ON THE WAKE OF A DARRIEUS TURBINE

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ABSTRACT

In the paper, the theory and experimental measurements on the aerodynamic decay of a wake from high performance vertical axis wind turbine will be discussed. In the initial experimental study, the wake downstream of a model Darrieus rotor, 28cm diameter and a height of 45.5cm, was measured in the University Boundary Layer Wind Tunnel. The wind turbine was run at the design tip speed ratio of 5.5. It was found that the wake decayed at a slower rate with distance downstream of the turbine, than a wake from a screen with similar troposkein shape and drag force characteristics as the Darrieus rotor. The initial wind tunnel results indicated that the vertical axis wind turbines should be spaced at least forty diameters apart to avoid mutual power depreciation greater than ten per cent.

INTRODUCTION

An important aspect concerning the wake of a Darrieus rotor, is the interaction of such turbines in widespread arrays. For the production of cheap electrical power from the wind, it has been proposed that large wind turbines could be placed in 'clusters' or 'arrays'. An initial study of this problem by R.T. Templin (1974), concluded that if the spacing between individual power units was more than about 30 rotor diameters, the power reduction per machine would not exceed 5 to 10 per cent. However, if the spacing is further reduced, there is a rapid increase in the wake interference. The report treated the rotors as an array of large-scale roughness elements added to the terrain surface that was already rough, and hence, the individual wakes of the turbine rotors were 'smeared out' by the large-scale turbulence shear layer, well within the average spacing between adjacent machines. Since this first paper on the Interaction of windmills in widespread arrays, several other studies have been made, notably by Craford (1975), Lissaman (1977), Newman (1977), Builtjes (1978), Bragg and Schmidt (1978), Faxén (1978), and Builtjes and Smit (1978).

In general, the development of the vortex wake downstream of a Darrieus rotor in the ambient boundary layer, may be divided into three regions. The first region represents the formation and rolling up of the concentrated vortex sheet shed from the rotating aerofoil blades, and then the decay of the coherent wake by vorticity amplification and molecular and turbulent diffusion and, finally, a turbulent wake uncorrelated with the initial disturbance. For a two-bladed Darrieus rotor when the vertical plane of the rotor is normal to the flow direction, the lift force on the blades is zero. For the time period for the rotor to rotate half a revolution, a closed vortex system is shed from the blades and is convected downstream, changing shape due to the mutual interaction, the effect of the ambient turbulent boundary layer, and the ground effect. In the case of the two-bladed Darrieus rotor, a 'packet' of vorticity is shed periodically.

The dominant factors involved in the description of the motion within the Darrieus rotor wake, are the initial properties of the wake, which governs the rate at which the wake entrains the surrounding atmosphere, and the rate and manner in which the entrained fluid is mixed into the wake. For example, it has been well illustrated (see Olsen, Goldburg and Rogers) that the two trailing vortices in the wake of an aircraft wing eventually break down to closed vortex configurations. The wake of the Darrieus rotor starts with a closed vortex structure. On the other hand, the wake behind a wire screen is already very turbulent, and shows very little evidence of any initial coherent vortex structure. It can be hypothesized therefore, that the wake of a horizontal axis wind turbine, which can be compared to that of an aircraft wing with the vortices forming a helix pattern, decays slower with distance downstream than the wake from a vertical axis wind turbine, when both turbines are tested at the same power coefficient and in the same turbulent boundary layer. This fact is important when discussing suitable spacing of turbines.

In the report, a vortex model of the near wake of the Darrieus rotor, will first be discussed, and finally, results of a study made on the simulation of the Darrieus wake by arrays of Troposkein shaped grids will be presented.

NEAR WAKE OF A DARRIEUS TURBINE - VORTEX MODEL

In the analysis, the Darrieus rotor troposkein shape was approximated by straight line elements. Since most of the torque is generated by the 'equatorial' element, the relevant theory will be discussed considering this element. In Fig. 1, a plan view of a typical straight bladed element is shown. The ambient airflow approaches from the right, and at the blade, the total velocity is the vector sum of the blade rotational speed and the free stream velocity corrected by a single inflow or blockage factor. The inflow factor takes into account the linear momentum change due to extraction of energy by the turbine.

