STATUS REPORT ON DEVELOPMENT OF A REDUCED BASIS TECHNIQUE FOR TRANSIENT THERMAL ANALYSIS

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INTRODUCTION

For some time researchers in structural analysis have recognized that the large number of degrees of freedom required in the solution of structural problems has often been the result of geometry and structural arrangement rather than complexity of the response behavior. This fact has led to considerable research into methods to reduce the degrees of freedom in structural problems and hence computer resources and costs. These methods have become known as reduction methods and are thoroughly reviewed in reference 1. One technique to reduce the degrees of freedom in static and dynamic problems is the reduced basis method which combines the classical Rayleigh-Ritz approximation with contemporary finite-element methods to retain modeling versatility as the degrees of freedom are reduced. The present paper reviews the reduced basis method and its applications to a nonlinear dynamic response problem presented in reference 1 and then summarizes the status of a research effort to apply the method to nonlinear transient thermal response problems.

SUMMARY OF METHOD FOR NONLINEAR DYNAMIC RESPONSE PROBLEMS

The equation of motion for a nonlinear dynamic response problem neglecting damping is shown at the top of figure 1. In the equation $[M]$ represents the mass matrix, $\{\ddot{X}\}$ is a vector of nodal accelerations and $\{Q\}$ and $\{F\}$ are the applied loads and internal nodal forces, respectively. The total number of degrees of freedom in the problem is denoted by m. The internal nodal forces are comprised of a linear portion and a vector of nonlinear displacement dependent terms as indicated by the expression for $\{F\}$. The essence of the reduction method is to use a few known modes or global basis vectors to represent the displacements in the structure. Thus, $\{X\}$ is replaced by the expression $[\Gamma]\{\Psi\}$ where $[\Gamma]$ is a matrix whose columns are the known structural mode shapes and $\{\Psi\}$ is a vector of modal participation coefficients which become the new unknowns in the problem. For practical application to dynamic response problems, $[\Gamma]$ is composed of only the first few vibration modes; thus, n is much smaller than m. To reduce the equations, the expression for $\{X\}$ is substituted into the equation of motion and both sides of the equation premultiplied by the transpose of $[\Gamma]$.

EQUATION OF MOTION: $\underset{m,m}{[M]}\underset{m}{\{\ddot{X}\}} = \underset{m}{\{Q\}} - \underset{m}{\{F\}}$

MASS MATRIX

INTERNAL NODAL FORCES

NODAL ACCEL

APPLIED LOADS

WHERE: $\{F\} = [K]\{X\} + \{G(X)\}$

NONLINEAR TERMS

LINEAR STIFFNESS MATRIX

REPLACE $\{X\}$ WITH REDUCED REPRESENTATION: $\underset{m}{\{X\}} = \underset{m,n}{[\Gamma]}\ \underset{n}{\{\psi\}}$; $n \ll m$

BASIS VECTORS (LOWEST VIBRATION MODES)

Figure 1

REDUCED EQUATIONS FOR NONLINEAR DYNAMIC RESPONSE PROBLEMS

The reduced equation of motion, expressed in terms of the unknown modal participation coefficients, is shown at the top of figure 2. In this equation n represents the number of basis vectors in $[\Gamma]$ and hence the number of unknowns in the reduced problem. The barred quantitites represent the reduced matrices or vectors and are obtained by the indicated matrix multiplications. As shown at the bottom of the figure, the solution process consists of solving eigenvalue problems to obtain the basis vectors, using the basis vectors to reduce the equations and then integrating the reduced equations to obtain the modal participation coefficients and thus, the dynamic response of the structure. This technique was applied to a shallow spherical cap subjected to a step load in reference 1 as described in the next two figures.

REDUCED EQUATION OF MOTION: $\underset{n,n}{[\bar{M}]}\ \underset{n}{\{\ddot{\psi}\}} = \underset{n}{\{\bar{Q}\}} - \underset{n}{\{\bar{F}\}}$

WHERE: $\underset{n,n}{[\bar{M}]} = \underset{n,m}{[\Gamma]^T}\ \underset{m,m}{[M]}\ \underset{m,n}{[\Gamma]}$

$\underset{n}{\{\bar{Q}\}} = \underset{n,m}{[\Gamma]^T}\ \underset{m}{\{Q\}}$

$\underset{n}{\{\bar{F}\}} = \underset{n,n}{[\bar{K}]}\ \underset{n}{\{\psi\}} + \underset{n,m}{[\Gamma]^T}\ \underset{m}{\{G(\psi)\}}$

$\underset{n,n}{[\bar{K}]} = \underset{n,m}{[\Gamma]^T}\ \underset{m,m}{[K]}\ \underset{m,n}{[\Gamma]}$

