Damping Seals for Turbomachinery

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a  gap velocity effect, Eq. (83)
A  relative seal gap velocity, Eq. (43)
a_o  unit matrix constant, Eq. (91)
b  gap acceleration effect, Eq. (79)
B  relative seal gap acceleration, Eq. (56)
b_o  right angle rotation matrix constant, Eq. (91)
c  Couette factor, Fig. 3 and Eq. (42)
C  induced seal damping, Eq. (103)
C_c  whirl coupling, Eq. (105)
C_o  steady-state induced seal damping, Eq. (100)
d  inlet effect, Eq. (77)
D  determinants, Eqs. (6) and (110)
e  relative eccentricity, Eqs. (1) and (18)
E  relative gap change, Eq. (19)
f  pipe friction factor
f_r  rotor friction, Eq. (29)
f_re  expanded rotor friction, Eq. (31)
f_s  stator friction, Eq. (28)
f_se  expanded stator friction, Eq. (30)
f_1  inlet loss factor, Eq. (60)
f_2  outlet loss factor, Eq. (61)
F  combined friction effect, Eq. (49)
F_y  force component in y, Eqs. (1) and (107)
F_z  force component in z, Eqs. (1) and (107)
\( g_r \)  
**rotor friction velocity effect**, Eq. (33)

\( g_s \)  
**stator friction velocity effect**, Eq. (32)

\( G \)  
**additive friction velocity effect**, Eq. (50)

\( h \)  
**radial seal gap**, Eq. (18)

\( h_o \)  
**average radial seal gap**, Eqs. (1) and (18)

\( h_r \)  
**rotor friction gap effect**, Eq. (35)

\( h_s \)  
**stator friction gap effect**, Eq. (34)

\( H \)  
**additive friction gap effect**, Eq. (51)

\( i \)  
**2x1 unit vector**, Eq. (3)

\( I \)  
**2x2 unit matrix**, Eq. (12)

\( k_r \)  
**rotor surface roughness**, Eq. (29)

\( k_s \)  
**stator surface roughness**, Eq. (28)

\( K \)  
**induced seal stiffness**, Eq. (102)

\( K_g \)  
**generalized modal stiffness**, Eq. (108)

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**steady-state induced seal stiffness**, Eq. (99)

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\( m \)  
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\( m_c \)  
**whirl rate coupling**, Eq. (106)

\( m_g \)  
**generalized modal mass**, Eq. (109)

\( m_o \)  
**steady-state induced seal mass**, Eq. (101)

\( m_r \)  
**resultant mass**, Eq. (109)

\( m_s \)  
**simple model rotor mass**, Fig. 1

\( N \)  
**inlet coupling transfer function**, Eq. (74)
p  seal gap pressure, Eq. (58)
$p_a$  average seal gap pressure, Eq. (73)
$p_o$  inlet cavity pressure, Eq. (60)
$p_1$  gap inlet pressure, Eq. (58)
$p_2$  gap outlet pressure, Eq. (59)
$p_3$  outlet cavity pressure, Eq. (61)
$q$  circumferential coordinate, Eq. (20)
$Q$  rotation matrix for angle $\beta-\alpha$, Eq. (11)
$Q_\alpha$  rotation matrix for angle $\alpha$, Eq. (2)
$Q_\beta$  rotation matrix for angle $\beta$, Eq. (90)
$r$  rotor radius, Fig. 2
$R$  stator radius, Fig. 2
$R_r$  Reynolds number relative to rotor, Eq. (27)
$R_s$  Reynolds number relative to stator, Eq. (26)
s  Laplace operator
t  time
$T$  transpose of vectors and matrices, Eqs. (13) and (89)
$u$  axial bulk flow velocity, Fig. 2
$u_a$  average axial bulk flow velocity at gap inlet, Eq. (63)
$u_b$  variable axial bulk flow velocity at gap inlet, Eq. (63)
$u_l$  gap inlet axial bulk flow velocity, Eq. (44)
v  Couette flow velocity, Fig. 2
$w$  rotor surface velocity, Fig. 2
$x$  axial coordinate, Fig. 2
$y$  first radial coordinate, Fig. 1
$z$  second radial coordinate, Fig. 1
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TECHNICAL PAPER

DAMPING SEALS FOR TURBOMACHINERY

INTRODUCTION

The stable operation of turbomachinery depends mainly on the damping of the rotor motion [1]. Squeeze film dampers are designed for this very purpose [2], and also shaft seals are employed [3,4]. Seals are most convenient, since they are already included in the turbomachine design. However, the damping of a seal also generates a whirl driver force as shown in the pioneering work of Henry Black [5] who died January 17, 1980. He and his service to all are well-remembered [6].

The seal damping, normally effective only to twice the first critical speed, becomes an unstable whirl driver at higher speeds. Thus, high performance turbomachinery which has to run above the first critical speed is limited by an upper bound twice the first critical speed unless additional damping is applied. The fluid shear forces between the rotating journal and the stationary bushing produce a circumferential bulk flow, known as Couette flow [7], at half the shaft speed. The fundamental relation between Couette flow and whirl forces was recognized by Black [5]. Black, Allaire, and Barrett [8] found for the circumferential flow, as an approximation, an exponential function with the axial position as the independent variable, the inlet swirl as the boundary condition, and the Couette flow at half speed as the asymptote. It is shown that the whirl force can be reduced with a low or reversed inlet swirl under the assumption that the swirl can be transferred from the inlet cavity to the narrow seal gap.

