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AN IMPROVED EMPIRICAL MODEL FOR DIVERSITY GAIN ON EARTH-SPACE PROPAGATION PATHS

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ABSTRACT

An empirical model has been generated to estimate diversity gain on earth-space propagation paths as a function of earth terminal separation distance, link frequency, elevation angle, and angle between the baseline and the path azimuth. This analysis utilized 34 diversity experiments which have been conducted in Canada, England, Japan, and the United States during the past decade. The resulting model reproduces the entire experimental data set with an RMS error of 0.73 dB.

The separation distance dominates the dependence of the diversity gain. The dependence on link frequency is small but significant. No identifiable dependence on baseline orientation was found.
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I. INTRODUCTION

Path diversity has been found to be a useful technique in some cases, necessary technique to provide acceptable reliability on earth-space communication links operating above 10 GHz. This technique takes advantage of the tendency of rainfall to occur in limited spatial regions in order to overcome the deleterious effects of rainfall attenuation. This is accomplished by placing two earth terminals such that the probability of rainfall attenuation occurring simultaneously on both paths is significantly less than the probability of rainfall attenuation occurring on either individual path. Although the redundant hardware normally associated with a single earth terminal may be removed to the remote diversity site in order to hold costs down, the costs of the additional site, the connecting link and the switching capability must be weighed against the resulting increase in link reliability.

Little progress has been made in the development of theoretical models applicable to this method. Therefore, a need still remains to establish an empirical model which will provide the link designer with sufficient information for design purposes. Such a model was developed earlier [1]; this model was based on the experimental results available in the early 1970's. This model established the dependence of diversity gain on the separation distance between the earth terminals. However, the question of the dependence of diversity gain on link frequency remained. And, furthermore, this early model predicted a fixed diversity gain for extremely deep fades which has been found to be incorrect in subsequent experiments.

The publication of results of a number of diversity experiments have now made it possible to reexamine and improve this empirical model. The results of thirty-four experiments were used in this study to establish the dependence of diversity gain on separation distance, link frequency, elevation angle, and the angle between the baseline and the path azimuth. The dependence on separation distance is strongest.
with the other dependences being weaker but significant. The model was also generalized to remove the constraint that the diversity gain increase as rapidly as the attenuation for deep attenuations.

II. DIVERSITY GAIN

Diversity gain, \( G_D \), is a measure of the reduction in attenuation exceeded for a fixed percentage of time as a result of selecting the larger of the two received signals at two separated earth terminals on an instantaneous basis. In order to determine the diversity gain it is first necessary to find the average single site attenuation associated with each fixed percentage of time as shown in Figure 1. Diversity gain is then the difference between the average single site attenuation and the diversity (joint) attenuation for each fixed percentage of time. Since the average single site attenuation distribution establishes a unique relationship between attenuation and percentage time, the diversity gain may be viewed as either a function of average single site attenuation or of percentage time. The former is generally found to be more useful and will be utilized in the following.

Diversity improvement, \( I_D \), is another parameter which may be useful for describing diversity performance. It can be defined as the ratio of the average single site percentage time to the diversity percentage time for fixed attenuation as shown in Figure 1. However, this approach suffers from two disadvantages. First, diversity improvement cannot be established for very deep fades, i.e., small percentages of times, due to experimental limitations. This is a consequence of the fact that, for very deep fade levels, the diversity attenuation distribution usually falls well below the temporal resolving capability of the experiment. Secondly, this parameter is based on measurements which occur over markedly different intervals of time. Thus, for experiments of limited duration, the uncertainties in these levels are quite different. This property leads to randomness which is not as evident in the use of diversity gain. Theoretically, of course, these two approaches are equivalent given ideal data sets.
Figure 1. Hypothetical example showing the definitions of diversity gain and diversity improvement.
III. EXPERIMENTAL DATA BASE

The results of thirty-four diversity experiments were utilized for the analysis presented below. These experiments are tabulated in Table 1 along with the references from which the data were obtained. The experiments utilized were restricted to direct satellite beacon measurements and radiometric emission measurements where no frequency scaling was utilized. The frequencies associated with these experiments ranged from 11.6 to 30 GHz, and the earth terminal separation distances ranged from 1.7 to 46.9 km. The conventional azimuth, $\alpha$, and elevation, $\beta$, angles of the propagation paths are also listed in Table 1 along with the orientation, $\gamma$, of the baseline separating the earth terminals. The orientation of the baseline is measured from true north as is conventional for the azimuth angle of the path. The orientation of the baseline relative to the propagation path, $\Delta$, will be defined as the difference between $\alpha$ and $\gamma$ taken such that

$$0 \leq \Delta \leq 90^\circ.$$  (1)

These definitions are shown in Figure 2.

