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A Design Methodology for Nonlinear Systems Containing Parameter Uncertainty: Application to Nonlinear Controller Design

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A Design Methodology for Nonlinear Systems Containing Parameter Uncertainty: Application to Nonlinear Controller Design

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A Design Methodology for Nonlinear Systems Containing Parameter Uncertainty

Application to Nonlinear Controller Design

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ABSTRACT

A design methodology capable of dealing with nonlinear systems containing parameter uncertainty is presented. Fundamental to this procedure is the mapping from the parameter space to the index (or indices) of performance. Most often, to obtain the mapping a set of differential equations must be numerically integrated.

For a fixed set of parameter values the system response (and subsequent performance measures) defines a behavior of the system. The behavior could be the satisfaction or nonsatisfaction of a design criterion or the occurrence or nonoccurrence of some qualitative system behavior. The parameter space is sampled a number of times resulting in m behaviors and n nonbehaviors. This binary classification and the Kolmogorov-Smirnov two-sample test statistic are the foundations for a generalized sensitivity analysis which is used to determine to what degree the behavior (or nonbehavior) is sensitive to the various parameters. This analysis depends upon the numbers m and n and is virtually independent of the number of uncertain parameters.

The parameters are categorized into two groups; those which are adjustable and those which are nonadjustable. For a system with j adjustable and k nonadjustable parameters an adaptive random search strategy is used to determine the combination of j adjustable parameter values which maximizes the probability of the performance indices simultaneously satisfying design criteria given the uncertainty in the k nonadjustable parameters. The sensitivity analysis is essential in determining what steps should be taken if the above probability is not sufficiently high.

The methodology was applied to the design of discrete-time nonlinear controllers. These nonlinear controllers can be used to control either linear or nonlinear systems. Several controller strategies were presented while illustrating this design procedure.

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CHAPTER 1

INTRODUCTION

The National Aeronautics and Space Administration (NASA) is currently pursuing a major new research program aimed at the development of life support systems for long-term space habitation. The objective of the "Controlled Ecology Life Support System" program (CELSS) is the integration of biological and physio-chemical subsystems into a stable and reliable system to provide human nutritional needs as well as handling waste products and maintaining atmospheric composition. The fundamental goal is the survival of the system components including the human inhabitants.

The mathematical models used to help design such CELSS systems will be nonlinear and poorly defined either structurally, parametrically, or both. With this interest in the design of nonlinear systems it is desirable to have a design method or strategy which does not depend upon the linearity of the system. Highly nonlinear systems (such as biological systems) can not be satisfactorily linearized with any meaningful conclusion attached to the results of such an analysis. In addition, when the system contains uncertain parameters the analysis becomes much

more complicated. The system response will no longer be deterministic and is usually interpreted in some sort of probabilistic manner.

In general, the nonlinear (more accurately, not necessarily linear) system to be considered will contain parameters which are adjustable and those which are nonadjustable. The adjustable parameters are those over which the designer has control. They are the design parameters since at the beginning of the process, the designer does not know what combination of adjustable parameter values is necessary to satisfy a given set of design criteria. The nonadjustable parameters are the inherent system parameters, initial state, and input variables which are uncertain. The designer is not free to select these parameters. He can only obtain better information about them.

Important to the design concept is the index (or indices) of performance. For a system containing no nonadjustable parameters, a given set of adjustable parameter values results in a unique deterministic state trajectory and subsequent measures of the performance. These measures need not be continuous functions of the parameters and are very often discrete as in the case of the occurrence or nonoccurrence of some behavior (such as survival). Assuming that these measures or indices can be obtained, the task of the designer is that of locating regions in the parameter space for which the performance

indices simultaneously satisfy some predetermined design criteria. Thus, vital to our efforts in the design stage is the mapping of the parameter space to the indices of performance. For a general system, however, this mapping is analytically unknown, complex, and not one-to-one. Therefore, the parameter space must be sampled.

Also important to the design process is the idea of a sensitivity analysis. Knowing to which parameters the performance is sensitive and being able to quantify it is crucial.

With this in mind the problem statement to be addressed by this thesis is as follows.

Consider a system with j adjustable and k nonadjustable uncertain parameters for which the mapping from the parameter space to the indices of performance can be obtained. For the above system, determine the combination of j adjustable parameter values which maximizes the probability of the performance indices simultaneously satisfying design criteria given the uncertainty in the k nonadjustable parameters.

The sensitivity analysis is essential in determining what steps should be taken next if the above probability of obtaining a desirable system response is not satisfactory.

The methodology used to solve this problem is discussed in the second chapter of this thesis.

As stated in the title, this design methodology is applied to nonlinear controller design. The resulting nonlinear controller can be used in conjunction with either linear or nonlinear systems to be controlled. There are many problems for which the designer of control systems would think that a nonlinear control strategy or scheme might perform better than a linear one. Among such problems are systems with asymmetric weighting functions and the control of an operating point near an unstable equilibrium point. The design of a controller for the survival goal of a CELSS system is another example. The problem that exists, however, is how to design such nonlinear controllers. This subject is addressed in Chapter 3.

The design methodology for the special case when all of the uncertain parameters are adjustable is presented in Chapter 4. Here, the number k of nonadjustable parameters is zero. The design methodology for the general case is considered in Chapter 5. Examples illustrating the methodology are given at the end of Chapters 4 and 5.

Concluding remarks are left for the final chapter of the thesis.

CHAPTER 2

FUNDAMENTAL CONCEPTS

Fundamental to the design process is the mapping from the parameter space to the indices of performance. In general, this mapping is some rule that allows us to obtain values for the performance measures for any given point in the parameter space and is usually not one-to-one. The problem facing the designer, however, is the fact that commonly this mapping is unknown. Most often, to obtain the mapping a set of differential equations must be solved. For the remainder of this thesis we consider systems described by sets of ordinary differential equations. Let these equations be of the form

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t),\underline{s},\underline{u}(t))$$

where $\underline{x}(t)$ is the state vector, $\underline{s}$ is the vector of parameters, and $\underline{u}(t)$ is the set of time-dependent functions which include input or forcing functions. $\dot{\underline{x}}(t)$ is the derivative of $\underline{x}(t)$ with respect to time.

The vector $\underline{s}$ consists of the inherent system parameters, initial state, and input variables which are uncertain as well as the design parameters. For specified

$\underline{s}$, $\underline{u}(t)$, and $\underline{x}(0)$ (if $\underline{x}(0)$ is not already contained in $\underline{s}$), $\underline{x}(t)$ is the solution of the set of differential equations and is deterministic or stochastic depending upon the nature of $\underline{u}(t)$. Parameters from $\underline{s}$ may be associated with the input functions, $\underline{u}(t)$. In this thesis we will consider cases where for fixed values $\underline{s}_k$ the functions $\underline{u}(t)$ are deterministic.

To each element of $\underline{s}$ we define a probability distribution which is a measure of our uncertainty in the value of the parameter. Thus, for a specific system all uncertainty is contained in the vector $\underline{s}$. As stated above, every sample of $\underline{s}$ taken from the a priori distributions results in a unique state trajectory, $\underline{x}(t)$.

We assume that there is a set of observed variables $\underline{y}(t)$, calculable from the state vector, which is important to the specific problem of interest. Then, for each randomly selected parameter set $\underline{s}_k$, there corresponds a unique observation vector $\underline{y}_k(t)$. This observation vector defines a behavior of the system. The behavior could be the satisfaction or nonsatisfaction of a design criterion or the occurrence or nonoccurrence of some qualitative system behavior such as the survival of a CELSS system. Since the elements of $\underline{y}(t)$ are observed it is sensible to define behavior in terms of $\underline{y}(t)$. Thus, we can think one step past the observation vector and define behavior in a binary sense, i.e., it either occurs or does not occur for a given parameter set $\underline{s}_k$. In the design process this

binary classification is such that for any $\underline{s}_k$ and subsequent $\underline{y}_k(t)$, the observation vector either passes or fails some design criteria.

Therefore, a random choice of the parameter vector $\underline{s}$ from the predefined distributions leads to a unique state trajectory $\underline{x}(t)$, an observation vector $\underline{y}(t)$, and, via the behavior-defining algorithm, to a determination of the occurrence or nonoccurrence of the behavior. Repeating this process for many sets of randomly chosen parameters, $\underline{s}_k$, results in a set of sample parameter vectors for which the behavior (B) was observed and a set for which the behavior was not observed ($\overline{B}$). The nature and degree of difference in these two parameter sets will form the basis for conclusions regarding the importance of particular elements of the parameter vector. This is the foundation for a generalized sensitivity analysis which will be discussed below. The generalized sensitivity differs from the classical point sensitivity in that the sensitivity is a function of the regions defined by the parameter space and not by specific values of the parameters.