This paper proposes the application of the seal surface roughness for stabilization and sealing. The distribution of the circumferential flow is approximated by the asymptotic state of the Couette flow which is assumed to prevail over the total seal length. The approximation is justified by the results of Black et al. [8], who show that seal friction advances the asymptotic approach upstream toward the inlet. The formulation treats the Couette flow generation by surfaces of different roughness without invoking a dominant axial flow as in References 5 and 8. This generalization is more representative of long seals. The turbulent seal friction is given by Moody’s pipe friction formula [9] for Reynolds numbers up to 10^5 and above. Linearization is applied at the steady flow point to aid the integration of the flow effects over the cylindrical surfaces. The steady flow is determined with the Newton-Raphson’s iteration method. The dynamic parameters are derived for small deflections about the seal’s center position. Numerical results are given for the oxygen and the hydrogen turbopumps of the Space Shuttle main engine.

ROTOR STABILITY

This section briefly reviews the purpose of the rotor damping with the help of a simple model. The model consists of a rotor (Fig. 1) with a mass m, eccentricity e, combined bearing and seal stiffnesses Ky and Kz, seal damping C, speed \( \Omega \), and Couette flow velocity v. The whirl cross coupling is given by c(\v/\r)C, with the Couette factor c and the seal radius r. The centrifugal force caused by the unbalance is given in equation (1). The components of the force are obtained with matrix Q\( \alpha \) of the equation (2) that rotates the unit vector i of equation (3) by the angle \( \alpha \). The force is balanced by the dynamic stiffness effect given in equation (4). The inverse is the response of equation (5).
Figure 1. Simple rotor model.

\[
\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} = e h o m_s \Omega^2 Q_\alpha i
\]  
\hspace{1cm} (1)

\[Q_\alpha = \begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix}\]  
\hspace{1cm} (2)

\[i = \begin{bmatrix}
1 \\
0
\end{bmatrix}\]  
\hspace{1cm} (3)

\[
\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
K_y + sC + s^2 m_s & c\Omega C \\
-c\Omega C & K_z + sC + s^2 m_s
\end{bmatrix}\begin{bmatrix}
y \\
z
\end{bmatrix}
\]  
\hspace{1cm} (4)

\[
\begin{bmatrix}
y \\
z
\end{bmatrix} = \begin{bmatrix}
K_z + sC + s^2 m_s & -c\Omega C \\
c\Omega C & K_y + sC + s^2 m_s
\end{bmatrix}\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} \frac{1}{D}
\]  
\hspace{1cm} (5)

\[D = K_y K_z + (c\Omega C)^2 + sC(K_y + K_z) + s^2 [C^2 + m_s(K_y + K_z)] + s^3 2Cm_s + s^4 m_s^2
\]  
\hspace{1cm} (6)

The common denominator of equation (5), given in equation (6), results from the determinant of equation (4). The characteristic equation \(D = 0\) determines the stability. The mapping of the contour of the right half of the complex s-plane (see Nyquist criterion [10]) yields the stability criterion. The limit is found by rearranging equation (6) for \(s = j\omega\) and setting \(D = 0\). The imaginary part of \(D\) yields the critical speed and rotor resonance [Eq. (7)]; and the real part of \(D\), the stability limit (inequalities 8 to 10).
\[ \omega_0 = \sqrt{\frac{(K_y + K_z)}{2m_s}} \]  
(7)

\[ K_yK_z + (c\omega C)^2 + \frac{(K_y + K_z)^2}{4} < \frac{K_y + K_z}{2m_s} \frac{C^2 + m_s(K_y + K_z)}{4} \]  
(8)

\[ (c\omega C)^2 < \frac{K_y + K_z}{2m_s} \frac{(K_y - K_z)^2}{4} \]  
(9)

\[ \Omega < \frac{\omega_0}{c} \]  
(10)

The stability condition of inequality (9) is obtained by rearranging the inequality (8). The special case of a uniform suspension \((K_y = K_z)\) yields the simplest stability criterion: inequality (10). The latter satisfies also the inequality (9) on the conservative side. Inequality (10) limits the speed of turbomachinery that operates above the critical speed. For example, a half-speed Couette flow has a \(c = 0.5\) and limits the speed at twice the critical speed \((\Omega < 2\omega_0)\) as, e.g., early in the development of the high pressure hydrogen turbopump of the Space Shuttle main engine [4]. If the Couette flow has a \(c = 0.25\), then the limit is extended to \(\Omega < 4\omega_0\). The objective of this study is to define a seal configuration with a Couette factor \(c < 0.5\).

**SEAL GEOMETRY**

Figure 2 shows the cross section of the seal for an exaggerated gap situation. The eccentricity \(e_{ho}\) \((0 < e < 1)\) points with the time dependent angle \(\alpha(t)\), and the gap is measured at an angle \(\beta\). Both angles are inertially referenced. The cylindrical coordinates are \(x\) (axial position) and \(q = r\beta\) \((r = R\) circumferential position). Equations (11) through (15) give, in respective order, the rotational transformation matrix \(Q\) for the angle \(\beta - \alpha(t)\), the right angle rotation matrix \(\Pi\), the unit vector \(i\), the partial derivative after \(\beta\), and the partial derivative after time. Equations (16) through (21) are, in respective order, the compact expressions of cosine and sine, seal gap \(h\), the differential relative to the gap, the circumferential partial derivative relative to the gap, and the time partial derivative relative to the gap. Equation (18) follows from Figure 2, \(R - r = h_0, R = r,\) and \(h = R - [r + e_{ho} \cos(\beta - \alpha)] = h_0 [1 - e \cos(\beta - \alpha)]\).

