NASA TM- 84481

NASA Technical Memorandum 84481

NASA-TM-84481 19820020185

SHOCK-FITTED EULER SOLUTIONS TO SHOCK-VORTEX INTERACTIONS

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

Manuel D. Salas
Langley Research Center, Hampton, Virginia

Thomas A. Zang
Department of Mathematics and Computer Science
College of William and Mary, Williamsburg, Virginia

M. Yousuff Hussaini
Institute for Computer Applications in Science and Engineering
Langley Research Center, Hampton, Virginia

MAY 1982

LIBRARY COPY

JUN 4 1982

LANGLEY RESEARCH CENTER
LIBRARY, NASA
HAMPTON, VIRGINIA

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

SUMMARY

The interaction of a planar shock wave with one or more vortexes is computed using a pseudospectral method and a finite difference method. The paper emphasizes the development of the spectral method. In both methods the shock wave is fitted as a boundary of the computational domain. The results show very good agreement between both computational methods. The spectral method is, however, restricted to smaller time steps and requires use of filtering techniques.

INTRODUCTION

In their recent paper, Pao and Salas (ref. 1) presented a finite difference solution to the Euler equations governing the phenomena of shock wave interaction with an isolated vortex. Their study emphasized the acoustic aspects of the problem. Zang, Hussaini, and Bushnell (ref. 2) extended this numerical approach to the problem of shock wave interaction with entropy spottiness (or hot spots), plane waves, etc. The present study is a continuation of this effort which concentrates on gaining insight into the nonlinear dynamics of the transient processes involved in the passage of a shock wave over a single vortex, a vortex street, and a hot spot. The governing equations are solved by two different numerical methods: the well known second-order, finite difference method originated by MacCormack, and a pseudospectral method

N82-28061#

of high accuracy. The finite difference solutions are calculated on a very fine grid and are used for comparison with the solutions obtained with the spectral method. The main emphasis of the paper is on developing a viable spectral technique.

Spectral methods have been demonstrated (refs. 3,4) to be powerful alternatives to finite difference methods for the numerical solution of smooth flows. Recently, the works of Gottlieb, Lustman, and Orszag (ref. 5) and Zang and Hussaini (ref. 6) have shown their applicability to one-dimensional compressible flows with shocks. However, for multi-dimensional flows in the presence of shock waves, spectral methods have not been developed to the same level of proficiency as finite difference methods. The present paper is thus an attempt to break new ground in this area.

STATEMENT OF THE PROBLEM

The physical problem that we model, the shock-vortex interaction problem, corresponds to an infinite, initially planar normal shock wave moving from left to right into a downstream region containing a flow field representative of one or more vortexes, or a hot spot. Since the shock wave travels at a speed greater than the local downstream speed of sound, the flow downstream of the shock wave remains unaware of the oncoming shock until it is overtaken by it. In order to model the interaction of the shock wave with some given flow field ahead of it, it is only necessary to compute the flow field upstream of the shock. The computational domain, therefore, consists of the region between some left boundary, judiciously chosen such that it will be far from the interaction region, and the shock wave front itself. A coordinate transformation is used to map the infinite lateral extent onto a finite

domain, thus concentrating the available grid points in the vicinity of the interaction.

If the relative shock Mach number, M_s, is sufficiently high ($M_s > 2.08$), the flow upstream of the shock remains supersonic and signals from the shock-vortex interaction will not reach the left boundary. In this case, the left boundary corresponds to a supersonic inflow, and all dependent variables can be prescribed on it. Most of the cases to be studied here will be of this type. However, if the relative shock Mach number is low, then the first signal from the interaction will reach the left boundary at some finite time t_o. For times greater than t_o, special procedures are required to prevent contamination of the solution in the region of interest by spurious wave reflections from the left boundary. Two approaches are available to achieve this end. Either radiation type boundary conditions can be applied at the left boundary, or the boundary can be allowed to move in such a way as to insure that it is inaccessible to the initial signal emanating from the interaction region. Both approaches have been tried for this problem and in a comparison with the experiment of reference 7 we will use the first method for illustration.

On the right, the computational region is bounded by the shock wave. Downstream of the shock the flow field is given analytically. The flow field immediately upstream of the shock, as well as the shape and velocity of the shock, are evaluated such that the Rankine-Hugoniot jump conditions and the compatibility condition reaching the shock wave from the upstream side are simultaneously satisfied.

