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COMPUTER PREDICTION OF DUAL REFLECTOR ANTENNA RADIATION PROPERTIES

by

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This research was supported by the National Aeronautics and Space Administration through grant NSG 1588.
A new program for calculating dual reflector antenna radiation patterns has been developed adding one more option to the original program developed jointly by NCSU and NASA. The previous program was capable of computing patterns for single reflector antennas with either smooth analytic surfaces or with surfaces composed of a number of panels.

Techniques based on the geometrical optics (GO) approach are used in tracing rays over the following regions:

1) From a feed antenna to the first reflector surface (subreflector).
2) From this reflector to a larger reflector surface (main reflector).
3) From the main reflector to a mathematical plane (aperture plane) in front of the main reflector.

The equations of GO are also used to calculate the reflected field components for each ray making use of the feed radiation pattern and the parameters defining the surfaces of the two reflectors. These resulting fields form an aperture distribution which is integrated numerically to compute the radiation pattern for a specified set of angles.

Spillover, diffraction and other factors [2] that affect the accuracy of the calculation of the far-out sidelobes, are neglected.
Examples and all test cases are mentioned to support the validity of the new algorithm.
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1. INTRODUCTION

The objective of the work reported herein was to develop an algorithm to calculate the radiation patterns of Cassegrain antennas, which belong to the general group of dual reflector antennas. (See Appendix A.) The approach taken is to adopt and extend an existing algorithm which was developed for single reflector antennas.

The original algorithm for single reflector antennas was published in 1976 [1]. Later on that year this program appeared as a NCSU report [2], but in a modified version. Between 1976 and 1978 this algorithm was extended to deal with new surfaces such as ellipsoids and spheres [3]. In 1980, Botula modified the algorithm giving it the capability to analyze antennas with either smooth analytic surfaces or with surfaces composed of a number of panels [5].

The method of the electric vector potential and the geometrical optics approach were used to compute the radiation field of the antenna in question.

This thesis includes:

1) All modifications and additions inserted into the program to increase the accuracy of the calculated results for multipanel single reflector antennas;

2) The equations written to describe hyperbolic surfaces; and

3) The equations used to describe all reflections of rays from both surfaces of a Cassegrain antenna and the
intersections of these rays with the two surfaces.

FORTRAN G level was the language used in writing the algorithm. The computing time was slightly increased due to the fact that more ray tracing is involved in a dual reflector antenna case.
2. ANALYSIS AND FORMULATION

2.1 Theoretical Development

The majority of operations in this algorithm are essentially the same as those in the single reflector algorithm. The GO approach is applied to calculate the reflected electric field using the feed radiation pattern and all parameters defining the surfaces comprising a reflector antenna. The electric field is computed over a planar aperture in front of the reflector surface. As a result, an integration over the aperture plane yields the radiation patterns of the antenna in question.

To understand the line of thought and development of the new algorithm it is necessary to review some aspects of the old program and see where the new additions appear. A more refined and detailed explanation of all equations in the old algorithm is given in references [1] to [5].

Figures 2.1 and 2.2 depict the coordinate systems used in the single and dual reflector algorithms.

The first difference is that the new algorithm has the capability of analyzing both dual reflector antennas and single reflector antennas, i.e., the old algorithm became part of the new one. The two reflector surfaces are described in terms of the reference coordinate system \((x, y, \text{ and } z)\) in which most of the mathematical operations are performed. The second difference between the old and new programs lies in the types of reflector surfaces that can
be analyzed. Previously, five types were available: planes, spheres, ellipsoids, paraboloids, and parabolic cylinders, whereas now hyperboloids can also be treated as another type of surface.

It should be stressed here that these six types of surfaces are available for each reflector for the case of dual reflector antennas.

Spherical coordinates are used for the radiation pattern calculations. The convention used concerning the angles $\theta$ and $\phi$ is shown in Figure 2.3.
The feed position is expressed in terms of the primed coordinates \( x', y', \) and \( z' \). The feed radiation pattern is expressed in spherical coordinates, based on the feed cartesian coordinate system using the same convention for the angles \( \theta' \) and \( \phi' \) as the reference spherical system. Here, \( \theta' \) and \( \phi' \) are referred to the feed coordinate system. The phase center of the feed antenna is the origin of its coordinate system.

The two coordinate systems are related to each other via a three-dimensional rotational matrix \([A]\), whose derivation can be found in [2]. The rotational operation of this matrix is used to make the feed system parallel to the reference system, making use of the three angles ALPHA, BETA, and GAMMA as shown in Figure 2.4. All counterclockwise rotations are defined as positive when looking in the negative direction along the axis of rotation.
Fig. 2.4. Feed rotation angles
ALPHA is the rotation about the $z'$-axis, BETA is the rotation about the $x'$-axis and GAMMA is the rotation about the $v'$-axis.

Each ray starts from the feed and is traced up to the aperture plane. Five pieces of information are associated with each ray: a set of angles $\theta'$ and $\phi'$, the appropriate $\theta'$ and $\phi'$ polarized electric field strengths and the initial phase, all taken from the feed antenna pattern. Figures 2.5 and 2.6 show all vector operations involved in ray tracing.

Fig. 2.5. Vector operations for a single reflector antenna

Fig. 2.6. Vector operations for a dual reflector antenna
The symbols in these figures are defined as follows:

1) $\hat{s}_i$ is a unit vector in the direction of an arbitrary ray incident on the reflector (or on the subreflector).

2) $R$ is the distance from the phase center of the feed to the point at which the incident ray strikes the reflector (or the subreflector).

3) $\hat{n}_o$ is the unit normal vector to the reflector surface (or the subreflector).

4) $\hat{s}_r$ is a vector in the direction of the reflected ray, (or reflected from the subreflector) and incident on the main reflector in the case of a dual reflector antenna.

5) $R_{M}$ is the distance from $(x_0, y_0, z_0)$ on the subreflector to $(x_{02}, y_{02}, z_{02})$ on the main reflector, i.e., the distance from a point on the subreflector to a point at which the reflected ray strikes the main reflector.

6) $\hat{s}_{r2}$ is a vector in the direction of the ray reflected by the main reflector.

7) $D$ is the distance from the point of reflection $(x_0, y_0, z_0)$ to the aperture plane for a single reflector or from the point $(x_{02}, y_{02}, z_{02})$ on the main reflector to the aperture plane for the dual reflector case.
The unit vector \( \hat{s}_i \) which is expressed in spherical feed coordinates is written in its cartesian coordinate system as:

\[
\hat{s}_i = s'_x \hat{x} + s'_y \hat{y} + s'_z \hat{z}
\]

where

\[
s'_x = \sin \theta' \cos \phi' \\
s'_y = \sin \theta' \sin \phi' \\
s'_z = \cos \theta'
\]

\( \theta' \) and \( \phi' \) are also expressed in terms of the feed cartesian coordinates. The feed system is not only rotated but translated with respect to the reference system. That means that a rotation as well as a translation should be performed to express the vector \( \hat{s}_i \) in the reference system. To achieve this task, the origin of the reference system must be known in the feed system.

The intersection of a ray having the unit vector \( \hat{s}_i \), with the reflector or subreflector surface is defined by a vector \( \vec{\Phi} \) as shown in Figure 2.7.
Thus $\hat{V} = r \hat{s}_i - 0^*0$ provided that $\hat{s}_i$ and $0^*0$ are expressed in the reference coordinate system. To accomplish the transformation a 3x2 matrix $[BB]$ is formed. This matrix has the ray unit vector ($\hat{s}_i$) and the translation vector as its columns. The rotational operation takes place by premultiplying $[BB]$ by the rotation matrix $[A]$.

$$[A] [BB] = [B]$$

Each ray is now described in the reference system by the parametric equations

$$x = B_{11}r - B_{12}$$
$$y = B_{21}r - B_{22}$$
$$z = B_{31}r - B_{32}$$

The point of intersection is found by solving simultaneously the equations mentioned above and the equation of the reflector surface. To find a vector ($\hat{s}_r$) in the direction of the reflected ray, the unit normal to the reflector surface, at the incident point is evaluated and Snell's Law is used, i.e.

$$\hat{s}_r = \hat{s}_i - 2(\hat{n}_0 \cdot \hat{s}_i) \hat{n}_0$$

Similarly, the reflected field except for phase, is given by

$$\hat{E}_r - 2(\hat{n}_0 \cdot \hat{E}_i) \hat{n}_0 = \hat{E}_i$$

where $\hat{E}_i$ is the incident field, attenuated, of course, by a factor $\frac{1}{R'}$, since we assume that the reflector is in the far field of the feed antenna.

All vector operations are the same for both the single and dual reflector antenna options.
The two options are now considered separately.

A) **Dual Reflector System**

The parametric equations for a ray along $\hat{s}_r$, which is treated now as the incident ray on the main reflector, are:

\begin{align*}
x &= x_0 + h \cos \alpha x \\
y &= y_0 + h \cos \alpha y \\
z &= z_0 + h \cos \alpha z
\end{align*}

where $h$ is the distance travelled from the point $(x_0, y_0, z_0)$ on the subreflector along the ray, and

\begin{align*}
\cos \alpha x &= \frac{s_{rx}}{s_r} \\
\cos \alpha y &= \frac{s_{ry}}{s_r} \quad \text{direction cosines} \\
\cos \alpha z &= \frac{s_{rz}}{s_r}
\end{align*}

and $s_{rx}$, $s_{ry}$, $s_{rz}$ are the components of the reflected vector $\hat{s}_r$. To find the intersections of the ray and the main reflector, simultaneous solution of the above parametric equations with the equations of the surface of the main reflector is required.

The unit normal to the surface is evaluated at this point and used to compute a vector in the direction of the reflected ray, i.e.,

$$\hat{s}_{r2} = \hat{s}_{i2} - 2 (\hat{n}_{02} \cdot \hat{s}_{i2}) \hat{n}_{02}$$

where $\hat{s}_{i2} = \hat{s}_r$ is a unit vector incident on the main reflector,
and $\hat{n}_{02}$ is the unit normal on the surface of the main reflector in cartesian components.

$$\hat{s}_{r2} = \hat{x} \left[ s_{ix2} - 2n_{x02}(n_{x02} \cdot 2i_{x2} + n_{y02} \cdot 2i_{y2} + n_{z02} \cdot s_{iz2}) \right] + \hat{y} \left[ s_{iy2} - 2n_{y02}(n_{x02} \cdot 2i_{x2} + n_{y02} \cdot 2i_{y2} + n_{z02} \cdot s_{iz2}) \right] + \hat{z} \left[ s_{iz2} - 2n_{z02}(n_{x02} \cdot 2i_{x2} + n_{y02} \cdot 2i_{y2} - n_{z02} \cdot s_{iz2}) \right]$$

where

$$s_{ix2} = s_{rx}$$
$$s_{iy2} = s_{ry}$$
$$s_{iz2} = s_{rz}$$

are the components of the ray vector reflected by the sub-reflector. Now if

$$s_{rx2} = s_{ix2} - 2n_{x02}(n_{x02} \cdot 2i_{x2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$
$$s_{ry2} = s_{iy2} - 2n_{y02}(n_{x02} \cdot s_{ix2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$
$$s_{rz2} = s_{iz2} - 2n_{z02}(n_{x02} \cdot s_{ix2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$

then

$$\hat{s}_{r2} = \hat{x} \ s_{rx2} + \hat{y} \ s_{ry2} + \hat{z} \ s_{rz2}$$

**Fig. 2.8. Electric field vectors**
Similarly, the reflected field (Figure 2.8), assuming a perfectly conducting reflector, is given by:

\[ \mathbf{E}_{r2} = 2 \left( n_{02} \mathbf{E}_{i2} \right) n_{02} \mathbf{E}_{i2} - \mathbf{E}_{i2} \]

where \( \mathbf{E}_{i2} = \frac{\mathbf{E}_r}{RM} \).

\( \mathbf{E}_{i2} \) is the incident electric field on the main reflector and \( \mathbf{E}_r \) is the electric field reflected by the subreflector. It is seen here that \( \mathbf{E}_r \) is multiplied by a factor \( 1/RM \) since the main reflector is assumed to be in the far field of the subreflector.

In component form,

\[ \mathbf{E} = x \frac{\mathbf{E}_{rx}}{RM} + y \frac{\mathbf{E}_{ry}}{RM} + z \frac{\mathbf{E}_{rz}}{RM} \]

\[ = x \mathbf{E}_{ix2} + y \mathbf{E}_{iy2} + z \mathbf{E}_{iz2} \]

and \( \mathbf{E}_{r2} \) becomes

\[ \mathbf{E}_{r2} = x \left[ 2n_{x02}(n_{x02} \mathbf{E}_{ix2} + n_{y02} \mathbf{E}_{iy2} + n_{z02} \mathbf{E}_{iz2}) - \mathbf{E}_{ix2} \right] \]
\[ + y \left[ 2n_{y02}(n_{x02} \mathbf{E}_{ix2} + n_{y02} \mathbf{E}_{iy2} + n_{z02} \mathbf{E}_{iz2}) - \mathbf{E}_{iy2} \right] \]
\[ + z \left[ 2n_{z02}(n_{x02} \mathbf{E}_{ix2} + n_{y02} \mathbf{E}_{iy2} + n_{z02} \mathbf{E}_{iz2}) - \mathbf{E}_{iz2} \right] \]

The procedure of finding the intersection of the reflected ray (by the main reflector) and the aperture plane is as follows:

Find the parametric equation for a line along \( \mathbf{s}_{r2} \) given by:
\[ \begin{align*}
x &= x_{02} + h' \cos \alpha' x \\
y &= y_{02} + h' \cos \alpha' y \\
z &= z_{02} + h' \cos \alpha' z
\end{align*} \]

where

\[ \begin{align*}
\cos \alpha' x &= \frac{x \cdot s_{r2}}{|s_{r2}|} = \frac{s_{rx2}}{|s_{r2}|} \\
\cos \alpha' y &= \frac{y \cdot s_{r2}}{|s_{r2}|} = \frac{s_{ry2}}{|s_{r2}|} \\
\cos \alpha' z &= \frac{z \cdot s_{r2}}{|s_{r2}|} = \frac{s_{rz2}}{|s_{r2}|}
\end{align*} \]

and \( h' \) is the distance travelled along the ray. The aperture plane is at \( x = x_c \), which defines \( h' = \frac{x_c - x_{02}}{\cos \alpha' x} \). The \((y, z)\) coordinates where this ray strikes the aperture plane are:

\[ \begin{align*}
y &= y_{02} + (x_c - x_{02}) \frac{\cos \alpha' y}{\cos \alpha' x} = y_{02} + (x_c - x_{02}) \frac{s_{ry2}}{s_{rx2}} \\
z &= z_{02} + (x_c - x_{02}) \frac{\cos \alpha' z}{\cos \alpha' x} = z_{02} + (x_c - x_{02}) \frac{s_{rz2}}{s_{rx2}}
\end{align*} \]

Then

\[ D = \left( (x_c - x_{02})^2 + (y - y_{02})^2 + (z - z_{02})^2 \right) \]

and the phase of the field upon reaching the aperture plane is given as:

\[ \psi_2 = \frac{2\pi}{\lambda} (R + R'M + D) + \text{Initial Phase.} \]
Thus, five parameters are computed for each ray at a point on the aperture plane: the y and z coordinates, the y and z components of the electric field, and the phase of the field.

B) Single Reflector System

In this case each ray is traced from the feed to the reflector up to the aperture plane in the same way as before. It is clear that in this case a smaller number of equations have to be written and the phase is given by

\[ \psi = \frac{2\pi}{\lambda} (R+d) + \text{Initial phase}. \]

A more detailed discussion of the above operation is provided by Kauffman [2].

2.2 Calculation of Radiation Patterns

In both cases, the tangent aperture field is given by:

\[ \hat{E}_{AP} = (\hat{y} E_{ry} + \hat{z} E_{rz}) e^{-j\psi} \]

for a single reflector

where \( E_{ry}, E_{rz} \) are the tangential components of the aperture electric field, or \( \hat{E}_{AP} = (\hat{y} E_{ry2} + \hat{z} E_{rz2}) e^{-j\psi} \) for a dual reflector.

In order to evaluate the secondary radiation pattern at a particular point in space, we integrate numerically over the aperture. The integrals to be evaluated are:

\[ E_\theta = \int \int_{\text{Aperture Surface}} E_r z \cos \phi e^{-j\psi} e^{jk[y \sin \phi \sin \phi + z \cos \phi]} dy dz \]

and
\[ E_\phi = \iiint_{\text{Aperture Surface}} \left[ E_r y \sin\theta + E_r z \cos\theta \sin\phi \right] e^{-j\psi} \]
\[ e^{jk [y \sin\theta \sin\phi + z \cos\theta]} dy dz \]

where the aperture surface is the area of the reflector aperture projected on the aperture plane. It is necessary to integrate only those points which result from reflections from the actual surface and not from its mathematical extension. This is achieved by interpolating a series of edge points on the boundary, using information from points which exist outside the aperture. All points then existing outside the reflector surface are disregarded.

Before the integration takes place, all points on the aperture plane are quantized in their y-coordinate. All details on quantization and integration are fully provided by Kauffman [2], Agrawal [3], and Botula [5].

2.3 Transition from the Old Algorithm to the New One

The block diagram in Figure 2.9 shows the locations where changes, additions and modifications were applied to the old algorithm to obtain the new one.

These general additions and changes, which will be explained later in more detail, are the following:

1. NPUT: Was enlarged to read in and print out data for both reflectors for a dual reflector antenna system. This feature
1. did not exist before. NPUT also calls an additional subroutine, named SUBPNT.

2. **SUBPNT:**
   Was added to determine the four extreme points on the subreflector, given the four extreme points on the main reflector.

