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NEUTRINO MASSES, NEUTRINO OSCILLATIONS, AND COSMOLOGICAL IMPLICATIONS

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NEUTRINO MASSES, NEUTRINO OSCILLATIONS, AND COSMOLOGICAL IMPLICATIONS

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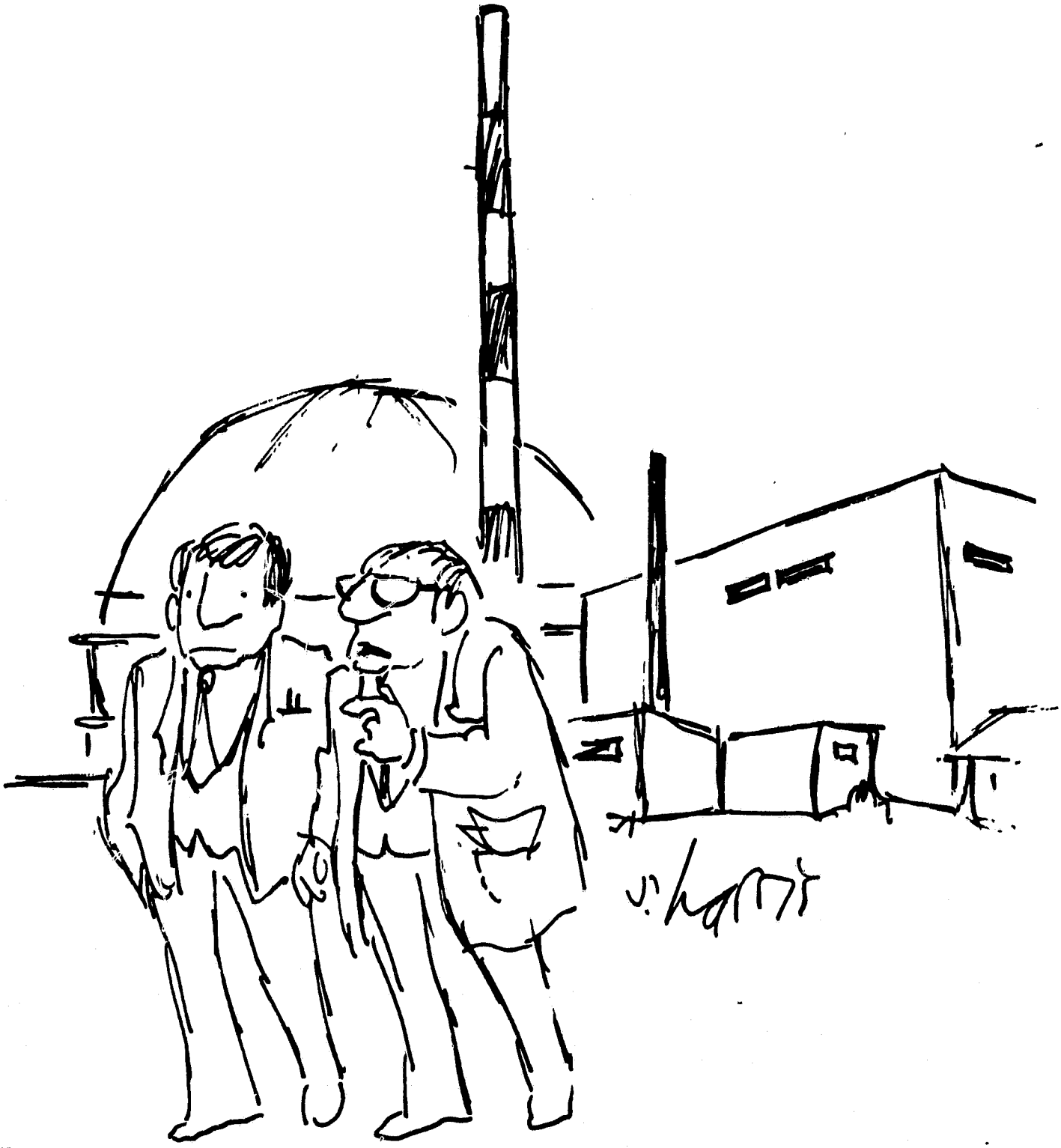

"THE QUESTION NOW IS, 'HOW MANY NEUTRINOS CAN DANCE ON THE HEAD OF A PIN?'"

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ABSTRACT

Theoretical concepts and motivations for considering neutrinos having finite masses are discussed first. Following this, the experimental situation on searches for neutrino masses and oscillations is summarized. This includes a discussion of the solar neutrino problem, reactor, deep mine and accelerator data, tritium decay experiments and double beta-decay data. Finally, the cosmological implications and astrophysical data relating to neutrino masses will be reviewed. Aspects of this topic include the neutrino oscillation solution to the solar neutrino problem, the missing mass problem in galaxy halos and galaxy clusters, galaxy formation and clustering, and radiative neutrino decay and the cosmic ultraviolet background radiation.

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I. INTRODUCTION:

Neutrinos, parity violation, Cabibbo mixing, neutral currents, unification and grand unification are some of the physical concepts which come to mind when one thinks of the subject of weak interactions. A surprisingly wide range of phenomena of unexpected sublety and beauty has unfolded in this century, leading to a deeper, but still incomplete, understanding of the subject. And now the possibility that neutrinos have mass has led to a new connection between particle physics and cosmology. These lectures will be concerned with the various aspects of neutrino mass and their broad and profound implications.

II. WHY SHOULD NEUTRINOS HAVE MASS?

In order to discuss why neutrinos may have mass and how they "get" mass, we must be more specific about defining the character of the neutrino. This character is determined by the field equations which the neutrinos obey and their couplings. In general, a spin one-half fermion obeys the Dirac equation and can be represented by a four-component spinor. The degrees of freedom represent both the particle and the antiparticle, each with two helicity states. For the neutrino, presently known phenomena only relate to two of these components, viz., the left-handed neutrino and the right handed-antineutrino. Thus, presently observed electroweak phenomena are satisfactorily described by the $SU(2)_L \times U(1)$ model of Glashow, Weinberg and Salam (GWS). There are three ways to account for this situation. Either

(1) there are no right-handed neutrinos and no left-handed antineutrinos.

This situation can only occur if neutrinos are massless. Otherwise a large enough Lorentz transformation could always transform a left-handed neutrino into a right-handed neutrino. Or

(2) right-handed neutrinos exist but don't participate in $SU(2)_L \times U(1)$ electroweak interactions ("sterile neutrinos"). If these neutrinos exist, we can construct Dirac mass terms of the form

$$\bar{\nu}_R M_D \nu_L + \bar{\nu}_L M_D \nu_R \tag{2.1}$$

Or

(3) Right-handed neutrinos exist and are really the antimatter (charge conjugate) counterparts of left handed neutrinos. As in the case of the π^0 boson, this requires that the neutrino be its own antiparticle. The fields which describe it are therefore real and lepton number is not a good (or well defined) quantum number. This is O.K. in modern gauge theory because there is no massless gauge boson associated with conservation of lepton number as the photon is associated with the conservation of charge. Neutrinos of this character are called **Majorana** neutrinos. Majorana mass terms are of the form

$$\begin{aligned} &\overline{(\nu^c)}_R M_L \nu_L + \overline{(\nu^c)}_L M_R \nu_R \\ &= \overline{\nu_L^c} M_L \nu_L + \overline{\nu^c}_R M_R \nu_R \\ &= \nu_L M_L \nu_L + \nu_R M_R \nu_R \end{aligned} \tag{2.2}$$

(This follows from eqs. (2.5) and (2.6) as we shall see.) Also, since ν_L is a two-component spinor field ν^α, $\alpha = 1,2$, "$\nu_L \nu_L$" denotes the antisymmetrized combination $\frac{1}{2}\varepsilon_{\alpha\beta}\nu_L^\alpha \nu_L^\beta$ where ε is the totally antisymmetric two dimensional tensor.)

The charge conjugate field in the four-component notation, ν^c, is defined by

$$\nu^c \equiv C\bar{\nu}^T \tag{2.3}$$

(4) In general, neutrinos can have both Dirac and Majorana masses. The general mass term therefore involves a mass matrix **M** and is of the form

$$\overline{(\nu_R^c,\ \nu_R)}\begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} \equiv \bar{\psi}_R\ \mathbf{M}\ \psi_L \tag{2.4}$$

It follows from equation (2.2) that M_L and M_R here are mass terms for left-handed and right-handed Majorana neutrinos, whereas M_D is a Dirac mass term. The equivalence of the off-diagonal terms follows from CPT Theorem. The matrix **M** in equation (2.4), being symmetric, can be diagonalized by an orthogonal transformation so that the two eigenvalues M_1 and M_2 are the masses of Majorana-type neutrinos and the neutrino states participating in the electroweak interactions are mixtures of these Majorana states.

In order to further clarify the interrelationships between the various neutrino fields, one can split up the four-component Dirac field into Weyl fields having chirality (handedness) as follows

$$\nu_{R,L}^{(W)} = \tfrac{1}{2}(1 \pm \gamma_5)\nu^{(D)} \tag{2.5}$$

Alternatively, we can define a Majorana (self-conjugate) two-component field from the Dirac field

$$\nu^{(M)} = 2^{1/2}(\nu^{(D)} + \nu^{(D)c}) \tag{2.6}$$

In the limit where the neutrinos are massless, neutrinos from Equation (2.4) with left handed chirality have left-handed helicity and the antineutrinos have right-handed helicity, i.e.,

$$(\nu_L)^c \equiv (\nu_L^{(W)})^c = (\nu^c)_R. \tag{2.7}$$

Since we have compared a Majorana neutrino to a π^0 boson, using the same analogy we can compare a Dirac neutrino to a K^0 boson which has a separate charge conjugate counterpart $\overline{K^0}$. The Dirac neutrino can be constructed from two independent Majorana neutrinos (Hereafter, we will define $\nu^{(D)} \equiv \Psi$ and $\nu^{(M)} \equiv \chi$.)

$$\begin{aligned} \Psi &= 2^{1/2}(\chi_1 + i\chi_2) \\ \Psi^c &= 2^{1/2}(\chi_1 - i\chi_2) \end{aligned} \tag{2.8}$$

so that (properly antisymmetrized)

$$\Psi^c\Psi = 1/2(\chi_1\chi_1 + \chi_2\chi_2) \tag{2.9}$$

All Majorana fields χ are real and self-conjugate, so that it is only necessary in what follows to denote their chirality, e.g., χ_L, χ_R.

In the past, it has usually been assumed that the mass of the neutrino is identically zero. This assumption was bolstered by the fact that no right-handed neutrinos have been seen. The argument was that if neutrinos had mass, right-handed neutrinos could be produced from left-handed neutrinos by a Lorentz transformation. However, if neutrinos have mass and Majorana

character, then such a transformation would be equivalent to changing left-handed neutrinos into right-handed "antineutrinos", which we do know exist. Also, as we have mentioned right-handed neutrinos could exist and be presently unobserved because they do not participate in standard GWS electroweak interactions.

Given then the possibility that neutrinos have mass, there are now several motivations for considering this possibility very seriously. They are:

A. Some recent experimental indications favoring a non-zero mass for the neutrino.

B. Observational results from astrophysical data and cosmological considerations which could be explained under the hypothesis that neutrinos have mass.

C. Theoretical considerations within the general framework of grand unified gauge theory leading to the ideas that (1) there is no general gauge principle leading to conservation of lepton number, and (2) grand unified models do not generally conserve lepton number, so that models can be constructed in which neutrinos have Majorana masses. Those of particular interest here contain very heavy right-handed Majorana neutrinos.

Let us now consider some of these grand unified models to see more specifically how neutrinos with non-zero masses* arise.

In the standard GWS model, the neutrino is part of a left-handed fermion SU(2) doublet and the right-handed electron comprises a singlet, i.e.,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad (\bar{e})_R \tag{2.10}$$

*We will henceforth refer to them as "massious" neutrinos, i.e., having the property of mass. (The word "massive" in English means heavy or bulky, a term not well suited for neutrinos with $m \approx 1$ ev.)

These arrangements are , of course, duplicated for the other lepton families, μ, ν_μ, τ, ν_τ. At this stage in the unification there is no need for ν_R's or approximately sterile ν's. as there are no right-handed currents. The simplest grand unified model of strong, weak and electromagnetic interactions, viz. the SU(5) model, has the families of left-handed fermions placed in the SU(5) representations of the form

$$\bar{5}: \begin{pmatrix} \bar{d}^r \\ \bar{d}^g \\ \bar{d}^b \\ e^- \\ \bar{\nu}_e \end{pmatrix}_L \qquad \mathbf{10}: \qquad 2^{1/2} \begin{pmatrix} 0 & -\bar{u}^b & -\bar{u}^g & u^r & d^r \\ \bar{u}^b & 0 & \bar{u}^r & u^g & d^g \\ \bar{u}^g & -\bar{u}^r & 0 & u^b & d^b \\ -u^r & -u^g & -u^b & 0 & e^+ \\ -d^r & -d^g & -d^b & -e^+ & 0 \end{pmatrix}_L \tag{2.11}$$

so that there are 15 fundamental fermions (per family) including only one neutrino. Again, this representation admits massless neutrinos. (There is no room for massious neutrinos unless an additional SU(5) singlet is added.)

