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A Normative Price for Energy from an Electricity Generation System: An Owner-Dependent Methodology for Energy Generation (System) Assessment (OMEGA)

Volume II: Derivation of System Energy Price Equations

Robert G. Chamberlain- Jet Propulsion Laboratory
Kirby M. McMaster- Loyola Marymount University, Los Angeles, California

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U.S. Department of Energy
Through an Agreement with
National Aeronautics and Space Administration
by
Jet Propulsion Laboratory
California Institute of Technology
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ABSTRACT

This report presents a generalization and updating of the Utility-Owned Solar Electric System (USES) methodology [Doane 1976], which is in widespread use throughout the energy generation systems analysis community. The major new contributions are these:

(1) Relaxation of the ownership assumption.

(2) Removal of the constraint that all systems compared must have the same system lifetime.

(3) Explicit treatment of residual system value at the end of system life. (Removal of the assumption that all components within a system have lifetimes commensurate with the system lifetime.)

(4) Explicit treatment of variations in system performance with time.

(5) Explicit treatment of tax incentives, including use of the investment as a tax shelter. Tax incentives incorporated include investment tax credits, solar tax credits, property tax rates, accelerated depreciation, and capital gains.

(6) Incorporation of financial benefits of usable thermal energy, utility buy-back (in parallel or simultaneous mode) of excess electricity generated, capacity displacement and fuel savings credits, and, where appropriate, roof credits.

The net present value of the system, viewed as an investment, is determined by consideration of all financial benefits and costs (including a specified return on investment). Along the way, life cycle costs, life cycle revenues, and residual system values are obtained. Break-even values of system parameters are estimated by setting the net present value to zero. While the model was designed for photovoltaic generators with a possible thermal energy byproduct, its applicability is not limited to such systems.

The resulting Owner-dependent Methodology for Energy Generation system Assessment (OMEGA) consists of a few equations that can be evaluated without the aid of a high-speed computer.

This report is published in two volumes. Volume I is a self-contained summary, and can be thought of as a user's guide to the application of OMEGA. Volume II gives the complete derivation.

ACKNOWLEDGEMENTS

The methodology presented in this report is a derivative of the Utility-owned Solar Electric Systems (USES) model [Doane 1976] created by **Dr. James W. Doane** of Science Applications Incorporated, Golden, CO, and **Dr. Richard P. O'Toole** of the Jet Propulsion Laboratory (JPL). It has also benefitted from the development of the Solar Array Manufacturing Industry Costing Standards (SAMICS) [Chamberlain 1979] methodology, which was based on the USES approach; discussions with **Robert W. Aster**, **Dr. James W. Doane**, **Chester S. Borden**, and **Dr. Richard B. Davis** were particularly relevant and helpful. **Mary Anne Fraresso** performed a yeoman's service in rewriting Volume I to express the OMEGA algorithm in its entirety.

Dr. Kirby M. McMaster (associate professor at Loyola Marymount University) reviewed the derivations and the pedagogical presentation from beginning to end. Although the first draft of this document was written before **Dr. McMaster** was at JPL, he contributed much to the content. The final draft, however, is the senior author's and responsibility for any obscurities or inaccuracies that may remain are his.

A preliminary draft was distributed to reviewers at JPL, at DOE, in industry, and at universities. Comments by **Tom W. Hamilton** were particularly helpful.

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ACRONYMS

APSEAM	Alternative Power Systems Economic Analysis Model
BBEC	Busbar Energy Cost
DOE	Department of Energy
FERC	Federal Energy Regulatory Commission
IPD	Implicit Price Deflator
LCP	Lifetime Cost and Performance (Model)
M&D	Marketing and Distribution (Model)
NASA	National Aeronautics and Space Administration
NPV	Net Present Value
OMEGA	Owner-dependent Methodology for Energy Generation (System) Assessment
PURPA	Public Utilities Regulatory Policies Act
PV	Photovoltaics (or Present Value, depending on context)
QIV	Fourth Quarter (of the Implicit Price Deflator published by the Department of Commerce)
SAMICS	Solar Array Manufacturing Industry Costing Standards
SYSGEN	Utility Generation System Production Costing Model
USES	Utility-owned Solar Electric Systems (Model)

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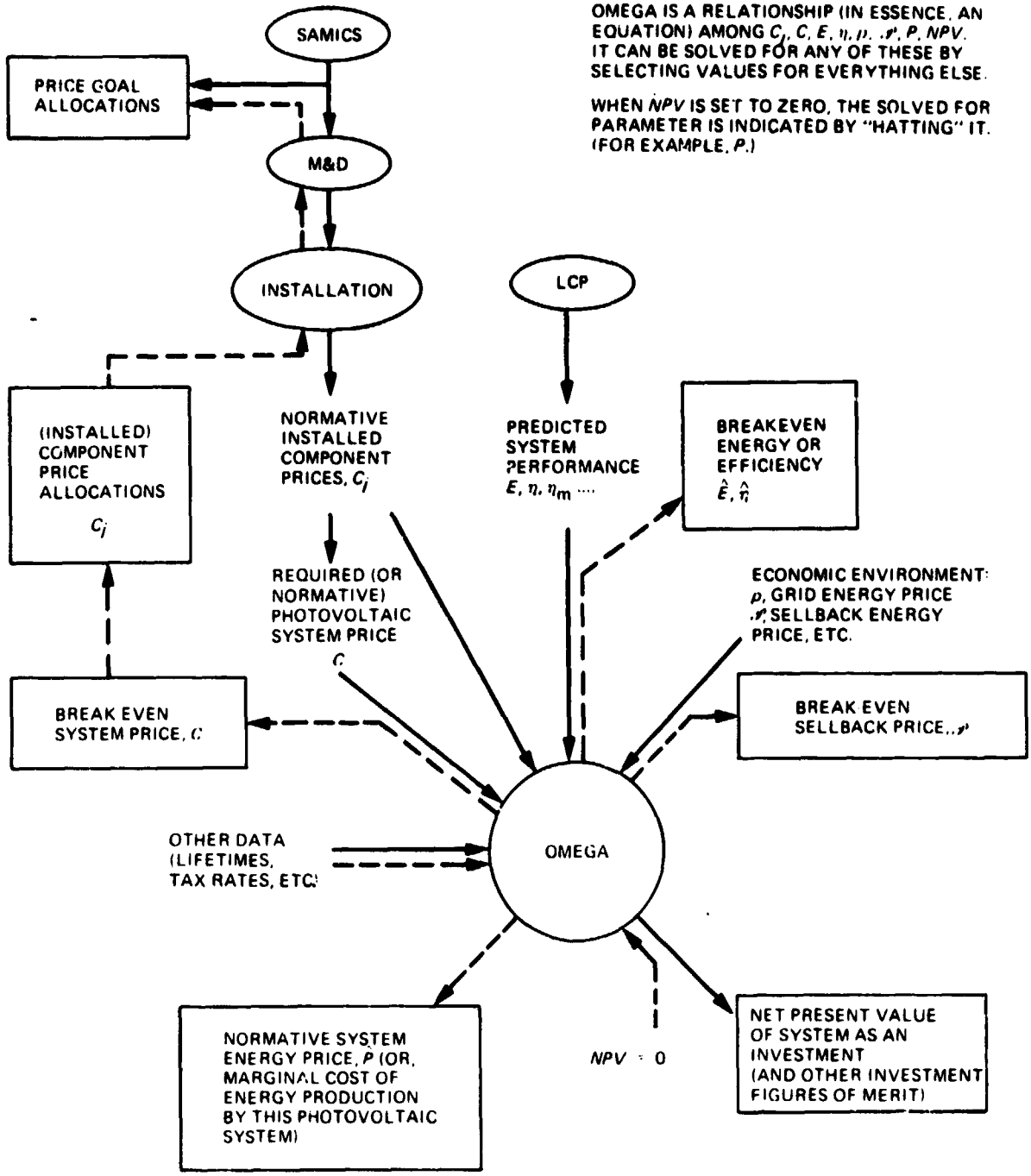

SOLID LINES PRODUCE A SUPPLY SIDE, NORMATIVE PRICE ANALYSIS.
DASHED LINES PRODUCE A DEMAND SIDE, BREAK EVEN PARAMETER ANALYSIS.
(MODELS ARE CIRCLED, POTENTIAL OUTPUTS ARE BOXED, INPUTS ARE BARE.)

MAJOR RELATIONSHIPS OF OMEGA

1. INTRODUCTION TO VOLUME II

Care was taken in preparing this document to separate the OMEGA algorithm from its rather lengthy derivation and the detailed discussion of issues, assumptions, and approximations which led to it.

The algorithm is given in Volume I, along with enough definitions and accompanying material to facilitate its use. This volume contains the background analyses and discussions. It is anticipated that the primary audiences of the two volumes will be quite different.

2. NET PRESENT VALUE

2.1 Definitions

The foundation of this methodology is the concept of the *net present value* of the system under consideration. Equal cash flows at different times do not have the same worth to the supplier (or recipient) of the cash flow; the present value operation adjusts for timing differences so that the worths can be compared. The *net* present value is obtained by subtracting the sum of the present values of all costs from the sum of the present values of all financial benefits.

Definition: The present value of a cash flow of an amount A occurring at time t is the amount of money which, if invested at a time arbitrarily called *the present* (to simplify notation, *the present* in this analysis is taken to be the time at which the system first starts producing usable energy; all times are measured relative to that time, hence: $t = 0$ at the start of usable energy production), earning interest compounded every period at the *discount rate* (see Section 4.4.1.3, II-37 for a discussion of the selection of a numerical value), would be worth A at time t. (If the cash flow occurs before *the present* the definition does not change – the investment is for a negative number at periods. An equivalent interpretation which may be easier to accept is that the amount A, earning interest compounded every period at the discount rate, would be worth the present value at *the present*.)

Not only does worth depend on the timing of cash flows, but the amount of money associated with a particular amount of worth also changes with time due to inflation and escalation.

Definition: *Inflation* is a change in the value of a given number of dollars, as approximated by the Implicit Price Deflator published by the Department of Commerce.

All present values are expressed in terms of *base year dollars*, denoted $\$_b$. The value of a dollar at time t is assumed to be equal to the ratio of the Implicit Price Deflator (IPD) at time Y_b, where Y_b is the *base year*, to its value at time t. The IPD is published quarterly; interpolation to finer resolution is not recommended – use of the IPD for the last quarter completed prior to the date of interest is preferred.

Note: The base year, Y_b, is not necessarily *the present*. (By way of illustration, a cost of A dollars on July 13, 1978, would be worth $A \times 121.60/150.98\ \$_{1975}$ because the IPD for the fourth quarter of 1974 is 121.60 and the IPD for the second quarter of 1978 is 150.98. It may be noted from this illustration that the symbol $\$_t$ is used to denote year t dollars and that the start of the year is to be assumed if no further information is given.)

Let

$\Phi_\$$ = Inflation factor from the base year, Y_b, to the start of system operation at $t = 0$; the ratio of IPD for $t = 0$ to IPD for t corresponding to Y_b. Note that $\Phi_\$$ will normally be greater than unity if the base year precedes the start of system operations.

The *present value computation* is given by

$$PV(A_t) = \frac{A_t}{\Phi_\$ (1+k)^t} \tag{1}$$

where[1]

$PV(A_t)$ = Present value of a cash flow of A_t (expressed in $\$_t$) occurring at time t, expressed in $\$_b$.

k = The discount rate, expressed in % (per year). (Note that percentages must always be converted to fractions (by dividing by 100%) before use in computations.)

The division by $(1 + k)^t$ produces the present value of the cash flow (as of $t = 0$, *the present*). The division by Φ_g deflates (or perhaps inflates) that present value to base year dollars, $\$_b$.

The IPD measures the change in the purchasing power of the dollar averaged over the entire gross national product. Prices of individual goods and services change at different rates. In addition, the amounts of goods and services required to achieve a given result may change. The combined effect of these two phenomena is called *escalation*. The following three *escalation factors* will be used later:

Φ_e = Escalation factor for the price of electrical energy from the base year to the start of system operation. (The ratio of the price of electrical energy to the prospective user of the system under consideration at time $t = 0$ to the price at Y_b.)

Φ_c = Escalation factor for the price of capital goods from the base year to the start of system operation.

Φ_x = Escalation factor for operating and maintenance expenses from the base year to the start of system operation.

In addition to these factors, which can incorporate non-constant percentage changes from year to year, the related *escalation rates*, assumed to be constant from $t = 0$ on (and in some cases before $t = 0$) for the sake of simplicity, will be used:

g_e = Electrical energy retail price escalation rate, expressed in % (per year).

g_c = Capital cost escalation rate, expressed in % (per year).

g_x = Operating and maintenance expense escalation rate, expressed in % (per year).

g_s = Escalation rate for energy sold back to the electrical utility, expressed in % (per year). This rate may be different from that for the retail price of electrical energy, g_e, as a result of changing penetration into the grid.

2.2 The *NPV* Equation

The *net present value* of the system is given by Equation 2:

$$\boxed{NPV = BENEFIT - COST} \tag{2}$$

[1] In this document, symbols used are completely defined when first introduced, usually immediately after an equation. Symbols defined earlier are not repeated. Note: The reader can find the definitions of previously defined symbols by looking for bold face type in the index.

where:

NPV = Net present value of all financial benefits and costs that would result from installing and operating the system under consideration, expressed in $\$_b$.

BENEFIT = Sum of the present values of all financial benefits (revenues and/or energy cost savings) resulting from system operation, expressed in $\$_b$.

COST = Sum of the present values of all costs resulting from system operation, expressed in $\$_b$.

2.3 The Required Revenue Condition

The basis for the determination of a normative system energy price is the *required revenue condition*:

> The price of energy shall be chosen so that the present value of all financial benefits is exactly equal to the present value of all costs (including a "reasonable" return on equity).

That is to say, the normative system energy price is that which makes the net present value of investment in the system exactly equal to zero.

The normative price that results from the application of this condition is the marginal cost of energy production by the system under consideration, and hence the minimum price of energy at which an investor considering the system would consider the investment economically viable.

The *normative system energy price* is also the *break-even system energy price*. In fact, break-even values of any of the system parameters are defined on the same basis:

> The break-even value of a system parameter is that value, if any, for which the present value of all financial benefits is exactly equal to the present value of all costs (including a "reasonable" return on equity).

Investment decisions will require consideration of additional information, such as the effects of interactions[2] between the system considered and other hardware already in place or scheduled, the relationship of this investment to the investor's total investment portfolio, and non-financial benefits and costs (such as energy independence, reduction of pollutant production, improvement of employees' working conditions, system aesthetics, and so on) relative to the competitive alternatives. The net present value itself is one of the most useful figures of merit available to the potential investor, and provides a measure in which the size of the system has not been eliminated.

2.4. Benefits and Costs

The two terms on the right hand side of Equation (2), II-3 may be written more explicitly as follows:

$$BENEFIT = \sum_{t=-\infty}^{N} PV(Benefit_t) + \sum_{t=N+1}^{\infty} PV(Benefit_t) \qquad (3)$$

[2] Some of the possible interactions can be evaluated within this methodology: see the discussions of *capacity credit*, *fuel savings credit*, *thermal credit*, and *roof credit*.

$$COST = \sum_{t=-\infty}^{N} PV(Cost_t) + \sum_{t=N+1}^{\infty} PV(Cost_t) \tag{4}$$

where

$Benefit_t$ = Total financial benefit received at t, expressed in nominal dollars ($\$_t$).

$Cost_t$ = Total cost paid at t, expressed in $\$_t$.

N = System lifetime, expressed in years. In the USES methodology, it is assumed that the end of the system lifetime ($t = N$) is a satisfactory planning horizon, and that either the present value computation (see Equation (1), II-2) reduces any benefits or costs that occur after $t = N$ to insignificance or that the system is decommissioned at $t = N$. The use of BBEC as a measure of the normative system energy price in USES led to the restriction that all systems to be compared had to have the same system lifetime, because the price BBEC does not show the effects of inflation on benefits. (The effects of inflation are fully accounted for in the USES determination of $BBEC_0$.) The restriction that all systems compared must have the same lifetime has been removed in OMEGA; in fact, the system lifetime need not even be related to the replacement lifetimes of any of the system components. In OMEGA, the system lifetime is rather arbitrary – it marks the dividing line between life cycle costs (and revenues) and the residual system value. If the system is resold for capital gains, it is sold at the end of the system lifetime – in that context, N should be considered to be the *investment lifetime.*

PV = Present value function defined by Equation (1), II-2 .

The careful reader will note in the following pages that although all the explanations are written in terms of annual cash flows, there is little in the mathematics to preclude the use of shorter time periods, such as months. Anyone willing to bear the additional computational burden can use the model with appropriately adjusted rates and lifetimes.

The first summation in both Equations (3) and (4), addresses the time period from the first system-resultant cash flow (which is very likely to occur before the start of system operations at $t = 0$, but much later than t = *negative infinity*, so the lower limit of the summation can be replaced by t_f, the time of that first cash flow) to the end of the system lifetime.

The present value operation causes the terms in the second summation in Equations (3) and (4), to be relatively small[3]. Furthermore, if the system under consideration is assumed to be a finite-lived project, then the last system-resultant cash flow occurs at or near the end of the system lifetime, so that there are no terms in the second summation. These arguments constitute the rationale for omitting the second summation altogether, as was done in USES. For calculation of the financial contribution to capital budgeting decisions, these conditions are usually met and the second summations can indeed be omitted.

[3]From Equation (1), II-2, if k = 10% and N = 30 years, a cash flow of \$1 at $t = N + 1$ contributes only \$0.05 to the present value.

However, this methodology has other[4] applications, and the necessary conditions are not always met. The concept of *residual system value* has been introduced to extend the methodology's applicability to situations in which component lifetimes are not necessarily submultiples of the system lifetime.

2.5. Life Cycle Revenue and Cost and Residual System Value

For convenience, associate names and symbols with the four summations in Equations (3) and (4), II-4:

$$REV = \sum_{t=t_f}^{N} PV\,(Benefit_t) \tag{5}$$

where

REV = Life cycle revenues; the sum of the present values of all financial benefits (not *just* revenues) resulting from system operation during the system lifetime, expressed in $\$_b$.

t_f = Time of the first system-resultant cash flow, expressed in years after the start of system operation. (Thus, t_f will normally be negative.)

$$LCC = \sum_{t=t_f}^{N} PV\,(Cost_t) \tag{6}$$

where

LCC = Life cycle cost; the sum of the present values of all costs resulting from system operation during the system lifetime, expressed in $\$_b$.

$$RSB = \sum_{t=N+1}^{\infty} PV\,(Benefit_t) \tag{7}$$

where

RSB = Benefit part of residual system value; the sum of the present values of all system-resultant financial benefits that occur after the end of the system lifetime, expressed in $\$_b$.

$$RSC = \sum_{t=N+1}^{\infty} PV\,(Cost_t) \tag{8}$$

[4]One of the most interesting uses of this methodology, for example, is the determination of the effect of component lifetimes on system energy price. Omission of the second summations can cause peculiar jumps in system energy price as the component lifetime is varied past submultiples of the system lifetime. By way of illustration, suppose the system lifetime is 30 years and array lifetimes of 29 and 31 years are considered. The 29-year array must be replaced at t = 29 in order to keep the system operational for 30 years, the 31-year array need not be replaced. The benefits from operating the system with the 29-year array for 27 more years than with the 31-year array would be ignored, but a major contributor to the cost of providing that capability would not be.

where

RSC = Cost part of residual system value; the sum of the present values of all system-resultant costs that occur after the end of the system lifetime, expressed in $\$_b$.

The net residual system value is of interest as well:

$$RSV = RSB - RSC \tag{9}$$

where

RSV = (Net) residual system value; the net present value of all system-resultant benefits and costs that occur after the end of the system lifetime, expressed in $\$_b$.

Evaluation of the four terms defined by Equations (5), through (8) II-6, requires consideration of what would really be done with the system at the end of the system lifetime. If the system were a "wonderful one-hoss shay"[5], there would be no further benefits or costs to be considered, and the residual system value would be zero. The same would be the case if, the system were resold, because the revenues from resale are (arbitrarily) treated as a reduction in life cycle cost, rather than as a residual value. (See Section 4.5.2.2, II-51.) If the system is decommissioned, on the other hand, the sum of the salvage values of the components, taking into account the fact that they may have different ages, should be used as the resale revenue, and the present value of the costs of system removal and site restoration should be inserted as the (negative) residual system value.

In many cases, the system, having been kept in good repair, would still have considerable value if maintenance and use were to be continued, or if those components that are still in good condition were to be used in other but similar systems. In these cases, it is the value remaining in the system that should be used. One way to estimate this value would be to sum the depreciated values of all of the system components. Depreciation, however, is merely a legally-approved approximation to that value and was designed to meet other[6] objectives. A better way to estimate the residual system value in these cases is to assume, *for the purpose of this calculation*, that the system will be maintained and operated in perpetuity.[7] The discounting that often makes the system lifetime a satisfactory planning horizon will greatly reduce the impact of ignoring the fact that obsolescence will eventually make continued operation uneconomic. The effect of this treatment is to move the planning horizon considerably farther into the future.

Strictly speaking, the summations in the definitions of *REV* and *LCC* should include all benefit and cost cash flows resulting from installation of the system and operation of it during the system lifetime, and the summations in the definitions of *RSV* should include all benefit and cost cash flows resulting either from removal, salvage, and restoration or from continuation of system operation beyond the specified lifetime. The distinction between "costs

[5] In Oliver Wendell Holmes' classic poem "The Deacon's Masterpiece or The Wonderful One-Hoss Shay", the deacon designed and built a horse-drawn carriage so exquisitely that no maintenance or repairs were ever required. After exactly 100 years of daily operation, the carriage was *worn out* – but so uniformly that it suddenly became "a heap or mound, as if it had been to the mill and ground".

[6] Tax depreciation rules are designed to encourage certain types of investment. Book depreciation conventions are sometimes designed to provide protection against some kinds of uncertainties and sometimes designed to estimate resale (i.e. salvage) values. Neither tax nor book depreciation usually accounts for inflation.

[7] It may be noted that extending the planning horizon to the least common multiple of all of the component lifetimes (and the system life) will give the same result. That, however, would be a little more cumbersome mathematically, and no more satisfactory from a philosophical point of view.

during the system lifetime" and "costs resulting from operation during the lifetime" is primarily of theoretical, not practical, importance: The timing of actual cash flows may not coincide exactly with the model of system operations, anyway. For example, receipt of payments for utility bills usually occurs a month or two after the electricity is delivered. There has been no attempt to model the real world to that level of detail.

3. BENEFITS

Financial benefits considered here come from three sources, which are of different relative importance to the different kinds of owners. The three sources are:

(1) Electrical energy generated by the system and used by the system owner. The *normative system energy price* is an estimate of the cost of producing this energy – it is analogous to the *busbar energy cost* (BBEC) determined by USES.

(2) Thermal energy generated by the system and used by the system owner. In the present context, thermal energy is treated as a byproduct of system operation. Its value (per unit of energy) is determined exogenously (by the user of OMEGA) from consideration of displacement of thermal energy purchases (e.g., fuel oil, coal, natural gas, etc.) and, possibly, from consideration of other effects on the owner's operations (e.g., more – or less – comfortable working conditions, reductions – or increases – in manufacturing process time, and so on).

(3) Electrical energy generated by the system and sold to the electrical utility. The price at which it is sold is determined by the combined effect of all such "cogenerators" on the utility [FERC 1980]. If *all* of the electrical energy generated by the system is sold to the utility, the system energy price is not meaningful; the *break-even sellback price* is then the estimate of the cost of producing this energy.

