise to interesting solutions of the frequency domain transfer functions of both the man-manipulator system and the closed loop pilot-aircraft control system. It can be shown that it is necessary to introduce the complete precision pilot-manipulator model into the closed loop pilot-aircraft transfer function in order to understand the well known handling quality criteria of MIL-F-8785B/C, and to derive these criteria directly from human operator properties.

INTRODUCTION

The pioneer work on the precision pilot model presented by D.T. McRuer and his co-workers (Refs. 1...3) has become the most important step towards our understanding of the role of man in manual control. It is the combination of neurophysiology, physical dynamics, and control theory which gives the fascinating aspects of how the several complicated problems of manual control should be solved. But, there is a little gap between the theory and practical solutions, because the precision theory always turns out to be too complicated if engaged to solve such problems as the question for the best manipulator characteristics in a high performance aircraft.

In fig. 1 the precision pilot model is shown as it was presented by McRuer and Magdaleno (Ref. 2), which enlightens schematically, what is running in the neuromotor system when the pilot puts his hand (or legs) on the manipulator. For practical use however, the human engineer wants to revise this model to also parametrically based but simpler facts.

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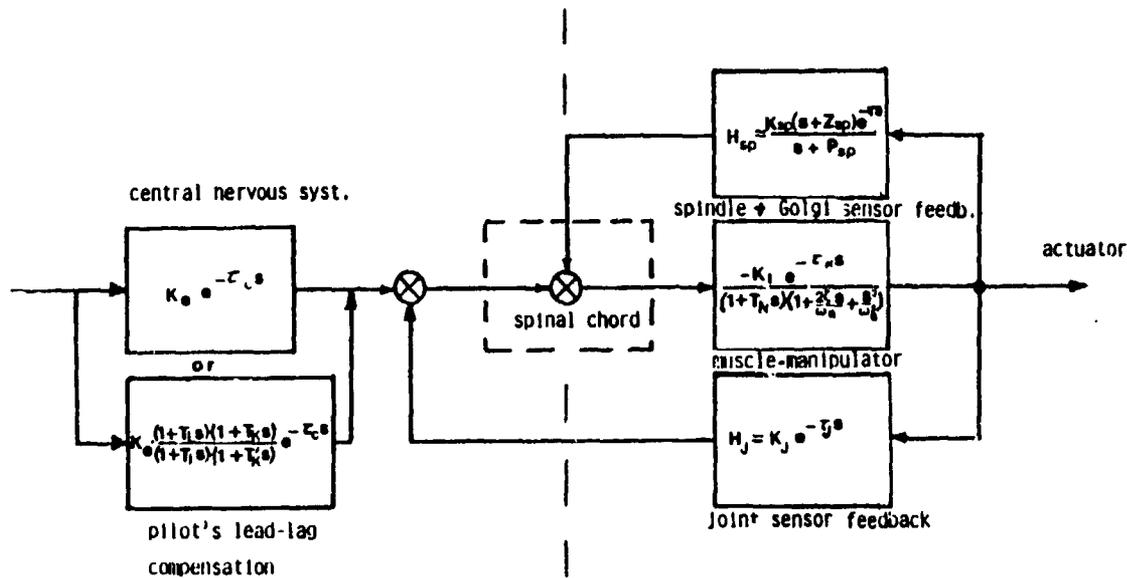


Fig. 1: Precision pilot model, after McRuer and Magdaleno (ref. 1)

If you ask the pilot about manual control qualities, he explains his wishes on

- stick forces
- aircraft response (as it is)
- predictability of response (as he expects it is)
- lead elements (after inquiry),

but he is not accustomed to consider his stick force and displacement feedback. If the pilot must consider these, the handling qualities of the system may be bad.