![Figure 2. Seal cross section.](image-url)
The flow friction is given by the pressure drop in pipes $\Delta p = \rho \frac{u^2}{2} (L/D)f$ with the friction factor $f$ [9], length $L$, hydraulic diameter $D$ ($D = 2h$ for seals with radial gap $h$), roughness $\kappa$ ($\kappa/h$ roughness relative to gap $h$), axial bulk flow velocity $u$, and fluid density $\rho$ [11]. The seal has the axial and circumferential bulk flow velocities $u$ and $v$ and the rotor surface velocity $w$ (Fig. 2). The fluid shear stresses on the surfaces of the stator (bushing) and the rotor (journal) are $\tau_s$ in equation (22) and $\tau_t$ in equation (23). The pressure gradient is given in equations (24) and (25) with the axial and circumferential components. The Reynolds
numbers are given in equations (26) and (27), the frictions in equations (28) and (29), the friction's Taylor expansions in equations (30) and (31), the friction velocity factors in equations (32) and (33), and the friction gap factors in equations (34) and (35); all equation pairs are for the stator and rotor, respectively. The fluid viscosity is $\mu$ in equations (26) and (27) and the surface roughnesses $k_s$ and $k_r$ in equations (28), (29), and (32) through (35) for the stator and rotor, respectively.

$$\tau_s = \rho \frac{u^2+v^2}{2} \cdot \frac{f_s}{4}$$

$$\tau_r = \rho \frac{u^2+(v-w)^2}{2} \cdot \frac{f_r}{4}$$

$$-\frac{\partial p}{\partial x} = \frac{\tau_{sx}+\tau_{rx}}{h} = \frac{\rho}{8h} \left[ uf_s \sqrt{u^2+v^2} + uf_r \sqrt{u^2+(v-w)^2} \right]$$

$$-\frac{\partial p}{\partial q} = \frac{\tau_{sq}+\tau_{rq}}{h} = \frac{\rho}{8h} \left[ vf_s \sqrt{u^2+v^2} + (v-w)f_r \sqrt{u^2+(v-w)^2} \right]$$

$$R_s = \frac{\rho}{\mu} \frac{2h}{\sqrt{u^2+v^2}}$$

$$R_r = \frac{\rho}{\mu} \frac{2h}{\sqrt{u^2+(v-w)^2}}$$

$$f_s = 0.0055 \left[ 1 + \left( \frac{10^4}{h} \frac{k_s}{R_s} + \frac{10^6}{R_s} \right)^{1/3} \right]$$

$$f_r = 0.0055 \left[ 1 + \left( \frac{10^4}{h} \frac{k_r}{R_r} + \frac{10^6}{R_r} \right)^{1/3} \right]$$

$$f_{se} = f_s - \frac{g_s}{u^2+v^2} \cdot \frac{h_s}{h}$$

$$f_{re} = f_r - \frac{g_r}{u^2+(v-w)^2} \cdot \frac{h_r}{h}$$

$$g_s = \frac{0.0055 \cdot 10^6}{3R_s \left( 10^4 k_s / h + 10^6 / R_s \right)^{2/3}}$$

$$g_r = \frac{0.0055 \cdot 10^6}{3R_r \left( 10^4 k_r / h + 10^6 / R_r \right)^{2/3}}$$
The Couette flow is generated by the rotor shear force and resisted by the stator shear force. An eccentricity causes an additional circumferential flow that produces the pressure gradient component of equation (25). The latter is assumed to be negligible, as expressed by the equation (36), which yields the Couette factor \( c = \frac{v}{w} \). The limits given in equations (37) and (38) follow from the circumferential and the axial flow dominances. The Couette factor varies little with \( u \), as Figure 3 shows.

\[
v f_s \sqrt{u^2 + v^2} + (v-w) f_r \sqrt{u^2 + (v-w)^2} = 0
\]  

(36)

\[
u = 0: \quad c = \frac{v}{w} = \frac{1}{1 + \sqrt{f_s/f_r}}
\]  

(37)

\[
u = \infty: \quad c = \frac{v}{w} = \frac{1}{1 + f_s/f_r}
\]  

(38)

Figure 3. Couette factor \( c = \frac{v}{w} \) versus the axial flow velocity ratio \( u/w \) for friction ratios \( f_s/f_r \).
FLOW CONTINUITY

The motion of the gap pumps the assumed incompressible fluid, according to equation (39), to preserve the volume. The equation is further simplified to equation (40) by canceling terms, assuming no gap taper \( \frac{\partial h}{\partial x} = 0 \), and by assuming a constant circumferential flow \( \frac{\partial v}{\partial q} = 0 \). Equation (41) is obtained by substituting equations (20) and (21) in equation (40), and by approximating \( 1/h \) with \( 1/h_0 \). The introduction of the definitions of the equations (42) and (43) and the integration of equation (41) yield the axial velocity of equation (44) with \( u_1 \) as the inlet velocity.

\[
\begin{align*}
\text{uhdq} + vhdx &= \text{uhdq} + h \frac{\partial u}{\partial x} \text{dx} + vhdx + h \frac{\partial v}{\partial q} \text{dq} dx + v \frac{\partial h}{\partial q} \text{dq} dx + \frac{\partial h}{\partial t} \text{dq} dx \\
\frac{\partial u}{\partial x} + \frac{v \partial h}{h \partial q} + \frac{\partial h}{h \partial t} &= 0 \\
\frac{\partial u}{\partial x} &= i \left( \dot{\varepsilon} I - \dot{\alpha} e \Pi \right) Q_i + \frac{v}{r} e_i T\Pi Q_i = A \\
\dot{\gamma} &= \dot{\alpha} - \frac{v}{r} = \ddot{\alpha} - c \Omega \\
A &= i \left( \dot{\varepsilon} I - \dot{\gamma} e \Pi \right) Q_i \\
u &= u_1 + xA
\end{align*}
\]