3. **APRTUR:**
   Was extended for the following reasons:
   - A) to incorporate hyperboloidal surfaces, as an addition to the previous list of surfaces.
   - B) To compute, automatically, the location of the aperture plane \( x_c \) in terms of parameters pertinent to the antenna under consideration. This is accomplished by calling the subroutine FINDXC.

4. **FINDXC:**
   FINDXC was added to provide APRTUR with an approximate value of \( x_c \). \( x_c \) is evaluated for both reflector systems, following different approximations depending on whether the antenna is a dual or a single reflector system.

5. **CASSA:**
   A new subroutine was inserted in APRTUR to account for all the tracing from the subreflector to the main reflector, up to
the aperture plane for the case of a dual reflector system.

The rest of the program is unchanged.
Fig. 2.9. Structure of new algorithm
3. STRUCTURE OF REFLECTR

3.1 New Variables

New variables were introduced to account for the increased complexity of the program. Some old variables and common storage blocks were changed to give the new algorithm a general character. Since the new variables come as a follow-up of the old ones, all common storage blocks and variables are introduced here.

1) BLOCKG/YCBL, ZCBL, HFMABL, HFMIBL (Aperture plane blockage information).

YCBL, ZCBL: y and z center coordinates of the aperture plane blockage ellipse.

HFMABL, HFMIBL:
Half-major and half-minor axes of the aperture plane blockage ellipse.

2) CASS/SR(3), XO, YO, ZO, Y, Z, RM, D, XO2, YO2, ZO2, ER2(3), ER(3) (Only for Cassegrain antennas).

XO, YO, ZO, A point where a ray emanating from the feed intersects the subreflector.

XO2, YO2, ZO2 A point of intersection of the main reflector and the ray.

Y, Z The y and z coordinates of each ray on the aperture plane.

RM Length of a ray from the subreflector to the main reflector.
D  Distance of aperture plane from main reflector.

SR(3)  A vector $\hat{s}_r$ in the direction of a ray reflected by the subreflector.

ER2(3)  The three components of the electric field reflected by the main reflector.

ER(3)  The three components of the electric field reflected by the subreflector.

3)  COLOS/DELT, XC, ANGING, PM(3,4), RS, XMX, ZMX, ZMN, YMX
(Parameters used for determining $x_c$.)

DELT  The $0^\circ$ angle subtended by the subreflector. (See Figure 2.2.)

ANGING  Angular increment. (See Botula [5] for more details.

PM(3,4)  Four extreme points on the main reflector.

RS  Distance from an extreme point on the subreflector to the origin.

XMX, YMX, ZMX  A point on the subreflector which is the closest point to the origin.

ZMN  The minimum Z coordinate of the subreflector.

4)  CONTRL/NOPT(3), NLIST, IOPT, ICASS, ILIST (100)

NOPT(3)  Three number specifying options regarding printer, plotter, and aperture plane, data output, respectively. (See [5] Section 6.)
NLIST  The number of panels for which the algorithm will print complete illumination and quantizing data.

IOPT  A variable which is zero when the program is to run normally, and one when the single-panel option is in effect.

ICASS  A variable which is one if a Cassegrain antenna is to be analyzed, and zero for a single reflector antenna.

ILIST(100)  The specific panels for which the algorithm is to provide complete illumination and quantizing data. (See Botula [5], Section 4.)

5) The common blocks: A) DIMENS, B) EXTENT, C) MATH and D) PATTERN, have remained the same as in [5].

6) FEED/EP(91), ET(91), NP, NT, XS, YS, ZS.

(Feed antenna parameters)

EP(91), ET(91)

Array containing the electric field strengths of the feed antenna in one-degree increments off-axis in the $\theta = 90^\circ$ and $\phi = 180^\circ$ planes, respectively.

NP, NT

The number of increments of phi and theta used in the illumination pattern, respectively.

XS, YS, ZS

A point on each panel which is the closest point to the origin of the reference coordinate system.
PARAMS/AORORF, BELLP, CELLP, DIST, PSI, PLNPNT (3), PLNORM (3), FEED (3), ALPHA, BETA, GAMMA, XLAM, AOROR2, BELLP2, CELLP2, PSI2, DIST2, POINT (3), NORM (3), SURFC1, NPNL, NPOINT, SURFC2. (Antenna system parameters.)

In the following, the variables that appear first are defined on the subreflector, and those that appear second are defined on the main reflector.

AORORF, AOROR2: The focal length of a paraboloidal reflector, the focal length of a parabolic cylindrical reflector, the radius of a spherical reflector, the semi-major axis of an ellipsoidal reflector along X, or half the transverse axis (x-direction) of a hyperboloidal reflector (Appendix B), depending on which surface is intended to represent the reflector.

BELLP, BELLP2: The semi-minor axes (along y and z, respectively) of an ellipsoidal reflector surface. Note that this does not define a completely arbitrary ellipsoid since the axes along y and z must be equal. For the case of a hyperboloidal reflector surface, this value represents the y semi-axis of the ellipse in the yz plane of the hyperboloid.

CELLP, CELLP2: Used only for a hyperboloid and stands for the z semi-axis of the ellipse in the yz plane of the hyperboloid.
DIST: A parameter used in translating the origin of the hyperbolic subreflector coordinate system so that it coincides with that of the main reflector. (See Appendix B.)

PLNPNT(3), POINT(3): The coordinates of a point on a planar reflector surface \((x, y, z)\).

PLNORM(3), NORM(3): The components of a unit normal vector to a planar reflector surface \((x, y, z)\).

FEED(3): The reference coordinate system origin as expressed in the feed coordinate system \((x, y, z)\).

XLAM: Wavelength of the feed antenna radiation.

ALPHA, BETA, GAMMA: Rotation angles mentioned before (Figure 2.4.)

SURFC1, SURFC2: Integer variables which determine the type of reflector surface. (This code is applied to the subreflector as well as the main reflector.)

1) Surface is a plane.
2) Surface is an ellipsoid.
3) Surface is a sphere.
4) Surface is a paraboloid.
5) Surface is a parabolic cylinder.
6) Surface is a hyperboloid.

NPNL: Determines the number of panels the reflector is made of. The value of one means that a list of perimeter points and other surface parameters for each panel must be supplied. In this case, the aperture boundary is approxi-
mated by a polygon. The value of zero means that the single-panel option is in effect and hence an ellipse is used to represent the boundary of that panel.

NPOINT: The number of rays stored for processing in the P array at any given time.

3.2 NPUT

This is an input/output routine. If ICASS = 0, the program is to analyze a single reflector antenna system with two options:

1) With IOPT = 1 for a single-panel option.

2) With IOPT = 0 for a multipanel option.

In both cases, the four extreme points of the reflector surface are required. If ICASS = 1 a dual reflector antenna is to be analyzed. For this case, the four extreme points of the main reflector are read in and used to find the four extreme points of the subreflector by calling subroutine SUBPNT. (SUBPNT explained later in this Section.)

NPUT also reads other parameters concerning the feed. This is important since all pieces of information read here are used in conjunction with the FILL routine which is called later in the program. The connecting agent in this operation is the common storage block, named FEED.

Previously, the four extreme points on the reflector were read into the P array only when NPNL was zero, and the variable $x_c$ was also provided by the user. In this algorithm the four extreme points are read regardless of the particular
value of NPNL. The reason for this is that the above points are needed to compute the variable \( x_c \) later in the program. Furthermore, new printing statements were added to be used for dual reflector antennas.

### 3.3 SUBPNT

Fig. 3.1. Finding the four extreme points

SUBPNT is called only for a dual reflector antenna. There is a "Do" loop which computes the distance (RR) from the extreme point on the main reflector to the reference point.

\[
RR = \left[ (PM(1,K))^2 + (PM(2,K))^2 + (PM(3,K))^2 \right]^{\frac{1}{2}}
\]

where PM(1,K), PM(2,K), and PM(3,K) are the coordinates of each extreme point on the main reflector. Then, the direction cosines are found as:

\[
\text{DIR1} = \frac{PM(1,K)}{RR} \text{ (direction cosine in the x-direction)}
\]
The parametric equations of a line passing through the origin (reference point), and a point on the main reflector are given by:

\[
P(1,K) = PM(1,K) - RR \cdot DIR1 \\
P(2,K) = PM(2,K) - RR \cdot DIR2 \\
P(3,K) = PM(3,K) - RR \cdot DIR3
\]

where \( P(1,K), P(2,K), P(3,K) \) is an extreme point on the reflector. To determine this point, the above parametric equations and the equation of the surface of the subreflector are solved simultaneously. (See Appendix C for details.)

This operation is repeated four times, i.e., once for each extreme point of the subreflector.

3.4 APRTUR, APRIN, AND FILL

APRTUR does all the ray tracing for the single reflector antenna and it calls a new subroutine named CASSA for additional tracing in the dual reflector case. Figure 11 shows the difference in approach between the old and new algorithms in determining the location of the aperture plane before integration for a multipanel, single reflector antenna.

This difference gives some increased accuracy in predicting the radiation pattern of a multipanel, single reflector antenna. (See results, Section 5.) In the case of a
single reflector, a short "Do" loop is used to find XMX, YMX, and ZMX, a point of the reflector which is the closest one to the origin.

Then a rotation matrix $A$ is computed from the rotation angles ALPHA, BETA, and GAMMA. The inverse of that matrix is also found. If the dual reflector option is in effect, the rotation matrix is calculated immediately skipping the above-mentioned "Do" loop. For single reflector antennas comprised of a number of panels, subroutine APRIN is called to provide data for each panel individually.

Two important additions have been made in APRIN: 1) For each plane reflector a normal is computed automatically using the principle of the CROSS product. (See Appendix D.) 2) Statements 20-28 make use of a "Do" loop to search for
\((x_s, y_s, z_s)\), a point on each panel, which is also the closest point to the origin. It is an important point because it is used later, in APTRUR, to find the location of the aperture plane \((x_{ci})\) for each panel individually. (See Figure 3.2 for geometry). For a complete discussion of APRIN, see [5].

From statements 50 to 65, APRTUR finds the angles subtended by the reflector or the reflector panel. Notice that in the dual reflector case, the angles subtended by the sub-reflector are the ones to be measured and not those for the main reflector. All points, either the perimeter points for a panel, or the four extreme points for a single panel option, are expressed as angles in the feed system. Then, a search for the maximum and minimum \(\theta^\prime\) and \(\phi^\prime\) angles represented by the above-mentioned set of points is performed to determine the angles subtended by a panel or a subreflector. (See Appendix B in [5].)

ILLUMINATION ARRAY - Statements 65-95 generate the appropriate illumination array to insure a well-ordered illumination of the chosen reflector option. The previous method of illumination has been kept the same since it serves the purpose of the new algorithm in a rather convenient way. (See Section 2.3 in [5].)

For the dual reflector case, the angles subtended by the subreflector are the ones to be considered instead of those of the main reflector. The reason for this is the fact that an overillumination of the subreflector results in an
overillumination of the main reflector. Overillumination is
desired so that the projected boundary of the main reflector
on the aperture plane can be defined before integration is
performed. The rays corresponding to the upper and lower
limits of $\theta$' miss the real subreflector. They get reflected
by its mathematical extension, and as a result, they miss
the main reflector too.

If a Cassegrain antenna is to be studied, as soon as
ANGINC is computed in APRTUR, subroutine FINDXC is called.
(See Section 3.5.) This is the first time where FINDXC ap-
ppears in the program to provide APRTUR with the location of
the aperture plane ($x_c$). APRTUR, with a "Do" loop in state-
ment 95, loads all illumination angles into the P array just
after the angle pairs corresponding to the perimeter points.
SUBROUTINE FILL is called to provide the angle pairs in the
P array with the field strength and phase values.

FILL - This routine is changed and adjusted to each
antenna whose radiation pattern is to be computed. A detail-
ed description of this subroutine and its various forms ap-
pear in [2], [3] and [5]. A new subroutine has been written
for a vertical polarization case. (See Appendix E.)

Furthermore, in APRTUR for single reflector antennas as
the ($x_0', y_0, z_0$) point is found, the location of a separate
plane are determined. This part of the algorithm is not
carried out for dual reflectors. The procedure for determin-
ing $xx$ and $x_{ci}$ is as follows:
If the single panel option is in effect, then subroutine FINDXC is called. This is the second location in the program where FINDXC appears. (See Section 3.5.) If a multipanel option is in effect, then $R_1$ (Figure 12) is expressed as:

$$R_1 = \left[ (x_s + B12)^2 + (y_s + B22)^2 + (z_s + B32)^2 \right]^{\frac{1}{2}} - 1.0 = R' - 1.0$$

where $R' = \left[ (x_s + B12)^2 + (y_s + B22)^2 + (z_s + B32)^2 \right]^{\frac{1}{2}}$ is the distance between $(x_s, y_s, z_s)$ and $(B12, B22, B32)$.

Fig. 3.3. Computation of $x_c$'s and $xx$

Also, the angle $\theta_{\text{max}}$ subtended by the reflector is expressed as:

$$\theta_{\text{max}} = \tan^{-1} \left( \frac{x_s + B32}{z_s + B12} \right), \text{ where } (x_s + B12) \text{ is a negative value and } (z_s + B12) \text{ a positive one. Hence, to obtain a positive } \theta_{\text{max}} \text{ angle, a negative sign is added. The}$$
angle $\theta_{\text{max}}$ is augmented by 2.5 ANGING, i.e., 2.5 times an angular increment. The reason that $R_1 = R' - 1.0$ and $\theta_{\text{aug}} = \theta_{\text{max}} + 2.5$ ANGING are used instead of $R'$ and $\theta_{\text{max}}$ is to make sure that the panel will be overilluminated. Thus $x_c$ is found as:

$$x_c = -(R_1 \cos(\theta_{\text{aug}}) + B(1,2)).$$

The distance between $x_c$ and $x_s$ for the first panel is computed as $\text{CONST} = |x_c - x_s|$. This number becomes an important factor in locating the aperture plane for the rest of the panels. The idea is to put an aperture plane in front of every panel and with a distance equal to $\text{CONST}$ away from it. This results in having an ordered arrangement of aperture planes in front of the reflector. So, the rest of the $x_c$'s are given as:

$$x_c = x_s + \text{CONST}$$

where $x_s$ is provided by APRIN, in advance. Once all $x_c$'s have been found, the location of a general plane ($xx$) is determined, using FINDXC. (See Section 3.5.) Each panel is first projected onto its own individual aperture plane, and then phase-referenced to the general aperture plane. Thus, the general plane sums up all these projections that comprise the total projection of the antenna on the aperture plane. This method of preparation of the aperture plane before integration yields better results compared with the previous method.

The difference in phase is written as:
DIFF = \left| x_c - x_s \right| and the PHASE = \frac{2\pi}{\lambda} (R+D+DIF) + Initial Phase

where R = distance from the feed to reflector.

D = a distance from the reflector to the individual aperture plane.

DIF = distance from the individual aperture plane to the general one.

If the dual reflector antenna option is in effect, subroutine CASSA is called by APRTUR to continue the ray tracing operation over the region lying between the subreflector and the main reflector. (See Section 3.6.)

3.5 FINDXC

This subroutine is called, as mentioned before, at two different locations in APRTUR.

\[ PM(1,M), PM(2,M), PM(3,M) \]

Fig. 3.4. Location of an aperture plane at \( x_c \) for a dual reflector
In the dual reflector antenna case, FINDXC is called immediately after ANGING is computed. In this case, $x_c$ is evaluated directly from the geometry of the two reflectors. From Figure 3.4, a point with the largest $z$ coordinate on the main reflector is determined and its distance ($R'$) from the reference system is computed. Then, new parameter RSM is computed as:

$$RSM = \left( (PM(1,M))^2 + (PM(2,M))^2 + (PM(3,M))^2 \right)^{\frac{1}{2}} \cdot 1.0 = R' - 1.0$$

where

$$R' = \left( (PM(1,M))^2 + (PM(2,M))^2 + (PM(3,M))^2 \right)^{\frac{1}{2}}$$

Also, $\theta_{\text{max}}$ the angle subtended by the main reflector is expressed:

$$\theta_{\text{max}} = \tan^{-1}\left( \frac{-PM(3,M)}{PM(2,M)} \right)$$

where the negative sign is provided here to obtain a positive $\theta_{\text{max}}$ angle, since $PM(3,M)$ is positive and $PM(1,M)$ is negative. In the reference system another angle, called $\theta_{\text{augmented}}$ is estimated as:

$$\theta_{\text{aug}} = \theta_{\text{max}} + 3.0 \cdot \text{ANGING (in radians)}$$

and $x_c$ is then calculated using the expression.

$$x_c = -RSM \cos (\theta_{\text{aug}})$$

The fact that RSM is used instead of $R'$ and $\theta_{\text{aug}}$ instead of $\theta_{\text{max}}$ is to insure overillumination and to make sure that this subroutine works for all sub and main reflector combinations, no matter what their geometrical relationships are. This subroutine could, if necessary, be changed to deal with each sub and main reflector combinations separately.
The second call of FINDXC by APRTUR is concerned with finding the location \((x, x)\) of the general aperture plane for the multipanel option reflector, or \(x_c\) for the single panel option. This task is accomplished as follows: (See Figure 3.5.)

\[
(x_{mx}, y_{mx}, z_{mx})
\]

First, find the distance \(R'\) between the point \((x_{mx}, y_{mx}, z_{mx})\), and the feed, i.e.,

\[
R' = \left[ (x_{mx} + B12)^2 + (y_{mx} + B22)^2 + (z_{mx} + B32)^2 \right]^{\frac{1}{2}}
\]

where \((x_{mx}, y_{mx}, z_{mx})\) is the point on the reflector which is the closest to the origin of the reference system. It should be noted that this point is computed at the beginning of the APRTUR routine. Second,
\[
\theta_{\text{max}} = \tan^{-1} \left( -\frac{(z_{mx} + B_{32})}{x_{mx} + B_{12}} \right)
\]
gives the maximum angle subtended by the reflector. This angle is increased by a 2.5 ANGING to give \( \theta_{\text{aug}} = \theta_{\text{max}} + 2.5 \text{ ANGING} \) (in radians) and third, to find \( x_c \), \( R' \) is reduced by 1.5 to yield
\[
R_{\text{SM}} = \left[ (x_{mx} + B_{12})^2 + (y_{mx} + B_{22})^2 + (z_{mx} + B_{32})^2 \right]^{1/2} - 1.5
\]
and hence
\[
x_c \text{ or } xx = -R_{\text{SM}} \cos(\theta_{\text{aug}}) + B(1,2) \text{ for a single panel or a multipanel antenna, respectively.}
\]

It is noted here that the distance \( R' \) is reduced by 1.5 instead of 1.0 (as was done in the case of individual panels) to insure that \( xx \) will be less than \( x_c \), in the multipanel case. The whole arrangement of separate aperture places and a general one is shown in Figure 11, Part B.

