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RELAXATION SOLUTION OF THE FULL EULER EQUATIONS

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SUMMARY

A numerical procedure for the relaxation solution of the full steady Euler equations is described. By embedding the Euler system in a second-order surrogate system, central differencing may be used in subsonic regions while retaining matrix forms well suited to iterative solution procedures and convergence acceleration techniques. Hence, this method allows the development of stable, fully-conservative differencing schemes for the solution of quite general inviscid flow problems. Results are presented for both subcritical and shocked, supercritical internal flows. Comparisons are made with a standard time-dependent solution algorithm.

INTRODUCTION

Techniques for the time-accurate solution of the unsteady Euler equations are well known and have relatively firm theoretical basis. The methodology for obtaining steady solutions to the Euler equations is in a more formative state and has undergone considerable evolution in the recent past.

Given steady boundary conditions and assuming that a unique steady solution exists, one may solve the unsteady Euler equations in a non-time-accurate fashion by means of an algorithm with a stability bound in excess of the CFL condition or by using local time stepping. These constitute the simplest form of pseudo-unsteady solution procedure. Other methods fix the total enthalpy at its steady-state value or use enthalpy damping, as proposed by Jameson, Schmidt and Turkel (1981), to accelerate convergence. Ni (1981) and Steger (1981) have developed multiple grid schemes for pseudo-unsteady solution of the time-dependent Euler equations. Approaches which modify the unsteady Euler equations by adding either time-dependent terms or time-dependent equations which enhance convergence have been developed by Essers (1980) and Viviand (1981).

Attempts to deal directly with the steady first-order Euler system meet with immediate difficulties. Only centered difference operators will lead to numerical schemes which are simultaneously stable for the entire system in subsonic flow. However, such schemes lead to ill-conditioned matrices which defeat most iterative

solution procedures.

Semi-direct methods using Newton iteration to avoid this numerical difficulty have been developed by Rizzi (1979) for the homoenthalpic Euler equations, by Rizzi and Sköllermo (1981) for the full potential equation written as a first-order system, and by Shubin, Stephens and Glaz (1981) for the quasi-one-dimensional Euler equations. The large matrices used by such methods may be an impediment when they are generalized to the full Euler system, or to higher dimensions.

An alternative approach to solving the steady Euler equations involves either embedding them in a higher-order steady system or preconditioning their finite difference representations to enhance their compatibility with iterative procedures. Chattot, Guiu-Roux and Laminie (1981) used a variational approach to transform the first-order system representing the potential equation into an equivalent second-order system which was solved by a conjugate gradient algorithm. Lomax (1981), Desideri and Lomax (1981) and Lomax, Pulliam and Jespersen (1981) described a strategy for solving the Euler equations by preconditioning the finite difference equations, choosing a stable relaxation procedure, and accelerating its convergence by a multiple grid technique. Preconditioned relaxation solutions were obtained for two-dimensional subsonic flow by Desideri and Lomax, while Lomax, Pulliam and Jespersen used a multiple grid procedure to solve a quasi-one-dimensional supersonic flow.

In Johnson (1981) we developed an approach whereby a first-order partial differential system is embedded in a second-order surrogate system which may then be solved by means of the same sort of numerical techniques routinely used on the potential equation. We obtained results with the full Euler system for both supersonic and subcritical, subsonic flow and with the transonic small disturbance equations for both subcritical and supercritical flow. The embedding used with the Euler equations assumed the invertability of flux-vector Jacobian matrices and was thus not suitable for use in transonic flow computations. The present paper presents the details of an improved surrogate equation technique which is capable of treating the full steady transonic Euler equations.

SURROGATE EQUATION TECHNIQUE

Given a first-order partial differential system, we embed this system in a second-order surrogate system, apply additional constraints to restrict the solution set of the surrogate, and solve the resulting partial differential problem by means of a conventional iterative procedure. This method maintains the generality of the Euler equations, while allowing the use of the same sort of relaxation procedures developed for the efficient solution of second-order equations.