PRESSURE DISTRIBUTION

The dynamic pressure gradient of equation (45) follows from equation (24) by adding the mass acceleration of equation (46). The equation (47) is obtained by expanding all factors of equation (45) with first order terms and by utilizing the expanded friction factors of equations (30) and (31). The multiplication of all factors and the neglecting of second order terms yield equation (48), which is the Taylor expansion of equation (45). The definitions of equations (49) through (51) lead to the compact form of the Taylor expansion, equation (52). The time derivative of equation (46) applied to equation (44) yields equation (53). The definitions of equations (42) and (56) result in a further simplification, equation (54). The pressure distribution of equation (58) is obtained by integrating equation (52); by substituting the definitions of equations (19), (43), (55), and (56); and by employing the approximation of equation (57).

\[
\frac{-\partial p}{\partial x} = \frac{\rho u}{\delta h} \left[ f_s \sqrt{u^2 + v^2} + f_r \sqrt{u^2 +(v-w)^2} \right] + \rho \frac{du}{dt}
\]
\[
\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial q}
\]

(46)

\[
\frac{-\partial p}{\partial x} = \frac{\rho u_1}{8h_0} \left( 1 + \frac{du}{u_1} - \frac{dh}{h_0} \right) \left[ f_s \left( 1 - \frac{g_s}{f_s} \frac{u_1 du}{u_1^2 + v^2} - \frac{h_s}{f_s} \frac{dh}{h_0} \right) \sqrt{u_1^2 + v^2} \left( 1 + \frac{u_1 du}{u_1^2 + v^2} \right) \right] + f_r \left( 1 - \frac{g_r u_1 du}{f_r u_1^2 + (v-w)^2} - \frac{h_r}{f_r} \frac{dh}{h_0} \right) \sqrt{u_1^2 + (v-w)^2} \left( 1 + \frac{u_1 du}{u_1^2 + (v-w)^2} \right) + \rho \frac{du}{dt}
\]

(47)

\[
\frac{-\partial p}{\partial x} = \frac{\rho u_1}{8h_0} \left[ f_s \sqrt{u_1^2 + v^2} \left( 1 + \frac{du}{u_1} \left( 1 + u_1^2 \frac{1-g_s/f_s}{u_1^2 + v^2} \right) - \frac{dh}{h_0} \left( 1 + \frac{h_s}{f_s} \right) \right) \right] + f_r \sqrt{u_1^2 + (v-w)^2} \left( 1 + \frac{du}{u_1} \left( 1 + u_1^2 \frac{1-g_r/f_r}{u_1^2 + (v-w)^2} \right) - \frac{dh}{h_0} \left( 1 + \frac{h_r}{f_r} \right) \right) + \rho \frac{du}{dt}
\]

(48)

\[
F = \frac{\rho}{8h_0} \left[ f_s \sqrt{u_1^2 + v^2} + f_r \sqrt{u_1^2 + (v-w)^2} \right]
\]

(49)

\[
G = \frac{\rho u_1^2}{8h_0} \left[ \frac{f_s g_s}{\sqrt{u_1^2 + v^2}} + \frac{f_r g_r}{\sqrt{u_1^2 + (v-w)^2}} \right]
\]

(50)

\[
H = \frac{\rho}{8h_0} \left[ h_s \sqrt{u_1^2 + v^2} + h_r \sqrt{u_1^2 + (v-w)^2} \right]
\]

(51)

\[
\frac{-\partial p}{\partial x} = u_1 F + (F+G) \Delta u_1 - \frac{dh}{h_0} u_1 (F+H) + \rho \frac{du}{dt}
\]

(52)

\[
\frac{du}{dt} = \frac{du_1}{dt} + xi^T \left[ (\tilde{c}-\alpha \tilde{e})I - (\alpha \gamma \tilde{e}+\alpha \gamma \tilde{e}) \right] Qi + u \frac{\partial u}{\partial x} + \frac{u^T v}{r} (\gamma e I + \tilde{e} \tilde{e}) Qi
\]

(53)

\[
\frac{dv}{dt} = \frac{dv_1}{dt} + xi^T \left[ (\tilde{c}-\gamma e^2)I - (\alpha \gamma e + 2 \gamma \tilde{e}) \right] Qi + u \frac{\partial u}{\partial x} = \frac{du_1}{dt} + xB + u \frac{\partial u}{\partial x}
\]

(54)
\[ \frac{dh}{h_0} = -E \]  
(19)

\[ \Delta u_1 = u - u_1 = xA \]  
(55)

\[ B = i \left[ (\partial^2 \gamma e) I - (\partial e + 2 \gamma \dot{e}) \Pi \right] Q_i \]  
(56)

\[ \frac{u_1^2}{2} - \frac{u_1^2}{2} = u_1(u - u_1) = u_1 xA \]  
(57)

\[ p_1 - p = xu_1 F + \frac{x^2}{2} A(F + G) + xu_1 F(F + H) + x\rho \frac{du_1}{dt} + \frac{x^2}{2} \rho B + x\rho u_1 A \]  
(58)

**BOUNDARY CONDITIONS**

The pressure drop over the total seal gap length \( L \) of equation (59) follows from \( x = L \) in equation (58). The inlet loss \( f_1 \) reduces the inlet cavity pressure \( p_o \) to the inlet pressure \( p_1 \) of equation (60), assuming \( u_o = 0 \). The outlet pressure \( p_2 \) equals the outlet cavity pressure \( p_3 \) of equation (61), assuming a sudden discharge into a large outlet cavity \( (f_2 = 1) \) and \( u_3 = 0 \). Equation (62) gives the pressure drop from one to the other cavity. Equation (63) describes the inlet flow velocity \( u_1 \) as the sum of the steady \( (u_a) \) and the oscillatory \( (u_b) \) velocities. Equation (64) results from equations (62) and (63), \( u_a >> u_b \), \( u_1 F = u_a F + u_b(F + G) \), and from neglecting second order terms. The steady-state part of equation (64) is given in equation (65) as a Taylor expansion, and in equation (66) in the iterative form after Newton-Raphson. Equation (67) is the oscillatory component of equation (64) with \( s \) replacing \( \frac{d(\cdot)}{dt} \) and tacitly assuming the Laplace transform for the variables. Equation (68) defines the time constant \( \tau \), and equation (69) the limit \( \tau_o \) that is found by conservatively neglecting \( \rho u_a(1 + f_1) \). Equation (70) follows from equations (58) and (60). Equation (71) follows from equations (63) and (70), \( u_a >> u_b \), \( u_1^2/2 = u_a^2/2 + u_a u_b \), \( u_1 F = u_a F + u_b(F + G) \), and by neglecting second order terms. The pressure distribution of equation (71), together with equations (66) and (67), expresses the effects of the boundary conditions and the seal gap.