In the SO(10) model, however, the picture changes. Here, the fermions are grouped into a total of 16 states so that in addition to the 15 fermions of SU(5) there is an additional neutral fermion which is an SU(5) singlet. In order to be consistent with experimental data, this new fermion must be quite heavy.

At the very heavy mass scale corresponding to grand unification, the $SU(2)_L \times U(1)$ electroweak symmetry should hold quite well. Therefore, a Majorana mass term for the neutrino must be constructed from an $SU(2)_L \times U(1)$ invariant. Since the ν_L field is part of an SU(2) doublet,

$$\psi \equiv \binom{\nu}{\ell^-}_L \tag{2.12}$$

it cannot by itself (i.e. $\nu_L \nu_L$) be used to construct a gauge invariant Majorana mass. However, by introducing the GWS scalar (Higgs) doublet

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \tag{2.13}$$

and by replacing ν_L by the SU(2) gauge invariant form, one can construct a Majorana mass term of the SU(2) singlet form:[1]

$$[\phi^T \varepsilon \psi]^2 = (\phi^o \nu_L - \phi^+ e_L^-)(\phi^o \nu_L - \phi^+ e_L^-) = \langle\phi^o\rangle^2 \nu_L \nu_L + \ldots \tag{2.14}$$

The operator shown in equation (2.14) is of dimension five, so that it is non-renormalizable.[2] It is therefore undesirable to have it as it stands in the fundamental Lagrangian. However, an operator of the form

$$\frac{f\langle\phi^0\rangle^2}{M} \nu_L \nu_L \tag{2.15}$$

can appear in the effective Lagrangian from the exchange of heavy particles of mass M with lepton-number violating couplings. The effective coupling constant is then of the form f/M. The effective theory versus renormalizable theory can be compared with the case of the Glashow-Weinberg-Salam theory (See Figure 1). The interaction terms of the fundamental Lagrangian are renormalizable and the corresponding Majorana neutrino mass is

$$m_\nu = \frac{f\langle\phi^0\rangle^2}{M} \tag{2.16}$$

In the GWS model, $\langle\phi^0\rangle \sim 300$ GeV. The mass M can have various values depending on the grand unified theory taken. If we take a typical scale $M \sim 10^{14}$ GeV,

then the corresponding mass for the neutrino is of the order of 10^{-5} eV.

One scheme for generating a neutrino mass in the SO(10) model was suggested by Gell-Mann, Ramond and Slansky[3]. For this model, the breakdown of SO(10) and the corresponding symmetry breakdown scenario, the relevant Higgs multiplet, and neutrino mass matrix are indicated in Figure 2.

The final neutrino mass matrix contains a heavy Majorana mass $M \sim 10^{15}$ GeV and a light Dirac mass $m \sim m_q$, the up quark mass induced by the Higgs field ϕ.

The mass matrix can be diagonalized to yield the Majorana mass terms

$$\lambda_1 \chi_1 \chi_1 + \lambda_2 \chi_2 \chi_2 \tag{2.17}$$

where

$$\lambda_{2,1} = \frac{M \pm (M^2 - 4m^2)^{1/2}}{2} \tag{2.18}$$

so that

$$\begin{aligned} \lambda_1 &\simeq m^2/M \\ \lambda_2 &\simeq M \qquad , \quad m^2 \ll M^2 \end{aligned} \tag{2.19}$$

are respectively the masses of a light left-handed and a heavy right-handed Majorana neutrino. Note that in the GWS theory, the quark masses are induced by Yukawa couplings of the form

$$L_Y = h\bar{\Psi}_q \phi \Psi_q \tag{2.20}$$

so that $m_q \sim h \langle\phi\rangle$ and

$$\lambda_1 = \frac{h^2 \langle\phi\rangle^2}{M} \tag{2.21}$$

is of the form of eq.(2.15) with $f \sim h^2$.

In the scheme of Gell-Mann Ramond and Slansky[3] (GRS), the superheavy right-handed Majorana neutrino obtains its mass from the vacuum expectation value of the **126**-plet of Higgs fields which breaks the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry and is an SU(5) and an SU(2) singlet. There are many other models within the context of grand unified theories which have been explored. One motivation for this has been the size of the neutrino mass obtained. A GRS mass is typically in the range 10^{-4} to 10^{-3} eV. While this range may be significant for the solar neutrino problem, as we shall see, such masses are not large enough to play a significant cosmological role, to account for the "missing mass" in galaxies or to account for some experimental results.

Because of the reasons mentioned above, masses in the 1-100 eV range are more "desirable". One possibility suggested by Witten[4] does not involve explicit Higgs fields at the 10^{15} GeV level. In this scheme M is not of the order of 10^{15} GeV because the right handed-Majorana neutrino, not being coupled to a 10^{15} GeV Higgs multiplet, remains massless at the level of SO(10) breakdown. However, the mixing of SO(10)-vector representation Higgs **10**-plets and **16**-plets in the spinor representation can induce an mass at the two-loop level. The effective mass is lower than $M = 10^{15}$ GeV, the level of SO(10) breaking, and the right-handed Majorana neutrino N is given a mass

$$M_N = (m_q/M_W)\varepsilon(\alpha/\pi)^2 M \qquad (2.22)$$

where M_W is the W-boson mass, ε is the ϕ_{10}-$\phi_{\overline{16}}$ mixing angle $\sim 10^{-1}$ and therefore the mass of the N_R would be in the 10^5 - 10^6 GeV range. The corresponding mass of the left-handed ν_e can then be obtained from equation (2.21) using M_N instead of M, since the form of the effective neutrino mass matrix is the same

as in the Gell-Mann, Ramond and Slansky model. Within different lepton families, the relevant masses are those of the up-quarks ($I_W = +1/2$). In the GRS case, $m_\nu \propto m_q^2$, from (eq.(2.21), whereas in the Witten Model $m_\nu \propto m_q$, as can be seen from combining eqs. (2.21) and (2.22). Other models[5] break $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry at a lower energy level, $M_R \simeq M_{B-L}$, which replaces M in equation (2.21). Within the context of SU(5) models, neutrinos cannot be given Dirac masses because there are no right-handed neutrino fields in the basic fermion representations. However, Majorana masses can be included in a non-minimal SU(5) which contains an SU(5) Higgs **15**-plet which transforms as an $SU(2)_L$ triplet (I_W=1).[6] The Majorana mass is induced by the vacuum expectation value of an $SU_L(2)$ triplet, which can also be introduced in the SO(10) model as part of a left-right symmetric theory[5].

The GRS mechanism, although it may not be the whole answer, provides a way of explaining, within the context of grand unified theories, why the neutrino mass is much less than other typical Dirac-type fermion masses obtained by Yukawa terms in the GWS Lagrangian involving the ϕ_L fields, i.e.

$$L = h\bar{\psi}_R \phi_L \psi_L \qquad (2.23)$$

$$m_f^D \sim h \langle\phi_L\rangle$$

At the same time, the GRS mechanism, through the heaviness of the right handed Majorana neutrino, $\nu_R \equiv N_L^c$, explains why right-handed neutrinos do not play a significant role in "low energy" physics.

We may generalize our discussion somewhat by noting that the mass matrix of equation (2.4) has Dirac type off diagonal terms

$$m = m^D \sim h \langle\phi_L\rangle \qquad (2.24)$$

and, with $\overline{\nu^c}_R \equiv \nu_L \equiv \chi_L$ and $\overline{\nu}_R \equiv \nu_L^c \equiv \chi_R$, Majorana type diagonal terms of the form $M_R \chi_R \chi_R$ and $M_L \chi_L \chi_L$. By diagonalization of the symmetric matrix we obtain the mass eigenstates of the two Majorana-type neutrinos whose wave functions (with m<<M) can be approximated by

$$\nu_L = \chi_L + \frac{m}{M} \chi_R \tag{2.25}$$

$$N_R = \chi_R - \frac{m}{M} \chi_L$$

with χ_L orginally assumed massless. Thus, the resulting left-handed Majorana neutrino gets a very light mass because of the small mixture of heavy right-handed χ_R neutrinos.

We may generalize the formalism of equations (2.4) (2.19) (2.21) and (2.24) to include the mixing of neutrino flavors by writing the mass terms as matrices which mix generations

$$m = m^D \rightarrow \mathbf{M}^D$$

$$M_R \rightarrow \mathbf{M}_R \tag{2.26}$$

$$\frac{m^2}{M_R} = \frac{h^2 \langle \phi_L \rangle^2}{M_R} \rightarrow \mathbf{M}_\nu^D \, \mathbf{M}_R^{-1} \, (\mathbf{M}_\nu^D)^T$$

so that

$$\mathbf{M}_{\nu_L} = \mathbf{M}_\nu^{(D)} \, \mathbf{M}_R^{-1} \, \mathbf{M}_\nu^{(D)T} \tag{2.27}$$

(See figure 3). Since the Majorana matrix $\mathbf{M}_R$ is real and symmetric and can therefore be diagonalized by an orthogonal tranformation **O**, we can rewrite equation (24) as

$$\mathbf{M}_{\nu L} = \mathbf{M}^{(D)} \, \mathbf{O} \, \mathbf{M}_R^{-1} \, \mathbf{O}^T \mathbf{M}^{(D)T} = \mathbf{M}^{(D)} \, \tilde{\mathbf{M}}_R^{-1} \, \mathbf{M}^{(D)T} \tag{2.28}$$

where $\tilde{\mathbf{M}}_R^{-1}$ is diagonal.

In many unified models where ϕ_L is a color singlet, the matrix

$$M_\nu^{(D)} = M_u \tag{2.29}$$

where M_u is the mass matrix of up-type quarks.

Thus, at least in some simple versions of grand unified models, the generation mixing in the left-handed neutral lepton sector can be the same as that in the up quark sector. However, this is not a necessary or proven condition. Such generation mixing brings us to the question of neutrino oscillations which we pursue in the next lecture. Some "predicted" neutrino masses are shown in Table I.

Table I. Neutrino Masses (eV)

Flavor	Observed	GRS Model[e]	Witten Model[f]	Left-Right Models[g]
ν_e	< 60[a] or 14-46[b]	$\sim 10^{-5}$	$\sim 5\times10^{-3}$	$\lesssim 1.5$
ν_μ	$< 5.2\times10^{5}$[c]		~ 1	$\lesssim 5.6\times10^{4}$
ν_τ	$< 2.5\times10^{8}$[d]		~ 30	$\lesssim 1.8\times10^{7}$

(a) Ref.20
(b) Ref.21
(c) Lu, D. C., et al., Phys. Rev. Lett 45, 1066 (1980).
(d) DELCO Collaboration
(e) Ref.3
(f) Ref.4
(g) Ref.5

III. NEUTRINO OSCILLATIONS

If neutrinos have mass and the masses of different eigenstates are different, oscillations can result either from (A) generational mixing ("first

class") or (B) doublet-singlet mixing ("second class"). Consider, for example, the case where two weak interaction eigenstates, e.g. ν_μ and ν_e, are mixtures of mass eigenstates ν_1 and ν_2 with masses m_1 and m_2. Then this mixing is given by a simple 2-dimensional or orthogonal matrix charactrized by a mixing angle θ

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \tag{3.1}$$

In general, we can have mixing of a larger number of generations. If we define the neutrino wave function $\psi_\nu(t)$ by an N-dimensional column vector in the case of N-generation mixing, and if we label the weak eigenstates by ν_α ($\alpha = e,\mu,\tau,\ldots$) and the mass eigenstates by ν_i ($i = 1,2,\ldots$), the general mixing matrix is an NxN unitary matrix. An NxN complex matrix has $2N^2$ independent parameters. The unitarity condition $U^\dagger = U^{-1}$ eliminates N^2 parameters. Of these $\frac{1}{2}N(N-1)$ can be placed in an orthogonal (rotation) Cabibbo matrix as independent mixing angles. In the the case involving Dirac neutrinos (as with quark mixing) 2N-1 relative phases can be absorbed into a redefinition of the fermion fields without any observable effect, leaving $\frac{1}{2}(N-1)(N-2)$ abritrary phases which can cause CP violation. In the case involving Majorana neutrinos there are N "reality" constraints in place of the (2N-1) relative phases of the Dirac case. (The "real" Majorana fields do not admit any relative phase tranformations). The result is that in the Majorana neutrino case, we are left with more arbitary CP violating phases[7], viz., $\frac{1}{2}N(N-1)$.