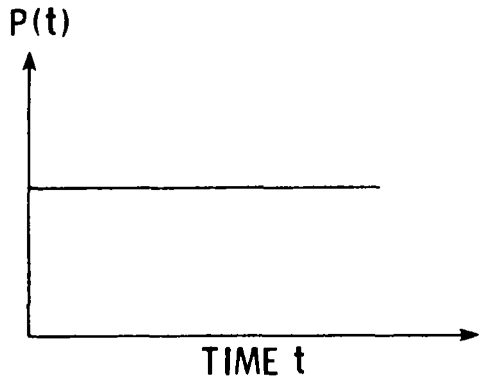
SOLUTION PROCESS
- SOLVE EIGENVALUE PROBLEM FOR BASIS VECTORS
- REDUCE EQUATIONS
- INTEGRATE REDUCED EQUATIONS TO OBTAIN DYNAMIC RESPONSE

Figure 2

SELECTION OF BASIS VECTORS

As indicated in figure 3, a combination of two sets of basis vectors were considered for step loaded dynamic response problems in reference 1. The first consisted of a few eigenvectors from the solution of a linear eigenvalue problem based on initial conditions. The second set was comprised of a few vectors from the linear problem and a few from the solution of a steady-state (static) nonlinear eigenvalue problem where the structural stiffness matrix has been modified to contain the nonlinear stiffness terms associated with the steady-state nonlinear deflections.

CASE OF STEP LOADING



BASIS VECTORS CONSIST OF:

- FEW EIGENVECTORS OF LINEAR PROBLEM

$$[K]\{X\} = \lambda[M]\{X\}$$

- FEW EIGENVECTORS OF STEADY-STATE (STATIC) NONLINEAR PROBLEM

$$\left[[K] + \left[\frac{\partial G_i}{\partial X_j}\right]\right]\{X\} = \lambda[M]\{X\}$$

Figure 3

CLAMPED SHALLOW SPHERICAL CAP

The problem shown in figure 4 consists of a clamped spherical cap subjected to a point load of 177.93 Newtons at the apex applied as a step function in time. The shell is axially symmetric and the meridian was modeled by 10 shear-flexible curved elements with quintic interpolation functions for each of the displacement and rotation components (for a total of 148 nonzero displacement degrees of freedom). Nondimensional motion histories for the shell apex from the full system equations (148 degrees of freedom) and two sets of reduced equations (10 initial modes and 5 initial + 5 steady-state modes) are shown on the right of figure 4. The 10 initial or linear modes track the full system solution for a short time but fail to duplicate the full response of the shell. The combined linear and steady-state nonlinear modes, however, do a very good job of duplicating the response except for a slight shift in phase after about 200 microseconds. This good agreement has led to consideration of the modal reduced basis technique for nonlinear transient thermal analysis as outlined in figure 5.

