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Raindrop oscillations

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Abstract

Raindrops measured by two orthogonal cameras were classified by shape and orientation to determine the nature of the oscillation. A physical model based on potential energy was then developed to study the amplitude variation of oscillating drops. The model results show that oscillations occur about the equilibrium axis ratio, but the time average axis ratio is significantly more spherical for large amplitudes because of asymmetry in the surface potential energy. A generalization of the model to oscillations produced by turbulence yields average axis ratios that are consistent with the camera measurements. The model results for average axis ratios have been applied to rainfall studies with a dual-polarized (vertical/horizontal) radar.

Introduction

The equilibrium shape of a falling drop is the result of a balance between surface tension, aerodynamic forces, and hydrostatic and internal pressures. Drops which have been observed in laminar flow wind tunnels[1] and in still air[2] assume shapes that are consistent with the equilibrium calculations of Pruppacher and Pitter[3]. Under turbulent conditions, however, drops are not quiescent since they undergo continuous oscillations[4,5]. In fact, photographs of raindrops in the atmosphere indicate that the average shape departs significantly from equilibrium[6]. Although these departures have usually been neglected, they take on a renewed importance with recent advances in radar meteorology that exploit the shape and orientation of scatterers to obtain a quantitative or qualitative description of precipitation[7].

The following study consisted of two major efforts. The first was to determine raindrop shape from the photographic record, and the second was to construct a physical model of the changing shape to compute the time average axis ratio for the analysis of dual-polarization radar data. The application of the model is discussed in another paper[8].

Data analysis

In the late 1950's Jones studied the shape of natural falling raindrops using two cameras with optical axes 90° apart[6]. From photographs he was able to determine the axial ratio and volume of 1783 drops. Jones' data has been replotted in Figure 1 to show the average axial ratio (vertical/horizontal) with the 95% confidence interval as a function of equivalent volume diameter. The solid line in Figure 1 represents the equilibrium drop shape calculated by Pruppacher and Pitter[3]. As the 95% confidence intervals shows, the mean axial ratio for raindrops is significantly more spherical than equilibrium. Raindrops larger than 4 mm were also found to be more spherical than equilibrium, but were not photographed in adequate numbers to yield a good estimate of the mean. Although the total sample of 1783 raindrops is relatively modest, a preliminary inspection of measurements using a single camera[9] with several hundred thousand raindrops between 2 and 6 mm diameter shows similar results.

Data from Jones was re-analyzed to determine the nature of raindrop distortions. A simple ellipsoidal shape was assumed which is consistent with an oscillation of the fundamental mode. Rotation was not considered as a cause of drop distortion since the energy required is much larger than for oscillations[10]. Each data point was analyzed for characteristic shape by comparing the measured vertical and horizontal dimensions. Smaller drops were found to be predominately spheres and larger drops predominantly ellipsoids. It was apparent from the shape analysis that raindrops do not oscillate just in the vertical axisymmetric mode because of the significant percentage of horizontal ellipsoids.

The type of oscillation was examined by combining shapes suggestive of the vertical axisymmetric oscillations and those shapes suggestive of horizontal oscillations. The result was consistent with the wind tunnel observations of Nelson and Gokale[11] that small drops ($D \lesssim 3$ mm) prefer the axisymmetric oscillation ($m = 0$ degeneracy of the fundamental mode) and of Blanchard[5] that large drops ($D \gtrsim 5$ mm) prefer the horizontal oscillation ($m = 2$).

The latter is apparently enhanced by aerodynamics since the external pressure should provide a positive feedback to the horizontal deformation. Support for an aerodynamic effect is found in the observation of stable horizontal "prolates" for very large drops[5].

The drop was assumed to have only surface, gravitational and kinetic energies. The minimum in the surface plus gravitational energy was calculated numerically. Agreement with wind tunnel data and the numerical result of Pruppacher and Pitter for the equilibrium axis ratio was within 1% for diameters less than 5 mm (Figure 1) demonstrating that the aerodynamic and hydrodynamic pressures are much less important than the hydrostatic pressure in determining the axis ratio.