Consider a first-order system written in conservation law form, such as

$$\left[\frac{\partial}{\partial x}(A\ ) + \frac{\partial}{\partial y}(B\ ) \right] q = 0$$

where q is an n-component vector and A and B are n x n matrices. We embed this system in a second-order surrogate of the form

$$\left[\frac{\partial}{\partial x}(M\ ) + \frac{\partial}{\partial y}(N\ ) \right] \left[\frac{\partial}{\partial x}(A\ ) + \frac{\partial}{\partial y}(B\ ) \right] q = 0$$

This system preserves the conservation law form of the original first-order system. The nature of the second-order partial differential operator is controlled by the choice of the matrices M and N. For example, the choice $M = A^T$, $N = B^T$ symmetrizes the coefficients of the terms of highest order and causes the surrogate system to be elliptic, while the choice $M = A^T$, $N = -B^T$ results in a non-elliptic system which may be upwind differenced. Alternative choices for M and N are possible but will not be discussed here. The problem specification is completed by requiring that, in addition to satisfying the original boundary conditions of the underlying first-order system, the solution to the surrogate system must also satisfy the first-order equations themselves at the boundaries. This is done to insure uniqueness. Additionally, in the case where we employ an elliptic surrogate system to compute a supercritical flow, this boundary treatment allows the introduction of dissipative terms for proper shock capture.

Observe that, by switching the second-order operator from elliptic to hyperbolic type when the flow changes from subsonic to supersonic, it is possible to create a type-dependent differencing scheme for the surrogate system. Such a scheme could provide an alternative means for insuring the correct shock capture and thus relax some of the constraints on the boundary treatment. While initial computations indicate that this may be a viable approach, the results to be presented subsequently were obtained using the choice $M = A^T$, $N = B^T$ everywhere in the domain. Notice that, in this case, the embedding operator is a close relative of the formal transpose of the Euler operator:

$$- \left[A^T \frac{\partial}{\partial x} + B^T \frac{\partial}{\partial y} \right]$$

A discrete representation of this operator has been independently proposed as a preconditioning operator by Desideri and Lomax. Because they operate on the finite difference equations, their approach is in several additional respects distinct from the one discussed here. For example, preconditioning the finite difference equations results in an effective non-compact differencing of the second-order system. Permutation of the resulting matrices is required to restore compact structure and reduce bandwidth. Furthermore, the surrogate equation technique

appears to offer more flexibility in the treatment of boundary conditions than is available with the preconditioned finite difference equation approach.

The second-order partial differential problem, being compatible with iterative techniques, may be solved by a variety of methods. For demonstration purposes, we use fully-conservative differencing together with the well-established successive line relaxation method.

RESULTS

We compute subcritical and shocked, supercritical flows in a straight channel with a 10% half-thick circular arc airfoil mounted on its lower wall. The second-order partial differential problem is illustrated in Fig.1. As physical boundary conditions, we require that flow tangency be satisfied at solid walls, we specify total pressure, total temperature and flow angle at the inlet, and we specify the exit static pressure. We require that the first-order Euler equations be satisfied at each boundary to provide the additional boundary conditions necessary to completely pose the problem and to insure the correct shock capture. As a standard of comparison for the accuracy of the results presented here, we have recomputed all cases using the explicit MacCormack (1969) algorithm.

The subcritical test case had an isentropic inlet Mach number of 0.5. Fig.2 shows the comparative upper and lower surface Mach number distributions. Isomach contours are plotted in Fig.3. The supercritical test case was run at an inlet Mach number of 0.675, producing a shocked but unchoked flow. The comparative surface Mach number distributions and isomachs are shown in Figs.4 and 5, respectively. The sonic line is dashed in the isomach plots. Comparison of the results of the surrogate equation algorithm with those of the MacCormack algorithm is encouraging. Minor discrepancies may be attributed, in part, to differences in the truncation error of the two algorithms or to the lack of complete annihilation of low frequency error modes.

