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The Processing of IMU Data in ENTREE- Implementation and Preliminary Results

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I. Summary

This study demonstrates that the Shuttle entry trajectory can be accurately represented in ENTREE with IMU data available post-flight. The IMU data consists of platform to body quaternions, and accumulated sensed velocities in mean of fifty (M50) coordinates approximately every 1 second. Described also is the preprocessing software required to incorporate the IMU data in ENTREE, as well as the relatively minor code changes to the ENTREE program itself required to process the IMU data.

The paper is divided into 6 sections. Section II contains a brief background to introduce the reader to the purpose of the study. Code changes to the ENTREE program proper are described in Section III, while input tape data format and content changes are described in Section IV. Section V reviews some results obtained from preliminary studies and Section VI presents conclusions and recommendations for future study. Additionally there are two appendices. Appendix A describes the IMU post-flight availability data rate, and the graphic output from the studies described in Section V is contained in Appendix B.

II. Background

In the event the primary inertial instrumentation, i.e., the Aerodynamic Coefficient Identification Package (ACIP), is degraded or unavailable for post-flight trajectory reconstruction, it would be desirable to be able to process vehicular sensed accelerations and angular rates as determined by the inertial measurement units. Indeed, if feasible, the IMU data might be used to aid in the determination of the accuracy of the ACIP data. The tri-redundant IMU's are gimballed inertial platforms whose orientations are skewed with respect to one another and are located at the nav base in the nose of the Shuttle vehicle. ENTREE is currently configured to
process sensed accelerations from a body mounted accelerometer and gyro set (e.g., ACIP). Section III will discuss code modifications necessary to allow ENTREE to process sensed accelerations and angular rates from the inertial platforms. As always, the attempt will be made to minimize required code changes.

III. Code Changes to ENTREE for IMU Processing

ENTREE will have knowledge of whether or not to expect inertial sensed accelerations and angular rate data via a flag IMU. IMU = 1 will imply the processing of IMU data, and will cause the execution of special sections of code at the integrator rate. IMU ≠ 1 defaults to the current ENTREE configuration. The flag IMU is defined in namelist PARAM.

ENTREE currently expects as input; 1.) $a_x^m$, $a_y^m$, and $a_z^m$ - the measured body mounted sensed accelerations, and 2.) $P_m$, $Q_m$, and $R_m$ - the instantaneous body roll, pitch, and yaw rates respectively as measured by the ACIP. If IMU = 1, it will be assumed that $a_x^m$, $a_y^m$, and $a_z^m$ will be the sensed acceleration as measured along the inertial IMU axes, and $P_m$, $Q_m$, and $R_m$ will be the inertial angular rate expressed about the X, Y, and Z IMU axes respectively. It should also be pointed out at this time that due to telemetry limitations ENTREE will process data from only 1 IMU at a time, i.e., 3 runs will be required to evaluate (or combine the results of) each IMU.

By modifying the sensed acceleration input tape to be consistent with the above assumptions, changes to ENTREE software proper will be minimal. For example, all acceleration parameter partials (pp. 4-39 to 4-41 in reference 1, subroutine FXXACC in the ENTREE program) and inertial angular rate parameter partials (pp. 4-38 and 4-39 reference 1; subroutine FXXAR in ENTREE) will remain unchanged. All bias, scale factor, and misalignment error terms are now with respect to the inertial instrument axes rather than the body axes as before.

Similarly, the translational and rotational equations of motion will remain intact (pp. 4-26). However, since they are performed in the "G-frame," care must be taken to transform the inertial data. For example, the G-matrix computed
in Eq. (50) must be pre-multiplied by a body to inertial platform rotation matrix. Likewise, the body relative center of gravity locations \( x_p, y_p, \) and \( z_p \) computed in Eq. (52) need to be rotated into the inertial platform frame of reference. Finally, the inertial angular rates must be rotated to body coordinates as expected in the rotational equations of motion on pp. 4-26. These straightforward and relatively minor modifications are all performed within subroutine MOTION.

With the c.g. locations now expressed in platform coordinates, one additional rotation needs to be performed in subroutine FXXCG, where the solve for or consider off - c.g. bias partials are computed.

IV. Modifying the PQR Input Tape

Currently, the PQR Data File contains the inertial angular rate expressed about the body axes and the sensed accelerations expressed in body axes along with time tags. This represents data output by the ACIP. This paper proposes to input the same quantities in the same format expressed, however, in IMU coordinates. (1) In addition, the 3 platform to body Euler angles valid at the same point in time will be appended to each PQR data record for reasons to be explained in the next section. These modifications require the use of a preprocessing program to convert the expected input data into the desired format.