Displaced expenditures for purchased electrical energy are considered to be in the first category of financial benefits. Displaced expenditures for energy generation capacity, for fuel, and for maintenance or other operating expenses are treated here as cost reductions, not as financial benefits. Any tax benefits are also treated as cost reductions.

Thus, for all kinds of owners,

$$Benefit_t = (1 - \sigma_t)\, P_t E_t + \alpha_t p_t E_t + \sigma_t\, \mathscr{P}_t E_t \tag{10}$$

where

σ_t = Fraction of the electrical energy produced by the system during year t which is sold to the electrical utility. If the system owner *is* the utility, this is the fraction sold to some other utility or the fraction put into storage.

P_t = Average value of electrical energy produced by the system during year t and used by the system owner, expressed in $\$_t$/kWh. When estimating the net present value of the system, P_t should be set to the average price of electrical energy obtained from alternative sources (e.g., the electrical grid). When determining the normative system energy price (this term is meaningful only if σ_t is not identically equal to unity), P_t is the value of that variable in year t.

E_t = Electrical energy (delivered to the "busbar" or to the meter) generated by the system during year t, expressed in kWh.

α_t = Value of system-produced thermal energy in year t, expressed as a fraction of the value that the system-produced electrical energy would have at the grid price of electricity. This parameter can be expressed as the product of a price fraction and an energy fraction:

$$\alpha_t p_t E_t = (\alpha_{1t} p_t) \times (\alpha_{2t} E_t) \tag{11}$$

p_t = Average retail price of electrical energy obtained from the utility grid in year t, expressed in $\$_t$/kWh.

α_{1t} = Price (or value) of thermal energy in year t, expressed as a fraction of the price of electrical energy obtained from the grid. Although the value of thermal energy is *expressed* relative to the grid price, the displaced *supplier* of thermal energy need not be the electrical grid.

α_{2t} = Thermal energy used by the system owner in year t, *expressed* (but not necessarily *produced*) as a fraction (or multiple) of the electrical energy produced by the system.

$\mathscr{P}_t$ = Average value of electrical energy produced by the system during year t and sold to the utility, expressed in $\$_t$/kWh. When estimating the net present value of the system, $\mathscr{P}_t$ should be set to the average sellback price for electrical energy (which must be determined exogenously). When determining the break-even sellback price (this term is only meaningful if σ_t is not identically equal to zero), $\mathscr{P}_t$ is the value of that variable in year t.

3.1 The Price Of Energy

Energy prices are assumed to escalate uniformly, but not necessarily at equal rates:

$$
\begin{aligned}
P_t &= P\,\Phi_e\,(1+g_e)^{t-1/2} \\
p_t &= p\,\Phi_e\,(1+g_e)^{t-1/2} \\
\mathscr{P}_t &= \mathscr{P}\,\Phi_e(1+g_s)^{t-1/2}
\end{aligned}
\tag{12}
$$

where

P = System energy price, the marginal cost of electrical energy produced by the system or the price of system-supplied electrical energy used by the system owner at the start of system operations ($t = 0$), expressed in $\$_b$/kWh.

p = Price of electrical energy obtained from the utility grid at the start of system operations ($t = 0$), expressed in $\$_b$/kWh.

$\mathscr{P}$ = Sellback price for electrical energy produced by the system and sold to the utility at the start of system operations ($t = 0$), expressed in $\$_b$/kWh.[8]

Strictly speaking, the averaging of energy prices called for by the definitions of P_t, p_t, and $\mathscr{P}_t$ should be weighted by energy production over the year, rather than merely being taken at the middle of the year, as is implied by the "-1/2" in the exponents in Equation (12). However, the error of approximation introduced by the assumption of uniform escalation, is probably more severe than that caused by failing to weight the average. Furthermore, the error of approximation due to assuming that the escalation rate is *known* is probably larger than either. (Ignoring escalation, on the other hand, would be tantamount to assuming that it is *zero*, which would lead to much poorer estimates.)

[8] A slight adjustment may be needed to account for transmission losses (or the lack thereof) if the system is installed on a residence. See also Footnote 11, II-11.

3.2 The Amount Of Net Electrical Energy

The net electrical energy produced by the system comes first from an initial period of building up to capacity;[9] then, after the plant has reached capacity, energy production is assumed to vary over a definable interval in a predictable way. If the system is assumed[10] to be "revitalized" by a block replacement of the energy collection components (i.e., the arrays), then the pattern of energy production during the first interval repeats itself until the system is decommissioned.

Determination of the predicted performance of the system is beyond the scope of this analysis; it is not a trivial task for weather dependent systems or for those that degrade with time. An hourly simulation over both the buildup period (called the *increment to capacity* period in [Borden 1980]) and the first of the repetitive system energy production cycles may be necessary. The following discussion is given to aid in the use of the results of such simulations; much of it would not be needed for weather independent systems which do not experience performance degradation.

The net electrical energy is thus assumed to vary with time as follows:

$$E_t = \begin{cases} E\, e_t & \text{for } t \leqslant t_o \\ E\, \eta_m & \text{for } t \geqslant t_o + 1 \end{cases} \tag{13}$$

where

t_o = Time at which the system starts full capacity operation. (Note that t_o could be *zero* if there were no *increments to capacity*, or it could be very large if there were no repetitive energy generation cycle.)

E = Net electrical energy produced by the system[11] during the *first year of capacity operation*, expressed in kWh. (If the possibility of "revitalization" is to be ignored, so that t_o is very large, some other year could be used to characterize the system's nominal output. Development of this approach is left as an exercise for the interested reader.)

e_t = Net electrical energy produced by the system in year t during the period of building up to capacity operation, expressed as a fraction of initial full-capacity output, E.

η_m = Relative performance, the net electrical energy produced by the system during the m-th year into a repetition of the full-capacity system energy production cycle, expressed as a fraction (or multiple) of the initial full-capacity output, E.

The index m is defined by the equation

$$m = L \times frac\ [(t - t_o)/L] \tag{14}$$

[9] This period of building up to capacity could, of course, be absent.

[10] It is recognized that the actual replacement scenario may not match this assumption. This scenario, however, should give a reasonably accurate formulation, and introduces the *system revitalization period* explicitly into the model. The effects of changes in the *expected value of the energy collection component lifetime* may be readily assessed by this scenario. The model does not depend on the validity of this scenario – the user may, if he wishes to supply sufficient data, define a *system revitalization period* so large that even the *residual system value* is unaffected by "revitalization" of the system.

[11] It should be noted, both here and particularly in the calculation of the value of electrical energy sold back to the grid, that transmission losses should be considered properly. "Proper consideration" depends upon how this analysis is being used.

where

$frac(A)$ = The *fractional part* of A.

L = System "revitalization" period, the length of the repetitive energy production cycle, expressed in years.

The annual energy produced by a solar energy system depends upon the local intensity of solar radiation, the installed area of solar collectors, and the system efficiency. Solar radiation depends upon location, time of year, and weather. The collector area depends upon how much of the system has been installed and is operating. System efficiency depends upon array pointing, cell temperatures, performance degradation (due to such factors as dirt accumulation, encapsulant yellowing, cell cracking, etc.), electrical losses, parasitic power requirements, replacement policies, and so on.

The annual net energy produced by a solar electric system can be estimated by Equation (15):

$$E_t = S_t Z_t \eta'_t \tag{15}$$

where

S_t = Total solar energy incident on the system during year t, expressed in kWh/kWp (which reduces to the rather obscure – in this context – unit *hours*). It should be noted that the value of this datum depends on the system design, as well as on location, because systems differ in their ability to use *diffuse* insolation (in addition to *direct* insolation). The value does not, however, depend on the area or the efficiency of the system.

Z_t = Average nameplate size of the system in year t, expressed in kWp. It may be noted that the use of the product of S_t and Z_t to represent the solar energy input to the system allows the system size to be expressed in terms of nameplate capacity, rather than area, imposing only the constraint that the same (perhaps rather arbitrary) definition of the insolation level required to produce one Wp be applied in determining both quantities.

η'_t = Average system efficiency, due to the combined effect of all factors, in year t.

Comparison of Equations (13), II-11, and (15) shows that

$$e_t = S_t Z_t \eta'_t / E \quad \text{for } t \leq t_o \tag{16}$$

$$E = S Z \eta \tag{17}$$

$$\eta_m = (S_t \eta'_t)/(S \eta) \quad \text{for } t_o + 1 \leq t \leq t_o + L \qquad m = t - t_o \tag{18}$$

where

S = Total solar energy incident on the system in year $t = 0$, expressed in kWh/kWp.

Z = Nameplate size of the system at capacity, expressed in kWp.

η = Average system efficiency (η_t') during the first year of full-capacity operation (that is, for t in the range $t_o < t \leq t_o + 1$).

Note that Equation (18) places a constraint on application of the methodology with monthly resolution: If the revitalization period does not correspond to an integral number of years, the degradation pattern would get out of synchronization with the insolation pattern. This lack of synchronization does not occur until t reaches $t_o + L + 1$; even then the impact on estimated energy production should not be large.

3.3 Life Cycle Revenues and Residual System Benefit

The year-by-year financial benefits from the system can be obtained by substituting the energy price equations (Equations (12), II-10) and the energy production equations (Equations (13), II-11) into the benefit equation (Equation (10), II-9). The result of these substitutions is:

$$Benefit_t = \left\{[(1-\sigma_t)\,P + \alpha_t p]\,(1+g_e)^{t-1/2} + \sigma_t \mathscr{P}(1+g_s)^{t-1/2}\right\} \Phi_e E\, e_t \quad \text{for } t \leq t_o \tag{19}$$

$$Benefit_t = \left\{[(1-\sigma_t)P + \alpha_t p]\,(1+g_e)^{t-1/2} + \sigma_t \mathscr{P}(1+g_s)^{t-1/2}\right\} \Phi_e E\, \eta_m \quad \text{for } t > t_o \tag{20}$$

where m is defined by Equation (14), II-11.

These benefit equations may now be substituted into the life cycle revenue Equation (5), II-6, and the benefit part of the residual system value Equation (7), II-6, to obtain, after defining some summary variables,

$$REV = [(1-\sigma)P\,spm + \alpha\, p\, tpm + \sigma \mathscr{P} bpm]\, E \tag{21}$$

$$RSB = [(1-\sigma)P\,rspm + \alpha\, p\, rtpm + \sigma \mathscr{P} rbpm]\, E \tag{22}$$

$$BENEFIT = REV + RSB = [(1-\sigma)P\,spm_v + \alpha\, p\, tpm_v + \sigma \mathscr{P} bpm_v]\, E \tag{23}$$

where

σ = Fraction of system-produced electrical energy which is sold back to the utility in the first year of full capacity operation ($t_o \leq t < t_o + 1$). Note that when $\sigma_t = 0$ for all t, the third term in Equations (21) to (23) need not be evaluated; when $\sigma = 1$ for all t, the first term may be omitted. (See also the discussion of σ_t on II-9.)

α = Value of system-produced thermal energy in the first year of full capacity operation ($t_o \leq t < t_o + 1$). (See also the discussion of α_t, II-9.)

spm = *System energy price multiplier*, a factor which accounts for the effects of changes in system capacity and performance and the discounting of escalating financial benefits *during the system lifetime*. See Equation (29), II-15.

rspm = *System energy price multiplier* for the ("residual") period *after the system lifetime*. See Equation (30), II-16.

spm_v = *System energy price multiplier both during and after the system lifetime*. This factor is the sum of *spm* and *rspm*, but it can be expressed more simply: see Equation (31), II-16.

tpm, *rtpm*, tpm_v = *Thermal energy price multipliers*, factors which account for the effects of changes in thermal energy production and the discounting of escalating financial benefits *during*, *after*, and *during and after* the *system lifetime*, respectively. See Equations (32), (33), (34), II-17 and II-18.

bpm, *rbpm*, bpm_v = *Sellback energy price multipliers during*, *after*, and *during and after* the system lifetime. See Equations (35), (36), (37), II-18 and II-19.

The energy price multipliers are conceptually simple, but their defining equations are somewhat complicated. If $N < t_o$ (that is, if the nameplate size of the system changes throughout the system lifetime),

$$spm = \begin{cases} \phi_e (1+g_e)^{-1/2} \sum_{t=t_f}^{N} \left(\frac{1-\sigma_t}{1-\sigma}\right)\left(\frac{1+g_e}{1+k}\right)^t e_t & \text{if } \sigma \neq 1 \\ 0 & \text{if } \sigma = 1 \end{cases} \tag{24}$$

where

ϕ_e = Φ_e/Φ_s, the relative inflation (or, as economists would say, the *real escalation*) of energy prices from the base year, Y_b, to the start of system operation, $t = 0$.

If $N > t_o$ (construction is completed before the end of the system lifetime), which will usually be the case, there are three time periods of interest: the build-up to capacity, the completed energy production cycles, and the partial cycle to the end of the system life. Thus, provided $\sigma \neq 1$, first calculate the number of completed cycles,

$$n = int\,[(N - t_o)/L] - 1 \tag{25}$$

where

$int\,(A)$ = the *integer part* of A. (26)

Then,

$$spm = \phi_e (1+g_e)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \left(\frac{1-\sigma_t}{1-\sigma}\right) \left(\frac{1+g_e}{1+k}\right)^t e_t \right.$$

$$+ \sum_{i=1}^{n} \sum_{m=1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma}\right) \left(\frac{1+g_e}{1+k}\right)^{t_o+(i-1)L+m} \eta_m \tag{27}$$

$$\left. + \sum_{m=1}^{N-(t_o+nL)} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma}\right) \left(\frac{1+g_e}{1+k}\right)^{t_o+nL+m} \eta_m \right\}$$

If $n = 0$, the summation on i is to be omitted. That summation, incidentally, can be simplified.[12] Omitting the intermediate algebra, and incorporating Equation (24), II-14, the *system energy price multiplier during the system lifetime* can be expressed as follows:

If $\sigma = 1$, $spm = 0$.

If $\sigma \neq 1$ and $N \leq t_o$, with $R_e = (1+g_e)/(1+k)$,

$$spm = \phi_e (1+g_e)^{-1/2} \sum_{t=t_f}^{N} \left(\frac{1-\sigma_t}{1-\sigma}\right) R_e^t e_t$$

If $\sigma \neq 1$ and $N > t_o$, with $n = int\,[(N-t_o)/L)] - 1$,

$$spm = \phi_e (1+g_e)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \left(\frac{1-\sigma_t}{1-\sigma}\right) R_e^t e_t \right. \tag{29}$$

$$+ \frac{R_e^{t_o}(1-R_e^{nL})}{1-R_e^L} \sum_{m=1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma}\right) R_e^m \eta_m$$

$$\left. + R_e^{t_o+nL} \sum_{m=1}^{N-(t_o+nL)} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma}\right) R_e^m \eta_m \right\}$$

[12] Series number 2 in [Jolley 1961] is

$$a + ar + ar^2 + \ldots n \text{ terms} = \sum_{i=1}^{n} ar^{i-1} = a\frac{1-r^n}{1-r} \tag{28}$$

Similarly, the *system energy price multiplier after the system lifetime* is given by:

If $\sigma = 1$,

$$rspm = 0.$$

If $\sigma \neq 1$ and $N < t_o$,

$$rspm = \phi_e (1+g_e)^{-1/2} \left\{ \sum_{t=N+1}^{t_o} \left(\frac{1-\sigma_t}{1-\sigma} \right) R_e^t e_t + \frac{R_e^{t_o+L}}{1-R_e^L} \sum_{m=1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma} \right) R_e^m \eta_m \right\} \tag{30}$$

If $\sigma \neq 1$ and $N > t_o$, with $n = int\,[(N-t_o)/L] - 1$,

$$rspm = \phi_e (1+g_e)^{-1/2} R_e^{t_o+nL} \left\{ \sum_{m=N-(t_o+nL)+1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma} \right) R_e^m \eta_m + \frac{R_e^L}{1-R_e^L} \sum_{m=1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma} \right) R_e^m \eta_m \right\}$$

The *system energy price multiplier during and after the system lifetime* is given by:

If $\sigma = 1$,

$$spm_v = 0.$$

If $\sigma \neq 1$,

$$spm_v = \phi_e (1+g_e)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \left(\frac{1-\sigma_t}{1-\sigma} \right) R_e^t e_t + \frac{R_e^{t_o}}{1-R_e^L} \sum_{m=1}^{L} \left(\frac{1-\sigma_{t_o+m}}{1-\sigma} \right) R_e^m \eta_m \right\} \tag{31}$$

The *thermal energy price multipliers* are just like the *system energy price multipliers*, except that α_t and α replace $(1 - \sigma_t)$ and $(1 - \sigma)$:

If $\alpha = 0$,

$$tpm = 0$$

If $\alpha \neq 0$ and $N \leq t_o$,

$$tpm = \phi_e (1 + g_e)^{-1/2} \sum_{t=t_f}^{N} \frac{\alpha_t}{\alpha} R_e^t e_t$$

If $\alpha \neq 0$ and $N > t_o$,

$$tpm = \phi_e (1 + g_e)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \frac{\alpha_t}{\alpha} R_e^t e_t + \frac{R_e^{t_o}(1 - R_e^{nL})}{1 - R_e^L} \sum_{m=1}^{L} \frac{\alpha_{t_o+m}}{\alpha} R_e^m \eta_m + R_e^{t_o+nL} \sum_{m=1}^{N-(t_o+nL)} \frac{\alpha_t}{\alpha} R_e^m \eta_m \right\} \tag{32}$$

If $\alpha = 0$,

$$rtpm = 0.$$

If $\alpha \neq 0$ and $N \leq t_o$,

$$rtpm = \phi_e (1 + g_e)^{-1/2} \left\{ \sum_{t=N+1}^{t_o} \frac{\alpha_t}{\alpha} R_e^t e_t + \frac{R_e^{t_o+L}}{1 - R_e^L} \sum_{m=1}^{L} \frac{\alpha_{t_o+m}}{\alpha} R_e^m \eta_m \right\}$$

If $\alpha \neq 0$ and $N > t_o$,

$$rtpm = \phi_e (1 + g_e)^{-1/2} R_e^{t_o+nL} \left\{ \sum_{m=N-(t_o+nL)+1}^{L} \frac{\alpha_{t_o+m}}{\alpha} R_e^m \eta_m + \frac{R_e^L}{1 - R_e^L} \sum_{m=1}^{L} \frac{\alpha_{t_o+m}}{\alpha} R_e^m \eta_m \right\} \tag{33}$$

If $\alpha = 0$,

$$tpm_v = 0.$$

If $\alpha \neq 0$,

$$tpm_v = \phi_e (1+g_e)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \frac{\alpha_t}{\alpha} R_e^t e_t + \frac{R_e^{t_o}}{1-R_e^L} \sum_{m=1}^{L} \frac{\alpha_{t_o+m}}{\alpha} R_e^m \eta_m \right\} \tag{34}$$

Also, the *sellback energy price multipliers* are just like the *system energy price multipliers*, except that the sellback price escalation rate must replace the energy price escalation rate, as well as the sellback fraction replacing its complement. Thus,

If $\sigma = 0$,

$$bpm = 0.$$

If $\sigma \neq 0$ and $N \leq t_o$,

with $\phi_s = \Phi_s/\Phi_g$ and $R_s = (1+g_s)/(1+k)$,

$$bpm = \phi_s (1+g_s)^{-1/2} \sum_{t=t_f}^{N} \frac{\sigma_t}{\sigma} R_s^t e_t$$

If $\sigma \neq 0$ and $N > t_o$,

$$bpm = \phi_s (1+g_s)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \frac{\sigma_t}{\sigma} R_s^t e_t + \frac{R_s^{t_o}(1-R_s^{nL})}{1-R_s^L} \sum_{m=1}^{L} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m + R_s^{t_o+nL} \sum_{m=1}^{N-(t_o+nL)} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m \right\} \tag{35}$$

If $\sigma = 0$,

$$rbpm = 0.$$

If $\sigma \neq 0$ and $N \leq t_o$,

$$rbpm = \phi_s (1 + g_s)^{-1/2} \left\{ \sum_{t=N+1}^{t_o} \frac{\sigma_t}{\sigma} R_s^t e_t + \frac{R_s^{t_o+L}}{1 - R_s^L} \sum_{m=1}^{L} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m \right\} \tag{36}$$

If $\sigma \neq 0$ and $N > t_o$,

$$rbpm = \phi_s (1 + g_s)^{-1/2} R_s^{t_o+nL} \left\{ \sum_{m=N-(t_o+nL)+1}^{L} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m + \frac{R_s^L}{1 - R_s^L} \sum_{m=1}^{L} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m \right\}$$

And, finally,

If $\sigma = 0$,

$$bpm_v = 0$$

If $\sigma \neq 0$,

$$bpm_v = \phi_s (1 + g_s)^{-1/2} \left\{ \sum_{t=t_f}^{t_o} \frac{\sigma_t}{\sigma} R_s^t e_t + \frac{R_s^{t_o}}{1 - R_s^L} \sum_{m=1}^{L} \frac{\sigma_{t_o+m}}{\sigma} R_s^m \eta_m \right\} \tag{37}$$

4. COSTS

System costs are of five basic types:

(1) Repayment of capital used to purchase capital goods:

REP_t = Amount of equity principal returned in year t, expressed in $\$_t$.

PDR_t = Amount of debt retired in year t, expressed in $\$_t$.

(2) Payment of a reasonable return on investment:

EQR_t = Amount of return paid to holders of equity in year t, expressed in $\$_t$.

INT_t = Amount of interest paid on debt in year t, expressed in $\$_t$.

(3) Payment of increased (or decreased) income taxes:

TAX_t = Amount of additional (over those which would be paid without the system under consideration) state and federal income taxes paid in year t, expressed in $\$_t$.

RSL_t = Gain from resale[13] of the system in year t, expressed in $\$_t$. This gain enters the cost equation as a negative term, because it results from selling the system at market value at the end of the *investment lifetime*, and produces a receipt.

(4) Payment of expenses that result from the presence of capital goods:

OTX_t = Other (than income) taxes (primarily property taxes) in year t, expressed in $\$_t$.

INS_t = Insurance premiums for year t, expressed in $\$_t$.

(5) Payment of expenses for operations, maintenance, fuel, etc.:

X_t = Operating expenses in year t, expressed in $\$_t$.

Adding up all of these costs gives the following expression for the annual cost:

$$Cost_t = REP_t + PDR_t + EQR_t + INT_t + TAX_t - RSL_t + OTX_t + INS_t + X_t \tag{38}$$

It should be noted that the actual purchase costs of capital goods are not explicitly included in Equation (38). It is assumed that capital goods are purchased with funds from equity investment and debt financing: Financial benefits (that is, explicit or implicit revenues) are not required to purchase capital goods, but they must cover equity and debt repayment (as well as return on equity and interest on debt), in addition to the costs of operating the

[13] A case could be made for calling RSL_t a *financial benefit*, rather than a *cost*. The rationale for considering it as a negative cost is that it is not a result of what the system is designed to do, but that it is the result of a financial manipulation involving the system. It is included as a cost of type (3), along with income taxes, because it is primarily the deductibility of depreciation (especially accelerated depreciation) that may make resale financially desirable. (Inflation increases its desirability.)

system. Thus, from the point of view of an investor, investment in the system under consideration may be thought of as a "financial machine" which converts investment cash flow streams into repayment cash flow streams. As will be seen in later sections, most of the cost components do depend on the amount of capital investment.