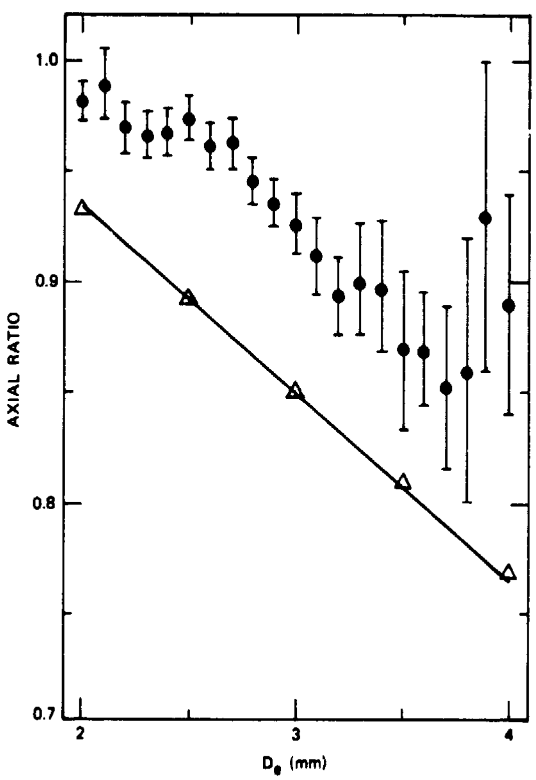

Figure 1. Drop axial ratio as a function of equivalent spherical volume diameter. Circles are for raindrops (Jones), and the curve for equilibrium shaped water drops (Pruppacher and Pitter). Confidence intervals for the mean have been added to the original raindrop data at the 95% significance level based upon the estimated standard deviation and sample size. Triangles are present model for equilibrium shape.

The surface plus gravitational energy function was used as a potential energy "well" for anharmonic oscillations of an ellipsoidal drop. The equation of axisymmetric motion for the fundamental mode was obtained from the energy relation

$$PE_0 - PE = KE = m(V_a^2 + V_b^2 + V_c^2)/5 \qquad (1)$$

by solving for the temporal change in axis ratio

$$\dot{AR} = 6AR^{4/3}\, V_a/D \qquad (2)$$

where

$$V_a = (5KE/m)^{1/2}\, (1 + 2AR^2)^{-1/2} \qquad (3)$$

and

$$AR = c/a = c/b \qquad (4)$$

with a constant volume constraint

$$D = 8abc. \qquad (5)$$

An analytical solution for small amplitude yielded the Rayleigh oscillation time (τ_R) using the factor of 1/5 in the kinetic energy relation. Linear dissipation was added to the calculation by assuming $\dot{KE}$ = B KE where B was determined from the result of Lamb[12] for a complete cycle.

Numerical integrations were made for D = 0.12 cm to compare with the Navier-Stokes simulation of Foote[13] for the axisymmetric oscillation without gravity with ρ_w = 1 g cm^{-3} and σ = 73 dynes cm^{-1}. The present model for the large amplitude oscillation of Foote (where AR_0 = 1/1.7) gives the same oscillation times (i.e., 1.04 τ_R for υ = 0.06 cm^2 sec^{-1} and 1.06 τ_R for υ = 0), and shows similar temporal variations in the surface and kinetic energies, and in axis ratio with 57% of the time spent as a prolate spheroid. The axis ratio averaged over one cycle for the above two cases, and also for D = 0.3 and 0.6 cm with gravity (g = 980 cm sec^{-2}, ρ_w = 0.998 g cm^{-2} and σ = 73 dynes cm^{-1}) for both υ = 0.01 cm^2 sec^{-1} and υ = 0, yielded values of $\overline{AR}$ only slightly less than the mean of the undamped turning points.

Asymmetric oscillations of the fundamental mode (m = 2 degeneracy) is currently under investigation. A set of equations similar to those given above is used to solve the temporal behavior of the axis ratio.

Analytical and numerical solutions are more difficult to obtain than the axisymmetric case since oscillations occur in the shape of a general ellipsoid. Preliminary analysis indicates that the time average axis ratio for the asymmetric oscillation of a raindrop can be approximated by the mean of the turning points defined by the potential energy well.

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Three candidates for the cause of raindrop oscillations were examined: drop interactions, turbulence in the air and pumping by wake shedding. A drop interaction model was used to obtain the oscillation energy from a balance between the average collisional energy and viscous dissipation. The time average axis ratio was found to be considerably larger than equilibrium for drops with $D \gtrsim 3$ mm in heavy rainfall. In contrast, preliminary results for wake forcing indicate that appreciable oscillations occure only for $D \lesssim 2$ mm.

For turbulent forcing the oscillation energy was related to turbulence in the inertial subrange using the distance traveled by a raindrop during an oscillation. This gave a unique scaling for the oscillation energy with raindrop size and dissipation rate. Mean axis ratios for the average oscillation energy were calculated from a balance between input of turbulent energy and dissipation by viscosity. The results provided a reasonable fit to the mean axis ratios of the camera data and a suitable envelope for the observed amplitudes. It was concluded that atmospheric turbulence and also drop interactions can produce significant increases in average axis ratio of raindrops resulting in an appreciable change in radar backscatter cross sections.

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