The diversity gains, $G_D$, associated with these experiments were extracted for average single site attenuation values of 2, 3, 4, ..., dB up to the highest value available for each experiment. This procedure yielded 312 values of diversity gain. These values along with the parameters shown in Table 1 were then stored in a disk file to permit convenient processing.
<table>
<thead>
<tr>
<th>Location</th>
<th>Ref.</th>
<th>Frequency, f</th>
<th>Separation Distance, d</th>
<th>Azimuth, α</th>
<th>Elevation, β</th>
<th>Baseline Orientation, γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columbus, OH</td>
<td>2</td>
<td>15.3 GHz</td>
<td>4.0 km</td>
<td>210°</td>
<td>38°</td>
<td>164°</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td>3</td>
<td>20.0</td>
<td>8.3</td>
<td>210°</td>
<td>38°</td>
<td>159</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td>3</td>
<td>20.0</td>
<td>13.2</td>
<td>197°</td>
<td>40°</td>
<td>151</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td>3</td>
<td>20.0</td>
<td>13.8</td>
<td>197°</td>
<td>40°</td>
<td>33</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td>3</td>
<td>20.0</td>
<td>14.0</td>
<td>197°</td>
<td>40°</td>
<td>90</td>
</tr>
<tr>
<td>Austin, TX</td>
<td>4</td>
<td>30.0</td>
<td>30.0</td>
<td>172°</td>
<td>55°</td>
<td>151</td>
</tr>
<tr>
<td>Blacksburg, VA</td>
<td>5</td>
<td>11.6</td>
<td>11.0</td>
<td>106°</td>
<td>11°</td>
<td>160</td>
</tr>
<tr>
<td>Yokohama, Japan</td>
<td>6</td>
<td>19.5</td>
<td>19.0</td>
<td>190°</td>
<td>48°</td>
<td>163</td>
</tr>
</tbody>
</table>

**SATELLITE BEACON EXPERIMENTS**

**RADIOMETER EXPERIMENTS**
### TABLE 1 (Contd.)

**DIVERSITY EXPERIMENTS**

<table>
<thead>
<tr>
<th>Location</th>
<th>Ref.</th>
<th>Frequency, f</th>
<th>Separation Distance, d</th>
<th>Azimuth, α</th>
<th>Elevation, β</th>
<th>Baseline Orientation, γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quebec, Canada</td>
<td>13</td>
<td>13.0 GHz</td>
<td>18.0 km</td>
<td>122°</td>
<td>19°</td>
<td>11°</td>
</tr>
<tr>
<td>Ontario, Canada</td>
<td>13</td>
<td>13.0</td>
<td>21.6</td>
<td>116</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>Atlanta, GA</td>
<td>14</td>
<td>17.8</td>
<td>15.8</td>
<td>228</td>
<td>38</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>31.0</td>
<td>228</td>
<td>38</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>46.9</td>
<td>228</td>
<td>38</td>
<td>146</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>14</td>
<td>17.8</td>
<td>33.1</td>
<td>197</td>
<td>43</td>
<td>86</td>
</tr>
<tr>
<td>Austin, TX</td>
<td>15</td>
<td>13.6</td>
<td>15.4</td>
<td>158</td>
<td>52</td>
<td>59</td>
</tr>
<tr>
<td>Etam, W.VA.</td>
<td>16</td>
<td>11.6</td>
<td>35.0</td>
<td>114</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>
Figure 2. Geometry of path diversity system.
IV. DATA ANALYSIS