GOVERNING EQUATIONS

The unsteady, two-dimensional, compressible, Euler equations are written in the form,

$$Q_t + \underline{A}'Q_x + \underline{B}'Q_y = 0 \tag{1}$$

where

$$Q = \left[P, u, v, S\right]^T \tag{2}$$

$$\underline{A}' = \begin{bmatrix} u & \gamma & 0 & 0 \\ \frac{a^2}{\gamma} & u & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{bmatrix} \tag{3}$$

and

$$\underline{B}' = \begin{bmatrix} v & 0 & \gamma & 0 \\ 0 & v & 0 & 0 \\ \frac{a^2}{\gamma} & 0 & v & 0 \\ 0 & 0 & 0 & v \end{bmatrix} \tag{4}$$

with subscripts denoting partial derivatives with respect to the independent variables. The natural logarithm of the pressure, the speed of sound, and the entropy are represented by P, a, and S, respectively, and γ is the isentropic exponent. The velocity in the x and y directions are u and v, respectively. All variables are normalized with respect to reference conditions at downstream infinity, as in reference 1.

The physical domain is defined by

$$h(t) \leq x \leq x_s(y,t) \tag{5}$$

$$-\infty \leq y \leq \infty \tag{6}$$

where h and x_s are the abscissas of the left boundary and the shock, respectively.

The physical domain is mapped onto a computational domain by the following transformation,

$$X = \frac{x - h(t)}{x_s(y,t) - h(t)} \tag{7}$$

$$Y = \frac{\tanh(\alpha y) + 1}{2} \tag{8}$$

$$T = t \tag{9}$$

The computational domain is thus

$$0 \leq X \leq 1 \tag{10}$$

$$0 \leq Y \leq 1 \tag{11}$$

and the governing equations (1-4) are rewritten as

$$Q_T + \underline{A}Q_X + \underline{B}Q_Y = 0 \tag{12}$$

where

$$\underline{A} = \begin{bmatrix} U & \gamma X_x & \gamma X_y & 0 \\ \frac{a^2 X_x}{\gamma} & U & 0 & 0 \\ \frac{a^2 X_y}{\gamma} & 0 & U & 0 \\ 0 & 0 & 0 & U \end{bmatrix} \tag{13}$$

$$\underline{B} = \begin{bmatrix} V & \gamma Y_x & \gamma Y_y & 0 \\ \frac{a^2 Y_x}{\gamma} & V & 0 & 0 \\ \frac{a^2 Y_y}{\gamma} & 0 & V & 0 \\ 0 & 0 & 0 & V \end{bmatrix} \tag{14}$$

here U and V are the contravariant velocity components defined by

$$U = X_t + uX_x + vX_y \tag{15}$$

$$V = Y_t + uY_x + vY_y \tag{16}$$

SOLUTION TECHNIQUE

Let k denote the time level and let Δt be the time step increment, the time discretization of equation (12) is then as follows:

$$Q^* = \left[1 - \Delta t\ (L_X + L_Y)\right] Q^k \tag{17}$$

$$Q^{k+1} = \frac{1}{2}\left[Q^k + (1 - \Delta t\ (L_X + L_Y))Q^*\right] \tag{18}$$

where the spatial operators L_x and L_y represent approximations to $\underline{A}\partial_X$ and $\underline{B}\partial_Y$, respectively. In the finite difference MacCormack method, these operators are evaluated as two points forward and two points backward differences in the predictor (Eq. (17)) and corrector (Eq. (18)) levels, respectively. In the pseudospectral method studied here, the solution Q is first expanded as a double Chebyshev series,

$$Q(X,Y,T) = \sum_{p=0}^{M} \sum_{q=0}^{N} Q_{pq}(T)\ \tau_p\ (\xi)\ \tau_q\ (\eta) \tag{19}$$

where

$$\xi = 2X - 1 \tag{20}$$

$$\eta = 2Y - 1 \tag{21}$$

and τ_p and τ_q are the Chebyshev polynomials of degree p and q defined by

$$\tau_p(\xi) = \cos(p \cos^{-1}(\xi)) \tag{22}$$

$$\tau_q(\eta) = \cos(q \cos^{-1}(\eta)) \tag{23}$$

The $(M+1)\cdot(N+1)$ Chebyshev coefficients Q_{pq} are such that at (X_m, Y_n, T),

$$Q(X_m, Y_n, T) = \sum_{p=0}^{M} \sum_{q=0}^{N} Q_{pq}(T) \cos\left(\frac{m\pi}{M} p\right) \cos\left(\frac{n\pi}{N} q\right) \tag{24}$$

and

$$X_m = \frac{1}{2} (\xi_m + 1) = \frac{1}{2} (\cos\left(\frac{\pi m}{M}\right) + 1) \tag{25}$$

$$Y_n = \frac{1}{2} (\eta_n + 1) = \frac{1}{2} (\cos\left(\frac{\pi n}{N}\right) + 1) \tag{26}$$