By way of comparison, the USES model includes capital investment as a cost (negative cash flow) and balances this by including equity investment (stock sales) and debt financing (bond sales) as benefits (positive cash flows). This is equivalent to the current model, because USES assumes that the amount of investment is equal (in a present value sense) to the combined revenue from stock sales and bond sales.

4.1. Capacity Credits and Fuel Savings Credits

Equation (38) allows for a reduction in costs that might occur when the proposed system reduces the need for conventional or alternative energy generation. The cost reductions are in the form of displaced or deferred capital investment (capacity credit) and avoided system operating expenses. The avoided operating expenses can include fuel (fuel savings credit), operations, maintenance, etc.

Credits for avoided operating expenses can be entered into the model as negative expenses through the X_t term in Equation (38).

Capacity credits can be viewed as negative capital investments. To include a capacity credit in the model, the full effect of a negative capital investment on cost must be entered. To do this, the method of financing and the book and tax depreciation schedules that would have been used must be detailed. Then the appropriate capital cost multipliers can be calculated for the displaced capital investments.

The capacity credits and fuel savings credits resulting from the displacement of expenditures on alternative energy generation equipment apply only if the system under consideration is evaluated by itself. If the alternative system design is also to be evaluated, and the resultant energy prices are to be compared, then these credits are not relevant, as they are intended to account for the difference. These credits would also not be relevant if there is no alternative system. Three examples to illustrate these points follow:

First example: Suppose a utility is considering adding capacity to its present plant to provide additional capacity in the face of a growing demand. One alternative might be to add 100 MWe of coal burning generators; the solar electric option might be 10 MWe of gas turbines plus 150 MWp of photovoltaics. (To make the comparison meaningful, both alternatives must be sized to provide the same level of service, which might be measured by the "loss of load probability" or by some other figure of merit.) Then, if the solar (plus gas turbine) system is priced by itself, the capital required for the 100 MWe coal plant may be taken as a *capacity credit*, and the cost of the coal that that plant would use may be taken as a *fuel savings credit*. The resultant price would be compared to the price of energy from the utility's current plant; a lower price for the solar system would indicate that the "solar plus gas turbine plus existing" option is cheaper than the "coal plus existing" option. On the other hand, the analyst may calculate a price for "solar plus gas plus existing" and a price for "coal plus existing"; in this case, the capacity credit and fuel savings credit would be "double counted" if they were included.

Second example: A solar (sub)system might be added to an existing plant, allowing the decommissioning of some existing equipment (while still providing the same performance) and/or allowing the utility to utilize less fuel. Then the capital that can be recovered by the decommissioning (if any) provides the *capacity credit*, and the reduction (if any) in fuel utilized provides the *fuel savings credit*.

Third example: Suppose a company is considering installing a photovoltaic system to supply part of its electrical and thermal energy needs. The photovoltaic system will not be connected to the grid, so no electricity will be sold back to the utility. However, any needs that the photovoltaic system cannot meet will be purchased from the electric and gas utilities. In this scenario, the *capacity credit* and either the *fuel savings credit* or the *thermal credit* (but not both) would be zero. Any effects that the company's reduced demand would have on retail electricity and gas prices would have to be dealt with exogenously or by modeling the larger system that includes the utilities.

4.2. Capital Investment

Funds for capital investment are assumed to be supplied by equity investment and debt financing, not by revenue from system energy generation. However, this statement, while true, can be misleading, for almost all of the cost components in Equation (38), II-20, are directly dependent on capital expenditures.

It is assumed that each kind of capital good has a known, deterministic lifetime, L_j, at the end of which it is replaced. Furthermore, some of the capital must be supplied quite a while before the start of system operation. Capital costs are assumed to grow at the rate of capital escalation. In some cases, replacement costs will be different than initial costs due to significant installation differences, and, possibly, due to differences in construction contingency allowances that must be made. Thus,

$$CI_t = \Sigma_j CI_{jt} \tag{39}$$

$$CI_{jt} = \begin{cases} C_j \Phi_c (1+g_c)^{t_j} & \text{for } t = t_j \\ C_j \Phi_c (1+g_c)^t (1+\delta_j) & \text{for } t = t_j + L_j, t_j + 2L_j, \ldots \end{cases} \tag{40}$$

where

CI_t = Capital investment at time t, expressed in $\$_t$.

CI_{jt} = Capital investment at time t in capital goods of type j, expressed in $\$_t$. The upper expression in Equation (40) applies to the initial installation, which occurs at $t = t_j$. The lower expression applies each time that capital good is replaced, which occurs at intervals of L_j.

C_j = Purchase cost of capital goods of type j, expressed in $\$_b$. This cost includes any allowance for construction contingencies that may be necessary (perhaps due partially to unfamiliarity among the construction trades with special handling required for this particular kind of hardware).

δ_j = Additional cost of replacement of capital goods of type j, expressed as a fraction of the capital cost at initial installation. This difference may be positive as might be the case if roof credits (see below) were claimed during initial installation, or it might be negative, as would be the case if the initial installation included a significant allowance for contingency costs. (*Zero* is also an acceptable value.)

t_j = Time at which funds for capital goods of type j must first be expended, expressed in years.

L_j = Lifetime (i.e., replacement period) of capital goods of type j, expressed in years.

4.2.1. Roof Credits

If any of the system components are designed to be part of some capital structure that would be built even if the system under consideration were not, then only the *additional* cost due to installation of the system should be charged to the system. (For example, if the energy collection components also serve as a residential roof covering, the appropriate cost to use is any excess over the cost of the conventional roof that would have been installed instead.) If the system component has the same lifetime as the structure that it supplants, the C_j in Equation (40) should simply be reduced in magnitude. If lifetimes are different, however, a capital category should be created for the supplanted capital good, and it should be included in the summation of Equation (39) with a negative capital cost (C_j).

4.2.2. Initial Capital

The initial cost of the system is somewhat difficult to define precisely, since the capital requirements may occur at different times. The liquidity requirement, discussed in Section 7.2, II-67, is a more meaningful measure of the funding that the owner must have in order to pursue the investment opportunity.

Nevertheless, the initial capital cost can be a useful measure of the state of the art, particularly if a broadened definition of "operations and maintenance" cost is added to the presentation. For this purpose, the simplest definition is

$$\boxed{K = \Sigma_j C_j / Z} \tag{41}$$

where

K = Initial system capital, expressed in base year dollars per unit of nameplate size (that is, in $\$_b$/kWp).

Z = Nameplate size of the system at full capacity, expressed in peak kilowatts II-12.

The broadened operations and maintenance cost is defined so that it accounts for everything else. This is,

$$NPV = (K + \bar{K})\, Z \tag{42}$$

where

NPV = Net present value of the system, expressed in base year dollars II-3.

$\bar{K}$ = Equivalent initial system non-capital cost, expressed in base year dollars per unit of nameplate size (that is, kWp). This is the broadened operations and maintenance cost term.

Equation (42) may be easily solved to give

$$\boxed{\bar{K} = \frac{NPV}{Z} - K} \tag{43}$$

Another definition of initial capital cost may be needed for tax purposes (note the similarity to, but difference from, Equation (39), II-22):

$$PUR = \Sigma_j CI_{j,t_j} \tag{44}$$

where

PUR = Purchase price of the system, expressed in nominal dollars of mixed vintage.

4.2.3. Capital Valuation and Depreciation

The value of capital goods changes over time as a result of wear-out, obsolescence, inflation, and other factors. These changes are approximated by use of *depreciation* schedules. Often, two, sometimes three, legally acceptable sets of books are kept, side by side: one for investors' accounting, two for tax purposes. All of the analytically tractable schedules can also be expressed by equations, as is done in Appendix C. The relationship between capital investment and valuation is given by

$$VAL_{jT} = CI_{jt}\, sch(T-t, DL_j, \mu_j, \theta_j) \text{ for } T = t, t+1, t+2, \ldots \tag{45}$$

where

VAL_{jT} = Investors' accounting book value ($BVAL_{jT}$) or tax book value based on accelerated depreciation ($TVAL_{jT}$) or tax book value based on straight line depreciation ($STVAL_{jT}$) at time T, expressed in $\$_T$, of capital goods of type j, purchased at time t. These values are often quite different due to the use of shortened tax lives and accelerated depreciation methods.

$sch(A,DL,\mu,\theta)$ = Schedule function for the depreciating value of capital of age A when the depreciable life is DL, the depreciation method is μ, and the salvage value fraction is θ, expressed as a fraction of the initial value. Schedule functions and related quantities are discussed in detail in Appendix B.

DL_j = Depreciable lifetime of capital goods of type j, expressed in years. Lifetimes used in the tax books may be shorter than those used in the investors' accounting books, which will normally be equal to L_j, the expected useful life.

μ_j = Depreciation method to be used for capital goods of type j (Appendix B), which may be different for the accounting books than for the tax books.

θ_j = Salvage value of goods of type j at the end of their lifetime, expressed as a fraction of purchase cost (that is, in $\$_t$ per $\$_t$).

Two characteristics of depreciation are of interest: (1) current value, and (2) change in value during the preceding year. The first is needed for use in computation of expenses that depend on depreciated value, such as property taxes and return on equity. The second is needed to determine the legally acceptable estimate of the depreciation "expense" that can be deducted from taxable revenue on the system owner's income tax return, and to determine the payments for return of equity principal.

The schedule function defined above gives the current value of capital goods for each year in the lifetime of the goods. The change in value from year to year is described by the depreciation function. The depreciation function for a capital asset can be defined in terms of the schedule function for each year after the initial purchase as:

$$dep(A, L, \mu, \theta) = sch(A - 1, L, \mu, \theta) - sch(A, L, \mu, \theta) \tag{46}$$

where

$dep(A, L, \mu, \theta)$ = Depreciation function, the change in value of capital during the preceding year, where the capital good is of age A out of a depreciable life of L, the depreciation method is μ, and the salvage value fraction is θ, expressed as a fraction of the initial value. Depreciation functions and related quantities are discussed in detail in Appendix B.

Then,

$$DEP_{jt} = CI_{jt}\, dep(T - t, DL_j, \mu_j, \theta_j) \text{ for } T = t + 1,\ t + 2, \ldots \tag{47}$$

where

DEP_{jT} = Book depreciation ($BDEP_{jT}$) or tax depreciation ($TDEP_{jT}$) during year T, expressed in $\$_T$, of capital goods of type j purchased at time t. Book and tax depreciation are often quite different due to use of shortened tax lives and "accelerated" depreciation methods. (Appendix B.)

Total system values and depreciations are obtained by summation over the different kinds of capital goods:

$$BVAL_t = \Sigma_j\, BVAL_{jt} \tag{48}$$

$$STVAL_t = \Sigma_j\, STVAL_{jt} \tag{49}$$

$$TVAL_t = \Sigma_j\, TVAL_{jt} \tag{50}$$

$$BDEP_t = \Sigma_j\, BDEP_{jt} \tag{51}$$

$$TDEP_t = \Sigma_j\, TDEP_{jt} \tag{52}$$

> It should be noted that if the owner of the system is a utility, current laws require that the same tax lives and depreciation methods be used for the tax and accounting books. This fact has not been built into the current model as a constraint in order to allow the option of evaluating alternative laws. The user of the methodology, however, should take care to satisfy the constraint when it is applicable.

The price for which the system can be sold will be needed in the evaluation of the use of investment in the system as a tax shelter. Since that price is a market-determined number, and therefore subject to such unmodelable factors as fashion, the uncertainty in any model must be large. *It is recommended that the user of OMEGA specify its value:*

$$RSL'_{t=N} = \text{exogenous.} \tag{53}$$

where

RSL'_t = The price at which the system could be resold at time t, expressed in $\$_t$.

If, however, the user would rather not do so, a reasonable estimate can be provided by assuming book depreciation inflated by the rate of general inflation:

$$RSL'_t = \Sigma_j BVAL_{jt} (1 + g_\$)^{t-t_j} \tag{54}$$

Another reasonable estimate could be provided by assuming that the system can be resold for its residual system value:

$$RSL'_{t=N} = RSV \tag{55}$$

Whether the resale price is obtained from Equation (53), (54), or (55), the resale revenue is defined by

$$RSL_t = \begin{cases} RSL'_N & \text{if } t = N \\ 0 & \text{if } t \neq N \end{cases} \tag{56}$$

where

RSL_t = Revenue from resale of the system at time t, expressed in $\$_t$.

4.3 INCOME TAXES

The income tax term can be eliminated from Equation (38), II-20, by expressing it in terms of revenues and costs. First, note that

$$TAX_t = \tau STI_t - ITC_t - STC_t \tag{57}$$

where

τ = System owner's marginal tax rate, which is assumed to be unaffected by investment in the system under consideration, expressed as a fraction of taxable income.

STI_t = System-resultant effect on the owner's taxable income in year t, expressed in $\$_t$.

ITC_t = Investment tax credit available to the system owner due to initial and replacement capital investments in system components in year t, expressed in $\$_t$.

STC_t = Special solar tax credit available to the system owner due to initial (but not replacement) capital investments in solar energy system components in year t, expressed in $\$_t$.

The effect of system ownership on taxable income depends on the owner's financial environment, and is discussed in the next three subsections.

4.3.1 Resale Of The System – A Possible Tax Shelter

If depreciation is deductible, which is allowable to whatever fraction of the system's operation contributes to taxable income, and the system owner sells the system for an amount which exceeds its depreciated value, the difference between the selling price and the depreciated value is subject to taxation.

The way in which this difference contributes to taxable income, STI_t, however, is quite complicated: part of it is considered to be *ordinary income*; the rest of it, if any, is considered to be a *capital gain*, which is taxed as if it were a smaller amount of ordinary income. The system owner benefits considerably from the deductibility of depreciation, but he benefits from the portion of the resale gain that is taxed as ordinary income only due to the time value of money – that is, income tax on the "recovered" portion of depreciation is deferred from the time it is taken as depreciation until the time the system is sold. The system owner benefits from the portion taxed as a capital gain in that way and, more importantly, by the fact that taxes on the untaxed portion of the capital gain are deferred forever! To illustrate the significance of these points, consider the following three examples.

Common to all three examples: Suppose an investor buys a solar energy system for $10,100 in 1980. Suppose the resale value of that system decreases linearly to $100 in 1990 when expressed in 1980 dollars, but that 10% inflation increases the resale value when expressed in nominal dollars. Suppose further that the investor's discount rate is 13% per year and his marginal income tax rate is 35%. The following table shows the year by year resale values (and summarizes the results of the three examples):

	Resale Value		(Partial) Net Present Value		
Year	In 1980 $	In Nominal $	Example 1	Example 2 Improvement over Example 1	Example 3 Improvement over Example 2
1980	10,100.00	10,100.00	0.00	0.00	0.00
1981	9,100.00	10,010.00	-1,241.59	27.87	140.93
1982	8,100.00	9,801.00	-2,424.38	117.58	233.13
1983	7,100.00	9,450.10	-3,550.61	256.34	235.03
1984	6,100.00	8,931.01	-4,622.44	433.35	303.86
1985	5,100.00	8,213.60	-5,641.99	639.55	295.74
1986	4,100.00	7,263.40	-6,611.25	867.34	265.90
1987	3,100.00	6,041.02	-7,732.20	1,110.37	218.77
1988	2,100.00	4,501.54	-8,406.70	1,363.39	158.09
1989	1,100.00	2,593.74	-9,236.58	1,622.04	87.02
1990	100.00	259.37	-10,023.59	1,882.76	8.22

Example 1: No depreciation. If the investor is a consumer and operates the system as a stand-alone system (that is, he sells no electricity to the grid), he may not deduct depreciation from his taxable income. The net present value of the investment (ignoring the financial benefits and all expenses that are common to all of the scenarios in these examples,– that is, considering only the effects of resale, depreciation, and capital gains) with resale in year t is simply the discounted value of the resale price minus the initial purchase price, and is shown in the above table. (In an analysis of all benefits and costs, these negative values are balanced, at least to some extent, by the system energy production. The full complexity is omitted here to illustrate the tax shelter effect.)

Example 2: Depreciation but no capital gains. If the investor sells all of the energy produced, he may deduct depreciation (of $1000 each year if he using the "straight-line" method) from his taxable income, thereby saving $350 each year. However, when he sells the system, he must pay 35% of the difference between the resale price and

the depreciated value. These calculations are shown in the following table, and the net effect is shown in the above table:

Year	Depreciation	Discounted Tax Savings Due to Deprec.	Cumulative Discounted Tax Savings	Taxable Income If Resold This Year	Present Value of Tax on Resale Income
1980	–	0.00	0.00	0.00	0.00
1981	1,000.00	309.73	309.73	910.00	-281.86
1982	1,000.00	274.10	583.83	1,701.00	-466.25
1983	1,000.00	242.57	826.40	2,350.10	-570.06
1984	1,000.00	214.66	1,041.06	2,831.01	-607.71
1985	1,000.00	189.97	1,231.03	3,113.60	-591.48
1986	1,000.00	168.11	1,399.14	3,163.40	-531.80
1987	1,000.00	148.77	1,547.91	2,941.02	-437.54
1988	1,000.00	131.66	1,679.57	2,401.54	-316.18
1989	1,000.00	116.51	1,796.08	1,493.74	-174.04
1990	1,000.00	103.11	1,899.19	159.37	- 16.43

Example 3: Capital gains. Suppose that all of the resale revenue in excess of the depreciated value is taxable as capital gains, rather than as ordinary income. Suppose further that the fraction of capital gains which is added to taxable income is one-half.[14] Then the two right-hand columns in the previous table would be halved, which produces the "Example 3" column in the table before that.

In these examples the benefits of depreciation seem to outweigh the benefits of the capital gains tax shelter, so that the "smart investor" would retain the system if the investment were properly described by the chosen numerical values of parameters. This is not necessarily the case, however, for the investor could use the resale revenues to purchase another system. It is not the intent of this analysis to investigate the optimal investment strategy; it does however, provide the information necessary to such an investigation.

The contribution of resale revenues to taxable income is described by the following equation:

$$RTI_t = \begin{cases} \Gamma CG_t + OI_t & \text{if} \quad t = N \\ 0 & \text{if} \quad t \neq N \end{cases} \tag{58}$$

where

RTI_t = Contribution of revenues from resale of the system to taxable income in year t, expressed in $\$_t$.

Γ = Capital gains fraction, the fraction of capital gains which is added to taxable income.

[14] "One-half" is used for simplicity in exposition. Currently, a consumer would be taxed on 40% of the capital gain; a company would pay 28% of the capital gain as (federal) income tax. Since companies with taxable income in excess of $100,000 pay a 46% marginal (federal) income tax rate, the effective fraction is 28%/46% = 61%. (Of course, the actual taxation algorithm is not that simple.)

CG_t = The portion of the revenues obtained from resale of the system at time t which would be designated a capital gain, expressed in $\$_t$.

OI_t = The portion of the revenues obtained from resale of the system at time t which would be designated ordinary income, expressed in $\$_t$.

The way in which the resale revenues are apportioned into capital gains and ordinary income is rather complicated. For example, if the system is considered to be "Section 1245 property."[15] then

$$OI_t = RF[min(RSL_t, PUR) - TVAL_t] \tag{59}$$

where

RF = Fraction of depreciation which is to be "recovered"[16] = 1.0.

$min(a, b)$ = The smaller of a or b.

$TVAL_t$ = Depreciated value (on the tax books) of the system at time t, expressed in $\$_t$.

If, on the other hand, the system is considered to be, for example, part of the homeowner's house (which might make it "Section 1250 property"):

$$OI_t = RF[min(RSL_t, STVAL_t) - TVAL_t] \tag{60}$$

where

$STVAL_t$ = Depreciated value (on the straight-line tax books) of the system at time t, expressed in $\$_t$.

Equations (59) and (60) can be combined into a single expression by introducing a new index variable, ρ:

$$\boxed{OI_t = RF\ \{min\ [RSL_t, \rho PUR + (1 - \rho) STVAL_t] - TVAL_t\}} \tag{61}$$

where

$$\rho = \begin{cases} 0 & \text{If only } \textit{accelerated} \text{ depreciation in excess of decrease of market value is to be recovered. ("Section 1250 property")} \\ 1 & \text{if } \textit{all} \text{ depreciation in excess of decrease in market value is to be recovered. ("Section 1245 property")} \end{cases}$$

[15] (The interested reader should consult a tax lawyer or a tax accountant before making any decisions based on the legal interpretations offered here.) "Section 1245 property" is depreciable property which has been held more than 12 months and "which is. . . used as an integral part of. . .[the]. . .furnishing of. . .electrical energy" [Tax Guide 1980, paragraph 989]. "Section 1250 property" is all other depreciable property held more than 12 months [Tax Guide 1980, paragraph 990].

While it is difficult to see how a system of the type under consideration in this document could be classified as anything other than "Section 1245 property," classification as "Section 1250 property" would increase the attractiveness of investment, perhaps considerably. Such a classification, then, might be made to provide an investment incentive.

[16] If the property is held longer than 100 months, some of the amount that would be treated as ordinary income will be treated as capital gains. *This* refinement is ignored.

Once the "ordinary income" portion has been determined, the rest is "capital gains":

$$CG_t = RSL_t - TVAL_t - OI_t \tag{62}$$

4.3.2 A Company-Owned System

If a utility owns the system, all of the financial benefits contribute to taxable income. Electrical energy produced by the system is sold (at the retail price, not the normative system energy price) to the utility's customers, thereby generating taxable revenues. Usable thermal energy reduces deductible expenses (perhaps by heating or cooling the control building, perhaps by preheating the water in the utility's conventional steam turbines, etc.), thereby increasing the taxable income. The "sellback" revenue term is retained because any "excess" energy might be sold ("wheeled," in utility jargon) to another utility (in which case, the *sellback price*[17], $\mathscr{P}$, should be based on an analysis of both utilities), stored for later use (in which case, the *sellback price* is the product of the retail price and all of the storage-destorage efficiencies), or dumped (in which case the *sellback price* is zero – or perhaps even negative if dumping of excess electrical energy is not free).

If a non-utility business owns the system, the effect on taxable income is the same, because utilities and businesses face the same[18] tax environment (that which applies to corporation). The electrical energy produced by the system does not necessarily create revenues, but it reduces deductible expenses, which has the same effect.

The effect on taxable income also includes allowable deductions and the possible gain from resale of the system. Thus, from the financial benefit Equation (10), II-9, deductible components from the annual cost Equation (38), II-20, the resale contribution to taxable income, and the tax depreciation, the system-resultant effect on taxable income is given by:

$$STI_t = [(1-\sigma_t)p_t + \alpha_t p_t + \sigma_t \mathscr{P}_t]\, E_t + RTI_t - (TDEP_t + INT_t + OTX_t + INS_t + X_t) \tag{63}$$

where all of the terms have already been defined.

Using the income tax Equation (57), II-26, and Equation (63) for the effect on taxable income, the TAX_t term may be removed from the annual cost Equation (38), II-20, to give the *year by year cost for a company-owned system*:

$$\begin{aligned} Cost_t = {} & EQR_t + \tau\,[(1-\sigma_t)p_t + \alpha_t p_t + \sigma_t \mathscr{P}_t]\, E_t + [REP_t + PDR_t - RSL_t - \tau\,(TDEP_t - RTI_t)] \\ & - (ITC_t + STC_t) + (1-\tau)\,[INT_t + OTX_t + INS_t + X_t] \end{aligned} \tag{64}$$

Inspection of this equation reveals that the cost is the sum of

(1) A return to the equity investors (that is, the stockholders) for the use of their money, EQR_t. Although this term appears first in the above equation, it bears essentially all of the risk in the real world, for it is computed last.

