STABILITY ANALYSIS OF AUTOMOBILE DRIVER STEERING CONTROL

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SUMMARY

In steering an automobile the driver must basically control the direction of the car's trajectory (heading angle) and the lateral deviation of the car relative to a delineated pathway. This paper considers a previously published linear control model of driver steering behavior which is analyzed from a stability point of view. A simple approximate expression for a stability parameter, phase margin, is derived in terms of various driver and vehicle control parameters, and boundaries for stability are discussed.

A field test study is reviewed that includes the measurement of driver steering control parameters. Phase margins derived for a range of vehicle characteristics are found to be generally consistent with known adaptive properties of the human operator. The implications of these results are discussed in terms of driver adaptive behavior.

INTRODUCTION

Analysis of the closed-loop dynamic behavior of the driver/vehicle system can give some insight into driver and model behavior, and provide simplified analytical expressions for relationships between various model parameters. Here we will use a linear, two degree of freedom vehicle model. The two degrees of freedom to be considered are heading, or direction of vehicle motion, and lateral position. These two variables are under direct control of the driver. A third basic vehicle mode not considered here is roll angle. Although roll angle may influence driver behavior, it is not controlled per se by the driver. The discussion here will include a summary of previously published work (References 1 and 2).

The two degree of freedom model should be considered as an equivalent or approximation to higher degree models. Thus it subsumes such things as roll steer and weight transfer effects to a first approximation, so that the heading response of the model is an adequate approximation to a real vehicle for similar inputs.

DRIVER/VEHICLE SYSTEM MODEL

A block diagram of the driver/vehicle system model is shown in Figure 1. The vehicle equations generate side velocity \(v\) and yaw (heading) rate as a function of steering inputs through the \(G_v\) and \(G_y\) transfer functions, respectively. Kinematic equations then compute vehicle heading angle and lateral lane position from side velocity and yaw rate inputs. The driver finally develops steering corrections based on perceived heading and lane position errors as processed by the behavioral transfer functions \(Y_y\) and \(Y_v\). For the closed-loop analysis reviewed here we will consider steering against disturbances applied at the steering point as shown in Figure 1.

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At this point a question might be raised as to why the \(Y_v\) block is not placed in the \(v_e\) pathway but rather is in the \(y_e\) pathway, which is the sum of the heading error and some function \(Y_v\) of lane position error \(y_e\). This arrangement is consistent with the perceptual information most readily available to the driver, which is further described in Ref. 3.
This disturbance input will be used to approximate the lateral response effects of
disturbances on the wheels, and various forces and moments caused by roadway
slope and lateral wind gusts.

The dynamics and stability of the Figure 1 system can best be analyzed by con-
sidering a steering disturbance signal \( \delta_d \) as the input to a closed-loop system, and
then analyzing the total open-loop transfer function between \( \delta_d \) passing through the
vehicle dynamics and driver behavior to the \( \delta_w \) point. In this way we subsume the
multi-path portions of the system as we progress from the single control input vari-
able \( \delta_e \) to the single control output variable \( \delta_w \).

Through simple block diagram algebra we can now derive an expression for the open-
loop transfer function \( \delta_e/\delta_w \) as follows. First the heading angle and lane position
errors can be expressed in response to \( \delta_e \) inputs as

\[
\psi_e = G_y \delta_e \quad ; \quad y_e = \frac{\delta_e}{s} (G_y + U_0 G_\psi) \tag{1}
\]

Next \( \delta_w \) steering response can be expressed in terms of perceived heading angle and
lane position errors as

\[
\delta_w = K_s \delta_{sw} = Y_e (\psi_e + y_e y_e) \tag{2}
\]

Combining Equations 1 and 2 we can then express the total open-loop transfer function
as

\[
\frac{\delta_w}{\delta_e} = Y_e \frac{K_s G_\psi}{s} \left\{ Y_y \left( \frac{G_y}{G_\psi} + \frac{U_0}{s} \right) + 1 \right\} \tag{3}
\]