The derivatives appearing in the operators L_X and L_Y are then evaluated as

$$Q_X = 2 \sum_{p=0}^{M} \sum_{q=0}^{N} Q_{pq} \frac{\partial \tau_p}{\partial \xi} \tau_q \tag{27}$$

The last relation can be expressed by another Chebyshev series

$$Q_X = 2 \sum_{p=0}^{M} \sum_{q=0}^{N} \tilde{Q}_{pq} \tau_p \tau_q \tag{28}$$

where

$$\tilde{Q}_{pq} = \frac{2}{c_p} \sum_{\substack{m=p+1 \\ m+p \text{ odd}}}^{M} mQ_{mq} \tag{29}$$

and

$$c_0 = 2 \tag{30}$$

$$c_p = 1, \; p > 0 \tag{31}$$

The Q_Y derivative is evaluated in a similar fashion.

Assuming the characteristic wave speed to be λ, the time step for the finite difference method with uniformally spaced N grid points over a unit interval is $\Delta t < \frac{1}{N\lambda}$, while for the spectral method with N modes the time step is $\Delta t < \frac{4}{N^2\lambda}$. This strong restriction on the time step is at present one of the main disadvantages of the spectral approach. Another disadvantage is its tendency to develop spurious oscillations. The exact cause of these oscillations is presently unknown, but its instantaneous spreading over the field must be caused by the global nature of the spectral method. Various filtering techniques are, however, available in order to recover the solution. The results presented here were obtained by applying a von Hann Window filter, see reference 6 for details, at every 160 time steps. A comprehensive study of various filters, as well as of the effect of filtering on the solution is under preparation.

The evaluation of the shock wave shape and velocity followed the same procedure described in Ref. 1, except that in the spectral formulation, the derivatives that must be evaluated on the downstream

side of the shock are expressed as Chebyshev expansions as given by Eq. (28).

At the left boundary, all variables were specified for supersonic inflow. For the case of subsonic inflow, the two velocity components and the entropy were specified, while the pressure was computed from a quasi-one-dimensional characteristic as described in Ref. 8.

RESULTS

Perhaps the simplest interaction to consider is that of a planar shock wave with a hot spot, as shown in figure 1. The flow field downstream of the shock wave situated at x=0 at t=0 is taken as a quiescent field whose temperature, σ, distribution is given by

$$\sigma = \frac{\kappa}{2\pi} \exp\left(-((x-x_o)^2 + (y-y_o)^2)/\ 2r^2\right) \tag{32}$$

where for this case $\kappa = 0.25$, $r = .125$, $x_o = 0.5$, $y_o = 0.0$, and $M_s = 3$ at $t = 0$. The flow field and vorticity distribution obtained by the finite difference method and the spectral method after the shock wave has passed over the hot spot is shown in figure 2. The finite difference solution presented here, and in all cases that follow, was obtained with 75 mesh points in the X direction and 60 mesh points in the Y direction. The spectral solutions were all obtained with 32 collocation points in the X direction and 16 collocation points in the Y direction. Qualitatively and quantitatively, there is very little difference between the two solutions. Both show the two counterrotating vortexes downstream of the shock, typical of this interaction.

Figure 3 shows the velocity field for a single vortex about to interact with a shock wave of the same initial strength as in the

previous case. The upstream conditions here are obtained by assuming a constant density field, calculating the velocity from the stream function,

$$\psi = \frac{\kappa}{2\pi} \log\sqrt{r^2+(x-x_o)^2+ (y-y_o)^2}, \tag{33}$$

the pressure from Bernoulli's relation, and the temperature from the equation of state. For the case shown in figure 3, the circulation $\kappa=2$ and the softening scale $r=0.1$. This model approaches an idealized incompressible point vortex at large distances from its center but is much smoother near the center. Figure 4 shows the resulting pressure field at two different time steps. At the later time the shock wave has passed over the vortex. For this case, the spectral method seems to be resolving the pressure field more accurately than the finite difference method. Overall, the results are qualitatively very similar. Finally, figures 5 and 6 show the results for the interaction with a Karman vortex street simulating the conditions of the experiment reported in Ref. 7. For this case, the stream function representing the vortex field is given by