Thus

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle \tag{3.2}$$
$$|\nu_i\rangle = U^{-1}_{i\alpha}|\nu_\alpha\rangle$$

For the $|\nu_i>$, the mass matrix can be diagonalized to a form $\mathbf{M}^{(d)} = m_i \delta_{ij}$ so that we obtain N independent Schrödinger equations

$$\mathbf{M}^{(d)} = \mathbf{U}\mathbf{M}\mathbf{U}^{-1} \tag{3.3}$$
$$\Psi_\nu^{(d)} = -i\ (\mathbf{M}^{(d)2} + p^2\ \mathbf{1})^{1/2}\Psi_\nu$$

or

$$\dot{\psi}_i = -i\ (p^2 + m_i^2)^{1/2}\psi_i \tag{3.4}$$

Consider again the two state case given by eq.(30). For a beam of neutrinos of momentum p produced in a weak eigenstate (say ν_e) at time t=0, defining $E_{1,2} = (p^2 + m_{1,2}^2)^{1/2}$, it follows that the probability to stay in the state ν_e at time t is

$$P(\nu_e \to \nu_e) = 1 - P\ (\nu_e \to \nu_\mu)$$
$$P(\nu_e \to \nu_\mu) = |<\nu_\mu(t)\ |\ \nu_e(0)>|^2$$
$$= |\cos\theta\ <\nu_\mu(t)\ |\ \nu_1> + \sin\theta\ <\nu_\mu(t)\ |\ \nu_2>|^2$$
$$= |\sin\theta\cos\theta e^{-iE_1 t} + \sin\theta\cos\theta e^{-iE_2 t}|^2 \tag{3.5}$$
$$= \sin^2\theta\cos^2\theta\,|e^{-iE_1 t} - e^{-iE_2 t}|^2$$
$$= \tfrac{1}{2}\sin^2 2\theta\ [1 - \cos\ (E_1 - E_2)t]$$

For

$p \gg m_1, m_2$ then $E_1 \approx E_2 \approx p$ and

$$E_1 - E_2 \approx p[(1 + \frac{m_1^2}{2p^2}) - (1 + \frac{m_2^2}{2p^2})] \approx \frac{m_1^2 - m_2^2}{2E} \tag{3.6}$$

Thus $$1 - \cos(E_1 - E_2\, t) \approx (1 - \cos \frac{m_1^2 - m_2^2}{2E} t)$$

$$= 2\sin^2(\frac{m_1^2 - m_2^2}{4E} t) \tag{3.7}$$

and $$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin(\frac{m_1^2 - m_2^2}{4E} t)$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2(\frac{m_1^2 - m_2^2}{4E} t) \tag{3.8}$$

Defining

$$\Delta^2 \equiv m_1^2 - m_2^2 \tag{3.9}$$

and with Δ^2 in eV^2, E in MeV and ct in meters, eq. (3.8) becomes

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2(1.27\, \Delta^2 \frac{L}{E}) \tag{3.10}$$

From equation (3.10), it follows that three conditions must exist in order for neutrino oscillations to occur: (A) there must exist at least one non-zero neutrino mass and (B) this mass must be different from the mass of at least one other mass eigenstate so that there exists a $\Delta^2 \neq 0$, and (C) there must be mixing between neutrino flavors so that at least one mixing angle $\theta \neq 0$.

Given these three conditions, there are three distinct ranges for the oscillation phenomena. In the above units eV^2 m MeV^{-1} they are:

1. $\Delta^2(L/E) \ll 1$. In this case an experiment at distance L with neutrino energy E will not detect oscillations ($\sin^2(1.27\Delta^2 L/E) \ll 1$).

2. $\Delta^2(L/E) \approx 1$. In this case, there will be significant changes in the detection probability with L provided $\sin^2 2\theta$ is moderately large.

3. $\Delta^2(L/E) \gg 1$. In this case, the oscillations will be on a scale small compared to L and the oscillations will average out to some constant probability < 1.

Oscillation experiments now exist in several ranges of L/E. They may be classified as follows:

A. Solar Neutrino Detection of ν_e

$$L = 1.5 \times 10^{11} m$$
$$E \approx 1 - 10 \text{ MeV}$$
$$L/E \approx 10^{10} - 10^{11} \text{ m/MeV}$$

B. Deep Mine Cosmic Ray ν_μ Detection

$$L \approx 10^6 - 10^7 \text{ m}$$
$$E \approx 10^4 - 10^6 \text{ MeV}$$
$$L/E \approx 1 - 10^3 \text{ m/MeV}$$

C. Reactor Experiments ($\bar{\nu}_e$)

$$L \sim 5 - 10 \text{ m}$$
$$E \sim 5 \text{ MeV}$$

$$L/E \simeq 1\text{-}2 \text{ m/MeV}$$

D. Accelerator Experiments

$$L \simeq 10^3 \text{ m}$$
$$E \simeq 2.5 \times 10^4 \text{ MeV}$$
$$L/E \simeq 4 \times 10^{-2} \text{ m/MeV}$$

The mininum Δ^2 for which an experiment is sensitive is $\Delta^2_{MIN}(eV^2) \sim E(Mev)/L(m)$ in the limit of moderately large mixing, so that

$$(\Delta^2_{MIN})_{SOLAR} < (\Delta^2_{MIN})_{COSMIC\ RAY} < (\Delta^2_{MIN})_{REACTOR} < (\Delta^2_{MIN})_{ACCEL.} \quad (3.11)$$

A. SOLAR NEUTRINOS

We first consider the data on solar neutrinos. The solar neutrino experiment[8] uses a large tank of CCl_4 in an underground mine to detect ν_e's via the reaction

$$^{37}Cl + \nu_e \rightarrow {}^{37}A + e^-, \; E_\nu \gtrsim 0.814 \text{ MeV}. \quad (3.12)$$

The ν capture rate is given in solar neutrino units (SNU's) defined such that 1 SNU $\equiv 10^{-36}$ captures/atom/s. Because of the relatively high threshold reaction for capture by ^{37}Cl, the CCl_4 experiment is most sensitive to ν_e's produced in the sun via the reaction

$$^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu_e \qquad (3.13)$$

(see Table II). The standard solar model predicts[9] a rate of 8 ± 3.3 (3σ) SNUs with the uncertainties in the calculation being due to the nuclear physics parameters (2.9 SNU), solar composition (1.3 SNU), solar opacity (0.5 SNU) and the neutrino cross section (0.7 SNU). However, the present data gives a capture rate of 1.9 ± 0.3 (1σ) SNU.

Table II. Solar Neutrino Rates[9]

Source Reaction	Predicted Flux ($10^{10}cm^{-2}s^{-1}$)	E_ν(MeV)	SNU(^{37}Cl)	SNU(^{71}Ga)
$p+p\rightarrow d+e^{+}+\nu_e$	6.1	0 - 0.42	0	65.1
$p+e^{-}\rightarrow p+d+\nu_e$	0.015	1.4	0.23	2.4
$^{7}Be+e^{-}\rightarrow {}^{7}Li+\nu_e$	0.34	0.86(90%)	1.03	27.6
		0.34(10%)		
$^{8}B\rightarrow {}^{8}Be^{*}+e^{+}+\nu_e$	6.0×10^{-4}	0-14	6.48	1.8
$^{13}N\rightarrow {}^{13}C+e^{+}+\nu_e$	0.045	0-1.2	0.07	2.4
$^{15}O\rightarrow {}^{15}N+e^{+}+\nu_e$	0.035	0-1.7	0.23	3.2
		Total:	8.04	102.4

Thus, the ratio of observed neutrinos to expected solar neutrinos is R = 0.31 ± 0.13(3σ). It was suggested by Gribov and Pontecorvo[10] that neutrino oscillations could account for this ratio. Such a scenario would require the mixing of at least three neutrino flavors with large mixing angles and $\Delta^2 \gtrsim 10^{-11}$ eV^2.

Because the ^{37}Cl experiment measures neutrinos from a relatively

insignificant solar reaction, it has been suggested that other materials such as ^{71}Ga and ^{115}In be used in order to detect the lower energy neutrinos from the basic reaction

$$p + p \rightarrow d + e^{+} + \nu_e \tag{3.14}$$

The threshhold energy for capture reactions on ^{71}Ga, i.e.

$$^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^{-}, \tag{3.15}$$

is only 0.236 MeV as compared with 0.814 MeV for ^{37}Cl. The total capture rate expected for ^{71}Ga is 102.4 SNU (see Table II) of which 65.1 SNU is expected from reaction (3.14). Thus a ^{71}Ga experiment will test solar theory and the neutrino oscillation hypothesis at a more sensitive and basic level.

B. REACTOR EXPERIMENTS

Reactor experiments have provided the next possible indication of neutrino oscillations, Reines, et al.[11] used a detector with D_2O to look for the charged current and neutral current reaction on deuterium induced by reactor generated $\bar{\nu}_e$'s from ^{235}U, ^{238}U and ^{239}Pu. The $\bar{\nu}_e$'s have a continuum energy spectrum with typical energies of a few MeV. The relevant reactions were

$$\bar{\nu}_e + d \rightarrow n + n + e^{+} \qquad \text{(CC)} \tag{3.16a}$$

$$\bar{\nu}_e + d \rightarrow n + p + \bar{\nu}_e \qquad \text{(NC)} \tag{3.16b}$$

$$\bar{\nu}_x + d \rightarrow n + p + \bar{\nu}_x \qquad \text{(NC)} \tag{3.16c}$$

Reaction (3.16a) is only induced by $\bar{\nu}_e$'s. However, reactions b and c are equivalent and can be induced by any neutrino flavors. Thus, if $P(\bar{\nu}_e \to \bar{\nu}_e) < 1$ owing to oscillations, $[R(CC)/R(NC)]_{obs}/[(R(CC)/R(NC)]_{theor} < 1$. Reines et al.[11] reported a depletion of $\bar{\nu}_e$'s to (0.40 ± 0.22) of the expected value. They interpreted this result as indicating $\Delta^2 \sim 1 eV^2$ and $\sin^2 2\theta \sim 1/2$ There has been controversy regarding this result, partly owing to an uncertainity in the $\bar{\nu}_e$ spectrum[12]. A more recent reactor experiment performed by a group at Grenoble[13] found $P(\bar{\nu}_e \to \bar{\nu}_e) \gtrsim 0.7$ by looking at the reaction $\bar{\nu}p \to ne^+$ at a distance of 8.7m from the reactor. These results are consistent with no oscillations $P(\bar{\nu}_e \to \bar{\nu}_e) = 1$ for $\Delta^2 \gtrsim 0.5$ eV^2 and large mixing angles. However, Silverman and Soni[14] have obtained solutions implying mixing (e.g. $\Delta^2 \sim 0.9$ eV^2, $\sin^2 2\theta \sim 0.4$) which they argue are a best fit to both reactor results. It should be noted that the solution sets given by these two experiments only overlap on the edges of the 90% confidence limits so that the probability of both results agreeing is $\lesssim 10\%$. Clearly, further work needs to be done to resolve this situation.

C. DEEP MINE EXPERIMENTS

There are two reported results from deep mine experiments looking at cosmic-ray ν_μ's. Here again, the results are mixed. The Kolar Gold field group[15] results give indiations of oscillations ($P(\nu_\mu \to \nu_\mu) = 0.62 \pm 0.17$) for $\Delta^2 \gtrsim 10^{-2}$ eV^2. However, the Baksan group[15] finds a result $P(\nu_\mu \to \nu_\mu) \gtrsim 0.8$.

D ACCELERATOR EXPERIMENTS

Finally, there have been a large number of accelerator results. One interesting type of experiment is the "beam dump" experiment which detects neutrinos from the decay of short lived ($<10^{-12}$s) charmed mesons (as opposed to π-decay and K-decay neutrinos). These are referred to as "prompt" neutrinos. At the source, the ratio $\nu_e/\nu_\mu = 1$ from the decay of charmed particles. Thus the ratio e/μ produced by prompt neutrinos should be 1 in the case of no oscillation. The measured ratios were reported as shown in Table III.

TABLE III. ACCELERATOR RESULTS[17]

RATIO	ERROR		GROUP
0.49	±0.21		CHARM
0.46	(+0.55,-0.22)		BEBC
0.77	±0.18 (STAT.)	±0.24 (SYST.)	CDHS

Here again the results are mixed. Many other results have been obtained by various groups. They have been reviewed by Baltay[17] and others. The remaining results are null results, placing limits on regions of the (Δ^2, sin 2θ) plane allowed to the oscillation parameters. These limits are shown in Figure 4 from Barger[18].

IV. OTHER NEUTRINO MASS EXPERIMENTS

There are two other types of experiments which have given indications of neutrino masses. They are the tritum β-decay endpoint experiment and searches

for neutrinoless double β-decay. These experiments both pertain to the mass of the neutrino mass eigenstate connected with ν_e. In the case of the neutrinoless double β-decay, violation of lepton number and therefore the Majorana character of the neutrino also come into the picture.