This expression can be further simplified if we now express the transfer function
between heading and lane position to control input in terms of the vehicle dynamics
and kinematic equations:

\[
\begin{align*}
G_\psi' & = \frac{\psi_e}{\delta} = \frac{G_\psi}{s} \\
G_y' & = \frac{y_e}{\delta} = \frac{G_y}{s} + \frac{(U_0/s)G_\psi}{s}
\end{align*} \tag{4}
\]
Combining Equations 3 and 4 we have
\[
\frac{\delta w}{\delta e} = Y_y K_y G_y^y \left\{ Y_y \frac{G_y^y}{G_y} + 1 \right\}
\]
(5)

Now consider models for each of the component transfer functions in Equation 5.

**Vehicle Dynamics**

Two degree of freedom vehicle dynamics have previously been analyzed in some detail (Reference 1) from which the following material has been summarized. In general the transfer functions between heading angle and lane position to steering control input can be expressed by second-order equations.

\[
G_y^x = \frac{N_y (s + 1/T_x)}{s(s^2 + 2z_1 \omega_s + \omega_f^2)}
\]

(6)

\[
G_y^y = \frac{Y_y (s^2 + 2z_1 \omega_s + \omega_f^2)}{s^2(s^2 + 2z_1 \omega_s + \omega_f^2)}
\]

(7)

where \(T_x\) is the basic time constant of the vehicle's heading response and the remaining coefficients are functions of various vehicle parameters.

Using the few basic vehicle parameters described in Figure 2 and some simple assumptions we can now express Equations 6 and 7 in terms of vehicle characteristics as follows. The inverse of the heading time constant is given by

\[
T_{x}^{-1} = \frac{2(a + b)}{m U_o a}
\]

(8)

If we now assume that the vehicle radius of gyration \(k_z\) is approximately equal to the geometric mean of the axle to c.g. distance:

\[
k_z = \frac{I_{zz}}{m} = \sqrt{ab}
\]

(9)

**Figure 2. Vehicle Parameters for Steering Dynamics**
and introduce two additional parameters, the stability factor $K$ ($\sec^2/\ell^2$), which is related to the SAE understeer/oversteer gradient (Reference 1)

$$K \ (\deg/sec) = 1847(a + b)K$$

(10)

and the "axle load ratio"

$$\nu = \frac{bv_{a2}}{aY_{a1}}$$

(11)

we can then write approximate expressions for the remaining coefficients in Equations 6 and 7.

$$Y_\delta = \frac{2}{m} Y_{a1}; \quad N_\delta = \frac{2a}{mk} Y_{a1}; \quad \frac{Y_\delta}{N_\delta} = \frac{k_2}{a} = b$$

$$\omega_1 = T_r^{-1} \frac{1 + KU_0}{\nu}; \quad \zeta_1 = \frac{\nu + 1}{2\sqrt{\nu(1 + KU_0)}}$$

$$2\zeta \omega_1 = T_r^{-1} \left(\frac{1}{\nu} + 1\right)$$

$$\omega_2^2 = \frac{T_r^{-1}}{b} U_0; \quad 2\zeta \omega_2 = T_r^{-1}$$

(12)

At best, the above expressions are good approximations for many cars. At worst the expressions should give us a qualitative feel for the lateral dynamic characteristics that are of importance to the driver.

Inspection of the above equations can give us insight into vehicle dynamic response characteristics that are important from a driver control point of view. It is obvious that the heading time constant ($T_r$) dominates the vehicle dynamics, and that it is a direct function of speed (Equation 8). In Equations 6 and 7 the numerator and denominator roots are an inverse function of speed (i.e., $T_r^{-1}$), so as the vehicle increases in speed the heading response becomes slower (lower frequency). The stability factor $K$ can also exert further influence as a function of speed. For an oversteering car ($K < 0$) the speed sensitivity of the heading mode is even further exaggerated, while the damping decreases, causing the car to become oscillatory. For an understeering car ($K > 0$) the speed sensitivity of the heading mode denominator is reduced and the damping increases with speed.

