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Solidification of Carbon-Oxygen white dwarfs

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**Abstract.** During cooling, a hail of oxygen can fall in the center of a Carbon-Oxygen white dwarf. Consequences for white dwarfs evolution are considered. Formation of pulsars or type I Supernovae production can be the result of a difference in chemical composition.

The following report is divided into four sections: (1) elementary information on the internal structure of white dwarfs, (2) basic information on highly correlated plasmas, (3) implications for phase separation in the core of cooling white dwarfs, and (4) consequences for evolution of white dwarfs.

The present report can be considered as a continuation of a paper of Canal, Isern and Labay (1981).

(1) Internal structure of white dwarfs. We just recall a few properties of white dwarfs which are relevant to the present problem.

We describe a white dwarf as a non-rotating, spherical star, in hydrostatic equilibrium. The main problem concerns the equation of state,

$$P = P(\rho, T)$$

which, in the major part of the star cannot be distinguished from the equation of state of a Fermi gas at zero temperature,

$$P = P(\rho, 0)$$

The region where the departure from  $T = 0$  influences the structure is extremely small, and represent a mass  $\Delta M / M = 10^{-4.58} T_{\text{eff}}^{20/3}$

If we ignore the properties of the nuclei and introduce in the equations of hydrostatic equilibrium the general relativity corrections to the Newtonian theory of gravitation, we write

$$\frac{dP}{dr} = - \frac{GM}{r^2} \rho \frac{(1 + (P/\rho c^2)) (1 + (4\pi P r^3 / M c^2))}{(1 - (2GM/c^2))}$$

where the term  $(1 + (P/\rho c^2))$  represents the increase of inertial mass, the next term the increase in gravitationally attractive mass and the denominator term represents the change of the metric for Euclidian to non Euclidian.

The knowledge of the equation of state is necessary in order to solve the equation of hydrostatic equilibrium. At high densities we can write the relativistic equation of state,

$$P = K_2 (\rho / \mu_e)^{4/3}$$

with  $\mu_e = (A/Z)$  and

$$K_2 = 1.23 \cdot 10^{15}$$

At lower densities, the non relativistic equation of state (neglecting electrostatic interactions) is

$$P = K_1 (\rho / \mu_e)^{5/3}$$

with  $K_1 = 9.91 \cdot 10^{12}$ . The change from one equation of state to the other takes place for a density of about  $10^8$  g cm<sup>-3</sup>.

For the kind of white dwarfs we are considering, the transition from the non relativistic to the relativistic equation of state takes place at a mass  $\Delta M_{\text{rel}}$ :

$$\frac{\Delta M_{\text{rel}}}{M} = (4\pi R^4 / GM^2) (\rho / \mu_e)^{5/3} K_1 = 8 \cdot 10^{-5}$$

The mass radius relation for white dwarfs shows the existence of a maximum mass. For  $\mu_e = 2$ , we have the following properties of a white dwarf with the maximum mass :

$$M = 1.36557 M (\text{Sun})$$

$$R = 1066 \text{ km}$$

$$\rho_c = 1.92 \cdot 10^{10} \text{ g cm}^{-3}$$

A white dwarf of high density, close to the limiting mass (we should call it the Chandrasekhar-Kaplan limiting mass), has no other energy source than its internal thermal energy. As long as the inside of the white dwarf is fluid, the internal thermal energy is the sum of the thermal energy of the ions and of the thermal energy of the electrons,

$$U = (\rho / A m_H) \left( (3/2) k T + (3/4) \left( Z \pi^2 k^2 T^2 / m_e c^2 x \right) \right)$$

where  $x$  is related to the density by

$$\rho = (8 \pi / 3) (m_e^3 c^3 / h^3) (A m_H / Z) x^3$$

The corresponding cooling time scale is given by the heat balance condition.

Again, for the kind of stars which we are considering, close to the Chandrasekhar - Kaplan limit, we have for the characteristic cooling time

$$t = 10^{6.056} (T / 10^8)^{1/2} \text{ years.}$$

This is valid however only as long as the white dwarf has not reached the solid state. If we consider the ratio of the electrostatic energy to the thermal energy,

$$\Gamma = (Z^2 e^2 / a k T)$$

where

$$a = (3 Z / 4 \pi N_e)^{1/3}$$

is the ionic radius, the solidification takes place when

$$\Gamma = 160.$$

During cooling, the first point where solidification takes place is in the center of the star. Two quantities are important to consider : (1) the melting temperature inside the star ; (2) the Debye temperature inside the star. It turns out that the heat content of the electrons is much larger than the heat content of the ions.

When solidification takes place, the latent heat of solidification is liberated, and the consequence is a slowing down of the cooling. For the kind of stars which we are considering, this is completely negligible.

