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7.7 A QUANTITATIVE ASSESSMENT OF RESAMPLING ERRORS

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INTRODUCTION

Applications associated with digital geographic imagery are subject to great diversity in required cell size, cartographic projection, etc. The need for resampling remote sensing scanner data is evident in all but the most undemanding cases. Past efforts [1] have shown that proper resampling of such data is dependant in important ways on the detailed knowlege of the original scanner's effective point-spread function and to the desired point-spread function of the resampled data. When both of these are known, it is relatively straightforward to compute the resampling coeffients which do the best job of approximating the shape and position of the synthesized point-spread function.

It is useful, however, to recognize that regardless of the rationale used to generate interpolation coefficients and apply them as a linear filter on a subset of the local data the result can be viewed as a synthesis of a new point spread function which is itself a linear combination of shifted positions of the original psf.

The resulting synthesized psf can be compared with an ideal psf located at various interpixel positions and any differences observed as errors.

ERROR MODEL

It is assumed that imaging scanners of interest are adequately modeled as a linear operator on the upwelling radiance and that additive noise is introduced. (The LANDSAT MSS does deliberately introduce a non-linear compression prior sampling but this effect is approximately removed in subsequent ground processing.) Thus, the scanner to be modeled is given by

$$y = Ax + n \qquad (1)$$

where x is a vector representing the two-dimensional upwelling radiance, A is a matrix describing the two-dimensional shape and location of the psf associated with each pixel sampled. The noise vector n is then added to produce the data vector y which can represent either all of the pixels in an image or only those in a locality of interest.

It should be noted that the dimensionality of x is very much greater than that of y in order to permit the A matrix to represent the subpixel detail of the point spread functions and the effect of the dimensionality reducing sampling process.

When an interpolation algorithm is used in the resampling process it is normally intended to estimate the data value that might have been obtained from a psf positioned at an intermediate location between the existing samples.

The desired result of the resampling process thus can be given by

$$z = Bx \qquad (2)$$

where B is a new spatial responsivity matrix with the point spread functions positioned correctly on the desired output grid and the vector z represents the resampled data. Note that the width and shape of the psf's in B need not be the same as that of the original scanner when it is desireable to alter the spatial resolution of the data as well as the location.

Restricting the interpolation process to a linear filter operating on the available data leads to an approximation of the desired scanner in (2) given by

$$\hat{z} = Cy$$

or

$$\hat{z} = (CA)x + Cn \qquad (3)$$

in which the interpolation coefficients C and the original psf matrix combine to produce a new psf matrix CA having spatial resolution properties which can be either better or worse than the original depending on the method used to determine the coefficients.

Any differences between the desired scanner (2) and the achieved scanner (3) can be regarded as an error given by

$$e = z - \hat{z} \qquad (4)$$

The mean squared error is easy to compute and relatively easy to defend as a performance criterion particularly for processes such as classification which may be sensitive to radiometric errors. The mean squared error

$$E = \langle e'e \rangle \qquad (5)$$

can be evaluated by substituting (2),(3), and (4) into (5). The result, after the elimination of terms with expected value of zero is

$$E = \mathrm{Trace}\,[CA\langle xx'\rangle A'C' - 2CA\langle xx'\rangle B' + B\langle xx'\rangle B' + C\langle nn'\rangle] \qquad (6)$$

This expression, when evaluated for various rules and interpixel positions yields the data shown in Table 1. The rules used were nearest-neighbor, bilinear, cubic convolution, and least square. The LANDSAT along-scan psf was used for matrix A and a rectangular psf with a width of 50 meters was chosen for the desired psf matrix B. A signal-to-noise ratio of ten was selected which established a ratio of 100 between the signal covariance matrix $\langle xx' \rangle$ and the noise covariance matrix $\langle nn' \rangle$. Finally, to restrict the error determination to the case of very fine local detail the subpixel correlations were set to zero by use of scalar matrices for $\langle xx' \rangle$ and $\langle nn' \rangle$.