Another factor to consider is the heading rate sensitivity to steering inputs. If we evaluate the derivative of Equation 6 at zero frequency we end up with simple expressions for steady-state heading rate and side acceleration:

$$G_\delta^r \ |_{s=0} = \frac{U_0}{t(1 + KU_0)}$$

$$G_\delta^r \ |_{s=0} = \frac{U_0}{t(1 + KU_0)}$$

(13)

Here we see that steering sensitivity is a function of speed, wheelbase ($a + b = t$), and the stability factor, $K$. At low speeds the car follows a path whose curvature ($C$) is proportional to wheel deflection and inversely proportional to wheelbase (i.e., Ackermann steering):

$$C = \delta_\omega/t$$

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Then the heading rate due to following this curved path is proportional to velocity

\[ r = CV_0 \]

Therefore,

\[ \frac{r}{V_0} = U_0 / t \]

At higher speeds the stability factor exerts additional speed effects (Equation 13), with understeering cars \((K > 0)\) having less speed sensitivity and oversteering cars \((K < 0)\) having increased speed sensitivity.

**Driver Behavior**

Given the above approximate lateral response dynamics let us now analyze the corresponding driver control dynamics for the \(Y_P\) and \(Y_K\) blocks of Figure 1. The \(Y_P\) block includes several components of driver behavior. First there is a component due to basic limitations in the driver's response properties. These limitations can be subdivided further into subcomponents: pure time delays due to central nervous system processing and neural conduction to the limbs; and interface effects due to the spring mass damping system formed by the driver's arms coupled to the steering system. Most of the neuromuscular effects are high frequency and can be approximated by a pure time delay, \(e^{-ts}\), in the frequency range of interest for car control (Reference 4).

The second \(Y_P\) component is a lead or anticipation term \((T_t s + 1)\) that the driver adopts to counteract vehicle response characteristics discussed above. The third component is a gain \(K_P\) which sets the magnitude of \(\Delta\), corrections for given heading errors \((\psi_o)\). Combining the above components we derive a heading response function that has been developed and used in a variety of past studies (e.g., References 2 and 5):

\[ Y_P = K_P(T_t s + 1)e^{-ts} \]  \(\text{(14)}\)

A pure gain feedback for lane position errors has been found satisfactory in previous research:

\[ Y_y = K_y \]  \(\text{(15)}\)

The addition of weighted lane position error and heading angle error can actually be considered as a composite angular error as shown in Figure 1:

\[ \psi_L = K_y \psi_e + \psi_o \]  \(\text{(16)}\)

It has been previously shown that this equivalent angular error can be interpreted as the angular error to a projected aim point located some distance down the road, as shown in Figure 3 (Reference 2). The aim point concept is appealing because it represents somewhat of a perceptual efficiency for the driver. Instead of separately

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*Figure 3. Aim Point Control Law. \(\psi_L = \psi_e + \psi_o K_y\) where \(\tan^{-1} \psi_o K_y \geq \psi_o K_y\)*
perceiving both lane position and heading errors, he need only perceive the angular error $\psi_L$ to the aim point down the road.

One additional term will be added here that has received only scant attention in the literature (References 6 and 7). Analysis and modeling of driver response at STI has shown the need for low-frequency characteristics or compensation that will act to reduce lane position error offsets and speed up the driver model’s transient response and error reduction in tasks such as lane changes. The need for low frequency compensation can be seen by considering the effect of constant input disturbances, $\delta_g$, to the Figure 1 block diagram. Given a constant $\delta_g$ input the driver model (i.e., $Y_L$ and $Y_y$) discussed so far would have to allow constant lane position errors in order to generate wheel angle $\delta_y$, at the differential summing block. In the real world this would mean that drivers’ subjected to steady crosswinds or crowned roadways (i.e., inputs causing a constant force input to the vehicle) would drive with a constant lane position error.

The above offset error effect is not very reasonable, and the model can be corrected to eliminate it by adding a parallel trimming integrator, as illustrated in Figure 4, somewhere in the driver model feedforward path. The effect of the parallel integrator is to continue increasing its output in the face of steady errors, and holding its value as the errors approach zero. This will then produce steady wheel angles, $\delta_y$, to compensate for steady disturbances, $\delta_g$, without requiring a steady offset error.