So for practical use we tried to reorganize the pilot model. Fig. 2 shows another pilot model less sophisticated as that of fig. 1 but more practical in use. Force and displacement feedback now feeds into the spinal chord without becoming conscious to the operator who is engaged in the compensation of system lags by lead. This model too can be fully identified by physical parameters. (Fig. 3) We now can divide the pilot model again into two parts: part A identifies the mental parameters (lead) while part B represents the neuromuscular-physical parameters of the combined man-manipulator system (lag). Now the intention of the following analysis is the formulation of the frequency domain transfer function of this quasilinear pilot model and the calculation of amplitude and phase shift of the man-manipulator combination; while further important investigations include the pilot-aircraft closed loop characteristics and their influence on handling qualities. (Ref. 5)

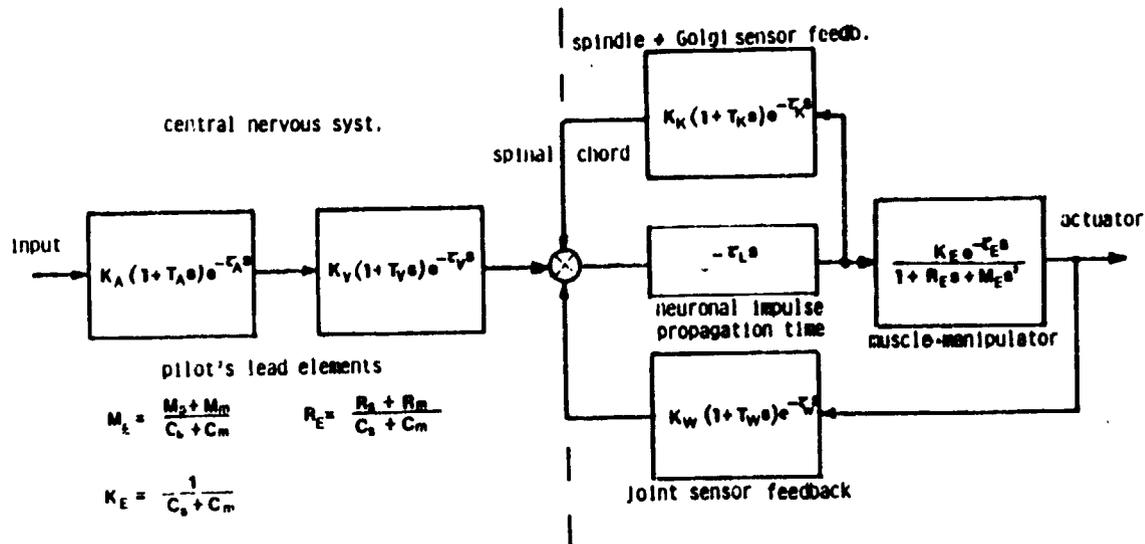


Fig. 2: Precision pilot model after P. Bubb (ref. 4), used in this report

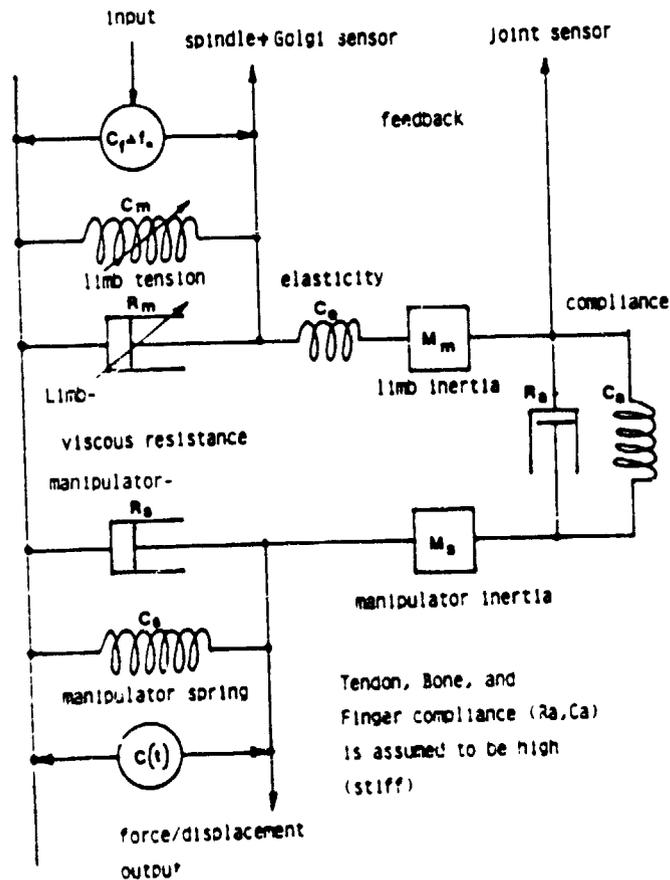


Fig. 3: Physical properties of the precision pilot model after P. Bubb (ref. 4)

ANALYSIS

Formal arrangement of the model.

The model (fig. 2) can be arranged formally alike

$$(1) \quad F_P(s) = \frac{F_A(s) \cdot F_V(s) \cdot F_E(s) \cdot F_L(s)}{1 + F_K(s) \cdot F_L(s) + F_W(s) \cdot F_L(s)}, \text{ where}$$

$$(1.1) \quad F_A(s) = K_A \cdot (1 + T_A s) e^{-\tau_A s} \quad (\text{information input term})$$

$$(1.2) \quad F_V(s) = K_V \cdot (1 + T_V s) e^{-\tau_V s} \quad (\text{information processing term})$$