[17] In OMEGA, the sellback price is always exogenous (that is, specified as input).

[18] It may be, however, that a solar energy system owned by a utility would be classed as electrical generation equipment ("Section 1245 property") so that ρ (II-29) = 1), while the same system owned by a non-utility would be classed as real estate ("Section 1250 property", so that $\rho = 0$).

(2) The government's share of the financial benefits, $\tau[(1-\sigma_t)p_t + \alpha_t p_t + \sigma_t \mathscr{P}_t]\, E_t$.

(3) Payment for the capital goods, consisting of

(A) A return of equity investment corresponding to the real reduction in their value as they wear out, REP_t.

(B) Repayment of loans by which the capital goods were purchased, PDR_t.

(C) A receipt due to resale of the entire system, $(-RSL_t)$.

(D) A government contribution to cost of capital goods, $[-\tau(TDEP_t - RTI_t)]$. In a sense, the existence of this term makes the government an investor in the system, just as the stockholders are; the government's return on that investment is its share of the financial benefits minus its contribution to the operating expenses, $(-\tau)[INT_t + OTX_t + INS_t + X_t]$. Curiously enough, the government's rate of return on investment does not depend (to first order – the tax rate is assumed constant) on the tax rate, τ.

(E) Government subsidies (transfer payments from all taxpayers to investors in capital goods) designed to encourage investment in capital equipment, $(-ITC_t)$, and in solar energy generation equipment, $(-STC_t)$.

(4) Payment of the operating expenses, $[INT_t + OTX_t + INS_t + X_t]$.

4.3.3 A Consumer-Owned System

If a consumer owns the system, the tax environment is quite different. All electrical and thermal energy that is supplied directly to the consumer, without being sold to the utility, $(1-\sigma_t)E_t$, reduces the electric bill (and possibly the gas, fuel oil, and/or coal bill, as well) but has no effect on taxable income, since these expenses are not deductible on personal income tax returns. Any sellback revenues are taxable, but all expenses, including depreciation associated with producing the taxable income are deductible. Consequently, the system-resultant effect on taxable income is given by:

$$STI_t = \sigma_t\,[\mathscr{P}_t E_t + RTI_t - (TDEP_t + OTX_t + INS_t + X_t)] - INT_t \tag{65}$$

(Interest expense is deductible even if the consumer sells no energy.)

Again, using the income tax Equation (57), II–26, but noting the only σ_t of the investment tax credit may be taken, and using Equation (65) for the effect on taxable income, the TAX_t term may be removed from the annual cost Equation (38), II-20, to give the *year-by-year cost for a consumer-owned system*:

$$\begin{aligned} Cost_t = {} & EQR_t + \tau\sigma_t \mathscr{P}_t E_t + (1-\tau)\,INT_t + [REP_t + PDR_t - RSL_t - \tau\sigma_t(TDEP_t - RTI_t)] \\ & - (\sigma_t ITC_t + STC_t) + (1-\tau\sigma_t)\,[OTX_t + INS_t + X_t] \end{aligned} \tag{66}$$

4.3.4 The Cost Equation

By comparing the year-by-year cost equations for a company-owned system, Equation (64), II-30, and for a consumer-owned system, Equation (66), II-31, it may be seen that by judiciously choosing an indicator variable, ω, a combined equation for the *year-by-year cost* (for the three owner types considered here) may be written:

$$
\begin{aligned}
Cost_t = {} & EQR_t + \tau\,[\omega(1-\sigma_t+\alpha_t)\,p_t + \sigma_t\,\mathscr{P}_t]\,E_t \\
& + [REP_t + PDR_t - RSL_t - \tau(\omega + (1-\omega)\,\sigma_t)\,(TDEP_t - RTI_t)] \\
& - [(\omega + (1-\omega)\,\sigma_t)\,ITC_t + STC_t)] + (1-\tau)\,INT_t \\
& + [1 - \tau(\omega + (1-\omega)\,\sigma_t)]\,[OTX_t + INS_t + X_t]
\end{aligned}
\tag{67}
$$

where

$$
\omega = \begin{cases} 1 & \text{if the owner is a company (utility or non-utility)} \\ 0 & \text{if the owner is a consumer} \end{cases}
$$

4.4 Cost Components

The next five subsections present models of the component terms of Equation (67).

4.4.1 Capital Recovery

An economically viable system must generate benefits of sufficient value to replenish equity investment and repay debt principal. In addition, the system must provide a reasonable rate of return on equity and pay the required interest on debt. However, the (discounted) costs of equity and debt repayment, return on equity, and interest on debt depend on the time schedule for the payments. For instance, in the usual case where a system owner's cost of capital is less than his rate of return on equity, but greater than the interest rate on debt, it would increase the net present value of the system to repay the equity investment as soon as possible and to delay debt repayment as long as possible. It is assumed, however, that the timing decisions are made for other reasons; companies are assumed to operate with constant financial leverage, consumers to use mortgage-type financing. The detailed implications of these assumptions[19] will be brought out below.

[19] The USES model assumed [Doane 1976, pages B-4 to B-7] that equity and debt principal were repaid by contribution to a sinking fund and that return on investment was paid at the owner's discount rate. Since the capital recovery factor is identically equal to the sum of the discount rate and the sinking fund factor, the USES' annual repayment of principal was equal to the product of the present value of capital investment and the capital recovery factor. By the definition of the capital recovery factor, the present value of this stream of repayments is indeed the present value of capital investment. However, while its present value is correct, the USES formulation incorrectly apportions funds between repayment *of* investment and payment of return *on* investment. The present formulation corrects that error.

Regardless whether book depreciation is used for such special purposes as to provide protection against possible early obsolescence, it is generally accepted[20] accounting procedure to calculate the book value of equity so that the following equation is satisfied at all times:

$$BVAL_t = DVAL_t + EVAL_t \tag{68}$$

where

$BVAL_t$ = Book value of capital investment[21] at time t, expressed in $\$_t$.

$DVAL_t$ = Value of the system debt at time t, expressed in $\$_t$.

$EVAL_t$ = Value of equity investment in the system at time t, expressed in $\$_t$.

Another way of expressing this requirement is that the combined payments toward debt and equity principal must equal the book depreciation[22].

$$BDEP_t = PDR_t + REP_t \tag{69}$$

where

$BDEP_t$ = Book depreciation in year t, expressed in $\$_t$.

The variation in book value (and the associated book depreciation) is determined (Equation (45), II-24) by the capital investments and the depreciation schedule function. The variation in debt value is determined by the capital investments and the loan repayment schedule, which is discussed for different kinds of owners in the next two subsections. The information required for these calculations is obtained exogenously, so Equation (68) can be solved to obtain the equity portion of the total value:

$$EVAL_t = BVAL_t - DVAL_t \tag{70}$$

Furthermore, Equation (69) can be solved to obtain the annual return of equity principal:

$$REP_t = BDEP_t - PDR_t \tag{71}$$

In addition to repayment of loans and compensation for reductions in the value of equity, a reasonable[23] (exogenously specified) return on investment must be paid to creditors and investors as long as their capital is in use.

[20] One purpose of this accounting convention is to provide a realistic measure of the market value of the equity-owned share of the assets. To the extent that the book valuation is designed to serve other purposes and to the extent that it does not account for inflation or for speculation, it does not achieve this purpose.

[21] It is assumed that there are no "intangible" assets, such as "customer good-will," associated with the system under consideration.

[22] The sinking fund approach of USES does not satisfy Equation (69).

[23] There is relatively little difficulty in determining an appropriate numerical value for the debt interest rate, but an appropriate numerical value for the rate of return on equity is more difficult to come by. For companies, the weighted average of reasonable rates of return associated with the various kinds of stock offered will suffice. For consumers, a reasonable rate of return is the opportunity cost – the rate that can be obtained on other long-term investments.

The general form of the equations for interest on debt and return on equity, which apply to all kinds of system owners, is

$$INT_t = i \quad DVAL_{t-1} \tag{72}$$

$$EQR_t = r \quad EVAL_{t-1} \tag{73}$$

where

i = Debt interest rate, expressed in %/year.

r = Required rate of return on equity, expressed in %/year.

4.4.1.1 Corporate Ownership

If a utility or a non-utility company owns the system, it is assumed that company management policies maintain a constant relationship among the different financial instruments. That is, the financial leverage[24] is constant. Hence,

$$DVAL_t = \left(\frac{\lambda - 1}{\lambda}\right) BVAL_t \tag{74}$$

$$EVAL_t = \frac{1}{\lambda} BVAL_t \tag{75}$$

where

λ = Financial leverage, the ratio of total capital to equity capital. The more familiar *debt-to-equity ratio* is given by $(\lambda - 1)$.

The assumption of constant leverage implies that the debt and equity repayment portions of book depreciation are

$$PDR_t = \left(\frac{\lambda - 1}{\lambda}\right) BDEP_t \tag{76}$$

$$REP_t = \frac{1}{\lambda} BDEP_t \tag{77}$$

Tax depreciation, deductible as an expense on the corporate income tax return, is obtained from Equation (52), II-25.

[24] In USES, the symbols C/V, P/V, D/V were used for *common stock as a fraction of total value*, *preferred stock as a fraction of total value*, and *debt as a fraction of total value*, respectively. These symbols are related to the leverage, λ, by

$$\frac{C}{V} + \frac{P}{V} = \frac{1}{\lambda}, \frac{D}{V} = 1 - \frac{1}{\lambda}.$$

From Equations (72) and (73), the returns *on* investment are given by

$$INT_t = i \frac{\lambda - 1}{\lambda} BVAL_{t-1} \tag{78}$$

$$EQR_t = \frac{r}{\lambda} BVAL_{t-1} \tag{79}$$

4.4.1.2 Consumer Ownership

Consumers (homeowners) are assumed to finance each capital investment separately with a mortgage type of loan. The initial value of the equity portion of the investment is whatever the homeowner must provide as a down payment, the rest is debt. The homeowner makes a uniform payment each year[25] until the mortgage is paid off. Part of the payment is interest on the loan, the rest reduces the loan balance, simultaneously increasing the equity value. Any decreases in equity value (which could happen if the book value declined faster than the loan balance) are repaid to the owner via the REP_t term.

At time t, an investment in the amount of CI_{jt} is made in capital goods of type j. (Section 4.2, II-22.) The mortgage taken out is

$$MORT_{jt} = (1 - \epsilon)\, CI_{jt} \tag{80}$$

where

$MORT_{jt}$ = Mortgage taken out on capital goods of type j at time t, expressed in $\$_t$.

ϵ = Down-payment fraction[26], the portion of a capital improvement that lending institutions will not cover.

The annual payment, which is the same every year, is

$$PAY_{jT} = MORT_{jT}\, crf_{i,MP} \quad \text{for } T = t+1, t+2, \ldots, t+MP \tag{81}$$

where

PAY_{jT} = Uniform annual payment (partially debt interest, the rest debt principal), expressed in $\$_T$, on the mortgage taken out at time t.

T = The dates of the payments resulting from the mortgage taken out at time t, expressed in years.

i = Mortgage interest rate, expressed in %/year.

[25] Mortgage payments are usually made monthly, rather than annually. However, the convention of annual cash flow is retained for consistency. Of course, as was noted on page II-5, the cash flows can be converted to a monthly basis if desired.

[26] The numerical value of ϵ will depend (in part) upon whether the system is *new* or *retrofit* construction.

MP = Mortgage period, expressed in years, not necessarily[27] assumed to be shorter than the component lifetime.

$crf_{i,MP}$ = Capital recovery factor; the uniform periodic payment, as a fraction of the original principal, that will fully repay a loan (including all interest) in MP periods at a periodic interest rate of i, with interest on the unpaid balance compounded every period.

The capital recovery factor [Doane 1976, p. B-14] is given by

$$crf_{i,MP} = \begin{cases} i/[1-(1+i)^{-MP}] & \text{if } i \neq 0 \\ 1/MP & \text{if } i = 0 \end{cases} \tag{82}$$

The mortgage payoff process is described by the following set of difference equations:

$$DVAL_{jt} = MORT_{jt} \tag{83}$$

$$\left.\begin{aligned} INT_{jT} &= i\ DVAL_{j,T-1} && (84) \\ PDR_{jT} &= PAY_{jT} - INT_{jT} && (85) \\ DVAL_{jT} &= DVAL_{j,T-1} - PDR_{jT} && (86) \end{aligned}\right\} \text{ for } T = t+1, t+2, \ldots, t+MP$$

These difference equations can be solved to give, for T as above,

$$DVAL_{jT} = MORT_{jt}\ \frac{1}{i}\ [crf_{i,MP} - (1+i)^{T}\ sff_{i,MP}] \tag{87}$$

$$INT_{jT} = MORT_{jt}\ [crf_{i,MP} - (1+i)^{T-1}\ sff_{i,MP}] \tag{88}$$

$$PDR_{jT} = MORT_{jt}\ (1+i)^{T-1}\ sff_{i,MP} \tag{89}$$

where

$DVAL_{jT}$, INT_{jT}, PDR_{jT} are defined like $DVAL_t$, INT_t, PDR_t but for capital goods of type j bought at time t.

$sff_{i,MP}$ = Sinking fund factor; the uniform periodic payment, as a fraction of the *final balance* (cf the definition of the capital recovery factor) that will accumulate (including all interest) in MP periods to that final balance at a periodic interest rate of i, with interest on the balance compounded every period.

[27] Lending institutions might constrain this period to be less than the expected lifetime (L_j) of the capital goods (of type j) purchased, in which case MP might have to be subscripted by j. It is likely, however, that the energy generation system and its components will be considered as part of the house, and paid for accordingly when installed as part of a *newly constructed* house. *Retrofit* installations, on the other hand, are likely to be financed by mortgages whose periods are more closely related to actual lifetimes of the equipment.

The sinking fund factor equals the capital recovery factor minus the interest rate; explicitly, it may be calculated by [Doane 1976, p. B-5]:

$$sff_{i,MP} = \begin{cases} i/[(1+i)^{MP}-1] & \text{if } i \neq 0 \\ 1/MP & \text{if } i = 0 \end{cases} \tag{90}$$

The cost components needed in Equation (67), II-32, are obtained by summing over system components:

$$\left.\begin{aligned} PDR_T &= \Sigma_j PDR_{jT} \\ INT_T &= \Sigma_j INT_{jT} \end{aligned}\right\} \quad T = t+1, t+2, \ldots, t+MP \qquad \begin{aligned} &(91) \\ &(92) \end{aligned}$$

where

PDR_{jT} is obtained from Equation (89).

INT_{jT} is obtained from Equation (88).

$MORT_{jt}$ is obtained from Equation (80), II-35.

The homeowner's decrease in equity is determined from Equation (71), II-33, which is repeated here as Equation (93).

$$REP_t = BDEP_t - PDR_t \tag{93}$$

The value of equity is obtained by applying Equation (70), II-33, repeated here as Equation (94).

$$EVAL_t = BVAL_t - DVAL_t \tag{94}$$

Finally, the return on equity is given by Equation (73), II-34, repeated here as Equation (95).

$$EQR_t = r \quad EVAL_{t-1} \tag{95}$$

4.4.1.3. The Discount Rate

The discount rate, denoted in this document by the symbol k, expresses the way in which the timing of a cash flow affects the value of that cash flow, and is essential to the calculation of present values.

Determination of an appropriate value to use in a particular context may require a great deal of subtle analysis. In the present context of determining a normative price, the task is much simpler: only a *reasonable* estimate of the discount rate consistent with the other numerical values used is required, rather than an estimate of the discount rate that various kinds of owners either *would* or *should*[28] use. A reasonable, consistent estimate can be obtained for the kinds of owners considered here from financial parameters already introduced.

[28] It may be noted that the basis for determination of the discount rate for deciding whether or not to invest in a particular system, for all types of owners, is the opportunity cost – the return that could be obtained if the funds were invested elsewhere. The two major difficulties in making such a determination are in measuring the returns (because non-financial benefits should also be considered) and in measuring and accounting for the uncertainties. Even identifying the best alternative can be very difficult.

First, when the owner is a regulated utility, we may note that one of the objectives of the regulatory bodies is to maintain equivalence between the utility's internal rate of return and the weighted average after-tax opportunity cost of investment. If it is assumed that the company has enough other projects (besides the system under consideration) to obtain or deposit funds to cover any temporary imbalances between current revenues and current expenses due to the project within the company (cf [Doane 1976, p. B-2]), then the discount rate – on the average – is the company's internal rate of return. Hence, for a regulated utility as owner of the system under consideration (cf [Doane 1976, Equation (B.1), p. B-2]),

$$k = (1 - \tau) \left(\frac{\lambda - 1}{\lambda}\right) i + \left(\frac{1}{\lambda}\right) r \tag{96}$$

A municipally-owned utility has no income tax liability, and capitalization is typically by debt only (that is, $\lambda = \infty$). Consequently, *unless the municipally-owned utility makes "payments in lieu of taxes"*, the discount rate is simply the cost of debt to the public borrower (cf [Doane 1976, Equation (B.1'), p. B-3]),

$$k = i \tag{97}$$

In a non-regulated company, a realistic estimate of the discount rate that should be used could be obtained by considering the expected internal rate of return of the best investments available to the company other than the system under consideration but of comparable size, taking into account the risk associated with those alternatives. However, while obtaining the discount rate in that manner would be appropriate to a venture analysis, it would not be appropriate in the context of normative pricing. Instead, the objective is very similar to that of the agencies responsible for setting prices for regulated utilities. Thus, Equation (96) gives the discount rate for *all* corporate owners, whether regulated or not.

The appropriate discount rate for consumers may be approximated by what it costs to obtain money.[29] There are, in general, two possible sources: savings (that is, bank deposits or the foregoing of other investment opportunities) and loans. Hence, for a private individual,

$$k = \textit{the larger of}\ (1 - \tau) r \text{ or } (1 - \tau) i \tag{98}$$

The multiplications by $(1 - \tau)$ are due to the facts that interest revenues are taxable and interest payments are deductible on individual tax returns.

4.4.2. Other (than Income) Taxes

Non-income taxes consist primarily of property (*ad valorem*) taxes, which are a constant fraction of the taxable value of goods. It is assumed that non-capital goods have negligible value and that taxes are paid on the current value:

$$OTX_t = \beta_1 \, TVAL_t \tag{99}$$

[29] Very peculiar results can be obtained if the discount rate, as calculated by Equation (96) or any other means, comes out to be less than or even equal to the largest escalation rate. Specifically, suppose the discount rate chosen is less than the general rate of inflation. Then, the investor is theoretically indifferent between receipt of a given amount of money now (say 1.00 $\$_{1980}$ in 1980) and a *smaller* amount of money *later* (say 0.98 $\$_{1980}$, which is, say, 1.06 $\$_{1981}$, in 1981). This is not realistic. The discount rate used *must* be at least as high as the general rate of inflation; perhaps it should be at least as high as the largest escalation rate faced by the investor.

where

β_1 = "Other" (non-income) tax rate, expressed as a fraction (per year) of taxable value.

The taxable value of capital, $TVAL_t$, is given by Equation (50), II-25, and depends upon the tax environment of the system owner:

(1) If a *utility* owns the system, the taxable value of capital is normally equal (II-25) to the accounting book value, and is also known as the "rate base."

(2) If a *non-utility business* owns the system, its tax books and accounting books may legally be different, due to differences between tax lives and accounting lives of capital goods and due to differences in depreciation methods used.

(3) The *consumer*'s taxable value of capital might vary only by reassessment, since the system is likely to be considered as part of the house.

4.4.3. Insurance Premiums

The cost of insurance is based on replacement values, rather than depreciated values. Hence, cf. Equations (39) and (40), II-22.

$$INS_{jt} = \beta_2 C_j \Phi_c (1+\delta_j)(1+g_c)^t \quad \text{for} \quad t \geq t_j \tag{100}$$

$$INS_t = \Sigma_j INS_{jt} \tag{101}$$

where

β_2 = Insurance rate, expressed in % (per year).

4.4.4 Investment and Solar Tax Credits

Investment tax credits are available as a rather complicated function[30] (see [Chamberlain 1979, pp. 32 to 33]) of the tax life of the capital goods purchased. For simplicity, it is assumed here that all capital goods have a sufficiently long tax life to obtain the full investment tax credit rate. Thus,

$$ITC_t = \beta_3 CI_t \tag{102}$$

where

β_3 = Investment tax credit rate,[30] expressed as a fraction of capital investment.

[30] According to current law, a tax life of 7 or more years will allow a credit of 10% (11% if certain profit sharing policies are followed in a company). A tax life between 5 and 7 years allows 2/3 of the full rate. A tax life between 3 and 5 years allows 1/3. No credit is allowed if the tax life is less than 3 years. A user of this methodology may modify the value of β_3 to account for these fractions, if desired.

The investment tax credits can be taken (to the extent that the investment be used to generate taxable revenues) whenever a capital investment is made, whether for initial purchase or for replacement of hardware.

In some locations and under some circumstances, additional tax credits are available to encourage investment in solar energy systems. When these credits can be obtained for replacement purchases, as well as for initial purchase, the tax credit rate should be included in β_3 and Equation (102) should be used.

Some solar tax credits, however, are available only for initial purchase. Thus,

$$STC_t = \beta_4 \sum_{\substack{\text{all } j \text{ such} \\ \text{that } t=t_j}} CI_{jt} \tag{103}$$

where

β_4 = Solar (one-time only[31]) tax credit rate, expressed as a fraction of capital investment.

"One-time only" tax benefits do not usually have to be taken as soon as possible, as is assumed by Equation (103). The APSEAM computer model [Davis 1980] does not make this simplification.

4.4.5. Expenses

Expenses include operations, maintenance, fuel, and so on. Expenses for operations consist mostly of labor. Maintenance expenses consist of a mixture of labor, supplies, and materials, but do not include the costs of replacement components, for these are assumed to be capital expenditures. These different kinds of expenses may have significantly different escalation rates, in which case the expense term should be replaced by a suitable summation.

Each kind of expense is assumed to increase with time at a constant escalation rate and to be proportional to the nameplate size of the systems:

$$X_t = \begin{cases} X_b \, \Phi_x \, (1+g_x)^t \, (Z_t/Z) + x_t \, \Phi_x \, (1+g_x)^t & \text{for } t < t_o \\ X_b \, \Phi_x \, (1+g_x)^t + x_t \, \Phi_x \, (1+g_x)^t & \text{for } t \geq t_o \end{cases} \tag{104}$$

where

X_b = Annual expenses when the system is at full capacity, expressed in $\$_b$.

Z_t/Z = Nameplate size of the system in year t, expressed as a fraction of the nameplate size at capacity. (II-12.)

t_o = Time at which the system starts full capacity operation. (II-11.)

x_t = One-time expense (if any) in year t, expressed in $\$_b$.

[31] Solar tax credits that may be taken on replacement as well as on initial purchase should be included in β_3, not β_4.