In more general terms, the basic idea underlying the analysis concerns the degree to which the a priori parameter distributions separate under the behavioral classification (see Figure 1). Given a behavior B and parameter element s_i, if an individual distribution does not separate, i.e.,

$$F(s_i) = F(s_i|B) = F(s_i|\overline{B})$$

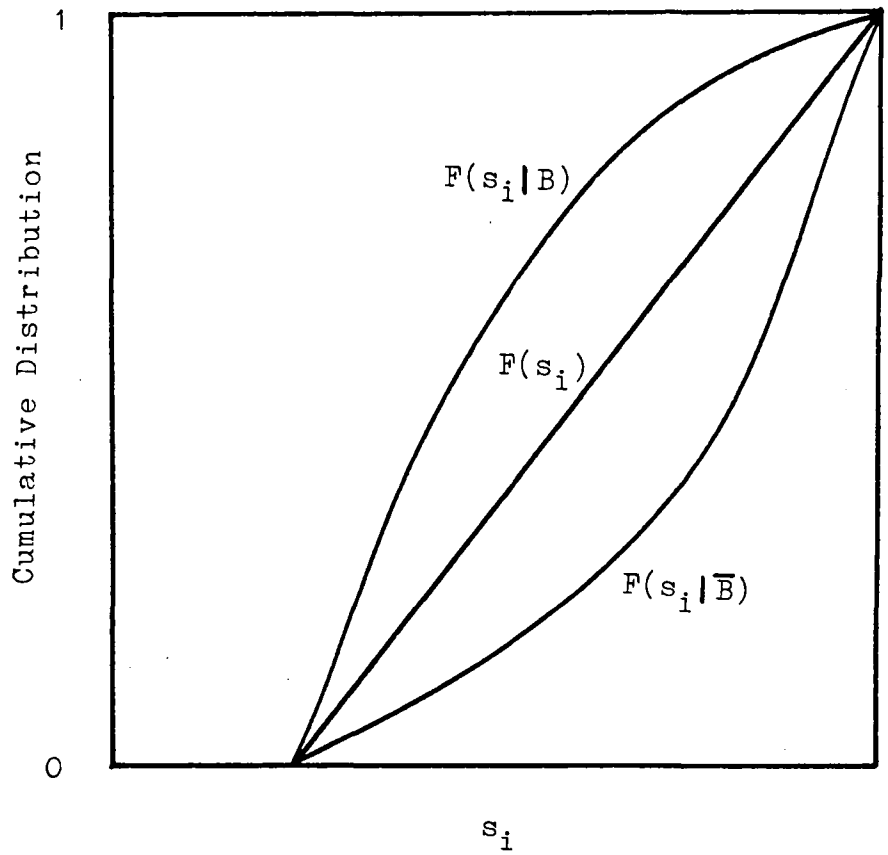

Figure 1. Cumulative distribution functions for parameter s_i. $F(s_i)$ = parent (a priori) distribution, $F(s_i|B)$ = distribution of s_i in the behavior category, $F(s_i|\bar{B})$ = distribution of s_i in the nonbehavior category.

then we argue that the parameter s_i, taken alone, appears to have no effect on the occurrence or nonoccurrence of the behavior. That is, the behavior appears to be insensitive to s_i over the multidimensional region of the parameter space defined by the a priori distributions. Unfortunately, the condition that

$$F(s_i|B) = F(s_i|\bar{B}) = F(s_i)$$

is a necessary but not sufficient condition for insensitivity as can be seen from the example shown in Figure 2. [1] Here, the regions of the two-dimensional parameter space associated with B and $\bar{B}$ are such that neither the distribution of s_1 nor s_2 separate under the behavioral classification but any pair of values uniquely determines the occurrence of B or $\bar{B}$. In this case it is the induced covariance between s_1 and s_2 that is of interest. Clearly, as a second step in our sensitivity analysis consideration must be given to the covariance induced by the behavioral mapping. [2]

The sensitivity ranking is based on a direct measure of the separation of $F(s_i|B)$ and $F(s_i|\bar{B})$. In particular, we utilize the Kolmogorov-Smirnov two-sample test statistic

$$d_{m,n} = \sup_x \left| S_m(x) - S_n(x) \right|$$

where S_m and S_n are the sample distribution functions corresponding to $F(s_i|B)$ and $F(s_i|\bar{B})$ for m behaviors and n

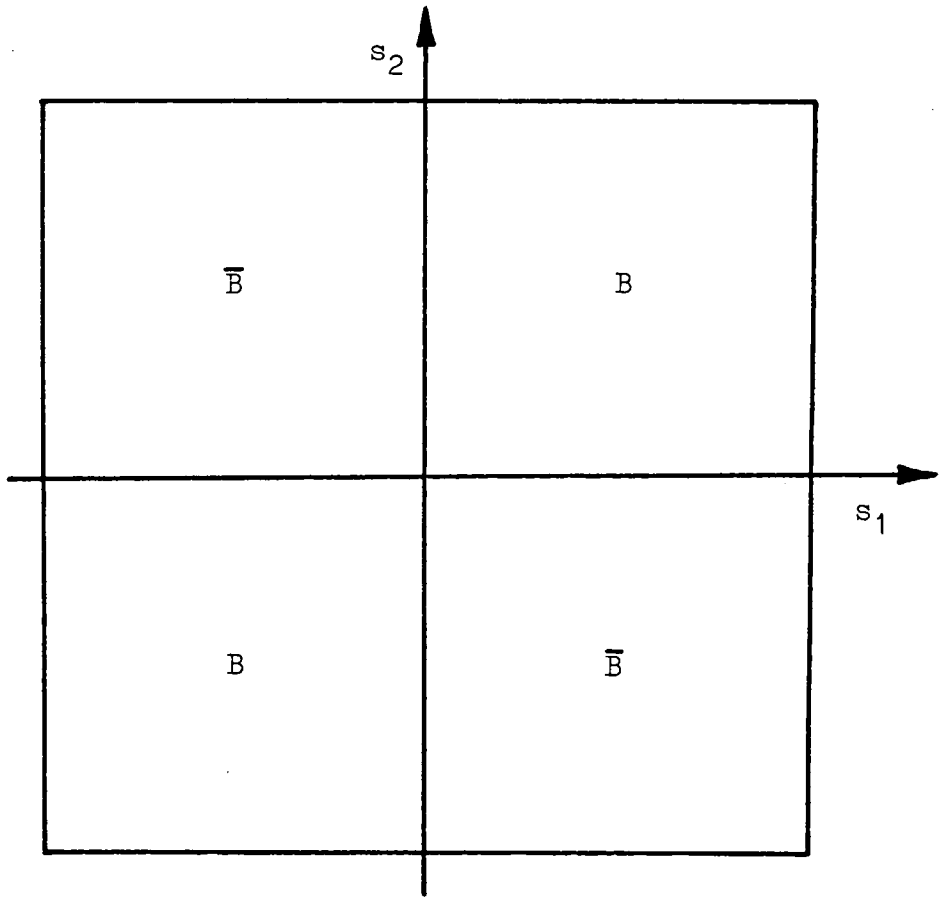

Figure 2. Schematic diagram of a two parameter case for which separation under the behavioral classification is total but for which discrimination by univariate tests is not possible.

nonbehaviors. Since the number of samples from the parameter space is finite, S_m and S_n are estimates of the unknown distributions $F(s_i \mid B)$ and $F(s_i \mid \bar{B})$ respectively. We see that $d_{m,n}$ is the maximum vertical distance between the two sample distributions. Large values of $d_{m,n}$ indicate that the parameter is important in obtaining the behavior (or not obtaining it) and, at least in cases where induced covariance is small, the converse is true for small values of the statistic.