There are two possible locations for the parallel integrator. One is in the $Y_y$ block that would operate only on lane position errors. Another possibility is the $Y_y$ block, where the parallel integrator would operate on the composite $\psi_L$ signal which is a combination of lane position and heading errors. The $Y_y$ location seems more reasonable for two reasons. First, the concept of perceiving a simple aim point error $\psi_L$ would still be valid, which would not be true if the parallel integrator were applied to just the lane position errors. Second, steady-state heading errors can also develop in situations such as following curved paths, and the $Y_y$ parallel integrator location would tend to accumulate these errors quicker than waiting for the heading error to accumulate into lane position errors which are operated on by the $Y_y$ block.

Although we have rationalized the inner-loop ($Y_y$ block) as the best location for the parallel integrator characteristic, we will analyze both possible locations ($Y_y$ and $Y_L$) below in order to accumulate further evidence for the best location. Summarizing the driver response behavior for both parallel integrator locations we have:

**Inner-Loop ($Y_y$) Parallel Integrator:**

$$Y_y = \frac{r + K_y}{s} K_p (T_L s + 1) e^{-Ts}$$

(16)

$$K_y = K_y$$

**Outer-Loop ($Y_L$) Parallel Integrator:**

$$Y_L = K_p T_L s + 1 e^{-Ts}$$

$$Y_L = \frac{r + K_L}{s} K_y$$

(17)
Driver/Vehicle Dynamics

Given the above dynamic characteristics for the vehicle and driver, let us now analyze the overall driver/vehicle system response given by Equation 5. The first term in Equation 5 \( Y_v \) is a driver characteristic, and the third is the car heading response \( (K_e) \) is simply the steering ratio. The last term in Equation 5 is a combination of driver and vehicle characteristics. The interaction of these characteristics when \( K_y \) is a pure gain has been considered previously (Reference 2). Here we will consider the case where the parallel integrator is in the outer loop.

Combining Equations 5, 12, and 17 for the \( Y \) parallel integrator we end up with the following expression for the bracketed term in Equation 5:

\[
Y_v \frac{G_y}{G_y^*} + 1 = \frac{b K_y (s + K')}{(s + T_y + \frac{1}{T_y})}
\]

This can be seen to be a classical root locus problem, with one first-order zero (at \( K' \)), a second-order zero pair, two poles at the origin and a pole due to the heading time constant \( (s = -\frac{1}{T_y}) \). We can now vary the lane position gain \( (K_e) \) and plot the locus of roots for Equation 18. Root locus plots for vehicle characteristics used in past research (Reference 8) are compared in Figure 5. Here we see that with the parallel integrator in the outer loop a complex pair of low-frequency zeros occurs at reasonable values of \( K_y \).

With the parallel integrator in the inner loop \( (Y_v \), Equation 16) the result is two real zeros, one at \( K' \) and the other closed-loop zero due to the Equation 18 expression without the parallel integrator term. However, as illustrated in Figure 5, we see that for a large heading time constant the outer-loop parallel integrator always results in a complex zero with fairly low damping. This implies that the driver's low-frequency phase curve has a steep slope. However, data considered below will show that the low-frequency phase curves are never very steep and can only be fitted with two real roots as opposed to the Figure 5 complex roots.

![Figure 5. Root Locus for Outer-Loop Parallel Integrator (Vehicle dynamics described in Figure 6)](image-url)
Phase Margin Approximation

The vehicle heading response (G,f) dictates the high-frequency lead compensation to be provided by the driver. Plots of G,f for various levels of vehicle heading time constant (T_m) studied in previous research (Reference 8) are given in Figure 6. The numerator zero and second-order denominator combine to give a net first-order-appearing transfer function combined with the kinematic integration, which gives an