Previous theory by Wilson and Lissaman (1974), and also by Base and Russell (1979) showed that the inflow factor (a) varied as the turbine solidity (Ω) and Tip Speed Ratio (TSR), so that it could be suitably represented by the following expression:

\[ a = \frac{\Omega}{2\pi} \sin \theta \quad (\text{For } 4^\circ \leq \theta \leq 6^\circ) \]  \hspace{1cm} (1)

Where \( \theta \) is the actual blade angle. By considering the geometry in Figure 1, then:
\[ \tan \alpha = \frac{U \sin \theta}{(U \cos \theta + U_t)} \]  
(2)

Defining the Tip Speed Ratio and inflow factor by:

\[ \text{TSR} \equiv \frac{\Omega R}{U_0} \]

and

\[ U \equiv U_0(1-a) \]

Then:

\[ \tan \alpha = \frac{(1-a)\sin \theta}{(1-a)\cos \theta + \Omega R} \]  
(3)

For the equatorial element \( x = 1 \), and also substituting Eqn. (1) into Eqn. (3), then the following equation is obtained for the angle of incidence of the aerofoil:

\[ \tan \alpha = \frac{1 - \sigma(\text{TSR}) \sin \theta}{(1 - \sigma(\text{TSR}) \sin \theta) \cos \theta + \text{TSR}} \]  
(4)

The aerofoil will experience a lift force whose magnitude will depend on the incidence of the total velocity vector and this lift force will be related to the circulation \( \Gamma \) developed about the aerofoil.

According to Lanchester's wing theory, line vortices may be considered to exist within the aerofoil. These are the so-called 'bound' vortices. However, the bound vortices for a finite wing cannot end in space, but become free trailing vortices ending downstream by a transverse vortex or the starting vortex. In plan view, the vortex array, in its simplest shape, is a rectangle with the trailing vortices linking the bound vortex to the starting vortex.

The concept of a line vortex is derived from a vortex tube by making the area of cross section of a vortex tube very small, while the circulation strength \( \Gamma \) remains unaltered. Glauert (1946) discussed the line vortex and also the equation for the induced velocity at a point \( P \) of an element of a line vortex at a point \( Q \), which is given by:

\[ dq = \frac{\Gamma ds}{4\pi r} \sin \theta \]  
(5)

where \( \Gamma \) is the circulation defined by \( \int_C \frac{U}{r} dr \) the line integral of the velocity around a closed curve 'c' and for a potential vortex is constant. This is the Biot-Savart Law. The diagram below shows an element of the line vortex where \( r \) is the distance between the field point \( P \) and the element at \( Q \), and \( \theta \) is the angle between the direction of the element and the line joining the element to the point \( P \). The velocity \( dq \) is normal to the plane containing \( r \) and \( ds \), and its 'sense' is the same as that of the circulation \( \Gamma \) of the elemental vortex.

In general, the vorticity has the same geometrical properties as the velocity field. The streamlines of an incompressible fluid therefore, correspond to vortex lines of the rotational flow of which the direction at every point is that of the vorticity vector. An element 'ds' of a line vortex therefore, cannot exist independently and the Biot-Savart Law, for an element of vortex, should only be used as "building bricks" for more complicated closed vortex loops.

Another expression for the "elemental vortex" is given by:

\[ dq = \frac{\pi h}{4\pi r} \]  
(6)

where 'h' is the length of the normal from the field point (P) to a line passing through the vortex line as shown in the figure.