The Kolmogorov-Smirnov test statistic is nonparametric so it is possible to assign a confidence measure to the estimate of the true distribution given only that it is continuous. Values of $d_{m,n}$ for which to accept the hypothesis of homogeneity of distributions for various confidence levels are given in Table 1. One important property to notice of $d_{m,n}$ is that the number of samples required to estimate the separation of $F(s_i \mid B)$ and $F(s_i \mid \bar{B})$ is independent of the number of parameters in the vector $\underline{s}$. [3] Thus, the number of samples from the parameter space necessary to obtain a given level of confidence of homogeneity is the same for one uncertain parameter or a thousand or more. This statement must be qualified, however. It is true that $d_{m,n}$ is a function only of the number of samples, m, leading to behaviors and the number of samples, n, leading to nonbehaviors. However, as the dimension of $\underline{s}$ increases (increasing the dimension of the parameter space), in general, the

Confidence Level (%)	Accept Homogeneity If
80	$d_{m,n} \leq 1.07 \sqrt{\frac{1}{m} + \frac{1}{n}}$
90	$d_{m,n} \leq 1.22 \sqrt{\frac{1}{m} + \frac{1}{n}}$
95	$d_{m,n} \leq 1.36 \sqrt{\frac{1}{m} + \frac{1}{n}}$
99	$d_{m,n} \leq 1.63 \sqrt{\frac{1}{m} + \frac{1}{n}}$

Table 1. Values of the Kolmogorov-Smirnov two-sample test statistic at which to accept the hypothesis of homogeneity between sample distributions for m behaviors and n nonbehaviors for various confidence levels.

fraction of the total number of samples leading to desirable system response tends to decrease. Thus, if desirable system response is associated with behavior, the fraction $(m/(m + n))$ tends to decrease with the increase in dimensionality of $\underline{s}$. So, to obtain a given level of statistical confidence the total number of samples $(m + n)$ may have to be increased to achieve acceptable individual sample sizes for S_m and S_n. This will depend, of course, upon the sensitivity of the behavior to the parameters in the parameter space defined by the a priori probability distributions. Experience has shown this not to be a major effect, however, and the above statement of independence to be nearly true.

As an aside, an interesting case occurs when there is a 'flat spot' in one of the sample cumulative distributions. This is shown in Figure 3 for hypothetical 'Pass' and 'Fail' distributions. In this region of the parameter space the behavior (passing) is dictated by s_j. A 'flat spot' in the 'Fail' distribution is a sufficient condition for obtaining a 'Pass' (given freedom to select the value s_j) but certainly not a necessary one. Although this situation has been observed [4], in the general case the behavior will not be ruled by the value of any one parameter.

As previously stated, the vector $\underline{s}$ consists of the inherent system parameters, initial state, and input variables which are uncertain as well as the design

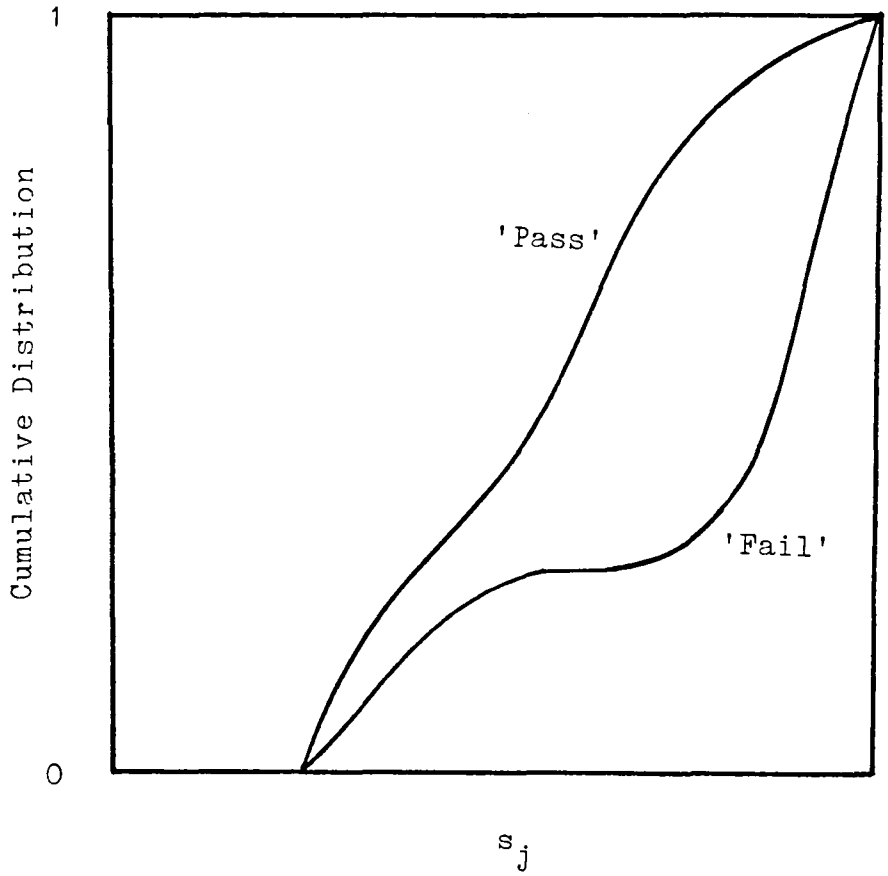

Figure 3. Hypothetical case showing 'flat spot' in the 'Fail' cumulative distribution for the parameter s_j.

parameters. However, a distinction must be made between those parameters which are adjustable and those which are nonadjustable. The adjustable parameters are the design variables over which the designer has control. The nonadjustable parameters are the uncertain system parameters. The designer is not free to choose these parameters and can only obtain better information about them. The importance of this distinction is explained as follows.

Consider a system for which there are no nonadjustable parameters. Here, the a priori probability distributions are a measure of the uncertainty of the combination of adjustable parameter values necessary to satisfy a given set of design criteria. The point to notice, however, is that for any random selection of the parameter vector, $\underline{s}$, information can be obtained as to the relative 'goodness' of the resulting solution. A sample point taken from this adjustable parameter space either results in a system response which satisfies some criteria or it doesn't. We can, therefore, search the parameter space for desirable solutions with information being obtained for each sample point.

Now consider a system for which there are both adjustable and nonadjustable parameters. In this case, one sample point taken from the parameter space provides no information as to the relative 'goodness' of the design. For a given set of adjustable parameter values

the satisfaction or nonsatisfaction of criteria will in general depend upon the values of the nonadjustable parameters. To what extent it depends upon these values is a function of how sensitive the satisfaction is to the nonadjustable parameters. So, rather than selecting the design variables that produce a given behavior, we must choose the adjustable parameters that maximize the probability of obtaining the behavior given the uncertainty in the nonadjustable parameters. Since we are concerned with binary criteria, the confidence limits for the binomial distribution [5] will be used to obtain estimates of this probability. For the given level of uncertainty in the nonadjustable parameters, however, the maximum probability may not be sufficiently high for a particular design problem. Then, the generalized sensitivity analysis must be utilized to determine which parameters and to what degree the behavior is sensitive. Better information might be obtained for these parameters and the design process repeated to locate regions in the adjustable parameter space providing higher probability estimates.

The details of this design methodology are given and illustrated through example in chapters 4 and 5. Chapter 4 considers the special case when there are no nonadjustable parameters present while Chapter 5 considers the general case.

CHAPTER 3

NONLINEAR CONTROLLER DESIGN

Historically, the approach to nonlinear controller design has been one of linearizing the nonlinearities in one way or another or of considering special cases. This is because, as stated by Garg in a survey article [6], at the present time no general approaches for controller synthesis exist. In many instances only the stability of the system is investigated. In this case, Garg states in another article [7] that "For handling generalized systems of any order and complexity the frequency-domain formulation is preferred since straightfoward synthesis techniques are available." In this thesis, a design methodology applicable to general nonlinear systems is presented based on ideas introduced in the previous chapter and some to follow.

The most general controller produces control actions based upon the state of the system to be controlled. In many cases these actions may be based upon output variables. Consider a controller which produces a control action, $\underline{u}$, which in a continuous-time system will take the form $\underline{u}(t)$, where t is time. In a discrete-time system $\underline{u}$ will be of the form $\underline{u}(nT)$, where n is the time step and T

is the sampling period. The input to the controller is the transformed state or partial state given by $(\underline{e}(t),\dot{\underline{e}}(t))$ in continuous-time and $(\underline{e}_n,(\underline{e}_n - \underline{e}_{n-1})/T)$ in discrete-time. Here, $\underline{e}$ denotes the error vector. First, consider the continuous-time case.

For some functions $\underline{F}_1 = \underline{F}_1(\underline{e},\dot{\underline{e}})$ and $\underline{F}_2 = \underline{F}_2(\underline{e},\dot{\underline{e}})$ the proposed structure for the controller is

$$\underline{u}(t) = \underline{F}_1 + \int_0^t \underline{F}_2 \, dt$$

Since $\underline{F}_1$ and $\underline{F}_2$ will in general be nonlinear functions of $\underline{e}$ and $\dot{\underline{e}}$, the above controller will also be nonlinear in general. For the special case when $\underline{F}_1$ and $\underline{F}_2$ are linear functions of $\underline{e}$ and $\dot{\underline{e}}$ we have

$$\underline{u}(t) = \underline{K}_1 \, \underline{e}(t) + \underline{K}_2 \, \dot{\underline{e}}(t) + \int_0^t (\underline{K}_3 \, \underline{e}(t) + \underline{K}_4 \, \dot{\underline{e}}(t))dt$$

where the $\underline{K}_i$ (i=1,4) are constant matrices. This reduces to

$$\underline{u}(t) = (\underline{K}_1 + \underline{K}_4)\underline{e}(t) - \underline{K}_4 \, \underline{e}(0) + \underline{K}_3 \int_0^t \underline{e}(t) \, dt + \underline{K}_2 \, \dot{\underline{e}}(t)$$

which is a proportional plus integral plus derivative (PID) controller where $-\underline{K}_4 \, \underline{e}(0)$ is a constant vector. The problem, of course, is to determine the functions $\underline{F}_1$ and $\underline{F}_2$ such that the controller produces desirable system response.