4.5. Life Cycle Cost and Residual System Cost

Now that expressions for all of the cost components in the cost Equation (67), II-32, are available, that equation may be used to determine life cycle cost from Equation (6), II-6, and the cost part of the residual system value from Equation (8), II-6. The result of substituting Equation (67) into Equations (6) and (8) will be a collection of sums of present values; let us define some summary variables, to facilitate dealing with those sums. Thus:

$$LCC = \tau \{\omega [(1-\sigma)\, spm + \alpha\, tpm]\, p + \sigma \mathscr{P}\, bpm\} E + LCC_{CR} - [\omega + (1-\omega)\sigma]\, (\tau\, LCC_{DEP} + LCC_{ITC})$$

$$- LCC_{STC} + \{1 - \tau [\omega + (1-\omega)\sigma]\}[LCC_{OTX} + LCC_{INS} + LCC_X] \tag{105}$$

Also,

$$RSC = \tau \{\omega [(1-\gamma)\, rspm + \alpha\, rtpm]\, p + \sigma \mathscr{P}\, rbpm\} E + RSC_{CR} - [\omega + (1-\omega)\sigma]\, (\tau\, RSC_{DEP} + RSC_{ITC})$$

$$- RSC_{STC} + \{1 - \tau [\omega + (1-\omega)\sigma]\}[RSC_{OTX} + RSC_{INS} + RSC_X] \tag{106}$$

And,

$$COST = LCC + RSC = \tau \{\omega [(1-\sigma)\, spm_v + \alpha\, tpm_v]\, p + \sigma \mathscr{P}\, bpm_v\} E + COST_{CR}$$

$$- [\omega + (1-\omega)\sigma]\, (\tau\, COST_{DEP} + COST_{ITC}) - COST_{STC} + \{1 - \tau [\omega + (1-\omega)\sigma]\}[COST_{OTX}$$

$$+ COST_{INS} + COST_X] \tag{107}$$

where the first term in each of these equations is similar to the benefit Equations (21 to 23), II-13, and

LCC_{CR} = Life cycle cost of capital recovery; the sum of the present values of return of and on capital investments during the system lifetime, expressed in $\$_b$.

RSC_{CR} = Residual cost of capital recovery; like LCC_{CR}, but *after* the system lifetime.

$COST_{CR}$ = Total cost of capital recovery; like LCC_{CR}, but *during and after* the system lifetime.

LCC_{DEP} = Sum of the present values of depreciation deductions during the system lifetime, expressed in $\$_b$.

RSC_{DEP}, $COST_{DEP}$ = Analogues to LCC_{DEP}.

LCC_{ITC} = Sum of the present values of investment tax credits during the system lifetime, expressed in $\$_b$.

RSC_{ITC}, $COST_{ITC}$ = Analogues to LCC_{ITC}.

LCC_{STC} = Sum of the present values of special solar tax credits during the system lifetime, expressed in $\$_b$.

RSC_{STC}, $COST_{STC}$ = Analogues to LCC_{STC}.

LCC_{OTX} = Sum of the present values of other (than income) tax payments during the system lifetime, expressed in $\$_b$.

RSC_{OTX}, $COST_{OTX}$ = Analogues to LCC_{OTX}.

LCC_{INS} = Sum of the present values of insurance premiums paid during the system lifetime, expressed in $\$_b$.

RSC_{INS}, $COST_{INS}$ = Analogues to LCC_{INS}.

LCC_X = Sum of the present values of system expenses incurred during the system lifetime, expressed in $\$_b$.

RSC_X, $COST_X$ = Analogues to LCC_X.

Comparison of Equation (105), II-41, with Equations (67), II-32, and (6), II-6, defines the life cycle cost components as follows:

$$LCC_{CR} = \sum_{t=t_f}^{N} PV \{EQR_t + REP_t + PDR_t - RSL_t + (1-\tau)\, INT_t\} \tag{108}$$

$$LCC_{DEP} = \sum_{t=t_f}^{N} PV \left\{ \frac{\omega + (1-\omega)\,\sigma_t}{\omega + (1-\omega)\,\sigma} (TDEP_t - RTI_t) \right\} \tag{109}$$

$$LCC_{ITC} = \sum_{t=t_f}^{N} PV \left\{ \frac{\omega + (1-\omega)\,\sigma_t}{\omega + (1-\omega)\,\sigma} ITC_t \right\} \tag{110}$$

$$LCC_{STC} = \sum_{t=t_f}^{N} PV \{STC_t\} \tag{111}$$

$$LCC_{OTX} = \sum_{t=t_f}^{N} PV \left\{ \frac{1-\tau\,[\omega + (1-\omega)\,\sigma_t]}{1-\tau\,[\omega + (1-\omega)\,\sigma]} OTX_t \right\} \tag{112}$$

$$LCC_{INS} = \sum_{t=t_f}^{N} PV \left\{ \frac{1-\tau\,[\omega + (1-\omega)\,\sigma_t]}{1-\tau\,[\omega + (1-\omega)\,\sigma]} INS_t \right\} \tag{113}$$

$$LCC_X = \sum_{t=t_f}^{N} PV \left\{ \frac{1-\tau\,[\omega + (1-\omega)\,\sigma_t]}{1-\tau\,[\omega + (1-\omega)\,\sigma]} X_t \right\} \tag{114}$$

Comparison with Equation (8), II-6, shows that the components of the residual system cost are defined analogously, with the sums running from $N + 1$ to *infinity*. From Equation (4), II-5, the total cost components are also analogous, with sums running from t_f to *infinity*.

The values of these summations are developed in the next seven subsections.

4.5.1. Capital Recovery

Capital recovery costs consist of the repayment of investment and of the payment of a normative return on investment. The financial environment of the owner affects the submodel for the return on investment, so it is convenient to express these two cost components separately:

$$LCC_{CR} = LCC_{OF} + LCC_{ON} \tag{115}$$

where

LCC_{OF} = Sum of the present values of all provisions for the return of equity principal and for debt retirement during the system lifetime, expressed in $\$_b$.

LCC_{ON} = Sum of the present values of the net (after taxes) costs of providing the normative (that is, exogenously specified as "reasonable") return on equity and interest on debt, expressed in $\$_b$.

Thus, from Equation (108), II-42,

$$LCC_{OF} = \sum_{t=t_f}^{N} PV\,\{REP_t + PDR_t - RSL_t\} \tag{116}$$

$$LCC_{ON} = \sum_{t=t_f}^{N} PV\,\{EQR_t + (1-\tau)\,INT_t\} \tag{117}$$

4.5.1.1 Return Of Investment

Equation (116) can be expressed in terms of the capital investments as follows. First, note that the sum of equity repayment (REP_t) and debt repayment (PDR_t) each year must equal (by Equation (69), II-33), the book depreciation ($BDEP_t$). Thus,

$$LCC_{OF} = \sum_{t=t_f}^{N} PV\,\{BDEP_t\} - \Delta\; PV\{RSL_{t=N}\} \tag{118}$$

where

$$\Delta = \begin{cases} 1 & \text{if the system is resold (at } t = N\text{).} \\ 0 & \text{if the system is not resold.} \end{cases}$$

From the definition of present value, Equation (1), II-2,

$$PV\{RSL_{t=N}\} = \frac{RSL_N}{\Phi_{\$}(1+k)^N} \tag{119}$$

From the depreciation Equations (51) and (47), II-25, and the present value Equation (1), II-2, the first term in Equation (118), II-43, may be expressed as

$$\sum_{t=t_f}^{N} PV\{BDEP_t\} = \sum_j \frac{1}{\Phi_{\$}} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=1}^{L_j} \frac{bdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{120}$$

The upper limit of L_j on the summation over T may cause the life cycle cost to include depreciation after the end of the system lifetime. If this occurs, it is appropriate, because the investors must be repaid their investments even if the system is decommissioned or sold. If the system is sold, the resale revenues provide some or all of the funds for the repayment; if it is decommissioned, financial benefits from operation of the system will have to provide all of the funds needed.

The summation over T can be evaluated explicitly if the depreciation method is known. (See Appendix B.)

$$\sum_{T=1}^{L_j} \frac{bdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} = bdep_{vj} \tag{121}$$

where

$bdep_{vj}$ = Present value, as of the time of purchase, of the book depreciation function, evaluated with the parameters of capital goods of type j, expressed as a multiple of the value at purchase.

The sum of the present values of the depreciable capital investments during the system lifetime will occur frequently. So, let

$$CI_{LCC,j} = \frac{1}{\Phi_{\$}} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \tag{122}$$

where

$CI_{LCC,j}$ = Sum of the present values of capital investments of type j made during the system lifetime and depreciated at least one time (hence the "N - 1"), expressed in $\$_b$.

Equations (119), (121) and (122) can be used to simplify (at least the appearance of) Equation (118), II-43, yielding the *life cycle cost of return of investment*:

$$\boxed{LCC_{OF} = \Sigma_j\, CI_{LCC,j}\, bdep_{vj} - \Delta \frac{RSL_N}{\Phi_{\$}(1+k)^N}} \tag{123}$$

Development of the contribution of return of investment to residual system cost is similar. The analogue of Equations (116) and (118), II-43, are

$$RSC_{OF} = \sum_{t=N+1}^{\infty} PV\{REP_t + PDR_t\} \tag{124}$$

$$RSC_{OF} = \sum_{t=N+1}^{\infty} PV\{BDEP_t\} \tag{125}$$

where

RSC_{OF} = Analogue of LCC_{OF}.

The analogue of Equation (120) is

$$\sum_{t=N+1}^{\infty} PV\{BDEP_t\} = \sum_j \frac{1}{\Phi_s} \sum_{t=N}^{\infty} \frac{CI_{jt}}{(1+k)^t} \sum_{T=1}^{L_j} \frac{bdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{126}$$

The analog to Equation (122) is

$$CI_{RSC,j} = \frac{1}{\Phi_s} \sum_{t=N}^{\infty} \frac{CI_{jt}}{(1+k)^t} \tag{127}$$

where

$CI_{RSC,j}$ = Sum of the present values of capital investments of type j, possibly made before, but not depreciated until after, the end of the system lifetime, expressed in $\$_b$.

And finally, the analogue to Equation (123) is the *residual system cost of return of investment*:

$$\boxed{RSC_{OF} = \Sigma_j \, CI_{RSC,j} \, bdep_{vj}} \tag{128}$$

The contribution of return of investment to the total system cost is also similar, but the resale revenues are not, of course, relevant:

$$COST_{OF} = \sum_{t=t_f}^{\infty} PV\{REP_t + PDR_t\} \tag{129}$$

where

$COST_{OF}$ = Sum of the present values of all provisions for the return of equity principal and for debt retirement, expressed in $\$_b$.

$$COST_{OF} = \sum_{t=t_f}^{\infty} PV\{BDEP_t\} \tag{130}$$

$$= \sum_j \frac{1}{\Phi_s} \sum_{t=t_f}^{\infty} \frac{CI_{jt}}{(1+k)^t} \sum_{T=1}^{L_j} \frac{bdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{131}$$

From Appendix B,

$$CI_{vj} = \frac{1}{\Phi_s} \sum_{t=t_f}^{\infty} \frac{CI_{jt}}{(1+k)^t} = \Phi_c R_c^{t_f} C_j \left[1 + (1+\delta_j) \frac{R_c^{L_j}}{1 - R_c^{L_j}} \right]$$

where

CI_{vj} = Sum of the present values of capital investments of type j, expressed in $\$_b$.

Finally, the *total cost of return of investment*:

$$\boxed{COST_{OF} = \Sigma_j CI_{vj}\, bdep_{vj}} \tag{133}$$

4.5.1.2 Return On Investment

For businesses, the cost of paying a return on investment [Equation (117), II-43] is simply the discount rate [see Equation (96), II-38] times the book value the previous year [from Equations (78) and (79), II-35]. Thus, using the book valuation Equation (48), II-28, and (45), II-24, and the present value Equation (1), II-2, Equation (117), II-43, may be rewritten as

$$LCC_{ON} = k \sum_j \frac{1}{\Phi_s} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{N-1-t} \frac{bsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{134}$$

where

$bsch(A, L, \mu, \theta)$ = The schedule function, $sch(A, L, \mu, \theta)$, evaluated with L = book life, μ = book depreciation method.

The result of the summation on T depends upon t, so it may not be factored out of the summation on t, as was done in the simplication of Equation (120), II-44. However, the derivation of $bsch_{LCC,j}$ in Appendix B takes that into account, so that, with the definition of $CI_{LCC,j}$ from Equation (122), II-44, Equation (134) may be rewritten, for *business-owned systems*:

$$LCC_{ON} = k\ \Sigma_j\ CI_{LCC,j}\ bsch_{LCC,j} \tag{135}$$

where

$bsch_{LCC,j}$ = Present value, as of the time of initial purchase, of the book values of goods of type j up to *one year before* the end of the system lifetime, taking into account that a non-integral number of replacement cycles may have been completed at that time, expressed as a multiple of the value at initial purchase.

Similarly, the contributions to residual system cost and total cost, *for business-owned systems*, are given by:

$$RSC_{ON} = k \ \Sigma_j \ CI_{RSC,j} \ bsch_{RSC,j} \tag{136}$$

$$COST_{ON} = k \ \Sigma_j \ CI_{vj} \ bsch_{vj} \tag{137}$$

where

$bsch_{RSC,j}$ = Present value, as of initial purchase, of the book values of goods of type j after the end of the system lifetime, taking into account that a non-integral number of replacement cycles may have been completed at the end of the system life, expressed as a multiple of the value at initial purchase.

$bsch_{vj}$ = Present value, as of initial purchase, of the book values of goods of type j, expressed as a multiple of the value at initial purchase.

For consumers, the return on investment is even more complicated, because the financial leverage is not constant and the discount rate does not have the same form as the two parts of Equation (117), II-43. However, by using Equations (95), II-37, and (94), II-37, to express EQR_t in terms of $BVAL_t$ and $DVAL_t$, and Equation (72), II-34, to express INT_t in terms of $DVAL_t$, Equation (117), II-43, may be rewritten as

$$LCC_{ON} = \sum_{t=t_f}^{N} PV\{r \ (BVAL_{t-1} - DVAL_{t-1}) + (1-\tau) \ i \ \ DVAL_{t-1}\} \tag{138}$$

In order to treat the book value and debt value parts of this equation separately, let us define $BVAL_{LCC}$ and $DVAL_{LCC}$ so that

$$LCC_{ON} = r \ BVAL_{LCC} + [(1-\tau) \ i - r] DVAL_{LCC} \tag{139}$$

where

$$BVAL_{LCC} = \sum_{t=t_f}^{N} PV\{BVAL_{t-1}\} \tag{140}$$

$$DVAL_{LCC} = \sum_{t=t_f}^{N} PV\{DVAL_{t-1}\} \tag{141}$$

The $BVAL_{LCC}$ term, however, was just calculated in the analysis of return on investment for business-owned systems (cf. Equation (135), II-46):

$$BVAL_{LCC} = \Sigma_j \, CI_{LCC,j} \, bsch_{LCC,j} \tag{142}$$

The debt value as a function of time is given by Equations (87), II-36, and (80), II-35. Using this information and Equation (1), II-2, Equation (141) may be expressed in terms of the capital investments:

$$DVAL_{LCC} = \sum_j \sum_{t=t_f}^{N-1} \frac{(1-\epsilon)CI_{jt}}{\Phi_s (1+k)^t \, i} \sum_{T=0}^{L_j-1} \frac{crf_{i,MP} - (1+i)^T \, sff_{i,MP}}{(1+k)^T} \tag{143}$$

Let

$$mort_{vj} = \sum_{T=0}^{L_j-1} \frac{crf_{i,MP} - (1+i)^T \, sff_{i,MP}}{i\,(1+k)^T} \tag{144}$$

Then, Equations (144) and (122), II-44, can be used to simplify Equation (143):

$$DVAL_{LCC} = \Sigma_j (1-\epsilon) \, CI_{LCC,j} \, mort_{vj} \tag{145}$$

where

$mort_{v,j}$ = Sum of the present values, as of the time of initial purchase, of the mortgage value function contained in the last summation of Equation (143) for goods of type j, expressed as a multiple of the value at initial purchase. (The value depends upon the type of capital goods only if the mortgage period also does.)

Inserting Equations (140), II-47, and (145), into Equation (139), II-47, gives, for *consumer-owned systems*:

$$LCC_{ON} = \Sigma_j \, CI_{LCC,j} \{ r \, bsch_{LCC,j} + (1-\epsilon)[(1-\tau)\, i - r] \, mort_{vj} \} \tag{146}$$

Using the indicator variable ω (1 for company-owned systems, 0 for consumer ownership), Equations (135), II-46, and (146) can be combined to give the *life cycle cost of return on investment*:

$$\boxed{\begin{aligned} LCC_{ON} = \Sigma_j \, CI_{LCC,j} \{ & [\omega k + (1-\omega) r] \, bsch_{LCC,j} \\ & + (1-\omega)(1-\epsilon)[(1-\tau)\, i - r] \, mort_{vj} \} \end{aligned}} \tag{147}$$

Similarly,

$$\boxed{\begin{aligned} RSC_{ON} = \Sigma_j \, CI_{RSC,j} \{ & [\omega k + (1-\omega) r] \, bsch_{RSC,j} \\ & + (1-\omega)(1-\epsilon)[(1-\tau)\, i - r] \, mort_{vj} \} \end{aligned}} \tag{148}$$

and,

$$\boxed{\begin{aligned} COST_{ON} = \sum_j CI_{vj} \{ & [\omega k + (1-\omega) r]\, bsch_{vj} \\ & + (1-\omega)(1-\epsilon)[(1-\tau) i - r]\, mort_{vj} \} \end{aligned}} \tag{149}$$

Finally, the return of investment, given by Equations (123), II-44, (127), II-45, and (132), II-46, can be added (Equation (114), II-43), to the return on investment, given by Equations (147), (148), (149), to obtain the costs of capital recovery:

$$LCC_{CR} = \sum_j CI_{LCC,j} \left\{ [\omega k + (1-\omega) r]\, bsch_{LCC,j} + bdep_{vj} + (1-\omega)(1-\epsilon)[(1-\tau) i - r]\, mort_{vj} - \Delta \frac{RSL_N}{\Phi_s (1+k)^N} \right\} \tag{150}$$

$$RSC_{CR} = \sum_j CI_{RSC,j} \{ [\omega k + (1-\omega) r]\, bsch_{RSC,j} + bdep_{vj} + (1-\omega)(1-\epsilon)\, [(1-\tau) i - r]\, mort_{vj} \} \tag{151}$$

$$COST_{CR} = \sum_j CI_{vj} \{ [\omega k + (1-\omega) r]\, bsch_{vj} + bdep_{vj} + (1-\omega)(1-\epsilon)\, [(1-\tau) i - r]\, mort_{vj} \} \tag{152}$$

4.5.2 Depreciation Deductions

Depreciation is a deductible "expense" only to the extent that the system contributes to the generation of taxable income, as shown in Equation (109), II-42. As was the case with the previous section, it is convenient to express the two components of this equation as separate terms:

$$LCC_{DEP} = LCC_{TD} - LCC_{RI} \tag{153}$$

where

LCC_{TD} = Sum of the present values of all depreciation tax deductions during the system lifetime, expressed in $\$_b$.

LCC_{RI} = Present value of the effect of resale of the system (recapture of depreciation) on taxable income, expressed in $\$_b$.

$$LCC_{TD} = \sum_{t=t_f}^{N} PV \left\{ \frac{\omega + (1-\omega)\sigma_t}{\omega + (1-\omega)\sigma}\, TDEP_t \right\} \tag{154}$$

$$LCC_{RI} = \sum_{t=t_f}^{N} PV\left\{\frac{\omega + (1 - \omega)\sigma_t}{\omega + (1 - \omega)\sigma} RTI_t\right\} \tag{155}$$

4.5.2.1 Deductions

The amounts of the depreciation deductions may be obtained from Equations (47) and (52), II-25. Using also the present value Equation (1), II-2, Equation (154), II-49, may be rewritten as

$$LCC_{TD} = \sum_j \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{\Phi_t(1 + k)^t} \sum_{T=1}^{N-t} \frac{\omega + (1 - \omega)\sigma_{t+T}}{\omega + (1 - \omega)\sigma} \frac{tdep(T, L_j, \mu_j, \theta_j)}{(1 + k)^T} \tag{156}$$

where

$tdep(A, L, \mu, \theta)$ = The depreciation function, $dep(A, L, \mu, \theta)$, evaluated with L = tax life, μ = tax depreciation method.

As with the return on investment (see II-46), the summation on T cannot be easily factored out of the summation on t, but $tdep_{LCC,j}$ can be defined (see Appendix B) to take that into account. Using Equation (121), II-44, the effect of depreciation deductions may be written as

$$LCC_{TD} = \Sigma_j \, CI_{LCC,j} \, tdep_{LCC,j} \tag{157}$$

where

$tdep_{LCC,j}$ = Present value, as of the time of initial purchase, of the depreciation deductions on goods of type j up to the end of the system lifetime, taking into account that a non-integral number of replacement cycles may have been completed at that time, expressed as a multiple of the value at initial purchase.

Similarly, the impacts on residual system cost and on total cost are given by:

$$RSC_{TD} = \Sigma_j \, CI_{RSC,j} \, tdep_{RSC,j} \tag{158}$$

$$COST_{TD} = \Sigma_j \, CI_{vj} \, tdep_{vj} \tag{159}$$

where

$tdep_{RSC,j}$ = Present value, as of initial purchase, of the depreciation deductions on goods of type j after the end of the system lifetime, taking into account that a non-integral number of replacement cycles may have been completed at the end of the system life, expressed as a multiple of the value at initial purchase.

$tdep_{vj}$ = Present value, as of initial purchase, of the depreciation deductions on goods of type j, expressed as a multiple of the value at initial purchase.

4.5.2.2 Resale Of The System

Equation (155), II-50, together with the cost Equation (67), II-32, provide for the recovery (by the government) of taxes that were not paid due to deducation of depreciation beyond[32] the decrease in market price. Strictly speaking, the fraction of that excess that would be recovered is *not* given by σ_t unless the sell-back fraction, σ_t, is constant for $t = 1, \ldots, N$. The fraction recovered should be the weighted average of σ_t over that period, but the additional complexity required to account for this nuance has been purposely omitted from this[33] model. A conservative result (conservative in the sense that it may slightly overestimate the effect on cost) can be obtained by using the sell-back fraction for the first year, σ. Now, obtaining the contribution to taxable income, RTI_t, from Equation (58), II-28, and using the definition of present value Equation (1), II-2, Equation (155), II-50, may be rewritten as:

$$LCC_{RI} = \frac{\Delta}{\Phi_s (1+k)^N} (\gamma CG_N + OI_N) \tag{160}$$

From Equation (61), II-29, the ordinary income is given by

$$OI_N = RF \{min [RSL_t, \rho PVR + (1-\rho) STVAL_N] - TVAL_N\} \tag{161}$$

and, from Equation (62), II-30, the capital gains income is given by

$$CG_N = RSL_N - TVAL_N - OI_N \tag{162}$$

Since resale of the system, if it occurs at all, takes place at the end of the system lifetime, there is no contribution to residual cost. Further, the total cost is calculated as if the system is never resold, so there is no contribution to it either.