<table>
<thead>
<tr>
<th>VEHICLE CONFIGURATION</th>
<th>STEERING RATIO, k_0^{-1}</th>
<th>UNDERSTEER/OVERSTEER GRADIENT, K \times 10^6 s^{-1}sec^{-2}/ft^2</th>
<th>\text{deg}/s</th>
<th>INVERSE HEADING TIME CONSTANT, T_h^{-1}(sec^{-1})</th>
<th>INVERSE EQUIVALENT TIME CONSTANT, T_{eq}^{-1}(sec^{-1})</th>
<th>HEADING RESPONSE PARAMET'S</th>
</tr>
</thead>
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<tr>
<td>A</td>
<td>23:1</td>
<td>1.1</td>
<td>1.9</td>
<td>4.0</td>
<td>5.0</td>
<td>4.9</td>
</tr>
<tr>
<td>B</td>
<td>17:1</td>
<td>1.3</td>
<td>2.2</td>
<td>1.6</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>C</td>
<td>9:1</td>
<td>3.3</td>
<td>5.7</td>
<td>2.6</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>D</td>
<td>11:1</td>
<td>3.4</td>
<td>5.8</td>
<td>4.4</td>
<td>7.2</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Figure 6. Test Vehicle Heading Response Transfer Functions

The additional free s in the denominator. Inspection of the Figure 6 phase curves shows that the heading response dynamics can be described by an equivalent time constant, T_{eq}, which should dictate required driver lead compensation (i.e., T_L \leq T_{eq}).

Given the above components the composite driver/vehicle open-loop transfer function can be written as follows:
Driver Operation on Aim Point Error

\[
\frac{\delta_w}{\delta_e} = \frac{K_aK_s(s + K')}{s(s + T_Ls + 1)} e^{-T_Ls}
\]

Vehicle Heading Response

\[
\frac{N_4(s + T^{-1}_f)}{s(s + 2\zeta_1\omega_1 s + \omega_1^2)} \times \frac{(s + (T_f^{-1}))}{s(s + T_Ls + 1)} e^{-T_Ls}
\]

Driver/Vehicle Interaction
Due to Outer-Loop Closure

where \(a = K_y U_0\). This equation can be rearranged according to frequency characteristics:

1. Low-Frequency Compensation
2. Low Frequency Vehicle Head Response
3. Mid-High Frequency Vehicle Head Response
4. Driver/Vehicle Interaction
5. Driver Lead Compensation for Frequency Delay
6. Driver High-Frequency Interaction Due to Driver/Vehicle Interaction

Now assume the driver adjusts \(T_f\) to compensate for combined phase lag characteristics of Term (3) and (4). The 45° phase lag point of (3) is somewhat higher than \(T_f\) and declines \(T_f\). Additional phase lead is derived from (4) so that the \(T_f\) frequency break may be somewhat higher than \(T_f\) or \(\omega_1\).

Given the complete transfer function expression above, it is now useful to consider an extended crossover model approximation. Assume that Expressions (3) and (4) combine into an equivalent first-order lag which is cancelled by the driver's lead term:

\[
\frac{s + T^{-1}_f}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2} \times \frac{(s + (T_f^{-1}))}{s(s + T_Ls + 1)} = \frac{T_f^{-1}}{\omega_f}
\]

Then the driver/vehicle equivalent open-loop transfer function reduces to

\[
\frac{\delta_w}{\delta_e} = \frac{(s + K')}{s^2} \times \frac{(s + a)}{s} \times e^{-T_Ls}
\]

where

\[
\omega_c = \frac{K_aK_sT_f^{-1}}{\omega_f} = \frac{K_aK_sU_0}{\omega_f}
\]
for a neutral steering car and \( \tau_w \) is an effective system time delay that accounts for the driver's time delay (\( t \)) and residual phase lags (or lead) left over from the approximations in Equation 21.