$$(1.3) \quad F_E(s) = \frac{K_E}{1 + R_E s + M_E s^2} \quad (\text{second order lag of man-manipulator})$$

$$(1.4) \quad F_L(s) = e^{-\tau_L s} \quad (\text{neuronal impulse delay})$$

$$(1.5) \quad F_K(s) = K_K (1 + T_K s) e^{-\tau_K s} \quad (\text{delayed force-feedback})$$

$$(1.6) \quad F_W(s) = K_W (1 + T_W s) e^{-\tau_W s} \quad (\text{delayed displacement feedback})$$

Rearranging the equations (1.1)...(1.6) into eq. (1) results in

$$(2) \quad F_P(s) = A(s) \cdot B(s) = \frac{K_A K_V K_E (1 + T_A s) (1 + T_V s) e^{-(\tau_A + \tau_V + \tau_L) s}}{(1 + R_E s + M_E s^2) \left\{ [1 + K_K (1 + T_K s) e^{-(\tau_K + \tau_L) s}] + K_E K_W (1 + T_W s) e^{-(\tau_W + \tau_L) s} \right\}}$$

and rewritten by the following introductions

$$(2.1) \quad K_P = K_A K_V K_E \quad (\text{effective pilot gain})$$

$$(2.2) \quad \tau_P = \tau_A + \tau_V + \tau_L \quad (\text{effective pilot delay time})$$

$$(2.3) \quad \bar{\tau} = \tau_K + \tau_L = \tau_W + \tau_L \quad (\text{feedback delay time, since } \tau_K \approx \tau_L)$$

yields

$$(3) \quad F_P(s) = A(s) \cdot B(s) = \frac{K_P (1 + T_A s) (1 + T_V s) e^{-\tau_P s}}{(1 + R_E s + M_E s^2) \cdot [1 + K_K (1 + T_K s) e^{-\bar{\tau} s}] + K_E K_W (1 + T_W s) e^{-\bar{\tau} s}}$$

To achieve the roots of this system, we first have to eliminate the delay terms as much as possible, which is done by multiplication of both the numerator and denominator of eq. (3) by $e^{+\bar{\tau}s}$. This results in the replacement of $e^{-\bar{\tau}e^s}$ by $e^{-\bar{\tau}e^s} = e^{-(\bar{c}e-\bar{\tau})s}$ in the numerator, and another form of the denominator $D_p(s)$ of eq. (3) like

$$(4) \quad D_p(s) = (1+R_E s + M_E s^2) \cdot [e^{+\bar{\tau}s} + K_K(1+T_K s)] + K_E K_W (1+T_W s)$$