In the vortex model of a straight bladed vertical axis wind turbine, the aerofoil blade was assumed to start at \( \theta = 0 \) as shown in Figure 1, rotating at a speed of \( \Omega \) in a free stream of velocity \( U_0 \) to achieve a particular Tip Speed Ratio. The lift coefficient of the aerofoil at this angle \( \theta = 0 \) is zero, and hence, the first shed vortex from the aerofoil has zero circulation. As the aerofoil rotates, by considering the angle of the local flow incidence to the aerofoil derived in Section 3 and the lift-incidence curve for the aerofoil, the circulation of the shed vortex may be determined as follows. The circulation is related to the aerodynamic lift on the aerofoil element by the Magnus effect equation and:

\[ \text{Lift Force} = \int_0^h \rho w \omega \, dz \]

Hence the lift force is given by the expression (assuming no tip losses)

\[ L = \rho U_0 \omega h \]

The lift coefficient is defined by the expression

\[ C_L = \frac{L}{\frac{1}{2} \rho U_t^2 h} \]

Where \( U_t \) is the total velocity of flow approaching the aerofoil and is the vector sum of the free stream velocity and the aerofoil blade rotation with allowance for the blockage factor. It can be shown that the aerofoil circulation is related to the aerofoil lift coefficient by the following expression:

\[ \Gamma = C_L \rho U_0 \frac{U_t}{2 \rho U_t^2} \]

The variation of lift coefficient with incidence (\( \alpha \)) is also known for the particular aerofoil section. Finally by previous theory (Eqn. 4), the angle of incidence is related to the blade angle (\( \theta \)), solidity \( (s) \), and Tip Speed Ratio (TSR).

\[ \text{and also: } U_t = (1-\sigma(\text{TSR}) \sin \theta) \frac{U_0 \sin \theta}{\sin \alpha} \]  
(7)

Figure 2 shows the initial rectangular vortex pattern and Figure 3 shows the complete vortex model. Further studies on this vortex model are currently in progress.
The main objective of this experiment was to determine the interaction that occurs within a widespread, two-dimensional array of Darrieus wind turbines. Figure 4 shows a sketch of a two-bladed turbine and some of the essential dimensions. The two-bladed turbine blades consisted of extruded aluminum foil section NACA 0012 which had the necessary high-lift, low drag characteristics, and were curved in the shape of a troposkein cantenary. The centre column served as a mounting point for the blades and also housed two low-friction roller bearings. This whole assembly was then mounted on a centre shaft which was braced by wires at the top to reduce turbine vibrations. Two vertical axis turbines used in the study were geometrically identical, being 45.5cm high, 28.0cm diameter, and with an aerofoil chord of 2.5cm. This gave a solidity which is defined as the ratio of blade area to wind turbine area of .36. Since the models used were operating with little load, the Tip Speed Ratio measured was approximately 5.5. The power coefficient for this Tip Speed Ratio and solidity was approximately 0.4. An important problem encountered when working with small models, is that bearing friction will often be greater than the power produced, and therefore, the windmill will not run. The Darrieus turbines used in this study were approaching the lower limit of geometric size. It is most difficult to construct working models of 2.5cm diameter, which would be required in the turbine array studies.

This particular experiment was an attempt to determine the special requirements for adjacent windmills, while allowing for a 10% drop in power. This 10% power loss applies to the last windmill of the array, so that if there were 10 rows of windmills, the windmill in the last row would be producing 10% less power than the windmill in the first row. In order to determine the power loss in a widespread array, it was necessary to model a large array, and then obtain velocity measurements downstream from the turbines. The power extracted by the turbine, is proportional to the velocity cubed, and so by obtaining the drop in velocity caused by the wind turbines, it was possible to calculate the percentage power loss.

In order to perform the experiment in the University Boundary Layer Wind Tunnel, with dimensions 2.4m x 1.8m x 24.0m, it was necessary to use very small models in order to obtain an array of acceptable size. It was not possible to construct working models of 2.5cm diameter, so 2.56cm screen discs were used.

A screen had to be selected, which had a porosity such that it would give the same effect on the momentum of the wind as the wind turbine itself. The following procedure was used to select the screen best suited for the experiment.

The two working models (28cm diameter) of the wind turbines were placed in the Boundary Layer Wind Tunnel, with one directly behind the other. Both were started, and their rotation speeds were measured, as the distance between them was varied from 4 diameters to 32 diameters. The rotational speed was measured with a Strobotac. Since the Tip Speed Ratio was constant for this particular turbine, the rotational speed \( \omega \) was directly proportional to the wind velocity \( U \).