Representative convergence histories are shown in Fig.6. The surrogate equation algorithm, using successive line overrelaxation on the second-order system, converges more rapidly than the MacCormack algorithm, using local time stepping at 0.9 of the local CFL limit. Since the residuals are defined quite differently for the two methods, the error measure used for this comparison is the correction to the vector of conservation variables. Consequently, the relative convergence rates are more significant than the indicated levels of error. Having demonstrated the capability of the surrogate equation technique to solve the full Euler equations by relaxation, it should prove relatively straightforward to further accelerate

convergence.

While all results presented here are two-dimensional, the extension of the surrogate equation technique to three dimensions presents no essential difficulties. Furthermore, the use of this technique as an inviscid component of a zonal procedure for the iterative solution of the steady Navier-Stokes equations appears feasible.

CONCLUSIONS

We may obtain a solution to the full steady transonic Euler equations by using a surrogate second-order system together with the original Euler physical boundary conditions and additional constraints obtained from the first-order Euler system.

This surrogate equation technique provides a means for formulating problems involving the full steady Euler equations in such a way as to allow the use of stable, fully-conservative differencing and relaxation solution procedures. Hence, we may solve either irrotational or rotational flow problems across the entire spectrum of subsonic, transonic and supersonic conditions without resort either to derived dependent variables, semi-direct methods, or to an unsteady formulation.

Embedding the Euler equations in a second-order system allows the application of the many convergence acceleration techniques which have been developed for other second-order systems. Thus, the surrogate equation technique provides an opportunity for the construction of fast and efficient numerical procedures for the solution of the full steady Euler equations.