\[ p_1 - p_2 = Lu_1 F + Lu_1 E(F + H) + \frac{L^2}{2} A(F + G) + L\rho \frac{du_1}{dt} + \frac{L^2}{2} \rho B + L\rho u_1 A \]  
(59)

\[ p_0 - \rho \frac{u_1^2}{2} \cdot f_1 = p_1 + \rho \frac{u_1^2}{2} \]  
(60)
\[ p_2 + \rho \frac{u_2^2}{2} = p_3 + \rho \frac{u_2^2}{2} \cdot f_2 \quad , \quad f_2 = 1 \]  

(61)

\[ p_0 - p_3 = p_1 - p_2 + \rho \frac{u_1^2}{2} (1+f_1) \]

\[ = \rho \frac{u_1^2}{2} (1+f_1) + L u_1 F + L u_1 E(F+H) + \frac{L^2}{2} A(F+G) + L \rho \frac{du_1}{dt} + \frac{L^2}{2} \rho B + L \rho u_1 A \]  

(62)

\[ u_1 = u_a + u_b \]

(63)

\[ p_0 - p_3 = \rho \frac{u_a^2}{2} (1+f_1) + u_a LF + u_a L[E(F+H) + \rho A] + \frac{L^2}{2} [A(F+G) + \rho B] \]

\[ + u_b[\rho u_a(1+f_1) + L(F+G) + s \rho L] \quad , \quad s = d(\cdot)/dt \]

(64)

\[ p_0 - p_3 = \rho \frac{u_a^2}{2} (1+f_1) + u_a LF + [\rho u_a(1+f_1) + L(F+G)] \Delta u_a \]

(65)

\[ u_{a,n+1} = u_{a,n} + \frac{p_0 - p_3 - \rho u_{a,n}(1+f_1)/2 - u_{a,n} LF}{\rho u_{a,n}(1+f_1) + L(F+G)} \]

(66)

\[ u_b = - \frac{u_a L[E(F+H) + \rho A] + L^2 [A(F+G) + \rho B]/2}{(1+s \tau)[\rho u_a(1+f_1) + L(F+G)]} \]

(67)

\[ \tau = \frac{\rho L}{\rho u_a(1+f_1) + L(F+G)} \]

(68)

\[ \tau_o = \frac{\rho}{F+G} \]

(69)
\[ p_0 - p = p_1 - p + \rho \frac{u_1^2}{2} (1+f_1) \]

\[ = \rho \frac{u_1^2}{2} (1+f_1) + xu_1 [F + E(F+H) + \rho A + sp] + \frac{x^2}{2} [A(F+G) + \rho B] \]

\[ p = p_0 - \rho \frac{u_a^2}{2} (1+f_1) - xu_a F - xu_a [E(F+H) + \rho A] - \frac{x^2}{2} [A(F+G) + \rho B] \]

\[ - u_b [\rho u_a (1+f_1) + x(F+G+s\rho)] \]

**DYNAMIC SEAL PARAMETERS**

The fluid forces are obtained by integrating the pressure distribution of equation (71). First, the axial integration gives the radial force in equations (72), (75), (81), and (87); and second, the circumferential integration gives the total fluid force in equations (88) and (107). Equation (73) defines the average pressure \( p_a \) which does not contribute to the side force. Equation (74) defines the factor \( N \) of the variable velocity \( u_b \) as derived from equations (67) and (72). Equation (74) is further simplified by substituting with equations (68) and (69). The substitution of equation (67) in equation (72) yields equation (75) and the definitions of equations (76) through (84). The time constants of equations (68), (69), (80), (84), and (85) are ordered like the inequalities (86). The upper limit of equation (85) is derived by selecting a \( v/u \) ratio that minimizes the stator and the rotor frictions in \( F+G \) [equations (49), (50), and (69)]. The circumferential integration of equation (88) is vectorially accomplished to advantageously use the matrices of equations (89) and (90). The integration of the functions \( E, A, \) and \( B \) [equations (19), (43), and (56)] involves the function type of equation (91). The latter employs equation (89) to separate the angles \( \alpha \) and \( \beta \) via the matrix product. The integrations of equations (92) lead to a diadic that, when integrated, yields a simple result. Equations (93) through (95) relate the polar and rectangular coordinates and their derivatives. Equations (96) through (98), the integrals of the functions \( E, A, \) and \( B, \) are transformed with equations (42) and (93) through (95); and \( s = d(\cdot)/dt \) to rectangular coordinates, tacitly assuming Laplace transformed variables. Equations (87) and (96) through (98) yield the definitions of equations (99) through (107). Equations (102) through (106) give the frequency response of the dynamic parameters. The frequency response is due to the time constants \( \tau, \tau_a, \) and \( \tau_b, \) which are small as indicated by the limit \( \tau_u \) of equation (85) and inequalities (86). The frequency response is obtained with \( s = j\omega \) and by replacing \( 1/(1+sr) = (1-sr)/(1+\omega^2 \tau^2) \) and \( s^3 = -s\omega^2. \) The \( (\omega \tau)^2 \) terms are usually negligible; however, \( (s\Omega)^2 m_c \) and \( \tau K_o \) are not [equations (102) and (107)]. The vector equation (107) is the dynamic model of the seal. The whirl driver is the off-diagonal elements that are proportional to the Couette factor \( c \) and the rotor speed \( \Omega. \)
\( p_a = p_0 - \rho \frac{u_a^2}{2} (1+f_1) - \frac{L}{2} u_a F \) \hspace{1cm} (73)