Thus, from Equation (153), II-49, and its unstated residual and total cost analogues,

$$\boxed{\begin{aligned} LCC_{DEP} &= \Sigma_j CI_{LCC,j}\, tdep_{LCC,j} - \frac{\Delta}{\Phi_s (1+k)^N} (\gamma CG_N + OI_N) && (163) \\ RSC_{DEP} &= \Sigma_j CI_{RSC,j}\, tdep_{RSC,j} && (164) \\ COST_{DEP} &= \Sigma_j CI_{vj}\, tdep_{vj} && (165) \end{aligned}}$$

4.5.3 Investment Tax Credits

Investment tax credits are a simple multiple of capital investments, but investments that are made precisely at $t = N$ contribute to residual system cost, rather than life cycle cost. Consequently, combining Equation (102), II-39, and Equation (110), II-42,

[32]The effect on capital gains income of a market price *below* the depreciated value of the system is ignored here. Input data could, however, cause that to happen; the resultant *capital loss* would be treated as (negative) ordinary income.
[33]This complication *is* included in the APSEAM computer model [Davis 1980].

$$LCC_{ITC} = \sum_j \sum_{t=t_f}^{N-1} \frac{\beta_3}{\Phi_\$} \frac{CI_{jt}}{(1+k)^t} \frac{\omega + (1-\omega)\sigma_t}{\omega + (1-\omega)\sigma} \tag{166}$$

If $\omega = 1$ or $\sigma_t = constant$, or both, this reduces quite simply (see Equation (121), II-44) to

$$LCC_{ITC} = \beta_3 \Sigma_j CI_{LCC,j} \tag{167}$$

If, on the other hand, $\omega = 0$ and $\sigma_t \neq constant$, a new term is needed:

$$LCC_{ITC} = \beta_3 \Sigma_j CI'_{LCC,j} \tag{168}$$

where

$$CI'_{LCC,j} = \frac{1}{\Phi_\$} \sum_{t=t_f}^{N-1} \frac{\omega + (1-\omega)\sigma_t}{\omega + (1-\omega)\sigma} \frac{CI_{jt}}{(1+k)^t} \tag{169}$$

Equation (168) applies, of course, whether Equation (167) does or not.

Similarly,

$$RSC_{ITC} = \beta_3 \Sigma_j CI'_{RSC,j} \tag{170}$$

$$COST_{ITC} = \beta_3 \Sigma_j CI'_{vj} \tag{171}$$

where

$$CI'_{RSC,j} = \frac{1}{\Phi_\$} \sum_{t=N}^{\infty} \frac{\omega + (1-\omega)\sigma_t}{\omega + (1-\omega)\sigma} \frac{CI_{jt}}{(1+k)^t} \tag{172}$$

$$CI'_{v_j} = \frac{1}{\Phi_\$} \sum_{t=t_f}^{\infty} \frac{\omega + (1-\omega)\sigma_t}{\omega + (1-\omega)\sigma} \frac{CI_{jt}}{(1+k)^t} \tag{173}$$

4.5.4 Solar Tax Credits

The "one-time only"[34] solar tax credits are given by Equation (103), II-40, so Equation (111), II-42, becomes

$$\boxed{LCC_{STC} = \beta_4 \sum_{\substack{\text{all } j \text{ such} \\ \text{that } t_j < N}} \frac{CI_{jt_j}}{\Phi_s (1+k)^{t_j}}} \tag{174}$$

Similarly,

$$\boxed{RSC_{STC} = \beta_4 \sum_{\substack{\text{all } j \text{ such} \\ \text{that } t_j \geq N}} \frac{CI_{jt_j}}{\Phi_s (1+k)^{t_j}}} \tag{175}$$

$$\boxed{COST_{STC} = \beta_4 \sum_{j} \frac{CI_{jt_j}}{\Phi_s (1+k)^{t_j}}} \tag{176}$$

4.5.5 Other (Than Income) Taxes

The "other taxes" submodel is given by Equation (99), II-38. It requires the taxable value, which may be obtained from Equations (48), II-25, and (45), II-24. Thus Equation (112), II-42, may be rewritten

$$LCC_{OTX} = \sum_{j} \sum_{t=t_j}^{N-1} \frac{\beta_1}{\Phi_s} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{N-1-t} \left(\frac{1 - \tau\,[\omega + (1-\omega)\,\sigma_{t+T}]}{1 - \tau\,[\omega + (1-\omega)\,\sigma]} \right) \frac{tsch\,(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{177}$$

where

$tsch\,(A, L, \mu, \theta)$ = The schedule function, $sch(A,L,\mu,\theta)$, evaluated with L = tax life, μ = tax depreciation method.

This equation is very similar to Equation (134), II-46. Consequently, it may be handled in the same way (that is, by deferring the complexity to Appendix A):

$$\boxed{LCC_{OTX} = \beta_1\, \Sigma_j\, CI_{LCC,j}\, tsch_{LCC,j}} \tag{178}$$

[34] Solar tax credits that can be taken on replacement investments as well as on initial investments are assumed to be included in the investment tax credit.

where

$tsch_{LCC,j}$ = Present value, as of the time of initial purchase, of the tax values of goods of type j up to *one year before* the end of the system lifetime, taking into account that a non-integral number of replacement cycles may have been completed at that time, expressed as a multiple of the value at initial purchase.

Similarly,

$$\boxed{\begin{aligned} RSC_{OTX} &= \beta_1 \ \Sigma_j \ CI_{RSC,j} \ tsch_{RSC,j} \\ COST_{OTX} &= \beta_1 \ \Sigma_j \ CI_{vj} \ tsch_{vj} \end{aligned}} \qquad \begin{aligned} &(179) \\ &(180) \end{aligned}$$

where $tsch_{RSC,j}$ and $tsch_{vj}$ are analogues of $tsch_{LCC,j}$ (or of $bsch_{RSC,j}$ and $bsch_{vj}$).

4.5.6 Insurance Premiums

The year by year cost of insurance premiums is given by Equations (100) and (101), II-39. Using those expressions in Equation (113), II-42, gives the life cycle cost of insurance premiums:

$$LCC_{INS} = \sum_j \sum_{t=t_j}^{N} \beta_2 \, C_j \, \phi_c \, (1+\delta_j) \, \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]} \left(\frac{1+g_c}{1+k}\right)^t \tag{181}$$

where

$\phi_c = \Phi_c/\Phi_S$, the *real escalation* of capital prices from the base year, Y_b, to the start of system operations, $t = 0$.

If $\omega = 1$ or σ_t = *constant*, or both, the summation over t can be expressed in closed form, so that Equation (181) may be reduced to

$$LCC_{INS} = \beta_2 \, \phi_c \sum_j C_j \, (1+\delta_j) \, R_c^{t_j} \left(\frac{1-R_c^{N-t_j+1}}{1-R_c}\right) \tag{182}$$

where

$R_c = (1+g_c)/(1+k)$, the discounted escalation rate for capital goods. (183)

If, on the other hand, $\omega = 0$ and $\sigma_t \neq constant$, another new term is needed:

$$LCC_{INS} = \beta_2 \, \phi_c \, \Sigma_j C_j (1 + \delta_j) \, ins_{LCC,j} \tag{184}$$

where

$$ins_{LCC,j} = \sum_{t=t_j}^{N} \frac{1 - \tau \, [\omega + (1 - \omega) \, \sigma_t]}{1 - \tau \, [\omega + (1 - \omega) \, \sigma]} R_c^t \tag{185}$$

Similarly,

$$RSC_{INS} = \beta_2 \, \phi_c \, \Sigma_j (1 + \delta_j) \, ins_{RSC,j} \tag{186}$$

$$COST_{INS} = \beta_2 \, \phi_c \, \Sigma_j C_j (1 + \delta_j) \, ins_{vj} \tag{187}$$

where

$$ins_{RSC,j} = \sum_{t=N+1}^{\infty} \frac{1 - \tau \, [\omega + (1 - \omega) \, \sigma_t]}{1 - \tau \, [\omega + (1 - \omega) \, \sigma]} R_c^t \tag{188}$$

$$ins_{vj} = \sum_{t=t_f}^{\infty} \frac{1 - \tau \, [\omega + (1 - \omega) \, \sigma_t]}{1 - \tau \, [\omega + (1 - \omega) \, \sigma]} R_c^t \tag{189}$$

4.5.7 Expenses

Year by year expenses are given by Equation (102), II-40. Inserting that expression into Equation (114), II-42, gives the life cycle cost of expenses:

$$LCC_x = \sum_{t=t_f}^{N} \phi_x R_x^t \left(X_b \begin{Bmatrix} Z_t/Z & \text{if } t < t_o \\ 1 & \text{if } t \geq t_o \end{Bmatrix} + x_t \right) \times \frac{1 - \tau \, [\omega + (1 - \omega) \, \sigma_t]}{1 - \tau \, [\omega + (1 - \omega) \, \sigma]} \tag{190}$$

where

ϕ_x = Φ_x / Φ_g, the real escalation of expenses from the base year, Y_b, to the start of system operations, $t = 0$. (191)

R_x = $(1 + g_x)/(1 + k)$, the discounted escalation rate for expenses. (192)

Once again deferring the complexity to Appendix B, Equation (190) may be rewritten as

$$LCC_x = X_b \phi_x Z_{LCC} + x_{LCC} + \phi_x \tag{193}$$

where

$$Z_{LCC} = \sum_{t=t_f}^{N} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]} \begin{Bmatrix} Z_t/Z & \text{if } t<t_o \\ 1 & \text{if } t \geq t_o \end{Bmatrix} \tag{194}$$

$$x_{LCC} = \sum_{t=t_f}^{N} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]}\, x_t \tag{195}$$

Similarly,

$$RSC_x = X_b \phi_x Z_{RSC} + x_{RSC} \phi_x \tag{196}$$

$$COST_x = X_b \phi_x Z_v + x_v \phi_x \tag{197}$$

where

$$Z_{RSC} = \sum_{t=N+1}^{\infty} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]} \begin{Bmatrix} Z_t/Z & \text{if } t<t_o \\ 1 & \text{if } t \geq t_o \end{Bmatrix} \tag{198}$$

$$x_{RSC} = \sum_{t=N+1}^{\infty} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]}\, x_t \tag{199}$$

$$Z_v = \sum_{t=t_f}^{\infty} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]} \begin{Bmatrix} Z_t/Z & \text{if } t<t_o \\ 1 & \text{if } t \geq t_o \end{Bmatrix} \tag{200}$$

$$x_v = \sum_{t=t_f}^{\infty} R_x^t \frac{1-\tau\,[\omega+(1-\omega)\,\sigma_t]}{1-\tau\,[\omega+(1-\omega)\,\sigma]}\, x_t \tag{201}$$

4.6. Cost Recapitulation

The seven subsections, 4.5.1 through 4.5.7, of the previous section were devoted to the determination of the elements of the cost Equations (105) and (107), II-41. Combining the results of those subsections and

collecting terms gives Equations (202), (206) and (209), for life cycle cost, residual system cost, and total system cost, respectively:

$$\boxed{\begin{aligned} LCC = \tau\,\{\omega\,[(1-\sigma)\,spm + \alpha\,tpm]\,p + \sigma\,\mathscr{P}bpm\}\,E \\ + \Sigma_j\,CI_{LCC,j}\,ccm_j + X_b\,xpm - \Delta\,NRG \end{aligned}} \tag{202}$$

where the following summary terms were introduced:

ccm_j = Capital cost multiplier for capital goods of type j; the factor by which the present value of capital investments of type j made during the system lifetime ($CL_{LCC,j}$, see Equation (122), II-44) must be multiplied to obtain the contribution of capital investment to life cycle cost.

xpm = Expense price multiplier; the present value of all expenses during the system lifetime, expressed as a multiple of annual expenses when the system is at full capacity (X_b).

NRG = Net resale gain; the present value of the after-tax gain from resale of the system at the end of the system lifetime.

The capital cost multiplier is given by the following collection of terms:

$$\begin{aligned} ccm_j = {} & [\omega\,k + (1-\omega)\,r]\;bsch_{LCC,j} + bdep_{vj} + (1-\omega)(1-\epsilon)\,[(1-\tau)\,i - r]\;mort_{vj} \\ & + (1 - \tau\,[\omega + (1-\omega)\,\sigma])\,\beta_1\;tsch_{LCC,j} - [\omega + (1-\omega)\,\sigma]\left[\tau\,tdep_{LCC,j} + \beta_3\,\frac{CI'_{LCC,j}}{CI_{LCC,j}}\right] \\ & + (1 - \tau\,[\omega + (1-\omega)\,\sigma])\,\beta_2\,\phi_c\,\frac{C_j\,(1+\delta_j)\,ins_{LCC,j}}{CI_{LCC,j}} - \frac{\beta_4}{\Phi_s\,(1+k)^{t_j}}\,\frac{CI_{pj}}{CI_{LCC,j}}\begin{Bmatrix} 1 \text{ if } t_j < N \\ 0 \text{ if } t_j \geq N \end{Bmatrix} \end{aligned} \tag{203}$$

The expense price multiplier is given by

$$xpm = (1 - \tau\,[\omega + (1-\omega)\,\sigma])\,\phi_x\left(Z_{LCC} + \frac{x_{LCC}}{X_b}\right) \tag{204}$$

The net resale gain is given by

$$NRG = RSL_N - \tau\,[\gamma\,CG_N + OI_N] \tag{205}$$

The expression for the residual system cost is

$$\boxed{\begin{aligned} RSC = \tau\,\{\omega\,[(1-\sigma)\,rspm + \alpha\,rtpm]\,p + \sigma\,\mathscr{P}rbpm\}\,E \\ + \Sigma_j\,CI_{RSC,j}\,ccm_{rj} + X_b\,rxpm \end{aligned}} \tag{206}$$

where

ccm_{rj} = Residual capital cost multiplier for capital goods of type j (multiplier of $CI_{RSC,j}$ – see Equation (123), II-44).

$rxpm$ = Residual expense price multiplier; the present value of all expenses after the end of the system lifetime, expressed as a fraction of annual expenses when the system is at full capacity.

The residual capital cost multiplier is given by

$$
\begin{aligned}
ccm_{rj} = {} & [\omega k + (1-\omega) r]\ bsch_{RSC,j} + bdep_{vj} + (1-\omega)(1-\epsilon)[(1-\tau) i - r]\ mort_{vj} \\
& + (1 - \tau[\omega + (1-\omega)\sigma])\beta_1\ tsch_{RSC,j} - [\omega + (1-\omega)\sigma]\left[\tau\, tdep_{RSC,j} + \beta_3 \frac{CI'_{RSC,j}}{CI_{RSC,j}}\right] \\
& + (1 - \tau[\omega + (1-\omega)\sigma])\beta_2 \phi_c \frac{C_j(1+\delta_j) ins_{RSC,j}}{CI_{RSC,j}} \\
& - \frac{\beta_4}{\Phi_g (1+k)^{t_j}} \frac{CI_{fr_j}}{CI_{RSC,j}} \begin{Bmatrix} 1 & \text{if } t_j \geq N \\ 0 & \text{if } t_j < N \end{Bmatrix}
\end{aligned}
\tag{207}
$$

The residual expense price multiplier is given by

$$
rxpm = (1 - \tau[\omega + (1-\omega)\sigma])\phi_x \left(Z_{RSC} + \frac{x_{RSC}}{X_b}\right) \tag{208}
$$

The expression for the total system cost is

$$
\boxed{\begin{aligned} COST = {} & \tau\{\omega[(1-\sigma) spm_v + \alpha\ tpm_v]p + \sigma\ \mathcal{P} bpm_v\}E \\ & + \Sigma_j\ CI_{vj}\ ccm_{vj} + X_b\ xpm_v \end{aligned}} \tag{209}
$$

where

ccm_{vj} = Total capital cost multiplier for capital goods of type j; the factor by which the present value of all capital goods of type j is multiplied to obtain the contribution of capital investment to total cost. (See Equation (132), II-46, for CI_{vj}.)

xpm_v = Total expense price multiplier; the present value of all expenses, expressed as a multiple of annual expenses when the system is at full capacity.

$$ccm_{vj} = [\omega k + (1-\omega) r]\, bsch_{vj} + bdep_{vj} + (1-\omega)(1-\epsilon)[(1-r) i - r]\, mort_{vj}$$

$$+ (1 - \tau[\omega + (1-\omega)\sigma])\, \beta_1\, tsch_{vj} - [\omega + (1-\omega)\sigma] \left[r\, tdep_{vj} + \beta_3 \frac{Cl'_{vj}}{Cl_{vj}} \right]$$

$$+ (1 - \tau[\omega + (1-\omega)\sigma])\, \beta_2 \phi_c \frac{C_j(1+\delta_j)\, ins_{vj}}{Cl_{vj}} - \frac{\beta_4}{\Phi_g (1+k)^{t_j}} \frac{Cl_{lr_j}}{Cl_{vj}} \tag{210}$$

$$xpm_v = (1 - \tau[\omega + (1-\omega)\sigma])\, \phi_x \left(Z_v + \frac{x_v}{X_b} \right) \tag{211}$$

5. NET PRESENT VALUE RECAPITULATION

It is now possible to evaluate the net present value of the system under consideration, which was defined in Equation (2), II-3.

The net present value includes consideration of all benefits and costs resulting from installing and operating the system. If the system is resold or decommissioned, those benefits and costs stop (from the point of view of the investor) abruptly. The net present values in these two scenarios ("no-resale" or "resale") are presented separately in the next two subsections, partly because doing so reduces the complexity of the resultant equations, but mainly because it is anticipated that most users will find only one of the two scenarios relevant.

5.1. Without Resale Of The System ($\Delta = 0$)

If the system is not resold, the net present value can be obtained by substituting Equations (23), II-13, and (209), II-58, directly into Equation (2), II-3, which produces

$$\boxed{\begin{aligned} NPV = {} & [(1-\sigma)(P-\tau\omega p)\,spm_v + \alpha(1-\tau\omega)\,p\,tpm_v \\ & + \sigma(1-\tau)\mathscr{F}\,bpm_v]\,E - \Sigma_j\,CI_{vj}\,ccm_{vj} \\ & - X_b\,xpm_v \end{aligned}} \tag{212}$$

Of course, unless the system owner can realize a different value for energy produced by this system (but not sold back to the grid) than the grid price, the numerical value of P should be the grid price (p) in all calculations except when the normative system energy price is the desired result of the computation.

5.2. With Resale Of The System ($\Delta = 1$)

If the system is resold, Equation (2), II-3, cannot be used directly. Instead the definitions of Equations (3), II-4, through (9), II-7, may be used in Equation (2), to obtain

$$NPV = REV - LCC + RSV \tag{213}$$

where, *in this context*, $RSV = 0$.

Then, Equations (21), II-13, and (202), II-57, can be substituted into Equation (213) to obtain

$$\boxed{\begin{aligned} NPV = {} & [(1-\sigma)(P-\tau\omega p)\,spm + \alpha(1-\tau\omega)\,p\,tpm \\ & + \sigma(1-\tau)\mathscr{P}\,bpm]\,E - \Sigma_j\,CI_{LCC,j}\,ccm_j \\ & - X_b\,xpm + NRG \end{aligned}} \tag{214}$$

5.3. Residual System Value

The residual system value, given by Equation (9), II-7, is also of significant interest. From Equations (22), II-13, and (206), II-57,

$$\boxed{\begin{aligned} RSV = {} & [(1-\sigma)(P-\tau\,\omega\,p)\,rspm + \alpha\,(1-\tau\,\omega)\,p\,rtpm \\ & + \sigma\,(1-\tau)\mathscr{P}\,rbpm]\;E - \Sigma_j\,CI_{RSC,j}\,ccm_{rj} \\ & - X_b\,rxpm \end{aligned}} \tag{215}$$

In this equation, P has still not been replaced by p, because it might be used to find the normative system energy price during the period after the end of the system lifetime, when solar tax credits and one-time costs do not appear, and the system has achieved a sort of steady-state operation.

6. BREAK-EVEN ANALYSIS

The term "break-even" is often used to mean that the benefits obtained from the system just equal the costs incurred. The general idea here is the same, except that

(1) a "reasonable" profit is considered to be one of the costs that must be recovered, so that "breaking-even" implies making the specified rate of return on equity, and

(2) the time value of money, due to both inflation and discounting, is taken into account, rather than ignored, so that an investment of D dollars must be repaid by more than D dollars to "break-even".

Explicitly, as discussed in Section 2.3, the system is considered to be a break-even investment if the *net present value* is zero.

The following subsections provide solutions or solution procedures for the values of various parameters that satisfy the equation $NPV = 0$. In some cases, it will be seen that there is no realizable solution (as, for example, when a break-even price comes out to be negative). The symbol "^" is used to indicate break-even values.

6.1. Normative System Energy Prices

The *normative system energy price* is hereby defined as the average inflating marginal price of energy produced by the system, assuming the system owner continues to own, maintain, and operate the system for an indefinitely long period of time. Thus, it corresponds to the Busbar Energy Cost levelized in real terms (BBEC) of USES, not the Busbar Energy Cost levelized in nominal terms ($\overline{\text{BBEC}}$). In this definition:

"Average" implies a "flat-rate" pricing structure. "Inflating" implies increases with time at the rate of energy price escalation. "Marginal" implies that, although the effect of the system on other operations is charged (or credited) to the system, system costs and benefits are not averaged with those of other operations. "Price" is used instead of "cost" because profit -- and other "general and administrative" expenses -- are included.

Setting Equation (212), II-60, to zero and solving for P gives the following expression for the *normative system energy price*:

$$\boxed{\hat{P} = \frac{\Sigma_j CI_{vj}\, ccm_{vj} + X_b\, xpm_v}{(1 - \sigma)\, E\, spm_v} + [\tau\,\omega - (1 - \tau)\,\alpha\, tpm_v]\, p - \sigma\,(1 - \tau)\,\mathscr{P}\, bpm_v} \qquad (216)$$

The *break-even sellback price* fulfills the same role as the normative system energy price if a major portion of the energy generated is sold to the utility. Since the PURPA regulations [FERC 1980] require the utility to pay a price that reflects its net avoided cost, that price could be considerably higher than the grid energy price, particularly if the system under consideration tends to produce energy at times when the utility's marginal cost of generation is high.