$$\psi = \frac{\kappa}{2\pi} \log(A/B) \tag{34}$$

where

$$A = \cosh\left(\frac{2\pi}{c}\sqrt{r^2 + (y - \tfrac{1}{2}b)^2}\right) - \cos\left(\frac{2\pi}{c}(x - \tfrac{1}{2}c)\right) \tag{35}$$

$$B = \cosh\left(\frac{2\pi}{c}\sqrt{r^2 + (y + \tfrac{1}{2}b)^2}\right) - \cos\left(\frac{2\pi}{c}(x + \tfrac{1}{2}c)\right) \tag{36}$$

To match the experiment, the circulation, core radius, shock Mach number and vortex separation parameters were determined as $\kappa = 0.186$, $r = 0.1$, $M_s = 1.3$, $c = .33$, and $b = 0.17$. For this calculation, the

inflow Mach number was subsonic and radiation boundary conditions were applied at the left boundary. The results shown in figure 6 are in agreement with the experimentally observed (ref. 7) longitudinal compression and lateral elongation of the vortex field after passage through the shock.

CONCLUSION

A pseudospectral technique has been presented for the shock-vortex interaction problem. The solutions obtained are in good agreement with those predicted by a standard finite difference method. The proper treatment of the shock wave, which is one of the major problems with any numerical technique, was resolved by fitting the shock wave as a boundary of the computational plane. The spectral technique seems capable of resolving features of the flow field which are difficult to resolve by finite difference. The spectral method is, however, presently restricted to very small time steps, and it requires the use of filtering techniques which need further investigation.

REFERENCES

1. Pao, S. P. and Salas, M. D.: A Numerical Study of Two-Dimensional Shock Vortex Interaction. AIAA Paper 81-1205. Presented at the AIAA 14th Fluid and Plasma Dynamics Conference, June 23-25, 1981, Palo Alto, California.

2. Zang, T. A.; Hussaini, M. Y.; and Bushnell, D. M.: Numerical Computations of Turbulence Amplification in Shock Wave Interactions. AIAA Paper 82-0293. Presented at the AIAA 20th Aerospace Science Meeting, January 11-13, 1982, Orlando, Florida.

3. Orszag, S. A. and Kells, L. C.: Transition to Turbulence in Plain Poiseuille Flow and Plain Couette Flow. Journal of Fluid Mechanics, vol. 96, 1980, pp. 159-205.

4. Wray, A. and Hussaini, M. Y.: Numerical Experiments in Boundary-Layer Stability. AIAA Paper 80-0275. Presented at the AIAA 18th Aerospace Science Meeting, January 14-16, 1980, Pasadena, California.

5. Gottlieb, D.; Lustman, L.; and Orszag, S. A.: Spectral Calculations of One-Dimensional Inviscid Compressible Flows. SIAM J. on Sci. and Statis. Computing, vol. 2, no. 3, Sept. 1981, pp. 296-310.

6. Zang, T. A. and Hussaini, M. Y.: Mixed Spectral-Finite Difference Approximations for Slightly Viscous Flows. Proceedings of the 7th International Conference on Numerical Methods in Fluid Dynamics, Stanford 1980, pp. 461-466.

7. Dosanjh, D. S. and Week, T. M.: Interaction of a Starting Vortex as well as a Vortex Street with a Traveling Shock Wave. AIAA J., vol. 13, 1965, pp. 216-223.

8. Oliger, J., and Sundström, A.: Theoretical and Practical Aspects of Some Initial Boundary Value Problems in Fluid Dynamics. SIAM J. Appl. Math., vol. 35, no. 3, Nov. 1978, pp. 419-446.