\[ N = \frac{1}{2(1+s\tau)} \cdot \frac{2 \rho u_a (1+f_1) + L(F+G+s\rho)}{\rho u_a (1+f_1) + L(F+G)} = \frac{2 + \tau(s-1/\tau_0)}{2(1+s\tau)} \] \hspace{1cm} (74)

\[ \int_{\sigma} \rho u_a d\sigma = L^2 u_a [E(F+H) + \rho A] [N-1/2] + \frac{L^3}{2} [A(F+G) + \rho B] [N-1/3] \] \hspace{1cm} (75)

\[ N-1/2 = \frac{d}{2(1+s\tau)} \] \hspace{1cm} (76)

\[ d = 1 - \frac{\tau}{\tau_0} = \frac{\tau}{L} u_a (1+f_1) \] \hspace{1cm} (77)

\[ N-1/3 = \frac{b}{2} \cdot \frac{1+s\tau b}{1+s\tau} \] \hspace{1cm} (78)

\[ b = \frac{4}{3} \cdot \frac{\tau}{\tau_0} = \frac{\tau}{3} \left( \frac{4}{3} \cdot \frac{3}{\tau_0} \right) = \frac{\tau}{3\tau_b} \] \hspace{1cm} (79)

\[ \tau_b = \frac{\rho L}{4 \rho u_a (1+f_1) + L(F+G)} \] \hspace{1cm} (80)

\[ \int_{\sigma} \rho u_a d\sigma = L^2 u_a \left[ \frac{F+H}{1+s\tau} + \frac{A}{1+s\tau} \cdot \frac{L^2}{2} \right] + B \frac{L^3}{4} \rho b \frac{1+s\tau b}{1+s\tau} \] \hspace{1cm} (81)

\[ \rho u_a d + \frac{L}{2} (F+G) b(1+s\tau b) = \tau \left[ \frac{\rho}{L} u_a^2 (1+f_1) + \frac{L}{6\tau_b} (F+G) + s \frac{L^3}{6} (F+G) \right] = a(1+s\tau_a) \] \hspace{1cm} (82)

\[ a = \frac{\tau}{\tau_a} \cdot \frac{L}{F+G} \] \hspace{1cm} (83)
\[ \tau_a = \frac{\rho L^2 (F+G)}{\rho u_a (1+\alpha)} [6\rho u_a + 4L(F+G) + L^2 (F+G)^2] \]  

(84)

\[ \tau_u = \frac{4h_o}{u_a [\sqrt{f_s (f_s - g_s)} + \sqrt{f_r (f_r - g_r)}]} \]  

(85)

\[ \tau_u > \tau_o > \tau > \tau_b > \tau_a \]  

(86)

\[ \int_0^L p dx = L p_a + \frac{E}{2} u_a d \cdot \frac{F+H}{1+sr} + A \frac{L^2}{2} a \frac{1+sr_a}{1+sr} + B \frac{L^3}{4} \rho b \frac{1+sr_b}{1+sr} \]  

(87)

\[ \begin{bmatrix} F_y \\ F_z \end{bmatrix} = r \int_0^L \left( \int_0^L p dx \right) Q_\beta \, d\beta \]  

(88)

\[ Q = Q_\alpha^T Q_\beta \]  

(89)

\[ Q_\beta = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \]  

(90)

\[ i^T (a_o I - b_o \beta) Q_i = a_o \cos (\beta - \alpha) + b_o \sin (\beta - \alpha) = i^T Q_\beta^T \begin{bmatrix} a_o \\ b_o \end{bmatrix} = i^T Q_\beta^T Q_\alpha \begin{bmatrix} a_o \\ b_o \end{bmatrix} \]  

(91)

\[ \int_0^{2\pi} i^T (a_o I - b_o \beta) Q_i \cdot Q_\beta \, d\beta = \int_0^{2\pi} Q_\beta i^T Q_\beta^T \begin{bmatrix} a_o \\ b_o \end{bmatrix} = \int_0^{2\pi} \begin{bmatrix} \cos^2 \beta \cos \beta \sin \beta \\ \sin \beta \cos \beta \sin^2 \beta \end{bmatrix} d\beta Q_\alpha \begin{bmatrix} a_o \\ b_o \end{bmatrix} = \pi Q_\alpha \begin{bmatrix} a_o \\ b_o \end{bmatrix} = \pi (a_o I + b_o \beta) Q_\alpha^i \]  

(92)

\[ \begin{bmatrix} Y \\ Z \end{bmatrix} = h_o e^{Q_\alpha i} \]  

(93)
\[
\begin{align*}
\begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix}
&= h_o(\dot{e}I+\dot{\alpha}e\Pi)Q_{\alpha}i \\
\begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix}
&= h_o[(\ddot{e}-\dot{\alpha}^2 e)I + (\ddot{\alpha}e+2\dot{\alpha}e\Pi)] \\
\int_0^{2\pi} E_o e_i d\beta &= \pi h_o e Q_{\alpha}i = \pi \begin{bmatrix}
y \\
z
\end{bmatrix} \\
\int_0^{2\pi} A_o e_i d\beta &= \pi h_o (\dot{e}I+\dot{\gamma}e\Pi)Q_{\alpha}i = \pi \begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix} - \pi c\Omega I \begin{bmatrix}
y \\
z
\end{bmatrix} = \pi (s-I-c\Omega\Pi) \begin{bmatrix}
y \\
z
\end{bmatrix} \\
\int_0^{2\pi} B_o e_i d\beta &= \pi h_o [(\ddot{e}-\dot{\gamma}^2 e)I + (\ddot{\gamma}e+2\dot{\gamma}e\Pi)]Q_{\alpha}i \\
&= \pi \begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix} - \pi (c\Omega)^2 \begin{bmatrix}
y \\
z
\end{bmatrix} - 2\pi c\Omega\Pi \begin{bmatrix}
\dot{y} \\
\dot{z}
\end{bmatrix} = \pi [(s^2 - (c\Omega)^2)I - s2c\Omega\Pi] \begin{bmatrix}
y \\
z
\end{bmatrix}
\end{align*}
\]