A. THE ^{3}H DECAY EXPERIMENTS

There have been several experiments to study the endpoint of the β-decay spectrum of tritium from the decay

$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e \tag{4.1}$$

Until recently, this type of experiment has only placed limits on the mass of $\bar{\nu}_e$. Bergkvist[19] obtained $m_\nu < 55$ eV (90%CL) and Simpson, et al.[20] found $m_\nu < 65$ eV (95%CL). However, one of the most stimulating results in the field has been the report by Lyubimov, et al.[21] that they had measured a neutrino mass

$$14 \text{ eV} \leq m_{\nu_e} \leq 46 \text{ eV} \tag{4.2}$$

The electron β-decay spectrum is of the form

$$\frac{dN}{dp} \equiv N_\beta(E;Z) = CF_c(E;Z)\, p^2(Q-E)[(Q-E)^2 - m_\nu^2]^{1/2} \tag{4.3}$$

where C is a constant, $F_c(E;Z)$ is the Coulomb factor

$$F_c(E;Z) \simeq \frac{2\pi Z}{(v/c)} \left[1-\exp\left(\frac{2\pi Z\alpha}{(v/c)}\right)\right]^{-1} \simeq F_c(Z) \tag{4.4}$$

Q is the total energy released in the decay. For ^3H decay, Q = 18.6 keV.

It follows from equation (4.3) that a convenient way to plot the β-spectrum is in the form of the "Kurie plot" K(E) such that

$$K(E) \equiv \left[\frac{N_\beta(E)}{F_c p^2}\right]^{1/2} \propto \left\{(Q-E)\left[(Q-E)^2 - m_\nu^2\right]^{1/2}\right\}^{1/2} \qquad (4.5)$$

Thus, for $m_\nu = 0$, the Kurie plot is a straight line

$$K(E) \propto (Q-E),\ m_\nu = 0 \qquad (4.6)$$

and K(Q) = 0. However for $m_\nu \neq 0$, K(E) takes on a modified form near E = Q as shown in Figure 5.

The shape of the endpoint spectrum is affected by other factors in addition to m_ν. For one thing, the finite energy resolution of the detector spreads out the observed electron energy spectrum and produces an artifical "tail". For another, there is the possibility that the ^3H decays into an excited state of energy ϕ of ^3He rather than the ground state. This will cause an effect similar to that of a finite neutrino mass, since the endpoint energy will be lowered from Q to $(Q-\phi)$. In atomic hydrogen, transitions to the ground state will occur 70% of the time. Another 25% probability is that the transition will be to an n=2 state with ϕ=41 eV <u>if the ^3H is in atomic form.</u> Note that ϕ is of the order of m_ν. Nobody has solved the molecular transition problem for a complex molecule such as valine, $NH_3CH_3CHCOOH$, so that this is a principal source of uncertainty for the experiment of Lyubimov, et al.[21] which used tritiated valine.

B. NEUTRINOLESS DOUBLE β-DECAY

The study of double β-decay has long been associated with a test for the Majorana character of the neutrino. The appropriate nuclides for study are those for which the single β-decay process is energetically suppressed. The double β-decay transitions looked for are the second order weak decay

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\,\bar{\nu}_e \qquad (4.7)$$

and the neutrinoless counterpart

$$(A,Z) \rightarrow (A,Z+2) + 2e^- \qquad (4.8)$$

which violates lepton number by two units.[22]

Reaction (4.8) can be looked at as the two stage sequence (in quark language)

$$d_1 \rightarrow u_1 + e^- + \nu^{(M)} \qquad (4.9a)$$

followed by

$$\nu^{(M)} + d_2 \rightarrow u_2 + e^- \qquad (4.9b)$$

involving two down quarks and a Majorana neutrino $\nu^{(M)}$ (see Figure 6).

The nuclide for which double β-decay is energetically favorable are even-even nuclides. The relevant transitions are $0^+ \rightarrow 0^+$ to the ground state of the daughter nuclide with also some possibility for $0^+ \rightarrow 2^+$ transitions to the excited state (see Fig 7). The relevant energy spectra, also shown in Fig. 7, indicate that the electrons carry off the total energy Q in the case of ν-less decay whereas they share the energy with the $\bar{\nu}_e$'s in the standard double β-decay.

It can be seen from Fig. 6 that in order for the neutrino which is

emitted in the first stage (4.9a) to be absorbed by the d quark in the second stage (4.9b) of the neutrinoless decay, d spin flip must occur. This can either be accomplished by the neutrino mass and/or by the existence of right-handed weak currents with a strength $j_R \equiv \eta\, j_L$. Transitions of the form $0^+ \rightarrow 2^+$ are produced solely by the right-handed current mechanism[23]. Thus, the study of ν-less double β decay provides not only a test for lepton number violation and neutrino masses, but also one for right-handed weak currents. The theory for this process has been given in great detail recently[22,23] and will not be detailed here.

There are two categories of double β decay measurements which have been carried out, viz., geochemical and laboratory. The geochemical measurements consist of the analysis of us ores known age ($\sim 10^9$ yr) which are rich in the parent nuclide where one looks for traces of the daughter nuclide. The daughter nuclides most amenable to analysis of this type are the noble gases. Thus, good measurements are available for the lifetimes of the decays $^{130}Te \rightarrow {}^{130}Xe$, $^{128}Te \rightarrow {}^{128}Xe$ and $^{82}Se \rightarrow {}^{82}Kr$. Of course, in this type of experiment only the lifetimes are measured, not the electron energy distribution, so that one cannot tell directly whether or not neutrinoless decay has occurred. However, different lifetimes are calculated for the $2\bar{\nu}_e$ and $0\bar{\nu}_e$ decays owing to the fact that the lifetime depends on a phase space factor involving a function $f(\hat{m},\eta)$ where $\hat{m} \equiv m_\nu/m_e$.

Several groups have measured $T_{1/2}(^{130}Te \rightarrow {}^{130}Xe)$ and obtained values in the range $\sim(2 \pm 1) \times 10^{21}$ yr both from geochemical data and laboratory data. For the decay of ^{82}Se, there appears to be an unfortunate conflict between the geochemically obtained lifetime ($\sim 2 \times 10^{20}$ yr) and that found experimentally ($\sim 10^{19}$ yr).

One method for determining the neutrino mass $m_\nu = \hat{m}\, m_e$ has been to study

the ratio of lifetimes of ^{128}Te and ^{130}Te. Letting ρ be the ratio of $2\bar{\nu}_e$ to $0\bar{\nu}_e$ decay matrix elements, Rosen[22] obtained the following condition on $\hat{m}$ and η

$$\hat{m}^2 + 0.093\ \hat{m}\eta + 0.15\ \eta^2 = 1.5 \times 10^{-9}\ \rho^2 \qquad (4.10)$$

Thus,

$$\hat{m} = \frac{m_\nu}{m_e} = 3.9 \times 10^{-5}\rho, \qquad \eta = 0$$

$$0 \leq \hat{m} \leq 4 \times 10^{-5}\rho, \qquad \eta \leq 10^{-4}\rho \qquad (4.11)$$

with estimate for ρ of 0.5 and 1.2. Such estimates give m_ν in the range of 10 to 40 eV with $\eta \lesssim 10^{-4}$. The limits on η could be greatly strengthed by non-observations of $0^+ \rightarrow 2^+$ transitions.

Various other calculations of m_ν from double β-decay have been reviewed by Rosen[22]. Here again, as in the case of neutrino oscillations, one finds conflicting results.

$m_\nu = 34$ eV (Ref.23)

$m_\nu \lesssim 15$ eV (Ref.24)

The experimental situation needs to be clarified.

C. INTERNAL BREMMSTRAHLUNG IN ELECTRON CAPTURE

De Rujula[25] has suggested a new method for obtaining m_{ν_e}. He has pointed out that radiative orbital electron capture reactions involving neutrino emission from neutron deficient nuclides could be used to determine m_ν. Here, the spectrum of the emitted photons would take the place of the electron spectrum in the ^3H decay experiment. No experiments of this sort have yet been attempted.

V. CONCLUSIONS REGARDING THE EXPERIMENTAL SITUATION

As we have seen, there are conflicting data within the various categories of neutrino mass experiments. Phillips[26] and Barger[18] have pointed out additional problems in reconciling the data among these categories.

Within the spirit of these discussions, one example of the type of puzzling relationships obtained is outlined below:

Suppose (A) $m_{\nu_e} > 14$ eV (Lyubimov,et al[21])

(B) ν-mass eigenstates are highly non-degenerate (as in grand unified models - see Section II)

Then (C) $\Delta^2 \gg 1\text{eV}^2$

But (D) from the Grenoble reactor experiment[13]

Δ^2 (probably) $< 1\text{eV}^2$

Unless (E) θ is small

But (F) if θ is small, oscillations don't solve the solar ν problem

However (G) there are still loopholes in these arguments

So (H) ?!?

VI NEUTRINOS AND COSMOLOGY

In this last lecture, I will discuss the possible role of neutrinos in cosmology. This is another quite active field of investigation at present, having many facets. I will stress here primarily the gravitational effects of a "neutrino dominated universe" within the context of the hot big-bang cosmology.

The hot big-bang model is now quite familar and the basic relations

describing it may be found in many places[27]. We will consider here that it rests on two main pieces of evidence (1) the Hubble relation showing that the distant galaxies are receeding from us at velocities proportional to their distance

$$v_{rec}(km/s) = H_0(km/s/Mpc)r(Mpc) \qquad (6.1)$$

where $50 \leq H_0 \leq 100$ in these units and 1 megaparsec (Mpc) $\approx 3 \times 10^{24}$ cm; and (2) the universe is filled with thermal blackbody radiation at a temperature T=2.8 0.1 K. From these two relations come the conclusions that (A) the universe is expanding (as implied also by the Einstein gravitational equations sans comological term) and (B) it was in a much hotter as well as denser state in the past. Most workers would also add (3) the data on the ^{4}He and ^{2}H abundances (implying primordial nucleosynthesis) as additional evidence of the hot big-bang model. This argument most likely has an "element" of truth. However, I do not consider this evidence to be on the same footing with (1) and (2) because it involves additional assumptions and may be inherently self-contradictory in its simplest form.[28] (Many things have been "deduced" from the ^{4}He and ^{2}H data, e.g., the number of neutrino flavors, and the student should approach these arguments with academic scepticism. I will, therefore, not repeat them here.)

The Einstein equations are second order differential equations. With a homogeneous isotropic metric (called the Robertson-Walker metric) they can be solved to give a scale size, R, as a function of cosmic time, t in terms of two parameters, the "deceleration parameter"

$$q_0 = -\frac{\ddot{R}R}{(\dot{R})^2} \qquad (6.2)$$

and the expansion rate

$$H_0 = \frac{\dot{R}}{R} \tag{6.3}$$

where the subscript 0 refers to the present time (redshift z=0). (Throughout this discussion, we will assume that the cosmological term, or equivalently Einstein's cosmological constant Λ=0.) The gravitational deceleration parameter can be replaced by a mass parameter

$$\Omega = 2q_0, \ \Lambda = 0 \tag{6.4}$$

and where

$$\Omega = \frac{\rho}{\rho_c} \tag{6.5}$$

the fraction of the critical mass density needed to close the universe gravitationally. The critical density can be determined in the Newtonian limit by equating the potential and kinetic energies of a test particle

$$G\left(\frac{4\pi}{3}R^3\right)\frac{\rho_c}{R} = \tfrac{1}{2}(H_0R)^2 \tag{6.6a}$$

or

$$\rho_c = \frac{3H_0^{\,2}}{8\pi G} \tag{6.6b}$$

so that

$$\Omega = \frac{8\pi G\rho}{3H_0^2} \tag{6.7}$$

Observationally, from studies of q_0, it is found that $\Omega < 2$. From studies of total matter density in galaxy clusters it is found that $\Omega > 0.02$.