ACKNOWLEDGEMENT

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FIGURES

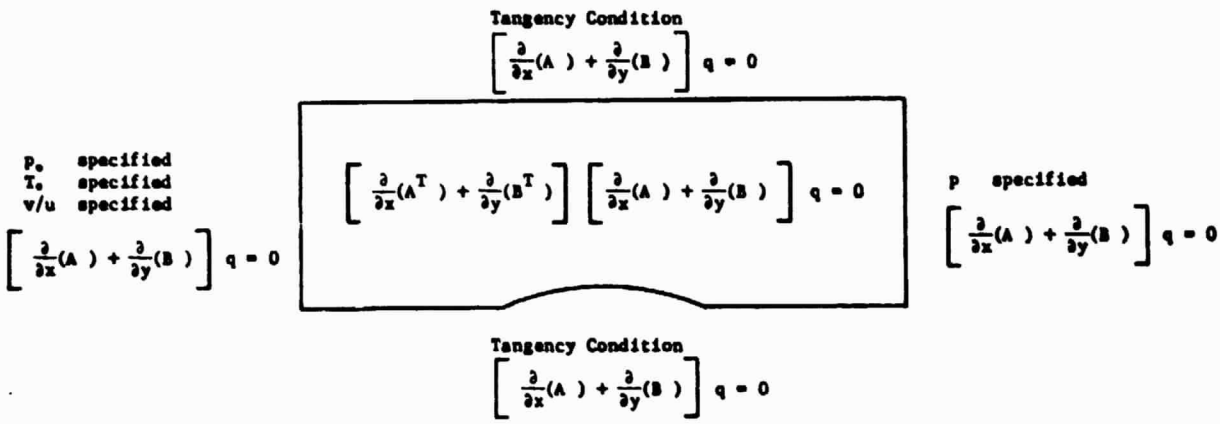

Fig. 1 Partial Differential Problem

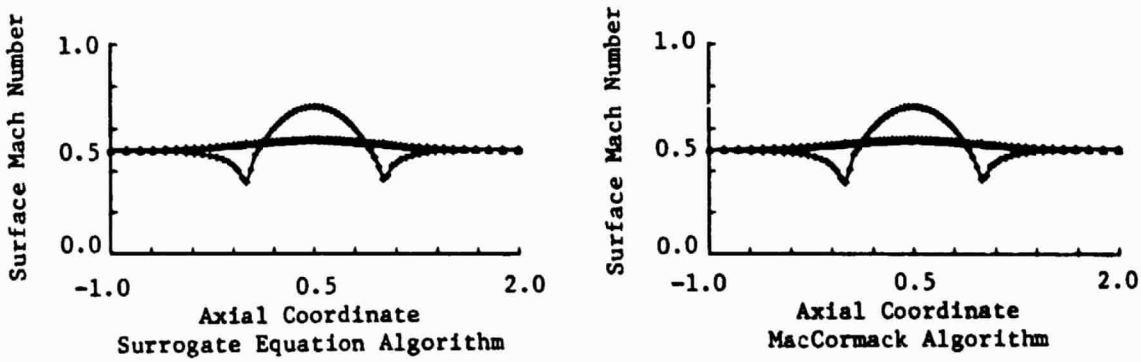

Fig. 2 Subcritical Mach Number Distributions

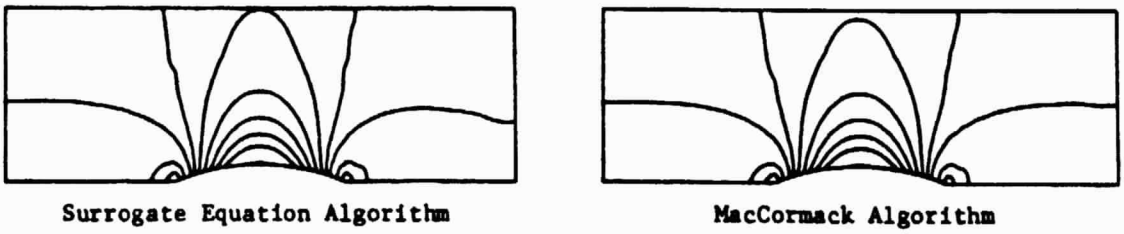