## 2. Highly correlated plasmas.

We have just seen that in a cold plasma, crystallization takes place when the ratio of the Coulomb energy to the thermal energy is about 160. We need not to consider in the following the quantum effects in the solid.

Let us consider now the case of a two component plasma, Carbon and Oxygen, which are the most abundant products of the late stages of stellar evolution.

There is a conjecture by Stevenson (1979) that an eutectic can exist, between Carbon and Oxygen, with the following characteristics :

$$\begin{aligned} X_O (\text{in number}) &= 0.332 \\ T_E &= 0.628 T_C (\text{melting of Carbo}) \\ &= 0.389 T_O (\text{melting of Oxygen}) \end{aligned}$$

with a density difference (Pollock and Hanse, 1974) :

$$\frac{\rho(\text{solid}) - \rho(\text{liquid})}{\rho} = 3 \cdot 10^{-4}$$

and with  $A(\text{Oxygen}) = 15.99468$  and  $A(\text{Carbon}) = 12.00000$ , we obtain :

$$\frac{\rho_0(\text{solid}) - \rho(\text{liquid})}{\rho} = \frac{3.10^{-4} - 3.24 \cdot 10^{-4} X}{1 - (X/4)} \quad ; X_C = X < 0.668$$

$$\frac{\rho_C(\text{solid}) - \rho(\text{liquid})}{\rho} = \frac{6.52 \cdot 10^{-4} - 4.07 \cdot 10^{-4} X}{(1 - (X/4))} \quad X > 0.668$$

As a consequence, we see that the density of the solid, either of pure carbon or of pure oxygen, is always larger than the density of the liquid.

During cooling, solid particles of Carbon, or solid particles of oxygen (according to the concentration) will appear and fall in the gravity field. When the chemical composition of the eutectic is reached, the solid particles of Carbon will be appreciably denser than the solid particles of Oxygen. In fact, we have :

$$(\Delta\rho/\rho) \text{ (Carbon to Eutectic)} = 3.91 \cdot 10^{-4}$$

$$(\Delta\rho/\rho) \text{ (Oxygen to Eutectic)} = 0.84 \cdot 10^{-4}$$

The argument of Stevenson is that, if the cooling is fast enough, the eutectic solidifies; if it is very slow, it will experience separation of Oxygen and Carbon. However, due to the large difference in the buoyancy forces, Carbon will settle faster.

### 3. Phase separation in white dwarfs.

We shall first consider the temperature excess of the condensing solid. Following the argument of Jeffreys (1918), and reproduced by Mason (1953), we assume the microscopic diffusion is the way by which the atoms join the growing crystal. We then have:

$$\rho(\text{solid}) R \frac{dR}{dt} = D (1-X) N_e (A/Z) m_H$$

Similarly, we write that the latent heat of crystallization, released on the crystal is carried away by thermal conductivity. We write,  $\lambda$  being the heat conductivity,

$$L R \frac{dR}{dt} = \lambda (T_S - T)$$

from which we derive the temperature excess :

$$T_S - T = \frac{D (1-X) \rho L}{\lambda}$$

$L$  is of the order of  $3 k T$  per ion. The microscopic diffusion coefficient is of the order of

$$D = a v_{th}$$

The thermal conductivity is large, heat being carried away by the relativistic electrons (Schwarzschild, White Dwarfs).

For  $\rho = 10^{10} \text{ g cm}^{-3}$ ,  $T = 10^8 \text{ K}$ , this gives

$$T_S - T = 1000 \text{ K} \ll T$$

Let us now consider the speed at which the crystals can fall inside the white dwarf. We consider that the force on the crystal is given by the Stokes formula. We then have:

$$-(GM/r^2) (4\pi a^3/3) (\rho_{\text{solid}} - \rho_{\text{liquid}}) = 6\pi \rho_S \nu a (dr/dt)$$

where  $\nu$  is the kinematic viscosity coefficient.

If we consider that the radius of the crystal,  $a$ , is time dependent,

$$(a^2/2) = D (1-X) t$$

we obtain,

$$(dr/dt) = - (16\pi/27) G \frac{\rho_{\text{solid}} - \rho_{\text{liquid}}}{\rho} (D/\nu) (1-X) t$$

we can write

$$r = r_0 \exp\left(-\left(t/\tau\right)^2\right)$$

and derive the time scale of the phase separation ,

$$\tau^2 = (27 / 8 \pi) (\rho / (\rho_{\text{solid}} - \rho_{\text{liquid}})) (1 / G \rho_{\text{liquid}}) (v / D) (1-X)^{-1}$$

The viscosity is due to the transfer of momentum to the ions . Therefore,  $v$  is of the order of  $D$  .  
With  $\rho = 2 \cdot 10^{10} \text{ g cm}^{-3}$ ,  $\Delta\rho / \rho = 3.9 \cdot 10^{-4}$ , we obtain ,

$$\tau = 6 \text{ sec}$$

With  $\Delta\rho/\rho = 0.84 \cdot 10^{-4}$ ,

$$\tau = 13 \text{ sec.}$$

We can consider that the cooling time is much longer than the crystallization time and the infalling time . Two cases have to be considered:

(1)  $X_C < 0.668$  : Oxygen is predominant .