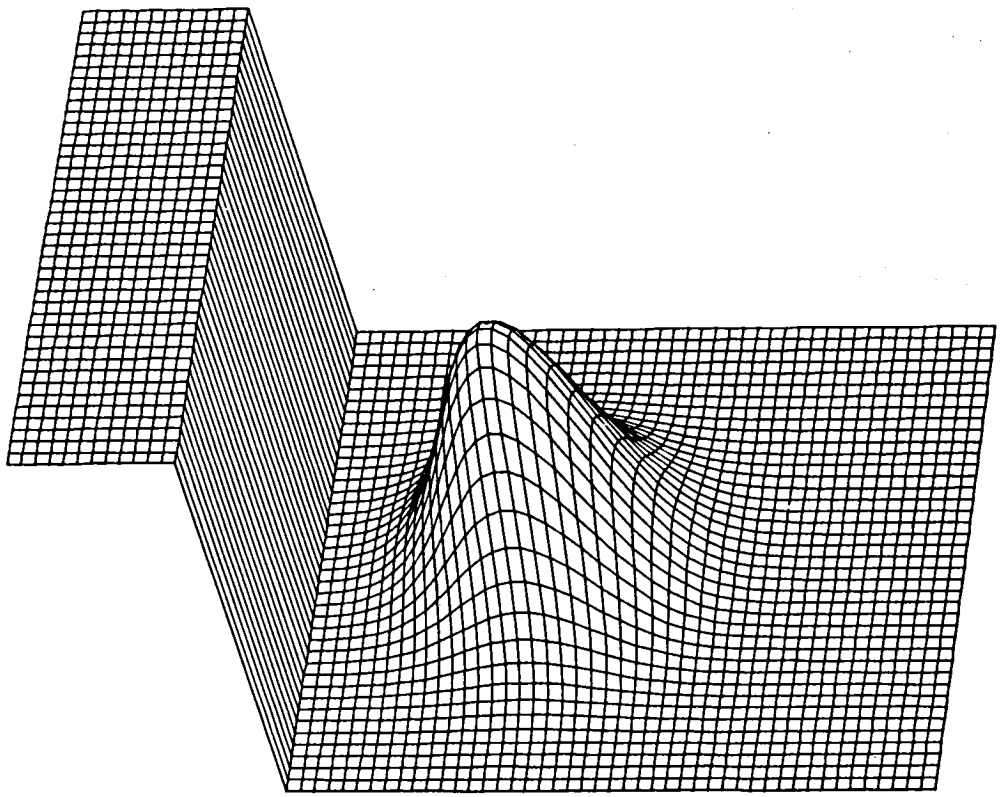


Figure 1.- Surface plot of temperature distribution for a hot spot about to be overtaken by a shock wave moving at $M_s = 3$.