Seven different types of screens, ranging from a very fine mesh to a quite heavy mesh, were then substituted for the upstream windmill. The distance between the screen and the downstream wind turbine was then varied from 4 to 32 diameters. At each position, the rotational speed of the down wind turbine was measured. It was then possible to select the screen which modelled the wind turbine drag, and this screen had a porosity of 0.48. Twenty screens, each with a diameter of 2.5cm and troposkein shape, were then manufactured.

The drag ratio of the screen was measured using a small balance that was constructed and calibrated using a solid disc which had a known drag coefficient of 1.17. The drag coefficient for the wind turbine was estimated from the power coefficient value, and was found to be about 0.8 for the range in which the windmill was operating. The screen was then mounted on the balance, and its coefficient of drag determined to be approximately 0.75.

The correct screen had now been selected, and it had been established by two methods that this screen had the same effect on the wind momentum as a wind turbine. Therefore, if the 2.5cm screen discs were placed in an array, the effect on the wind velocity should be similar to the effect caused by an array of wind turbines.

Four sheets of masonite were placed in the tunnel and secured to the floor, and the array of screens were then placed in the Boundary Layer Wind Tunnel. The masonite had holes drilled every ten centimeters in each direction, so that the size of the array could be varied from 4 diameters to 36 diameters. In all cases, measurements were obtained using an array consisting of five rows perpendicular to the wind flow, and three parallel to the wind flow.

Velocity values were recorded downstream from the array as the distance between adjacent windmills was changed from 4 to 32 diameters. The velocity was measured at the location where the sixth row of the array would have been placed. The vertical height of the velocity measurements was equal to the mean height of the porous screen models. Measurements were taken every ten centimeters along the horizontal, by using transversing gear located in the Boundary Layer Wind Tunnel. By entering the required information into the computer, the data was automatically obtained for a sample time of 60 seconds. A hot wire anemometer was used to obtain the values of the velocity, which were compared to the values of velocity when no models were present. The value for no models present was obtained from the velocity profile which was taken at each location before the array was placed in the wind tunnel. The fact that the profiles taken at each location were similar, was also an indication of the fact...
that the turbulent boundary layer did not deteriorate over the masonite.

Discussion of Wakes

In the analysis of the wake downstream of a Darrieus rotor, two distinct approaches may be used. In the near field, the decay may be considered to be that of an interacting vortex. For a single two-dimensional viscous vortex, the circulation ($\Gamma$) decays according to the following equation:

$$\Gamma = \frac{\Gamma_0}{2\pi} \left[1 - \exp \left(-\frac{r^2}{4\nu}\right)\right]$$

and this would infer that for the vortex wake from a turbine, 'time' was more important in the decay process than distance, although both are related by the convection velocity of the vortex.

Batchelor (1964) discussed the axial flow in trailing line vortices with application to trailing vortices from lifting surfaces. An important result of this study showed that the axial velocity deficit varied as:

$$(\bar{u}_v - U)_{max} = \frac{C_0}{V} \log \frac{x}{2}$$

Where $C_0$ is $(2\pi)^{-1}$ times the vortex circulation at large values of radius. For a turbulent wake downstream of a circular disc, using the mixing length theory (see Schlichting, 1968), the velocity deficit varies as:

$$(\bar{u}_v - U) = \frac{x^2}{3}$$

Grainger (1966) studied three-dimensional vortex flow and approximated the vorticity by an infinite power series of the form:

$$\Omega(r, \xi) = f(\xi) e^{-\xi(r)^2}$$

An interesting solution to the steady flow vorticity equation was then obtained, and in particular, the variation of the vorticity at the centre line of the vortex was shown to vary in the form of the elliptical function of the Weierstrass canonical form. One disadvantage of this type of solution when applied to the study of the decay of a vortex sheet shed from a Darrieus rotor, is that the mutual vortex effects which could also cause vorticity amplification, have been ignored. Another effect also excluded in these studies, which is important when considering the turbine wake decay, is the ground effect. At large distances downstream of the turbine, the vortex wake approaches an established turbulence flow state.