It can be seen that \( xx \) has to be behind all individual aperture planes. If the multipanel option is not in effect, \( xx \) becomes \( x_c \).

3.6 CASSA

This subroutine accomplishes all the ray tracing from the subreflector to the main reflector up to the aperture plane. It starts with finding the direction cosines of a vector along the ray reflected by the subreflector. Parametric equations of a line are expressed as:

\[
x_{02} = x_0 + R.M\cdot DC(1)
\]
\[
y_{02} = y_0 + R.M\cdot DC(2)
\]
\[
z_{02} = z_0 + R.M\cdot DC(3)
\]
where \((x_{02}, y_{02}, z_{02})\) is a point on the main reflector,
\((x_0, y_0, z_0)\) is a point on subreflector, RM distance between these two points and DC(1) DC(2) DC(3) are the direction cosines with respect to x, y and z axes, respectively. The solution of simultaneous equations consisting of the above parametric equations and the equation of the reflector surface yield the point \(x_{02}, y_{02}, z_{02}\). Although this subroutine has been written to deal with six analytical surfaces, it could be extended to incorporate any other number of types of surfaces, if desired. Surfaces expressed numerically could also be added to this algorithm, especially for the dual reflector antenna option, where shaping of one or both of the reflectors is now widely used in their actual design.

Once the point \(x_{02}, y_{02}, z_{02}\) is evaluated, the normal \((\hat{n}_{02}(1), \hat{n}_{02}(2), \hat{n}_{02}(3))\) on the surface at that point is computed as follows:

Let the surface be represented as \(g(x, y, z) = C\).

Then \[
\hat{n}_{02} = \frac{\nabla g(x_{02}, y_{02}, z_{02})}{|\nabla g|}
\]

A detailed explanation of computing normals and intersections of rays with surfaces is not given in this thesis, since a complete discussion can be found in all references from \([1]\) to \([5]\), in their description of subroutine APRTUR. The only difference lies in the fact that the parameters used
in CASSA are pertinent to the surface of the main reflector and not the subreflector.

The normal on the main reflector is used to apply Snell's law of reflection to find a vector in the direction of the reflected ray (SR2(1), SR2(2), SR2(3)). This part of the algorithm is described in Section 2.1. A point, \((y, z)\) on the aperture plane is then computed, and passed over to APRTUR where it is stored, to be retrieved later by QUANTZ.

The principles of geometrical optics are used to determine the electric field during these two phases of ray tracing. All equations in this part of the algorithm are mentioned in Section 2.1. In general, all operations taking place in CASSA are depicted in Figures 2.6 and 2.8.

3.7 Main Procedure and the Utility Routines

The main procedure and all the rest of the utility subroutines were kept the same as before with a minor change in their storage blocks. A complete development of these subroutines and the main procedure is provided by Botula in [5].
4. EXAMPLES AND TEST CASES

4.1 Introduction

Two test cases on the Cassegrain antennas are provided here to demonstrate the use of the program and support the validity of the algorithm. These cases are the following:

FIRST, a classical Cassegrain antenna which was used to check the algorithm in the case of uniform illumination, but with no blockage.

SECOND, a dual offset reflector antenna, used to check the results obtained by this algorithm against calculated data obtained from two other algorithms.

4.2 Example and First Test Case

The classical Cassegrain antenna, shown in Figure 4.1 employs a hyperboloid for the subreflector and a paraboloid for the main reflector. One of the two foci of the hyperboloid is the real focal point of the system, and is located at the origin of the feed coordinate system; the other is a virtual focal point which is located at the focus of the paraboloid which coincides with the origin of the reference system. As a result, all rays originating from the real focus and reflected from both surfaces travel equal distances to a plane in front of the antenna. (See Figure 4.1.)
Fig. 4.1. Classical Cassegrain antenna system

Table 4.1 gives a number of parameters that define completely the geometry of the antenna system. All parameters required by NPUT will now be evaluated from this Table.

**TABLE 4.1.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main reflector focal length</td>
<td>(F_m) 100.0 in.</td>
</tr>
<tr>
<td>Main reflector illumination angle</td>
<td>(θ_2) 60.785</td>
</tr>
<tr>
<td>Eccentricity of subreflector</td>
<td>(e) 1.50177</td>
</tr>
<tr>
<td>Distance between two foci</td>
<td>(F_c) 91.0 in.</td>
</tr>
<tr>
<td>Wavelength</td>
<td>(λ) 4.734 in.</td>
</tr>
</tbody>
</table>
FEED PARAMETERS

Since the origin of the feed coordinate system is located at the real focus of the hyperboloid and at distance $x = -91.0$ from the origin of the reference system, the feed parameters can be given as:

1) Feed (1) = -91.0 in., Feed (2) = 0.0, Feed (3) = 0.0
2) ALPHA = 0.0, BETA = 0.0, GAMMA = -180.0

MAIN REFLECTOR PARAMETERS

SURFC2 is set equal to 4, since a paraboloidal reflector is to be used as a main reflector.

$F_m = 100.0$, as was given in the Table.

The four extreme points of that reflector to be read in are:

Upper point

$$r = \frac{2F_m}{1 + \cos \theta_{\text{max}}} = \frac{2(100.0)}{1 + \cos(60.785^\circ)} = 134.4$$

$$x = r \cos \theta_{\text{max}} = -r \cos(60.785^\circ) = -65.599$$

$$y = 0.0$$

$$z = r \sin \theta_{\text{max}} = r \sin(60.785^\circ) = 117.304$$

Lower point

$$r = \frac{2F_m}{1 + \cos \theta_{\text{min}}} = \frac{2 \cdot 100.0}{1 + \cos(-60.785^\circ)} = 134.4$$

$$x = -r \cos \theta_{\text{min}} = -65.599$$

$$y = 0.0$$

$$z = r \sin \theta_{\text{min}} = r \sin(-60.785^\circ) = -117.304$$
These two points correspond to the 0° extrema in the feed system. Also, the two points representing the y - extrema are almost exactly the φ° extrema as well. The z coordinates of these points are identical.

\[ z = \frac{z_{\min} + z_{\max}}{2} = \frac{117.304 - 117.304}{2} = 0.0 \]

The reflector, as seen from the geometry of the antenna system, is 234.608 inches wide and symmetric with respect to the xz plane, hence \( y = \pm \frac{234.608}{2} = \pm 117.304 \) in.

Finally, the paraboloid equation provides the x coordinates

\[ x = \frac{y^2 + z^2}{4F_m} - F_m = 65.599 \]

Thus, the four aperture points become:

Upper point: \((-65.599,0.0,117.304) = \text{PM}(1,1), \text{PM}(2,1), \text{PM}(3,1))\)

Lower point: \((-65.699,0.0,-117.304) = \text{PM}(1,2), \text{PM}(2,2), \text{PM}(3,2))\)

Leftmost point: \((-65.599,-117.304,0.0) = \text{PM}(1,3), \text{PM}(2,3), \text{PM}(3,3))\)

Rightmost point: \((-65.599,117.304,0.0) = \text{PM}(1,4), \text{PM}(2,4), \text{PM}(3,4))\)

It should be noted here that the diameter of the main reflector can also be found from the relationship given in Appendix A as follows:

\[ \tan \frac{\theta}{2} = \frac{1}{4} \frac{D_m}{F_m} + D_m = 4F_m \tan \frac{60.785^\circ}{2} = 234.608 \]
**SUBREFLECTOR PARAMETERS**

SURFC1 is set equal to 6, since a hyperboloidal surface is to be used for a subreflector. NPNL takes the value of zero, since neither the subreflector nor the main reflector is composed of panels.

The parameters $a, b,$ and $c$ (semi-transverse axis along $x = AORORF),\ b (semi-axis along the y direction = BELLP),$ and $c (semi-axis along z direction = CELLP)$ are computed as follows: (See Appendix B for details.)

$$a = \frac{F_c}{2} = \frac{91.0}{1.50177} = 30.2976$$

$$c = b = a \sqrt{\epsilon^2 - 1} = 30.2976 \sqrt{(1.50177)^2 - 1} = 33.95$$

Also, $DIST = \frac{F_c}{2} = \frac{91.00}{2} = 45.0$ which is a parameter used in translating the origin of the subreflector coordinate system so that it coincides with that of the main reflector. (See Appendix B for details.)

There is no need to read in $x_c$, since this value is computed in the program as a function of the antenna system parameters. In this case, the FILL routine was not used, and no data for the E and H plane patterns of the feed were used in the input file.
### 4.3 General Input File

For format information, refer to the program listing, Appendix F.

#### A) Dual Reflector Cases

<table>
<thead>
<tr>
<th>Cards</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>Title Cards</td>
</tr>
<tr>
<td>5</td>
<td>Feed (1-3), ALPHA, BETA, GAMMA, XLAM</td>
</tr>
<tr>
<td>6</td>
<td>SURFC2, AOROR2, BELLP2, CELLP2, DIST2, PSI2</td>
</tr>
<tr>
<td>7</td>
<td>POINT(1-3), NORM(1-3)</td>
</tr>
<tr>
<td>8</td>
<td>SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI</td>
</tr>
<tr>
<td>9</td>
<td>PLNPNT(1-3), PLNORM(1-3)</td>
</tr>
<tr>
<td>10,11,12,13</td>
<td>Four extreme points $(x_{02}, y_{02}, z_{02})$, on the edge of the main reflector. One point goes on each card.</td>
</tr>
<tr>
<td>14</td>
<td>YCBL, ZCBL, HFMABL, HMIBL (Blockage of main reflector by subreflector)</td>
</tr>
<tr>
<td>15-N</td>
<td>Any data required by the FILL routine</td>
</tr>
<tr>
<td>N+1</td>
<td>NOPT, NLIST</td>
</tr>
<tr>
<td>N+2</td>
<td>MAJOR, AMAJOR, MINOR, AMINOR(1-3) (Pattern request cards)</td>
</tr>
<tr>
<td>N+3</td>
<td>DONE typed in the first four columns of the card</td>
</tr>
</tbody>
</table>
B) Single Reflector Cases

<table>
<thead>
<tr>
<th>Cards</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>Title Cards</td>
</tr>
<tr>
<td>5</td>
<td>Feed (1-3), ALPHA, BETA, GAMMA, XLAM</td>
</tr>
<tr>
<td>6</td>
<td>SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI</td>
</tr>
<tr>
<td>7</td>
<td>PLNPNT(1-3), PLNORM(1-3)</td>
</tr>
<tr>
<td>8,9,10,11</td>
<td>Four extreme points (x, y, z) on the reflector. One point goes on each card (single panel option only)</td>
</tr>
<tr>
<td>12</td>
<td>YCBL, ZCBL, HFMABL, HMIBL (Blockage of reflector feed)</td>
</tr>
<tr>
<td>12-N</td>
<td>Any data required by FILL ROUTINE</td>
</tr>
<tr>
<td>N+1</td>
<td>NOPT, NLIST (if NOPT specifies that only certain panels are to be printed or plotted, cards containing the list of these panels follow this card)</td>
</tr>
<tr>
<td>N+2</td>
<td>MAJOR, AMAJOR, MINOR, AMINOR(1-3)</td>
</tr>
<tr>
<td>N+3</td>
<td>DONE is typed in the first four columns of the card</td>
</tr>
</tbody>
</table>

These cards are followed by the panel data. The organization of the panel data is as follows:
4.4 Development of a Uniformly Illuminated, Classical Cassegrain Antenna

All parameters needed for this case were computed in Section 4.2. None of the available FILL subroutines was used and the H and E plane patterns (for the feed) were not read in as data in this particular case. The reason for that was to insure uniform illumination over the main reflector. The procedure adopted to achieve this task was as follows:

1. FILL is not called in APERTUR.
2. All lines in APERTUR related to the amplitude and phase of the E field were moved to subroutine CASSA.
3. In subroutine CASSA the following modifications took place:
   \[ P1 = R \cdot RM \text{ and } P2 = 0.0 \]
\[ E_{T_1} = \frac{P_1}{R} \quad \text{(i.e., equal to } RM) \quad \text{and} \quad E_{p_1} = \frac{P_2}{R} = 0.0 \]

where \( E_{T_1} \) and \( E_{p_1} \) are the \( \theta \) and \( \phi \) electric field components of the incident (on the subreflector) ray, respectively.

From \( E_{T_1} \), and applying Snell's law to rays reflected by the two surfaces, \( E_r \) and \( E_{i_2} \) were evaluated, where \( E_r \) is the electric field vector along a ray reflected by the subreflector, and \( E_{i_2} \) is the electric field vector along a ray incident on the main reflector.

It is obvious that in the far field, \( E_{i_2} = \frac{E_r}{RM} \)

Also, \( E_r/RM = \frac{E_{T_1}/R}{RM} = \frac{P_1/R}{RM} = \frac{P_1}{R \cdot RM} = \frac{R \cdot RM}{R \cdot RM} = 1.0 \)

which means that the \( E \) field was kept constant at the value of one along every ray. Thus, the constant amplitude requirement for uniform illumination was met.

4. The constant phase requirement was also satisfied by the above arrangement, since

the phase was set equal to:

\[
\text{PHASE} = \frac{2\pi}{\lambda} (R + RM + D).
\]

Notice that \( R + RM + D \) is always constant for a focused Cassegrain antenna. (See Appendix A.)

Table 4.2 shows the input file for Case A. The first four cards contain title information which is also reproduced at the printout. Information about the feed coordinate system (FEED, ALPHA, BETA, GAMMA, and XLAM) appear on Card 5.
Cards 6 and 7 contain information about the surface of the main reflector. Card 6 is for SURFC2, AOROR2, BELLP2, CELLP2, DIST2, PSI2 and Card 7 is for POINT, NORM. For this main reflector, SURFC2 = 4 and AOROR2 = 100.0. None of the other parameters is required for this surface, so all are given the value of zero. Cards 8 and 9 contain information for the subreflector surface. Card 8 is for SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI and Card 9 for PLNPNT and PLNORM. For that type of subreflector surface SURFC1 = 6, NPNL = 0.0, AORORF = 30.2976, BELLP = 33.95, CELLP = 33.95, and DIST = 45.500. The rest of the other parameters are given the value zero, since none of them is required for this surface. Cards 10, 11, 12 and 13 contain the four extreme points (x02, Y02, Z02) of the main reflector. Card 14 carries the required blockage information, i.e., YCBL, ZCBL, HFMABL, and HFMIBL. In this case, aperture blockage is not considered and so all the above parameters are set equal to zero. Since the FILL routine is not used in this case and no data for the feed radiation patterns are needed, Card 15 is used to determine the output option code. Here the computer is instructed to print and plot information about the two surfaces, as follows:

\[
\begin{align*}
\text{NOPT}(1) & = 2 \quad \text{(print all results)} \\
\text{NOPT}(2) & = 2 \quad \text{(plot aperture after quantizing)} \\
\text{NOPT}(3) & = 1 \quad \text{(print aperture array onto a disc file at the end of QUANTZ).}
\end{align*}
\]
NLIST is equal to zero since the antenna in question is not divided into panels. Cards 16 and 17 are the radiation pattern requests. One pattern is required in $\phi = 0^\circ$ plane for $\theta$ from 85.0$^\circ$ to 95.0$^\circ$ by increments of 0.5$^\circ$, and another one in the $\theta = 90.0^\circ$ plane for $\phi = -4.0^\circ$ to 4.0$^\circ$ by 0.5$^\circ$. The next and last card (No. 18) has DONE typed in the first four columns, which signifies the end of the pattern requests and the end of the input file. The result of this check case are shown in Appendix G.

Figure 4.2 shows a comparison of the results obtained by this algorithm with those results reported by Silver for a uniformly illuminated circular aperture [6].
UNIFORMLY ILLUMINATED
CLASSICAL CASSEGRAIN ANTENNA
(CIRCULAR APERTURE)

E-PLANE

- CALLED...UTED
- SILVER'S DATA[6]

0

-10

-20

-30

0 1 2 3 4

THETA (DEG)

Fig. 4.2. Classical Cassegrain antenna radiation pattern
(Due to the symmetry, only one-half of
the pattern is shown)
<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CASSEGRAIN ANTENNA EXAMPLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A PARABOLOID-HYPERBOLOID COMBINATION</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>FEBRUARY 13, 1981, NCSU</td>
<td>PGMR:CHRISTOS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>FCLTY:RD-DAN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PRT:HILLSBORO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(A BLANK CARD)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-91.05</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-180.0</td>
<td>4.734</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>030.2976</td>
<td>33.95</td>
<td>33.95</td>
<td>45.50</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>-65.5997</td>
<td>-117.304</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-65.5997</td>
<td>117.304</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-65.5997</td>
<td>0.0</td>
<td>-117.304</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-65.5997</td>
<td>0.0</td>
<td>117.304</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>221</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>PHI</td>
<td>0.0</td>
<td>THETA</td>
<td>85.0</td>
<td>95.0</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>THETA</td>
<td>90.0</td>
<td>PHI</td>
<td>-4.0</td>
<td>4.0</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>DONE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.5 Second Test Case

Dual Offset Reflector Antenna

Here, the algorithm is tested with calculated data reported by TICRA A/S [8], and C. C. Chen [9]. The reason for choosing an offset case as a second test case is the fact that offset geometry does not have the symmetry of the first test case, which can sometimes mask errors.

