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Information Theory Lateral Density Distribution for Earth Inferred from Global Gravity Field

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INFORMATION THEORY LATERAL DENSITY DISTRIBUTION FOR EARTH INFERRED FROM GLOBAL GRAVITY FIELD

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ABSTRACT

Information Theory Inference, better known as the Maximum Entropy Method, is used to infer the lateral density distribution inside the earth. The approach assumes that the earth consists of indistinguishable Maxwell-Boltzmann particles populating infinitesimal volume elements, and follows the standard methods of statistical mechanics (maximizing the entropy function). The GEM 10B spherical harmonic gravity field coefficients, complete to degree and order 36, are used as constraints on the lateral density distribution. The spherically symmetric part of the density distribution is assumed to be known. The lateral density variation is assumed to be small compared to the spherically symmetric part. The resulting information theory density distribution for the cases of no crust removed, 30 km of compensated crust removed, and 30 km of uncompensated crust removed all give broad density anomalies extending deep into the mantle, but with the density contrasts being the greatest towards the surface (typically ±0.004 g cm⁻³ in the first two cases and ±0.04 g cm⁻³ in the third). None of the density distributions resemble classical organized convection cells. The information theory approach may have use in choosing Standard Earth Models, but, the inclusion of seismic data into the approach appears difficult.
INTRODUCTION

The problem addressed here is inferring via information theory the lateral density variation inside the earth from the observed external anomalous gravity field. Information theory is used because it is the least subjective way to deal with inverse problems (Baierlein, 1971). The motivation for this study is the relationship of the lateral density variation to tectonics and convection.

The nature of the problem is the following. The lateral density variation inside the earth generates the observed gravity anomalies. Hence information about the lateral density variation is provided by examining the anomalies. However, the observed gravity anomalies cannot be inverted to recover the actual density variation. The problem is nonunique: there are an infinite number of density distributions which can generate the observed gravity field. This is unfortunate, since it is desirable to know the lateral density variation, especially with regard as to how it relates to tectonics and convection (Phillips and Lambeck, 1980).

The nonuniqueness can be dealt with by various approaches in order to obtain insight into the physics of the earth. Of these modeling is by far the most common approach. Here extra assumptions are introduced until the solution to the problem becomes unique. Kaula (1963), for instance, assumes that the shear strain energy of the mantle is minimized. This key assumption, and other minor ones, together with the constraints of the observed gravity field determine a unique lateral density variation. Phillips and Lambeck (1981) review many papers which use modeling. Another approach is the Backus-Gilbert method (Backus and Gilbert, 1967, 1968; Parker, 1977), which studies all possible solutions consistent with the given data. This study is called the geophysical inverse problem (Backus and Gilbert, 1967, p. 249). The Backus-Gilbert method has been used
extensively in seismology (e.g., Jordan and Franklin, 1971). Burkhard and Jackson (1976) apply the method to gravity data. There are also other approaches to data inversion; see, e.g., Parker (1977), Sabatier (1977), and references contained therein.

The approach to the lateral density variation used here is that of information theory (Jaynes, 1957, 1963, 1967). This approach is commonly known as the Maximum Entropy Method, or MEM for short. A better name would be Information Theory Inference (ITI for short), since information theory is its basis and the name avoids confusion with thermodynamic entropy (Baierlein, 1971, pp. 473-478). It will be called ITI here.

ITI is a probabilistic approach to nonuniqueness. Each possible answer (labelled i) to a non-unique problem is assigned a probability \( P_i \) that it is the correct answer. The probabilities \( P_i \) are assigned numerical values so as to maximize Shannon's (1948) information measure

\[
\text{MI} = - \sum P_i \ln P_i
\]  

subject to the constraints of the known data. MI in (1) stands for "Missing Information," i.e., the amount of information needed to determine which answer is correct (Baierlein, 1971, p. 64). In practice the expectation value of the desired unknown quantity is taken as the inferred answer to the problem. The information theory approach thus provides a solution to what will be called the geophysical inference problem: picking one answer out of a number of possible answers as the most likely to be true. ITI may hence be regarded as a complementary method to the Backus-Gilbert method (Gull and Daniell, 1978), which investigates the geophysical inverse problem.

The rationale for using ITI is that it picks the "best" answer, dictated by the information at hand, out of the many possible answers. "Best" here means "least subjective," i.e. the number of unconscious assumptions in choosing an answer is minimized (Tribus and Rossi, 1973). See Baierlein (1971, pp. 11-89) for an excellent introduction to ITI.
ITI (or MEM) has been used with great success in several different fields. One is statistical mechanics (Jaynes, 1957; Tribus, 1961; Katz, 1967; and Baierlein, 1971). In fact, (1) is the entropy function of statistical mechanics. The only difference between ITI and statistical mechanics lies in ITI's powerful information theory foundation, which allows the approach to be applied to a wide variety of problems, and not just to statistical mechanics. It has been so applied to spectral analysis (Burg, 1967, 1968, 1972) and to radio brightness maps of the sky (Gull and Daniell, 1978). In solid earth geophysics ITI has been applied to the spectral analysis of polar motion (e.g., Smylie et al., 1973; Graber, 1976), and to the radial density distribution of the earth (Rietsch, 1977; Rubincam, 1978, 1979; Graber, 1977) and of the planets (Koyama, 1979).

The present work is an extension of Rubincam (1979) to the lateral density variation. ITI is used to infer the lateral density structure based on the spherical harmonic coefficients of the gravity field and on an assumed spherically symmetric density distribution (also called the radial density distribution). The gravity field coefficients are those of GEM 10B (Lerch et al., 1981), complete to degree and order 36. The hydrostatic equilibrium bulge is subtracted out of the \( R = 2,4, m = 0 \) terms using the hydrostatic coefficients of Nakiboglu (1979). Also subtracted from the GEM 10B terms when the need arises are the gravity field coefficients of a crustal model. Two different crustal models are used: one a 30 km-thick isostatically compensated crust and the other an isostatically uncompensated crust, also 30 km thick. Carl Wagner supplied the spherical harmonic coefficients for these models (Wagner, private communication, 1976). Both sets of coefficients are complete to degree and order 36. The radial density distribution is the average structure Parametric Earth Model (PEM) of Dziewonski et al., (1975). Further, the earth is assumed to be a sphere and that the lateral density variation is small compared to the radial density distribution.

The principal results are as follows. The information theory density distribution can be written as a spherical harmonic expansion. The equation for the density variation is similar in form
to that of the equation which gives density contrasts due to lateral temperature differences.

For the cases where no crust or the 30 km thick compensated crust is removed the density contrasts are greatest near the earth's surface and have typical magnitudes of \( \pm 0.004 \text{ g cm}^{-3} \). The density contrasts are also greatest near the surface for the case of 30 km of uncompensated crust removed and are typically a factor of 10 larger than in the two other cases. In all three cases the density contrasts decrease with depth but significant density anomalies still extend deep into the mantle. None of the three density distributions look like classical convection patterns, i.e., organized cells with columns of low density where material is rising and columns of high density where material is sinking. No attempt has been made in any of the cases to compute stresses or stress-differences.

DERIVATION OF THE INFORMATION THEORY DENSITY DISTRIBUTION

The information theory density distribution is derived from the following considerations.

It is first assumed that the data consist of the known values \( I_q \) for Q integrals of the form

\[
I_q = \int_V \rho(\vec{r}) f_q(\vec{r}) \, dv, \quad (q=1, 2, \ldots, Q),
\]

where \( \rho(\vec{r}) \) is the earth's density distribution, \( f_q(\vec{r}) \) is a function which depends on position \( \vec{r} \) inside the earth, \( dv \) is a volume element, and \( V \) is the volume of the earth. Examples of integrals of this form are the mass and moment of inertia of the earth. The gravity field coefficients also have this form, since they can be written (Phillips and Lambeck, 1980, p. 30)

\[
\bar{C}_{\ell m_1} = \frac{\int_V \rho(\vec{r}) r^\ell \, \bar{Y}_{\ell m_1}(\theta, \lambda) \, dv}{(2\ell + 1) M_E a_E^6}
\]

where \( \bar{C}_{\ell m_1} = \bar{C}_{\ell m} \) and \( \bar{C}_{\ell m_2} = \bar{S}_{\ell m} \) are the normalized coefficients of degree \( \ell \) and order \( m \), \( r = |\vec{r}| \), and the \( \bar{Y}_{\ell m_1}(\theta, \lambda) \) are surface spherical harmonics using Kaula's (1967) \( 4\pi \) normalization.
with $\theta$ being colatitude and $\lambda$ longitude. $M_E$ and $a_E$ are the mass and radius of the earth, respectively. Obviously in this case $F_q = \tilde{C}_{gml}$ and

$$f_q(\vec{r}) = \frac{r^6 Y_{gml}(\theta, \lambda)}{(2l+1) M_E a_E^6}$$  \hspace{1cm} (4)

The next consideration in using ITI is to set up the earth models which constitute the various possible answers to the problem. The task is then to choose the "best" model based on the data of the form (2). The earth models are set up as follows: the earth is divided up into infinitesimal cubes, all with equal volume $dv = dx dy dz$. Each cube is labelled with the running subscript $j$. The position vector from the center of the earth to the $j$th cube is $\vec{r}_j$. The cubes are populated with indistinguishable particles of mass $m$. Each earth model $i$ has $n_{ji}$ particles in the $j$th cube where $n_{ji}$ is an integer $\geq 0$. The density of the earth at position $\vec{r}_j$ in the $i$th model is then $\rho_i(\vec{r}_j) = n_{ji} m / dv$. The integrals (2) for each earth model $i$ become the sums

$$F_{qi} = \sum_j \left( \frac{n_{ji} m}{dv} \right) f_q(\vec{r}_j) \ dv = m \ \sum_j n_{ji} f_q(\vec{r}_j).$$  \hspace{1cm} (5)

The expectation values

$$\langle F_q \rangle = \sum_i P_i F_{qi}$$  \hspace{1cm} (6)

are assumed to constitute the observed values of $F_q$, where $P_i$ is the probability that the $i$th model is in fact the correct model.

The problem so formulated is analogous to the standard statistical mechanics problem of determining the population numbers of indistinguishable particles following Bose-Einstein statistics using the grand canonical ensemble (Rubincam, 1979). (Indistinguishable particles are chosen since the interchanging of particles does not affect the density distribution, which is the topic under discussion.) The solution can thus be carried out in the usual statistical mechanics fashion.
(Morse, 1969, pp. 316-319; Reif, 1965, pp. 346-349). (1) is maximized subject to the constraints of the data (6) and \( \Sigma P_i = 1 \):

\[
\frac{\partial}{\partial P_i} \left[-\Sigma P_i \ln P_i + \alpha_0 \Sigma P_i + \alpha_1 \Sigma P_i F_{11} + \ldots + \alpha_Q \Sigma P_i F_{Q1}\right] = 0
\]

where \( \alpha_0, \ldots, \alpha_Q \) are Lagrange multipliers, yielding \( P_i = \exp(\Sigma \alpha_q F_{qi})/Z \), where

\[
Z = e^{1-\alpha_0} = \sum_i \exp(\Sigma \alpha_q F_{qi})
\]

is the grand partition function. Using (5) in (7) gives

\[
Z = \sum_i \exp \left[ \alpha_1 \sum_j n_{ij} f_1(\vec{r}_j) + \ldots + \alpha_Q \sum_j n_{ij} f_Q(\vec{r}_j) \right]
\]

for \( Z \), where \( \max_q \) has been redefined as \( \alpha_q \). Note that the partial derivative of the logarithm of (8) with respect to \( \alpha_1 f_1(\vec{r}_j) \) gives

\[
\frac{\partial \ln Z}{\partial (\alpha_1 f_1(\vec{r}_j))} = \sum_j n_{ij} \exp \left[ \alpha_1 \sum_j n_{ij} f_1(\vec{r}_j) + \ldots + \alpha_Q \sum_j n_{ij} f_Q(\vec{r}_j) \right]
\]

\[
= \sum_j n_{ij} P_i = \langle n_j \rangle
\]

which is the expectation value of the number of particles in the cube at position \( \vec{r}_j \). This result will be used shortly.