IMU input data is expected to be provided in the following form (as described in the Master Measurement List of the Downlink Telemetry Document): Accumulated sensed velocities in M50 coordinates, and stable member to body quaternions \( Q \), all at about 1 Hz. (2) To provide the IMU axis accelerations at the desired output rate, the accumulated sensed velocity data will be fitted with a cubic spline curve. The spline fit can either be smoothed or forced to pass through all the data points, and is both 1st and 2nd derivative continuous throughout. The slope (first derivative) of the cubic will then provide the acceleration data, which is multiplied by the M50 to IMU (REFSMAT) matrix to obtain the accelerations at the desired times in the proper coordinates.

(1) i.e., inertial sensed accelerations in IMU platform coordinates, and inertial body attitude rate about the IMU platform axes.

(2) The actual input rate is described in Appendix A.
The input quaternions \( Q \) will first be checked for normality, and then used to construct the IMU to body Euler angles. The Euler angle data will then be spline fitted, as is the accumulated velocity data. The first derivative is evaluated to obtain the Euler angle rates, which in turn are used to compute the instantaneous rotation rate about the (inertial) IMU platform axes at the desired times. (See Figure IV 1.)

As stated earlier, the Euler angles themselves are appended to each PQR data record. This is done so that the appropriate body to platform rotations required for the code changes described earlier can be calculated. More importantly, if the integrator rate requires data at times not stored on the PQR data tape, the built-in ENTREE interpolator can obtain the correct angles at the required time. (Rotations computed from interpolated angle data are always orthogonal; interpolated quaternion data do not necessarily yield orthogonal rotations.)

Thus, the PQR tape is generated.

V. Preliminary Results

The objective of these initial studies was to determine how accurately the Shuttle trajectory could be resurrected using simulated error-free IMU data. Hence, the deterministic program DETRAJS was used (Ref. 2) since no measurement processing was required.

The simulated IMU data was constructed as follows: The state vector (expressed in ECI coordinates), the external sensed accelerations (expressed in body coordinates), and the instantaneous pitch, roll, and yaw angles (defined with respect to the local horizontal) were read from a reference trajectory tape generated by R. Powell, VAB/SSD of LaRC. An initial (inertial) IMU platform orientation was arbitrarily chosen. The platform to body Euler angles were then determined, from which the platform to body quaternions were extracted. Likewise, the accelerations were rotated to platform coordinates, summed, and then rotated to M50 coordinates. This data was then time-tagged, and stored on a magnetic tape at every point (25 Hz.).

Initially, there are 3 variables which determine the accuracy of the IMU based trajectory reconstruction: Input rate, output rate, and integrator stepsize.
FIGURE IV-1 EULER ANGLE AND EULER ANGLE RATES

\[ X_B, Y_B, Z_B = \text{BODY AXES} \]
\[ X_P, Y_P, Z_P = \text{PLATFORM AXES} \]
\[ \psi, \theta, \phi = \text{BODY TO PLATFORM EULER ANGLES} \]
\[ \dot{\psi}, \dot{\theta}, \dot{\phi} = \text{EULER ANGLE RATES} \]

Define \( P, Q, R \) to be the Angular Rotation Rate expressed about the \( X_P, Y_P, \) and \( Z_P \) Axes respectively. Then

\[ P = \dot{\psi} \cos \theta \cos \phi + \dot{\theta} \sin \phi \]
\[ Q = -\dot{\psi} \cos \theta \sin \phi + \dot{\theta} \cos \phi \]
\[ R = \dot{\phi} + \dot{\psi} \sin \theta \]
Input rate means "how frequently are the accumulated velocities and quaternions provided to the preprocessor?" Output rate means "at how many points along the spline fit do you desire to output platform accelerations and body attitude rates?" The resultant performance is a function not only of absolute value of each of the 3 parameters, but of their relative size as well. For example, generally speaking, the smaller the integration stepsize, the more accurate the integration (up to within round-off considerations). However, for relatively sparse input data, a smaller stepsize cannot "make up" for a basic lack of input information. Another example: Generally, the more frequent the input data the better, except here again, if the spline passes through "too many" closely spaced data points exactly, the slope (and hence the computed rates) tend to oscillate rather rapidly. So the overall performance depends on a rather complex interrelationship between input, output, and integration stepsize rates.