Fig. 4.3. Dual offset antenna geometry
TABLE 4.3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>2.47</td>
</tr>
<tr>
<td>$F_c$</td>
<td>33.07 in</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.98425 in</td>
</tr>
<tr>
<td>$F_m$</td>
<td>69.685 in</td>
</tr>
<tr>
<td>Offset angle ($\theta^0$)</td>
<td>37.6°</td>
</tr>
<tr>
<td>Aperture diameter ($D_m$)</td>
<td>64.8 $\lambda$</td>
</tr>
<tr>
<td>Tilted angle of feed axis ($\alpha_1$)</td>
<td>16.4°</td>
</tr>
<tr>
<td></td>
<td>-11 dB taper was used.</td>
</tr>
</tbody>
</table>
Using the relationships between the hyperboloid and paraboloid from Appendix B, and using the given data in Table 4.3, one can estimate AORORE, BELLP, CELLP and DIST. Furthermore, in this case, ALPHA = 0.0, BETA = 0.0 and GAMMA = -163.6, since the axis of the feed makes an angle (α₁) of 14.6° with the x axis of the reference system, as shown in Figure 4.3. Feed (1), Feed (2), and Feed (3), as well as the four extreme points of the main reflector are easily calculated. The input file is shown in Table 4.4. In this case, the input file is arranged in the same way as before up to the fourteenth card. Cards 15 to 52 contain information about the feed radiation pattern. Card 53 contains NOPT, NLST, and Cards 54 and 55 are used for the pattern requests. Finally, DONE is typed on Card 56. The secondary radiation pattern is shown in Figure 4.4, and compared with data obtained from the other two algorithms.
<table>
<thead>
<tr>
<th></th>
<th>1 OFFSET CASSEGRAIN ANTENNA EXAMPLE</th>
<th>2 A PARABOLOID - HYPERBOLOID COMBINATION</th>
<th>3 FEBRUARY 19, 1981 NCSU PGMR-CHRISTOS FCLTY:RD-DAN</th>
<th>4 TICRA AP/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-31.725344 0.0 9.337276 0.0 0.0 0.0 0.0 0.0  -163.6  .98425</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4 69.685 0.0 0.0 0.0 0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>6 6.694507 15.119643 15.119643 16.535433 0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0 0.0 0.0 0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-57.18974 -31.88699 49.66035</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-57.18974 31.88699 49.66035</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-45.82776 0.0 81.54734</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-68.55171 0.0 17.77336</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.000000 0.99053 0.96266 0.91793 0.85878</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.78830 0.70997 0.62737 0.54392 0.46269</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.38617 0.31623 0.25407 0.20029 0.15491</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.11756 0.08755 0.06394 0.04563 0.03223</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19-32</td>
<td>0.00000 0.00000 0.00000 0.00000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.00000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>1.0000 0.99053 0.96266 0.91793 0.85878</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>35</td>
<td>0.78830 0.70997 0.62737 0.54392 0.46269</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.38617 0.31623 0.25407 0.20029 0.15491</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.11756 0.08755 0.06394 0.04563 0.03223</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.4 (continued)

<p>| | | | | | |</p>
<table>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>38-51</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>52</td>
<td>0.00000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>221</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>PHI 0.0</td>
<td>THETA 87.0</td>
<td>93.0</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>THETA 90.0</td>
<td>PHI -3.0</td>
<td>3.0</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>DONE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4.4. Offset Cassegrain E-plane pattern
5. A SINGLE REFLECTOR ANTENNA EXAMPLE
   (A SEGMENTED SPHERICAL REFLECTOR)

5.1 Description of the Problem

A single reflector composed of 54 panels was constructed and tested by NASA at the Langley Research Center. Its measured radiation patterns were compared twice: First, with calculated results obtained by using the old version [5]; and second, with calculated results obtained via the modified version incorporated in the new algorithm. A complete description of the antenna and its parameters is provided by Botula in [5]. Here, the input file and the results only are given.

5.2 Results and Comments

Figures 5.1 and 5.2 depict the projections on all panels on the aperture plane. The result obtained by the old version is shown in Figure 5.1, whereas the result from the revised algorithm is shown in Figure 5.2.

Figures 5.3-5.6, inclusively, show the secondary radiation pattern for both versions. The reason for this discrepancy in the above results lies in the amount of overlapping between the projected panels on the aperture plane. The more the overlapping, the less accurate results are obtained compared to measured data.

The reason for this overlapping is due to the fact that the rays reflected by the perimeter points of each panel tend to diverge on their way to the aperture plane. To reduce their divergence, the aperture plane is brought closer to
each panel so that the rays travel over shorter distances before they strike the aperture plane. Once this occurs, the projected panel is then phase referenced to the general aperture plane.

This procedure, which is summarized in Figure 3.2, yields less overlapping and better results than the old version.

5.3 Input File

<table>
<thead>
<tr>
<th>TABLE 5.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT FILE FOR A SINGLE REFLECTOR ANTENNA</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Faceted Spherical Reflector Test Case (LSST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Surface composed of 54 panels, three perimeter points per panel,</td>
</tr>
<tr>
<td>3</td>
<td>no blockage, Feed phase center 0.5 lambda inside horn aperture, E-plane only</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Blank Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>9.441 0.00 8.026 0.0 0.0 -40.0 0.3335</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3 5424.0 0.0 0.0 0.0</td>
</tr>
<tr>
<td>7</td>
<td>0.0 0.0 0.0 0.0 0.0 0.0</td>
</tr>
<tr>
<td>8</td>
<td>-19.6617 0.0 13.7628</td>
</tr>
<tr>
<td>9</td>
<td>-20.7015 5.0252 11.0542</td>
</tr>
<tr>
<td>10</td>
<td>-22.2326 -5.0581 7.4916</td>
</tr>
<tr>
<td>11</td>
<td>-23.8206 0.0 2.9285</td>
</tr>
<tr>
<td>12</td>
<td>0.0 0.0 0.0 0.0</td>
</tr>
<tr>
<td>13-50</td>
<td>Illumination data for FILL routine</td>
</tr>
<tr>
<td>51</td>
<td>101 3</td>
</tr>
</tbody>
</table>
Table 5.1 (continued)

52  1  15  51
53  THETA  91.0  PHI  0.0  15.0  0.5
54  PHI  0.0  THETA  81.0  100.0  0.5
55  DONE
56  3  1  150
57  X1  Y1  Z1  Three points for the first panel on reflector
58  X2  Y2  Z2
59  X3  Y3  Z3
60  3  1  150
61  X1  Y1  Z1  Three points for the second panel on reflector
62  X2  Y2  Z2
63  X3  Y3  Z3
64-273  This process is repeated for all 54 panels.
MAP OF PANEL PROJECTIONS

Fig. 5.1. Old algorithm
Fig. 5.2. New algorithm
SPHERICAL FACETED REFLECTOR

E-PLANE

--- CALCULATED
--- MEASURED

Fig. 5.3. Sphere E-plane pattern (old algorithm)
Fig. 5.4. New algorithm
Fig. 5.5. Sphere H-plane (old algorithm)
Fig. 5.6. New algorithm
6. CONCLUSIONS

An algorithm capable of computing radiation patterns of single reflector antennas has been modified and extended to analyze dual reflector antennas. A new technique for determining the aperture plane for multipanel single reflector antennas has been incorporated into the new program. The location of any aperture plane and the normals on each plane panel are computed automatically. Furthermore, equations for hyperbolic surfaces have been added.

The capability of expressing any non-analytic surface numerically will render the present algorithm very versatile. This fact will make the analysis of dual reflector antennas with shaped surfaces possible.

Presently, the algorithm requires that the feed center coincide with the real focus of the hyperboloid for a Cassegrain antenna, but modifications could be inserted to deal with any off-focus applications.

The results for the dual reflector antennas obtained by this algorithm show good agreement with those obtained by other algorithms. It is believed that a direct comparison with measured patterns will give a better estimate of the accuracy of the present algorithm.
7. LIST OF REFERENCES


8. APPENDICES
8.1. APPENDIX A

CASSEGRAIN ANTENNA GEOMETRY

The classical Cassegrain geometry shown above employs a parabolic contour for the main reflector and a hyperbolic contour for the subreflector. One of the foci of the hyperboloid is the **real focal point** of the system and is located at the origin of the feed coordinate system; the other is a **virtual focal point** which is located at the focus of the paraboloid. As a result, all parts of a wave emanating from the real focal point and then reflected from both reflector
surfaces, travel equal distances to a plane in front of the antenna.

Four fixed parameters are adequate to completely describe a Cassegrain system, two for each reflector. In Figure 8.1, seven parameters are shown. If four are known, the other three can be derived from the mathematical relationships between the two reflector surfaces. For the main reflector,

\[ \tan \frac{1}{2} \theta_2 = \frac{1}{4} \frac{D_m}{F_m} \]

and for the subreflector:

\[ \frac{1}{\tan \theta_1} + \frac{1}{\tan \theta_2} = \frac{2 F_c}{D_s} \]

\[ 1 - \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} = 2 \frac{f_1}{f_2} \]

where: 
- \( F_c \) - distance between two foci,
- \( f_1, f_2 \) = focal lengths of hyperboloid,
- \( D_m \) = diameter of main reflector,
- \( D_s \) = diameter of subreflector,
- \( F_m \) = focal length of paraboloid
- \( \theta_2 \) = one-half of the angle subtended by the main reflector
- \( \theta_1 \) = one-half of the angle subtended by the subreflector.

For example, if \( D_m, F_m, F_c \) and \( \theta_1 \) are determined by considerations of antenna performance and space limitations, then \( \theta_2, D_s, \) and \( f_2 \) can be derived.
Note $\theta$, which determines the beamwidth required of the feed radiation pattern, may be determined independently of the ratio $F_m/D_m$ which specified the shape of the main reflector.

The surface of the main reflector is given by:
$$y^2 + z^2 = 4 F_m (F_m + x),$$
and the surface of the subreflector is expressed as:
$$\left(\frac{x + \text{DIST}}{a}\right)^2 - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

where $\text{DIST} = \frac{F_c}{2} = a + |x_0|$ (See Figure A.2) is the distance used to translate the origin of the hyperbola coordinate system so that it coincides with the origin of the referenced system.

Fig. 8.2. Subreflector coordinate system

\begin{itemize}
\item $a =$ half the transverse axis (along $x$-axis)
\item $b =$ semi-axis along the $y$ direction in the ellipse lying in the $yz$ plane.
\item $c =$ semi-axis along the $z$ direction in the ellipse lying in the $yz$ plane.
\end{itemize}
If $\epsilon$ (eccentricity) of the hyperboloid is known, the following equations can be used:

$$\epsilon = \frac{\sinh(\theta_2 + \theta_1)}{\sinh(\theta_2 - \theta_1)}$$

$$a = \frac{F_c}{2\epsilon}, \quad b = a \sqrt{\epsilon^2 - 1}, \quad \text{and} \quad \frac{f_2}{f_1} = \frac{\epsilon + 1}{\epsilon - 1} = M$$

where $M$ is the magnification factor of the hyperboloid.
ADDITION OF HYPERBOLOID

The equation of the hyperboloid, depicted in Figure 8.3, in the cartesian system is given as:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \text{ where } a = \text{ half the transverse axis along } x \\
b = \text{ semi-axis of the ellipse in the } yz \text{ plane} \\
c = \text{ semi-axis of the ellipse in the } yz \text{ plane}
\]

In this equation, the hyperboloid is expressed in the \(x_s, y_s\) and \(z_s\) coordinate system. To express the same surface in the \(x, y\) and \(z\) system, a translation has to take place along the \(x\) axis, so that the origins of the two systems \(O_s\) and \(O\) coincide. It is clear that \(y_s = y\) and \(z_s = z\), and hence no
charge is needed to be made in the y and z directions.

If DIST is the distance between O and O, then x can be expressed as \( x = x_s - \text{DIST} \), or \( x_s = x + \text{DIST} \), and hence the hyperboloid equation in the x, y, z system becomes:

\[
\left( \frac{x}{a^2} \right)^2 - \left( \frac{y}{b^2} \right)^2 - \left( \frac{z}{c^2} \right)^2 = 1, \text{ where } \text{DIST} = \frac{F_c}{2} \quad (8.1)
\]

and \( F_c = \) distance between the two foci of the hyperboloid.

The parametric equations for a ray are:

\[
\begin{align*}
  x &= R_B^{11} - B_{12} \\
y &= R_B^{21} - B_{22} \\
z &= R_B^{31} - B_{32}
\end{align*}
\quad (8.2)
\]

Substitute Equation (8.1) back into the equation of the hyperboloid to obtain:

\[
\left( \frac{R_B^{11} - B_{12} + \text{DIST}}{a^2} \right)^2 - \left( \frac{R_B^{21} - B_{22}}{b^2} \right)^2 - \left( \frac{R_B^{31} - B_{32}}{c^2} \right)^2 = 1 = 0
\]

or

\[
\begin{align*}
  \frac{R^2 B_{11}^2}{a^2} + \frac{B_{12}^2}{a^2} + \frac{\text{DIST}^2}{a^2} - \frac{2R B_{11} B_{12}}{a^2} + \frac{2R B_{11} \text{DIST}}{a^2} - \frac{2B_{12} \text{DIST}}{a^2} \\
  - \frac{R^2 B_{21}^2}{b^2} + \frac{B_{22}^2}{b^2} + \frac{2R B_{21} B_{22}}{b^2} - \frac{R^2 B_{31}^2}{c^2} + \frac{B_{32}^2}{c^2} + \frac{2B_{31} B_{32}}{c^2} - 1 = 0
\end{align*}
\quad (8.3)
\]

Equation (8.3) is of the form
\[
AR^2 + BR + C = 0 \\
\text{(8.4)}
\]

where
\[
A = \frac{B_{11}^2}{a^2} - \frac{B_{21}^2}{b^2} - \frac{B_{31}^2}{c^2} \tag{8.5}
\]
\[
B = -2 \left( \frac{B_{11}B_{12}}{a^2} - \frac{B_{11}\text{DIST}}{a^2} - \frac{B_{21}B_{22}}{b^2} - \frac{B_{31}B_{32}}{c^2} \right) \tag{8.6}
\]
\[
C = \frac{B_{12}^2}{a^2} + (\text{DIST})^2 \frac{B_{12}^2}{a^2} - \frac{B_{22}^2}{b^2} - \frac{B_{32}^2}{c^2} - 1 \tag{8.7}
\]

Equations (8.5), (8.6), and (8.7) are evaluated by the program and (8.4) is solved to find the intersection point of the ray with the surface.

Now, to find the inside normal of the surface, the gradient of Equation (8.1) is taken as:
\[
\nabla g(x, y, z) = \hat{n}(x, y, z) \tag{8.8}
\]

where
\[
g(x, y, z) = \frac{(x + \text{DIST})^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 \tag{8.9}
\]

it follows that:
\[
\nabla g = x \frac{\partial g}{\partial x} + y \frac{\partial g}{\partial y} + z \frac{\partial g}{\partial z} = x \frac{2(x + \text{DIST})}{a^2} - y \frac{2y}{b^2} - z \frac{2z}{c^2} \tag{8.10}
\]
or
\[
\frac{\partial g}{\partial x} = \frac{2(x + \text{DIST})}{a^2} \tag{8.11}
\]
\[
\frac{\partial g}{\partial y} = -\frac{2y}{b^2}
\]
\[
\frac{\partial g}{\partial z} = -\frac{2z}{c^2}
\]
Normalization results in obtaining the unit vector $\hat{n}$ as:

$$\hat{n} = \frac{\mathbf{V}_g(x,y,z)}{\mathbf{V}_g} = \frac{x\, 2(x + \text{DIST})/a^2 - \hat{y}(2/b^2) - \hat{z}(2z/a^2)}{\left(\frac{4(x + \text{DIST})^2}{a^4} + \frac{4y^2}{b^4} + \frac{4z^2}{c^4}\right)^{1/4}} \quad (8.12)$$

The factor 2 cancels out from both numerator and denominator.

Let the denominator be expressed as:

$$\text{DEN} = \left[\frac{(x + \text{DIST})^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right]^{1/2} \quad (8.13)$$

then

$$n_x = \frac{(x + \text{DIST})/a^2}{\text{DEN}}$$

$$n_y = \frac{-y/b^2}{\text{DEN}}$$

$$n_z = \frac{-z/c^2}{\text{DEN}}$$
In this subroutine, the four extreme points of the main reflector are used to find the four extreme points on the subreflector. This task is accomplished as follows:

Take a given extreme point on the main reflector and write the parametric equations of the line ($RR$) connecting that point to the origin of the reference system ($O$).

Express the direction cosines as:

$$DIR_1 = \cos A = \frac{PM(1,K)}{RR} \quad (8.14)$$
$$DIR_2 = \cos B = \frac{PM(2,K)}{RR} \quad (8.15)$$
$$DIR_3 = \cos C = \frac{PM(3,K)}{RR} \quad (8.16)$$
Hence, the parametric equation of that line is given by:

\[ x_0 = P(1,K) = PM(1,K) - RR \cdot DIR1 \]  
(8.17)

\[ y_0 = P(2,K) = PM(2,K) - RR \cdot DIR2 \]  
(8.18)

\[ z_0 = P(3,K) = PM(3,K) - RR \cdot DIR3 \]  
(8.19)

where \((P(1,K), (P(2,K) and P(3,K))\) is a point on the sub-reflector which is to be found.