If there are no limits to the number of particles occupying each cube, then (8) can be factored as (Morse, 1969, p. 326; Reif, 1965, p. 347)

\[
Z = \prod_j Z_j
\]
where

$$Z_i = \sum_{n_j=0}^{\infty} \exp \left\{ [\alpha_1 f_1(\vec{r}_j) + \ldots + \alpha_Q f_Q(\vec{r}_j)] n_j \right\}$$

(10)

But this has the form $$\sum_{n=0}^{\infty} x^n$$, which is equal to $$1/(1-x)$$. Hence (10) becomes

$$Z_i = \frac{1}{1-\exp \left[ \alpha_1 f_1(\vec{r}_j) + \ldots + \alpha_Q f_Q(\vec{r}_j) \right]}$$

(11)

Therefore

$$\langle n_j \rangle = \frac{1}{e^{\alpha_1 f_1(\vec{r}_j)} - \ldots - e^{\alpha_Q f_Q(\vec{r}_j)} - 1}$$

(12)

by (9). If it is now assumed that $$\langle n_j \rangle \ll 1$$, so that the Bose-Einstein statistics pass over to Maxwell-Boltzmann statistics (Reif, 1965, p. 352; Morse, 1969, p. 329), then the exponential term in the denominator of (12) is much greater than 1, and (12) simplifies to

$$\langle n_j \rangle \approx e^{\alpha_1 f_1(\vec{r}_j)} + \ldots + e^{\alpha_Q f_Q(\vec{r}_j)}$$

Multiplying this by $$m/dv$$ and dropping the subscript $$j$$ gives the expression

$$\rho(\vec{r}) = \frac{m}{dv} \langle n_j \rangle = \frac{m}{dv} e^{\alpha_1 f_1(\vec{r})} + \ldots + e^{\alpha_Q f_Q(\vec{r})}$$

(13)

as the information theory density distribution. This is a most important result: it gives the form of the information theory density distribution for particles following Maxwell-Boltzmann statistics where the constraints on the density distribution have the form (2). The variance of the distribution is discussed in Appendix A.

The $$m/dv$$ appearing in (13) is a troublesome factor. No commitment has been made either to the value of $$m$$ or $$dv$$; and there seems to be no clear guidance on how to choose their values. As it turns out, this problem may be avoided by absorbing the factor into the spherically symmetric part of the density distribution, as shown next.
The task now is to simplify (13). This involves two assumptions: first, that $f_1(\vec{r})$ pertains solely to the spherically symmetric part of the density distribution (explained below); and second, that

$$|\alpha_2 f_2(\vec{r}) + \ldots + \alpha_Q f_Q(\vec{r})| \ll 1$$

so that

$$\alpha_2 f_2(\vec{r}) + \ldots + \alpha_Q f_Q(\vec{r}) \approx 1 + \alpha_2 f_2(\vec{r}) + \ldots + \alpha_Q f_Q(\vec{r})$$

where the $f_2(\vec{r}), \ldots, f_Q(\vec{r})$ are the appropriate functions (4) for the gravity field (i.e. the anomalous density variation is small compared to the radial density.) With these two assumptions (13) becomes

$$\rho_1(\vec{r}) \approx \frac{m}{dV} e^{\alpha_1 f_1(\vec{r})} \left[ 1 + \sum_{\ell m i} \frac{\alpha_{\ell m i} r^\ell Y_{\ell m i}(\theta, \phi)}{(2\ell+1)M_E a_E^\ell} \right]$$

which may be written

$$\rho_1(\vec{r}) \approx \rho_o(\vec{r}) + \sum_{\ell m i} \frac{\alpha_{\ell m i} \rho_o(\vec{r}) r^\ell Y_{\ell m i}(\theta, \phi)}{(2\ell+1)M_E a_E^\ell}$$

where

$$\rho_o(\vec{r}) = \frac{m}{dV} e^{\alpha_1 f_1(\vec{r})}$$

with $\rho_o(\vec{r})$ being the spherically symmetric part of $\rho(\vec{r})$ and where subscripts $\ell m i$ have been substituted for subscript $q$. Obviously taking $f_1(\vec{r})$ to be

$$f_1(\vec{r}) = \frac{1}{\alpha_1} \ln \left[ \rho_o(\vec{r}) dV/m \right]$$
results in (15). This is artificial, to be sure; no value \( f_1 \) in (2) is known for the earth where \( f_1(\vec{r}) \) has the form given above. However, it has two advantages: the troublesome factor \( m/dv \) disappears, and any desired \( \rho_0(\vec{r}) \) can be used in (14). The \( \rho_0(\vec{r}) \) for the earth is known to a high degree of accuracy from other data: it cannot differ greatly from the PEM \( \rho_0(\vec{r}) \) of Dziewonski et al., (1975). Hence it will be assumed here that the integral \( F_1 \) is known for the earth where \( f_1(\vec{r}) \) is given by the above equation, in order to use Dziewonski et al.'s (1975) radial density distribution.

All that remains to find the information theory density distribution is to evaluate the \( \alpha_{\xi m l} \) using the gravity field coefficients. The raw gravity field coefficients given by (2) will not be used, however, for two reasons. The first reason is that the contribution of the earth's hydrostatic equilibrium rotational bulge to the gravity field must be subtracted out. The second reason is that the gravity field of the crust must also be subtracted from the coefficients when a crustal model is used. In this case the gravity field data become

\[
\hat{c}_{\xi m l} = \tilde{c}_{\xi m l}^{GEM} - \tilde{c}_{\xi m l}^{CR} - \tilde{c}_{\xi m l}^{HE}
\]

for the \( \xi = \text{even}, \ m = 0, \ i = 1 \) terms and

\[
\hat{c}_{\xi m l} = \tilde{c}_{\xi m l}^{GEM} - \tilde{c}_{\xi m l}^{CR}
\]

for the other coefficients. The superscripts GEM, HE, and CR stand for “Goddard Earth Model,” “Hydrostatic Equilibrium,” and “Crust” respectively. Actually only the \( \xi = 2 \) and \( \xi = 4 \) hydrostatic equilibrium coefficients computed by Nakiboglu (1979) for Dziewonski et al.'s (1975) PEM will be used here; the higher degree hydrostatic equilibrium terms are assumed to be zero. Strictly, these terms should be included, but their computation is difficult and the error in ignoring them is probably small. All of the crustal terms up to and including degree and order 36 will, however, be subtracted from the GEM 10B coefficients when a crustal model is used.
Substituting (14) for \( \rho(\mathbf{r}) \) and \( \hat{C}_{\ell m} \) for \( \hat{C}_{\ell m} \) into (2) to evaluate the \( \alpha_{\ell m} \) yields

\[
\rho_I(R) = \rho_o(R) \left[ 1 + \sum_{\ell m} \delta_{\ell m} \hat{C}_{\ell m} R^\ell \nabla_{\ell m}(\theta, \lambda) \right] \tag{16}
\]

as the information theory density distribution, where the

\[
\delta_{\ell m} = \frac{(2\ell + 1) \overline{\rho}_E}{3 \int_0^{R_U} \rho_o(R) R^{2\ell + 2} dR} \tag{17}
\]

are found by using the orthogonality properties of the \( \nabla_{\ell m}(\theta, \lambda) \), where the earth is assumed to be a sphere, and where the variable \( r \) has been replaced by \( R = r/\overline{\rho}_E \) for convenience so that \( 0 \leq R \leq 1 \). Also, \( \overline{\rho}_E \) is the average density of the earth and \( \overline{\rho}_E = 6371 \) km is the radius of the earth. \( R_U \) is the radius of the sphere in which the unknown density distribution to be inferred resides (the subscript \( U \) standing for "Upper.") For example, \( R_U = 1 \) if no crust is stripped off the earth and \( R_U = 6341/6371 \) if a 30 km thick crust is stripped off. The integral in the denominator of (17) can be evaluated analytically, since Dziewonski et al. (1975) break up the earth into eight shells, with \( \rho_o(R) \) being given as a polynomial in \( R \) in each shell. Table 1 gives the \( \delta_{\ell m} \) resulting from this computation for \( R_U = 1 \) and for \( R_U = 6341/6371 \).

RESULTS AND COMPARISONS WITH OTHER STUDIES

The fundamental equations of this paper are (16) and (17). What they give is, in a sense, the broadest possible density anomalies; more localized anomalies are not warranted by the data. Some general features of these equations should be noted before examining specific density distributions.

The information theory density distribution \( \rho_I(\mathbf{r}) \) (the subscript \( I \) standing for "Information Theory") given by (16) is obviously a spherical harmonic expansion of the form

\[
\rho_I(\mathbf{r}) = \rho_o(\mathbf{r}) + \Delta \rho_I = \rho_o(\mathbf{r}) + \sum_{\ell m} \hat{\rho}_{\ell m}(\mathbf{r}) \nabla_{\ell m}(\theta, \lambda) \tag{18}
\]
where

\[ \Delta \rho_1 = \sum_{\ell m l} \tilde{p}_{\ell m l}(R) \widetilde{V}_{\ell m l}(\theta, \lambda), \quad \tilde{p}_{\ell m l}(R) = \delta_\ell C_{\ell m l} R^\ell \rho_0(R). \]

(19)

Note that \( \Delta \rho_1 \) changes discontinuously when \( \rho_0(R) \) does. Also, (16) has a form similar to that of the equation giving a density variation due to lateral temperature differences (e.g., Phillips and Lambeck, 1980, p. 32):

\[ \rho = \rho_0 \left[ 1 - \alpha(T - T_0) \right] \]

(20)

Here \( \alpha \) is the coefficient of thermal expansion, \( \rho_0 \) and \( T_0 \) are the reference density and reference temperature, respectively, while \( T \) is the temperature. (The reason (16) has this form is due to the assumption that \( \Delta \rho_1 \) is small compared to \( \rho_0(R) \)). So (16) is consistent with the idea that the density anomalies are due to lateral temperature differences—but does not necessarily imply that the anomalies are so caused.

It should further be noted that there is no point in computing the power spectrum of the anomalous potential \( V_k^2(\Delta U) \) (e.g., Phillips and Lambeck, 1980, equation 11) generated by the density distribution (16), since the field coefficients are the given data, which (16) automatically satisfy. Moreover, the information theory density distribution (16) does not have a white noise spectrum, which Lambeck (1976) (see also Phillips and Lambeck, 1980) found will reproduce the observed anomalous potential spectrum (i.e., Kaula's 1967 rule-of-thumb). Instead the information theory density distribution \( \rho_1(\vec{R}) \) concentrates the density anomalies towards the earth's surface, due to the \( R^k \) behavior of \( \tilde{p}_{\ell m l}(R) \) and the \( \ell^2 \) behavior of \( \delta_\ell \) in (19). (The \( \ell^2 \) behavior may be seen by substituting \( \tilde{p}_\ell \) for \( \rho_0(R) \) in (17) and evaluating the integral.) Also, it is clear from (19) that the lower degree anomalies are spread more evenly throughout the earth than the higher degree anomalies. This concentration of density anomalies towards the surface is in contrast to the findings of the Monte Carlo studies of Kaula (1977) and the mass-point studies of Lowrey (1978),
who indicate that the anomalies may increase with depth. Dziewonski et al. (1977) and Julian and Sengupta (1973), among others, also indicate that large anomalies are to be found deep within the mantle on the basis of seismic travel time studies. The statistical gravity study of Khan (1977), however, places most of the anomalies in the upper mantle. The seismic studies of Romanowicz (1979) and Cara (1979) and others show considerable upper mantle lateral structure, while the analysis of satellite-to-satellite tracking data by Marsh et al. (1981) indicates that most of the gravity anomalies in the Pacific can be explained by lithospheric sources.