For the discrete-time case, the control action is

$$\underline{u}_n = \underline{u}(nT) = \underline{F}_1(n) + \sum_{i=1}^{n} \underline{F}_2(i)\ T$$

When $\underline{F}_1$ and $\underline{F}_2$ are linear functions of $\underline{e}_n$ and $(\underline{e}_n - \underline{e}_{n-1})/T$ we have

$$\underline{u}_n = \underline{K}_1\ \underline{e}_n + \underline{K}_2\ \Delta\underline{e}_n + \sum_{i=1}^{n} (\underline{K}_3\ \underline{e}_i + \underline{K}_4\ \Delta\underline{e}_i)T$$

where

$$\Delta\underline{e}_n = (\underline{e}_n - \underline{e}_{n-1})/T$$

This reduces to

$$u_n = (\underline{K}_1 + \underline{K}_4)\underline{e}_n - \underline{K}_4\ \underline{e}_0 + \underline{K}_3 \sum_{i=1}^{n} \underline{e}_i\ T + \underline{K}_2\ \Delta\underline{e}_n$$

which is a discrete PID controller with the constant vector $-\underline{K}_4\ \underline{e}_0$. For the general controller, then, $\underline{F}_1$ and $\underline{F}_2$ will be nonlinear functions of the state.

In the remainder of this thesis we will concentrate on discrete-time controllers. The concepts and procedures presented, however, are equally applicable to continuous-time controllers.

Consider a nonlinear control strategy obtained by discretizing the pseudostate space $(\underline{e},\Delta\underline{e})$. Figures 4 and 5 show such a discretization for a two-dimensional state space to determine the scalars F_1 and F_2. The discretizations used to determine F_1 and F_2 need not be the same (and generally are not). The values $F_1(n)$ and

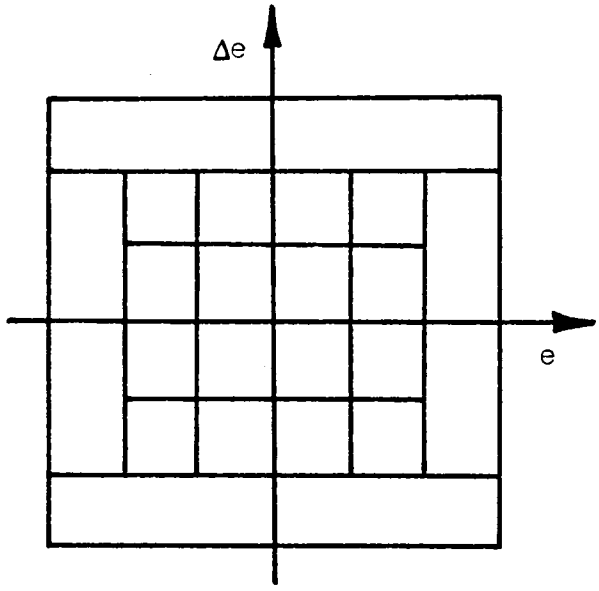

Figure 4. Discretized 'State Space' used to determine the value of F_1 at time step n.

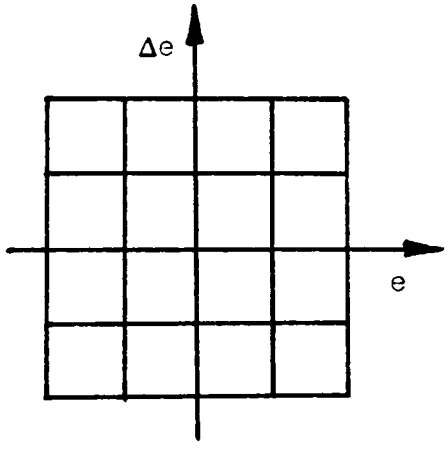

Figure 5. Discretized 'State Space' used to determine the value of F_2 at time step n.

$F_2(n)$ at time step n are those associated with the region which contains the 'state' $(e_n, \Delta e_n)$. This illustrates a noninterpolating controller since only one parameter is associated with each region. The control problem, then, is one of determining these parameters such that the design criteria will be satisfied.

If no inherent offset exists in the particular system of interest we could take $F_2 \equiv 0$ without altering the performance of the controller. However, this term is usually taken as nonzero to offset unknown loads or disturbances. We call the graph in Figure 5 an integrating table. We could also take F_2 to be a function of e only; disregarding information contained in Δe. For the scalar case shown in Figure 5 this would mean replacing the two-dimensional integrating table with a one-dimensional table. Since a two-dimensional table can always be constructed from a given one-dimensional table, information has been lost in the determination of the value for F_2 at time step n. If F_2 is taken as a linear function of e (and F_1 is again taken as a linear function of e and Δe) this is equivalent to setting $\underline{K}_4 = \underline{0}$ in the expressions above. The interesting point to notice is that a linear PID controller still results. Thus, incorporating the Δe information in the determination of F_2 only 'makes sense' for a nonlinear control strategy.

Next, an interpolating control scheme will be considered. Here, the values of F_1 and F_2 at time step n

are obtained by interpolating between the parameters associated with the region which contains the 'state' (e_n, Δe_n). Figures 6 and 7 show such a controller for a two-dimensional state incorporating rectangular regions. In this case we must interpolate between the four parameters associated with the particular region containing the 'state'. This region is shown in Figure 8 where the horizontal and vertical axes have been normalized so that the ranges of x and y are [0,1]. The parameters are the f_i (i=1,4). It is desired that we obtain the interpolated value, f. First, considering uniqueness and symmetry requirements, we assume a polynomial of the form

$$f = a_1 + a_2 x + a_3 y + a_4 xy$$

Then, using the constraints that $f = f_i$ on the corners (i=1,4) the a_i (i=1,4) can be obtained. This results in the expression for f as a function of the f_i given by

$$f = \underline{N}^T \underline{f}$$

where the f_i are contained in the vector, $\underline{f}$, and the transposed vector $\underline{N}$ is

$$\underline{N}^T = [(1 - x)(1 - y),\ x(1 - y),\ xy,\ y(1 - x)]$$

The reason for contemplating this interpolating strategy is that a 'smoother' control action will result and possibly better system response. Of course, a price

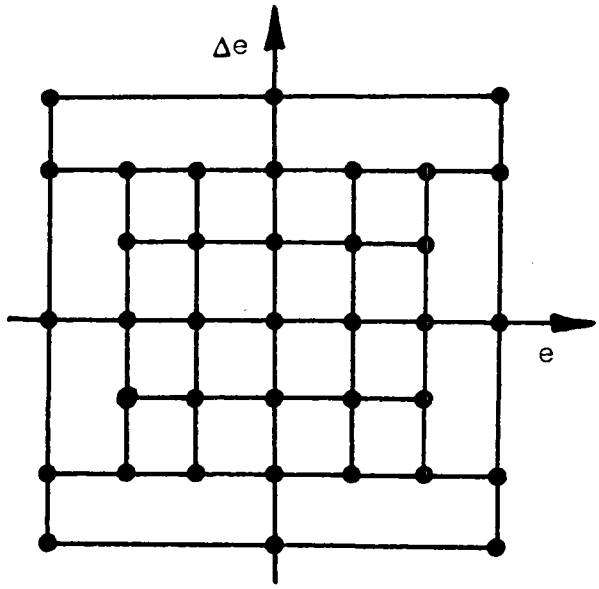

Figure 6. Interpolating control scheme used to determine the value of F_1 at time step n.

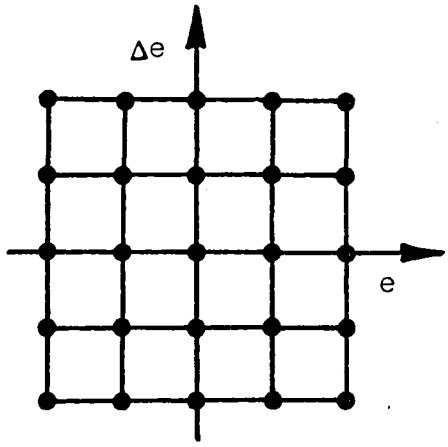

Figure 7. Interpolating control scheme used to determine the value of F_2 at time step n.