While the numerator has already its final root arrangement, the denominator has not. For better handling, the term $e^{+\bar{\tau}s}$ should be approximated by a Taylor series

$$(5) \quad e^{+\bar{\tau}s} = 1 + \bar{\tau}s + \frac{\bar{\tau}^2 s^2}{2!} + R(s)$$

$$(5.1) \quad R(s) = \frac{\bar{\tau}^n s^n}{n} \quad n = 2, 3, \dots, 6$$

the frequency range of which is $0 < (\omega = j) < 70$ rad/sec. with an amplitude and phase error below 20 %, for $n = 2$:

$$(6) \quad F_p(s) = A(s)B(s) =$$

$$= \frac{K_p (1+T_A s) (1+T_V s) e^{-\bar{\tau}e^s}}{(1+R_E s + M_E s^2) \left[1 + s + \frac{\bar{\tau}^2 s^2}{2} + \frac{\bar{\tau}^1 s^1}{n} + K_K (1+T_K s) \right] + K_E K_W (1+T_W s)}$$

The roots of the denominator polynomial are solved by the calculation of the denominator polynomial form

$$(7) \quad D_p = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5$$

and by comparison of the coefficients a_i and the coefficients b_i of another, well-known polynomial the roots of which meet the 5th order arrangement

$$(1+b_1 s) (1+b_2 s + b_3 s^2) (1+b_4 s + b_5 s^2)$$

The necessary arithmetics are described and discussed in ref.5. It has to be mentioned, that an exact solution of a 5th order linear arithmetic equation does not exist.

SOLUTION

The best solution of eq. (6) is the following

$$(9) \quad F_p(s) = A(s) \cdot B(s) =$$

$$\frac{K_p (1+T_A s) (1+T_V s) e^{-\bar{\tau}e^s}}{K (1+T_W s) \left(1 + \frac{1+K K}{K} \frac{1+K K}{R E S} + \frac{1+K K M E S^2}{K} \right) \left(1 + \frac{1+K K}{K} \frac{1+K K}{R E S} + \frac{1+K K M E S^2}{K} \right)}$$

which describes the required roots of a 5th order lag system $B(s)$, if

$$(9.1) \quad A(s) = K_p (1+T_A s)(1+T_V s)e^{-\bar{\tau}e^s}.$$

The conditions for the solution of $B(s)$ within eq. (9) are:

$$(10.1) \quad \bar{K} = 1+K_K+K_E K_W \quad (\text{combined feedback gain})$$

$$(10.2) \quad T_W = R_E = \beta \bar{\tau} \quad (\text{neuromuscular lag time constant})$$

$$(10.3) \quad \frac{1+K_K}{\bar{K}} R_E = \frac{2\zeta_E}{\omega_E} \quad (\text{2nd order lag time constant})$$

$$(10.4) \quad \frac{1+K_K}{\bar{K}} M_E = \frac{1}{\omega_E^2} \quad (\text{man-manipulator resonant frequency})$$

$$(10.5) \quad \gamma_E = \frac{R_m+R_s}{2\sqrt{R_m R_s}} \geq 1.0 \quad (\text{damping of the 2nd order lag term})$$

$$(10.6) \quad M_E = \frac{M_m+M_s}{C_m+C_s} \quad (\text{Inertial resistance of the manipulator})$$

$$(10.7) \quad R_E = \frac{R_m+R_s}{C_m+C_s} \quad (\text{viscous resistance of the man-manipulator})$$

$$(10.8) \quad \delta = \frac{+\sqrt{n+2 + \left[\frac{(R_m+R_s)^2}{2k\beta R_m R_s} \right]^2} - \frac{(R_m+R_s)^2}{2K\beta R_m R_s}}{2nk}$$