Now, substitute Equations (8.17), (8.18), and (8.19) in the equation for the surface of the hyperboloid, that is in

\[ \frac{(x + DIST)^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]  
(8.20)

to obtain:

\[ \left[ \frac{PM(1,K) - RR \cdot DIR1 + DIST}{a} \right]^2 - \left[ \frac{PM(2,K) - RR \cdot DIR2}{b} \right]^2 - \left[ \frac{PM(3,K) - RR \cdot DIR3}{c} \right]^2 = 1 \]  
(8.21)

or

\[ \frac{(PM(1,K))^2}{a^2} + \frac{(DIST)^2}{a^2} + \frac{(RR)^2(DIR1)^2}{a^2} - \frac{2RR \cdot DIR1 \cdot PM(1,K)}{a^2} - \frac{2RR \cdot DIR1 \cdot DIST}{a^2} + \frac{2PM(1,K) \cdot DIST}{a^2} - \frac{(PM(2,K))^2}{b^2} - \frac{(RR)^2(DIR2)^2}{b^2} + \frac{2PM(2,K) \cdot RR \cdot DIR2}{b^2} - \frac{(PM(3,K))^2}{c^2} - \frac{(RR)^2(DIR3)^2}{c^2} + \frac{2 \cdot RR \cdot PM(3,K) \cdot DIR3}{c^2} - 1 = 0 \]  
(8.22)
This equation is of the form \((ARR) (RR)^2 + BRR \cdot RR + CRR = 0\) \(\tag{8.23}\)

where \(ARR = \frac{(DIR1)^2}{a^2} - \frac{(DIR2)^2}{b^2} - \frac{(DIR3)^3}{c^2}\) \(\tag{8.24}\)

\[
BRR = 2 \left[ (-PM(1,K) \cdot DIST) \cdot DIR1/a^2 + PM(3,K) \cdot DIR2/b^2 \\
+ PM(3,K) \cdot DIR3/c^2 \right] \tag{8.25}
\]

\[
CRR = \left[ \frac{(PM(1,K))^2 + (DIST)^2 + 2.0 \cdot PM(1,K) \cdot DIST}{a^2} \\
- \frac{(PM(2,K))^2}{b^2} - \frac{(PM(3,K))^2}{c^2} - 1 \right] \tag{8.26}
\]

Equations (8.24), (8.25), and (8.26) are evaluated by the program and (8.23) is solved to find \(RR\). Substituting for the value of \(RR\) in Equations (8.14), (8.15), and (8.16), a point on the subreflector is obtained.
8.4. APPENDIX D

DEVELOPMENT OF NORMALS ON A PLANE PANEL

In the APRIN routine a certain number of perimeter points for each panel are read in. To determine a unit normal on each panel, the following procedure is applied:

1) Any three perimeter points are used to form two vectors, as shown in Figure 8.2.

\[
\vec{A} = (x_1 - x_2) \hat{i} + (y_1 - y_2) \hat{j} + (z_1 - z_2) \hat{k}, \quad \text{and} \quad \vec{B} = (x_1 - x_3) \hat{i} + (y_1 - y_3) \hat{j} + (z_1 - z_3) \hat{k}
\]

2) The cross product operation is used to find a vector normal \( \hat{N} \) to the plane defined by the vectors \( \vec{A} \) and \( \vec{B} \):

\[
\hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \end{vmatrix} = (y_1 - y_2)(z_1 - z_3) - (y_1 - y_3)(z_1 - z_2) \hat{i} + (x_1 - x_3)(z_1 - z_2) - (x_1 - x_2)(z_1 - z_3) \hat{j} + (x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2) \hat{k}
\]
3) The unit normal $\hat{N}$ is computed by: 
$$N = \frac{\hat{N}}{|\hat{N}|}$$

4) If this normal on the surface of the panel has a negative x component, then the vector is inverted to yield a positive x component, since any normal vector on the surface of the reflector should be directed toward the origin of the reference system, i.e., along the positive x axis. (See Figure 2.5).
8.5. APPENDIX E

FILL ROUTINE FOR A VERTICALLY POLARIZED FEED

The basis of this subroutine can be found in (5). To use it, the E- and H-plane patterns of the feed must be provided by the programmer in increments of 1°.

Fig. 8.6. Definition of angles δ and ε

Fig. 8.7. Ellipse used for interpolation
In Figure C.3, the angles used in FILL are shown.

(From (5).)
\[ \delta = \cos^{-1} \left( \sin \theta' \cos \phi' \right) \]  
where \( \theta' \) and \( \phi' \) are angles in the feed system.

and \( \epsilon = \tan^{-1} \frac{\cos \theta'}{\sin \theta' \sin \phi'} \)  

Figure 8.4 depicts the interpolation ellipse which is given by:

\[ \frac{u^2}{E^2} + \frac{v^2}{H^2} = 1 \]  

where

\[ u = r \cos \epsilon, \quad v = r \sin \epsilon, \]  

\[ E = E_{\theta'} = 90^\circ, \text{ and } H = E_{\phi'} = 180 \]  

Hence:

\[ r = \frac{E_{\theta'} = 90 \cdot E_{\phi'} = 180}{E_{\theta'} = 90 \sin^2 \epsilon + E_{\phi'} = 180 \cos^2 \epsilon} \]  

The code of this subroutine is shown in Appendix F. In that code, PROJX = \( \cos \delta \) and PROJEX = \( \sin \epsilon \). To insure vertical (i.e., \( \theta' \)) polarization, PROJEX is set equal to zero. That means \( u = r \cos \epsilon \) and \( v = 0 \). Substitution for \( u \) and \( v \) is Equation (8.32).

\[ \text{Etot} = \frac{E_{\theta'} = 90 \cdot E_{\phi'} = 180}{E_{\phi'} = 180 \cdot \cos^2 \epsilon} \]  

where \( \cos^2 \epsilon = 1 - \sin^2 \epsilon \). Since a \( \theta' \) polarized feed is associated with the \( z \) component of a cartesian system \( P(3,I) \), and \( P(4,I) \) are given as:

\[ P(3,I) = \text{Etot} \text{ (along } z) \]
\[ P(4,I) = 0.0 \text{ (along } y) \]
8.6. APPENDIX F

LISTING OF THE CODE FOR REFLECTR
MAIN

IMPLICIT REAL (A-H,O-Z)
REAL*8 MAJOR(8), MINOR(8), NORM
COMPLEX*16 ETOT(2,400), FIELDY(400), FIELDD(400)
INTEGER SURFC1, SURFC2
COMMON/PARAM/AGROUP, BELLP, CELLP, DIST, PS1, PLNPSH(3), PLNNUM(3),
       FEED(3), ALPHA, BETA, GAMMA, XLEN, XX, AUROR, BELLPS, CELLP2,
       PS12(3), ST2, POINT(3), NORM(3), SURFC1, NPNL, NPOINT, SURFC2
COMMON/APPRPRM/APTRPL, NPERIM
COMMON/COLOS/DELTA, XC, ANGINC, PM(3, +), R, XX, XMAX, ZMIN, ZMAX
COMMON/NORT/NOPT(3), NLST, IOPT IF, ICAB, NSLIST(100)
COMMON/DMLN/VDIM, JDIM, YCT, ZCT
COMMON/EXTENT/YNIN, YMAX, ZMIN, ZMAX
COMMON/HATH/P1, P2, PI2, P102, YTO, RTOD
COMMON/PATRN/ETOT, ANINO(3, 8), MAJOR(8), MINOR, MAJOR, NANGLE(5)
DIMENSION P(2, 750), YPLD(75), XPLD(75), PR(2, 500)
DATA DONE / T(M), NPART / G0, 7.0
DATA VLO, YMI, ZLO, ZMI / 1.00*10.0, 1.00*10.0
FOSI X = 10.04*LOG10(X)
P064X1 = 10.0*LOG10(X)
MAXPTS = 2750
CALL NPUT(P, NPAT)
DO 400 I = 1, NPAT
CALL APTRP(P, I)
PRINT 777
CALL QUANTZ(P, NPERIM, I)
PRINT 778
IF (I01(1, I). EQ. 0) GO TO 80
I$ = 1
IF (IOPT. EQ. 1) I$ = -1
CALL APPRT(P, NPOINT, I$)
PRINT 780
CONTINUE
IF (YMNI.LT.YLO) YLO = YMIN
IF (YMAX.LT.YHI) YHI = YMAX
IF (ZMIN.LT.ZLO) ZLO = ZMIN
IF (ZMAX.LT.ZHI) ZHI = ZMAX
IF (ICASS.EQ.1) NPERIM = 4
DO 50 L = 1, NPERIM
PR(1, L+MLYL), P(1, L+NPOINT)
MLYL = KVL + NPERIM + 1
PR(1, MLVL) = 1.0D+40
I JUNK = 0
DO 200 K = 1, NPAT
CALL INTGR(P, MAJOR(K), MAJOR(K), MINOR(I, K), FIELDY, FIELDD)
PRINT 779
NANG = NANGLE(K)
DO 150 L = 1, NANG
ETOT(L + ISUM) = ETOT(L + ISUM) + FIELDY(L)
150 ETOT(2 + L + ISUM) = ETOT(2 + L + ISUM) + FIELDD(L)
200 ISUM = ISUM + NANG
CONTINUE
PRINT 781
IF (IOPT. EQ. 1) GO TO 420
IF (I01(1, I). EQ. 0) GO TO 420
YMIN = YHI - YLO
ZD1M-L11I-LLD
YCT\(^+(YHI+YLO)/2.0\)
ZCT\((ZHI+ZLO)/2.0\)
CALL APRMAP(CR,NPNL,-1)
PRINT 762
420 ISUM=0
DO 770 I=1,NPAT
NANG=NANGLE(I)
FMAXY=-1.00+40
FMAXZ=-1.00+40
DO 450 J=1,NANG
YFLD(J)=CDABS(ETOT(1,J+ISUM))
ZFLD(J)=CDABS(ETOT(2,J+ISUM))
FMAXY=DMAX(FMAXY,YFLD(J))
FMAXZ=DMAX(FMAXZ,ZFLD(J))
ISUM=ISUM+NANG
D=AMIND(1.1)
FMYDB=-60.000
FMZDB=-60.000
PWZDB=-60.000
PWDB=FMAYX*FMAXZ+FMAXY*FMAXZ
IF (FMAXY.GT.1.00-10) FMYDB=FOB(FMAXY)
IF (FMAXZ.GT.1.00-10) FMZDB=FOB(FMAXZ)
IF (PWZDB.GT.1.00-10) PWZDB=DBD(PWDB)
PRINT 600,MAJOR(I),MINOR(I),(AMIND(1.1),J=1,3)
600 FORMAT(I1,'///24X,\n**TABLE OF ELECTRIC FIELD STRENGTHS (DB)**///23X,\n**-------- -------- -------- -------**///24X,\n**//19X,'ANGLE',AS,'PHOM',F8.3,' TO',F8.3,' BY',F6.3,* DEG**///24X,\n**PRINT 666, MINOR(I)\n**\n666 FORMAT('/13X,AS,4X,'DB(Z/Z)',4X,'DB(Y/Z)',4X,'DB(Z/Y)',5X,\n'DF/1/Y/15X,'PWZDB','///24X,\nDO 700 K=1,NANG
PWZ(K)=PWZDB-100.000
DBY =FMYDB-100.000
DBZ =FMZDB-100.000
PWZ=ZFLD(K)*YFLD(K)+YFLD(K)*ZFLD(K)
IF (YFLD(K).LT.1.00-15) DBY=FBY(YFLD(K))
IF (ZFLD(K).LT.1.00-15) DBZ=FBZ(ZFLD(K))
IF (PWZ.GT.1.00-20) PWZ=DBD(PWZ)
IF (FMYDB.EQ.-60.000) DBY=-60.000
IF (FMZDB.EQ.-60.000) DBZ=-60.000
DBZ=0.000
DBY=0.000
DBZ=0.000
DBZ=DBZ-PWZDB
DBY=DBY-PWZDB
DBZ=DBZ-PWZDB
PWZ=DBDB
PRINT 690, D,DBZ, DBY, DBZ, DBY, DBZ, DBZ
690 FORMAT(10X,F9.3,5F11.5)
D=AMIND(1,1)
YFLD(K)=D3Y
ZFLD(K)=DBZ
700 CONTINUE
PRINT 750, FMAXZ,FMZDB,FMAXY,FMYDB
750 FORMAT(///13X,'MAXIMUM FIELD VALUES=**///15X,
20LOG(MAX(FIELD-Z))=20LOG(*1PE15.7,*)=.0PF12.7//16X,
20LOG(MAX(FIELD-Y))=20LOG(*1PE15.7,*)=.0PF12.7
PRINT 755, NPARTS

INTERPOLATION NUMBER USED FOR INTEGRATION IS
PRINT 765, MAJOR(1), MINOR(1)

CALCULATE NORMALIZED Z-COMPONENT OF SECONDARY PATTERN (DB)
PRINT 765, MAJOR(1), MINOR(1)

CALCULATE NORMALIZED Y-COMPONENT OF SECONDARY PATTERN (DB)
PRINT 765, MAJOR(1), MINOR(1)

CALL PLOT4(64H, NORMALIZED SECONDARY PATTERN Z
PRINT 765, MAJOR(1), MINOR(1)

CONTINUE
IF (101(4,1).EQ.1) WRITE(7,775)
IF (101(5,1).EQ.0) STOP
REWIND 7
CALL PLOT3

FINISHED INPUT 
FINISHED APERTUR 
FINISHED QUANTIZ 
FINISHED INTGR 
*** EXECUTED APRPLT *** 
--- PATTERN COMPUTATIONS COMPLETE ---
*** EXECUTED APRMAP *** 
STOP
SUBROUTINE NPUT(P,NPAT)

IMPLICIT REAL*(A-H,O-Z)
REAL*8 MAJOR(5),MINOR(5),NORM
INTEGER SURFC1,SURFC2
COMMON/BLOCK/YCBL,YZBL,NPMABL,HPMABL
COMMON/FEED/EP(I),ET(91),NP,NT,XS,YS,ZS
COMMON/COLUS/DELT,XC,ANGINC,PNI(4),RS,XMX,ZMX,ZNN,YMX
COMMON/CTRL/NOPT(4),NLIST(:),ICASS,ILIST(100)
COMMON/PARAMS/ADORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3).

FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,ADORF,BELLP2,CELLP2,
PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPINT,SURFC2
COMMON/PATTRN/ETOT,AMINOR(3,5),AMAJOR(5),MINOR,MAJOR,NANGLE(5)
COMMON/MATH/P1,P2,P12,DOTU,HT6,G
DIMENSION P(5,2750),TITLE(40)
DATA DONE/SHOUNE,ICASS=0

READ 5,TITLE
FORMAT (10A8)
READ(1,10) FEED,ALPHA,BETA,GAMMA,XLAM
FORMAT (1F10.4)
IF (ICASS.NE.1) GO TO 39
READ(1,20) SURFC2,ADORF,BELLP2,CELLP2,DIST2,PSI2,POINT,NORM
FORMAT (11,9X,5F10.4/6F10.4)
READ(1,37) SURFC1,NPNL,ADORF,BELLP,CELLP,DIST,PSI,PLNPNT,PLNORM
FORMAT (11,7X,12,5F10.4/6F10.4)
IF (ICASS.NE.1) GO TO 40
READ(1,39) (PNI(I),J=1,3),J=1,4)
FORMAT (3F10.4)
END OF MAIN REFLECTOR INPUT DATA
CALL SUBPNT(P)
GO TO 43
READ (1,41) (PNI(I,J),J=1,3),J=1,4)
FORMAT (3F10.4)
END OF SUB REFLECTOR INPUT DATA
READ (1,50) XX,YCBL,YZBL,NPMABL,HPMABL
FORMAT (5F10.4)
READ (1,60) NOPT,NLIST
FORMAT (3F15.5)
READ (1,60) NOPT,NLIST
FORMAT (1F10.4)
IF (NOPT(I).EQ.1.OR.NOPT(2).EQ.1) READ (1,70) (LIST(I),I=1,NLIST)
60 FORMAT (16I5)
70 IF (NPA=1)

77 READ (1,80) MAJOR(NPAT),AMAJOR(NPAT),MINOR(NPAT),AMINOR(1,NPAT),

1=1,3)

80 FORMAT (5X,5F10.4,5X,3F10.4)
IF (MAJOR(NPAT)-EDONE) GO TO 86
NANGLE(NPAT):(AMINOR(2,NPAT)-AMINOR(1,NPAT))/AMINOR(3,NPAT)+1.5
111=NANGLE(NPAT)*GT,75)
GO TO 85
111=SUM*NANGLE(NPAT)
NWRITE(NPAT)
IF (NPAT.LT.6) GO TO 77
PRINT 330
STOP
85 PRINT 335
STOP
88 IF (ISUM.LE.400) GO TO 95
PRINT 340.ISUM
STOP
95 NPAT=NPAT-1
DO 98 L=1,ISUM
ETOT(1,L)=(-0.000,0.000)
98 ETOT(2,L)=(0.000,0.000)
PRINT 577.XC,YCBL,ZCBL,HFMBL,HPMBL,NPML
IF (ICASS.NE.1) GO TO 180
PRINT 578
GO TO (120,130,140,150,160,161),SURFC2
120 PRINT 579,POINT,NORM
GO TO 179
130 PRINT 580,AORQR2,BELLP2
GO TO 179
140 PRINT 581,AORQR2
GO TO 179
150 PRINT 582,AORQR2
GO TO 179
160 PRINT 583,AORQR2,PSI2
GO TO 179
161 PRINT 584,AORQR2,BELLP2,CELLP2,DIST2
170 PRINT 585,((PM(1,J),I=1,3),J=1,4)
PRINT 586
180 GO TO (220,230,240,250,260,270),SURFC1
220 PRINT 579,PLNPNT,PLNORM
GO TO 300
230 PRINT 580,AORQRF,BELLP
GO TO 300
240 PRINT 581,AORQRF
GO TO 300
250 PRINT 582,AORQRF
GO TO 300
260 PRINT 583,AORQRF,PSI
GO TO 300
270 PRINT 584,AORQRF,BELLP,CELLP,DIST
300 IF (NPNL.GE.1) GO TO 310
1OPT=1
NPNL=1
310 PRINT 585,((P(1,J),I=1,3),J=1,4)
PRINT 587
PRINT 588
PRINT 589
PRINT 599,LP
PRINT 587
PRINT 588
PRINT 660,ET
PRINT 490,NPAT
GO 320 M=1,NPAT
320 PRINT 500,MAJOR(M),MINOR(M),AMINOR(KK,M),MINOR(KK,M)
330 FORMAT(*???????? ERROR-MORE THAN 5 PATTERN *)
ORIÇIAI
PAGE IS 91 OF POOR QUALITY
CALCULATIONS REQUESTED.
ERROR - REQUESTED*15* ANGLES TO BE *
CALCULATED EXCEEDS AVAILABLE STORAGE

NUMBER OF PATTERN REQUESTS

FORMAT(//

APERTURE PLANE LOCATION(XC)......
SUB DISH SHADOW CENTER COORDINATES IN APERT. PL.
HALF MAJOR AXIS OF SUB DISH SHADOW
HALF MINOR AXIS OF SUB DISH SHADOW
NUMBER OF PANELS IN REFLECTOR

MAIN DISH DESCRIPTION AND ITS PARAMETERS-

IT IS A PLANAR REFLECTOR
APERTURE PLANE LOCATION(XC).