It should be mentioned that the density anomalies given by (16) and (17) are not confined to the crust and mantle, but extend into the core as well. To exclude them from the core the lower limit of the integral in (17) would have to be replaced by $R_L$ (the subscript L standing for "Lower"), where $R_L = 3485.7 / 6371.0$, the radius of the core being 3485.7 km. In practice excluding the density anomalies from the core makes little difference in the resulting density distribution for the crust and mantle. Finally, the boundaries where $\rho_o(R)$ changes discontinuously are assumed to be spherical, so that there is no possibility of density anomalies arising from bumps on these boundaries in the manner of Hide and Horai (1968) and McQueen and Stacey (1976), for example.

The above constitute the general remarks on the information theory density distribution. Specific examples are discussed next.

A computer program was written to produce maps in order to examine specific information theory density distributions.* All of the maps are based on the GEM 10B gravity field (Lerch et al., 1981). The GEM 10B field is based on satellite, surface gravity, and GEOS-3 altimetry data. Since GEM 10B is complete to degree and order 36, the limit of resolution is about 5 degrees of arc, or about 550 km on the earth's surface. All of the higher degree terms ($\ell \gtrsim 16$) are assumed to be meaningful, although Phillips and Lambeck (1980, p. 44) warn that these terms may largely be noise.

*See Appendix B for a listing of the program.
Plate 1 shows the density variation $\Delta \rho_1$ given by (19) overlaid on the global tectonic and volcanic activity map of Lowman (1981). The density variation is given on the surface of a sphere with radius 6368.0 km. It is at this depth, 3 km, that the oceans leave off and the rock surface begins in Dziewonski et al.'s (1975) FFM. No crust has been stripped off. The interval between contour lines is 0.002 g cm$^{-3}$. This map looks quite similar to the GEM 10B free-air gravity anomaly map (S. Klosko, private communication, 1980). It shows such typical features as the lows at Hudson Bay, Fennoscandia and some of the abyssal plains (e.g., Somali, Hatteras); and highs at some slow-moving ocean ridges (e.g., Mid-Atlantic, Southwest Indian Ocean), subduction zones (e.g., Peru-Chile, Tonga-Kermadec), and hot spots (e.g., Hawaii, Iceland). Hence for qualitatively relating density and tectonics, similar to relating gravity to tectonics as done by Kaula (1972), a free-air gravity anomaly map might just as well be used.

The map shows regions of artificial densities, due to the assumption that the earth is a sphere; the topography has been flattened. So at the Mid-Atlantic Ridge south of Iceland, for example, the effect of topography more than cancels the effect of the low density material upwelling beneath the ridge (assuming the basic validity of plate tectonics), producing a positive anomaly (Lambeck, 1972) and hence an artificially high density.

Plate 2 shows the density distribution at 30 km depth ($r = 6341$ km), where 30 km of isostatically uncompensated crust has been stripped off the earth (giving essentially the Bouguer anomalies). The interval between contour lines is 0.02 g cm$^{-3}$. Note that it gives low densities at some subduction zones. Removing the Andes, for example, completely erases the positive anomaly of Plate 1, so that low densities prevail. There is no sign of a high density subducting slab (which is probably too small to be seen in any case with the resolution employed here).

Figures 1, 2, and 3 show $\Delta \rho_1$ on a plane which slices through the center of the earth in the equatorial plane for the cases of no crust removed, 30 km of isostatically compensated crust removed, and 30 km of isostatically uncompensated crust removed, respectively. The equatorial plane was
chosen because it illustrates typical features of such slices, plus one atypical feature which appears in Figure 2.

All three figures illustrate a remark made earlier: that the information theory density anomalies extend deep into the earth, but the greatest density variation occurs near the surface. This is in qualitative agreement with Arkani-Hamed’s (1970) minimum shear strain energy density distribution, but his density anomalies are much larger than those shown here. The figures also show regions where $\Delta \rho_1$ changes sign with depth. This is also in qualitative agreement with Sanchez’s (1980) density distribution which minimizes the sum of the mantle shear strain energy plus gravitational potential energy. The size of the density variation shown in Figures 1 and 2 is in fair agreement with Sanchez (1980). However, the density distribution across Sanchez’s (1980) slice through the equatorial plane looks nothing like those shown in the figures. It should be mentioned that the minimum energy solutions of Kaula (1963), Arkani-Hamed (1970), and Sanchez (1980) give nonhydrostatic stresses which probably exceed the finite strength of the mantle, indicating that the assumed elastic rheology is unrealistic (Lambeck, 1976, p. 6333). The alternation of the sign of $\Delta \rho_1$ with depth is also in qualitative agreement with Lewis and Dorman (1970), who used a communications theory approach to the relation of density to topography.

The information theory density distributions shown in Figures 1 to 3 clearly tend to form "pockets" of high and low density extending downwards from the surface. There is no obvious convection pattern shown in any of the figures. None of them show what look like classical convection cells; that is, organized columns of low density material moving upwards and columns of high density material moving downwards. Pockets which slant at an angle to the local normal (such as the low density region at 305 degrees east longitude shown in Figure 2, for example) look like they might indicate some sort of horizontal as well as vertical motion of material. But closer examination of the region around such features generally reveals that there are other pockets of similar density slanting towards them (as is obvious with the two low density pockets between 60
and 90 degrees east longitude shown in Figure 1, for example) which are not contoured. Hence the apparent “motion” is probably an artifact of the contouring process.

The one atypical feature mentioned earlier is the high density “blob” located at 285 degrees east longitude shown in Figure 2. Closer examination of this feature shows that it is a “tube” of high density material connecting the Peru-Chile Trench with the Middle America Trench. This feature is atypical in that the greatest density contrast occurs beneath the surface, while with the pockets the greatest density contrast occurs at the surface of the sphere inside which the density distribution is inferred.

Figures 1 and 2 are quite similar to each other. The reason for this is that the removal of a 30 km-thick isostatically compensated crust affects mostly the high degree spherical harmonic terms, and not the low degree terms considered here. The density distribution shown in Figure 3 is of course dominated by the removal of the uncompensated topography, and not the GEM 10B gravity field.

If the density anomalies of Figures 1 and 2 are assumed to be due to lateral temperature variation, then typical temperature differences of 100 K are required, assuming $\alpha$ in (20) is about $3 \times 10^{-5}$ K$^{-1}$ (e.g., Phillips and Lambeck, 1980, p. 53). The temperature differences must be about a factor of 10 greater to explain the density anomalies of Figure 3, assuming the same value for $\alpha$.

Maps showing the density distribution with the degree of the gravity and crustal fields restricted to $\ell < 16$ also give pockets similar to those shown in the figures. Hence it appears that the qualitative behavior of the information theory density distribution will not change if terms of higher degree ($\ell > 36$) than those considered here are included in the fields.
DISCUSSION

One question which arises is which of the three density distributions considered here is to be preferred. Obviously the one in which no crust is removed is over-abstracted: no account is taken of the topography. So it is not the preferred density distribution. The density distribution in which 30 km of uncompensated crust is removed has intriguing consequences for the deep structure of continents (e.g., Jordan, 1975; but see Anderson, 1979): very deep indeed. However, the geophysical evidence favors the density distribution in which 30 km of compensated crust is removed; so it is preferred. But it is not ideal: it assumes that the topography is isostatically supported everywhere over the earth; and, moreover, with a depth of compensation of 30 km. This is certainly not the case. To cite just one example where the assumptions fail, no account is taken of thermal isostacy at the ocean ridges (Haxby and Turcotte, 1978), where the effective depth of compensation is greater than 30 km. Hence the density distribution still gives a high density for the Mid-Atlantic Ridge south of Iceland, for example, after the crust is removed, when it should be low density. Hence better models of the crust are needed in order to use ITI to infer the density distribution below it.

Another question which arises is why the information theory density distribution disagrees with the results of Lambeck (1976), Kaula (1977), and Lowrey (1978) which are also based solely on the relationship of density to gravity (and not on seismic travel times), and which give large density anomalies in the lower mantle. The answer appears to be that these studies examine only a limited number of models which mimic the external gravity field, while the information theory density distribution is a weighted average over all possible density models and reproduces the external gravity field exactly.

The reason why the information theory density distribution disagrees with the seismic evidence of Julian and Sengupta (1973), Dziewonski et al. (1977) and others, which also give large anomalies in the lower mantle, seems clear enough: the seismic data have not been included in the information theory approach. Their inclusion presumably would show large anomalies in the lower mantle.
The dominant impression from the foregoing remarks on the problem addressed here is one of simplicity. More data must be introduced to obtain results which are closer to the actual state of affairs inside the earth. Taking the earth to be made up of indistinguishable particles following Maxwell-Boltzmann statistics is an obviously simplifying, if fundamental assumption which must also be dealt with in order to obtain more realistic results.

Introducing seismic travel times into the approach appears to be an obvious next step to take in examining the density anomalies. However, including the seismic travel time data into ITI appears to be difficult. It is not obvious how to put such information into an approach which sums over all possible models. Numerical models are out of the question: dividing up the earth into 10 volume elements each of which may be occupied by up to 10 particles gives $10^{10}$ models to consider — far too many already for a computer. Thus the problem must be done analytically. But even such a seemingly simple task as using free oscillation periods to obtain a spherically symmetric earth model has analytical difficulties: the data do not have the simple form (2), and the elastic parameters vary as well as the density. These difficulties make the evaluation of the partition function $Z$ troublesome (Graber, 1977). The variance (see Appendix A) and choosing $m/dv$ in (13) also post difficulties in applying ITI.

On the positive side is the heart of the method: ITI (MEM) minimizes subjectivity. To illustrate, there is no need in ITI to decide which lower degree harmonics to ignore in examining density anomalies in the lithosphere (e.g., Marsh et al., 1981), which is a highly subjective procedure. ITI weights all the harmonics automatically. And since ITI gives the one “best” (i.e., least subjective) model, it may well have use in choosing a Standard Earth Model, although the mathematical obstacles mentioned earlier would have to be overcome.
ACKNOWLEDGMENTS

I wish to thank David E. Smith for many helpful discussions and other valuable assistance. Discussions with Mike Graber were also very useful. Carl Wagner supplied the crustal models and the rock-equivalent topography. I thank Barbara Putney and Tom Odt for programming help. The support of the Director's Discretionary Fund of the Goddard Space Flight Center is gratefully acknowledged. The Technical Memorandum of Rubincam (1979) was erroneously numbered 80549. It should be 80586.
REFERENCES


McQueen, H. W. S., and F. D. Stacey, Interpretation of low degree components of gravitational potential in terms of undulations of mantle phase boundaries, *Tectonophysics*, 34, T1-T8, 1976.


Plate 1. Lateral density variation $\Delta \rho$, at 3 km depth (no crust removed). Note that no zero line is shown, which would make the map too busy.
Figure 1. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of no crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.
Figure 2. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of 30 km of compensated crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.
Figure 3. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of 30 km of uncompensated crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.
FIGURE CAPTIONS

Plate 1. Lateral density variation $\Delta \rho_1$ at 3 km depth (no crust removed). Note that no zero line is shown, which would make the map too busy.

Plate 2. Lateral density variation $\Delta \rho_1$ at 30 km depth for 30 km of uncompensated crust removed. Note that no zero line is shown, which would make the map too busy.

Figure 1. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of no crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.

Figure 2. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of 30 km of compensated crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.

Figure 3. Lateral density variation $\Delta \rho_1$ in the equatorial plane of the earth for the case of 30 km of uncompensated crust removed. Sea level is displaced from the earth's surface for clarity, and the rock-equivalent topography is greatly exaggerated. Anomalies in the core are not shown.
Table 1

The Coefficients $\delta_k$ for No Crust Removed ($R_u = 1$) and 30 km of Crust Removed ($R_u = 6341/6371$).