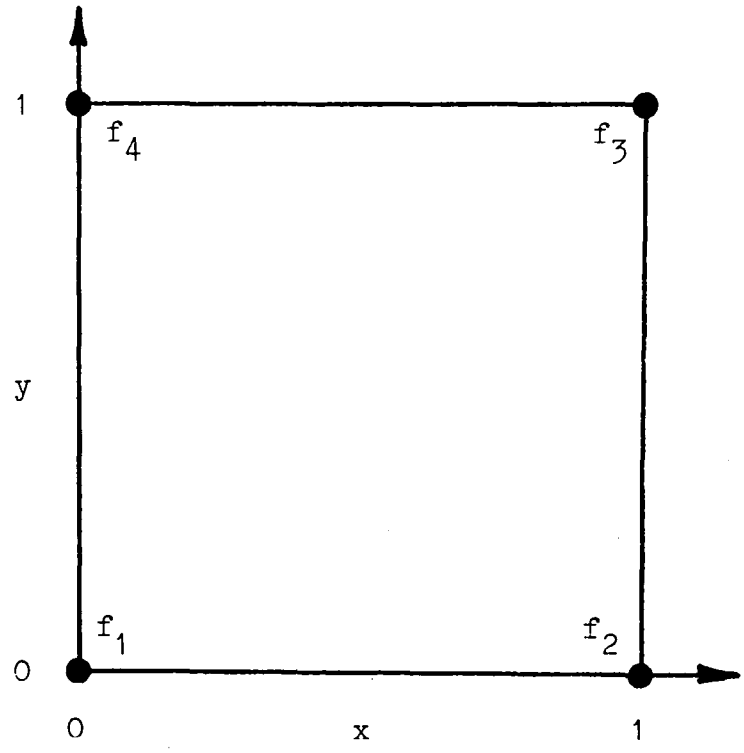

Figure 8. Two-dimensional rectangular region used in interpolating control scheme.

has been paid. First, since four variables are associated with a given region rather than one, the dimension of the parameter space will be greater for the interpolating controller than the noninterpolating controller. Second, the control algorithm for the interpolating scheme requires multiplication and division. Thus, if the controller is implemented with a microprocessor, increased overhead in computation will be incurred over that for the noninterpolating controller. Finally, since the interpolating control algorithm requires more computation time, for a given microprocessor the sampling period, T, may have to be increased to accomodate these extra calculations.

A one-dimensional integrating table can also be used to determine F_2 in Figure 7. In this case, the two-dimensional interpolation will reduce to a one-dimensional or linear interpolation.

Examples of these control strategies are given in the following chapter.

CHAPTER 4

DETAILS OF THE METHODOLOGY WHEN ONLY ADJUSTABLE PARAMETERS ARE PRESENT

When the system we are concerned with contains only adjustable parameters (i.e., no nonadjustable uncertain parameters) our goal is that of locating regions in the parameter space which lead to desirable system response. Here, the adjustable parameters are treated as uncertain since we do not know at the beginning of the design process what combination of adjustable parameter values will lead to favorable response. A priori probability distributions are assigned as a measure of this uncertainty. Selection of these distributions (which have been taken as uniform in the examples to follow) should be such that extreme values in the sampled parameter space produce undesirable solutions. The parameter space under consideration should not be unduly restricted. Such restriction could mean the overlooking of regions which produce highly desirable response.

As was previously stated, of vital importance to the design process is the mapping from the parameter space to the indices of performance. If this mapping or rule were known for a particular system of interest the design task

would be virtually accomplished. All that remains is to analytically determine a satisfactory design. However, for a general nonlinear system this mapping is not known and can be very complicated. Therefore, the parameter space must be sampled and the resulting performance measures obtained (usually through a numerical algorithm). Since the number of adjustable parameters could be large with an apparent design unclear, the space will be sampled randomly. This procedure may at first seem inefficient but as will be seen in the examples produces excellent solutions in very short time. An example taken from Richard Dawkins' The Selfish Gene illustrating why this random sampling method works follows.

"One oarsman on his own cannot win the Oxford and Cambridge boat race. He needs eight colleagues. Each one is a specialist who always sits in a particular part of the boat--bow or stroke or cox etc. Rowing the boat is a cooperative venture, but some men are nevertheless better at it than others. Suppose a coach has to choose his ideal crew from a pool of candidates, some specializing in the bow position, others specializing as cox, and so on. Suppose that he makes his selection as follows. Every day he puts together three new trial crews, by random shuffling of the candidates for each position, and he makes the three crews race against each other. After some weeks of this it will start to emerge that the winning boat often tends to contain the same individual men.

These are marked up as good oarsmen. Other individuals seem consistently to be found in slower crews, and these are eventually rejected. But even an outstandingly good oarsman might sometimes be a member of a slow crew, either because of the inferiority of the other members, or because of bad luck--say a strong adverse wind. It is only on average that the best men tend to be in the winning boat."

So, a procedure that could be followed is one of sampling the parameter space until a satisfactory design is found. However, experience has shown that there exist two types of solutions. First, there are responses which basically do the 'right' thing and may or may not satisfy the design criteria. Second, there are those system responses which are qualitatively unacceptable. Thus, an alternative procedure is one of initially accepting less restrictive criteria and subsequently reducing the size of the parameter space about the 'best' solution and 'converge' to a design which satisfies the original criteria. This could be accomplished by first sampling a given number of times. The parameter space would then be reduced in size and centered about the best solution found. This process would be repeated until a satisfactory design was found. Since the 'volume' in the parameter space leading to desirable response for the less restrictive criteria is greater than or equal to that for the original criteria one would expect solutions to be

found more readily with this procedure. This was, in fact, the case when the procedure was applied. How much more readily solutions are found using this procedure will depend upon the particular system and mapping. However, through experience, initially accepting criteria which were about 50% less restrictive provided ultimate designs much more rapidly than maintaining the original criteria from the start. No absolute quantities can ever be specified since a satisfactory design could be achieved after only one sample of the parameter space. This should not be counted on, however.

Several examples will now be presented where the adjustable parameters are the 'state-dependent' controller outputs.

Example 1

Consider the system shown in Figure 9. The mass, m, is to be positioned by a servosystem modeled as a velocity source with first order dynamics. The equations of motion are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 1/m\ [k(x_3 - x_1) - b\ x_2]$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = c(VEL - x_4)$$

where (assuming a compatible set of units)

$$m = 1$$

$$k = 3$$

$$b = 1$$

$$c = 5$$

VEL = desired velocity from controller

$$|x_4| \leq 1$$

A schematic of the control system is shown in Figure 10. The set point, R, is taken to be 3 while the sampling period, T, is taken as 0.4. The initial conditions, $x_i(0)$, (i=1,4) are zero. Although linear damping and a linear spring force are incorporated, the saturation condition on x_4 makes this a nonlinear system regardless of the form of the controller.

The design criteria are overshoot and settling time. Overshoot should be such that the controlled position is less than or equal to R + 0.1 (3.1). This simulates position control near a wall. The settling time, T_s, is that time for which

$$|x_1 - R| \leq 0.05 \text{ for all } t \geq T_s$$

where t is time. Here, we would like to make the settling time as small as possible while still satisfying the overshoot criterion.

First, let us consider a linear discrete PID (Proportional + Integral + Derivative) controller. For an error at time step n defined as

$$e_n = R - x_1(nT)$$

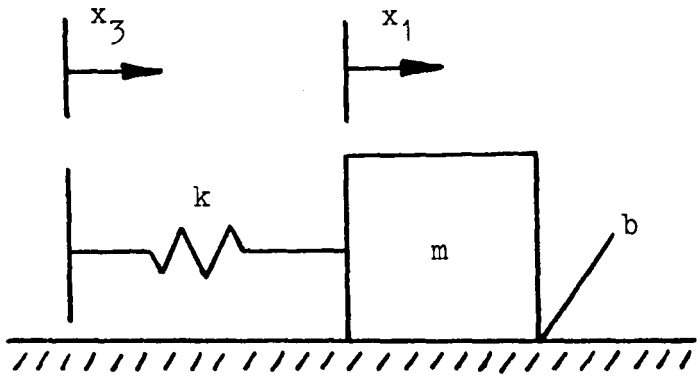

Figure 9. Schematic of controlled system for Example 1.