δ is a "high frequency weighting factor", in which β , k , and n are defined as follows

$$\beta = \frac{1+(K_K \frac{T_K}{\bar{\tau}})}{1+K_K}; \quad k = \frac{1+K_K}{\bar{K}}, \quad n = 2 \quad (\text{see eq. 5.1.})$$

The 5th order solution (9) degrades to a 3rd order solution, if $\delta \gg 0$. A limit of δ is given by $\delta < 1$: If $\delta < \epsilon$, set $\delta = 0$, while ϵ is defined by

$$(10.9) \quad \epsilon = \frac{1}{70 \sqrt{k \cdot M_E}} \quad \text{i.e. if}$$

$$\omega'_E = \frac{1}{\sqrt{k\delta^2 + M_E}} > 70 \text{ rad/sec which is the limit in } \omega \text{ de-}$$

fined by the approximator in eq. (5.1)

Numerical evaluation.

While the numerical values of M_m (limb inertia), M_s (manipulator inertia), C_s (feel spring constant), and R_s (manipulator viscous resistance) can be achieved by measurement - or effectively are known - the values of C_m (limb muscular tension at operating point) or R_m (limb viscous resistance at operating point) have to be assumed in order to achieve reasonable results.

The same is done for the values of K_k and K_w which only can be defined statically as in the following way

$$K_k = \frac{F_s(\text{oper. point})}{F_s \text{ max}} \quad (F_s = \text{control stick force})$$

$$K_w = \frac{\delta_s(\text{operat. point})}{\delta_s \text{ max}} \quad (\delta_s = \text{control stick displacement})$$

while K_E might be defined as

$$K_E = \frac{F_s \text{ max}}{F_s/n} \quad (F/n = \text{stick force gradient at operating point, eg. } n = n_z \text{ for pitch axis})$$

For force-displacement-manipulators, the product $K_E K_W$ is defined to be unity at every operating point.

RESULTS

Comparison with experimental data

Some calculations of T_w , \dot{y}_E , ω_E , and δ are made based on experimental data published in ref. 2. Most interesting data were those of ref. 2 which have been evaluated from tracking experiments with all manipulator characteristics but only the simplest controlled element (pitch axis)

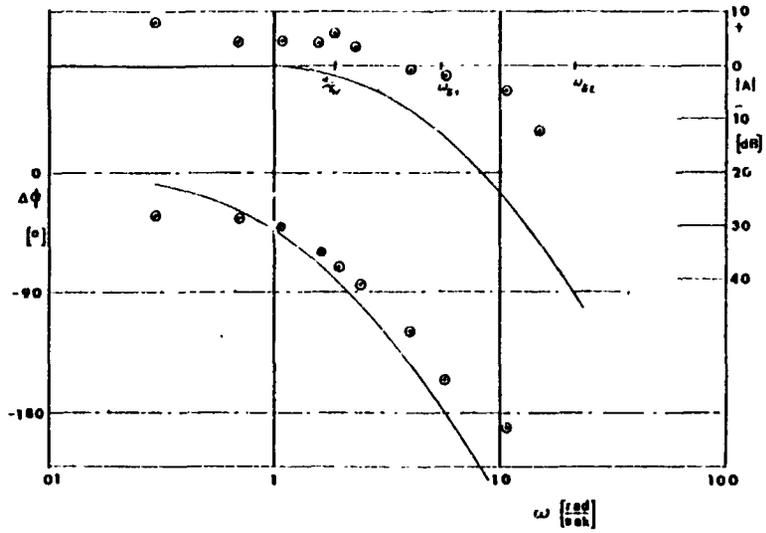
$$F_C(s) = K_C,$$

because these data are most characteristic for the man-manipulator system alone. If one assumes $K_A = K_V = 1$ and $T_A = T_V \rightarrow 0$ for a controlled element without the necessity of lead compensation, which will be almost true for open loop control, the resulting transfer function is