\[K_o = \frac{\pi r}{h_o} u_a^2 (1 + f_1) \frac{L}{2} (F+H)\]
\[C_o = \frac{\pi r}{h_o} \frac{\tau}{r_a} \frac{L^3}{12} (F+G)\]
\[m_o = \frac{\pi r}{h_o} \frac{\tau}{r_b} \frac{L^3}{12} \rho = C_o \frac{\tau_a}{r_b} \tau_o\]
\[K = \frac{K_o}{1 + \omega^2 r^2}\]
\[
C = \frac{C_0(1+\omega^2 \tau \tau_d) + m_0\omega^2(\tau-\tau_b) - \tau K_0}{1+\omega^2 \tau^2}
\]

\[
m = \frac{m_0(1+\omega^2 \tau \tau_d) - C_0(\tau-\tau_a)}{1+\omega^2 \tau^2}
\]

\[
C_c = \frac{C_0(1+\omega^2 \tau \tau_d) + 2m_0\omega^2(\tau-\tau_b)}{1+\omega^2 \tau^2}
\]

\[
m_c = \frac{m_0(1+\omega^2 \tau \tau_b)}{1+\omega^2 \tau^2}
\]

\[
\begin{bmatrix}
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
K-(c\Omega)^2 m_c + sC + s^2 m & c\Omega[C_c + s(m+m_c)] \\
-c\Omega[C_c + s(m+m_c)] & K-(c\Omega)^2 m_c + sC + s^2 m
\end{bmatrix} \begin{bmatrix}
y \\
z
\end{bmatrix}
\]

**STABILITY**

The dynamic models of turbomachines are far more complex than the simple model of Figure 1. The dynamic models consist of a sum of normal modes with gyroscopic moments and fluid forces added. The normal modes are the sum of resonators. The resonators are tuned to individual resonances, such as the critical speeds of the rotor. At one critical speed, one resonator dominates; and thus, the simple model of Figure 1 appears to be representative of the rotor behavior when the rotor mass is replaced by the generalized mass \(m_g\) and the suspension stiffness by the generalized stiffness \(K_g\) at the seal location. The stability is assessed by adding \(K_g + s^2 m_g\) to the diagonal of the matrix in equation (107). The combined parameters are defined in equations (108) and (109). The determinant of the modified matrix of equation (107) gives the characteristic equation (110). (See Eq. 6.) The imaginary part of \(D = 0\) yields the resonance of equation (111); and the real part, the stability limit of the inequalities (112) through (114). The ratio \(\Omega_L/\omega_o\) of equation (115) is approximately \(1/c\), because \(C = C_c\) and \(\omega_o^2(m+m_c)\) is small. In other words, the Couette factor controls the speed limit.

\[
K_r = K + K_g
\]

\[
m_r = m + m_g
\]
\[ D = [K_r - (c\Omega)^2 m_c]^2 + (c\Omega)^2 C_c^2 + s^2[C(K_r - (c\Omega)^2 m_c) + (c\Omega)^2 C_c(m + m_c)] \]

\[ + s^2 [C^2 + 2(K_r - (c\Omega)^2 m_c)m_r + (c\Omega)^2 (m + m_c)^2] + s^3 2m_r C + s^4 m_r^2 = 0 \] (110)

\[ \omega_0^2 = \frac{K_r}{m_r} - \frac{(c\Omega)^2 m_c}{m_r} \] (111)

\[ 2[K_r - (c\Omega)^2 m_c]^2 + (c\Omega)^2 C_c^2 < [K_r - (c\Omega)^2 m_c] C^2/m_r + 2[K_r - (c\Omega)^2 m_c]^2 \]

\[ + [K_r - (c\Omega)^2 m_c] (c\Omega)^2 (m + m_c)^2/m_r \] (112)

\[ (c\Omega)^2 C_c^2 < \omega_0^2 [C^2 + (c\Omega)^2 (m + m_c)^2] \] (113)

\[ \Omega < \frac{\omega_0}{c} \frac{C}{\sqrt{C_c^2 - \omega_0^2 (m + m_c)^2}} = \Omega_L \] (114)

\[ \frac{\Omega_L}{\omega_0} = \frac{C/c}{\sqrt{C_c^2 - \omega_0^2 (m + m_c)^2}} \] (115)

**NUMERICAL RESULTS**

Table 1 compares a Rocketdyne analysis after Black for a uniform Couette flow factor \( c = 0.5 \) with the new analysis for equally polished seal surfaces of relative roughnesses \( k_s/h_0 = k_r/h_0 = 0.002 \). The results are similar and differ mostly at the fluid film stiffness. The difference is most likely due to Black’s assumption of a dominant axial flow, while the new formulation is not thus restricted. Table 2 lists the recent design parameters for seals of the high pressure fuel turbopump’s (HPFTP) first and second pump interstages, and the high pressure oxidizer turbopump’s (HPOTP) inducer shroud. Table 3 gives the results for damping seals with rough strators of \( k_s/h_0 = 0.2 \) and polished rotors of \( k_r/h_0 = 0.002 \). A high roughness ratio is required because of the cubic root in Moody’s friction equations (28) and (29). Table 3 demonstrates that speed limits of 2.8 and 3.9 times the first critical speed are feasible. The ratios \( \Omega_L/\omega_0 \) are calculated with equation (115) for first critical speeds of 18,000 RPM and 15,000 RPM of the HPFTP and the HPOTP, respectively. An inlet loss factor of \( f_I = 0.5 \) is applied in all cases. A comparison of Tables 1 and 3 shows that damping seals also have a lower leakage velocity \( u \) (70 percent).
TABLE 1. COMPARISON BETWEEN THE BLACK AND THE NEW MODEL