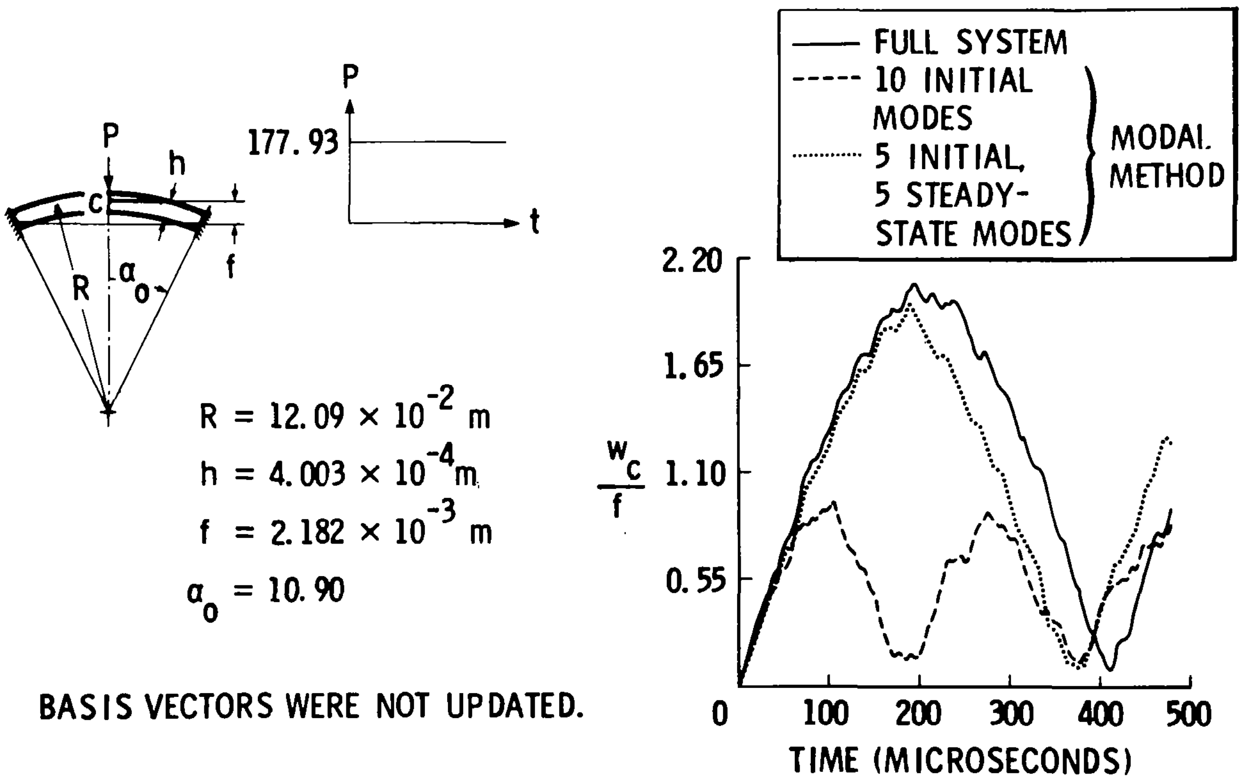


Figure 4

APPLICATION TO NONLINEAR TRANSIENT THERMAL ANALYSIS

Matrix equations describing heat transfer in a heated structure are shown at the top of figure 5. In the equations $[K]$ is the conductance matrix, $\{T\}$ the nodal temperatures, $[C]$ the capacitance matrix, $\{\dot{T}\}$ the time rate of change in the nodal temperatures and $\{Q\}$ the applied heat load. The total number of degrees of freedom is denoted by m. To reduce the equations, $\{T\}$ is replaced by a modal representation where $[\Gamma]$ contains vectors of thermal mode shapes and $\{\Psi\}$ is a vector of unknown modal participation coefficients. The vectors in $[\Gamma]$ may be obtained from solution of two thermal eigenvalue problems associated with the full system of equations. When $\{T\}$ is replaced with the modal representation in the heat transfer equation and both sides of the equation multiplied by the transpose of $[\Gamma]$, a set of reduced equations in terms of the unknown modal participation coefficients is obtained. The barred quantities represent the reduced matrices and vectors obtained by the indicated matrix multiplications. Similar to the dynamic response problem, it is assumed that local temperatures can be represented by a few global modes or basis vectors so that n will be much smaller than m.

HEAT TRANSFER EQUATIONS: $\underset{m,m}{[K]}\,\underset{m}{\{T\}} + \underset{m,m}{[C]}\,\underset{m}{\{\dot{T}\}} = \underset{m}{\{Q\}}$

CONDUCTANCE MATRIX — $[K]$

NODAL TEMPERATURES — $\{T\}$

APPLIED HEATING — $\{Q\}$

CAPACITANCE MATRIX — $[C]$

REPLACE $\{T\}$ WITH $\underset{m}{\{T\}} = \underset{m,n}{[\Gamma]}\;\underset{n}{\{\psi\}}$

$[\Gamma]$ — BASIS VECTORS FROM SOLUTION OF

$$\underset{m,m}{[K]}\,\underset{m}{\{T\}} = \lambda\,\underset{m,m}{[C]}\,\underset{m}{\{T\}}$$

REDUCED EQUATIONS: $\underset{n,n}{[\bar{K}]}\;\underset{n}{\{\psi\}} + \underset{n,n}{[\bar{C}]}\;\underset{n}{\{\dot{\psi}\}} = \underset{n}{\{\bar{Q}\}}$

WHERE: $\underset{n,n}{[\bar{K}]} = \underset{n,m}{[\Gamma]^T}\;\underset{m,m}{[K]}\;\underset{m,n}{[\Gamma]}$

$$\underset{n,n}{[\bar{C}]} = \underset{n,m}{[\Gamma]^T}\;\underset{m,m}{[C]}\;\underset{m,n}{[\Gamma]}$$

$$\underset{n}{\{\bar{Q}\}} = \underset{n,m}{[\Gamma]^T}\;\underset{m}{\{Q\}}$$

AND: $n \ll m$

Figure 5

IMPLEMENTATION OF REDUCED BASIS TECHNIQUE

Implementation of the reduced basis technique for thermal problems is outlined on figure 6. The SPAR Finite Element Thermal Analyzer (ref. 2) was used to generate full system conductance and capacitance matrices and heat load vectors and save them for use in auxiliary computer programs. An existing eigenvalue extraction routine was used to solve the thermal eigenvalue problems to obtain thermal mode shapes used as basis vectors. These basis vectors were then used in a pilot computer program to reduce the full system equations and integrate them using the Crank-Nicholson algorithm to obtain the unknown modal participation coefficient $\{\Psi\}$ and thus the thermal response. This process was evaluated by applying it to the sample problem described in figure 7.