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<th>Degree $k$</th>
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APPENDIX

Finding the variance $\sigma_n^2 = \langle n_j^2 \rangle - \langle n_j \rangle^2$ from (8) closely follows the standard statistical mechanics treatment (e.g., Reif, 1965, pp. 336-337). It is given by

$$\sigma_n^2 = \frac{\partial^2 \ln Z}{\partial \left( \alpha_i \tau_i \langle \tau_j \rangle \right)^2}$$

which gives

$$\frac{\sigma_n}{\langle n_j \rangle} = \sqrt{1 + \frac{1}{\langle n_j \rangle}} = \frac{\sigma_\rho}{\rho(r)}, \quad \sigma_\rho = \frac{m}{d\nu} \sigma_n$$

for particles following Bose-Einstein statistics (Reif, 1965, p. 346; Morse, 1969, p. 333). Because it is assumed that $\langle n_j \rangle \ll 1$, all that can be said about $\sigma_\rho$ is that $\sigma_\rho > \rho(r)$, since no commitment to the value of $\langle n_j \rangle$ has been made. Hence the data do not greatly constrain the density distribution.
The computer program used to generate the plates and figures is listed here. Plates 1 and 2 use subroutine VANDER. Figures 1, 2, and 3 use subroutine SLICE.

\*INFORMATION THEORY CENSITY DISTRIBUTION\*

THIS PROGRAM COMPUTES THE INFORMATION THEORY DENSITY DISTRIBUTION. IT ASSUMES MAXWELL-BOLTZMANN STATISTICS AND THAT THE SPHERICALLY SYMMETRIC PART OF THE DENSITY DISTRIBUTION IS KNOWN.

THE PROGRAM USES SPHERICAL HARMONICS. THE LATERAL DENSITY VARIATION IS ASSUMED TO BE A SMALL PERTURBATION ON TOP OF THE SPHERICALLY SYMMETRIC DENSITY DISTRIBUTION.

THE PROGRAM COMPUTES THE DENSITY VARIATION, RELATIVE DENSITY, OR ACTUAL CENSITY INSIDE THE EARTH, WHICH MAY BE DISPLAYED IN VARIOUS WAYS ACCORDING TO A CHOSEN SUBROUTINE. THE DISPLAY IS IN ONE OF TWO MODES: PRINT-OUT PICTURES OR DECOPED PHOTOGRAPHS.

THE DENSITY IS DEFINED AS THE SPHERICALLY SYMMETRIC PART (RHO-ZERO) PLUS THE DENSITY VARIATION (DELTA RHO). THE RELATIVE CENSITY IS DEFINED AS DELTA RHO DIVIDED BY RHO-ZERO.

CRUSTAL MODELS (SUCH AS AN ISOSTATICALLY COMPENSATED CRUST OR AN ISOSTATICALLY UNCOMPENSATED CRUST) MAY BE SUBTRACTED OFF IF DESIRED AND THE INFORMATION THEORY ALGORITHM APPLIED TO THE REMAINING GRAVITY SIGNAL.

ALSO, THE DEGREES USED BY THE SPHERICAL HARMONIC FIELDS CAN BE RESTRICTED TO A CERTAIN RANGE (BETWEEN LMIN AND LMAX INCLUSIVE) IF SO DESIRED.

NOTATION

GENERAL DATA

PI = 3.1415926535897900
PIDIV2 = PI/2.000
RIOW = MINIMUM DEGREE OF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD RECORDS READ
LUP = MAXIMUM DEGREE OF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD RECORDS READ
LPIH = MINIMUM DEGREE OF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD TO BE USED IN PROGRAM
LPMAX = MAXIMUM DEGREE OF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD TO BE USED IN PROGRAM
SUBNPE = NAME OF SUBROUTINE CALLED
COMENT = GENERAL COMMENTS ABOUT PARTICULAR RUN (SUCH AS THE NAME OF THE REGION BEING EXAMINED, ETC)
NPRINT = 1 IF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD COEFFICIENTS ARE TO BE LISTED, 0 IF NOT

EARTH DATA

ROAVG = AVERAGE DENSITY OF THE EARTH IN GRAMS/CM**3
RADOV = AVERAGE RADIUS OF EARTH IN KILOMETERS
RCORE = RADIUS OF OUTER CORE IN KILOMETERS
KLLOW = LOWER RACIAL LIMIT ALLOWED FOR DENSITY ANOMALIES, IN KILOMETERS
KLOWER = LOWER RADIUS OF GRAVITY/TOPOGRAPHY/CRUSTAL FIELD RADIUS允许的最低密度异常
KUPPER = UPPER RACIAL LIMIT ALLOWED FOR DENSITY ANOMALIES, IN KILOMETERS

GRAVITY FIELD

GRAV = DESCRIPTION OF GRAVITY FIELD

CRUST

CRUST = DESCRIPTION OF CRUSTAL MODEL

LATERAL DENSITY DISTRIBUTION

LATERAL = REMARKS ABOUT THE COEFFICIENTS (SUCH AS THE DEPTHS FOR WHICH

33
THEY WERE WOVED) MAR 6)

ONCCE - INFOMATION-THEORETIC COEFFICIENTS BASED ON THE
SPHERICALLY SYMMETRIC DENSITY DISTRIBUTION

CLM - INFORMATION-THEORETIC RELATIVE DENSITY/DENSITY/DENSITY
VARIATION SPHERICAL HARMONIC COEFFICIENTS

NDEN = 0 IF RELATIVE DENSITY DISTRIBUTION IS PLOTTED, +1 IF
ACTUAL DENSITY IS PLOTTED, -2 IF DENSITY VARIATION IS
PLOTTED

NEDEN = REFERENCE DENSITY, USED ONLY IF NODEN

PICTORIAL DATA

PRINTLOT = 0 IF PRINT-OUT PICTURE IS DESIRED, +1 IF DISCRETE
PICTURE OR DISCRETE PHOTOGRAPH IS COMPLETED

MAXSYM = NUMBER OF ALPHANUMERIC SYMBOLS USED FOR PRINT-OUT

ALPHA = ARRAY IN WHICH ALPHANUMERIC SYMBOLS USED FOR PRINT-OUT
PICTURES ARE STORED, IN ORDER FROM LOW TO HIGH

BETA = ARRAY IN WHICH DISCRETE COLOR CODING NUMBERS ARE
STORED, USED FOR DISCRETE PHOTOGRAPH, IN ORDER FROM LOW TO HIGH

SCALE = RELATIVE DENSITY/DENSITY/DENSITY VARIATION INTERVAL
BETWEEN ALPHANUMERIC SYMBOLS (OR COLORS)

BOPEX = 0 IF PRINT-OUT PICTURE IS DESIRED, +1 IF DISCREED

SUBROUTINES CALLED:

1. CUTS (IF SUNXKE .EQ. CUT)
2. PIECE (IF SUNXKE .EQ. PIE)
3. POLES (IF SUNXKE .EQ. POL)
4. SECTIONS (IF SUNXKE .EQ. SEC)
5. SLICE (IF SUNXKE .EQ. SLI)
6. VANDER (IF SUNXKE .EQ. VAND)

NOTES

1. ALL SPHERICAL HARMONIC FIELDS (GRAVITY, TOPOGRAPHIC, CRUSTAL) USE KALAMS AND NORMALIZATION.

2. TOPOGRAPHIC FIELD IS NORMALIZED TO THE RADIUS OF THE EARTH (IN KILOMETERS)

3. THE FOLLOWING CONDITIONS MUST BE SATISFIED:
   LPBC=0, LLDF
   LPAX=0, LLDF

4. RELATIVE DENSITY VARIATION OR RELATIVE DENSITY IS MORE CONVENIENT TO DISPLAY THAN THE ACTUAL DENSITY.

5. ALPHABETICAL SYMBOLS WHERE JNUM .GT. MAXSYM, THEN THE PROGRAM SETS JNUM=MAXSYM, ALSO, IF JNUM .LT. 1, THEN WE SET JNUM=1, LIMESI FOR RETAINJNUM.

6. IN OTHER WORDS, IF THE RELATIVE DENSITY/DENSITY/DENSITY VARIATION GOES OFF SCALE AT EITHER END, THEN THE LIMIT AT THAT END IS USED.

7. PROGRAM NORMALIZES ALL RADIAL DISTANCES BY DIVIDING RADIAL DISTANCE (IN KILOMETERS) BY RADAVG.

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ALPF=(2)=BLNK

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inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
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PI12=PI/2.000
ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

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inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
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inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
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inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
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CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
PI2=2.00000
PI12=PI/2.000
ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
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ALPA=(12)*=ODDLAR
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CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
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PI12=PI/2.000
ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
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ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
PI2=2.00000
PI12=PI/2.000
ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)

inizIALize BAsIC ConStANTs
PI=3.141592653589793CO
P=PI/180.000
PI2=2.00000
PI12=PI/2.000
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ALPF=(2)=BLNK

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P=PI/180.000
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ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)

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ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

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PI2=2.00000
PI12=PI/2.000
ALPA=(12)*=ODDLAR
ALPF=(2)=BLNK

CALL ERSET(208,256,-1,11)
CUTS

FIND THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION ON EACH CUT.

COMPUTE THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION AT POINTS ON CUT.

THIS SUBROUTINE COMPUTES THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION ON VERTICAL RECTANGLES (CUTS) WHICH ARE PERPENDICULAR TO A GREAT CIRCLE CONNECTING TWO POINTS ON THE EARTH'S SURFACE AT INTERVALS EQUALLY SPACED ALONG THE GREAT CIRCLE SEGMENT.

CUTS WHEN LOOKING DOWN ON THE SURFACE OF THE EARTH IT LOOKS LIKE THE LATTICE OF A FOOTBALL (WITH THE CUTS EXTENDING VERTICALLY) OR THE LATTICE OF A BALL MINUS ONE.

CUTS 1, C, GREAT CIRCLE 9

EACH CUT IS DISPLAYED ALONG WITH THE TOPOGRAPHY ALONG THE TRACK OF THE CUT.

ALL CUTS HAVE THE SAME LENGTH AND DEPTH, ALL POINTS ON A VERTICAL CUT PRINT-OUT PHOTO LINE HAVE THE SAME LATITUDE AND LONGITUDE.

THE CUTS ARE DISPLAYED AS RECTANGLES, SO THAT THERE IS A CERTAIN AMOUNT OF DISTORTION. WE ARE LOOKING AT THE CUTS IN THE FOLLOWING WAY:

CUT 1 ARE CHOSEN WHERE THE GREAT CIRCLE INTERSECTS THE EQUATOR AND GO NORTH ALONG THE GREAT CIRCLE UNTIL THE CUTS ARE REACHED.

THE TWO POINTS CONNECTED BY THE GREAT CIRCLE SEGMENT MAY BE CHOOSED AT WILL, THE NUMBER OF CUTS, THEIR LENGTH, AND THEIR DEPTH MAY BE ALSO CHOSEN AT WILL. THERE IS ALWAYS A CUT AT EACH ENDPOINT OF THE GREAT CIRCLE SEGMENT.

NAMING CONVENTIONS CALLED:

SUBRoutines CALLED:

1. ANGLE
2. DENSITY
3. DEPTH
4. VEG

NOTES

1. XLMA1 = X - XLMA2.
2. DEPTH -5 DEPTH1.
3. NDME = 1.0 AND = 1.0.
4. DEPTH IS USUALLY CHANGED SLIGHTLY BY THE SUBROUTINE TO MAKE THE VERTICAL LENGTH OF THE CUTS AN INTEGER NUMBER OF SPACES LONG ON THE PRINT-OUT PHOTOGAPHR.
5. NCUT = CUTOV.