Again setting Equation (212), II-60, to zero, and replacing P by p, then solving for the *break-even sellback price* gives:

$$\boxed{\hat{\mathscr{P}} = \frac{\Sigma_j CI_{vj}\, ccm_{vj} + X_b\, xpm_v}{\sigma(1-\tau)\, E\, bpm_v} - \frac{(1-\sigma)(1-\tau\,\omega)\, spm_v + (1-\tau\,\omega)\,\alpha\, tpm_v}{\sigma(1-\tau)\, bpm_v}\, p} \tag{217}$$

Resale of the system could affect these break-even prices. The *normative temporarily-owned-system energy price* can be obtained by setting Equation (213), II-60, to zero and solving for P.

$$\hat{P} = \frac{\Sigma_j CI_{LCC,j}\, ccm_{vj} + X_b\, xpm}{(1-\sigma)\, E\, spm} + [\tau\,\omega - (1-\tau)\,\alpha\, tpm]\, p - \sigma(1-\tau)\,\mathscr{P}\, bpm - \frac{NRG}{(1-\sigma)\, spm} \tag{218}$$

Also from Equation (214), II-60, the *break-even sellback price for a temporarily-owned-system* is given by

$$\hat{\mathscr{P}} = \frac{\Sigma_j CI_{LCC,j}\, ccm_{vj} + X_b\, xpm - NRG}{\sigma(1-\tau)\, E\, bpm} - \frac{(1-\sigma)(1-\tau\,\omega)\, spm + (1-\tau\,\omega)\,\alpha\, tpm}{\sigma(1-\tau)\, bpm}\, p \tag{219}$$

From the residual system value Equation (215), II-61, we can obtain the *normative residual-system energy price*:

$$\hat{P} = \frac{\Sigma_j CI_{RSC,j}\, ccm_{rj} + X_b\, rxpm}{(1-\sigma)\, E\, rspm} + [\tau\,\omega - (1-\tau)\,\alpha\, rtpm]\, p - \sigma(1-\tau)\,\mathscr{P}\, rbpm \tag{220}$$

and the *residual-break-even sellback price*:

$$\hat{\mathscr{P}} = \frac{\Sigma_j CI_{RSC,j}\, ccm_{rj} + X_b\, rxpm}{\sigma(1-\tau)\, E\, rbpm} - \frac{(1-\sigma)(1-\tau\,\omega)\, rspm + (1-\tau\,\omega)\,\alpha\, rtpm}{\sigma(1-\tau)\, rbpm}\, p \tag{221}$$

6.2. Component Prices, Energy Production, etc.

Equation (212), II-60, with $P = p$, can be solved for a few other parameters whose break-even values are of interest:

The break-even price of component J is found by first solving for CI_{vJ}, then solving Equation (132), II-46, for C_J. Thus,

$$\begin{aligned}\hat{CI}_{vJ} = (\{[(1-\sigma)(1-\tau\,\omega)\, spm_v + (1-\tau)\,\alpha\, tpm_v]\, p + \sigma(1-\tau)\,\mathscr{P}\, bpm_v\}\, E - X_b\, xpm_v \\ - \Sigma_{j\neq J}\, CI_{vj}\, ccm_{vj})/ccm_{vJ}\end{aligned} \tag{222}$$

It is shown in Appendix A that Equation (132), II-46, can be written as

$$CI_{vj} = \phi_c R_c^{t_j} \left[1 + (1+\delta_j)\, \frac{R_c^{L_j}}{1 - R_c^{L_j}} \right] C_j \tag{223}$$

Consequently, the *break-even price of component J* may be found by substituting the result of evaluating Equation (222) into

$$\hat{C}_J = \hat{Cl}_{vJ} \Big/ \phi_c R_c^{t_J} \left[1 + (1 + \delta_J) \frac{R_c^{L_J}}{1 - R_c^{L_J}} \right] \tag{224}$$

The *break-even energy production in the first year of capacity operation* is

$$\hat{E} = \frac{\Sigma_j \, Cl_{vj} \, ccm_{vj} + X_b \, xpm_v}{[(1 - \sigma)(1 - \tau\,\omega)\, spm_v + (1 - \tau)\,\alpha\, tpm_v]\, p + \sigma\,(1 - \tau)\,\mathscr{P}\, bpm_v} \tag{225}$$

The *break-even efficiency* (which may exceed limits imposed by physical laws) is obtained from Equation (17), II-12.

$$\hat{\eta} = \hat{E}/S\,Z \tag{226}$$

The *break-even grid energy price* is

$$\hat{p} = \frac{\Sigma_j \, Cl_{vj} \, ccm_{vj} + X_b \, xpm_v - \sigma\,(1 - \tau)\, bpm_v \mathscr{P}\, E}{[(1 - \sigma)(1 - \tau\,\omega)\, spm_v + (1 - \tau)\,\alpha\, tpm_v]\, E} \tag{227}$$

6.3. Initial System Capital

The net present value Equation (212), II-60, contains a weighted sum of component prices, rather than the unweighted sum implied by Equation (41), II-23:

$$C = \Sigma_j \, C_j \tag{228}$$

where

C = Initial system capital, expressed in $\$_b$.

If we assume, however, that component prices are exogenously known fractions[35] of the total price – that is:

$$C_j = f_j \, C \tag{229}$$

where

f_j = Price of capital goods of type j, expressed as a fraction of the initial system capital,

[35] One possible source for values of those fractions is the goals of the research and development program.

then Equation (212), II-60, after P is replaced by p and NPV is set to zero, may be rewritten and solved for C to give the *break-even initial system capital*:

$$\hat{C} = (\{[(1-\sigma)(1-\tau\,\omega)\,spm_v + (1-\tau)\,\alpha\,tpm_v]\,p + \sigma\,(1-\tau)\mathscr{P}\,bpm_v\}\,E - X_b\,xpm_v)/\Sigma_j f_j\,ccm_{vj} \tag{230}$$

The component prices associated with the break-even initial system capital are obtained by applying Equation (229), II-65:

$$\hat{C}_j = f_j\,\hat{C} \quad \text{for all } j \tag{231}$$

Note that the $\hat{C}_J$ given by Equation (224), II-64, requires component J to provide all of the savings needed (if any); the $\hat{C}_j$ given here shares the "burden" among all of the components.

6.4. Other Exogenous Variables

Break-even values of other exogenous variables, such as *component lifetimes* (one of which may determine the *system revitalization period*) enter Equation (212), II-60, in a complex fashion. (It is not valid, for example, to obtain $\hat{L}_J$ by solving Equation (224), II-64, for L_J, because L_j enters into ccm_{vj} as well.) They can be determined by multiple evaluations of Equation (212), II-60, starting from scratch each time, using an appropriate procedure for selecting successive values of the parameter of interest, until the computed value of NPV is sufficiently close to zero. (In performing these calculations, note that P should be set equal to p.)

7. OTHER FINANCIAL CONSIDERATIONS

The net present value of a potential investment is one of the most important financial estimates available to a venture capitalist. Its components – the life cycle revenues, the life cycle costs, and the residual system value – are also of considerable interest. These are not, however, the only criteria on which investment decisions are based. The liquidity requirement – the maximum cumulative negative cash flow (that is, how "deep in the hole" the investment gets before it starts paying off), or the present value of the liquidity required (the equivalent amount that would have to be in the bank at some point in time) – is also crucial. Non-quantitative criteria – such as the esthetic appeal of the system – are also relevant. Risk and uncertainty have been assumed in the present analysis to be accounted for by the numerical values of the required rate of return on equity and the debt interest rate; a venture capitalist would be more comfortable if these factors were explicit.

The normative system energy price is one of the most important financial estimates available for assessing the state of the art. The total *initial* cost of the system is also very important.

These issues, and others, are discussed in the remainder of this section.

7.1. Working Capital

One of the kinds of capital required is working capital, the amount of capital which must be injected into the system to get it operating once it exists. Working capital may be modeled as follows:

$$\boxed{C_w = \Sigma_j s_j C_j + l\, OPR} \tag{232}$$

where the summation on j covers all kinds of capital goods *except* working capital, and

s_j = Spares fraction for capital goods of type j, expressed in dollars per dollar (a dimensionless number in computation). The numerical value depends on the specific system design; a small system will often have no spares.

l = Operating period coverage fraction, the fraction (of a year) for which the system owner must be able to cover operating expenses out of working capital. The numerical value of this factor is determined by the "lumpiness" of revenue and cost cash flows and by the differences in their timing. If the owner has sufficient flexibility to defer payment of costs, without penalties, the numerical value could be zero.

OPR = Net annual operating expenses of the system under consideration, expressed in base year dollars (per year). Equation (233) approximates its value in terms of more fundamental parameters.

The net annual operating expense can be approximated by multiplying the non-capital portions of Equation (216), II-62, which gives the marginal cost of energy generation and takes into account the effect of the system on other expenses that the owner might have, by the annual amount of energy produced:

$$OPR = \frac{X_b\, xpm_v}{(1-\sigma)\, spm_v} + \{[\tau\,\omega - (1-\tau)\,\alpha\, tpm_v]\, p - \sigma(1-\tau)\mathscr{P}\, bpm_v\}\, E \tag{233}$$

It should be noted that working capital does not depreciate, but that the amount required does increase with time due to inflation. Thus,

$$DL_w = \infty \tag{234}$$

$$\mu_w = \text{"RA" with } g = g_\$ \tag{235}$$

where

DL_w, μ_w = Parameters DL_j and μ_j (see II-24) for capital goods of type j = w (*working capital*).

"RA" = Depreciation method "reassessment". (See Appendix B.)

g = Growth rate parameter associated with the "reassessment" depreciation method.

$g_\$$ = General inflation rate, expressed in % (per year).

7.2. Liquidity Requirement

The owner of the system incurs costs before starting to receive revenues. This fact implies that the owner must be able to survive a deficit of some amount for an investment in the system to be financially viable. The maximum size of that deficit is the *liquidity requirement*.

$$Q_t = \sum_{T=t_f}^{t} (Cost_T - Benefit_T) \tag{236}$$

where

Q_t = Liquidity requirement at time t, expressed in year t dollars.

$Cost_T$ = Cost in year T, expressed in year T dollars. (See Equation (67), II-32.)

$Benefit_T$ = Financial benefit in year T, expressed in year T dollars. (See Equations (19) and (20), II-13.)

Note that Equation (236) contains the assumption that all financial benefits reduce liquidity requirements. (If it did not contain that assumption, the resultant liquidity requirements would be likely to grow without bound.) It also assumes that costs and benefits occur at the times called for by their submodels. If this assumption provides too coarse an estimate in a particular application of the present model, the user is hereby directed to [Davis 1980]. Application of the current model with monthly resolution (see II-5) rather than yearly resolution could reduce that coarseness considerably.

The maximum liquidity requirement is obtained from the liquidity requirement at time t by finding that time, say t^*, for which Q_t takes on its largest value. Thus,

$$Q_{max} = max_t Q_t = Q_{t^*} \tag{237}$$

where

Q_{max} = Maximum liquidity requirement, expressed in year t^* dollars.

A more meaningful definition of the *initial capital requirement* than that provided in Section 4.2.2, II-23, and used in Section 6.3, II-65, is the funding that the owner would need at the time of the first cash flow, t_f, so that his funds in hand would never be entirely gone. Its value could be determined by finding that value of Q_f in the following equation that makes the minimum value of BAL_t equal to zero.

$$BAL_t = Q_f(1+k)^t + \sum_{T=t_f}^{t} (Benefit_T - Cost_T) \times (1+k)^{t-T} \tag{238}$$

where

BAL_t = Balance in a hypothetical investment account that earns at the discount rate and is used to meet all costs and receive all financial benefits, expressed in year t dollars.

Q_f = Initial capital requirement, the amount that must be put into that hypothetical investment account just before the first cash flow at time t_f so as to have that account never go negative, expressed in year t_f dollars.

Determination of the value of Q_f requires a search over all values of t in the range $t_f \leqslant t \leqslant t^*$; the value of t that implies Q_f is likely to be closer to t^* than to t_f.

7.3. Time-Variable Rates

All of the analyses presented here assume constant inflation, escalation, interest, and required return on equity rates. It is likely that these rates will actually change with time. To the extent that they all change in exact synchronism with the rate of general inflation, such variations will have no effect on the results[36]. To the extent that they vary independently, the model will give less accurate results.

[36]This is due to the fact that results are stated in (base year) constant dollars and that none of the costs or benefits are non-linear with the value of a dollar. If the income tax model had not assumed a fixed marginal tax rate, this would not have been true.

8. SIMPLIFICATION AND SYNTHESIS

Finally, everything needed to determine normative system energy prices, net present values, break-even parameter values, etc. has been delineated. All that remains is to put it together into a usable computational algorithm. Even if that were done next, however, the result would be so formidable that few of the intended users would bother with it, so that some simplification is required before proceeding (back) to Volume I.

The following simplifications will be made *before* synthesizing the algorithm:

(1) The sellback fraction and thermal energy value will be assumed constant:

$$\sigma_t = \sigma \text{ and } \alpha_t = \alpha \text{ for all } t > 0 \tag{239}$$

(2) The system will be assumed to be a quickly constructed, "turn-key" installation:

$$t_o = 0 \text{ and } t_f = 0 \tag{240}$$

(3) Only "straight line" depreciation will be considered.

$$\mu_j = \text{"SL" for all } j \text{ if the owner is a company} \tag{241}$$

(4) Possible resale of the system will be ignored.

$$\Delta = 0 \tag{242}$$

The general algorithm in Appendix A of Volume I was developed by following the same approach as in this section but without these simplifying assumptions.

8.1. Simplified Net Present Value

Equation (212), II-60, with $P = p$, is

$$NPV = \{[(1-\sigma)(1-\tau\,\omega)\,spm_v + (1-\tau)\,\alpha\,tpm_v]\,p + \sigma\,(1-\tau)\,\mathscr{P}\,bpm_v\}\,E$$
$$- \Sigma_j\, CI_{v_j}\, ccm_{v_j} - X_b\, xpm_v \tag{243}$$

The following elements are exogenous (input):

σ = Sellback fraction. See Equation (239), II-69, II-13, and II-9.

τ = Owner's marginal income tax rate. See II-26.

ω = Ownership indicator: zero if consumer, unity if company. See II-32.

α = Value of system thermal energy as fraction of value of system electrical energy if bought from the grid. See II-13.

p = Price of grid electricity. See II-10.

$\mathscr{P}$ = Price paid (by the utility) for electricity sold to the grid. See II-10.

E = Net electrical energy produced by the system during the first year of capacity operation. See II-11.

X_b = Annual expenses during first year of capacity operation. See II-40.

The following elements are endogenous; the equations that determine them follow:

spm_v = Total system energy price multiplier. See II-14.

tpm_v = Total thermal energy price multiplier. See II-14.

bpm_v = Total sellback energy price multiplier. See II-14.

ccm_{vj} = Total capital cost multiplier for capital goods of type j. See II-58.

xpm_v = Total expense price multiplier. See II-58.

Equation (31), II-16, using the simplifications of Equations (239) and (240), becomes

If $\sigma = 1$, $spm_v = 0$; if $\sigma \neq 1$:

$$spm_v = \frac{\phi_e (1+g_e)^{-1/2}}{1-R_e^L} \sum_{m=1}^{L} R_e^m \eta_m \tag{244}$$

Almost all of the elements of this equation are (essentially) exogenous:

ϕ_e = Relative inflation (i.e., real escalation) of energy prices from the base year to the start of system operation. See II-14.

g_e = Energy price escalation rate. See II-3.

L = System revitalization period. See II-12.

η_m = Degradation efficiency during the m-th year of the system energy production cycle. See II-19.

The endogenous element is

R_e = Discounted energy price escalation rate. From Equation (29), II-15:

$$R_e = (1+g_e)/(1+k) \tag{245}$$

The remaining element in this equation is

k = System owner's discount rate. See Section 4.4.1.3, II-37 ff.

Equation (34), II-18, using the simplifications of Equations (236), II-64, becomes

If $\alpha = 0, tpm_v = 0$; if $\alpha \neq 0$:

$$tpm_v = \frac{\phi_e (1 + g_e)^{-1/2}}{1 - R_e^L} \sum_{m=1}^{L} R_e^m \eta_m \tag{246}$$

Note that the *expressions* for spm_v and tpm_v are identical (with the assumed simplifications), but that the *conditions* are different.

Equation (37), II-20, using the simplifications of Equations (239), II-69, becomes

If $\sigma = 0, bpm_v = 0$; if $\sigma \neq 0$:

$$bpm_v = \frac{\phi_s (1 + g_s)^{-1/2}}{1 - R_s^L} \sum_{m=1}^{L} R_s^m \eta_m \tag{247}$$

The exogenous element is

ϕ_s = Relative inflation (real escalation) of sellback prices from the base year to the start of system operation. See II-18.

The as-yet undefined endogenous element is

R_s = Discounted sellback price escalation rate. From Equation (35), II-18:

$$R_s = (1 + g_s)/(1 + k) \tag{248}$$

The as-yet undefined exogenous element is

g_s = Sellback energy price escalation rate. See II-3.

Equation (210), II-59, is considerably simplified if split into separate cases depending on ownership:

If $\omega = 0$ (the owner is a consumer):

$$ccm_{vj} = r\, bsch_{vj} + bdep_{vj} + (1 - \epsilon)\,[(1 - \tau)\, i - r]\, mort_{vj} + (1 - \tau\sigma)\,\beta_1\, tsch_{vj} - \sigma\,[\tau\, tdep_{vj}$$
$$+ \beta_3 \frac{Cl'_{vj}}{Cl_{vj}} + (1 - \tau\sigma)\,\beta_2\, \phi_c \frac{C_j (1 + \delta_j)\, ins_{vj}}{Cl_{vj}} - \frac{\beta_4}{\Phi_s (1 + k)^{t_j}} \frac{Cl_{jt_j}}{Cl_{vj}} \tag{249}$$

If $\omega = 1$ (the owner is a company – utility or non-utility):

$$ccm_{vj} = k\, bsch_{vj} + bdep_{vj} + (1-\tau)\,\beta_1\, tsch_{vj} - \left[\tau\, tdep_{vj} + \beta_3 \frac{CI'_{vj}}{CI_{vj}}\right]$$

$$+ (1-\tau)\,\beta_2\,\phi_c \frac{C_j(1+\delta_j)\, ins_{vj}}{CI_{vj}} - \frac{\beta_4}{\Phi_s (1+k)^{t_j}} \frac{CI_{jt_j}}{CI_{vj}} \tag{250}$$

The exogenous elements not previously identified in this section are:

r = Required rate of return on equity. See II-34.

ϵ = Down-payment fraction. See II-35.

i = Debt or mortgage interest rate. See II-34 and II-35.

β_1 = Other (non-income) tax rate. See II-39.

β_2 = Insurance rate. See II-39.

β_3 = Investment tax credit rate. See II-39.

β_4 = Solar (one-time only) tax credit rate. See II-40.

ϕ_c = Relative inflation (i.e., real escalation) of capital prices from the base year to the start of system operations. See II-54.

Φ_s = Inflation (of dollars) from the base year to the start of system operation. See II-2.

C_j = Purchase cost of capital goods of type j. See II-22.

δ_j = Additional cost of replacing capital goods of type j. See II-22.

t_j = Time of payment ("lead time") for capital goods of type j. See II-22.

Several new (to this section) endogenous elements were introduced in Equations (249-250). It will be seen, however, that the simplifications of Equations (239), II-69, will simplify some of the capital ratios. The new endogenous elements are:

$bsch_{vj}$ = Present value of book values of capital goods of type j as a multiple of initial price. See II-47.

$bdep_{vj}$ = Present value of book depreciation functions. See II-44.

$mort_{vj}$ = Present value of mortgage value function. See II-48.

$tsch_{vj}$ = Present value of tax valuation function. See II-53.

$tdep_{vj}$ = Present value of tax depreciation function. See II-50.

ins_{vj} = Present value of replacement values. See II-55.

CI_{vj} = Present value of all capital investment in goods of type j. See II-46.

CI^f_{vj}/CI_{vj} = 1.

From Equation (A-4), A-2,

$$bsch_{vj} = \sum_{T=0}^{L_j} \frac{bsch\,(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{251}$$

The new exogenous elements are

L_j = Lifetime of capital equipment of type j. See II-22.

θ_j = Salvage value fraction. See II-24.

By simplifying Equation (241), II-69, the schedule function is given by Equation (B-6), B-2:

$$sch\,(T, L_j, \mu_j, \theta_j) = 1 - (1 - \theta_j)\,\frac{T}{L_j} \qquad (\text{for } 0 \leq T \leq L_j) \tag{252}$$

From Equations (241), II-69, and (C-7), C-2,

$$bdep_{vj} = \sum_{T=1}^{L_j} \frac{bdep\,(T, L_j, \mu_j, \theta_j)}{(1+k)^T} = \sum_{T=1}^{L_j} \frac{(1-\theta_j)/L_j}{(1+k)^T} \tag{253}$$

By Equations (28), II-15, and (82), II-36, this can be rewritten as

$$bdep_{vj} = \frac{1-\theta_j}{L_j}\, crf_{k,L_j} \tag{254}$$

The crf function was used to represent the summation because that particular combination of two arguments will be needed frequently. Rewriting Equation (82), II-36,

$$crf_{k,L} = \begin{cases} k/[1-(1+k)^{-L}] & \text{if } k \neq 0 \\ 1/L & \text{if } k = 0 \end{cases} \tag{255}$$

(In this equation, k and L are the arguments of the function, not necessarily the *discount rate* and the *revitalization period*.)

From Equation (A-10), A-4,

$$mort_{vj} = \sum_{T=0}^{L_j-1} \frac{crf_{i,MP} - (1+i)^T \, sff_{i,MP}}{i(1+k)^T} \tag{256}$$

The new exogenous element is

MP = Mortgage period. See page II-36.

The endogenous element is the sinking fund factor function given by Equation (90), II-37:

$$sff_{k,L} = crf_{k,L} - k \quad \text{or} \quad = \begin{cases} k/[(1+k)^L - 1] & \text{if } k \neq 0 \\ 1/L & \text{if } k = 0 \end{cases} \tag{257}$$

(As in the definition of crf, Equation (255), II-73, k and L in this equation are arguments, not necessarily to be interpreted as the k and L defined earlier.)

From Equation (A-7), A-2, using simplification Equation (239), II-69,

$$tsch_{vj} = \sum_{T=0}^{L_j-1} \frac{tsch\,(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{258}$$

The tax schedule function depends on who owns the system, in spite of Equation (241), II-69. Furthermore, the tax life is shorter than the expected life, by Equation (B-17), B-4. Thus, from Equations (B-6), B-2, and (B-14), B-4,

$$tsch\,(T, L_j, \mu_j, \theta_j) = \begin{cases} (1+g_a)^T & \text{if } \omega = 0 \text{ and } T < tf\,L_j \\ 1 - (1-\theta_j)T/(tf\,L_j) & \text{if } \omega = 1 \text{ and } T < tf\,L_j \\ \theta_j & \text{if } tf\,L_j \leq T \leq L_j \\ 0 & \text{if } L_j < T \end{cases} \tag{259}$$

The new exogenous elements are

g_a = Assessment escalation rate. See B-4.

tf = Tax life fraction (normally, 2/3). See B-4.

From Equation (A-9), A-3, using simplification Equation (239), II-69.

$$tdep_{vj} = \sum_{T=1}^{L_j} \frac{tdep\,(T, L_j, \mu_j, \theta_j}{(1+k)^T} \tag{260}$$

By Equations (241), II-69, (B-17), B-4, (B-7), B-2, and (B-15), B-4, and using Equations (28), II-15, and (82), II-36,

$$tdep_{vj} = \frac{1-\theta_j}{tf\,L_j}\; crf_{k,\,tf \times L_j} \tag{261}$$

From Equations (189), II-55, (239), II-69, (241), II-69, and (28), II-15,

$$ins_{vj} = 1/(1-R_c) \tag{262}$$

The new endogenous element is

R_c = Discounted capital goods escalation rate. From Equation (183), II-54:

$$R_c = (1+g_c)/(1+k) \tag{263}$$

The new exogenous element is

g_c = Capital cost escalation rate. See II-3.

From Equation (A-1), A-1,

$$CI_{vj} = \phi_c R_c^{t_j} C_j \left[1 + (1+\delta_j)\frac{R_c^{L_j}}{1-R_c^{L_j}}\right] \tag{264}$$

There are no new exogenous or endogenous elements in this equation.

Finally, the last endogenous element in the net present value Equation (243), II-69, is the expense price multiplier, given by Equation (211), II-59:

$$xpm_v = (1 - \tau\,[\omega + (1-\omega)\,\sigma])\,\phi_x \left(Z_v + \frac{x_v}{X_b}\right) \tag{265}$$

The new exogenous element is

ϕ_x = Relative inflation (i.e., real escalation) of system expenses from the base year to the start of system operation. See II-55.

Two new endogenous elements were introduced:

Z_v = Present value of total repetitive expenses, as a multiplier of the first year repetitive expenses. See II-56.

x_v = Present value of total non-repetitive expenses. See II-56.

From Equation (200), II-56, using Equation (28), II-15, and simplification Equations (239), II-69, and (238), II-69,

$$Z_v = 1/(1 - R_x) \tag{266}$$

The new endogenous element is

R_x = Discounted expense escalation rate. From Equation (192), II-55:

$$R_x = (1 + g_x)/(1 + k) \tag{267}$$

The new exogenous variable is

g_x = Expense escalation rate. See II-3.

From Equation (101), II-56, using the simplification of Equation (239), II-69,

$$x_v = \sum_{t=t_f}^{\infty} R_x^t x_t \tag{268}$$

The new exogenous element is

x_t = One-time (non-repetitive) cost in year t. See II-40.