- OBTAIN FULL SYSTEM MATRICES WITH SPAR THERMAL ANALYZER
- SOLVE EIGENVALUE PROBLEMS TO OBTAIN BASIS VECTORS
- USE TEST CODE TO REDUCE EQUATIONS
- USE A CRANK-NICHOLSON ALGORITHM TO INTEGRATE REDUCED EQUATIONS

Figure 6

SAMPLE THERMAL PROBLEM

The problem shown in figure 7 represents a 147.32 cm segment of the lower surface of the Space Shuttle wing and consists of a 0.39 cm thick aluminum skin covered by a 3.81 cm thick layer of Reusable Surface Insulation (RSI). The combined structure was modeled with two-dimensional finite elements as shown on the left of figure 7. The RTV adhesive-Strain Isolator Pad (SIP)-RTV adhesive bonding mechanism used to attach the RSI to the aluminum was also included in the model. The grid shown has 84 node points and hence 84 degrees of freedom since the elements used to model the structure have only temperature as the nodal degrees of freedom. The edges and aluminum structure were assumed to be adiabatic and the surface was heated by the heat pulse shown on the right of figure 7. The heat pulse is reasonably representative of Shuttle reentry and is sufficient to produce surface temperatures where radiation becomes appreciable and, thus, causes the heat transfer equations to become highly nonlinear. Thermal properties of the RSI are also nonlinear as indicated in figure 8.