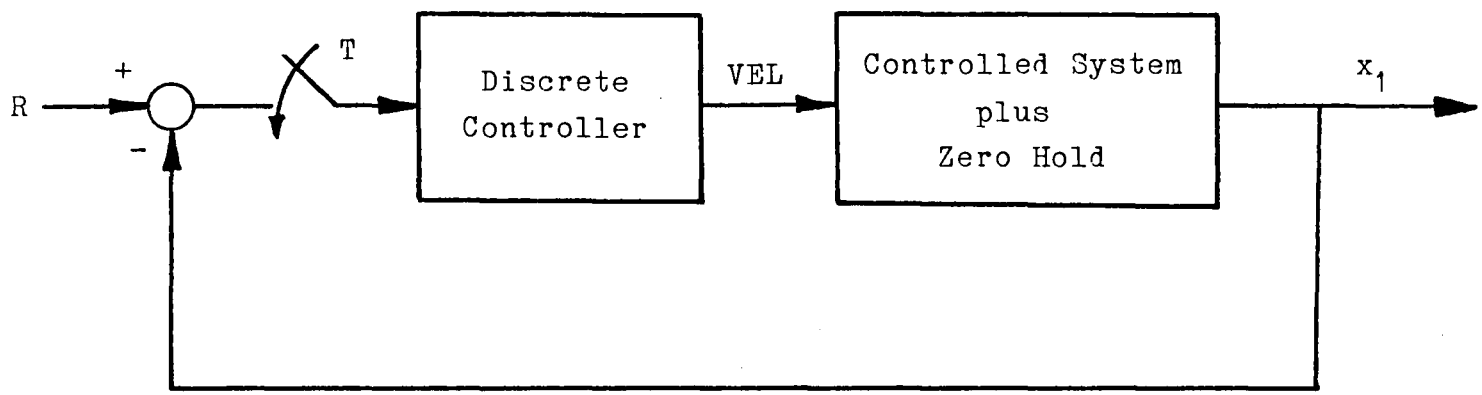

Figure 10. Schematic of control system.

the PID control algorithm is given by

$$VEL_n = K_c\ e_n + T_i \sum_{k=1}^{n} e_k\ T + T_d(e_n - e_{n-1})/T$$

where VEL_n is the desired servosystem velocity at time step n and K_c, T_i, and T_d are the controller gains to be determined. The best design found was obtained as given in [4] by randomly sampling the three-dimensional parameter space. The equations of motion were then numerically integrated with a time step of 0.05 until a time of 20.4 was reached. The process was repeated one thousand times taking approximately ten minutes on a PDP-11/60 minicomputer. This gave a best case with an overshoot of 0.05 and a settling time of 9.65. The values for the controller gains which produced this response to three significant figures are $K_c = 3.31$, $T_i = 0.218 \times 10^{-1}$, and $T_d = 0.228$.

We will now consider a discrete nonlinear controller. One of the infinite number of ways to section the x_1-x_2 space is shown in Figure 11. Four of the design parameters indicated by circled numbers are the locations of boundaries between regions in the space. The various regions (1 through 17) are indicated by the uncircled numbers. Since there is no inherent offset, an integrating table is not used for this example. The value of VEL_n will be that associated with the region which contains the pseudostate of the mass $(x_1, \Delta x_1)$ at time step n. Since the controller is discrete only an

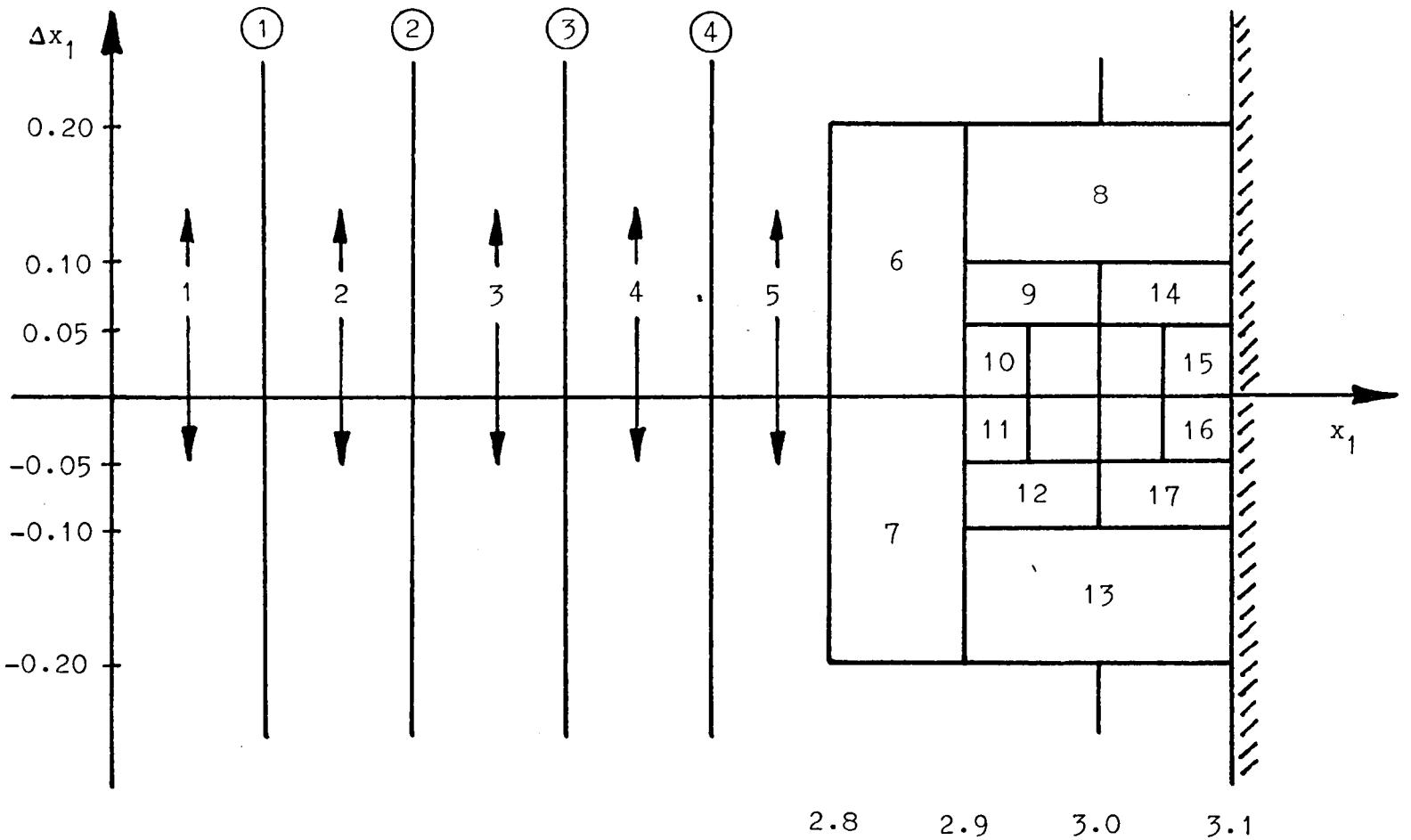

Figure 11. 'State'-dependent discrete nonlinear controller for Example 1.

approximation to the state is available. The pseudostate at time step n is $(x_1(nT), (x_1(nT) - x_1((n-1)T))/T)$. For simplicity we let $\Delta x_1 = (x_1(nT) - x_1((n-1)T))/T$. Here, we take VEL = 0 in the region defined by $2.95 \leq x_1 \leq 3.05$ <u>and</u> $-0.05 \leq \Delta x_1 \leq 0.05$. With the initial settling time criterion equal to about 6 the 21-dimensional parameter space was sampled (with subsequent integration) until a design was found whose performance indices satisfied the criteria. The ranges of the a priori uniform distributions where then halved and centered about this solution. A number (100) of samples were then taken in this restricted parameter space. Again, the ranges were halved and centered about the best solution found. This process was repeated until taking an additional 100 samples did not result in a better controller design. The final design was obtained after approximately 500 samples. For this system this took about 5 minutes using a PDP-11/60 minicomputer. The solution gave an overshoot of 0.05 and a settling time of 3.70. This is 2.6 times faster than the PID controller. Although the time-optimal control has not been calculated for this problem the minimum time to reach the set point with maximum control is approximately 3.25. The time-optimal solution will give a settling time greater than 3.25, of course, since maximum control will result in the nonsatisfaction of the overshoot criterion.

Boundary	Location	Region	VEL X 10
1	1.00	1	9.97
2	1.93	2	7.29
3	2.50	3	3.11
4	3.54	4	3.66
		5	-0.50
		6	0.30
		7	1.65
		8	0.83
		9	-0.73
		10	-1.21
		11	0.10
		12	-0.68
		13	1.61
		14	-2.29
		15	0.36
		16	-0.21
		17	-0.16

Table 2. Final design for the nonlinear controller of Example 1.

The final design for this nonlinear controller (to three digits) is given in Table 2. The controller output (VEL) times 10 is shown for each of the 17 regions. A value of 10.00 corresponds to VEL = 1.00 which, due to the saturation condition on x_4, is the maximum obtainable velocity for the velocity source. The four boundaries and design locations are also given. Notice that the location for Boundary 4 (3.54) eliminates Region 5 since its value is greater than the set point, R = 3. This could not have occurred if the a priori probability distributions on boundary locations were unduly restricted at the beginning of the problem.