The neutrino contribution to Ω, which we will call Ω_ν, can be calculated

in the context of the hot big-bang model. We assume that at some time $t < t_\nu$, corresponding to a temperature $T > T_\nu$, photons, electrons, positrons and neutrinos were in thermal equilibrium. The temperature T_ν when this situation last occurred was when the ν-e interaction rate was equal to the expansion rate of the universe, $T_\nu \sim 1$ MeV. Shortly thereafter at $T \sim m_e \simeq 1/2$ MeV, the electrons and positrons went out of thermal equilibrium and annihilated

$$e^+ e^- \rightarrow 2\gamma \tag{6.8}$$

with all of the energy release going into the photons, the neutrinos having decoupled. At $T > T_\nu$, the ratio of neutrinos to photons was

$$\frac{n_\nu}{n_\gamma} = \frac{\omega_\nu}{\omega_\gamma} \frac{\int_0^\infty F dE}{\int_0^\infty B dE} = 3/4 \ f \tag{6.9}$$

where ω_γ=2 is the number of photon degrees of freedom, ω_ν is the number of neutrino degrees of freedom (taken to be 2 per flavor x f = the number of flavors) which were in thermal equilibrium with the photons at $T_\nu \simeq 1$ MeV (only ν_L and $\overline{\nu}_R$ meet this criterion) and the factor of 3/4 comes from the ratio of the integrals over the Fermi-Dirac function F(E;T) and the Planck function B(E;T) and equation (6.9). For $T < m_e$, additional photons are added from the e^+e^- annihilation. The new factor multiplying the photon number is determined by the additional entropy per unit volume added to the photon component and is 11/3. Thus, for $T_\gamma < m_e$

$$\frac{n_\nu}{n_\gamma} = \frac{3/4 \ f}{11/4} = \frac{3}{11} f \tag{6.10}$$

At the present time

$$n_\gamma = 400 \left(\frac{T}{2.7}\right)^3 \text{cm}^{-3} \tag{6.11}$$

a number which is obtained from the fact that the effect of redshift $z \equiv \Delta\lambda/\lambda$ on the Planck function is to shift the temperature

$$B\,[(1+z)T] \rightarrow B(T) \tag{6.12}$$

Thus, from equations (6.10) and (6.11)

$$n_\nu \approx 110\ f\ \left(\frac{T}{2.7K}\right)^3\ \mathrm{cm}^{-3} \tag{6.13}$$

and the total mass density divided by the closure density is (from equations (6.7) and (6.13)

$$\Omega_\nu \approx 0.01\ h_0^{-2} \sum_f m_f\ (\mathrm{eV}) \tag{6.14}$$

where $h_0 \equiv H_0/100$ km/s/Mpc so that $1/2 \lesssim h_0 \lesssim 1$.

We may compare Ω_ν with the various values of Ω deduced from the gravitational dynamics of galaxies and groups of galaxies at various scales. From these measurements, it has been found that the ratio of gravitational mass (i.e. all mass) to luminosity M/L scales roughly linearly with scale size r over a wide range of r up to ~1Mpc. Figure 8 shows some results together with an analytic fit to M/L of the form

$$\frac{M}{L} = \mu_0\,[1 - \exp\,(-r/\Lambda)] \tag{6.15}$$

which serves roughly to define a scale size ~ 3 Mpc which appears to be characteristic of the non-luminous mass in the universe.[29] This size is interestingly close to the gravitational clustering size.[30] As we shall see, it is characteristic of the Jeans mass one obtains from neutrinos

with $m_\nu \sim$ 10-30 eV.

We note that it also follows from equation (6.14) that $\Omega_\nu = 1$ for $25 \lesssim \sum m_\nu \lesssim 100$ eV and, from the lower limit on Ω in baryons, it is possible for ν's to gravitationally dominate the universe[31] if $1/2$ eV $\lesssim \sum m_\nu \lesssim 2$ eV. Hereafter we will assume that this is the case. And we will further assume for simplicity (and also because grand unified models favor a neutrino mass heirarchy similar to that in other fermion families) that one neutrino mass eigenstate dominates, i.e.

$$\sum_f m_f = \sum_i m_i \simeq \text{"}m_\nu\text{"} \tag{6.16}$$

The neutrino masses similar to those which we discussed in previous lectures could gravitationally dominate or even close the universe. It has also been pointed out by various workers that massious neutrinos could play an important role in producing the largest scale structure in the universe.[32] This is basically because perturbations of neutrinos on a large enough scale (see below) can survive and grow, whereas in a hot dense universe plasma baryon perturbations are damped by the high viscosity of the thermal blackbody radiation.

For a collisionless gas of neutrinos, the gravitational trapping scale is determined by the virial theorem with a thermal velocity dispersion. Gravitational trapping occurs for scales greater than the Jeans length λ_J such that

$$\frac{G\rho_\nu \lambda^3}{\lambda} > \langle v^2 \rangle \simeq 3.6\ T_\nu/m_\nu \tag{6.17a}$$

$$\text{or} \quad \lambda > \lambda_J = \left(\frac{\langle v^2 \rangle}{G\rho_\nu} \right)^{1/2} \tag{6.17b}$$

with the corresponding Jeans mass

$$M_J \equiv \frac{4\pi}{3} \rho_\nu \left(\frac{\lambda_J}{2}\right)^3 \tag{6.18}$$

For relativistic neutrinos, perturbations can exist on the scale of the horizon size

$$\lambda_{\nu J} \simeq \lambda_H \propto ct \tag{6.19}$$

below which they decay by collisionless (Landau) damping owing to the fact that the thermal motion of the neutrinos smears out irregularities. This process is effective until the ν's become non-relativistic at $T_{NR} \propto M_\nu/3$, below which pressure effects become unimportant.

Since, in the hot big-bang model for $\Omega z >> 1$, $\lambda_H = ct \propto T^{-2/3}$, it follows that $t_{NR} \propto m_\nu^{-2/3}$ and the maximum neutrino Jeans mass

$$M_\nu^{max} \propto \lambda_H^3 \propto t_{NR}^3 \propto m_\nu^{-2} \tag{6.20}$$

Plugging in the numbers, one finds

$$M_\nu^{max} = 4 \times 10^{18}\ m_\nu^{-2}\ (\mathrm{eV})\ M_\odot \tag{6.21}$$

in solar mass units.

If this mass scale is the size of galaxy clusters, 10^{15}-10^{16} $M_\odot$, the corresponding neutrino mass required is in the range 20 eV $\lesssim m_\nu \lesssim$ 65 eV.

Tremaine and Gunn[33] have related the observational parameters of non-luminous mass in galaxy "halos" and rich clusters of galaxies to derive another astrophysically related requirement on m_ν. For simplicity, let us

assume non-degeneracy and consider only the heaviest mass eigenstate. Then, from Fermi-Dirac statistics

$$n_\nu \leq 2 \int^{m_\nu v_{esc}} \frac{p^2 dp}{2\pi^2} = \frac{m_\nu^3 v^3_{esc}}{3\pi^2} \tag{6.22}$$

where v_{esc} is the gravitational escape velocity. Thus

$$\rho_\nu \leq \frac{m_\nu^4 \; v^3_{esc}}{3\pi^2} \tag{6.23}$$

and the maximum neutrino mass density is proportional to m_ν^4. This sets a lower limit requirement on m_ν in order to account for non-luminous ("missing") mass. Tremaine and Gunn have modified this argument by considering the neutrinos to be distributed in isothermal gas spheres with Maxwellian velocities and central density ρ_0. The numbers are basically the same (within a factor of $2^{1/4}$) but the descriptive parameters are now the core radius r_c and maximum velocity, where r_c is given by

$$r_c = \frac{9\sigma^2}{8\pi G\rho_0} \tag{6.24}$$

σ being the 1-dimensional velocity dispersion. Numerically, one obtains

$$m_\nu \gtrsim 30 \text{ eV} \left(\frac{\sigma}{300 \text{ km/s}}\right)^{1/4} \left(\frac{r_c}{10 \text{ kpc}}\right)^{1/2} \tag{6.25}$$

To explain the rotation curve of (velocity versus galactocentric distance) of our own galaxy[34] and others[35] with a massious neutrino halo would then require $m_\nu \gtrsim$ 15-30 eV, and a typical galaxy cluster mass distribution could be explained by neutrinos with $m_\nu \gtrsim$ 4-8 eV.

Finally, we note one other possible piece of astrophysical evidence

regarding neutrino mass from observations of the cosmic ultraviolet background spectrum at high galactic latitudes.[29,36] De Rujula and Glashow[37] pointed out that the decay of a massious neutrino from a heavier mass eigenstate ν' to a lighter one ν, i.e.,

$$\nu' \to \nu + \gamma \tag{6.26}$$

could be detectable through the decay of cosmic neutrinos producing photons in the ultraviolet range. The photon energy

$$E_0 = \frac{m'^2 - m^2}{2m} \tag{6.27}$$

or, in the hierarchy approximation $m' \gg m$,

$$E_0 \simeq \frac{m'}{2} \tag{6.28}$$

The diffuse line intensity of ν-decay photons from the galactic halo neutrinos is given by the integral along the line of sight[38] of the telescope

$$I_\lambda = \frac{1}{4\pi\tau\Delta\lambda} \int n' d\ell \quad cm^{-2}s^{-1}sr^{-1}\text{Å}^{-1} \tag{6.29}$$

where τ and n' are the lifetime and density of ν' neutrinos. The line width $\Delta\lambda$ is

$$\Delta\lambda = \frac{\langle v^2 \rangle^{1/2}}{c} \lambda_0 \ , \ \lambda_0 = E_0^{-1} \tag{6.30}$$

so that $\Delta\lambda/\lambda_0 \sim 10^{-3}$ for galactic halo neutrinos.

If the mass of a galaxy cluster is assumed to be mainly from ν'

neutrinos, then the number of neutrinos in the source is given by

$$N' = \frac{2 \times 10^{66}(M_s/M_\odot)}{m' \ (eV)} \tag{6.31}$$

and the flux from the source is

$$F_\lambda = \frac{N'}{4\pi R_s^2 \tau \Delta\lambda} \ \ cm^{-3} s^{-1} A^{-1} \tag{6.32}$$

where R_s is the distance of the source.

There should also be a cosmic isotropic background component of radiation from the decay of ν's at all redshifts. The spectrum is a smeared out continuum which is roughly a power-law in wavelength for $\lambda \geq \lambda_0$ and which vanishes for $\lambda < \lambda_0$. More precisely

$$I_\lambda = 7.8 \times 10^{28} \ h_0^{-1} \ \tau^{-1} \ \frac{\lambda_0^{3/2}}{\lambda^{5/2}} \ [1-(\Omega-1)(1-\frac{\lambda_0}{\lambda})]^{-1/2} \ \ cm^{-2} s^{-1} sr^{-1} A^{-1} \qquad \lambda \geq \lambda_0 \tag{6.33}$$

as obtained by taking various cosmological factors into account.[36,38,39] Lower limits on $\tau(m_\nu)$ obtained from astrophysical data[29] using equation (88) are shown in Fig. 9.

It turns out that there is an enhancement in the cosmic ultraviolet spectrum at high galactic latitudes which as been observed at $\lambda_0 \sim 1700$ A. This would correspond to $E_0 \approx 7$eV and $m \approx 14$ eV from equation (6.28). The implied neutrino lifetime of 2×10^7 yr is higher than that predicted for the standard GWS model, however, such lifetimes are possible within the context of composite models of quarks and leptons. A detailed discussion is given elsewhere.[29] Further measurements of this ~1700 A feature with much higher

wavelength resolution will be required in order to determine if this feature is indeed from neutrino radiative decay.

To sum up this section, we see that the astrophysical data all hint at (but do not prove) cosmological neutrino masses in the 10-100 eV range. Note the similar numbers given below:

A) From the "missing mass" in galaxy clusters ($\Omega \approx \Omega_\nu \gtrsim 0.4$)
$m_\nu \gtrsim$ 10-40 eV

B) From the Jeans mass for galaxy cluster formation
$m_\nu \approx$ 20-65 eV

C) To explain the galaxy cluster mass distribution
$m_\nu \gtrsim$ 4-8 eV

D) To explain the galaxy halo mass distribution
$m_\nu \gtrsim$ 15-30 eV

E) To explain the 1700A ultraviolet background feature
$m_\nu \approx$ 14-15 eV

Of course, all of these indications are consistent with the mass results obtained by Lyubimov, et al.[21] 14 eV $\lesssim m_{\bar{\nu}_e} \lesssim$ 46 eV. But again we have a puzzle because the simplest grand unified models would predict that the mass eigenstate associated with $\bar{\nu}_e$ would have the lightest mass whereas the cosmological interpretations would pertain to the eigenstate with the heaviest mass.

VII CONCLUSIONS

Many avenues of investigation have opened up for addressing the problem of neutrino masses with a whole host of future investigations planned and perhaps new surprises to come. This is as it should be considering the great

importance of this topic for many basic questions ranging from unifed field theories to cosmology. The phenomena involved indeed range from structure on the smallest scales - composite models of quarks and leptons - to those on the largest scales - clustering and "superclustering" of galaxies.