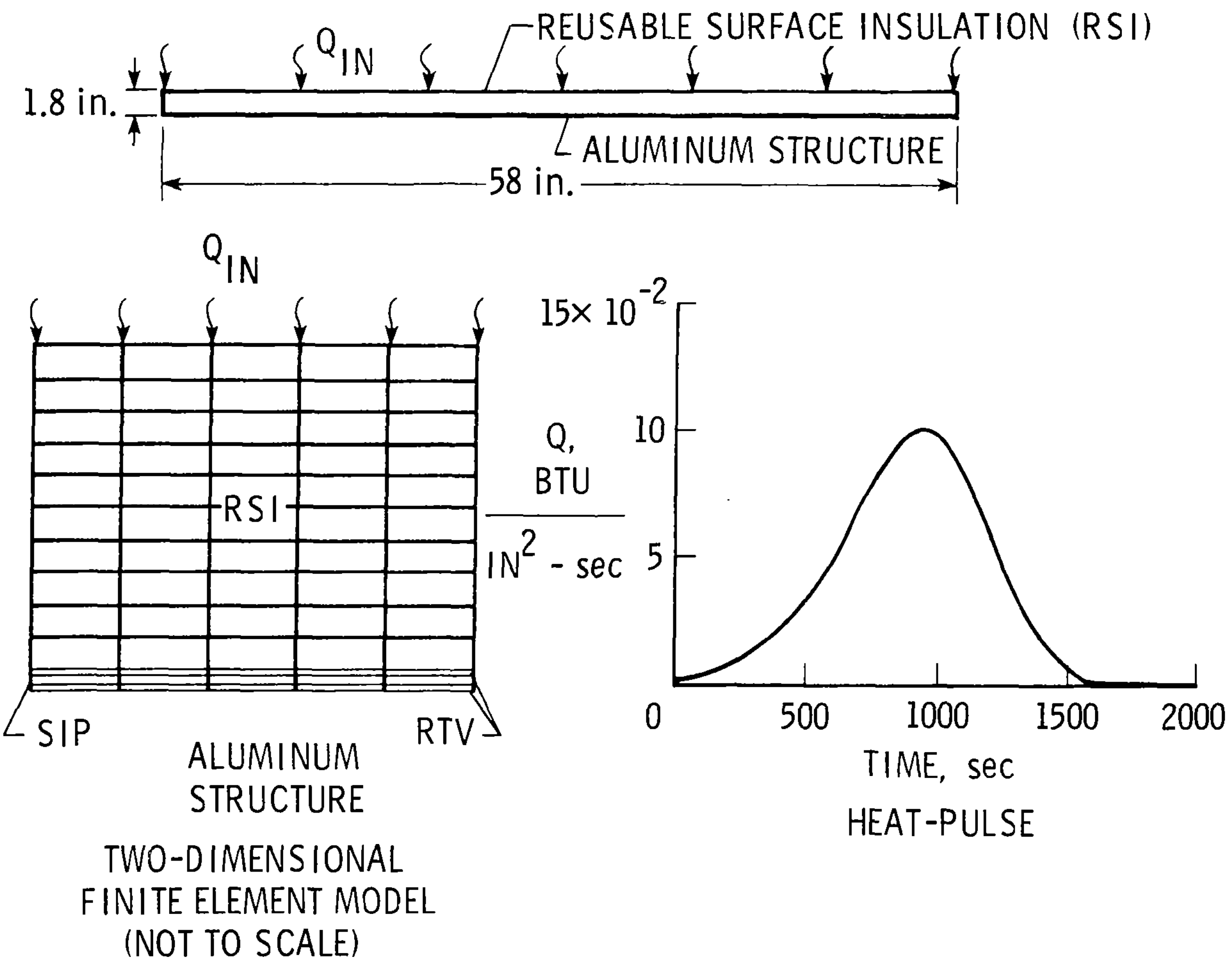


Figure 7

RSI THERMAL PROPERTIES

Specific heat and conductivity for the RSI are shown as functions of temperature in figure 8. The specific heat varies with temperature and because the RSI is very porous, the conductivity varies with pressure as well as temperature. The version of the SPAR Thermal Analyzer used in this investigation accomodates only temperature and time dependent properties. Consequently, the pressure dependency was converted to a time dependency by utilizing the known pressure history for a typical Shuttle reentry trajectory. Thus, the nonlinear material properties of the RSI also contribute to the overall nonlinearity of the problem.

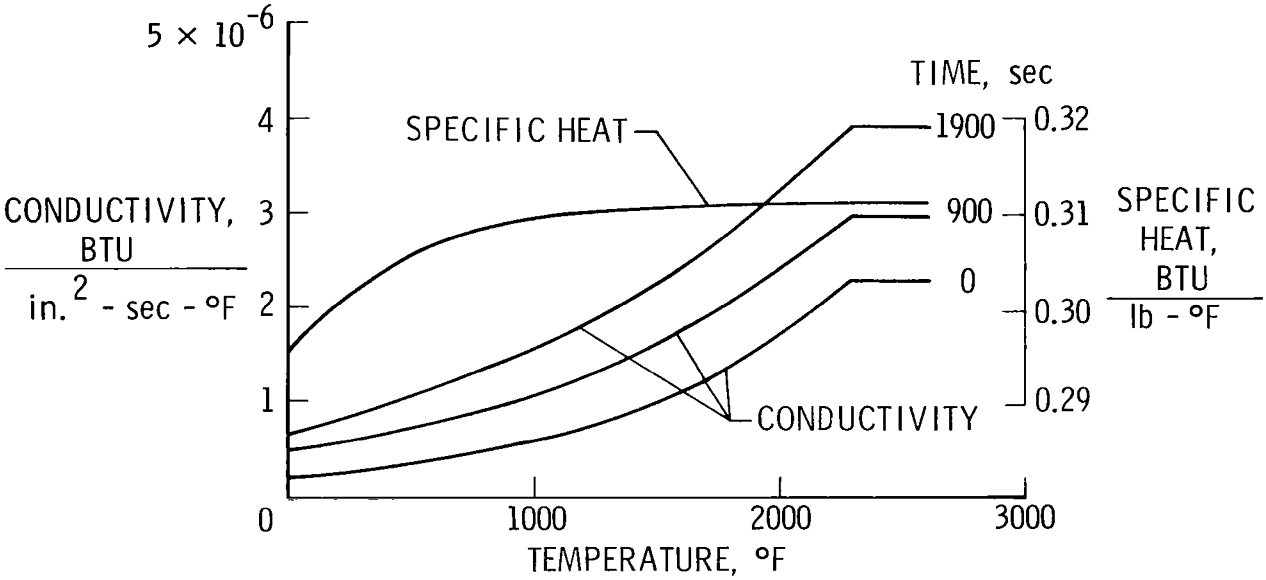


Figure 8

TEMPERATURE PROFILES FOR SAMPLE PROBLEM

A series of temperature distributions through the depth of the sample problem from a full SPAR analysis are shown in figure 9 for several discrete times during the heat pulse. These distributions indicate the type of behavior the basis vectors must approximate to be useful. Initially the entire structure is at a constant temperature of 311 K. As heating is applied, the RSI surface experiences a rapid temperature rise which gradually diffuses through the RSI and SIP to the aluminum skin. After peak heating occurs, the surface begins to cool while the interior of the RSI and the aluminum skin continue to experience a temperature increase. To be useful, the basis vectors used to reduce the degrees of freedom must characterize this nonlinear response, give accurate solutions and be easily and inexpensively generated. The approach used to generate basis vectors for this problem is shown in figure 10.