SAMPLE INPUT DATA (COLUMN 1 OF INPUT STARTS IN COLUMN 2 HERE)

SUBRoutines (FORMAT: 61)

CUTS

(ORTHODOX) (FORMAT: 62)

CUTS

(ORTHODOX) (FORMAT: 13AB42)

(ORTHODOX) (FORMAT: 13AB42)

(ORTHODOX) (FORMAT: 13AB42)

PHI1 XLM1A1 PHI2 XLM2A2 DEPTH1 DEPTH2 (FORMAT: 13AB42)
CCSXI = 1.000
SINXX = 0.000
OPEGA = 0.000
CCCG = 0.000
PS11 = XLMOAI
PS12 = XLMOA2
GC TC 60
80 TCLLP = CARSXLMOA2 - XLMOA1
C IS THE GREAT CIRCLE A MERIDIAN?
IF IT IS A MERIDIAN (TCLLP = -TOLER) (XLL, XI, YL)
82 XLL = PI01V2
CCSXI = 0.000
SINXX = 0.000
OPEGA = XLMOA1
CCCG = -CCOEG(MEGA)
SINCG = CCOSINEG(MEGA)
PS11 = PIH2
PS12 = PIH2
GC TC 60
83 CONTINUE
CCOEG = XCM1(B1) + C0EXP1(11)
CCOEG = CARSCXCMG
SINP = 0.9000 - C0EXP5(92)
CALL ANGLE (CCOEG, SINP, P1)
IF (P1 = XLMOA2) 51, 52, 53
54 CCNTINUE
XLMOA1
PH11 = PIH2
XLMOA2
PH12 = PIH2
GC 60 = 1.2
XLMOA1 - XLMOA2
IF (P1 = 59, 57, 59)
56 X = P12
57 CONTINUE
CCCG = C0EXP5(92) + C0EXP1(11)
CCOEG = CARSCXCMG
SINP = 0.9000 - C0EXP5(92)
CALL ANGLE (CCOEG, SINP, P1)
IF (P1 = XLMOA2) 51, 52, 53
58 CONTINUE
IF (P1 = PI01V2) 54, 54, 54
64 PS11 = P5
61 PS11 = P5
60 GC TC 60
59 CONTINUE
IF (P1 = PI01V2) 62, 62, 62
62 PS11 = P5
61 GC TC 60
60 CONTINUE
OPEGA = XLMOA1/PI
61 XLMOA1
60 CONTINUE
C PRINT XXI AND OMEGA IN DEGREES
C XXI = ANGLE OF GREAT CIRCLE WITH EQUATOR
C OMEGA = ANGLE OF ASCENDING NODE OF GREAT CIRCLE, MEASURED
C EASTWARD FROM GREENWICH
WRITE (6, 315)
315 FORMAT (2/55, 'GREAT CIRCLE LONGITUDE, INCLINATION TO EQUATOR:',
'AND TOLERANCE */', '')
WRITE (6, 77) OMEGA, XXI, TOLER
77 FORMAT (2X, 'OMEGA=', I6, F5.1, 'DEGREES', X0, 'XXI=', I6, F5.1, 'DEGREES', X0,
' TOLERANCE=', F10.6, 'DEGREES', 'XXI=', I6, F5.1, 'DEGREES', 'OMEGA=', I6, F5.1, 'DEGREES', '')
C
AND R2. WITH THE OTHERS (IF ANY) SICHANCED IN BETWEEN THESE TWO.
THE SPHERICAL RECTANGLES ARE DISPLAYED AS RECTANGLES WITH THE
EAST-WEST DIRECTION RUNNING HORIZONTALLY ACROSS THE PAGE (PHOTO), SO
THERE IS A CERTAIN AMOUNT OF DISTORTION.

ROTATION

R1 = RADIAL DISTANCE OF LOWEST RECTANGLE IN KILOMETERS
R2 = RADIAL DISTANCE OF HIGHEST RECTANGLE IN KILOMETERS
THETA1 = COLATITUDE OF NORTHERN BOUNDARY OF THE RECTANGLES IN
DEGREES
THETA2 = COLATITUDE OF SOUTHERN BOUNDARY OF THE RECTANGLES IN
DEGREES
XLMDA1 = LONGITUDE OF THE WESTERN BOUNDARY OF THE RECTANGLES IN
DEGREES
XLMDA2 = LONGITUDE OF THE EASTERN BOUNDARY OF THE RECTANGLES IN
DEGREES
NSPACE = LENGTH OF THE NORTHERN AND SOUTHERN BOUNDARIES IN
HORIZONTAL SPACES ACROSS THE PAGE (PHOTO).
NSURF = NUMBER OF RECTANGLES DESIRED

SUBROUTINES CALLED
1. CENSTY

NOTES
1. R2 < T R1,
2. THETA2 < THETA1,
3. XLMDA2 < XLMDA1,
4. NSPACE < NSURF,
5. NSURF < 1.

SAMPLE INPUT DATA (COLUMN 1 OF INPUT STARTS IN COLUMN HERE)
SUBNAME (FORMAT: A6) PIE 35
PANAM (FORMAT: 13.4, A2) PIE 36
(COMPENT(J, J+1, 14)) PIE 37
R1 R2 THETA1 THETA2 XLMDA1 XLMDA2 NSPACE NSURF PIE 38
J121.0 6321.0 90.0 0.0 180.0 210.0 80 40 39

IMPLICIT REAL*4 (A-H, O-Z)
INTEGER*2 TRAP(121), ETA(111)
DIMENSION ALPHA2(2), LCWAP(205, 21)
DIMENSION ALPHAP(121)
DIMENSION COMPENT(14)
COMMON/PLKG/PLKG
COMMON/PLKG/FPI, P12, PCIV2, SCALE, NPHOTO, MAXSYM
COMMON/PLKG/ALPHA-LCPW
COMMON/PLKG/PLKG/PLKG
COMMON/PLKG/LPI1, LMD1, LLON, LUP
COMMON/PLKG/RADANG, PCCRED, MLOWER, NUPPER
COMMON/PLKG/PLKG/PLKG
COMMON/PLKG/PLKG/PLKG

PRINT INPUT DATA
WRITE (6, 300)
300 FORMAT (13, 12X, 'SUBROUTINE CALLED: PIECE*////') PIE 31
WRITE (6, 311) (COMPENT(J, J+1, 14)) PIE 32
311 FORMAT (10X,13A4, '///') PIE 33

C FIND FIELD OF SYMPLCCLER RANGE
SYMPAS = SYMPAS + TSRTH
SYMPAS = SYMPAS / 2.0+ G.100
C STRETCH=5.000/4.000
IF (PHOTO .EQ. .0) TSRTCH=1.000
C WRITE (6, 11)
1 FORMAT (11X)
WRITE (6, 12)
C 61 FORMAT (12X, 'S', a, 12X) PIE 54
C WRITE (6, 13)
C 62 FORMAT (12X, 'S', a, 12X) PIE 55
C WRITE (6, 14)
C 63 FORMAT (12X, 'S', a, 12X) PIE 56
C WRITE (6, 15)
C 64 FORMAT (12X, 'S', a, 12X) PIE 57
C WRITE (6, 16)
C 65 FORMAT (12X, 'S', a, 12X) PIE 58
C WRITE (6, 17)
C 66 FORMAT (12X, 'S', a, 12X) PIE 59
C WRITE (6, 18)
C 67 FORMAT (12X, 'S', a, 12X) PIE 60
C WRITE (6, 19)
C 68 FORMAT (12X, 'S', a, 12X) PIE 61
C WRITE (6, 20)
C 69 FORMAT (12X, 'S', a, 12X) PIE 62
C WRITE (6, 21)
C 70 FORMAT (12X, 'S', a, 12X) PIE 63
C WRITE (6, 22)
C 71 FORMAT (12X, 'S', a, 12X) PIE 64
C WRITE (6, 23)
C 72 FORMAT (12X, 'S', a, 12X) PIE 65
C WRITE (6, 24)
C 73 FORMAT (12X, 'S', a, 12X) PIE 66
C WRITE (6, 25)
C 74 FORMAT (12X, 'S', a, 12X) PIE 67
SUBROUTINES CALLED
1. ANGLE
2. CIRC
3. CENSTY

NOTES
1. ANCMAX < 90 DEGREES.
2. NRAC > 90.

SAMPLE INPUT DATA (COLUMNS 1 OF INPUT STARTS IN COLUMN 3 HERE)

SAMPLE INPUT DATA (COLUMNS 1 OF INPUT STARTS IN COLUMNS 3 HERE)

SUBNAME (FORMAT: 46)

C POLES

(COMPEN(I,J), J=1,14)

C SURFACE CENSITY DISTRIBUTION

C NRAD ANCMAX NRAC

PCL 14
PCL 15
PCL 16
PCL 17
PCL 18
PCL 19
PCL 20
PCL 21
PCL 22
PCL 23
PCL 24
PCL 25
PCL 26
PCL 27
PCL 28
PCL 29
PCL 30
PCL 31
PCL 32
PCL 33
PCL 34
PCL 35
PCL 36
PCL 37
PCL 38
PCL 39
PCL 40
PCL 41
PCL 42
PCL 43
PCL 44
PCL 45
PCL 46
PCL 47
PCL 48
PCL 49
PCL 50
PCL 51
PCL 52
PCL 53
PCL 54
PCL 55
PCL 56
PCL 57
PCL 58
PCL 59
PCL 60
PCL 61
PCL 62
PCL 63
PCL 64
PCL 65
PCL 66
PCL 67
PCL 68
PCL 69
PCL 70
PCL 71
PCL 72
PCL 73
PCL 74
PCL 75
PCL 76
PCL 77
PCL 78
PCL 79
PCL 80
PCL 81
IF (INPOLE .EQ. 2) WRITE (6,61)
36 FORMAT (D0,10,H2,DE*)
37 FORMAT (D0,10,H2,DE* )

C FIND THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION ON EACH
C HORIZONTAL LINE.
DC 2 LP,LX
1,2 NAY = NP + 1
FLX(LX)
KRI=KLNH(I+1,1)
KPA=KLNH(I+1,2)
DC 3 KP,NK,MAX

C FIND THE X,Y,Z COORDINATES OF EACH POINT ON THE LINE
KPNX = 0.0,0,0
STRECH=1.0002,0,0
IF (INPOLE .EQ. 1) KMAX=1.000
Y=L(E1) = RT(PRE*STRECH)

C FIND THE SPHERICAL POLAR COORDINATES OF POINT
CALL ANCLCC(X,Y,ALPHA)
IF (INPOLE .EQ. 2) ALPHA=MZ - ALPHA
XSOY=CSORT(4000,8000) - YPAT
SINTHP=SINTYP+SPCTH*PSEC
IF (SINTHP < 1.DOC) GOTO 821
WRITE (6,203) NRAC,RECTHER,NRY,NAY,LP,LX,K
203 FORMAT (9X,13I9)
WRITE (6,204) NOSFEC,RT,FLX,FK,NY,XSOY,SINTH
204 FORMAT (9X,1I10,5F10.5)
GCTC = 19

C CONTINUE
GOTO 25
202 CONTINUE
C CONTINUE
CSTHP=CSORT1,1,000 - SINTHE#2)
CALL ANCLICOSIHE,SINTHP,THEITA
IF (INPOLE .EQ. 2) THEITA=PI - THEITA

C COMPUTE THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION AT POINT HP.
C CALL ENSITY (X,THEITA,ALPHA,ADEN,DERE,DERH)
C PUT THE RIGHT SYMPLIS/NUMBER IN XPAPIK(I,THAPIK)
C X=CONS/SCALE + C,100
JNP=SYMPH * XS
IF (JNP) = NC.1 + JNP=1
IF (JNP) = NC.1 + JNP=1