IT IS AN ELLIPTICAL REFLECTOR
MAJOR AXIS OF THE ELLIPTICAL REFLECTOR
MINOR AXIS OF THE ELLIPTICAL REFLECTOR

IT IS A SHERICAL REFLECTOR
RADIUS OF REFLECTOR SHAPE
FOCAL LENGTH OF THE REFLECTOR

IT IS A PARABOLIC CYLINDRICAL REFLECTOR
FOCAL LENGTH OF PARABOLIC CYLINDER
ANGLE OF ROTATION ABOUT X-AXIS (PSI)

IT IS A HYPERBOLIC REFLECTOR
MAJOR AXIS OF REFLECTOR IN X DIRECTION
AXIS OF REFLECTOR IN Y DIRECTION
AXIS OF REFLECTOR IN Z DIRECTION
DISTANCE USED FOR TRANSLATION OF ORIGIN OF AXES

PROGRAM IN SINGLE PANEL MODE

MINIMUM-Y POINT ON THE REFLECTOR
MAXIMUM-Y POINT ON THE REFLECTOR
MINIMUM-Z POINT ON THE REFLECTOR
MAXIMUM-Z POINT ON THE REFLECTOR

SUBDISH DESCRIPTION AND ITS PARAMETERS-
PATTERN OF FEED IN ONE DEG INCREMENTS OFF-AXIS-

SUB-O-PLANE

FORMAT(2X,5F16.10)
PI=3.141592653589793
PI2=PI*PI
P102=10.*PI2
DTR=PI/180.
RTOO=180./P1
RETURN

END

C-2
APRTUR

SUBROUTINE APRTUR(P, ICALL)

IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 NMAT(3), NMA = NORM
INTEGER SUNFC1, SUNFC2
COMMON/APPRM/NPTPLL, NPERIN
COMMON/CASS/AR(I), EN(J), X0, Y0, Z0, RM, D* X02, Y02, Z02, ER2(I)
COMMON/FLEED/EPR(I), ET(I), nP, nAT, X5, Y5, Z5
COMMON/MATH/P1, P12, P1D2, DTOR, NTD
COMMON/COLOD/DELT, XG, ANGINC, PM(I, J), RS, XM, XM, ZM, YM
COMMON/CONTR/NOPI(I), NLIST(I), OPT, T, CASS, LIST(100)
COMMON/PMAN/AORDR, BELLP, CELLP, DIST, PSI, PLNPNT(I), PLNORMAL(I)

IF (ICASS.EQ.1) GO TO 10

M = 1
DO 1 I = 2, 4
IF (P(3, M) - P(3, 1)) 3, 2, 2
1 M = 1
CONTINUE

XX = P(1, M)
YY = P(2, M)
ZZ = P(3, M)

IF (ICALL.EQ.1) GO TO 50
ALPHAR = ALPHAR + DOT
BETAR = BETAR + DOT
GAMMAR = GAMMAR + DOT

A(1, 1) = COS(ALPHAR) * COS(GAMMAR) - DSIN(ALPHAR) * DSIN(BETAR)
A(1, 2) = DSIN(ALPHAR) * COS(GAMMAR) + COS(ALPHAR) * DSIN(BETAR)
A(1, 3) = DSIN(ALPHAR) * DSIN(GAMMAR)
A(2, 1) = -DSIN(ALPHAR) * COS(BETAR) + COS(ALPHAR) * DSIN(BETAR)
A(2, 2) = COS(ALPHAR) * COS(BETAR) + DSIN(ALPHAR) * DSIN(BETAR)
A(2, 3) = COS(BETAR)
A(3, 1) = DSIN(ALPHAR) * COS(GAMMAR) + DSIN(ALPHAR) * DSIN(BETAR)
A(3, 2) = DSIN(ALPHAR) * DSIN(GAMMAR) - COS(ALPHAR) * DSIN(BETAR)
A(3, 3) = COS(ALPHAR) * COS(GAMMAR)

DO 40 I = 1, 3
DO 40 J = 1, 3
40 AINV(I, J) = A(J, I)
NPERI = 4
NPTPLL = 2000

IF (IDPT.EQ.0) CALL APRIN(P, ICALL)

TMA = 0.000
TMN = P1
PR = P1
PD2 = P1D2

DO 60 I = 1, NPERI
DO 60 J = 1, 3
60 X(J) = AINV(J, 1) * P(1, 1) + AINV(J, 2) * P(2, 1) + AINV(J, 3) * P(3, 1)
R = DSORT((X(1) * FEED(1)) * (X(2) + FEED(2)) * (X(3) * FEED(3)) * (X(1) + FEED(3)) * (X(2) + FEED(3)) * (X(1) + FEED(3)))
P(1, 1) = DARCUS((X(3) * FEED(3)) / R)
SINTH=DSIN(P(1,1))
IF ( SINTH.LT.1.0) SINTH=1.0
P(Z+1)=0-DSIN((X(2)+FEED(2))/(NP*SINTH))
61 IF (P(1,1).GT.PMAX) PMAX=P(1,1)
IF (P(1,1).LT.TMIN) TMIN=P(1,1)
IF (P(2,1).LT.PMIN) PMIN=P(2,1)
65 CONTINUE
DEL=PMAX-PMIN
DELT=MAX-TMIN
NP=NPINT(DEL+/=DELTP+NPINT(NP)/(NPINT+1.0)
ANGINC=DELTP)/(NPINT+2.0)
IF (ICASS.EQ.1) CALL FINDC(P,B)
ND2=DELT/(2.0*ANGINC)+1.0
NT=2*ND2+1
P=0.0,0.0,TANGINC
PMAX=0.0,0.0,TANGINC
TMAX=0.0,0.0,TANGINC
DO 95 J=1,NT
95 P=0.0,0.0,TANGINC
P=0.0,0.0,TANGINC
P=0.0,0.0,TANGINC
95 TMIN=TMIN/NTD2
TMAX=TMAX/NTD2
PMAX=PMAX/NTD2
TANGINC=TANGINC/NTD2
IF (PMIN.EQ.PMIN) PRINT 107,TMIN,TMAX,PMIN,PMAX,
+ ,ANGINC,NTD2,NPINT
107 FORMAT(/"I ILLUMINATION DATA-I/
+ , "THE FIRST ILLUMINATION DATA FROM F9.3 TO
+ , F9.3/",
+ , "THE SECOND ILLUMINATION DATA FROM F9.3 TO
+ , F9.3/",
+ , "THE INCREMENTAL ANGLE (DEG) FROM F7.4 TO
+ , F7.4/",
+ , "THE TOTAL NUMBER OF GENERATED RAYS IS 17/",
+ , "THE TOTAL NUMBER OF APERTURE PLANE POINTS IS 17/"
IF (SURFICNE.EQ.1) GO TO 114
CPSI=UCOS(P51@T0R)
SNPSI=DSIN(P51@T0R)
DO 110 I=1,NTD2
110 SIN=DSIN(P32(1,1))
COS=UCOS(P32(1,1))
SINT=DSIN(P1,1))
COST=UCOS(P1,1))
D1(I,1)=INT*COSP
D1(2,1)=INT*SINP
D1(3,1)=COST
D1(2,2)=FEED(1)
D1(2,2)=FEED(2)
BB(3,2)=+FEED(3)
CALL MULT32(BA,BB)
GO TO (120,130,140,150,160,161),SURFC1
120
AR=0.0
BR=0.0
CR=0.0
GO TO 160
130
AR=(1.1)*#2/AORORF#2*(B(2.1)*#2*B(3.1)*#2)/BELLP#2
BR=-2.0*(B(1.1)+AORORF#2*(B(2.1)*#2*B(3.1)*#2))/BELLP#2
CR=(1.0)/BELLP#2
GO TO 160
140
AR=(B(2.1)*#2*(B(1.1)*#2+B(1.2)+B(2.1)+B(3.1)+B(3.2))/BELLP#2
BR=2.0*(B(1.1)*#2+B(1.2)+B(2.2)+B(3.1)*#2+B(3.2)*#2)
CR=(B(1.2)*#2+B(2.2)+B(3.1)*#2+B(3.2)*#2)/BELLP#2
GO TO 160
150
AR=(1.0)*#2/AORORF#2*(B(2.1)*#2*B(3.1)*#2)/BELLP#2
BR=-2.0*(B(1.1)*#2+B(1.2)+B(2.1)+B(3.1)*#2+B(3.2)*#2)
CR=(1.0)*#2/AORORF#2*(B(1.2)+#2+B(3.1)*#2+B(3.2)*#2)
GO TO 160
160
AR=(B(1.1)*#2/AORORF#2*(B(2.1)*#2*B(3.1)*#2)/BELLP#2
BR=-2.0*(B(1.1)*#2+B(1.2)+B(2.1)+B(3.1)*#2+B(3.2)*#2)
CR=(B(1.2)*#2+B(2.2)+B(3.1)*#2+B(3.2)*#2)/BELLP#2
GO TO 160
170
IF (ICASS.NE.1) GO TO 181
IF (UABS(AR)+LT.1.00-10) R=CR/BR
IF (UABS(AR)+LT.1.00-10) GO TO 185
R=(BR-DGRT(BR*BR-4.0)*#AR*CR)/(AR+AR)
GO TO 185
181
IF (ICASS.EQ.1) GO TO 190
IF (UABS(AR)+LT.1.00-10) R=CR/BR
IF (ICALL+LT.1.00-10) GO TO 185
R=VBR*BR-4.0*AR*CR
R=(UR+DSQRT(V))/(AR+AR)
185
CONTINUE
X0=B(1.1)+R-B(1.2)
Y0=B(1.1)+R-B(1.2)
Z0=B(1.1)+R-B(1.2)
IF (ICALL16 GT.1) GO TO 219
IF (ICASS.EQ.1) GO TO 219
IF (ICALL+LT.1.0) GO TO 189
IF (ICALL+E.G.1) GO TO 190
R=DSQRT((X5+B(1.1))/#2+(Z5*B(3.2))/#2-1.0)
THMAX=DATAN(-45+8(B(3.2))/#2)/X5+B(1.1))
THANG=DTHMAX+2.5*ANGINC*DGRT

XC = (R1 * DCOS(THTAUG) + B(1,2))
CONST = DCOS (XC - XS)
GO TO 190
189
X = X + CONST
X = 2 * M
IF (ICALL.GT.1) GO TO 219
190
CALL FINDXC(P,B)
X = 2 * M
IF (OPT.EQ.1) XC = X
219
GO TO (220, 230, 240, 250, 260, 261), SURF C
220
NHAT (1) = PLNORM (1)
NHAT (2) = PLNORM (2)
NHAT (3) = PLNORM (3)
GO TO 288
230
NHAT (1) = 40 * BELLP**2 / DSQRT(X0**2 * BELLP**4 + Y0**2 * Z0**2) * AGRDF**4
NHAT (2) = -Y0 * AGRDF**2 / DSQRT (X0**2 * BELLP**4 + Y0**2 * Z0**2) * AGRDF**4
NHAT (3) = -Z0 * AGRDF**2 / DSQRT (X0**2 * BELLP**4 + Y0**2 * Z0**2) * AGRDF**4
GO TO 288
240
NHAT (1) = -XO / AGRDF
NHAT (2) = -Y0 / AGRDF
NHAT (3) = -Z0 / AGRDF
GO TO 288
250
NHAT (1) = 2.0 * AGRDF / DSQRT (4.0 * AGRDF**2 + Y0**2 + Z0**2)
NHAT (2) = -Y0 / DSQRT (4.0 * AGRDF**2 + Y0**2 + Z0**2)
NHAT (3) = -Z0 / DSQRT (4.0 * AGRDF**2 + Y0**2 + Z0**2)
GO TO 288
260
NMAG = DSQRT (1.0 * AGRDF**2 * (Z0 * CPS1 * SNPS1 - Y0 * SNPS1 * SNPS1)**2 +
(Y0 * SNPS1 * CPS1 - Z0 * CPS1 * CPS1)**2 /
NMAG = DSQRT (X0 * DIST)**2 + AGRDF**2)**2 + Y0**2 / BELLP**4)**2 + (Z0**2 /
CELLP**4)**2)
NHAT (1) = X0 / DIST / (1.0 * AGRDF**2 + DEN)
NHAT (2) = -Y0 / (CELLP**2 + DEN)
NHAT (3) = -Z0 / (CELLP**2 + DEN)
GO TO 288
288
IF (ICASS.NE.1) GO TO 289
NHAT (1) = NHAT (1)
NHAT (2) = NHAT (2)
NHAT (3) = NHAT (3)
289
SCALAR = 2.0 - (BIL(1,1)*NHAT(1) + B(2,1)*NHAT(2) + B(3,1)*NHAT(3))
DO 295 L = 1, L + 3
295
SCALAR = 2.0 - (BIL(1,1)*NHAT(1) + B(2,1)*NHAT(2) + B(3,1)*NHAT(3))
DO 295 L = 1, L + 3
400
SUBROUTINE SUBPNT(P)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 NORM
INTEGER SURFI,SURFC2
COMMON/COLDY/DELT,XC,ANGINC,PN(3,4),RS,XMK,ZMK,ZNK,YMK
COMMON/PARAMS/ADGOF,BELLP,CELLP,DIST,PS1,PLPN(3),PLNORR(3),
+ FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,ADGOR2,BELLP2,CELLP2,
+ PS12,DISTZ,POINT(3),NGAMM(3),SURFC1,PNPL,NPOINT,SURFC2
DIMENSION P(5,2750)
DO 33 K=1,4
RA=DSQRT(PM(1,K)*PM(1,K)+PM(2,K)*PM(2,K)+PM(3,K)*PM(3,K))
DIR=PM(1,K)/RA
DIR2=PM(2,K)/RA
DIR3=PM(3,K)/RA
ARH=DIR*2/(ADGOF+2)-(DIR*2/(CELLP+2))
BKH=PM(1,K)*DIST&DIR1/(ADGOF+2)+(PM(2,K)*DIST*2/)
+ (BELLP+2))+(PM(3,K)*DIST*2/(CELLP+2))
CRH=PM(1,K)*DIST*2*2.0+PM(1,K)*DIST*2/(ADGOF+2)-
- (PM(2,K)*DIST*2/(CELLP+2))-(PM(3,K)*DIST*2/(CELLP+2))&-1.0
KH=-(RAH+DSQRT(BKH+2.0*ARR+ARR))/ARR
P(1,K)=PM(1,K)-KH&DIR1
P(2,K)=PM(2,K)-KH&DIR2
P(3,K)=PM(3,K)-KH&DIR3
33 CONTINUE
RETURN
END
SUBROUTINE CASSA(P)

IMPLICIT REAL*8(A-M,O-Z)
REAL*8 NMAT(3), MGRS, NMG2, NLAT(3)
INTEGER SURFC1, SURFC2

COMMON/PARAMS/ADG0R2, BELLP, CELLP, DIST, PS1, PLNMPT(3), PLNORM(3),
* FEED(3), ALPHA, BETA, GAMMA, XM, YX, ADG0R2, BELLP, CELLP
* PS1, DIST, Point(3), NORN(3), SURFC1, NPN1, NPOINT, SURFC2

COMMON/COLOA/DEL1, XM, XZ, YX, ZK, XM
COMMON/CASS/SR(3), ER(3), XO, YO, ZO, RM, 01, Y02, Z02, ERZ(3)
COMMON/MATH/PI, P12, PI02, JTOR, RTUD

DIMENSION DC(3), E1(2), PI(5, 2750), SR2(3), C(3)
MAGSR, OSQRT(SR(1)) OR(1) SR(2) SR(3) CM(3))