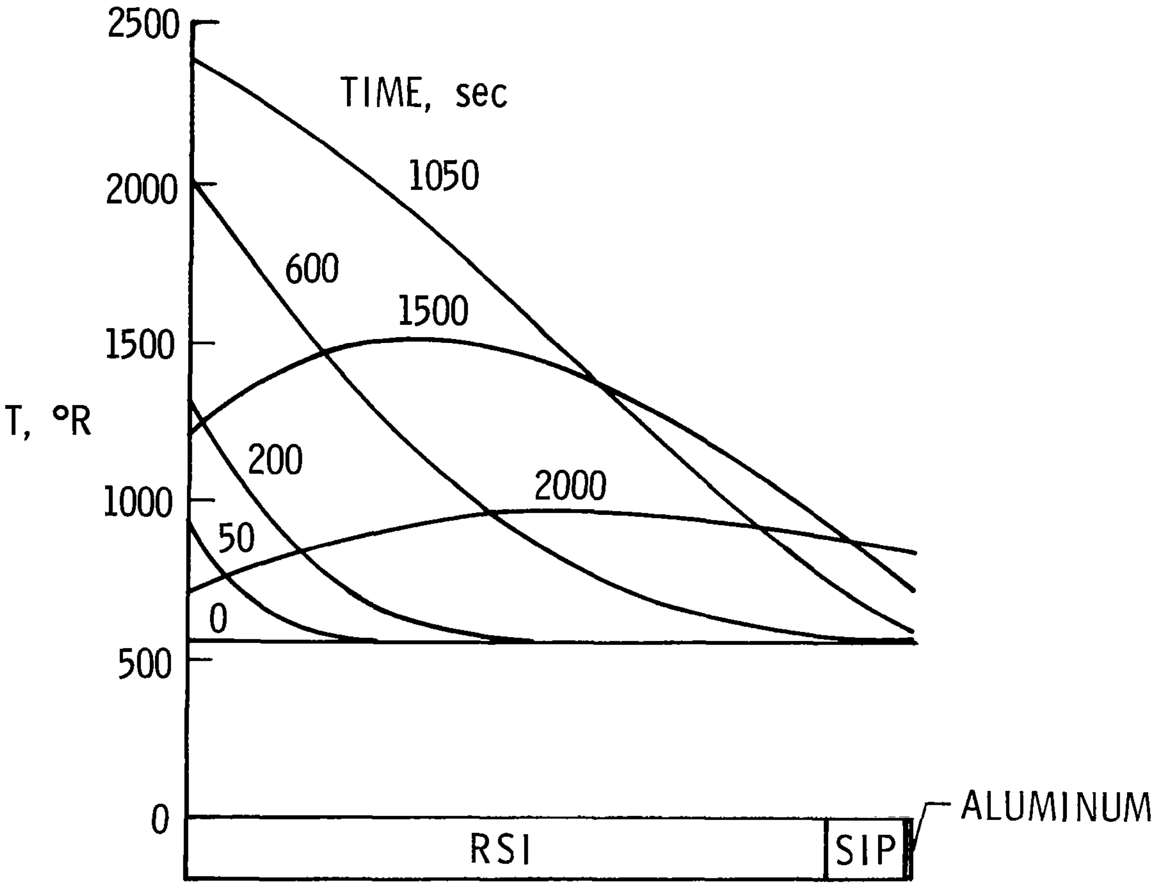


Figure 9

GENERATION OF BASIS VECTORS

Since the use of eigenvectors from the structural eigenvalue problem proved to be a useful set of basis vectors in the dynamic response problem, a similar approach was taken to generate basis vectors for the thermal response problem. In general, the thermal eigenvalue problem indicated in figure 10 would be solved for two temperature states of the system. The first state corresponds to the initial temperature condition and the second state corresponds to a temperature distribution from a "pseudo" steady-state problem for time averaged thermal properties and heating where the aluminum temperature was held constant at some selected value. A few thermal mode shapes from the first eigenvalue problem and a few from the second eigenvalue problem (which include the nonlinear temperature effects) would be combined to form a set of basis vectors. Additionally, for reasons which are explained subsequently, the reciprocal of the first vector from the two eigenvalue problems and a constant vector might also be included as basis vectors. Thermal mode shapes from the eigenvalue problem based on initial conditions are shown in figure 11.

- SOLVE THE EIGENVALUE PROBLEM: $[K]_{m,m}\{T\}_{m} = \lambda [C]_{m,m}\{T\}_{m}$
- USE THE THERMAL MODE SHAPES AS BASIS VECTORS

  (1) FIVE VECTORS FROM PROBLEM INITIAL CONDITIONS

  (2) FIVE VECTORS FROM PSEUDO STEADY STATE SOLUTION

  (3) RECIPROCAL OF FIRST VECTOR FROM (1) AND (2)

Figure 10

THERMAL MODE SHAPES (BASIS VECTORS)

Normalized thermal mode shapes from the linear eigenvalue problem (in which matrices were evaluated at an initial temperature of 311 K) are shown in figure 11. Although numbered sequentially, these modes do not, in fact, represent the first five modes from the two-dimensional eigenvalue problem associated with the finite element model shown in figure 7. Because of the two-dimensional nature of the eigenvalue problem, most of the lower modes involve multiple waves in the lateral direction. A total of 84 eigenvalues were extracted and the five modes shown have only a single wave in the lateral direction with multiple waves through the depth of the structure. As a first attempt to approximate the temperature distributions shown in figure 9, twelve modes from the eigenvalue problem for the initial temperature condition were selected as basis vectors. Additionally, to enhance the representation of the diffusion character of the temperature distributions up to 600 sec (see fig. 9), the reciprocal of the first mode shape was also used as a basis vector. Finally, to accommodate a uniform temperature change, a constant vector was included for a total of 14 basis vectors. Temperatures from the reduced basis method are compared with temperatures from full SPAR analysis in figure 12.