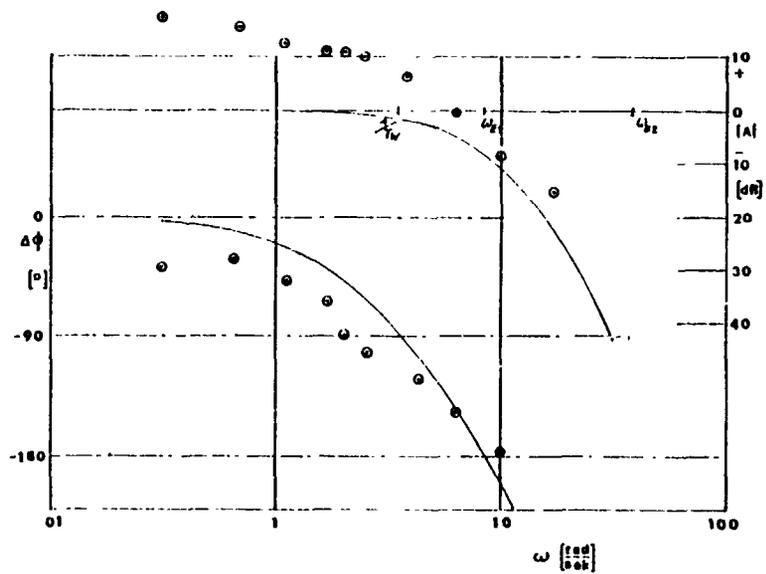
$$F(s) = B(s) \cdot K_C, \quad K_C = 1.$$

Using the data of experimental runs no. 3, 8, and 23 of ref. 2 for M_s , R_s , C_s and assuming:

run no. 3



run no. 8



run no. 23

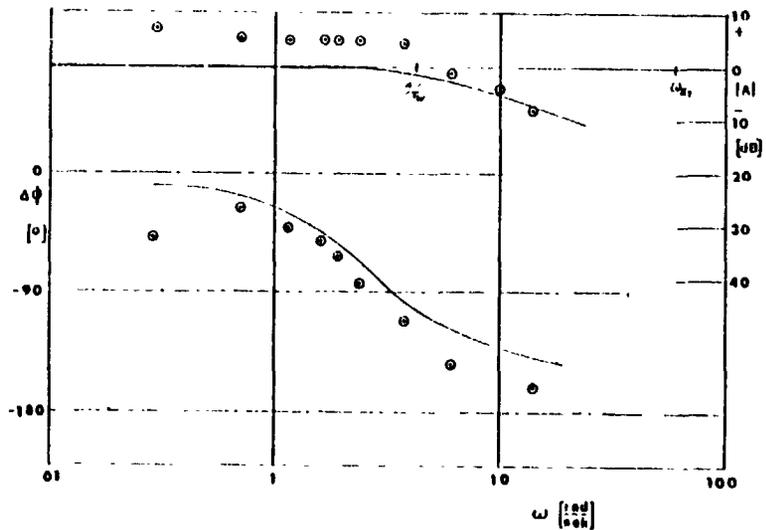


Fig. 4: Comparison between experimental data (o) measured by Magdaleno and McRuer (ref. 2) and theoretical data (-)
 - pitch axis, open loop -

$M_m = 0.00199 \text{ [kgm}^2\text{]} = \text{hand-to elbow inertia for side stick}$

$C_m = 1.5 \text{ [Nm}^2\text{/sec}^2\text{]}$

$R_m = \frac{(M_m + M_s)(C_m + C_s)}{2 \zeta_s \sqrt{M_s C_s}} \quad (\text{see ref. 5) for } \zeta_s = 0.7$

the calculated open loop Bode characteristics are plotted against the experimental data from ref. 2 in fig. 4. As one can recognize, only the amplitude differences are quite noticeable, because the calculation was based upon $K_C = 1$ and

$K_E = 0,666$ for run 3 (force-displacement stick)

0,666 " " 8 (displacement stick)

0,00913 " " 23 (force stick)

Nevertheless, the calculated plots in fig. 4 clearly show the same characteristics as the experimentally evaluated ones.