In the examples to follow in this chapter only the results will be presented. The actual designs will not be given due to the number of parameters associated with the control schemes. We will next look at an example which incorporates an integrating table and examine various nonlinear controller strategies.

Example 2

Consider the system shown in Figure 12. The mass m_2 is to be positioned by a servosystem modeled as an effort or force source. The equations of motion are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 1/m_1 \; [F - (k_1 + k_2)x_1 + k_2 \, x_3 - b_1 \, x_2]$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = 1/m_2 \; [k_2 \, x_1 - k_2 \, x_3 - b_2 \, x_4 - F_u]$$

where (assuming a compatible set of units)

$$m_1 = m_2 = 1$$
$$b_1 = b_2 = 1$$
$$k_1 = 1$$
$$k_2 = 3$$

F = desired force input from controller with

$$|F| \leq 5$$

F_u = 0.25 (unknown but constant force)

F_u is modeled as a constant wind load which is unknown to the controller. Thus, an integrating scheme for the controller is necessary.

The schematic of the control system is the same as that shown in Figure 10 with the exception that the output is x_3 rather than x_1. Also, the controller output, F, replaces VEL. As in the previous example the set point, R, sampling period, T, and initial conditions, $x_i(0)$, (i=1,4) are 3, 0.4, and 0, respectively. The design criteria are overshoot of the controlled position, x_3, to be less than or equal to 0.1 while minimizing the settling time as defined in Example 1. The numerical integration time step and duration are 0.05 and 20.4.

The best linear PID controller was designed as in [4] by sampling the parameter space a thousand times. This gave an overshoot of 0.02 and a settling time of 10.8.

Now consider the nolinear controller termed a noninterpolating controller illustrated in Figures 13 and

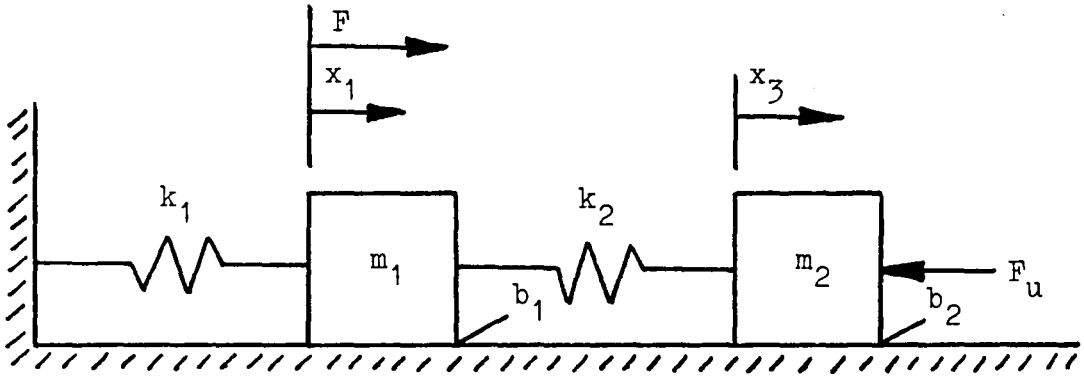

Figure 12. Schematic of controlled system for Example 2.

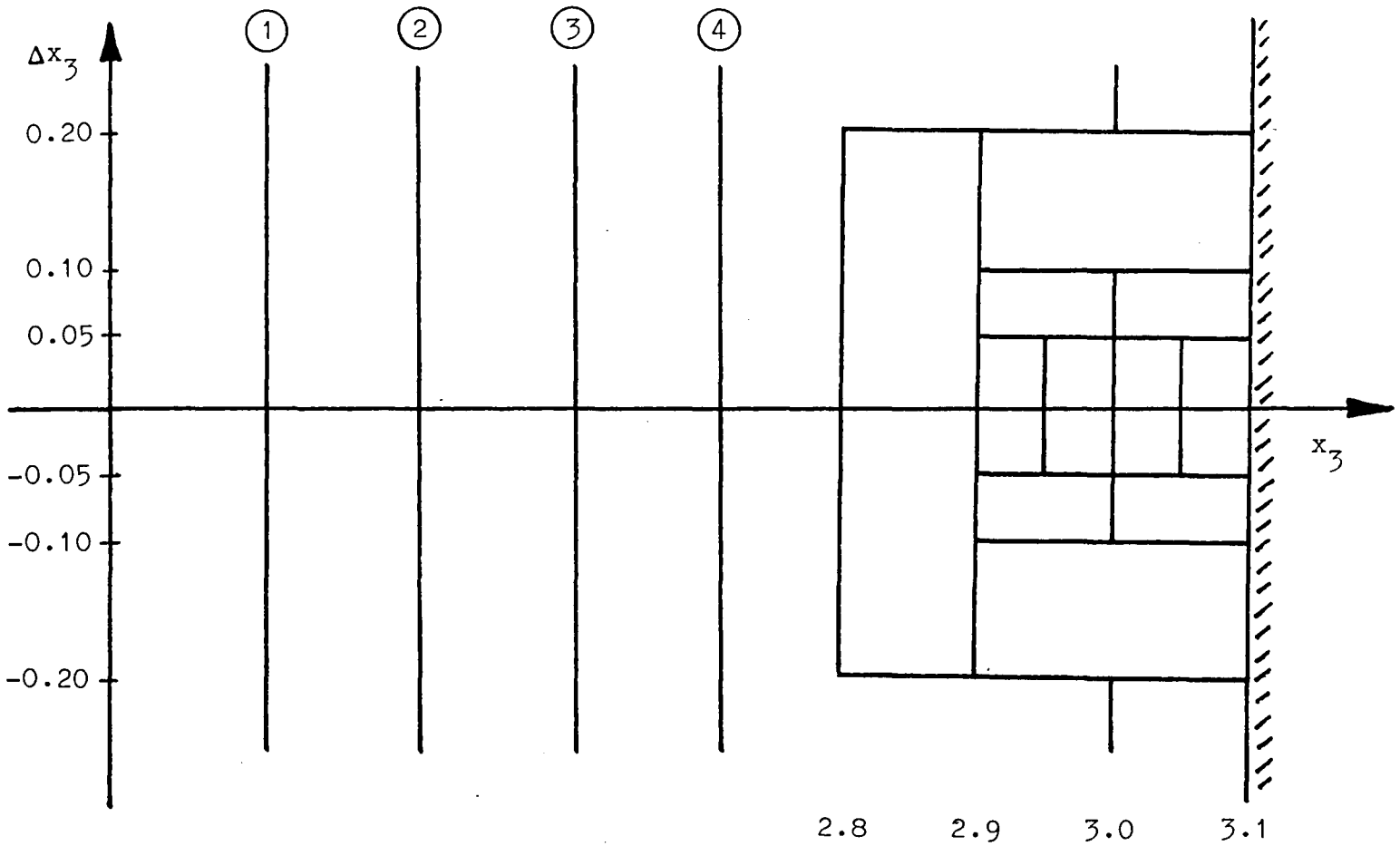

Figure 13. 'State'-dependent noninterpolating table used to determine the value of F_1 at time step n for Example 2.