FROM

$$[K]_{m,mm}\{T\} = \lambda [C]_{m,mm}\{T\}$$

EVALUATED AT $\{T\} = 560^{\circ}$ R

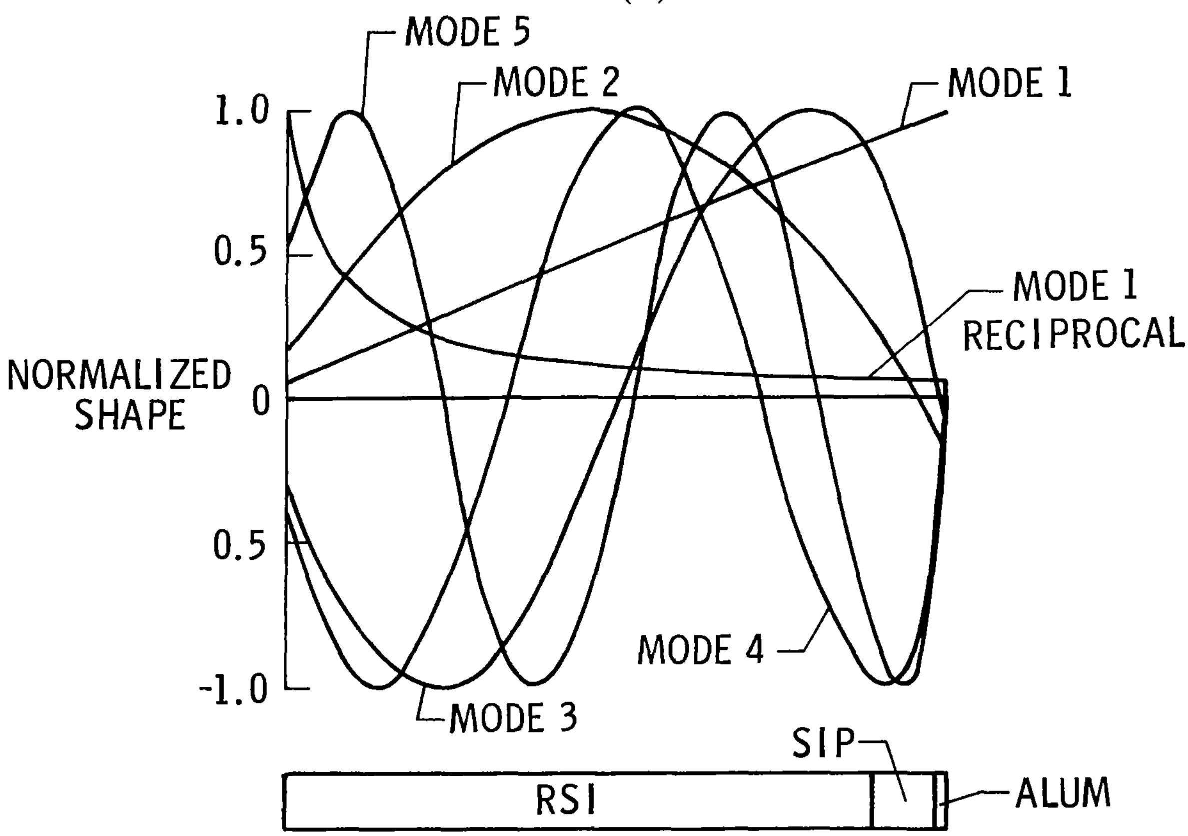


Figure 11

COMPARISON OF REDUCED BASIS AND FULL SYSTEM RESULTS

Temperature histories for the sample problem are shown for the RSI surface, RSI mid-point, and the aluminum structure in figure 12. The solid curves represent results from the full system of equations obtained with the SPAR Thermal Analyzer and the solid symbols are the reduced basis results based on the 14 modes discussed in figure 11. The results from the reduced basis method agree very well with those from the full SPAR analysis. However, it should be noted that the uniform heating and symmetry of the sample problem result in a one-dimensional problem in which a 14 degree of freedom model (i.e., a single vertical slice through the model in figure 7) would be sufficient for the problem. Thus, use of 14 basis vectors would be expected to give excellent results.