As in the earlier analysis, [1], a nonlinear regression was performed to find the coefficients, a and b, which provided the least mean square error fit between the experimental data set and the analytic form,

\[ G_d = a(1-e^{-bd}) \]  

(2)

where \( d \) is expressed in km. This analysis was performed as a function of average single site attenuation, \( A \), so that two families of coefficients, \( a(A) \) and \( b(A) \), were obtained for integer values of \( A \) from 2 to 10 dB. The resulting coefficients are shown in Figures 3 and 4. The coefficients shown in these figures are actually the final result of the iterative application of the procedure described in this section. The process was, however, very stable with only minor changes occurring after the first iteration.

These coefficients were then modeled by the following analytical expressions:

\[ a = 0.64A - 1.6 \left(1-e^{-0.11A}\right) \]  

(3)

\[ b = 0.585 \left(1-e^{-0.098A}\right) \]  

(4)

where \( A \) is expressed in dB and the numerical constants were determined by a regression analysis. These expressions are of the same forms as those used in the earlier model except that the leading coefficient in the expression for \( a \) was not forced to be unity. This eliminates the problem in the earlier model associated with deep fades.

Having found the dependence, \( G_d \), of the diversity gain on single site fade depth, the experimental data set was normalized to remove
Figure 3. Coefficient $a$ versus average single site fade depth, $A$. 
Figure 4. Coefficient $b$ versus average single site fade depth, $A$. 
its dependence on separation distance, \( d \), and average single site fade depth, \( A \). The resulting values,

\[
G_f = \frac{G_d}{G_d + d}
\]  

shown in Figure 5, were then used to establish the coefficients in the following analytic expression by regression analysis:

\[
G_f = 1.64 e^{-0.025f}
\]  

where \( f \) is expressed in GHz. This expression shows moderate frequency dependence, declining with increasing frequency. Equation (6) is plotted in Figure 5 along with the normalized data.

Next, the frequency dependence was also normalized out of the original data set and the dependence on elevation angle, \( \beta \), was sought:

\[
G_\beta = \frac{G_d}{G_d G_f}
\]  

The values of \( G_\beta \) along with the model resulting from a linear regression analysis

\[
G_\beta = 0.00492 \beta + 0.834
\]  

are shown in Figure 6. \( \beta \) is expressed in degrees. This regression indicates a weak increasing dependence on elevation angle.

Finally, this process was repeated seeking the dependence on the orientation of the baseline relative to the propagation path,

\[
G_\Delta = \frac{G_d}{G_d G_f G_\beta}
\]  

A very weak increasing linear dependence was found as shown in Figure 7:
Figure 5. Normalized diversity gain, $G_f$, versus link frequency, $f$. 
Figure 6. Normalized diversity gain, $G_\beta$, versus elevation angle, $\beta$. 
Figure 7. Normalized diversity gain, $G_{\Delta}$, versus orientation $\Delta$, of the baseline relative to the propagation path.
\[ G_\Delta = 0.00177 \Delta + 0.887 \]  

(10)

where \( \Delta \) is expressed in degrees.

The dependence of the diversity gain on the orientation of the baseline itself was also examined. However, no discernible dependence was found.

The entire preceding analysis was performed iteratively. The changes found during the second iteration were quite small, and those found during the third iteration were negligible.

Finally, the resulting model,

\[ G_D = G_d G_f G_\beta G_\Delta \]  

(11)

was compared with the original data set. The resulting RMS error was 0.73 dB. This value indicates that the model produces a satisfactory characterization of diversity gain over a wide range of separation distances, frequencies, elevation angles, and relative baseline orientations.

V. CONCLUSIONS

Thirty-four diversity experiments were studied to establish an empirical model for the prediction of diversity gain. The resulting model characterizes diversity gain as a function of average single site fade depth, separation distance, link frequency, path elevation angle, and orientation of the baseline relative to the propagation path. The model reproduces the original data set with an RMS error of 0.73 dB.
VI. RECOMMENDATIONS

It is recommended that the improved empirical model presented here be used for the calculation of diversity gain on earth-space propagation paths.
REFERENCES


15. W. J. Vogel, Private Communications.