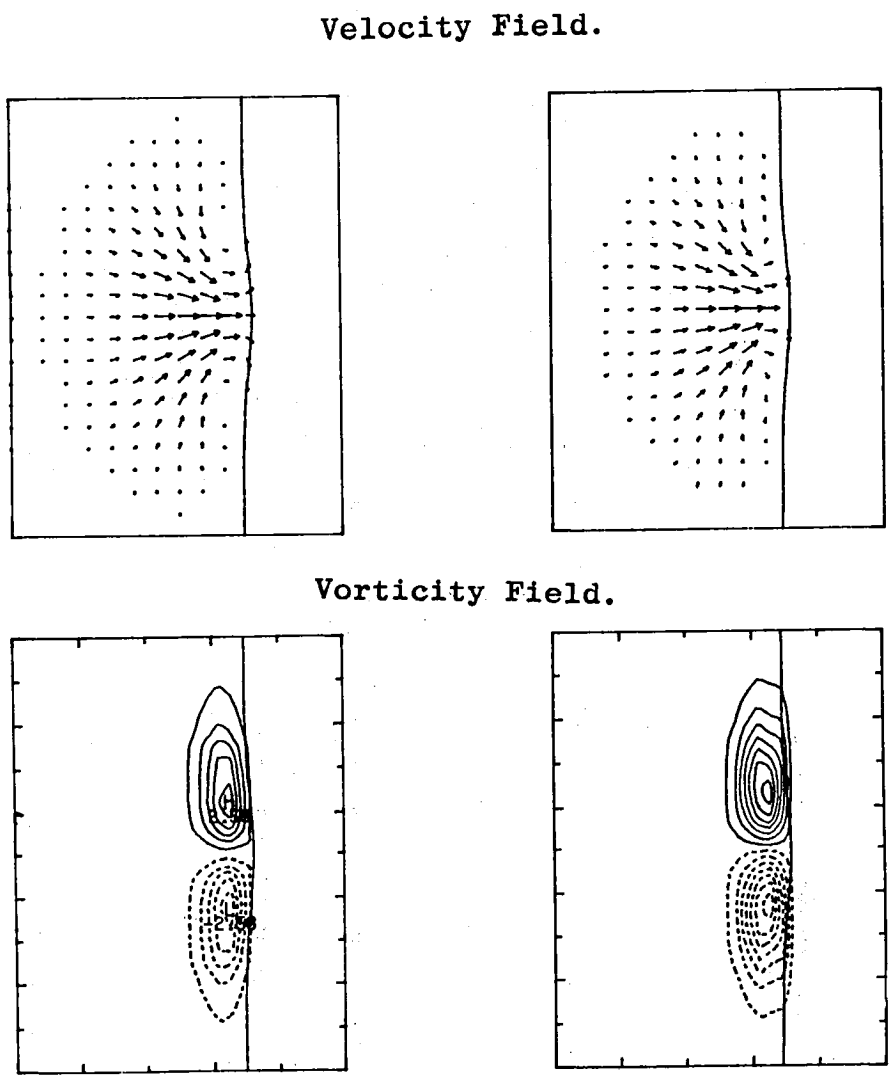


Figure 2.- Velocity and vorticity fields computed by spectral and finite difference method for a hot spot overtaken by a shock wave.

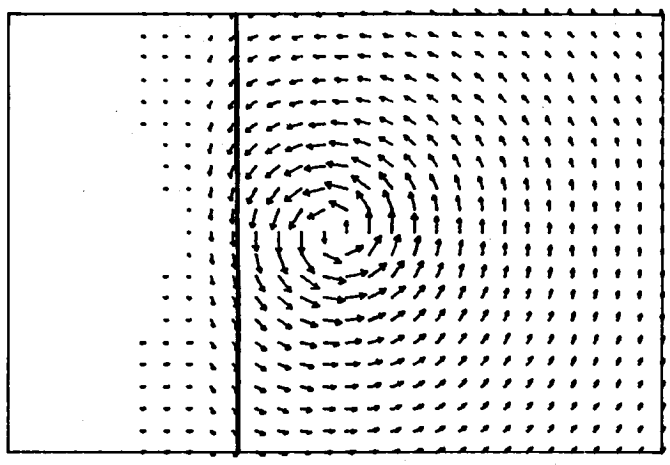


Figure 3.- Velocity field depicting single vortex about to interact with a shock wave.