Handling quality requirements

According to the Handling quality specification MIL-F-8785-C (ref. 6) the phase angle between F_s (stick force) and δ_c (control area displacement) should not exceed 35 degrees, for level 1, related to the axis resonant frequency, ω_n . The minimum requirements for $1/T_W$ and ζ_E/ω_E are, while other phase shift between F_s and δ_c be excluded,

$$\frac{1}{T_W} \geq 3,5 \omega_n$$

$$\omega_E = (3,5 \dots 7) \omega_n \text{ for } \zeta_E \geq 1.0$$

Assume, the pitch axis resonant frequency $\omega_{nsp} = 3 \dots 5$ rad/sec, these minimum requirements mean:

$$\frac{1}{T_W} = (3 \dots 5) \cdot 3,5 = 10 \dots 17,5 \text{ rad/sec } (T_W = 0,057 \dots 0,1)$$

$$\omega_E = (3 \dots 5) \cdot (3,5 \dots 7) = 10 \dots 35 \text{ rad/sec}$$

As an example, we should remember the high resonant frequency of the T33 manipulator system, used by CAL during the Neal Smith experiments, which was $\omega_c = 31$ rad/sec (ref. 7). The low values of T_W as well as the high values of ω_E from the above assessment are realized only by light, stiff manipulators (hand controls) used aboard fighter aircraft with high resonant frequencies ($\omega_{nsp} = 3 \dots 10$ rad/sec).

Handling quality criteria derivations

The derivation of well known handling quality criteria is possible by the introduction of the solution of eq. (9) into the closed-loop transfer function of the system pilot-aircraft. With the assumption that ω_E is high enough to satisfy the requirement

$$\omega_E \cong 3,5 \dots 7) \cdot \omega_{nsp} \quad \text{- pitch axis -}$$

the solution of eq. (9) is simplified to

$$(11) \quad F_p(s) = \frac{K_p (1+T_A s) (1+T_V s) e^{-\bar{\tau} s}}{\bar{K} (1+T_W s)}$$

If this pilot model is used in a closed loop together with the well known pitch axis aircraft transfer function

$$(12) \quad F_{nsp}(s) = \frac{\theta}{F_s} = \frac{K_c K_e (1+T_\theta s)}{s(1+T_{nsp} s + T'_{nsp} s^2)}$$

$$T_{nsp} = \frac{2 \zeta_{nsp}}{\omega_{nsp}}, \quad T'_{nsp} = \frac{1}{\omega_{nsp}^2}$$

the closed loop response is

$$(13) \quad F_G = \frac{F_p F_n}{1 + F_p F_n} = \frac{\theta}{\theta_c}$$

Again, $e^{-\bar{\tau} s}$ has to be substituted. In this case the 1st order Padé approximation

$$(14) \quad e^{-\bar{\tau} s} = \frac{1 - \frac{\bar{\tau} s}{2}}{1 + \frac{\bar{\tau} s}{2}} = \frac{1 - T_e s}{1 + T_e s}$$

is the best substitution. The solution of the closed loop problem (13) can be achieved by using a method similar to that applied to eq. (6). We may suggest

$$(15) \quad F_G(s) = \frac{\theta}{\theta_c} = \frac{(1+T_A s) (1+T_V s) (1+T_\theta s) (1 - T_e s)}{(1+T_W s) (1 + [\frac{\bar{K}}{K_e} - \frac{T_\theta T_e}{T_n}] s + \frac{\bar{K}}{K_e} T_e s^2) (1+T_{nsp} s + T'_{nsp} s^2)}$$

with the conditions

$$(15.1) \quad \begin{aligned} T_{nsp} &= T_\theta - T_e \text{ for } T_\theta > T_{nsp} & \bar{K} & \text{ as defined above} \\ T_V &= \frac{T'_{nsp}}{T_{nsp}} = \frac{1}{2 \zeta_{nsp} \omega_{nsp}} & K_e &= K_p K_c K_\theta \\ T_A &= T_W & T_e &< T_\theta \text{ (PIO)} \end{aligned}$$