C XPAPIK(1,NUMP) = NUMP
29 XPAPIK(ALPHA,THETA) = NUMP

C CONTINUE
C PRINT THE LINE
WRITE (6,141) (XPAPIK(IP), IP=1,121)
141 FORMAT (1227I1)
C CONTINUE
C CONTINUE
C CONTINUE
15 CONTINUE
C CONTINUE
R E T U R N
C S U B R U T I N E  S E C T I O N S ( I N T E G R A R A D )
S E C 1

*SECTIONS* SEC 2
C THIS SUBROUTINE COMPUTES THE RELATIVE DENSITY, DENSITY, OR
C DENSITY VARIATION ON A NUMBER OF PLANES WHICH SLICE THROUGH THE EARTH
C PARALLEL TO THE EQUATOR. THE INTERSECTION OF THE EARTH WITH A PLANE
C IS CALLED A SECTION.
C ONE SECTION ALWAYS CONTAINS THE EQUATOR, THE SECTIONS ARE EQUALLY SEC
C SPACED BETWEEN THE NORTH POLE AND THE SOUTH POLE. FOR EXAMPLE, IF SEC
C THREE SECTIONS ARE CHOSEN, ONE CONTAINS THE EQUATORIAL PLANE AND THE
C OTHER TWO SECTIONS ARE SPACED ONE-HALF EARTH RADIUS NORTH AND SOUTH
C OF THE EQUATOR (ASSUMING NO CRUST IS STRIPPED OFF).
C ACTATION SEC 12
C NSEC = NUMBER OF SECTIONS CHOSEN SEC 13
C NRAC = RADIUS OF EQUATORIAL SECTION IN HORIZONTAL SPACES ACROSS SEC 14
C PAGE (PHRST) SEC 15

S U B R U T I N E S CALLED:
1. ANGLE SEC 16
C 1. ANGLE SEC 17

47
CIRC  SEC 18
3. CENSTY  SEC 19

1. NSCC IS CCC.  SEC 20
2. MAC =LE. 56.  SEC 21

SAMPLE INPUT DATA (COLUMNS 1 OF INPUT STARTS IN COLUMN 2 HERE)  SEC 22

C SUBRNV  (FORMAT: A  SEC 23
C SECTIONS  SEC 24
L
C (COPERTET,J.), J=1,14)  (FORMAT: 33B2.61)  SEC 25
C FIVE SECTIONS CHOSEN  SEC 26
C NSCC RAD  (FORMAT: 4(5))  SEC 27
5 60  SEC 28

[NP]PLICIT REAL*(4-1),0-21  SEC 29
INTEGER IMQEC(23), PETAI(11)  SEC 30
DIMENSION ALPHAI(23), LCPI(2052)  SEC 31
DIMENSION CCPIETE(14)  SEC 32
CCPECM/SLKAG/PPI+PI1,P12,PI3,PI4,SCALE,MPHOTO,MACSY  SEC 33
C3PC/M/EK/LV/H+LCPI-LCPI  SEC 34
C3PC/SLKH/AAGC,PCCM,HICHER,KUPPER  SEC 35
C3PC/REL,PCOH,PC12,PC23  SEC 36
C3PC/SLKH/PCOLPH,KOH,PC14  SEC 37
C3PC/3,PCOLPH,PC13  SEC 38
C PRINT INPUT DATA  SEC 39
WRITE (6,100)  SEC 40
WRITE 16,300)  SEC 41

301 FORPAP (104)*,NSCC =-1,1x14,10X,* (NUMBER OF SECTIONS)*,///  SEC 42
WRITE (6,301) NSCC  SEC 43
302 FORPAP (104),NRAD =-1,1x14,10X,* (NUMBER OF SECTIONS)*,///  SEC 44
WRITE (6,302) NRAD  SEC 45

C FIND PIECCE OF SYMPOL/SCALE RANGE  SEC 46
SYMPAK=MASYP  SEC 47
SYMPMAT=MAMPAT/2.000 + CI,00  SEC 48
C NPEP+INSEC = 1/2 + 1  SEC 49
FRAC=NRAD  SEC 50
HEMP=HEM/D  SEC 51
C DO EACH SECTION  SEC 52
DC IC = J,1,NSCC  SEC 53
FJH=JEP- JJ  SEC 54
IF (JJ .LE. NPEP) CO IC 21  SEC 55
SQURE=LCPI**2.115.000 -.FJU**21/(HEM**2))  SEC 56
RASEC=CSCRT(SQURE)  SEC 57
ZPC=FU1J/HMPPI  SEC 58
NPC=SACSEC  SEC 59
RASEC+RASEC  SEC 60
GC IC 22  SEC 61
21 NACSEC=RASSEC  TEMPLTIPIIV2  SEC 62
RASEC+RASSEC  SEC 63
C CONTINUE  SEC 64
C COMPUTE NUMBER OF HORIZONTAL LINES IN MAP  SEC 65
NRV=MNRSEC/5  SEC 66
IF (NRV>5)  SEC 67
NRV=NRE1  SEC 68
NRV=NRE2  SEC 69
NRV=NRE3  SEC 70
C FIND LEFT AND RIGHT LIMITS OF MAP BY CALLING CIRC  SEC 71
CALL CIRC(NMRSEC)  SEC 72
C INITIALIZE LINE VALUES  SEC 73
C XPAK IS FOR PRINT-OUT PICTURE  SEC 74
TPAP IS FOR DEKEP PRINT  FILE  SEC 75
UGA A R=1.121  SEC 76
TPAPK = BETA11  SEC 77
+XPAKP=AALPA123)  SEC 78
C SPACE DOWN THE PAGE  SEC 79
SUBRUTINE SLICE (PF, XLMDA1, PH12, XLMDA2, TOLERD, NRAD, SLICE)

*SLICE* SLICE

**SLICE**

THE SUBROUTINE COMPUTES THE RELATIVE DENSITY/DENSITY/VARIATION ON A PLANE WHICH PASSES THROUGH THE CENTER OF THE EARTH.

THE INTERSECTION OF THE PLANE WITH THE EARTH IS CALLED A SLICE.

THE SLICE IS ORIENTED SC TO CONTAIN THE TWO POINTS

*SLICE* SLICE

*SLICE* SLICE

THE SLICE IS ORIENTED SC TO CONTAIN THE TWO POINTS

THE SLICE IS ORIENTED SC TO CONTAIN THE TWO POINTS

THE INTERSECTION OF THE PLANE WITH THE EARTH IS CALLED A SLICE.

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THE INTERSECTION OF THE PLANE WITH THE EARTH IS CALLED A SLICE.
### Intersection of the Slice with the Equator

The figure shows the intersection of a slice with the equator. Key parameters include:

- **Latitude of First Point**: \( \phi_1 \)
- **Longitude of First Point**: \( \lambda_1 \)
- **Latitude of Second Point**: \( \phi_2 \)
- **Longitude of Second Point**: \( \lambda_2 \)
- **Tolerance Limit**: \( \Delta \)
- **Radius of Slice**: \( R \)

These parameters are crucial for determining the spatial relationship between the slice and the equator. The tolerance limit, \( \Delta \), is used to avoid singularities that might occur when the two points are very close to each other.

### Sample Input Data

Sample input data for the program includes:

- **Column 1**: Starts in column 3
- **Column 2**: "S" subname, IFORMAT1 A61
- **Column 3**: SLICE

### Subroutines

The program includes subroutines for handling various inputs and calculations, such as converting angles from degrees to radians, finding angles on the plane, and handling spherical coordinates.

### Sample Code

```c
#include <stdio.h>

#define PI (3.14159265358979323846)

int main() {
    // Program logic
    return 0;
}
```

This code snippet demonstrates the use of constants and includes necessary headers for the program to function correctly.

---

The document contains detailed descriptions of the intersection of a slice with the equator, along with sample input data and a C program for processing the data. The code includes subroutines for handling various inputs and calculations, ensuring accurate and efficient handling of the data.
SINC0G-0.000  SLI  76
GC  IC  81  SLI  74
60  TCLP+CADSXLIPCA2  -  XLPDA1)  SLI  80
C  IS THE GREAT CIRCLE A MERIDIAN?  SLI  81
IF  (ICLAM  =  TCLAM)  SLI  82
82  XXI+FIC1V2  SLI  84
CSSLXG=0.000  SLI  84
SINX1=0.000  SLI  85
OPECA+LAMX1  SLI  86
CCS+CCSIC(CPECA)  SLI  87
SINC+CCS(CPECA)  SLI  88
GC  IC  81  SLI  89
83  CONTINUE  SLI  90
TANCPG+(IDINSXLMPCAI)+(IDTAN(THZ2))  -  (IDINSXLMPCAI)+(IDTAN(THZ1)))  SLI  91
/ (IDINSXLMPCAI)+(IDTAN(THZ2))  -  (IDINSXLMPCAI)+(IDTAN(THZ1)))  SLI  92
OPECA+DITAN(TANCPG)  SLI  93
TANH+(IDTAN(THZ2))/IDINSXLMPCAI  SLI  94
IF  (TANKI)  SLI  95
52  OPECA+OPECA  SLI  96
TANH=TANKII  SLI  97
53  CONTINUE  SLI  98
IF  (ICPECA)  SLI  99
55  OPECA+OPECA  SLI  100
54  CONTINUE  SLI  101
XR+C*XR1  SLI  102
SINXR+SINXR1  SLI  103
OPECA+OPECA/F  SLI  104
C  PRINT  XR  AND  OPECA  IN  DEGREES  SLI  105
C  XR  =  ANGLE  OF  GREAT  CIRCLE  WITH  EQUATOR  SLI  106
C  OPECA  =  LONGITUDE  OF  ASCENDING  NODE  OF  GREAT  CIRCLE,  MEASURED  SLI  107
C  EASTWARD  FROM  GREENWICH  SLI  108
WRITE  (6,303)  XR  SLI  109
301  FORMAT  (10X,**1X,10.5,1X,*DEGREES,1X,**TAN**INCLINATION  TO  EQUATOR)  / SLI  110
WRITE  (6,303)  OPECA  SLI  111
303  CONTINUE  SLI  112
ST  FROM  GREENWICH(II)/*)  SLI  113
C  READ  ARC  SLI  114
WRITE  (6,302)  NRAC  SLI  115
302  FORMAT  (10X,1X,4,1X,*RADIUS  OF  SLICE  IN  HORIZONTAL  SPACE)  SLI  116
+*,1X14,1X,*1X/  SLI  117
C  CCSPSI  =  COS(SI1)  SLI  118
SINPS1-CSINCSI1  SLI  119
XGCCSPS1*COSOMG  =  CCSXIA*SINCMG*SINPSI  SLI  120
Y=CCSPS1*SINOMG  +  CCSXIA*COSCMG*SINPSI  SLI  121
Z=SINXIA*SINPSI  SLI  122
Write  (6,201)  SLI  123
C  CC  SLI  124
C  CC  SLI  125
C  RADMAP  +  RADIUS  OF  VAN  DER  GRINNEN  MAP  IN  INCHES  SLI  126
RADMAP+1.07927  SLI  127
C  RADSEC  =  RADMAP/3.1416  SLI  128
CONVER  +  INCH  IC  KM  RATIO  USED  IN  TOPOGRAPHY  PLOT  SLI  129
CONVER=1.00  SLI  130
RADSEC=RADSEC/3.1416  SLI  131
RING+RING+1.00  SLI  132
NCEG=1  SLI  133
WRITE  (6,301)  SLI  134
265  PRINT  (////,/TX1,265)  SLI  135
WRITE  (6,302)  RADMAP  SLI  136
264  PRINT  (12X,RADIUS  OF  VAN  DER  GRINNEN  MAP  **ST.*,1X,1X,*INCHES*)  SLI  137
WRITE  (6,303)  RADSEC  SLI  138
265  CONTINUE  SLI  139
WRITE  (6,16)  SLI  140
WRITE  (6,201)  SLI  141
266  PRINT  (////,12X,1X,3X,,1X,**TOPO  HEIGHT**+,1X,5X,*INCHES*)  SLI  142
WRITE  (6,16)  SLI  143
WRITE  (6,201)  SLI  144
267  CONTINUE  SLI  145
IF  (SINX1  =  SINCMG  /  SINPS1)  SLI  146
IF  (SINX1  =  SINEG)  SLI  147
IF  (SINEG  =  SINCMG)  SLI  148
IF  (SINEG  =  SINEG)  SLI  149
IF  (SINEG  =  SINEG)  SLI  150
IF  (SINEG  =  SINEG)  SLI  151
IF  (SINEG  =  SINEG)  SLI  152
IF  (SINEG  =  SINEG)  SLI  153
IF  (SINEG  =  SINEG)  SLI  154
IF  (SINEG  =  SINEG)  SLI  155
IF  (SINEG  =  SINEG)  SLI  156
IF  (SINEG  =  SINEG)  SLI  157
IF  (SINEG  =  SINEG)  SLI  158
IF  (SINEG  =  SINEG)  SLI  159
IF  (SINEG  =  SINEG)  SLI  160
IF  (SINEG  =  SINEG)  SLI  161
CONTINUE
CALL TC 68