DO 5 N=1,3
C FIND DIRECTION COSINES
5 DC(1)=SR(1)/MAGSR
GO TO (10, 20, 30, 40, 50, 60) SURFC2
10 AA=0.0
BB=NR(1)*DC(1) + NR(2)*DC(2) + NR(3)*DC(3)
CC=(X0-POINT(1)) + NORM(1) + (Y0-POINT(2)) + NORM(2) +
* (Z0-POINT(3)) + NORM(3)
GO TO 100
20 AA=(DC(1)**2/ADG0R2**2) + (DC(2)**2/BELLP**2) + (DC(3)**2/CELP**2)
BB=2.0*4(X0*DC(1)/ADG0R2**2) + (Y0*DC(2)/BELLP**2) +
* (Z0*DC(3)/CELPP**2)
CC=(X0**2/ADG0R2**2) + (Y0**2/BELLP**2) + (Z0**2/CELPP**2)
GO TO 100
30 AA=DC(1)*DC(1) + DC(2)*DC(2) + DC(3)*DC(3)
BB=2.0*4(X0*DC(1) + Y0*DC(2) + ZO*DC(3))
CC=X0*Y0*Z0-ADG0R2**2
GO TO 100
40 AA=DC(2)*DC(2) + DC(3)*DC(3)
BB=2.0*4(Y0*DC(2) + ZO*DC(3) - 2.0*ADG0R2*DC(1))
CC=Y0*Y0*Z0-4.0*ADG0R2**2 - 4.0*ADG0R2*X0
GO TO 100
50 SNPS12=SNPS12**2/*DOR
CSP12=CDCE(PS12**2/DOR)
AA=(DC(3)**3/DC(3)**2)*CSP12**2 + (DC(2)**3/DC(2)**2)*SNPS12**2 -
* (DC(1)**3/DC(1)**2)*SNPS12**2 - (Y0**3/DOR**2)
BB=2.0*4(Y0*DC(3) + ZO*DC(2))
CC=(X0**3/ADG0R2**2) + (Y0**2/SNPS12**2) - 2.0*4(Y0*SNPS12**2)
GO TO 100
60 AA=(DC(1)**2/ADG0R2**2) + (DC(2)**2/BELLP**2) + (DC(3)**2/CELP**2)
BB=2.0*4(X0*DC(1) + ADG0R2**2) + (DIST**2/DC(1) + ADG0R2**2) -
* (Y0*DC(2)/BELLP**2) + (DIST**2/CELPP**2)
CC=(X0**2/ADG0R2**2) + (DIST**2/ADG0R2**2) - (Y0**2/BELLP**2)

IF(DABS(AA).LT.1.0D-10) RM=CC/BB
IF(DABS(AA).LT.1.0D-10) GO TO 110
V2=BB**4.0**4.0**CC
IF(V2.*LT.0.0) V2=0.0
RM=V2+4.0**4.0**CC
GO TO 110
110 COMMON XO2=X0*RM**DC(1)
Y02=Y0*RM**DC(2)
Z02 Z00-RCSEOC(3)
GO TO 120, 130, 140, 150, 160, 170, SURFC2
120 NHAT2(I)=NORM(1)
NHAT2(2)=NORM(2)
NHAT2(3)=NORM(3)
GO TO 130
130 NHAT2(I)=-X02*BELLP2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
  *AQROR2**4)
NHAT2(2)=-Y02*AQROR2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
  *AQROR2**4)
NHAT2(3)=-Z02*AQROR2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
  *AQROR2**4)
GO TO 200
140 NHAT2(I)=-X02/AQROR2
NHAT2(2)=-Y02/AQROR2
NHAT2(3)=-Z02/AQROR2
GO TO 200
150 NHAT2(I)=2.0*AQROR2/DSQRT(4.0*AQROR2**2+Y02**2+Z02**2)
NHAT2(2)=-Y02/DSQRT(4.0*AQROR2**2+Y02**2+Z02**2)
NHAT2(3)=-Z02/DSQRT(4.0*AQROR2**2+Y02**2+Z02**2)
GO TO 200
160 NMA=DSQRT(4.0*AQROR2*AQROR2+Z02*CSPS12*SNPS12-
  *Y02*SNPS12*SNPS12)**2+(Y02*SNPS12*CSPS12-Z02*CSPS12*CSPS12)**2)
NHAT2(I)=2.0*AQROR2/NMA
NHAT2(2)=SNPS12*(Z02*CSPS12-Y02*SNPS12)/NMA2
NHAT2(3)=CSPS12*(Y02*SNPS12-Z02*CSPS12)/NMA2
GO TO 200
170 DEN2=DSQRT(4.0*X02*O1ST2)**2/(AQROR2**2)+(Y02*Y02/BELLP2**4)*
  *(Z02*Z02/CLELP2**4))
NHAT2(I)=(X02*O1ST2)/(AQROR2**2+DEN2)
NHAT2(2)=-Y02/(BELLP2**2+DEN2)
NHAT2(3)=-Z02/(CLELP2**2+DEN2)
200 SCALA2=2.0*(O1**2+DEN2)+NHAT2(1)+O1**2+NHAT2(2)+O1**2+NHAT2(3)
DO 250 L=1,3
250 SR2(L)=SCALA2-NHAT2(L)
E12(N)=0.0
DO 300 N=1,3
300 E12(N)=ER(N)/RM
DO 350 K=1,3
SCALA3=2.0*((E12(1)+NHAT2(1)+E12(2)+NHAT2(2)+E12(3)+NHAT2(3)))
350 EH(K)=SCALA3*NHAT2(K)-E12(K)
IF(DABS(SR2(1}}>LT1.0D-5) SR2(1)=1.0D-5
Y=Y02+(X02-SR2(1))/SR2(1)
Z=Z02+(X02-SR2(1))/SR2(1)
U=DQSORT(XC-X02)**2+(Y-Y02)**2+(Z-Z02)**2
RETURN
END
APRIN

SUBROUTINE APRIN(P, ICALL)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 NORM
INTEGER SURFC1, SURFC2
COMMON/APPROM/NPTPPL, NPERIM
COMMON/PARAMS/AURURF, BELLP, CELLP, Dist, PSI, PLPNIT(3), PLNORM(3),
    FEED(3), ALPHA, BETA, GAMMA, LLAM, XC, ADROR2, BELLP2, CELLP2,
    PSI2, DIST2, POINT(3), NORM(3), SURFC1, NPNL, NPOINT, SURFC2
COMMON/FEED/EP(91), ET(91), NP, NT, X, Y, Z
COMMON/CNTRL/NOPT(3), MLIST, LCPT, ICASS, ILIST(100)
DIMENSION P(8, 2750)
READ(1, 10) NPERIM, SURFC1, NPTPPL
10 FORMAT (3I5)
IF (NPERIM. LE. 2) GO TO 250
IF (NPERIM. LT. 60) GO TO 260
IF (SURFC1. LT. 6) GO TO 270
IF (NPTPPL. LT. 2500) GO TO 270
IF ((NPERIM. LE. SURFC1).LE. 0) GO TO 250
READ(1, 20) ((P(I, J), I=1, J=1, J=NPERIM)
20 FORMAT (F10.6)
N=1
DO 2 I=2, NPERIM
IF (P(3, M).EQ. P(3, 1)) I3, 2, 2
3 M=1
2 CONTINUE
X=P(1, M)
Y=P(2, M)
Z=P(3, M)
28 GO TO (30, 40, 50, 60, 61) SURFC1
30 PLNORM(1)=(P(2, 1)-P(2, 2))**2*(P(3, 1)-P(3, 3))-
    *(P(2, 1)-P(2, 3))**2*(P(3, 1)-P(3, 2))**2*
    PLNORM(2)=(P(3, 1)-P(3, 2))**2*(P(1, 1)-P(1, 3))-
    *(P(3, 1)-P(3, 3))**2*(P(1, 1)-P(1, 2))**2*
    PLNORM(3)=(P(1, 1)-P(1, 2))**2*(P(2, 1)-P(2, 3))-
    *(P(1, 1)-P(1, 3))**2*(P(2, 1)-P(2, 2))**2*
    VMAG=DSQRT(PLNORM(1)##2+PLNORM(2)##2+PLNORM(3)##2)
DO 35 K=1, 3
    PLNORM(4-K)=PLNORM(4-K)/VMAG
35 CONTINUE
PLPNIT(1)=P(1, 1)
PLPNIT(2)=P(2, 1)
PLPNIT(3)=P(3, 1)
GO TO 100
40 READ(1, 45) AURURF, BELLF
45 FORMAT(2F10.3)
GO TO 100
50 READ(1, 55) AURURF
55 FORMAT(F10.3)
GO TO 100
60 READ(1, 65) AURURF, PSI
65 FORMAT(F10.3)
GO TO 100
61 READ(1, 70) AURURF, BELLF, CELLP, DIST
70 FORMAT(F10.3)
100 CONTINUE
199 IF(IO11:ICALL).EQ.0) RETURN
200 PRINT 200, ICALL
201 GO TO (320, 330, 340, 350, 360, 370), SURF1
250 PRINT 252, ICALL
252 FORMAT(*'********** INPUT ERROR ON CARD ONE FOR PANEL NUMBER', I4,'**********
', 'EXECUTION TERMINATING **********')
260 STOP
262 FORMAT(*'********** STORAGE DOES NOT EXIST FOR NUMBER OF', I4,'**********')
280 STOP
270 PRINT 272, ICALL
272 FORMAT(*'********** MAXIMUM ILLUMINATION REQUEST IS 2500', I4,'**********')
280 /NTPPL=2500 GO TO 28
320 PRINT 401, PLENPT, PLNORM, NPERIM
330 PRINT 402, AORCF, BELLP, NPERIM
340 PRINT 403, AORCF, NPERIM
350 PRINT 404, AORCF, NPERIM
360 PRINT 405, AORCF, PSI, NPERIM
370 PRINT 406, AORCF, BELLP, CELLP, DIST, NPERIM
RETURN
401 FORMAT('///IOX,*PANEL IS A PLANAR SURFACE', I4,'///
', 'A POINT ON THE REFLECTOR SURFACE (X,Y,Z)***********, F7.2
', 'COMPONENTS OF UNIT NORMAL TO SURFACE (X,Y,Z)***********, F7.2
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
402 FORMAT('///IOX,*PANEL IS AN ELLIPTICAL SECTION', I4,'///
', 'MAJOR AXIS OF ELLIPTICAL REFLECTOR*****************, F7.2/
', 'MINOR AXIS OF ELLIPTICAL REFLECTOR***************, F7.2/
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
403 FORMAT('///IOX,*PANEL IS A SPHERICAL SECTION', I4,'///
', 'RADIUS OF REFLECTOR SPHERE*****************, F7.2/
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
404 FORMAT('///IOX,*PANEL IS A PARABOLIC SECTION', I4,'///
', 'FOCAL LENGTH OF THE PARABOLA*****************, F7.2/
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
405 FORMAT('///IOX,*PANEL IS A SECTION OF A PARABOLIC CYLINDER', I4,'///
', 'FOCAL LENGTH OF THE PARABOLA*****************, F8.3/
', 'FOCAL LINE ROTATION FROM Y-AXIS (PS1)************, F8.3/
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
406 FORMAT('///IOX,*PANEL IS A HYPERBOLIC SECTION', I4,'///
', 'MAJOR AXIS OF REFLECT IN X DIRECTION************, F8.3/
', 'AXIS OF REFLECTOR IN Y DIRECTION************, F8.3/
', 'AXIS OF REFLECTOR IN Z DIRECTION************, F8.3/
', 'NUMBER OF USER-SUPPLIED EDGE POINTS***********, I7)
END
SUBROUTINE FINDXC(P,B)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/COLCLS/DELT,XT,ANINC,P(3,6),RS,XXZ,ZMX,ZMN,VMX
COMMON/HATH/P1,P2,PID2,DTOR,RTOD
COMMON/CTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
DIMENSION P(5,2750),B(3,2)
IF (ICASS.NE.1) GO TO 15
N=1
DO 2 I=2,4
1 IF(PM(3,N)-PM(3,1)) .GE.2,2 N=1
2 CONTINUE
IF(SMOSORT(PM(1,N)+2+PM(2,N)+2+PM(3,N)+2-1.0)
THMAX=DATAN(-PM(3,N)/PM(1,N))
THMAG=THMAX*3.0*ANGINC
XC=HSMDCOS(THMAG)
RETURN
15 RSMOSORT((MX+0(1,2))$2+(YM+0(2,1))$2+(ZM+0(3,2))$2-1.5
THMAX=DATAN(-(ZM+0(3,2))/(XM+0(1,2)))
THMAG=THMAX*3.0*ANGINC
ZMN=HSMDCUS(THMAG)*B(1,2)
RETURN
END

FUNCTION IU1 (INTENT,ITER)
IMPLICIT REAL*8(A-H,O-Z)
INTEGER SURFC1,SURFC2
COMMON/CTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
GO TO (20,30,40,50,60,70,80,90)
20 IF (NOPT(1).EQ.0) GO TO 90
21 IF (NOPT(1).EQ.2) GO TO 91
22 DO 25 I=1,NLIST
23 IF (ILIST(1).EQ.ITER) GO TO 91
24 CONTINUE
GJ TU 90
30 IF (NOPT(2).GT.0) GO TO 91
GJ TU 90
40 IF (NOPT(2).EQ.0) GO TO 90
41 IF (NOPT(2).EQ.2) GO TO 91
GJ TU 22
50 IF (NOPT(3).GE.1) GO TO 91
GJ TU 90
60 IF (NOPT(3).GE.2) GO TO 91
90 I=0
RETURN
91 I=1
RETURN
END
INTEGER INTG

SUBROUTINE INTG(P, MAJOR, MAJOR, AMINOR, FIELD1, FIELD2)

IMPLICIT REAL*8 (A-H, O-Z)

REAL*8 MAJOR, MINOR

REAL*8 NORM, NORM

COMPLEX*16 CTENP, CT2, CT2, CT2, TSZ, TSY, DTZ, DTI, ZIUL, YIUL

ZT, T, FLDY, FLDY(200), NORM(200)

INTEGER SURFC1, SURFC2

COMMON/MATH/P1, P12, P12, DTO, DTO

COMMON/CONT/L/NOPT(3), NLIST, ICASS, ICASS(100)

COMMON/PKANS/AURDF, BELL1, CELLP, CELLP, DIST, PSI, PNPNF(3), PILNORM(3)

FIELD(3), ALPHA, BETA, GAMMA, XAN, XX, AORDZ, BELL2, CELLP2

PS12, DIST2, POINT(3), NORM(3), SURFC1, NPNI, NPPOINT, SURFC2

DIMENSION AMINOR(3), P(5*2750)

DATA HPH1, HPHI1, HPHI2, HPHI3

SEN#99.0

NPARTS=7

ZMAT=PLZ/XAN

CALL SETH(SER#, P1, NPPOINT+1, 3)

DEG#MAJOR

DEG#=DEGATOR

DL0R=AMINOR(1)*DTO

D1CH#AMINOR(3)*DTO

DSTOR=AMINOR(2)*DTO+OCR#0.5

NP#0

N1=0

IF (MAJOR#MAJOR) GO TO 3400

400 COS=CCOS(DIST)

SIN=CCOS(DIST)

GO TO 3425

J400 COS=CCOS(DIST)

SIN=CCOS(DIST)

GO TO 3425

J3400 NPTH=TH1

COS=CCOST+S/IN

ZK=ZLAM#CST

YN=ZLAM#SIN#SINT

IOLD=1

INE#2

FLDZ=(0.0, 0.0)

FLDZ=(0.0, 0.0)

YLD=SEN

Y1=(0.0, 0.0)

Z1=(0.0, 0.0)

3450 CONTINUE

IF (P(I1, IOL0).NE.P(I1, INEW)) GO TO 4000

Z=P(2, IOL0)

FTR=P(3, IOL0)

ERZ=P(4, IOL0)

PR=P(5, IOL0)

JZ=P(2, INEW)-1*NPART

REV=P(3, INEW)-1*RPART

REV=P(4, INEW)-1*RPART

REV=P(4, INEW)-1*RPART
DPH=P(S1,INE1-PH)*PART
CTEMP=CDEXP(DCMPLX(0.000,0K*Z-PH))
CZ1=ERZ*COSD*CTEMP
CY1=(ERY*SINT*ERZ*CTSP)*CTEMP
TSV=(0.000.00)
TSZ=(0.000.00)
DO 3700 N=1,NPARNTS
Z=Z+DZ
ERY=ERY*DERY
ERZ=ERZ*ERZ
PH=PH*DPM
CTEMP=CDEXP(DCMPLX(0.000,0K*Z-PH))
CZ2=ERZ*COSD*CTEMP
CY2=(ERY*SINT*ERZ*CTSP)*CTEMP
TSZ=TSZ+DZ+CY2
TSY=TSY+CY1+CY2
Z1=Z2
CY1=CY2
3700 CONTINUE
ZI=ZI+TSZ*(0.5*DZ)
YI=YI+TSY*(0.5*DZ)
3900 IULD=IULD+1
INEX=INEX+1
GO TO 3400
4000 CONTINUE
YNEW=P(1.10LD)
IF (YULD.EQ.0.SEN) GO TO 4400
4200 DZI=(ZI-1.10LD)*PART
DYI=(YI-Y10LD)*PART
Y=(YNEW-YULD)*PART
CTEMP=CDEXP(DCMPLX(0.000,0K/Y0LD))
CZI=1.10LD*CTEMP
CYI=Y0LD*CTEMP
TSY=(0.000.00)
TSZ=(0.000.00)
DO 4300 N=1,NPARNTS
YULD=YULD+DY
ZIULD=ZIULD+DZI
YIULD=YIULD+DYI
CTEMP=CDEXP(DCMPLX(0.000,0K*Y0LD))
CZ2=Y0LD*CTEMP
CY2=Y0LD*CTEMP
TSZ=TSZ+CY2+Z2
TSY=TSY+CY1+CY2
Z1=Z2
CY1=CY2
4300 CONTINUE
FLDZ=FLDZ+TSZ*(1.5*DY)
FLDY=FLDY+TSY*(1.5*DY)
4400 CONTINUE
YULD=YNEW
ZIULD=ZI
YIULD=YI
ZI=(0.000.00)
IF (P(1.INE1),NE.,SEN) GO TO 3000
FIELDY(NTH)=FLDY
FIELDZ(NTH)=FLDZ
D=D+D1CR
IF (D=.GE.DSTOPR) GO TO 5000
IF (MAJOR.EQ.HPLL) GO TO 400
GO TO 3400
5000 CONTINUE
RETURN
END