These conditions show a very strong dependence between the aircraft parameters T , ζ_{nsp} , ω_{nsp} , the most critical parameter of the man-manipulator T_w , and the lead time constants generated by the pilot himself in order to stabilize the closed loop. If level 1 proven aircraft parameters are used, the closed loop response satisfies the Neal and Smith tracking criterion (ref. 7) for good handling, if the Bandwidth of the "band-pass filter" described by eq. (15) is restricted to

$$BW = 3,5 \text{ rad/sec}$$

$$\log |A| (BW) = -3 \text{ dB}$$

The pilot conditions are then:

$$\bar{t}_e < 0,4 \text{ sec (PIO-criterion)}, \quad \frac{\bar{K}}{K_e} \sim \frac{F_s}{n_z}$$

$$T_v + T_A < 2,0 \text{ sec (ref. 8)}$$

This "band-pass filter criterion" is shown by fig. 5. While T_v is the indicator for critical a/c parameters, T_A is that for critical man-manipulator parameters. Both should be as small as possible for good handling qualities.

The addition of the 2nd order term $(1+kR_E s+kM_E s^2)$ disturbs this Bandwidth conditions only, if $1/\sqrt{kM_E} \approx \omega_{nsp}$. In this case, another lead time constant T_{A1} has to be generated by the pilot in order to compensate the lag time constant kR_E , resulting in a higher order pilot lead element

$$(1+T_A s)(1+T_{A1} s)(1+T_v s),$$

the sum of which again must satisfy

$$T_A + T_{A1} + T_v < 2.0 \text{ (Arnold, ref. 8)}.$$

The higher order of the lead element surely will also degrade the pilot rating further.

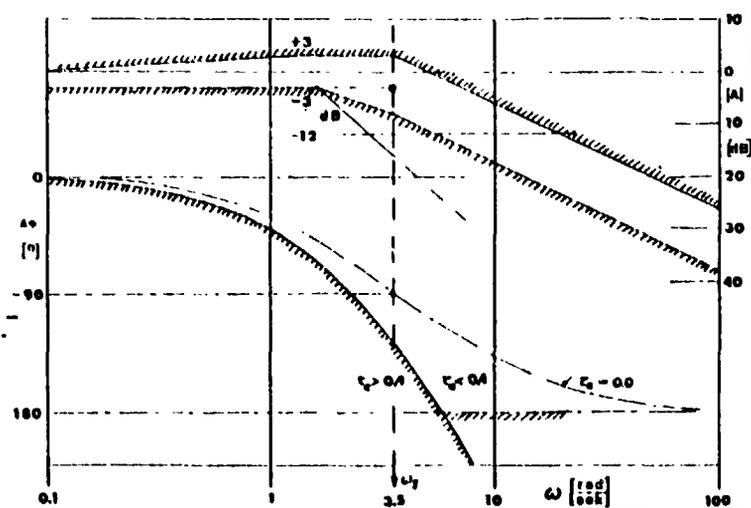


Fig. 5: Handling quality criterion proposed by Neal and Smith (ref. 7). Bode diagram of closed loop response of system pilot-aircraft

CONCLUSIONS

The formal arrangement of the full quasilinear pilot model first proposed by McRuer et. al. into both open loop and closed loop transfer functions of the manual aircraft control is a powerful means to assess aircraft handling quality criteria. The derivation of known criteria from lead term and delay term limits of the pilot has been shown clearly.

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