Batchelor (1970), Counihan (1971) and Lemberg (1973), considered turbulent wall wakes behind various shaped obstacles. The main conclusion of these studies, was that for a turbulent wake, the centre line velocity deficit varied inversely as the distance downstream of the turbine ($r^{-1}$). These established results for the decay of wakes, will be used for comparison with measured decay rates in the following section.

Results

Near Wake

A vortex model describing the flow field about a Darrieus rotor was introduced. In the model, the flow field was assumed to consist of a regular intermittent coherent pattern of finite vortex elements, shed from the rotating blades every half a revolution. The circulation strengths of the vortices in the vortex model depended on the turbine parameters, such as Tip Speed Ratio, solidity, aerofoil blade section, and blade position relative to the incident airstream. The vortex model also satisfied Kelvin's theorem concerning continuity of vortices. Figures 2 and 3 show typical vortex arrays used in the model. In this initial study, it was shown that for a two-bladed Darrieus rotor, the vortices shed from the 'equatorial' element of the upstream blade intersected the downstream blade after $150^\circ$ of rotation, when the rotor Tip Speed Ratio had a value of $\pi$. Further computer studies on this vortex model of the Darrieus rotor are planned in the future.

Far Wake

The first tests were to determine the correct screen with a given porosity, to model the Darrieus rotor operating at the design Tip Speed Ratio of 5.5. Two similar turbines were placed in a line parallel to the ambient air flow, so that the downstream turbine was affected by the wake of the upstream turbine. The downstream turbine was then moved downstream in increments of four diameters, and the various rates of rotation of the downstream turbine recorded. The forward turbine was then replaced by troposkein shaped screens of different porosity, and the rate of rotation was again recorded for the downstream wind turbine. Figure 5 shows the test results for the rate of rotation of the downstream turbine when initially a Darrieus turbine was upstream, creating blockage, and also with a screen with a porosity of 0.48, which was used in subsequent experiments placed upstream.

Tests were now made on an array of 'wind turbines' modelled by one inch diameter porous screens. Figure 6 shows the $(3 x 5)$ array of fifteen screens in a square pattern array. The velocity measurements were made, as shown, in the sixth row. Figure 7 shows a typical wake traverse along the sixth row, when the screen array spacing was eight diameters. The maximum velocity deficit was taken as a measure of the wake effect. For example, when the square spacing was eight diameters and the free stream velocity 18.9 ft/sec., the velocity deficit was 4.4 ft/sec. This implied a percentage velocity deficit based on the free stream velocity of 23.4%, and a power deficit of 54.8%. Figures 8 and 9 show the estimated velocity and power deficits respectively, due to the wake effects of the five rows of screens, representing an array of Darrieus turbines for various square spacing, with the measurements being made in the sixth row position. Also in Figure 8, the wake decay is compared with the wake decay predicted by the various theories discussed in the previous section.

Conclusions

The theory and experimental measurements on the wake of a Darrieus turbine and an array of turbines were studied. In general, the development of the turbine wake may be classified into 3 regions: The first region consisted of the for-
mation and rolling up of the concentrated vortex sheet, and then the decay by molecular and turbulent diffusion. Finally, a homogeneous, fully developed turbulent state was achieved far downstream of the turbine. For a two-bladed Darrieus rotor or straight-bladed vertical axis turbine, when the vertical plane of the rotor was normal to the flow direction, the lift force on the blades was zero. At any other angle, the local incidence of the aerofoil blades at the 'equator' for instance, could be calculated by the following expression:

\[
\tan \alpha = \frac{(1 - \frac{\tau S}{\rho} |\sin\beta| - \frac{1}{\rho} |\cos\beta| + \frac{\tau S}{\rho})}{(1 - \frac{\tau S}{\rho} |\sin\beta| |\cos\beta| + \tau S)}
\]

For the time period for the rotor to rotate half a revolution, a closed vortex array was shown to be shed from the aerofoil blades, and was convected downstream, changing shape due to the mutual vortex interaction and the effect of the ambient air flow.