Now we can analyze Equation 22 to determine the stability limit for the drive - \( K \) gain. For stability the unity magnitude of Equation 22 must occur below the 180 deg phase lag point. At high frequencies \( \omega \gg K' \) and \( a \) the unity gain point is given by \( \omega_c \) (i.e., the unity gain crossover frequency) and the Equation 22 phase is given by

\[
\delta \omega = - \frac{\omega}{2} - \tan^{-1} \frac{K'}{\omega_c} - \tan^{-1} \frac{a}{\omega_c} - \tau_w \omega
\]  

(23)

Phase margin is defined at the gain crossover frequency \( \omega_c \), which occurs at relatively high frequency, so that

\[
\tan^{-1} \frac{K'}{\omega_c} \approx \frac{K'}{\omega_c}; \quad \tan^{-1} \frac{a}{\omega_c} \approx \frac{a}{\omega_c}
\]

Thus the phase margin, or amount of extra phase shift allowable before instability is reached, is given by

\[
\phi_N = 2\pi - \frac{\delta \omega}{\omega_c} = \frac{\pi}{2} - \frac{K' + K_y U_c}{\omega_c} - \tau_w \omega_c
\]  

(24)

The second term on the right side of Equation 24 is the phase lag due to the driver's low frequency behavior in controlling lane deviations. The third term is the phase lag due to the driver's basic time delay limitation, which defines the limiting bandwidth he/she can achieve. Note that the outer-loop operations \( K' \) and \( K_y \) add phase lag and further limit the achievable bandwidth, so that the driver must trade off inner- and outer-loop gains (i.e., \( K_y \) and \( K' \)) in order to optimize performance.

Model Validation

A field study using an instrumented car on a closed test course has been previously reported on in Reference 8. The dynamics of the test vehicle could be easily modified, and the conditions given in Figure 6 were included in a test program involving 8 males and 8 females, ages 23-40, with an average of 13 years driving experience. Describing functions were obtained for each set of vehicle dynamics using a measurement technique reported previously (Reference 2).

Averaged driver describing function data for each set of vehicle dynamics are shown in Figure 7, along with curve fits according to the model of Equations 19 and 20. Some data reinterpretation over what was reported in Reference 8 was necessary in order to obtain detailed model fits. Although each vehicle data set was fit individually, some constraints were observed across vehicles in order to obtain parameter values that changed in an orderly manner with vehicle heading time constant.

Model parameters are plotted as a function of inverse equivalent vehicle time constant (i.e., the basic bandwidth of vehicle heading or yawing response) in Figure 8. In general, the trends shown in Figure 8 are consistent with the known behavior of the human operator (Reference 4), to wit:

- Lead generation (\( T_L \)) increases with increasing system lag (\( T_{eq} \)), although the cancellation is not complete.
- Operator time delay (\( t \)) increases with lead generation (\( T_L \)).
- Open-loop gain (\( K_xK_C \)) decreases with increasing system lag to maintain stability.

In this case of multiple-loop dynamics we also note that the outer-loop gain (\( K_y \)) decreases with increasing vehicle lag, which according to Equation 24 also tends to maintain stability.

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The equivalent crossover model parameters plotted in Figure 9 give further insight into the driver's adaptation to different vehicle dynamics. Here we see that the phase lag component due to the driver's lane control behavior \( \frac{(K' + K_y U)}{\omega_n} \) is maintained relatively constant. This is accomplished by reducing \( K_y \) as \( \omega_c \) is reduced in response to increased system time delay (\( \tau_g \)) which in turn results from increased vehicle lags.

**Concluding Remarks**

The approximate driver/vehicle steering dynamics analysis developed herein provides insight into the driver's adaptive behavior. The driver offsets increased vehicle lags with anticipation or lead behavior, but in doing so incurs additional time delay penalty. The driver then compensates for the phase lag due to extra time delay by reducing his/her gain.

The effect of the driver's inherent time delay penalty on stability is analyzed with a phase margin approximation. This approximation shows that both crossover frequencies and outer-loop gain affect steering stability. Thus, the driver's behavior in controlling lane position results in a phase lag penalty which influences the directional stability of the driver/vehicle system.

The analysis in this paper relates to directional control stability independent of the path the driver is commanded to follow. Path commands due to roadway curvature evoke additional driver behavior which has been considered previously in References 3 and 5.
Figure 8. Effects of Vehicle Dynamics on Driver Model Parameters

Figure 9. Effects of Vehicle Dynamics on Driver Crossover Model Parameters
REFERENCES