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>10^3 lb/in.</th>
<th>lb-s/in.</th>
<th>lb</th>
<th>ft/s</th>
<th>( \frac{1}{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPFTP Interstage Seals</td>
<td>K</td>
<td>cΩm_c</td>
<td>C</td>
<td>cΩm_c</td>
<td>m</td>
</tr>
<tr>
<td>Black Model Rocketdyne Data 7/13/76</td>
<td>285</td>
<td>209</td>
<td>107</td>
<td>10.7</td>
<td>2.11</td>
</tr>
<tr>
<td>New Model</td>
<td>404</td>
<td>205</td>
<td>93</td>
<td>6.5</td>
<td>0.56</td>
</tr>
</tbody>
</table>

The comparison is for the straight smooth seal with a 3.14-in. diameter, 1.5 in. length, 37,360 RPM, and a relative roughness of \( k_s/h_o = k_r/h_o = 0.002 \) for the stator and the rotor.

TABLE 2. DESIGN PARAMETERS OF SEALS FOR THE HIGH PRESSURE FUEL TURBOPUMP (HPFTP) AND THE HIGH PRESSURE OXIDIZER TURBOPUMP (HPOTP) OF THE SPACE SHUTTLE MAIN ENGINE (SSME)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>inch</th>
<th>psia</th>
<th>RPM</th>
<th>lb/ft-s</th>
<th>lb/ft-s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seals</td>
<td>Gap</td>
<td>Dia.</td>
<td>Length</td>
<td>In</td>
<td>Out</td>
</tr>
<tr>
<td>HPFTP First Interstage</td>
<td>0.0055</td>
<td>3.56</td>
<td>1.8</td>
<td>2432</td>
<td>288</td>
</tr>
<tr>
<td>HPFTP Second Interstage</td>
<td>0.0055</td>
<td>3.56</td>
<td>1.8</td>
<td>4810</td>
<td>2557</td>
</tr>
<tr>
<td>HPOTP Inducer Shroud</td>
<td>0.01</td>
<td>4.70</td>
<td>1.5</td>
<td>1824</td>
<td>433</td>
</tr>
<tr>
<td>HPOTP Inducer Shroud</td>
<td>0.02</td>
<td>4.70</td>
<td>1.5</td>
<td>1824</td>
<td>433</td>
</tr>
</tbody>
</table>
### TABLE 3. DAMPING SEAL PARAMETERS

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>$10^3$lb/in.</th>
<th>lb-s/in.</th>
<th>lb</th>
<th>ft/s</th>
<th>$10^3$</th>
<th>$10^{-6}$s</th>
<th>$10^{-3}$</th>
<th>$\frac{1}{c}$</th>
<th>$\frac{\Omega L}{\omega_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seals</td>
<td>$K$</td>
<td>$c_\Omega C_c$</td>
<td>$C$</td>
<td>$c_\Omega m_C$</td>
<td>$m$</td>
<td>$u$</td>
<td>$v$</td>
<td>$w$</td>
<td>$R_s$</td>
</tr>
<tr>
<td>HPFTP First Interstage</td>
<td>340</td>
<td>182</td>
<td>182</td>
<td>4.9</td>
<td>1.3</td>
<td>660</td>
<td>142</td>
<td>573</td>
<td>271</td>
</tr>
<tr>
<td>HPFTP Second Interstage</td>
<td>358</td>
<td>179</td>
<td>181</td>
<td>4.5</td>
<td>1.2</td>
<td>703</td>
<td>140</td>
<td>573</td>
<td>333</td>
</tr>
<tr>
<td>HPOTP Inducer Shroud, 0.01 in. Gap</td>
<td>168</td>
<td>240</td>
<td>202</td>
<td>37</td>
<td>7.3</td>
<td>154</td>
<td>204</td>
<td>623</td>
<td>343</td>
</tr>
<tr>
<td>HPOTP Inducer Shroud, 0.02 in. Gap</td>
<td>116</td>
<td>123</td>
<td>101</td>
<td>23</td>
<td>3.2</td>
<td>217</td>
<td>195</td>
<td>623</td>
<td>783</td>
</tr>
</tbody>
</table>

The stator and rotor roughnesses are $k_s/h_0 = 0.2$ and $k_r/h_0 = 0.002$, respectively, $\omega = 0$; and the critical speeds for the HPFTP and the HPOTP are 18,000 RPM and 15,000 RPM, respectively.

### CONCLUSION

The dynamic and static seal parameters of seals with rough surfaces are derived without invoking a dominant axial flow. Whirl stability and leakage are shown to be controllable by employing a stator with a high surface roughness. The feasibility of damping seals is analytically demonstrated, but experimental verification is needed. The proposed seals are simple and should be readily applicable to turbomachinery. Speed limits can thus be raised, rotor vibrations reduced, bearing life improved, and costly shutdowns avoided.
REFERENCES


A rotor seal is proposed that restricts leakage like a labyrinth seal, but extends the stabilizing speed range beyond twice the first critical speed. The dynamic parameters are derived from bulk flow equations without requiring a dominant axial flow. The flow is considered incompressible and turbulent. Damping seals are shown to be feasible for extending the speed range of high performance turbomachinery beyond the limit imposed by conventional seals.