FILLP

SUBROUTINE FILLP(P,NPT)
IMPLICIT REAL*A—N,O-Z
COMMON/FEED/EP(91),ETF11),NP,MT,X5,YS,Z5
COMMON/MATH/P1,P12,P1D2,DTOR,RTOD
COMMON/CTRL/IOPT(3),NLIST,IOPT,ICASS,ILIST(100)
DIMENSION P(N,NPT)
DO 100 I=1,NPT
PROJX=DIN(P(1.l))=DCOS(P(2.I))
PROJEX=0.000
IF (DABS(P(2.I)—PII).GT.1.D-5)
$1
PROJEX=DSIN(DATAN(DCOS(P(1.1))/DS1N(P(1.1))/DSIN(P(2.I))))
ANGLX=DARCOS(DABS(PROJX))=RTOD
LO=ANGLX+1.000
IH=LO+1
PPFGLD=(ANGLX—DFLOAT(LO—1))(*(EP(IHl)—EP(LO))*EP(LO)
TPFLO=(ANGLX—DFLOAT(LO—1))*(ET(IHl)—ET(LO))*ET(LO)
SINE2=PROJX*PROJEX
C05E2=1.000—SINE2
P(3.I)=PPFGLD*TFPLD/(DSQRT(TPFLD*TPFLD*C05E2+*
PPFLD*PPFLD*SINE2))
P(4.I)=0.000
P(5.I)=0.000
100 CONTINUE
RETURN
END
SUBROUTINE QUANTZ(P,NPERIM,ICALL)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 NORM
INTEGER SURFC1,SURFC2
COMMON/BLKC/GLBL,2CBL,2MBL,2MSCBL,2MCBL
COMMON/DIMENS/VOIM,2DIN,YCT,2CT
COMMON/EXTENT/YMIN,YMAX,ZMIN,ZMAX
COMMON/CONTROL/UPTR(3),HIST(100),OPT1,ICASS1,ICASS1(100)
COMMON/FEED/EPTR(3),ET(91),NP,NT,AS,YS,ZS
COMMON/PARAMS/AURURF,SCALEL,CELLP,DIST,PSI,PNPNT(3),PLNORM(3)
* FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,ADORR,BELLP2,CELLP2
PSLZ,DISTZ,POINT(3),NORM(3),SURFC1,NPNT,SURFC2
DIMENSION PS(NPNT),PI(100),PR(5,41),(Z(2,101)
IF(1CASNEQ.1) NPERIM=4
NBARS=NP-2
YMIN=1.0D+10
YMAX=-1.0D+10
ZMIN=1.0D+10
ZMAX=-1.0D+10
NUS=2*NBAR
DO 20 I=NPERIM
CALL SETM(1.0D+20,2,NOS)
20 CONTINUE
IF (P(I,1).GT.YMAX) YMAX=P(I,1)
IF (P(I,1).LT.YMIN) YMIN=P(I,1)
IF (P(I,1).GT.ZMAX) ZMAX=P(I,1)
IF (P(I,1).LT.ZMIN) ZMIN=P(I,1)
CALL MOVLMP(I,1),PR(1,1),5)
YMIN=YMAX-YMIN
ZMIN=ZMAX-ZMIN
YCT=YMAX+YMIN)/2.
ZCT=ZMAX+ZMIN)/2.
GRID=(YMAX-YMIN)/(DFLOAT(NBARS)-0.5)
GRID0=(YMAX-YMIN)/GRID/5.00
GRID1=(YMAX-YMIN)/GRID/5.00
31 CONTINUE
IBGN=NPERIM+1
NUEX=NPERIM
GO TO 98
IF (P(I,1).GT.YMAX) GO TO 98
IF (P(I,1).LT.YMIN) GO TO 98
NGRID=(P(I,1)-GRID0)/GRID0.5
P(I,1)-GRID0+FLOAT(NGRID)*GRID
CALL MOVELMP(I,1,P(I,1),NDEX),5)
GO TO 100
GO TO 100
GO TO 100
CONTINUE
NPOINT=NPOINT-NUEX
CALL PTSORT(P5,NPOINT)
IF (IUPTEQ.1) GO TO 422
CALL MOVELMP(P5(1),P5(1,NPERIM+1),5)
KDEX=2
Y2=PNS(1,1)
Z2=PNS(2,1)
200 CONTINUE
Y1=PNS(1,KDEX)
Z1=PNS(2,KDEX)
IF (DABS(Y1-Y2).LT.1.0D-5) GO TO 400
SLOPE=(Z2-Z1)/(Y1-Y2)
B=2-SLOPE*Y2
IF (Y1-Y2) EQ 0.0 GO TO 330

Y1=Y2
GO TO 240

Y1=V1
YLO=Y2

INDEX=INDEX+1
YV=GRID+GRID+INDEX-1*GRID
IF (YV+GRID) EQ INDEX GO TO 409
YLO=1
ZEE=SLOPE*YQ+B
IF (Z1+INDEX) LT 1.00+10) ILAD=2
Z1=ILAD+INDEX-ZEE
GO TO 250

V2=Y1
Z2=Z1

KDE=KDE+1
IF (KDE.LE.NPERI+1) GO TO 200

VO 420 0=1, NBARS
IF ((Z1*Z2), GT 1.00+10) GO TO 100B
IF (Z1-Z2) EQ 1.00+10) 410, 420, 420

Z2=Z2-1)
Z1=Z1+1)

Z1(1)=Z1(1)
Z1(1)=Z2

CONTINUE

GO TO 444

HFMAX=YV*1M/2.000

HFMAX=YV*1M/2.000

GRID=GRID+GRID+GRID

GRID=DALT/-1)*GRID
V3=1.000-(Y-YC)/HFMAX)*2
IF (V3 LT 0.000) V3=0.000
Z2=HFMAX*6.000
Z1=Z1-Z2*Z3
Z1(1)=Z1-Z2(1)

CONTINUE

CONTINUE

L=0

N=1

CALL SLT(0.0, P1, K, 5)

YV=P(1,1)

INDEX=INDEX+(YV-GRID) /GRID+1.001

GRID=INDEX+1, NPRI
IF (P1,1)=EQ.YV) GO TO 490
IF (L+GT 2) GO TO 470

N=N-L

L=0

INDEX=INDEX+(YV-GRID) /GRID+1.001

P1(1)=P(1,1)

P1(1)=P(1,1)

TEST=1.0

IF (Y(2,1)+EQ.Z(1,1)*INDEX)+P(1,1)=EQ.Z(2,1)*INDEX) TEST= 0,0

IF (P2,1)+LT.Z(1,1)*INDEX)+AND.P(1,1)=LT.Z(2,1)*INDEX) TEST=1.0

TESTUL=*MAUL+*MAUL+*MAUL+*MAUL+*MAUL
* 
* -HFMAUL*HFMAUL*(P(2.1)-ZCBL)*(P(2.1)-ZCBL)
* -HFMAUL*HFMAUL*(P(2.1)-ZCBL)*(P(2.1)-ZCBL)

IF (TEST) 701,501,501
501 IF (TESTBL.LE.0.0) GO TO 510
CALL MOVE(MBLK,P(1,N),5)
GO TO 515
510 CALL MOVE(P(1,1),P(1,N),5)
515 N=N+1
L=L+1
IF (TEST.EQ.0.0) GO TO 800
701 IF (L.EQ.0.0) GO TO 800
IF (TEST*TEST) 705,800,800
704 CALL INTL(P1,MLK,P(1,1),P(1,1),INDEX)
NMC=G
IF (INDEX*INDEX) GO TO 711
CALL MOVE(MBLK,P(1,N-NCM),5)
GO TO 720
711 CALL MOVE(P(1,N),P(1,N),2)
TEST*:HFMAUL*HFMAUL*HFMAUL*HFMAUL
* -HFMAUL*HFMAUL*(P(2.1)-ZCBL)*(P(2.1)-ZCBL)
* -HFMAUL*HFMAUL*(P(2.1)-ZCBL)*(P(2.1)-ZCBL)
IF (TEST.EQ.0.0) GO TO 720
CALL MOVE(MBLK,P(1,1),NCM,5)
GO TO 725
720 CALL MOVE(P(1,1),P(1,N),5)
725 N=N+1
L=L+1
800 CALL MOVE(P(1,1),P(1,N),5)
TEST:TEST
900 CONTINUE
NPUN=NPUN-1
NLUC=NPUN
CALL MOVE(PR,P(1,N),NLUC)
IF(LU(1,1).EQ.0) WRITE (29,600) NPUN,CALL
IF(LJ(1,1).EQ.0) WRITE (29,620) (PLJ,I=1,5),J=1,NPUN
IF(LJ(1,1).EQ.0) RETURN
PRINT 900, YMIN, YMAX, ZMIN, ZMAX, GRID1, GRID2, NBAR, NPUN
950 FORMAT(10//QUANTIZATION DATAF//
  * POINT PATTERN EXTENTS ON APERTURE PLANE...............YMIMA,F7.2/
  * ............YMAX,F7.2/
  * ............ZIMA,F7.2/
  * ............ZMAX,F7.2/
  * GRID RANGES FROM.................................,F8.3* TO/
  * ,F8.3/
  * SPACING BETWEEN GRID BARS IS...............................*,F8.4/
  * THE HORIZONTAL NUMBER OF GRID BARS........................*,F8.4/
  * THE VERTICAL NUMBER OF GRID BARS........................*,F8.4/

951 FORMAT(11D12)
952 FORMAT(5D16.14)
RETURN
1000 PRINT *,NBAR
1000 FORMAT(/'*'************ ARRAY FOR RESIDUE NOT FILLED CORRECTLY
 ************,10X,-* STOP EXECUTION -*
STOP
END
8.7. APPENDIX G

OUTPUT FOR TEST CASES A AND B
INPUT PARAMETERS:

- **WAVELENGTH OF ELECTRIC FIELD**
  - Value: 4.7340

- **LOCATION OF ORIGIN WRT FIELD (X,Y,Z)**
  - Values: -65.600, -117.304, 0.0

- **FEED ROTATION ANGLES (ALPHA, BETA, GAMMA)**
  - Values: 0.0, 0.0, -180.000

- **APERTURE PLANE LOCATION (XC)**
  - Value: 0.0

- **SUB DISH SHADOW CENTER COORDINATES IN APERTURE PLANE**
  - Values: 0.0, 0.0

- **HALF MAJOR AXIS OF SUB DISH SHADE**
  - Value: 0.0

- **NUMBER OF PANELS IN REFLECTOR**
  - Value: 0

MAIN DISH DESCRIPTION AND ITS PARAMETERS:

- **IT IS A PARABOLIC REFLECTOR**

- **FOCAL LENGTH OF THE REFLECTOR**
  - Value: 100.000

- **MINIMUM Y POINT ON THE REFLECTOR (X,Y,Z)**
  - Values: -65.600, -117.304, 0.0

- **MAXIMUM Y POINT ON THE REFLECTOR (X,Y,Z)**
  - Values: -65.600, 117.304, 0.0

- **MINIMUM Z POINT ON THE REFLECTOR (X,Y,Z)**
  - Values: -65.600, 0.0, -117.304

- **MAXIMUM Z POINT ON THE REFLECTOR (X,Y,Z)**
  - Values: -65.600, 0.0, 117.304
SUBDISH DESCRIPTION AND ITS PARAMETERS:

IT IS A HYPERBOLIC REFLECTOR
MAJOR AXIS IN X-DIRECTION: 70° 950
AXIS OF REFLECTOR IN Y-DIRECTION: 33° 950
DISTANCE USED FOR TRANSLATION OF ORIG. OF AXES: 45° 500

MINIMUM-Y POINT ON THE REFLECTOR (x,y,z): -10° 712 -19° 156 0° 0
MAXIMUM-Y POINT ON THE REFLECTOR (x,y,z): -10° 712 19° 156 0° 0
MINIMUM-Z POINT ON THE REFLECTOR (x,y,z): -10° 712 0° 0 -19° 156
MAXIMUM-Z POINT ON THE REFLECTOR (x,y,z): -10° 712 0° 0 19° 156

2 PATTERN GROUP REQUESTED

THETA = 90° 0000
PHI FROM = -4° 0000 TO 4° 0000 BY 0° 2500
THETA FROM = 36° 0000 TO 6° 0000 BY 0° 2500

ILLUMINATION DATA:

THETA ILLUMINATION FROM = 76° 075 TO 103° 525
PHI ILLUMINATION FROM = 186° 075 TO 123° 525
INCREMENTAL ANGLE (DEG) = 0° 6325
TOTAL NUMBER OF GENERATED POINTS: 2025
TOTAL NUMBER OF APERTURE PLANE POINTS: 2025

FINISHED APERTURE

---
<table>
<thead>
<tr>
<th>PHI</th>
<th>PHI/Z/Y</th>
<th>PHI/Y/Y</th>
<th>PHI (Z/Y)</th>
<th>PHI (Y/Y)</th>
<th>PHI (Z/Y)</th>
<th>PHI (Y/Y)</th>
<th>PHI (Z/Y)</th>
<th>PHI (Y/Y)</th>
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</thead>
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<td>-1.250</td>
<td>-31.27215</td>
<td>-152.69030</td>
<td>121.41815</td>
<td>0.0</td>
<td>-31.27215</td>
<td>0.0</td>
<td>-31.27215</td>
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<td>96.03906</td>
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<td>0.0</td>
<td>-28.3043</td>
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<td>-152.69030</td>
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<td>-24.12722</td>
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<td>-24.12722</td>
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**Maximum Field Values**

\[
\text{20log}(\text{max}(\text{FIELD}=7)) = 20\text{log}(4.3103377) = 92.6502594
\]

\[
\text{20log}(\text{max}(\text{FIELD}=y)) = 20\text{log}(1.1134110-13) = -60.0000000
\]
### TABLE OF ELECTRIC FIELD STRENGTHS 1921

**Principal Plane of Cut is PHI = 0.0 Deg**

**Angle Theta from 56.000 to 54.000 by 0.250 Deg**

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APERTURE PLANE AFTER QUANTIZING
FAR FIELD RADIATION PATTERN CALCULATION

OFFSET CASSEGRAIN ANTENNA EXAMPLE...
A PARABOLOID- HYPERBOLOID COMBINATION
MARCH 27 1981
LUCAS POURARIKHAN
TICRA API

INPUT PARAMETERS-

WAVELENGTH OF ELECTRIC FIELD
LOCATION OF COORDINATE ORIGIN OF FIELD (x,y,z)
FIELD ROTATION ANGLES (ALPHA, BETA, GAMMA)
APERTURE PLANE LOCATION (x,y,z)
SIDE DISH SHADOW CENTER COORDINATES IN APR T. PL.
HALF MAJOR AXIS OF SIDE DISH SHADOW
HALF MINOR AXIS OF SIDE DISH SHADOW
NUMBER OF PANELS IN REFLECTOR

\[ 0.9843 \]
\[ -3.725 \ J, \ 9.337 \]
\[ 0.0 \ 0.0 \ -153.600 \]
\[ 0.0 \ 0.0 \]
\[ 0.5 \ 0.0 \]
\[ 0.0 \]
### Paraboloid Description and Its Parameters

**It is a parabolic reflector.**

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### Spherohiph Description and Its Parameters

**It is a hyperbolic reflector.**

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PATTERN OF FEED IN ONE DEG DEGREEMENTS OFF-AXIS

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\text{\textdegree} & \text{0.0000000000} & \text{9.999999999} & \text{9.000000000} & \text{9.999999999} & \text{9.000000000} \\
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### Pattern of Field in One Deg Increments Off-Axis -

**E-Plane**

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**Number of Pattern Grounds Rejected**

---

**Phi = 0.00**
**Theta = 90.0000**

**Phi from -270.000 to 270.000 by 0.000**
**Theta from 0.0000 to 90.0000 by 0.0000**
ILLUMINATION DATA:

THETA ILLUMINATION FROM: 79.379 TO 101.848
PHI ILLUMINATION FROM: 108.750 TO 191.235
INCREMENTAL ANGLE (DEG): 0.5127

TOTAL NUMBER OF GENERATED RAYS: 2725
TOTAL NUMBER OF APERTURE PLANE POINTS: 2029

--------------- FINISHED APERTURE ---------------

QUANTIZING DATA:

POINT PATTERN EXTENTS ON APERTURE PLANE:

YMIN = -3.087
YMAX = 3.087
ZMIN = 1/7.77
ZMAX = 3/7.73

GRID RANGES FROM:

-3.506 TO 3.508

SPACING BETWEEN GRID BARS AS:

4.504

RESULT: NUMBER OF GRID BARS: 43

NUMBER OF POINTS SUPPLIED TO RADPAT: 1479

--------------- FINISHED QUANTIZ ---------------

--------------- FINISHED INTENS ---------------

--------------- FINISHED INTENS ---------------

--------------- PATTERN COMPUTATIONS COMPLETE ---------------
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**Table of Experimental Data Showing**

**Principal Plane of View: 20°**

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### Table of Electrical Field Dispersions (cont.)

**Principal Plane of Cut Is Theta = 90.00 Deg**

**Angle Phi From -2.00 Deg to 2.00 Deg by 0.080 Deg**

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**Maximum Fill Values:**

\[ L = \text{MAX}(\text{FILL} - c), L = \text{MIN}(\text{FILL} + c) = -0.0013347 \]

\[ L = \text{MIN}(\text{FILL} + \gamma), L = \text{MIN}(\text{FILL} + \gamma) = 1.0023547 \]

*Interpolation number used for integration is***********