66 UC=0.05D0*1
PH=1.0*PH
TH=1.0*TH
ADD=FILL
CONTINUE
CALL ANGLE (TH,PH,ALPHA)
XLP=ALPHA+LPD
IF (THAND) IFX1,2,3
CONTINUE
CALL TCPOCM (THE1,PLP1,D1)
PH+POCM
CALL VDC (PH1,XLPDP,D1,N,P,HG,V1)
XHP=1.0+PHMS(NO)
YHP=1.0+PHMS(NO)
CONTINUE
CALL FCPM (PH1,XLPDP,D1,N,P,HG,V1)
WRITE (7,203) (X(NP),Y(NP),Z(NP))
CONTINUE

C FING PIONCLE OF SYMPC/ECNRL RARGE
SYMP=SYMPL

C COMPUTE NUMBER OF HORIZONTAL LINES IN MAP
NHOSE=NC
RADSE=NC
NR=1.0+NR
IF (NCHT0 .EQ. 1) NR=NC
NR=2.0+NR
NR=N+NR
CONTINUE

C FING LEFT AND RIGHT LIMITS OF MAP BY CALLING CRIC
CALL CIRCINC

C DC IC NUMPA=1+1
CONTINUE

C INITIALIZE LINE VALUES
DC IF PA IS FCX PRINT-DCT PICTURE
PCAP IS FCX DISCRED PHIC
DC 4X,1,121
TPAP(I)=0

4 XAPPMX(ALPHA(I))

C SPACE DOWN THE PAGE
WRITE (6,163)

16 FCAPPM (161)
NSPACE=12M-NN
DC 17 NN=NN+NSPACE
WRITE (6,183)
CONTINUE

18 WRITE (6,183) ALPHA(1)

19 FORMAT (10,31)

C FING THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION ON EACH HORIZONTAL LINE
DC 2 XLP1,1,1Y
EHPY=LP+1
FLPPL
HP=LOMHP(LP,1)
KPRAX=LOMHP(LP,2)
CONTINUE

C DC 3 X=MINI,MAX
C FING THE X,Y,Z COORDINATES OF EACH POINT ON THE LINE
AX=AX+AX
VX=VX+VX
STRECH=STRECH/A
CONTINUE

C FIND THE SPHERICAL POLAR COORDINATES OF POINT
X=OSCRTX(161,2)+YPM(2)
C IS THE POINT IN THE CORE?
IF (IC - NCORE) 28,27,27
CONTINUE
C IS THE POINT BELOW KLOWER?
IF (IC - KLOWER) 101,102,102
CONTINUE
C IS THE POINT BELOW HLOWER?
CONTINUE
C IS THE POINT BELOW THE LOWER LIMIT?
CONTINUE
C IS THE POINT BELOW THE LOWER LIMIT?
CONTINUE
C IS THE POINT BELOW THE LOWER LIMIT?
CONTINUE
C IS THE POINT BELOW THE LOWER LIMIT?
CONTINUE
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C IS THE POINT BELOW THE LOWER LIMIT?
CONTINUE
C IS THE POINT BELOW THE LOWER LIMIT?
```plaintext
102 CONTINUE
  KEVSC=SORTK(I*2 + Y+2)
  IF (I)  P4,P27,P4
  GC TC 24
  24 UN = SC2(2)
  THETA = CATAN(THETA)
  IF (I)  P4,P27,P4
  44 THETA = THETA
  20 CONTINUE
  CALL ANOMAL(X,Y,ALPHA)
  CALL ANOMAL(Y,X,ALPHA)

  C COMPUTE THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION AT POINTS
  CALL DENSITY(G,THETA,ALPHA,ALPHA,ALPHA)
  103 CONTINUE
  C PUT THE RIGHT SYMBOL/NUMBER IN XMAP(K)/IMAP(X)
  X=$CEN/SCLH$ + 0.0C
  JNUP = SIMB + X
  IF (JNUP .LE. 1) JNUP = 1
  IF (JNUP .GE. MAXSH) JNUP = MAXSH
  29 XMAP(K) + ALPHA(JNUP)
  JNUP = JNUP + 1
  3 CONTINUE
  C PRINT THE LINE
  WRITE (6,141) (XMAP(K), K=1,121)
  14 FCRPAT(56,121)
  WRITE (110,121) (IMAP(K), K=1,121)
  C 100 FCRPAT(121,121)

  C RESET THE LINE
  OC 15 XMAX,MAX
  XMAP(K) = XMAX(121)
  JNUP = JNUP + 1
  19 CONTINUE
  2 CONTINUE

  C CONTINUE
  SLT 273
  RETURN
  END
  SUBROUTINE VANDER (PKP,VPAP,MPAP,RPAP)

  C SUBROUTINE COMPUTES THE RELATIVE DENSITY, DENSITY, OR DENSITY VARIATION ON A SURFACE OF FIXED RADIUS USING THE VAN DER GRINTEN PROJECTION.
  IT DIVIDES THE MAP INTO TWO HEMISPHERES TO MAKE THE MAP BIG ENOUGH TO OVERLAY ON THE CARTONIC TECTONIC ACTIVITY MAP.
  NOTATION
  RRM = RADIAL DISTANCE OF MAP SURFACE IN KILOMETERS
  HMAP = TOTAL HEIGHT OF OVERLAY MAP IN CM
  WMAP = TOTAL WIDTH OF OVERLAY MAP IN CM
  NPAC = RADIUS OF MAP IN HORIZONTAL SPACES
  SUBROUTINES CALLED:
   1. CIRC
   2. CUBIC
   3. ELLIP
   4. QUARC

  NOTES
   1. NPAC .LE. 102
   2. WMAP .LE. 57 CM

  SUFFIX INPUT DATA (COLUMN 1 OF INPUT STARTS IN COLUMN 3 HERE)
  SUBRNAME (FORMAT A6)
  VANDER
  (FORMAT A11)
  (FORMAT 1346,A2)
  (FORMAT 15.3F10.5)
  (FORMAT 15.3F10.5)

  IMPLICIT REAL*8(A-H,Z)
  INTEGER(2) (I,J)
  REAL*8 (K)
  REAL*8 (L,M)
  REAL*8 (P)
  REAL*8 (Q)
  REAL*8 (R)
  REAL*8 (T)
  REAL*8 (U)
  REAL*8 (V)
  REAL*8 (W)
  REAL*8 (X)
  REAL*8 (Y)
  REAL*8 (Z)

  53
```
CIPENSICN ALPH4I21140 VAN 31
CIPENSICN XPAP11211 VAN )2
CIPENSICN CDPENT114) VAN 33
CIPENSICN/ILKAlXPAP VAN )4
CIPENSICN!lLkClFrPI•P12rPIEIYIrSCAIErNPHCfO.MAXSYP VAN J5
CIPENSICN/FLKf/OEfA VAN 41
PRINT INPLT DATA VAN 42
WRITE 16016) VAN 43
WRITE (6300) VAN 44
ED FCPAT W h IC90 SUERCUIINE CALLEDI VAN 45
WRITE	 14#1111	 ICO
WRITE 160011
MMP
FCAPAt	 I10X•1)A6rA2.1!!1 VAN 47
WRITE I60011
FCAPAt	 I10X•
•
APIAL DISTANCE OF SURF
1

ACE1
WRITE 14002) NRAC VAN 51
FCPAT IIOWNRAO •
•
APIUS OF PAP IN HCRIUNTAL SPACE
5)
M-RKP/RADAVG VAN 54
C FINC LEFT AND RIGHT LIMITI OF PAP BY CALLING CIRC CALL CINC(NHSEC)

C CCMPUIE NLMRER OF HCRIICN IAL LINES IN MAP VAN 54
C XPAP 1S FCA PRINT-OLT PICIUAC VAN 73
OC 4 9-1r121 VAN 74
TFAPIK)	 - RCTAIII) VAN 15
4 XPAPIKIoALPHAl23) VAN 76
WRITE 16161 VAN 77
It FCRPAT	 I1111) VAN 78
NV AVI - I.CCO)*HMlP/WPAP VAN 79
LPMIX
NV - IPPIh • I VAN 81
C FINC THE RELATIVE OEN!ITY. DENSITYr OR DENSITY VARIATION CN EACH
C HCRII[ATAL LINE VAN 83
C 2 LP+LPMINrL MIX VAN 14
IF	 IhMEP15	 .EC.	 2)	 CO	 TC 51 VAN 65
TERP
a
B VAN 86
KMA9-LOWHI1LTh24 - t0 VAN 11
GC IC $2 VAN 19
TERP • RACSEC VAN 90
%PIN
LO%HIILP.1) - t0 • NROSEC VAN 91
KPAX-MICSEC VAN 92
52 CCNflhUE VAN 93
LE-hV -LP 1 VAN 94
X
•
Fk - 1.000 - TERM VAN 99
SIRECH-5.00014.00C VAN 100
IF INPHCTO .EC.	 11
Y-IFLI- RVINSTRECI• VAN 102
C fINC THE SPHERICAL POLAR COORDINATES OF POINT VAN 103
CALL CUSICIX.V.RAC$IC.T ETA) VAN 104
CALL CUAORCIX.Y.RAOSEC.)LMOAI VAN l05
C CCM
THE RELATIVE (EhS11Y. DENSITY. OR DENSITY VARIATION AT POINfVAN 106
CALL CENSIV (R.THETErXLPPA.NDEN.REFOEN.DEN) VAN 107
C PUT THE RIGHT SVMDOL0UPOER IN XPAP)K)/TMAPIKI VAN 100
AS-ICEN/SCALE) • C'.00 VAN 109
JNUP-SVPRL • XS VAN 110

54
IF JNUP .LE. 1 )NLUP=1
IF JNUP .GE. PARSY )JNLP=PARSY

THUP(I) = THETAY(I)

CONTINUE

C PRINT THE LINE
WRITE (2,L4) (THUP(I), H=1,121)

LCONTINUE (56,121)
C WRITE (10,LOC1)(THUP(I), H=1,121)
C 100 ICONTUP (1211)

C RESET THE LINE
CC 35 K=MIN(KMAX)
THUP(I) = THETAY(I)

THUP(I) = THETAY(I)