The time of the first cash flow, t_f, (see II-6) was left in Equation (266), rather than being omitted by virtue of simplifying Equation (240), II-69, because one-time expenses are quite likely to occur before the start of system operations.

8.2 Simplified Residual System Value

Computation of the residual system value, even using the simplification of Equations (239), II-69, to (242), II-69, would be a formidable task, and is not included. It would not be correct, however, to conclude that the residual system value is being ignored. On the contrary, exclusion of the residual system value from the definition of the net present value would have contributed significantly to the complexity of its calculation.

The starting point for a calculation of the residual system value would be Equation (215), II-61.

8.3. Simplified Normative System Energy Prices

From this point on it gets considerably easier because there is only one new endogenous element and only three new exogenous elements to be required. That is, everything else (except the *initial system capital*) can be calculated from information already available in this section.

The normative system energy price is given by Equation (216), II-62:

$$\hat{p} = \frac{\Sigma_j CI_{vj}\, ccm_{vj} + X_b\, xpm_v}{(1-\sigma)\, E\, spm_v} + [\tau\,\omega - (1-\tau)\,\alpha\, tpm_v]\, p - \sigma\,(1-\tau)\hat{\mathscr{P}}\, bpm_v \tag{269}$$

This is also the marginal price of system electrical energy generation (averaged – with discounting – over time), and corresponds to the constant (in constant energy dollars) busbar energy price ($\mathbf{BBEC_0}$) of USES.

The break-even sellback price is given by Equation (217), II-63,

$$\hat{\mathscr{P}} = \frac{\Sigma_j CI_{vj}\, ccm_{vj} + X_b\, xpm_v}{\sigma\,(1-\tau)\, E\, bpm_v} - \frac{(1-\sigma)(1-\tau\,\omega)\, spm_v + (1-\tau)\,\alpha\, tpm_v}{\sigma\,(1-\tau)\, bpm_v}\, p \tag{270}$$

8.4. Other Simplified Break-Even Values

The break-even initial system capital is given by Equation (230), II-65:

$$\hat{C} = (\{[(1-\sigma)(1-\tau\,\omega)\, spm_v + (1-\tau)\,\alpha\, tpm_v]\, p + \sigma\,(1-\tau)\hat{\mathscr{P}}\, bpm_v\}\, E - X_b\, xpm_v)/\Sigma_j f_j\, ccm_{vj} \tag{271}$$

The new exogenous element is

f_j = Fraction of system price due to capital goods of type j. See II-65.

The component prices that correspond to the break-even system price is given by Equation (231), II-65.

$$\hat{C}_j = f_j\, \hat{C} \qquad \text{for each } j \tag{272}$$

If, on the other hand, all of the burden of breaking even is placed on a single component, say J, then Equation (224), II-64, gives the break-even price of component J:

$$C_J = \frac{CI_{vJ}}{\phi_c\, R_c^{t_j}\;\; 1 + (1+\delta_J)\, R_c^{L_J}/(1-R_c^{L_J})} \tag{273}$$

The new endogenous element,

$\hat{CI}_{vJ}$ = Present value of all expenditures on capital goods of type J,

is given by Equation (222), II-64:

$$\hat{CI}_{vJ} = (\{[(1-\sigma)(1-\tau\,\omega)\,spm_v + (1-\tau)\,\alpha\,tpm_v]\,p + \sigma\,(1-\tau)\,\mathscr{P}\,bpm_v\}\,E - X_b\,xpm_v$$

$$- \Sigma_{j \neq J}\,CI_{vj}\,ccm_{vj})/ccm_{vJ} \tag{274}$$

The break-even system energy production is given by Equation (225), II-64:

$$\hat{E} = \frac{\Sigma_j\,CI_{vj}\,ccm_{vj} + X_b\,xpm_v}{[(1-\sigma)(1-\tau\,\omega)\,spm_v + (1-\tau)\,\alpha\,tpm_v]\,p + \sigma\,(1-\tau)\,\mathscr{P}\,bpm_v} \tag{275}$$

The break-even system efficiency can be found from Equation (227), II-64:

$$\hat{\eta} = \hat{E}/S\,Z$$

The final two exogeneous elements are

S = Annual energy density in kWh/kWp. See II-12.

Z = Nameplate size of the system in kWp. See II-12.

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APPENDIX A
PRESENT VALUE FORMULAS

APPENDIX A. PRESENT VALUE FORMULAS

A.1 Present Values of Capital Investments

For Equation (132) II-46, and (223), II-63, using Equation (40) II-22:

$$CI_{vj} = \frac{1}{\Phi_s} \sum_{t=t_f}^{\infty} \frac{CI_{jt}}{(1+k)^N}$$

$$= \frac{1}{\Phi_s}\left[\Phi_c C_j \left(\frac{1+g_c}{1+k}\right)^{t_j} + (1+\delta_j) \sum_{n=1}^{\infty} \Phi_c C_j \left(\frac{1+g_c}{1+k}\right)^{t_j+nL_j}\right]$$

$$= \phi_c R_c^{t_j} C_j \left[1 + (1+\delta_j) \sum_{n=1}^{\infty} \left(R_c^{L_j}\right)^n\right]$$

$$= \Phi_c R_c^{t_j} C_j \left[1 + (1+\delta_j) \frac{R_c^{L_j}}{1 - R_c^{L_j}}\right] \tag{A-1}$$

A.2. Present Values Of Schedule Functions

In Appendix B, schedule functions for four depreciation methods are given:

"SL" – Straight line

"SYD" – Sum of the Years' Digits

"DDB" – Double Declining Balance

"RA" – Reassessment

For Equation (135), II-46,

$$bsch_{LCC,j} = \frac{1}{\Phi_s} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{N-1-t} \frac{bsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{LCC,j} \tag{A-2}$$

For Equation (136), II-47,

$$bsch_{RSC,j} = \frac{1}{\Phi_s}\left(\sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=N-t}^{L_j} + \sum_{t=N}^{\infty} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{L_j}\right) \frac{bsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{RSC,j} \tag{A-3}$$

For Equation (137), II-47,

$$bsch_{vj} = \sum_{T=0}^{L_j} \frac{bsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{A-4}$$

(The fact that this equation is so much simpler than its two predecessors illustrates the artificiality and arbitrariness of choosing a finite planning horizon.)

For Equation (178), II-53:

$$tsch_{LCC,j} = \frac{1}{\Phi_s} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{N-1-t} \frac{1-\tau[\omega+(1-\omega)\sigma_{t+T}]}{1-\tau[\omega+(1-\omega)\sigma]} \frac{tsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{LCC,j} \tag{A-5}$$

For Equation (179), II-54,

$$tsch_{RSC,j} = \frac{1}{\Phi_s} \left(\sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=N-t}^{L_j} + \sum_{t=N}^{\infty} \frac{CI_{jt}}{(1+k)^t} \sum_{T=0}^{L_j-1} \right)$$

$$\times \frac{1-\tau[\omega+(1-\omega)\sigma_{t+T}]}{1-\tau[\omega+(1-\omega)\sigma]} \frac{tsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{RSC,j} \tag{A-6}$$

For Equation (180), II-54,

$$tsch_{vj} = \sum_{T=0}^{L_j-1} \frac{1-\tau[\omega+(1-\omega)\sigma_T]}{1-\tau[\omega+(1-\omega)\sigma]} \frac{tsch(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{A-7}$$

A.3. Present Values Of Depreciation Functions

For Equation (157), II-50,

$$tdep_{LCC,j} = \frac{1}{\Phi_s} \sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=1}^{N-t} \frac{\omega+(1-\omega)\sigma_{t+T}}{\omega+(1-\omega)\sigma} \frac{tdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{LCC,j} \tag{A-8}$$

For Equation (158), II-50,

$$tdep_{RSC,j} = \frac{1}{\Phi_s} \left(\sum_{t=t_f}^{N-1} \frac{CI_{jt}}{(1+k)^t} \sum_{T=N-t}^{L_j} + \sum_{t=N}^{\infty} \frac{CI_{jt}}{(1+k)^t} \sum_{T=1}^{L_j} \right)$$

$$\times \frac{\omega+(1-\omega)\sigma_{t+T}}{\omega+(1-\omega)\sigma} \frac{tdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} / CI_{RSC,j} \tag{A-9}$$

For Equation (159), II-50,

$$tdep_{vj} = \sum_{T=1}^{L_j} \frac{\omega+(1-\omega)\sigma_T}{\omega+(1-\omega)\sigma} \frac{tdep(T, L_j, \mu_j, \theta_j)}{(1+k)^T} \tag{A-10}$$

APPENDIX B
DEPRECIATION

APPENDIX B. DEPRECIATION

As noted in the body of the text in Section 4.2.3, II-24, on capital valuation, the change in value of capital goods with time can be represented, as in Equation (45), II-24, by a function. That is,

$$VAL_A = VAL\ sch\ (A, DL, \mu, \theta) \tag{B-1}$$

where

VAL_A	=	Value of capital goods of age A, expressed in nominal dollars.
VAL	=	Purchase price of the capital goods, expressed in nominal dollars.
A	=	Age of the capital goods, expressed in years.
DL	=	Depreciable lifetime of the capital goods, expressed in years.
μ	=	Depreciation method to be used.
θ	=	Salvage value, expressed as a fraction of the purchase price.
$sch\ (A, DL, \mu, \theta)$	=	Schedule function for capital goods of age A when the depreciable life is DL, the depreciation method is μ, and the salvage value fraction is θ. This function expresses the value of the capital goods as a fraction of the purchase price, and is identically equal to zero if the age is negative (because the capital goods have not yet been purchased.)

As pointed out in the discussion leading to Equation (46), II-25, *depreciation* is the change in capital value from one year to the next:

$$DEP_A = VAL_{A-1} - VAL_A \tag{B-2}$$

where

DEP_A = Depreciation of capital goods now of age A, expressed in nominal dollars.

By substituting Equation (B-1) into Equation (B-2), we have

$$DEP_A = VAL\ [sch\ (A\text{-}1, DL, \mu, \theta) - sch\ (A, DL, \mu, \theta)] \tag{B-3}$$

Replacing the content of the brackets by the *depreciation function*, we have

$$DEP_A = VAL\ dep\ (A, DL, \mu, \theta) \tag{B-4}$$

where

$$dep\ (A, DL, \mu, \theta) = sch\ (A\text{-}1, DL, \mu, \theta) - sch\ (A, DL, \mu, \theta) \tag{B-5}$$

B.1. Straight Line Depreciation

The simplest model of decrease in value is called "straight line". This model assumes that value decreases linearly over its lifetime from an initial fraction of *unity* to the final salvage value fraction θ. This model accurately describes the amount of usefulness left in the capital goods if that usefulness occurs uniformly over time. The coresponding schedule function is:

$$sch\,(A, DL, \text{"SL"}, \theta) = \begin{cases} 0 & \text{if } A < 0 \\ 1 - (1-\theta)\dfrac{A}{DL} & \text{if } 0 \leq A \leq DL \\ \theta & \text{if } DL \leq A \leq \textit{actual life} \\ 0 & \text{if } \textit{actual life} < A \end{cases} \tag{B-6}$$

The associated depreciation function is:

$$dep\,(A, DL, \text{"SL"}, \theta) = \begin{cases} 0 & \text{if } A < 1 \\ (1-\theta)/DL & \text{if } 1 \leq A < DL \\ 0 & \text{if } DL < A \end{cases} \tag{B-7}$$

B.2. Sum Of The Years' Digits Depreciation

"Accelerated" depreciation methods correspond to the assumption that value decreases more rapidly at first than it does later. The schedule function for the "sum of the years' digits" method[1] is quadratic:

$$sch\,(A, DL, \text{"SYD"}, \theta) = \begin{cases} 0 & \text{if } A < 0 \\ 1 - \dfrac{2(1-\theta)}{DL\,(DL+1)}\left[(2\,DL+1)A + A^2\right] & \text{if } 0 \leq A \leq DL \\ \theta & \text{if } DL \leq A \leq \textit{actual life} \\ 0 & \text{if } \textit{actual life} < A \end{cases} \tag{B-8}$$

The associated depreciation function is

$$dep\,(A, DL, \text{"SYD"}, \theta) = \begin{cases} 0 & \text{if } A < 1 \\ (1-\theta)\,\dfrac{2\,(DL+1-A)}{DL\,(DL+1)} & \text{if } 1 \leq A < DL \\ 0 & \text{if } DL < A \end{cases} \tag{B-9}$$

[1]The name derives from the form of the *dep* function. The ratios of the depreciation the first year to that of the second year to that of the next year, etc., is $(DL) : (DL-1) : (DL-2) : \ldots : 1$. To make the total depreciation equal the total depreciable value, these numbers must each be divided by the sum of all of them: $DL\,(DL+1)/2$.

B.3. Double Declining Balance Depreciation

Another class of accelerated depreciation methods has an exponential schedule function: the depreciation each year is a fraction of the current depreciated value (the "declining balance"). The fraction taken must be between one and two times that which would be taken if the straight line method were used. The greatest shift in timing is obtained by using the factor of two (hence, "double"). When the age reaches half the lifetime, however, straight line depreciation of the remaining depreciable value would give a larger depreciation deduction than continuing to use the same fraction of the declining balance. Switching to straight line is allowed by law and is assumed to be done. The "double declining balance" schedule function is given by

$$sch\,(A, DL, \text{"DDB"}, \theta) = \begin{cases} 0 & \text{if } A < 0 \\ (1-\theta)\left(1-\frac{2}{DL}\right)^{A} + \theta & \text{if } 0 \leq A \leq M \\ (1-\theta)\left(1-\frac{2}{DL}\right)^{M}\left(\frac{DL-A}{DL-M}\right) + \theta & \text{if } M \leq A \leq DL \\ \theta & \text{if } DL \leq A \leq \textit{actual life} \\ 0 & \text{if } \textit{actual life} < A \end{cases} \tag{B-10}$$

where

$$M = \begin{matrix}\text{Time at which} \\ \text{switch is made} \\ \text{to straight line}\end{matrix} = \begin{cases} DL/2 & \text{if } DL \text{ is even} \\ (DL+1)/2 & \text{if } DL \text{ is odd} \end{cases} \tag{B-11}$$

The associated depreciation function is

$$dep\,(A, DL, \text{"DDB"}, \theta) = \begin{cases} 0 & \text{if } A < 1 \\ (1-\theta)\left(\frac{2}{DL}\right)\left(1-\frac{2}{DL}\right)^{A-1} & \text{if } 1 \leq A \leq M \\ (1-\theta)\left(1-\frac{2}{DL}\right)^{M}\left(\frac{1}{L-M}\right) & \text{if } M < A \leq DL \\ 0 & \text{if } DL < A \end{cases} \tag{B-12}$$

B.4. Reassessment (and No) Depreciation

Some capital goods, such as land and working capital (and the entire energy generation system if owned by a consumer who derives no taxable income from its operation – that is, who sells no energy back to the utility), are not depreciable, but they are subject to property taxation. Thus, the book schedule function is given by

$$bsch\,(A, DL, \text{"RA"}, \theta) = \begin{cases} 0 & \text{if } A < 0 \\ 1 & \text{if } 0 \leq A \leq \textit{actual life} \\ 0 & \text{if } \textit{actual life} < A \end{cases} \tag{B-13}$$

and the tax schedule function is given by

$$tsch\,(A, DL, \text{"RA"}, \theta) = \begin{cases} 0 & \text{if } A < 0 \\ (1 + g_a)^A & \text{if } 0 \leq A \leq DL \\ \theta & \text{if } DL \leq A \leq \textit{actual life} \\ 0 & \text{if } \textit{actual life} < A \end{cases} \tag{B-14}$$

where

g_a = Assessment escalation rate, expressed in % (per year). (Note that g_a is not necessarily equal to g_c.)

Income tax laws do not require reporting of increases in capital values until the property is sold, which is covered in Section 4.3.1. Consequently,

$$dep\,(A, DL, \text{"RA"}, \theta) = 0 \tag{B-15}$$

B.5. Tax Depreciation

When depreciation is computed for tax purposes, the system owner is normally allowed to use a tax life which is shorter than the expected useful life. Hence,

$$tsch\,(A_j, DL, \mu_j, \theta_j) = sch\,(A_j, tf \times L_j, \mu_j, \theta_j) \tag{B-16}$$

and

$$tdep\,(A_j, DL_j, \mu_j, \theta_j) = dep\,(A_j, tf \times L_j, \mu_j, \theta_j) \tag{B-17}$$

where

tf = Tax life fraction (normally, 2/3).

APPENDIX C
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APPENDIX C. INDEX

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$dep(A, DL, \text{"RA"}, \theta)$	*II-B-3*
$dep(A, DL, \text{"SL"}, \theta)$	*II-B-2*
$dep(A, DL, \text{"SYD"}, \theta)$	*II-B-2*
$dep(A, L, \mu, \theta)$	II-25; *II-25*
DL	II-B-1
DL_j	II-24
DL_w	II-67; *II-67*
$DVAL_{jt}$	II-36; *II-36*
$DVAL_{LCC}$	*II-47*
$DVAL_t$	II-33; *II-34*
Δ	II-43; *II-69*
δ_j	II-22
E	II-11; *II-12*
$\hat{E}$	*II-64*
E_t	II-9; *II-11*
e_t	II-11; *II-12*
EQR_t	II-20
$EVAL_t$	II-33; *II-33*
ϵ	II-35
η	II-13
$\hat{\eta}$	*II-64*
η_m	II-11; *II-12*
η'_t	II-12

f_t	*II-64*
f_j	*II-65*
$frac(A)$	II-12
g	II-67
g_a	II-74
g_c	II-3
g_e	II-3
g_s	II-3
g_x	II-3
$g_\$$	II-67
i	II-34
$ins_{LCC,j}$	*II-55*
$ins_{RSC,j}$	*II-55*
ins_{vj}	*II-55*
$int(A)$	II-14
INS_{jt}	*II-39*
INS_t	II-20; *II-39*
INT_{jT}	II-36; *II-36*
INT_t	II-20; *II-34*
ITC_t	II-26; *II-39*
K	II-23; *II-23*
$\bar{K}$	II-23; *II-23*
k	II-3; *II-38*
L	II-12
L_j	II-22
l	II-66
LCC	II-6; *II-6*
LCC_{CR}	II-41; *II-42*

LCC_{DEP}	II-41; *II-42*	OTX_t	II-20; *II-38*
LCC_{INS}	II-42; *II-42*	ω	II-32
LCC_{ITC}	II-41; *II-42*		
LCC_{OF}	II-43; *II-43*	P	II-10
LCC_{ON}	II-43; *II-43*	$\hat{P}$	*II-62*
LCC_{OTX}	II-42; *II-42*	P_t	II-9; *II-10*
LCC_{RI}	II-49; *II-50*	p	II-10
LCC_{STC}	II-41; *II-42*	$\hat{p}$	*II-64*
LCC_{TD}	II-49; *II-49*	p_t	II-9; *II-10*
LCC_X	II-42; *II-42*	$\mathscr{P}$	II-10
LCC_x	*II-56*	$\hat{\mathscr{P}}$	*II-63*
Γ	II-28	$\mathscr{P}_t$	II-10; *II-10*
λ	II-34	PAY_{jt}	II-35; *II-35*
		PDR_{jT}	II-36; *II-36*
M	II-B-3; *II-B-3*	PDR_t	II-20; *II-34*
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$min(a,b)$	II-29	$PV(A_t)$	II-3; *II-2*
$MORT_{jt}$	II-35; *II-35*	Φ_c	II-3
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MP	II-36	Φ_x	II-3
μ	II-B-1	Φ_s	II-2
μ_j	II-24; *II-69*	ϕ_c	II-54
μ_w	II-67; *II-67*	ϕ_e	II-14
		ϕ_s	II-71; *II-18*
N	II-5	ϕ_x	II-55
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NPV	II-4; *II-3*	Q_f	II-68
NRG	II-57; *II-57*	Q_{max}	II-68; *II-68*
		Q_t	II-67; *II-67*
OI_N	*II-51*		
OI_t	II-29; *II-29*	r	II-34
OPR	II-66, *II-66*	R_c	II-54; *II-54*

R_e	II-15; *II-70*	S_t	II-12
R_s	II-71; *II-71*	$sch(A, DL, \text{"DDB"}, \theta)$	*II-B-3*
R_x	II-55; *II-55*	$sch(A, DL, \text{"SL"}, \theta)$	*II-B-2*
"RA"	II-67	$sch(A, DL, \text{"SYD"}, \theta)$	*II-B-2*
rbpm	II-14; *II-19*	$sch(A, DL, \mu, \theta)$	*II-24*
REP_t	II-20; *II-33*	$sch(T, L_j, \mu_j, \theta_j)$	*II-73*
REV	II-6; *II-6*	$sff_{k,L}$	*II-74*
RF	II-29	$sff_{i,MP}$	II-36; *II-37*
RSB	II-6; *II-6*	"SL"	II-A-1
RSC	II-7; *II-6*	*spm*	II-13; *II-14*
RSC_{CR}	II-41; *II-49*	spm_v	II-14; *II-16*
RSC_{DEP}	II-41	STC_t	II-26; *II-40*
RSC_{INS}	II-42; *II-55*	STI_t	II-26; *II-30*
RSC_{ITC}	II-41; *II-52*	$STVAL_t$	II-25
RSC_{OF}	*II-45*	"SYD"	II-A-1
RSC_{ON}	*II-47*	σ	II-13
RSC_{OTX}	II-42; *II-54*	σ_t	II-9; *II-69*
RSC_{STC}	II-42; *II-53*		
RSC_{TD}	*II-50*	T	II-35
RSC_X	II-42	t_f	II-6
RSC_x	*II-56*	t_j	II-22
RSL_t	II-20; *II-26*	t_0	II-11; *II-69*
RSL'_t	II-26; *II-26*	TAX_t	II-20; *II-26*
$RSL'_{t=N}$	II-25; *II-26*	$TDEP_t$	II-25
rspm	II-14; *II-16*	$tdep(A, L, \mu, \theta)$	II-50
RSV	II-7; *II-7*	$tdep(A_j, DL_j, \mu_j, \theta_j)$	*II-B-4*
RTI_t	II-28; *II-28*	$tdep_{LCC,j}$	II-50
rtpm	II-14; *II-17*	$tdep_{RSC,j}$	II-50
rxpm	II-58; *II-58*	$tdep_{vj}$	II-50; *II-75*
ρ	II-29	*tpm*	II-14; *II-17*
		tpm_v	II-14; *II-18*
S	II-12	$tsch(A, DL, \text{"RA"}, \theta)$	*II-B-4*
s_j	II-66	$tsch(A_j, DL, \mu_j, \theta_j)$	*II-B-4*

$tsch(A, L, \mu, \theta)$	**II-53**	X_t	**II-20**; *II-40*
$tsch_{LCC,j}$	**II-54**	x_{LCC}	*II-56*
$tsch(T, L_j, \mu_j, \theta_j)$	*II-74*	x_{RSC}	*II-56*
$tsch_{vj}$	*II-74*	x_t	**II-40**
$TVAL_t$	**II-25**	x_v	*II-56*
τ	**II-26**	xpm	**II-57**; *II-57*
θ	**II-B-1**	xpm_v	**II-58**; *II-59*
θ_j	**II-24**		
VAL	**II-B-1**	Z	**II-12**
VAL_A	**II-B-1**; *II-B-1*	Z_{LCC}	*II-56*
VAL_{jT}	**II-24**; *II-24*	Z_{RSC}	*II-56*
		Z_t	**II-12**
X_b	**II-40**	Z_v	*II-56*