Fig. 3 Subcritical Isomach Contours

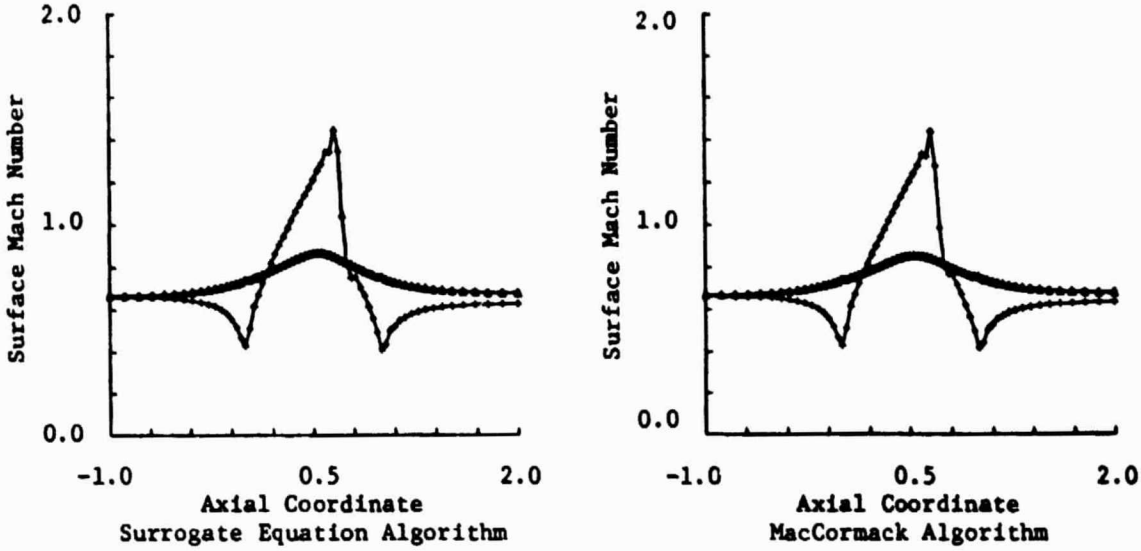

Fig. 4 Supercritical Mach Number Distributions

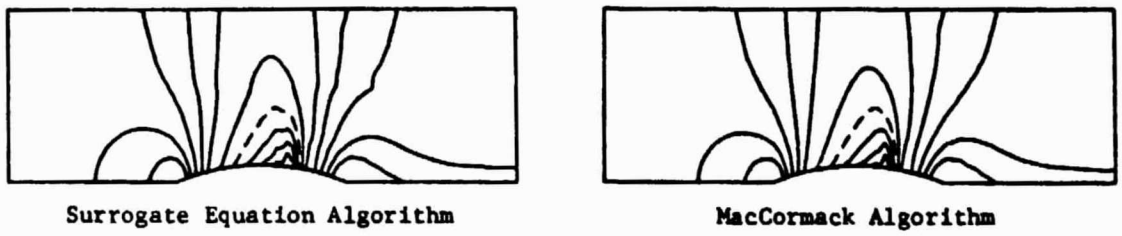

Fig. 5 Supercritical Isomach Contours

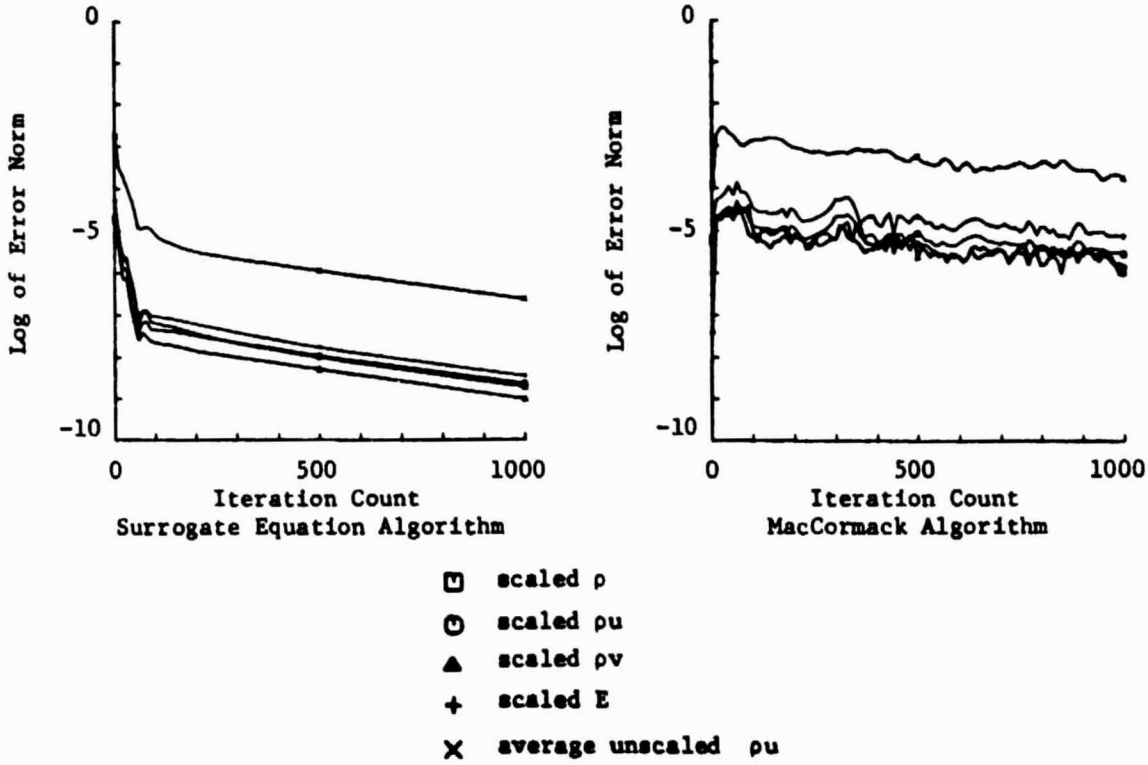

Fig. 6 Convergence Histories