CONTINUE

C 2 CONTINUE
C 10 CONTINUE
RETURN
END

SUBROUTINE DENSITY (F,THETA,ALPHA,NOEN,REFDEN,DEN)
DEN 1

SUBROUTINE CALLS:
DEN 14

1. LGEN
DEN 15
2. CZIECN (IF ADEN=1 OR NOEN=2)
DEN 16

IMPLICIT REAL*(E,R,E,D,R)
DIMENSION CLM2(36,37,21),CCE(36)
DIMENSION PRA(101)
CCPNA/RLH/CLM/CEE
CCPNA/RLH/CLM/FJ/PI,PIZIV2,SCALE,PHOTO,MAXSYM
CCPNA/RLH/CLM/LLL2LIUP
CCPNA/RLH/CLM/LLL2LUP
CCPNA/RLH/CLM/LLL2LPRR
CCPNA/RLH/CLM/DENSITY
C DIA VARIABLES FROM THETA TC PHI
PI=PIZIV2 - THETA
C COMPUTE LEGENDRE POLYNOMIALS
CALL LEGENDR(PI,LPAY1)
C COMPUTE RELATIVE DENSITY
SUPC.G,COOA
DC 5 L=LPN,L=MAX
RL=L
MAX=L
DC 5 P=1,MAX
X=P1 - 1
ARG=2NP1+1
SCSIN(ARG)
CC=CCSIN(ARG)
SINC=CCSIN(ARG)
IP=IP + CLM(L,PI)/2
SUM+SUM + (CLM(L,PI)+CLM(L,2)+S5)*(PBAR(IND1))*RL
CONTINUE
DEN-SUM
CONTINUE
C SEE IF ACTUAL DENSITY IS WANTED
IF ADEN .EQ. 1 GO TO 7
C SEE IF DENSITY VARIATION IS WANTED
IF ADEN .EQ. 2 GO TO 9
CONTINUE
CALL CENZEN (R,SENO)
C =CENZEN*1.000 + DEN - REFDEN
CONTINUE
RETURN
END
END  
SUBROUTINE ANGLE(X,Y,XLMDA)  
ANG 1  

THIS SUBROUTINE COMPUTES THE ANGLE WITH THE X-AXIS FROM THE (X,Y) AND  
COORDINATES OF A POINT.  
ANG 2  

ACTION  
ANG 3  

X = X-COEFFICIENT OF POINT  
ANG 4  

Y = Y-COEFFICIENT OF POINT  
ANG 5  

XLMDA = ANGLE WITH X-AXIS  
ANG 6  

SUBROUTINES CALLED:  
ANG 7  

NONE  
ANG 8  

NOTES  
ANG 9  

1. XLMDA IS MEASURED IN RADIANS.  
ANG 10  

2. IF X=0 AND Y=0 THEN XLMDA=0.  
ANG 11  

P0LICY REAL*(4,0:2)  
ANG 12  

PI4=3.14159265358979323  
ANG 13  

PI012=PI/2.000  
ANG 14  

IF (X).EQ.0.0 AND (Y).EQ.0.0  
ANG 15  

1=0  
ANG 16  

U=V=X  
ANG 17  

XLMDA=DATAN1(U)  
ANG 18  

IF (Y).LE.24.25+0.000  
ANG 19  

1=0  
ANG 20  

XLMDA=XLMDA+PI  
ANG 21  

GC IC 30  
ANG 22  

XLMDA+PI012  
ANG 23  

GC IC 30  
ANG 24  

22  
ANG 25  

CONTINUE  
ANG 26  

IF (Y).GT.31.32+0.000  
ANG 27  

1=0  
ANG 28  

XLMDA+XLMDA+PI  
ANG 29  

GC IC 30  
ANG 30  

32  
ANG 31  

CONTINUE  
ANG 32  

RETURN  
ANG 33  

END  
ANG 34  

SUBROUTINE LEGENDRE(NPAX,NMAX)  
ANG 35  

\( \text{THIS SUBROUTINE COMPUTES THE NORMALIZED ASSOCIATED LEGENDRE POLYNOMIALS.} \)  
ANG 36  

ACTION  
ANG 37  

PHI = GEOCENTRIC LATITUDE IN RADIANS  
ANG 38  

NMAX = MAXIMUM DEGREE AND ORDER OF THE POLYNOMIALS  
ANG 39  

PRAX = ARRAY IN WHICH THE POLYNOMIALS ARE STORED  
ANG 40  

SUBROUTINES CALLED:  
ANG 41  

NONE  
ANG 42  

NOTES  
ANG 43  

1. COMPUTES POLYNOMIALS USING KAULA'S AMPL NORMALIZATION.  
ANG 44  

2. NMAX IS LESS THAN OR EQUAL TO 120.  
ANG 45  

3. DIMENSION OF PRAX IS 1+NMAX*(NMAX+31/2).  
ANG 46  

4. POLYNOMIAL OF DEGREE N AND ORDER M IS STORED IN PRAX(INDEX).  
ANG 47  

WHERE INDEX = \( M + M \times N \times (N+1)/2 \).  
ANG 48  

5. SUBROUTINE HAS UNDERFLOWS WHEN PHI IS NEAR THE POLES.  
ANG 49  

\( \text{EXPLICIT REAL*(4,1:2)} \)  
ANG 50  

\( \text{LOGICAL NOTIST} \)  
ANG 51  

\( \text{CIPRAX} = \text{PRAX}+1.0 \)  
ANG 52  

\( \text{COPPER} = \text{KUFRAX} \)  
ANG 53  

\( \text{DATA NOTIST,.FALSE.} \)  
ANG 54  

98
I = 1; SUCCEEDUCING CENSORES THE SPHERICALLY SYMMETRIC PART OF THE
LENSE R DISTRIBUTION. IT USES THE AVERAGE STRUCTURE PARAMETERIZED
BY EARTH MODEL OF A.P. DEIWINIKAI, ET AL., "PARAMETRICALLY SIMPLE EARTH
MODELS CONSISTENT WITH ECLIPSE PHYSICAL DATA, PHYSICS OF THE EARTH AND
PLANETARY INTERIORS, VOLUME 10. PP. 12-48, 1974. (SEE ESPECIALLY
TABLE 1.)

ACTIVATION

a = DENSITY IN GMAPS/CM**3

b = RADIAL DISTANCE, COMPARED TO THE RADIUS OF THE EARTH

p = RADIAL DISPLACEMENT OF DENSITY DISCONTINUITIES, ALSO
NORMALIZED TO THE RADIUS OF THE EARTH

SUBCLINES CALLED

ACNL

:PHICLI1 REAL**R(A=1.0,H=2)

DIMENSION PI11

CCPCQ/UL/RH/RADAVG,HCERE,HECWER,HEPHER

F11+10.0

P: T1+1.0,E,RANW

P: T2+365.000,RANW

P: 4+100.000,RANW

P: 5+90.000,RANW

P: 6+145.000,RANW

P: T1+291.000,RANW

P: E8+326.000,RANW

P: 10+357.000,RANW

P: 10+368.000,RANW

P: 111+1.000

IF (R = F11) 1.12

C: REGION 1

1 1 GENZ1+1.021900+R.45232500*(R**2) 31

G: IC 19 31

CONTINUE 32

IF (R = F31) 3.3.4

C: REGION 2

3 GENZ1+1.2589100+1.6492900*(R**2)+7.1125100*R**3 36

G: IC 19 31

CONTINUE 32

IF (R = F41) 5.5.6

C: REGION 3

5 GENZ1+1.413000+1.4673000+1.1853100*(R**2) 40

G: IC 19 31

CONTINUE 32

IF (R = F51) 7.7.8

C: REGION 4

7 GENZ1+1.1197800+7.7054000**R 47

G: IC 19 31

CONTINUE 32

8
SUBRoutines CALLED:

ACNE

NOTES

1. LCNF1(LP,13) STOPS THE SPACE NUMBER WHERE THE MAP STARTS ON LINE LP+AND LCONF1(LP,2) WHERE IT STOPS (INCLUSIVE).
2. ALL MAPS ARE CENTERED ON SPACE 61.

[CORRECTED ]

SUBROUTINE CLBIC3(X,Y,Z,HETA)

NOTATION

X = X-COORDINATE OF POINT
Y = Y-COORDINATE OF POINT
S = SCALE LENGTH OF MAP, EQUAL TO 100 DEGREES OF LONGITUDE MEASURED ALONG THE EQUATOR
THETA = COLATITUDE IN RADIANS

SUBRoutines CALLED:

ACNE

NOTES

2. S*2*HID IN THE NOTATION OF THE PAPER CITED.
THIS SUBROUTINE COMPUTES THE LONGITUDE OF A POINT FROM ITS X-Y COORDINATES IN THE VAN CAR GRINTEJ PROJECTION.

**Declaration**

- \( X \) = X-COORDINATE OF POINT
- \( Y \) = Y-COORDINATE OF POINT
- \( S \) = SCALE LENGTH OF MAP, EQUAL TO 100 DEGREES OF LONGITUDE MEASURED ALONG THE EQUATOR
- \( XLMCA \) = LONGITUDE IN RADIANS

**Subroutines Called:**

- ACME

**Notices**

1. For further details see D.P. KURINCA, "INVERTING X-Y GRID COORDINATES TO EARTH LATITUDE AND LONGITUDE IN THE VAN DER GRINTEJ PROJECTION", NASA TP 81998, August 1980.
2. S-ZWHO IN THE NOTATION OF THE PAPER CITED.

**APPLCIT REHEAIA-H:O=O-Z**

\[
P = 3.141594671652507764
\]

1. IF \( |X| > 1.2 \)
   \( XLMCA = 0.000 \)
   \( GC = IC \)
2. CONTINUE

\[
X2 = X^2
Y2 = Y^2
S2 = S^2
R2 = X^2 + Y^2 + 2.0000*S2*I*X - 2.0000*S2*Y + 2.0000I*Y + 2.0000Y^2
\]

1. CONTINUE

**RETURN**

END

**THIS SUBROUTINE COMPUTES THE X-Y GRID COORDINATES OF A POINT IN THE VAN CAR GRINTEJ PROJECTION FROM ITS LATITUDE AND LONGITUDE.**

**Declaration**

- \( PH1 \) = LATITUDE OF POINT (IN DEGREES OR RADIANS)
- \( XLMCA \) = LONGITUDE OF POINT (IN DEGREES OR RADIANS)
- \( RADMAP \) = RADIUS OF MAP
- \( NDEG \) = 1 IF PH1 AND XLMCA ARE IN DEGREES; 0 IF IN RADIANS
- \( X \) = X GRID COORDINATE
- \( Y \) = Y GRID COORDINATE

**Subroutines Called:**

- ACME

**Notices**

- 2. S-ZWHO IN THE NOTATION OF THE PAPER CITED.
1. THE GREENWICH MERIDIAN (XLMDA=0) RUNS DOWN THE CENTER OF THE MAP.

2. THE ORIGIN X=0, Y=0 OF THE GRID COORDINATE SYSTEM IS LOCATED AT THE CENTER OF THE MAP (XMDR=0, XLMDA=0).

3. IF THE ABSOLUTE VALUE OF BEE OR ELL, SHE, EPSLN, THEN THE VALUE IS ASSUMED TO BE ZERO. THIS IS DONE TO AVOID INFINITIES.

```
IMPLICIT REAL*8 (A-H, O-Z)
PI=3.1415926535897932
EPSLN=1.00E-10
IF (NCDC .EQ. 1) CO TC 1
ELL=XLMDA/180.000
BEE=PHI/PI
GC TC 2
ELL=XLMDA/180.000
BEE=PHI/180.000
CONTINUE
END
```

1. THE GREENWICH MERIDIAN (XLMDA=0) RUNS DOWN THE CENTER OF THE MAP. THE ORIGIN X=0, Y=0 OF THE GRID COORDINATE SYSTEM IS LOCATED AT THE CENTER OF THE MAP (XMDR=0, XLMDA=0). IF THE ABSOLUTE VALUE OF BEE OR ELL, SHE, EPSLN, THEN THE VALUE IS ASSUMED TO BE ZERO. THIS IS DONE TO AVOID INFINITIES.