As we have seen, despite all of the many areas of investigation, we only possess hints rather than answers to our questions. While ideas such as the Gell-Mann, Ramond and Slansky model may be pointing us in the right direction theoretically, it is far from a complete picture. In addition, the generation problem is at least as puzzling here as it is for the other fermions. Questions have been raised regarding the standard solar neutrino model and detection of the dominant pp neutrinos must await a new generation of experiments. Reactor, deep mine and accelerator experiments have defined limits to the oscillation parameters, but mixed, possibly conflicting, results in these areas leave us with more unanswered questions. The double β-decay experiments also give conflicting results among themselves. The ^3H decay results are indeed exciting. But here there is uncertainty in the molecular physics and the results themselves raise questions about the theoretical framework and the neutrino mass heirarchy. Long lived neutrinos with masses above 100 eV would create conflicts with the astrophysical observation that $q_0 < 1$. If ν_e has an associated mass $\sim 30 \pm 16$ eV, what of ν_μ? Or ν_τ? Finally, the astrophysical data provide only hints. The existence of 10-100 eV neutrinos could help provide many answers to cosmological questions - but do such neutrinos exist?

ACKNOWLEDGMENT:

I would like to thank Robert W. Brown, Richard Holman, and S. Peter Rosen for helpful discussions. Special thanks to Sidney Harris for providing me with the cartoon which appears at the front of these lectures and for permission to reproduce it.

SUPPLEMENTAL READING

It has been my purpose in these lectures to try to present a large number of the basic arguments pertaining to various aspects of the neutrino mass problem. For this reason, I have tried to limit the number of specific references rather than compile an extensive review of the literature. Thus, many significant papers have not been referenced explicitly. (I hope that my colleagues will bear my purpose in mind so that little offense will be taken.)

However, more specialized recent papers and reviews cover specific parts of the literature more intensively. Further details of the topics discussed here may also be found in these works. I list below a few of these by topic (again not a complete listing) as recommended supplemental reading.

A) Theory of Neutrino Mass:

Langacker, P. "Grand Unified Theories and Proton Decay" (Ref.6)

Wetterich, C. "Neutrino Masses and the Scale of B-L Violation," Nucl. Phys. B187, 343 (1981).

Wolfenstein, L. "Lepton and Baryon Number Nonconservation Neutrino Mass," Proc. 1981 Intl. Conf. on Neutrino Physics and Astrophysics ("ν81"), Maui, Hawaii, Ed. R. J. Cence, E. Ma and A. Roberts, Univ. Hawaii Press 2, 329 (1981).

B) Neutrino Oscillations and Solar Neutrinos

Bahcall, J. N. "Solar Neutrinos: Rapporteurs Talk" Proc "ν81" ibid. 2, 253 (1981).

Baltay, C. "Experimental Results on Neutrino Oscillations and Lepton Non-conservation" (Ref. 16).

Barger, V., "Neutrino Oscillation Phenomena" (Ref 17).

Silverman, D. and Soni, A. "Reactor Experiments and Neutrino Oscillations" (Ref. 13).

C) Double Beta Decay

Rosen, S. P. "Lepton Non-Conservation and Double Beta Decay: Constaints on the Masses and Couplings of Majorana Neutrinos" (Ref. 21).

D) Cosmology and Background Radiation

Weinberg, S. Gravitation and Cosmology (Ref. 26).

Stecker, F. W. Cosmic Gamma Rays (Ref. 37).

Stecker, F. W. and Brown, R. W. "Astrophysical Tests for Radiative Decay of Neutrinos and Fundamental Physics Implications" (Ref. 28).

E) Neutrino Dominated Universe

Dorochkevich, et al. "Cosmological Impact of the Neutrino Rest Mass" (Ref. 31).

Sato, H. "The Early Universe and Clustering of Relic Neutrinos" (Ref. 31).

Peebles, P. J. E. "The Mass of the Universe" (Ref. 29).

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FIGURE CAPTIONS

Fig. 1. Effective weak interaction and neutrino mass terms and renormalizable theories.

Fig. 2. Scheme for breakdown of SO(10) and Gell-Mann, Ramond and Slansky model.

Fig. 3. Solution regions for the Reines, et al. (UCI) and Kwon et al. (ILL) data and the best fit solutions (black areas) obtained by Silverman and Soni (Ref. 13).

Fig. 4. Limits on $\sin^2 2\theta$ and Δ^2 as summarized by Barger (Ref. 17).

Fig. 5. Kurie plots for $m_\nu = 0$ and $m_\nu \neq 0$ shown with and without tails (T) owing to the energy resolution of the detector.

Fig. 6 Feynman diagram for neutrinoless double β decay.

Fig. 7 (a) Transition level diagram and (b) electron energy spectral for double β decay. For neutrinoless double β decay the spectrum of $E_1 + E_2$ is a spike at $E_1 + E_2 = Q$ as shown. With accompanying neutrinos sharing the energy, the spectrum is spread out owing to the phase space factor.

Fig. 8 Plot of M/L as a function of astronomical distance scale showing data on a fit to the analytic form of equation (70).

Fig. 9 Theoretical model predictions for $\tau(m_\nu)$ and astrophysical lower limits on $h_0\tau(E_0)$.[28] (It is assumed that $m_\nu = 2E_0$). The limits marked SB_F (Stecker and Brown) were obtained directly from cosmic photon fluxes. The limits MS_I (Melott and Sciama) and SB_I (Stecker and Brown) are from ionizing flux limits. The point S is obtained from the ~ 1700 A feature. The limits marked SCC and SCV were obtained by Shipman and Cowsik from observations of the Coma cluster and Virgo cluster. Limits obtained from other observations of Coma and Virgo by Henry and Feldman are labled HC and HV, respectively. (See Ref. 28 for complete reference list.)

FIG. 1

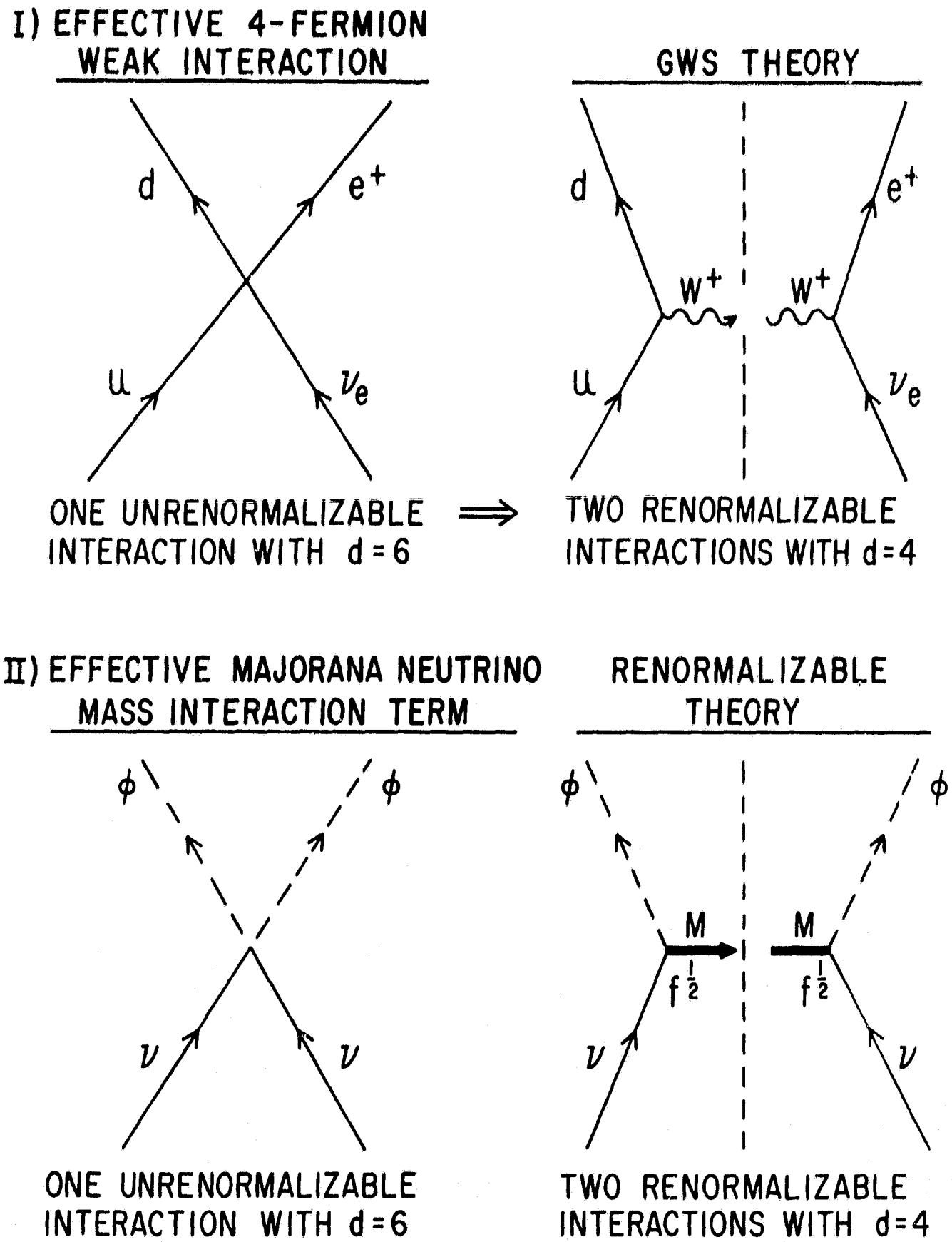

I) EFFECTIVE 4-FERMION WEAK INTERACTION
GWS THEORY
d
e+
u
νe
W+
W+
ONE UNRENORMALIZABLE INTERACTION WITH d = 6
⟹
TWO RENORMALIZABLE INTERACTIONS WITH d=4
II) EFFECTIVE MAJORANA NEUTRINO MASS INTERACTION TERM
RENORMALIZABLE THEORY
φ
φ
ν
ν
M
M
f 1/2
f 1/2
ONE UNRENORMALIZABLE INTERACTION WITH d=6
TWO RENORMALIZABLE INTERACTIONS WITH d=4

FIG. 2

$$SO(10)$$

$\sim 10^{15}$ GeV $\quad \Big\downarrow \; M_X \simeq \langle \Phi_X \rangle$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\sim 10^{13}$ GeV $\quad \Big\downarrow \; M_R \simeq \langle \Phi_R \rangle \simeq \langle \Phi_{B-L} \rangle$

$$SU(2)_L \times U(1)_Y \times SU(3)_C$$

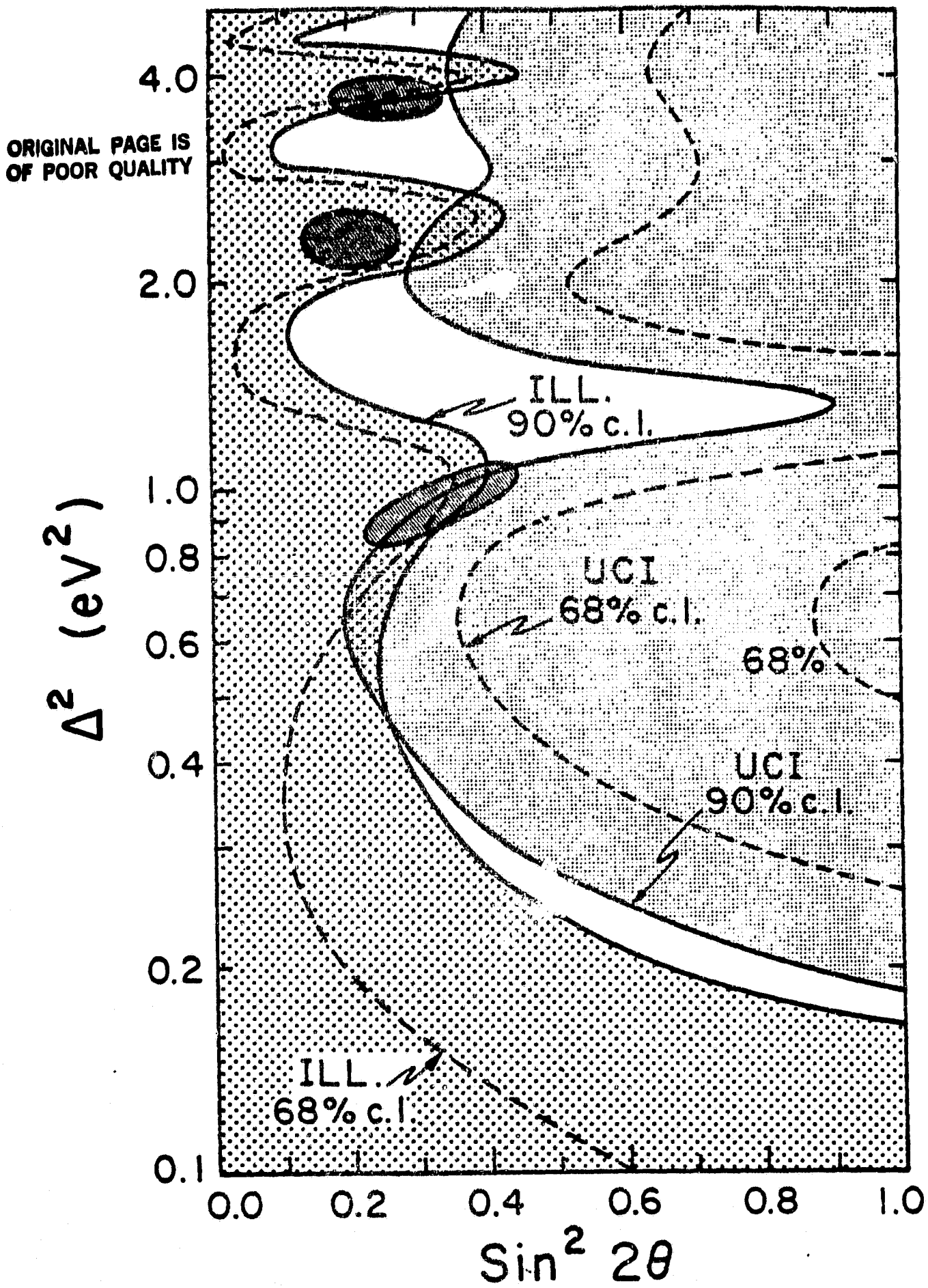
$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}$$