The numbers tabulated are relative errors, giving a maximum of 100 for the worst possible case of no corresponance at all between psf matrices (CA) and B. The relative error is defined as

$$\text{r.e.} = 100\ \text{SQRT}(E/2B'\langle x'x \rangle B) \qquad (7)$$

The algorithms for generating coefficients for the first three interpolation rules are well known and will not be repeated here. The least square coefficients are obtained from

$$C = B\langle xx' \rangle A'[A\langle xx' \rangle A' + \langle nn' \rangle]^{-1} \qquad (8)$$

In the table, positions 1 and 17 correspond to the cases of the desired psf falling exactly on one of the original data samles. All others are located at interpixel locations differing by multiples of 1/16 of a pixel.

All four methods show the worst error near the center of the interpixel interval where a narrow psf is the most difficult to synthesize- that it is not the exact center is due to the asymmetry introduced by the lowpass presampling filter used in the LANDSAT MSS.

It is recognized that all of the errors might be regarded as undesirably large. Had the sampling rate been even slightly better than the 56 meters used the errors for LS would have been substantially reduced while the errors for the other methods would, of course, remain unchanged.

To explore in the other direction, the case of wider synthesized psf's may be considered. Such resolution-reducing resampling would be the preferred method for the generation of large area, small scale images. As the desired psf is increased in width the errors for all four methods would decrease until an approximate match with the original psf is reached. After this the LS error would continue to decrease while the other three errors would increase again because of their failure to synthesize a psf suited to the resampling interval.

As should be expected NN, BL, and CC all have the same error at positions 1 and 17 since all three merely reproduce the original data when no interpolation is required. By contrast, the LS solution has synthesized a psf more closely approximating the desired psf and hence exhibits a smaller error. The largest error occurs for NN at position 8, where the psf is not only the wrong shape and width but also in the wrong place by a half pixel. Perhaps it could be observed that the BL and CC algorithms do rather well for procedures which provide no opportunity introduce information about the original and desired psf's and the signal and noise statistics.

Another measure of image quality is the modulation transfer function. Although the MTF is merely the magnitude of the Fourier Transform of the psf and hence carries less information than the psf itself, it frequently invoked when image quality is considered. The normalized width at the half amplitude of the MTF at positions 1 and 8 for each resampling scheme is shown in Table 2. It can be seen that LS shows a broader MTF than the other methods for both positions and that both BL and CC have degraded MTF's at the midpixel location.

[1] R. Dye, "Restoration of LANDSAT Images by Discrete Two-Dimensional Deconvolution", Proceedings of the Tenth International Symposium on Remote Sensing of the Environment. ERIM, Ann Arbor,MI, October 1975.

TABLE 1. RELATIVE ERRORS FOR VARIOUS INTERPIXEL POSITIONS

POSITION	NN	BL	CC	LS
1	40.1	40.1	40.1	34.6
2	41.4	41.8	40.7	35.9
3	43.1	43.4	41.5	37.4
4	45.3	44.9	42.4	38.7
5	47.7	46.1	43.2	39.8
6	50.4	47.0	43.8	40.4
7	53.2	47.5	44.1	40.6
8	56.0	47.7	44.2	40.2
9	49.8	47.5	43.9	39.4
10	47.1	47.0	43.4	38.2
11	44.6	46.1	42.7	36.8
12	42.5	45.1	41.9	35.3
13	40.9	43.9	41.1	34.1
14	39.8	42.6	40.5	33.2
15	39.3	41.5	40.0	33.0
16	39.4	40.6	39.9	33.3
17	40.1	40.1	40.1	34.3

TABLE 2. RELATIVE WIDTHS OF MODULATION TRANSFER FUNCTIONS

POSITION	NN	BL	CC	LS
1	100.0	100.0	100.0	142.6
9	100.0	76.6	95.0	127.8