For the Shuttle entry trajectory reconstruction there exist certain constraints on the above parameters. The input rate, for example, is limited to the downlink data telemetry availability (described in Appendix A.) With ENTREE's 4th order Runge-Kutta integrator, the output rate should be set to one half times the integration stepsize required by the integrator to map to the mid-points. More frequent output results in unused data, and less frequent output causes the ENTREE interpolator to be invoked. Ideally, one would want the largest stepsize possible consistent with desired accuracies.

The graphic plots contained in Appendix B represent the results of studies performed which identify the effects of changes in the input, output, and integration stepsizes consistent with the previously stated constraints. The first 4 figures represent the difference between the reference trajectory (which was generated using body accelerations and rates at 25 Hz) and the IMU determined trajectory. All runs began with no initial condition errors at time $t = 0$ ($h \approx 569,000$ ft.) and ran for 2000 seconds (the plots show only the results between $t = 600$ to $t = 2000$ seconds). Data are plotted every 2 seconds. For a more detailed description of the plot package, see Reference 3.
In Figure 1, a .5 second preprocessor generated PQR data output interval was arbitrarily chosen. DETRAJS integrated the trajectory at half second step-sizes. Thus, the ENTREE (linear) interpolator was invoked to determine accelerations and rotation rates at the midpoints.

In Figure 2, with the identical downlist input rate, and the same .5 second integrator stepsize, the preprocessor output data at .25 sec. Thus, Figs. 1 and 2 demonstrate the effects of different preprocessor output rates. The mathematical distinction between Fig. 1 and Fig. 2 is that in the former a linear interpolation scheme was used to evaluate the midpoints, whereas in the latter, the midpoints were evaluated along the spline fit. In all difference plots, the results of Fig. 2 are more accurate than those of Fig. 1, generally by about a factor of 2 for this particular combination of input, stepsize, and output.

Figure 3 is identical to Fig. 2, except the stepsize has been increased to 1 second (input = downlist; output = .25 seconds.) Comparing Figures 2 and 3 gives a measure of the effect of integration stepsize for this particular relative value of input. A close examination does not reveal any clear-cut universal trend as far as accuracy is concerned. The reader should be cautioned however not to conclude that the resultant accuracy is relatively insensitive to integration stepsize in genera. Other studies were performed in which the input rate was artificially increased and thus more frequent as compared to the integrator step-size. In these cases, the resultant accuracy was highly dependent on stepsize.

Figure 4 is a repeat of Fig. 3, except the preprocessor output interval has been increased to .5 seconds. Note that Figs. 3 and 4 are identical in all components. This proves that the output data need not be generated more frequently than one half times the integrator stepsize.

In an absolute sense, it is the opinion of the author that the results indicate that any of the combinations of input, output, and integrator stepsize shown are sufficiently accurate for Shuttle entry trajectory analysis. Any small errors introduced using the IMU data available at about a 1 Hz rate via telemetry will be masked by the processing of external measurements in the post-flight reconstruction process.
For purposes of comparison, an additional study was performed whereby the ACIP accelerometer and rotational rate data, available at 25 Hz, was integrated with a 1 second stepsize. The results are plotted in Figure 5. Note how unacceptably large the errors are compared with the 1 second stepsize IMU runs. (These results were also generated in a study by J. T. Findlay, documented in Ref. 4. In the study, Findlay showed that simple integration of instantaneous ACIP data at stepsizes larger than about .4 seconds yielded unacceptably large trajectory errors.)

The fact that the IMU data can integrate accurately with 1 second stepsizes, while the ACIP based data cannot, is explained as follows. In the accelerometer channel, the IMU output consists of accumulated sensed velocity, which, although not capable of reproducing exact instantaneous accelerations, maintains the net average acceleration accurately. With the addition of the smoothing effect of the spline fit, essentially a double integration effect is obtained. The 1 second stepsize ACIP data, however, are local instantaneous accelerations used over entire integration half steps.

Furthermore, in the attitude channel, the spline curve is fitting body attitude angles exactly, even though here again instantaneous angular rates may not be perfect. The net average computed rate, however, keeps attitude accurate with the IMU input data. On the other hand, ACIP is using local instantaneous rates over the entire integration half step, and has no angle data except initially.