~ 300 GeV $\quad \Big\downarrow \; M_W \simeq \langle \phi \rangle$

$$U(1)_{em} \times SU(3)_C$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

FIG. 3



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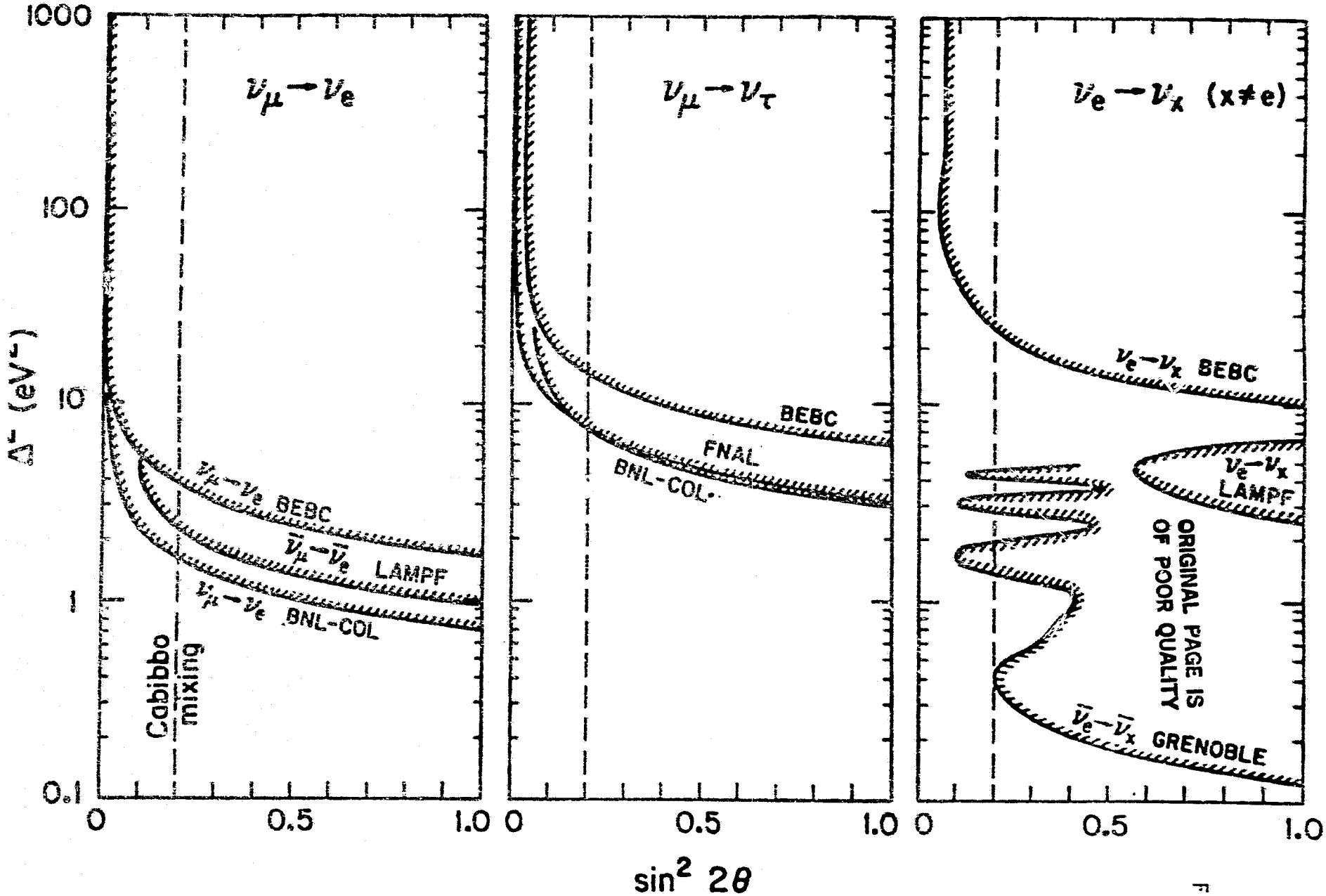
$\nu_\mu \to \nu_e$
$\nu_\mu \to \nu_\tau$
$\nu_e \to \nu_x$ (x≠e)
$\nu_\mu \to \nu_e$ BEBC
$\bar{\nu}_\mu \to \bar{\nu}_e$ LAMPF
$\nu_\mu \to \nu_e$ BNL-COL
Cabibbo mixing
BEBC
FNAL
BNL-COL
$\nu_e \to \nu_x$ BEBC
$\nu_e \to \nu_x$ LAMPF
$\bar{\nu}_e \to \bar{\nu}_x$ GRENOBLE
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Δ^2 (eV^2)
1000
100
10
1
0.1
0
0.5
1.0
$\sin^2 2\theta$

FIG. 4

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FIG. 5

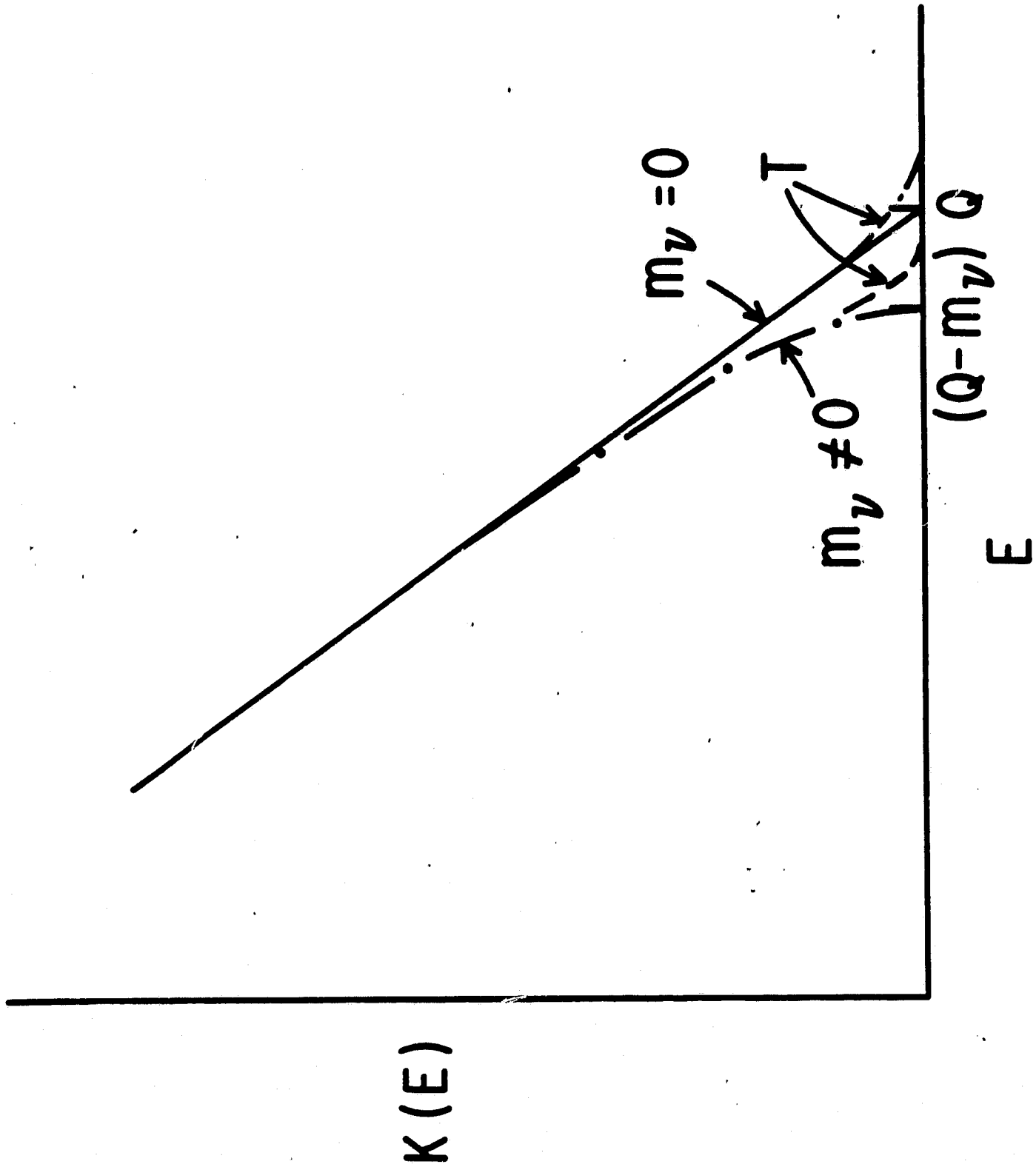
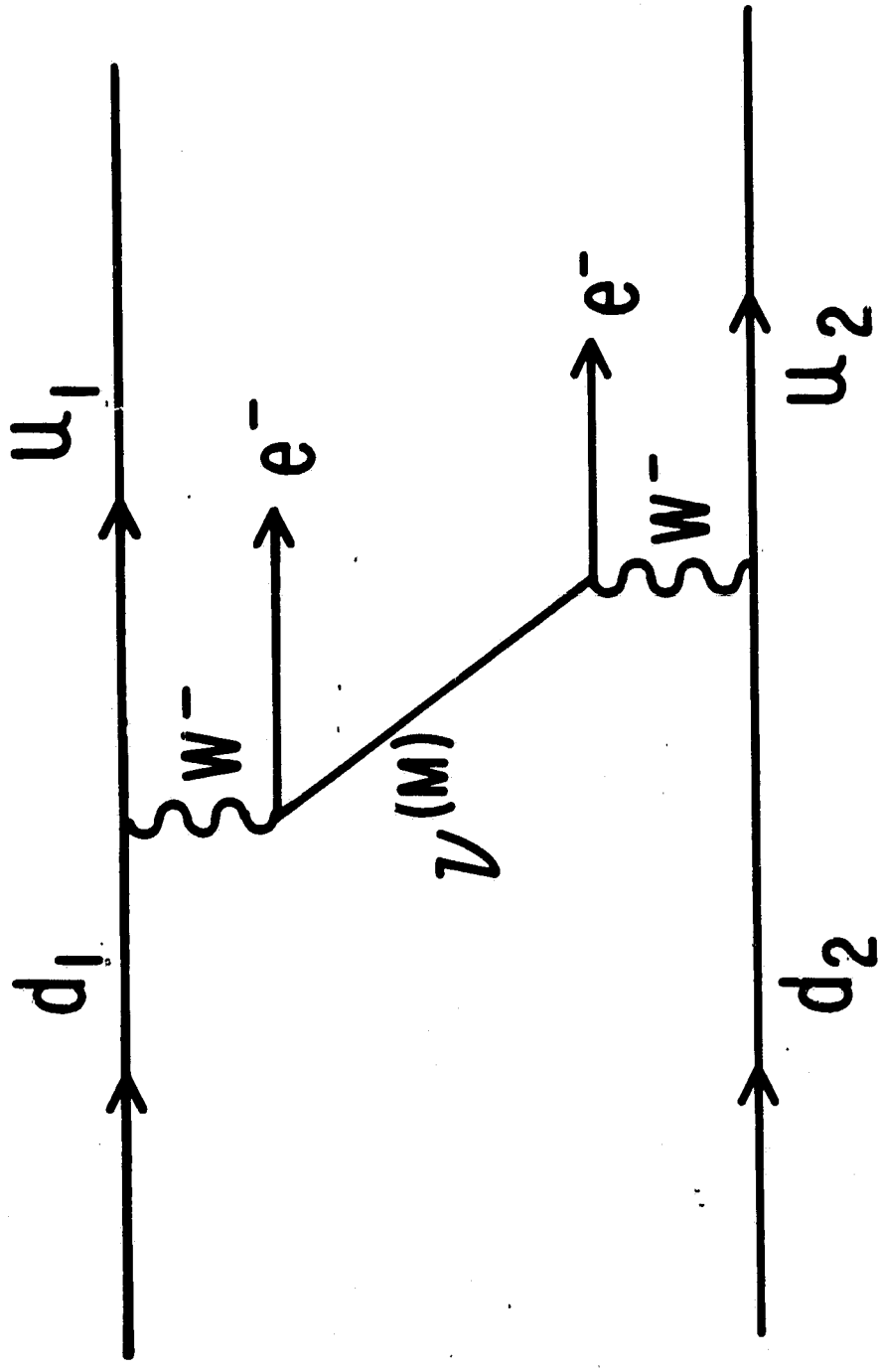