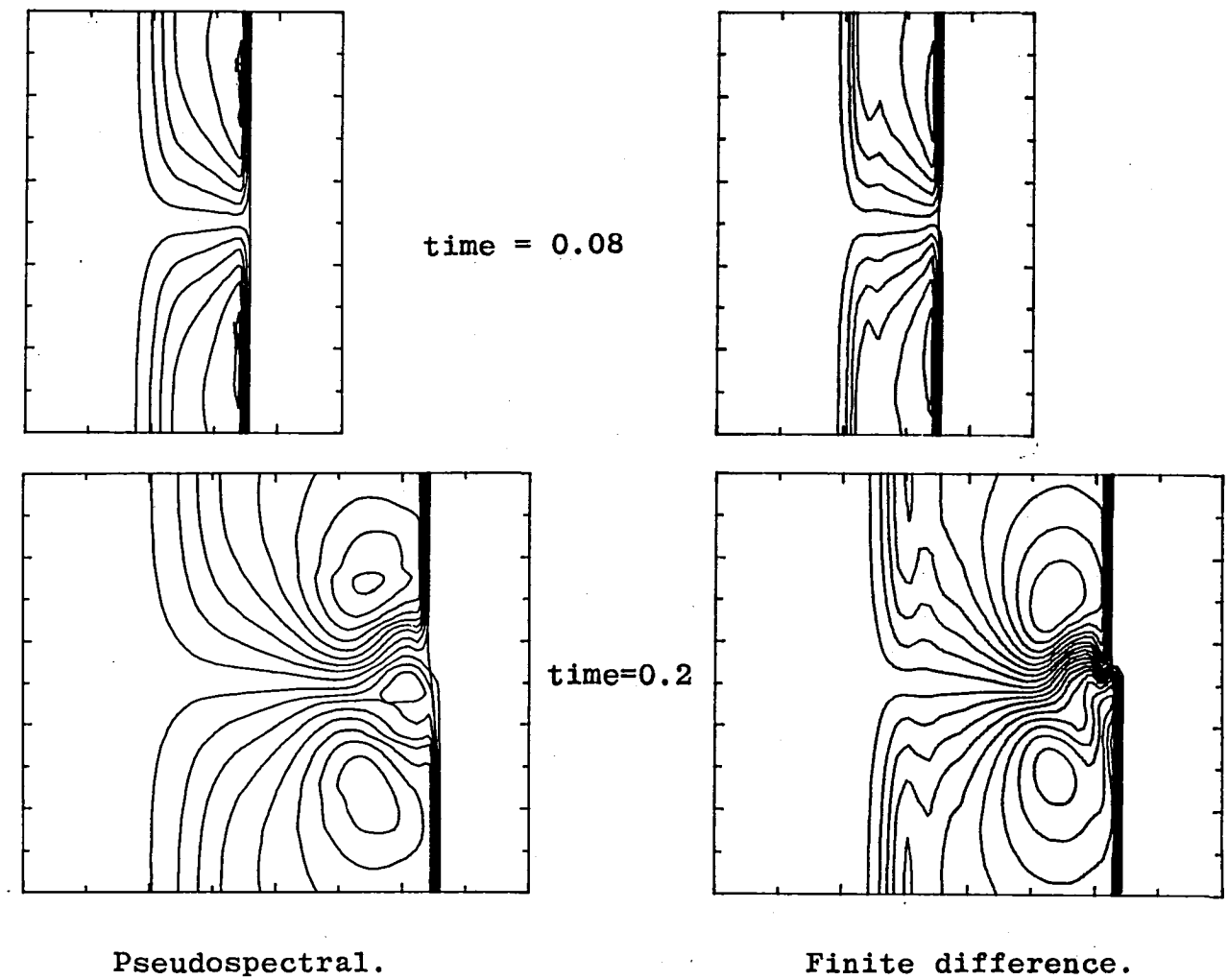
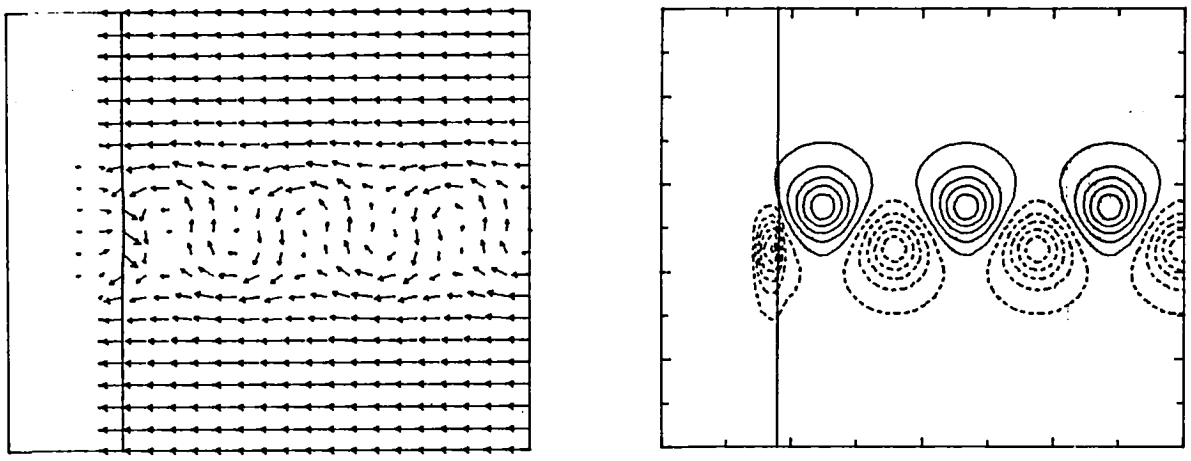