In the experimental study of the wake far downstream of an array of Darrieus wind turbines, represented by troposkein shaped porous screens, the results indicated that the 'turbines' should be spaced at least forty diameters apart to avoid mutual power depreciation greater than ten per cent. This predicted value of suitable spacing of turbines, compares favourably with the analytical estimates of Templin (1974).

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REFERENCES


NOMENCLATURE

a interference or blockage factor for vertical axis wind turbine
b number of windmill blades
c blade chord
c\text{\textsubscript{c}} vortex circulation times (2\pi)^{-1}
C\text{\textsubscript{L}} blade section lift coefficient
d diameter of wind turbine
r radius of a point in a vortex from the centre line or radius of a point on a turbine
h height of blade element of wind turbine
t time
\text{\textsubscript{\text{TSR}}} Tip Speed Ratio (\text{\textsubscript{R}}/\text{\textsubscript{U}})
\text{\textsubscript{\text{U}}} total local airspeed relative to blade
\text{\textsubscript{\text{W}}} ambient (approaching) wind velocity
\text{\text{\alpha}} angle of attack
x distance downstream of turbine
\text{\text{\theta}} angle of blade rotation for vertical shaft windmill
\nu kinematic viscosity
\rho density of fluid
\text{\text{\sigma}} solidity (BC/2\text{\textsubscript{R}})
\Omega blade rotational speed or vorticity
$\Gamma$ circulation
$\Gamma_0$ initial circulation
FIGURE 1  VELOCITY DIAGRAM FOR ELEMENT OF DARRIEUS TURBINE BLADE.
FIGURE 2 A SKETCH OF A RECTANGULAR VORTEX PATTERN SHED FROM A VERTICAL ROTATING AEROFOIL
FIGURE 3  TRAILING RECTANGULAR VORTEX SYSTEM SHED FROM A SINGLE ROTATING AEROFOIL BLADE IN A CROSS FLOW
FIGURE 4  SCHEMATIC VIEW OF TWO BLADED DARRIEUS ROTOR
Figure 5: Effect of an Upwind Disturbance on a Turbine Speed.
FREE STREAM FLOW \( U_\infty \)

Measurements made along this line.

**FIGURE 6** SCHEMATIC OF 'WINDTURBINE' SCREEN ARRAY

- **Schematic Description:**
  - **FREE STREAM FLOW**: Arrows indicating the direction and magnitude of the flow.
  - **Dia. 2.54cm**: Diameter of the screen elements.
  - **\( \Delta x = \Delta y \)**: Indicates square spacing.
  - **\( \delta_y = \delta_x \)**: Likely referring to equal spacing or some other measurement characteristic.

**Details:**
- **Ax:** Abcissa.
- **Ay:** Abscissa.
- **Schematic Array:** Diagram showing the arrangement of screen elements with specified dimensions and spacings.
Turbine Position

Turbine Spacing = 8d

U = Undisturbed Flow Velocity 5.8 m/s

Vertical Velocity Profile \( \frac{U}{U_\infty} = \left( \frac{Z}{6} \right)^{0.15} \)

FIGURE 7 VELOCITY MEASUREMENTS AT SIXTH TURBINE ROW POSITION
FIGURE 8  VELOCITY DEFICIT DUE TO TURBINE WAKES
MEASUREMENTS MADE IN SIXTH ROW
Figure 9  Power Loss Due to "Turbine" Wakes
Measurements Made in Sixth Downstream Row
QUESTIONS AND ANSWERS
T.E. Base

From: T. A. Egolf

Q: Isn't it true that if you reduce your time step interval to a sufficiently small $\Delta t$, that the unsteadiness due to the blade impact with the shed vortex sheet is significantly reduced, and if unsteady aerodynamics were used, the effect would be further reduced?

A: If the time increment is too short, the shed vortices from the blade will not be formed into a coherent structure.

From: T. Sullivan

Q: How do you determine wake position for the near wake model? Are any measurements of this wake position available?

A: The wake is shed from the wing and the subsequent position is determined from the induced flow field of the other vortex elements and are free stream, i.e.

$$u_p = \sum_{i=1}^{N} u_i + u_m$$

I do not know of any published detailed near wake velocity traces or vorticity distributions or flow visualization studies.