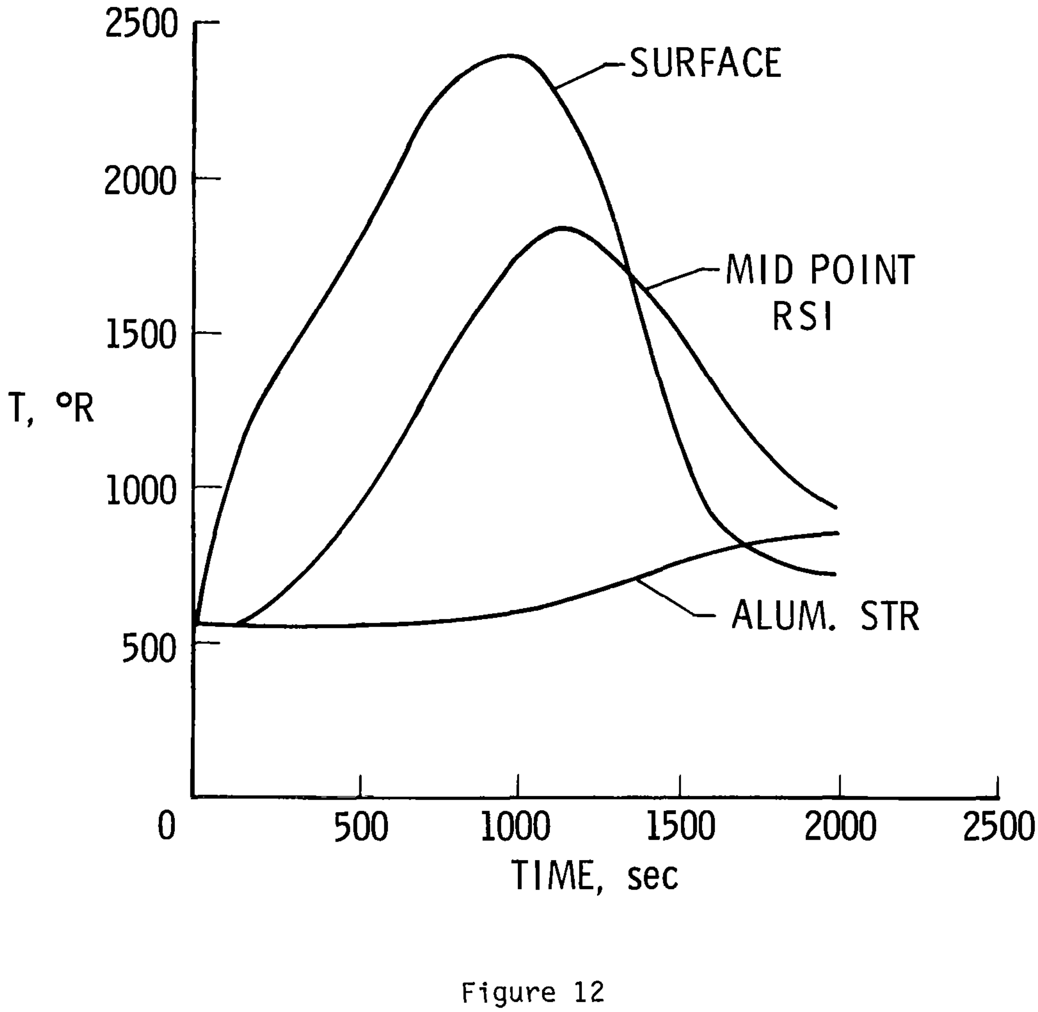


Figure 12

SUMMARY

The effort described in the current paper is directed toward applying the reduced basis method to nonlinear transient thermal analysis. Quite obviously the success of the method depends on the choice of basis vectors used to reduce the system of equations. Initial efforts used a set of 14 basis vectors consisting of modes from a thermal eigenvalue problem where the matrices were evaluated at the initial temperatures. This set of basis vectors gave excellent results for a one-dimensional 14 degrees of freedom thermal problem. Future work will focus on use of additional or alternate basis vectors including modes from the previously described eigenvalue problems, time derivatives of such eigenvectors, and possibly one-dimensional eigenvectors (analogous to the use of beam vibration modes in plate vibration problems). The type and number of basis vectors needed for approximate solutions to more complex problems beginning with two-dimensional nonsymmetric transient thermal problems will be studied.

REFERENCES

1. Noor, Ahmed K.: Recent Advances In Reduction Methods For Nonlinear Problems. Computers & Structures, Vol. 13, pp 31-44, 1981.

2. Marlowe, M. B.; Moore, R. A.; and Whetstone, W. D.: SPAR Thermal Analysis Processors Reference Manual, System Level 16, NASA CR-159162, 1979.