FIG. 6



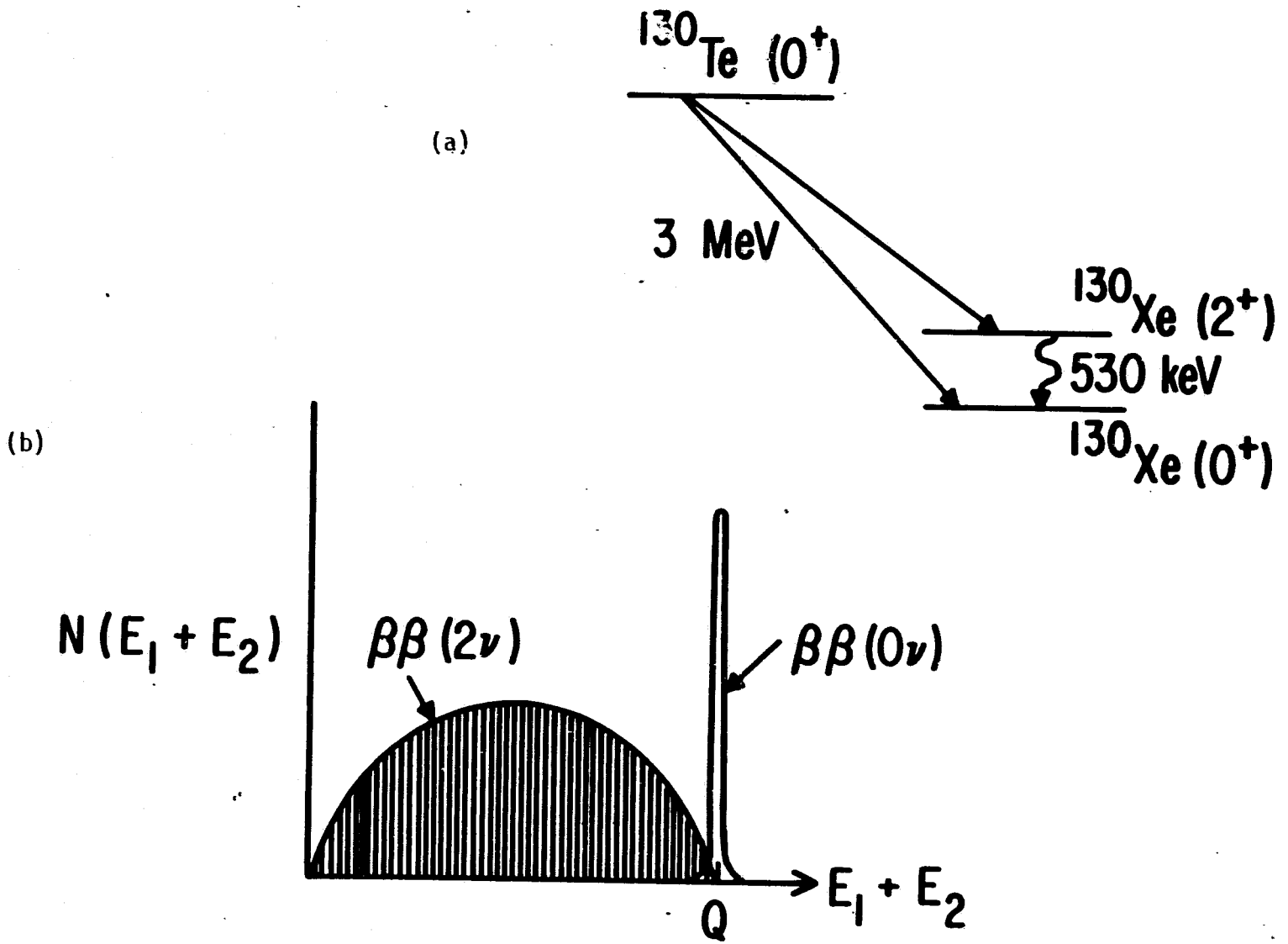


FIG. 7

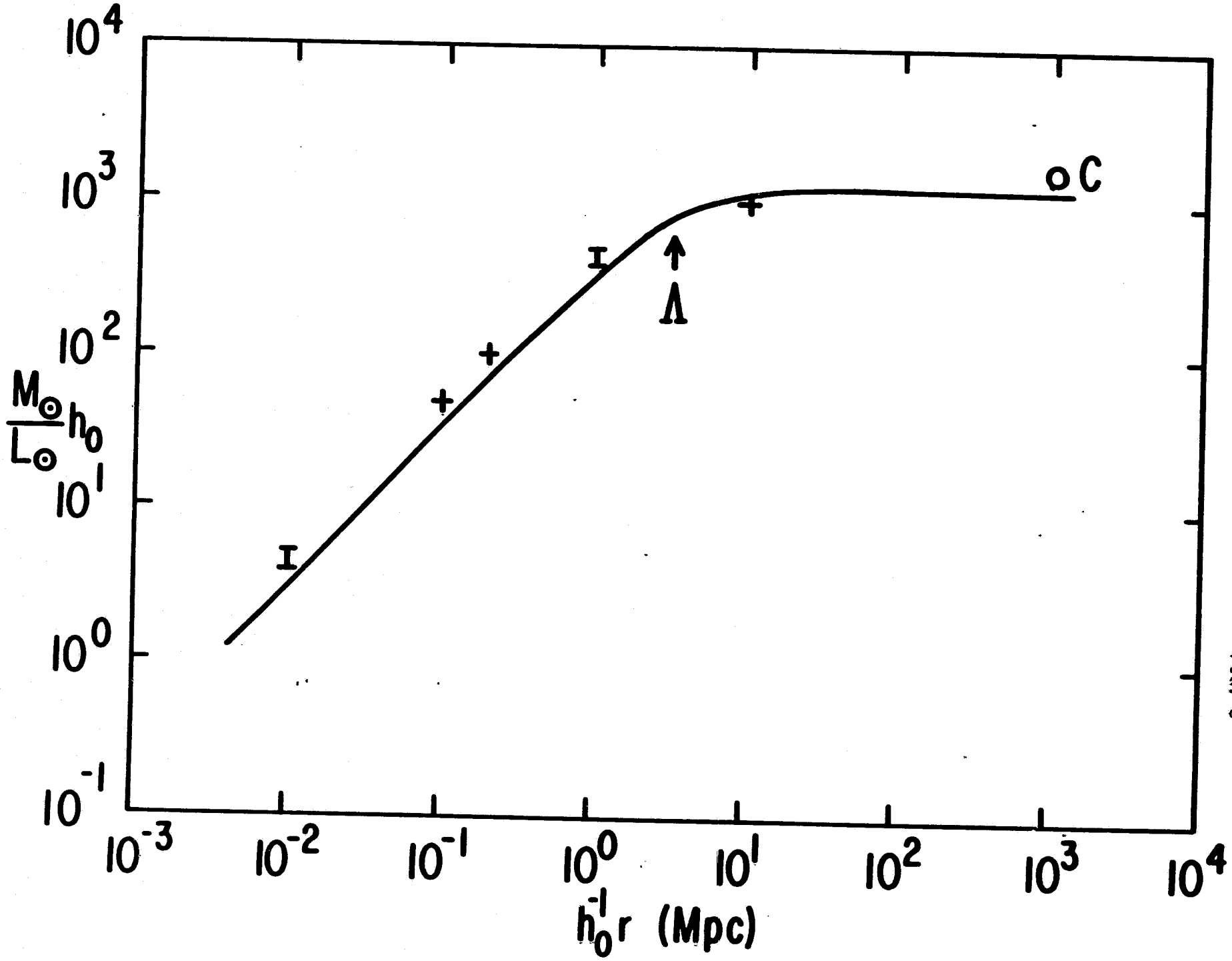
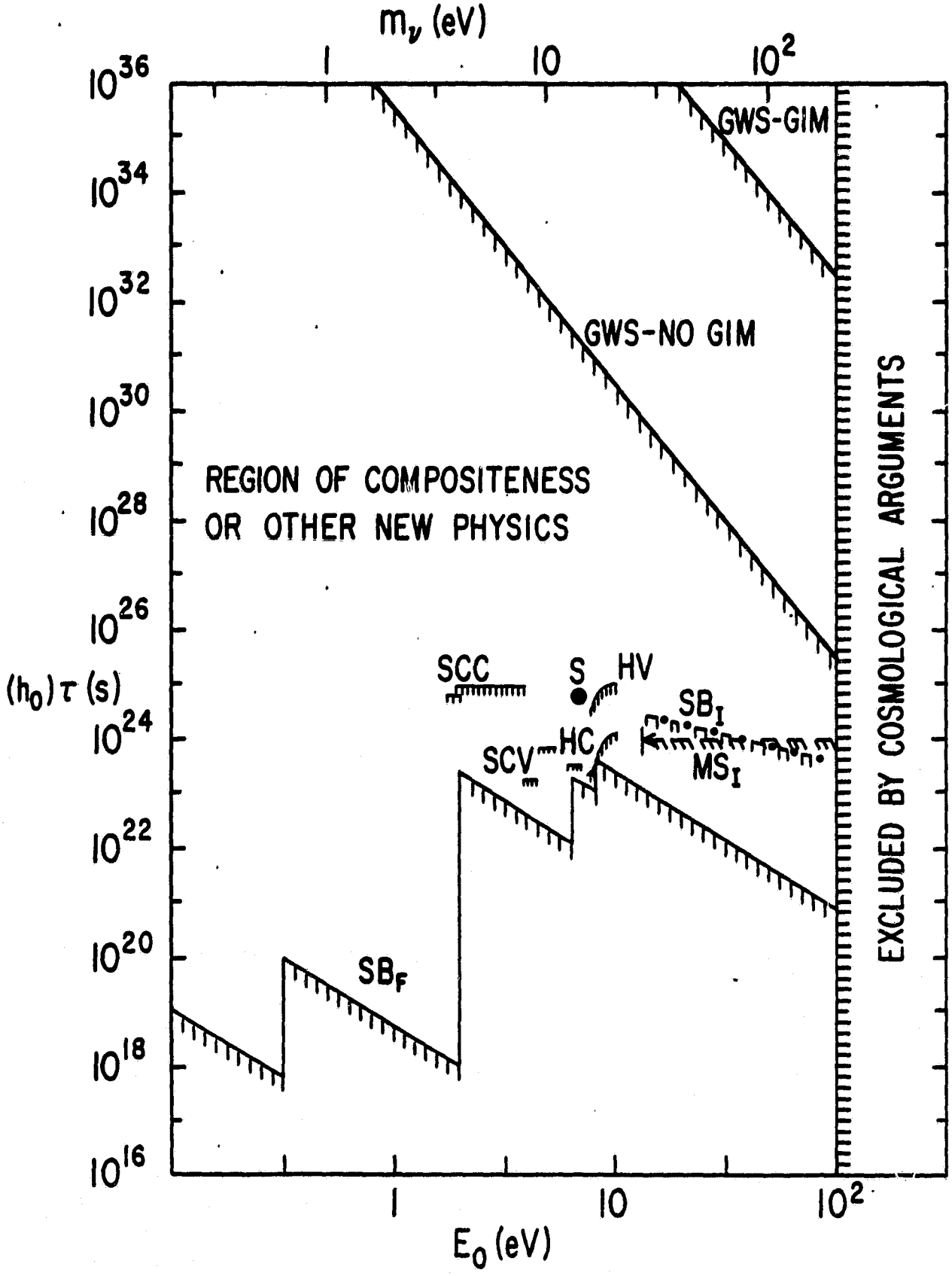

10^4
10^3
10^2
10^1
10^0
10^{-1}
$\frac{M_\odot}{L_\odot} h_0$
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4
$h_0^{-1} r$ (Mpc)
C
Λ

FIG. 8

FIG. 9


m_ν (eV)
1
10
10^2
10^{36}
10^{34}
10^{32}
10^{30}
10^{28}
10^{26}
10^{24}
10^{22}
10^{20}
10^{18}
10^{16}
$(h_0)\tau$ (s)
GWS-GIM
GWS-NO GIM
REGION OF COMPOSITENESS
OR OTHER NEW PHYSICS
EXCLUDED BY COSMOLOGICAL ARGUMENTS
SCC
S
HV
SB_I
SCV
HC
MS_I
SB_F
1
10
10^2
E_0 (eV)