The fact that the IMU data is able to maintain a small net mean rate and acceleration error is demonstrated in Figure 6, where the differences between the true (ACIP instantaneous) and IMU computed $P$, $Q$, $R$, $a_x$, $a_y$, $a_z$ data are plotted. Also shown are the mean and standard deviation of the differences in the upper right hand corner of each plot. (For a more detailed discussion of the plot description, see Ref. 5.) The small computed means illustrate the point of the preceding paragraphs.

Obviously, in a manner analogous to the IMU data preprocessing, the ACIP data could be summed and spline fitted if so desired in order to obtain comparable performance. In fact, the results of this study suggests that such a procedure might be the most prudent course of action for the processing of ACIP data. That proposal should be evaluated in a separate study.
VI. Conclusions and Recommendations for Future Studies

With the use of a preprocessor to spline fit and interpolate the expected IMU input data, and otherwise relatively minor code changes to the ENTREE code proper, it has been shown that the Shuttle entry trajectory can be deterministically generated quite accurately, even given the ~1 second downlist input data rate limitation. It is thus the conclusion of the author that IMU data processing with an up to 1 second integration stepsize (implying a preprocessor output rate of every .5 seconds) is a viable backup to ACIP data processing.

In addition, the preprocessed IMU data can be used as a tool to evaluate the accuracy of the ACIP data. Direct comparison plots similar to that shown in Figure 6 can be generated and used to detect any appreciable bias, scale factor, etc., errors which might be present.

The next logical step in the study of IMU data processing is to implement into ENTREE the code changes made to DETRAJS, and run some IMU error cases to see if the filter can correctly identify and solve for the errors.
References:


APPENDIX A
Description of IMU Data: Post Flight Availability

The basic IMU output rate is 6.25 Hz. The relevant downlist telemetry rate is 1.0 Hz. This means that every 1 second, the downlist dispatcher buffers off and telemeters the current IMU time tag and associated data (quaternions, accumulated sensed velocities per IMU). Thus the telemetered data is timewise homogenous (since all of the IMU data is valid at the time tag time) but is asynchronous relative to the downlist time.

The following table illustrates the point. (Recall that the IMU data is only output every 160 m sec).

<table>
<thead>
<tr>
<th>Downlist Time</th>
<th>IMU Time Tag</th>
<th>Δ Time Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>.96</td>
</tr>
<tr>
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<td>.96</td>
</tr>
<tr>
<td>5.0</td>
<td>4.96</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B: RESULTS
FIGURE 2: State difference plots versus time IMU Input = 0.25 sec.
Stepsize = 0.5 sec.
FIGURE 3: State difference plots versus time IMU Input = 0.25 sec.
Stepsize = 1.0 sec.
FIGURE 4: State difference plots versus time. IMU Input = 0.5 sec.
Stepsize = 1.0 sec.
\( \Delta P, \text{deg/sec} \)

\(~2.00\)
\(~0.67\)
\(~-0.67\)
\(~-2.00\)

\( \text{TIME, 100sec} \)

(a) Roll rate differences with time

\( \Delta Q, \text{deg/sec} \)

\(~0.10\)
\(~0.03\)
\(~-0.03\)
\(~-0.10\)

\( \text{TIME, 100sec} \)

(b) Pitch rate differences with time

\( \Delta R, \text{deg/sec} \)

\(~0.20\)
\(~0.07\)
\(~-0.07\)
\(~-0.20\)

\( \text{TIME, 100sec} \)

(c) Yaw rate differences with time

\( \Delta x, \text{ft/sec}^2 \)

\(~2.00\)
\(~0.67\)
\(~-0.67\)
\(~-2.00\)

\( \text{TIME, 100sec} \)

(d) X-body acceleration differences with time

\( \Delta y, \text{ft/sec}^2 \)

\(~1.00\)
\(~0.33\)
\(~-0.33\)
\(~-1.00\)

\( \text{TIME, 100sec} \)

(e) Y-body acceleration differences with time

\( \Delta z, \text{ft/sec}^2 \)

\(~1.00\)
\(~0.33\)
\(~-0.33\)
\(~-1.00\)

\( \text{TIME, 100sec} \)

(f) Z-body acceleration differences with time

\( \sigma = 0.08468 \quad \mu = -0.00002 \quad \sigma = 0.02468 \quad \mu = 0.00006 \)

FIGURE 6: TRUE minus IMU COMPUTED ANGLE RATES and ACCELERATIONS