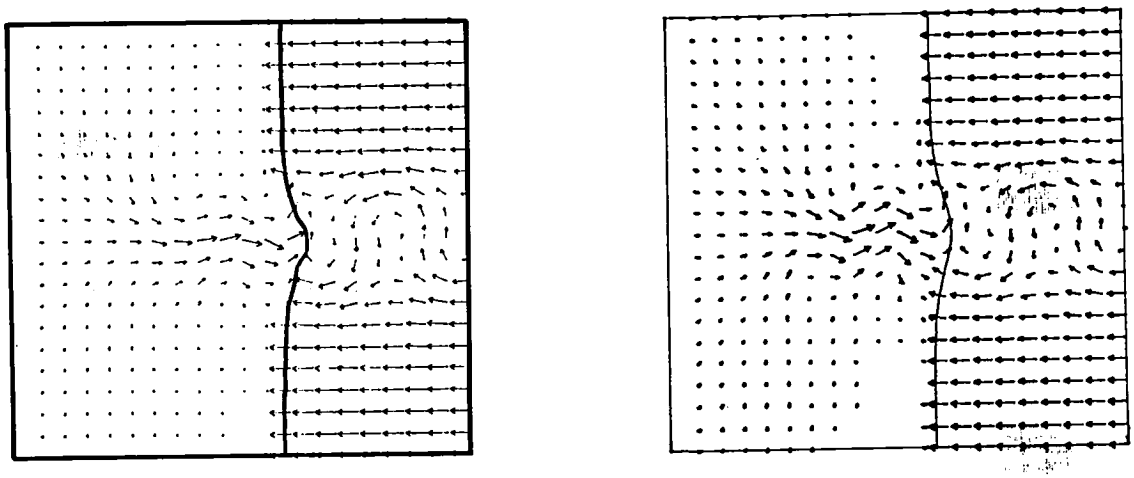
Figure 4.- Isobars computed by spectral and finite difference method for the interaction of a single vortex with a shock wave.



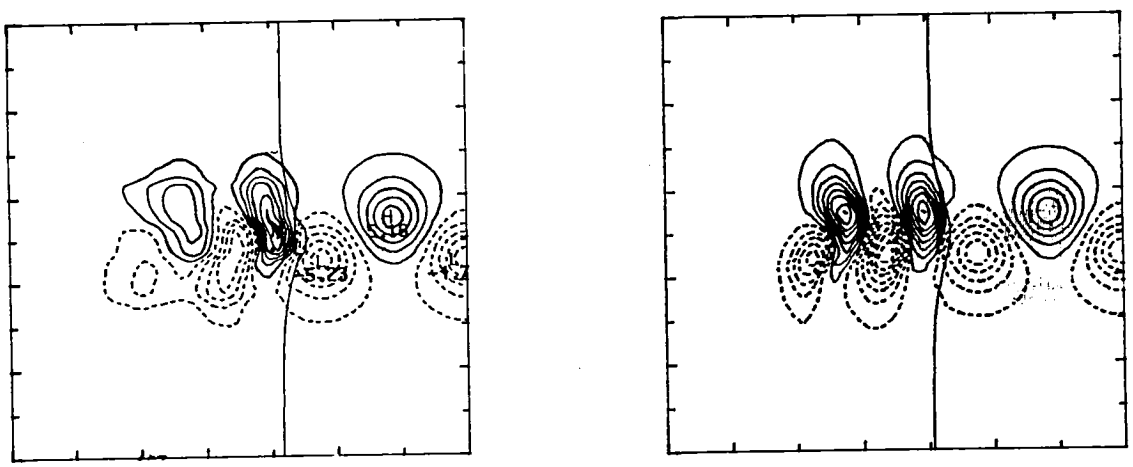
Velocity Field. Vorticity Field.

Figure 5.- Shock wave about to interact with Karman vortex street. The flow field is representative of experiment reported in reference 7.

Velocity Field.



Vorticity Field.



Pseudospectral.

Finite difference.

Figure 6.- Velocity and vorticity field for shock-Karman vortex street interaction after four vortexes have passed through the shock wave, time = 0.36.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM 84481		

4. Title and Subtitle	5. Report Date
SHOCK-FITTED EULER SOLUTIONS TO SHOCK-VORTEX INTERACTIONS	May 1982
	6. Performing Organization Code 505-31-13-01
7. Author(s) Manuel D. Salas, Thomas A. Zang, and M. Yousuff Hussaini	8. Performing Organization Report No.
	10. Work Unit No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The interaction of a planar shock wave with one or more vortexes is computed using a pseudospectral method and a finite difference method. The paper emphasizes the development of the spectral method. In both methods the shock wave is fitted as a boundary of the computational domain. The results show very good agreement between both computational methods. The spectral method is, however, restricted to smaller time steps and requires use of filtering techniques.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Shock-Vortex Interaction Finite Difference Method Spectral Method Time Dependent Euler Equations	Unlimited - Unclassified Subject Category 64

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	20	A02

N-305

For sale by the National Technical Information Service, Springfield, Virginia 22161

LANGLEY RESEARCH CENTER
3 1176 00504 4442