Oxygen crystallizes immediately, as soon as the eutectic curve is reached. Carbon being soluble in oxygen, the temperature and the oxygen concentration decrease until the chemical composition of the eutectic is reached. The liquid, with the eutectic composition, which is left behind moves up and mixes with the liquid above, making it more rich in carbon. Further separation takes place until the whole solid oxygen corer is surrounded by the eutectic.

Due to the slow cooling, crystals of C and O will form, but we can expect Carbon to fall faster than Oxygen. The melting temperature will have a tendency to rise above the melting temperature of the eutectic, with a tendency of the mixture to solidify quickly. The most reasonable assumption is that the oxygen corer is surrounded by a solid eutectic.

(2)  $X_C > 0.668$  : Carbon is predominant .

We have a similar picture, but, instead of an oxygen core, we have a carbon core, surrounded by the solid eutectic.

#### 4. White dwarf evolution .

(a) thermal effects.

The infall of solid Oxygen (or solid Carbon) makes available a gravitational energy which is of the same order of magnitude as the internal heat content of the star, and larger than the latent heat which is made available during crystallization .

(b) role of accretion.

For a white dwarf, close to the Chandrasekhar - Kaplan limit, the capture of a small amount of matter can make the star unstable. Capture at an appreciable rate can take place when the white dwarf belongs to a binary system. The effect is a pressure increase in the core of the star.

Two cases have to be considered.

(i) the white dwarf has an oxygen core. In that case, electron capture on oxygen can take place.

The decrease in the number of free electrons per unit of mass generates instability . The star contracts about a speed which is sort of average between the free fall time and the characteristic time of electron capture, until it reaches dynamical instability .

What is expected is the formation of a neutron star. Mass ejection taking place at a later stage of collapse can lead to the escape of the star from the binary system. This could explain the large value of the space velocities of pulsars in the galaxy .

(ii) The white dwarf has a carbon core . In that case, dynamical instability would take place before electron capture . In fact the explosive reaction ,



would take place even before the star has begun to collapse. The present conjecture is that a thermal runaway would take place and the star would explode as a type I supernova.

## Bibliography

The theory of the internal structure of White dwarfs can be found in

S.Chandrasekhar : *Stellar Structure*, Chicago University Press, 1939

There are several monographies and review articles :

E.Schatzman , *White Dwarfs*, North Holland Publishing Comp. 1958

E.Schatzman , in *Handbuch d.Phys.* , Vol LI , 1958 , p.723

L.Nestel , in *Stars and Stellar Systems*, *Stellar Structure*, L.H.Aller and D.B.Mc Laughlin Ed., Chicago University Press, 1965, p.297.

A recent review paper is :

H.Van Horn , *Physics Today* 1979.

Recent developments on the theory of highly correlated plasmas can be found in :

Strongly coupled plasmas, Kalman G. and Cavini P. Ed. Plenum Press, N.Y. and London 1977 . See especially the papers by

Ichimaru S. p.189

Schatzman E. p.407

A colloquium , devoted to the problems of dense matter, includes many aspects of the properties of white dwarfs matter :

*Physique de la matière dense* , *Journal de Physique* , Tome 41 , C2 , 1980 . See especially the papers by

D.J.Stevenson , p.61

H.Van Horn , p.97.

The conductivity of degenerate matter has been studied by several authors . See especially :

A.Kovetz and G.Shaviv, *Astronomy and Astrophys.* 28, 315 , 1973.

Flowers E. and Itoh N. , *Astrophys. J.* 206 , 218, 1976.

The viscosity due to ions has been estimated by

H.Van Horn , in *Low Luminosity stars*, Kumar S. Ed. Gordon and Breach , N.Y. , London and Paris 1968, p.297.

Concerning the idea of non-explosive collapse, see :

R.Canal and E.Schatzman , *Astronomy and Astrophys.* 46, 229, 1976.

The colloquium :

*White dwarfs and degenerate stars*, I.A.U. Colloquium N° 53 , H.M.Van Horn and V.Weidemann Ed. See especially

R.Canal and J.Isern , p.52.

R.Mochkovitch , *Thèse de troisième Cycle*, 1980 (to be published )

The question of the effect of accretion on white dwarfs has been considered again during the I.A.U.

Symposium N° 93 :

*Fundamental problems in the theory of stellar evolution* , D.Sugimoto, D.Q.Lamb and D.N.Schramm Ed. Reidel 1981