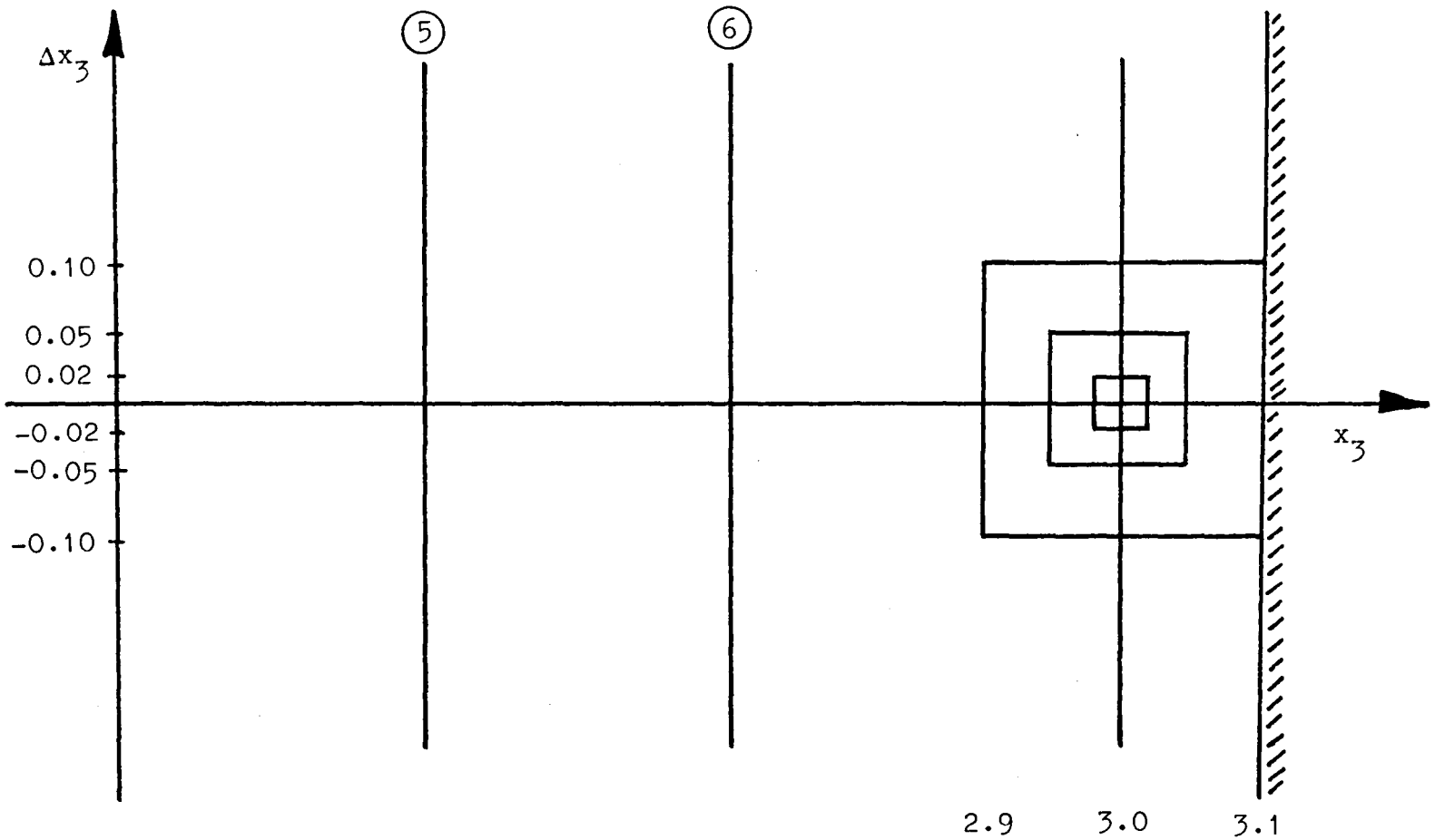

Figure 14. 'State'-dependent noninterpolating table used to determine the value of F_2 at time step n for Example 2.

14. The control signal, F, at time step n is

$$F_n = F_1(n) + \sum_{i=1}^{n} F_2(i)\ T$$

where F_1 and F_2 are determined from tables illustrated in Figures 13 and 14. Again, the circled numbers in these figures represent locations of boundaries which are allowed to vary. Since the F_2 sequence is summed to offset the unknown load, F_u, Figure 14 is called an integrating table. A total of 34 parameters are associated with this particular controller.

With the initial settling time criterion equal to 5 the 34-dimensional space was sampled until a response was found which satisfied both the settling time and overshoot criteria. The ranges on the 34 uniform probability distributions were then halved and centered about this design. The parameter space was then sampled 100 times. Again the ranges were halved and centered about the best solution found. This process was repeated until taking an additional 100 samples did not result in a better controller design. This design produced an overshoot of 0.05 and a settling time of 3.35 (3.2 times better than the linear controller). It should be noted that due to the constraint on the input force, the minimum time for the mass m_2 to reach the set point is approximately 2.75.

When the settling time criterion was set to 10 and an integrating table was not used, i.e., $F_2 \equiv 0$, no satisfactory response was obtained, as expected.

The role of the two-dimensional integrating table was investigated next. Projecting the integrating table of Figure 14 onto the x_3 axis, the number of controller parameters is reduced to 30. Thus, information of Δx_3 is disregarded in determining the value of F_2. Applying the above process a design was found for which the overshoot was 0.03 and the settling time was, again, 3.35. Thus, so far it does not appear that the two-dimensional integrating table provides better results than the one-dimensional table.

Finally, a two-dimensional interpolating scheme was considered. Here, F_1 and F_2 are determined by interpolating between the parameters associated with the region containing the pseudostate $(x_3, \Delta x_3)$. The control strategy is shown in Figures 15 and 16 in which there are 49 design parameters. This increase in the number of controller parameters is solely due to the fact that interpolation is being used. The basic framework of the controller is the same as that in Figures 13 and 14. The best design was found to give an overshoot of 0.05 and a settling time of 3.05. This is about 1.1 times better than the noninterpolating controller and 3.5 times better than the linear controller. Although a slight improvement in performance has been achieved a price has been paid. The interpolating control strategy is more complicated than its noninterpolating counterpart and requires more computational time to determine the controller output.

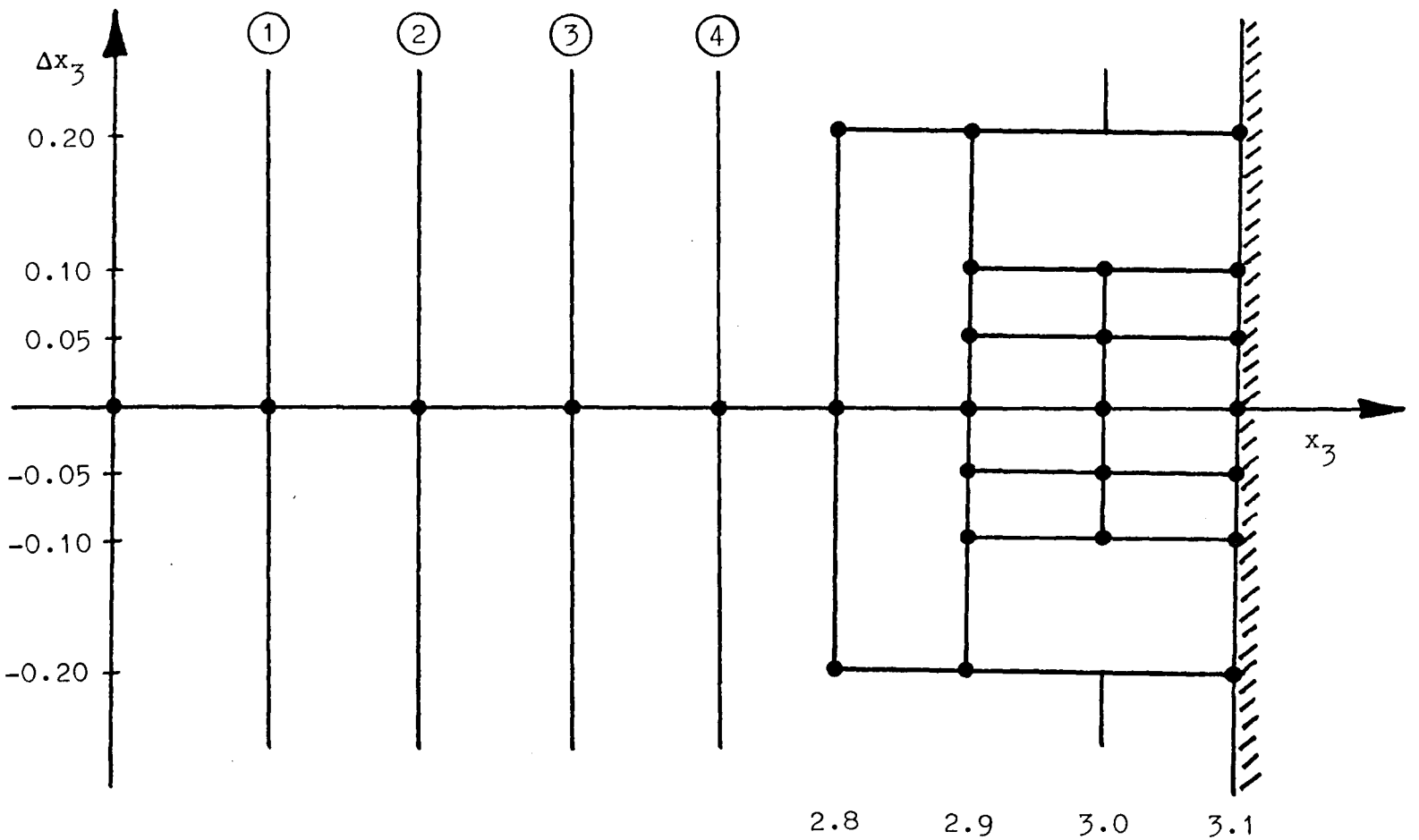

Figure 15. Interpolating table used to determine the value of F_1 at time step n for Example 2.

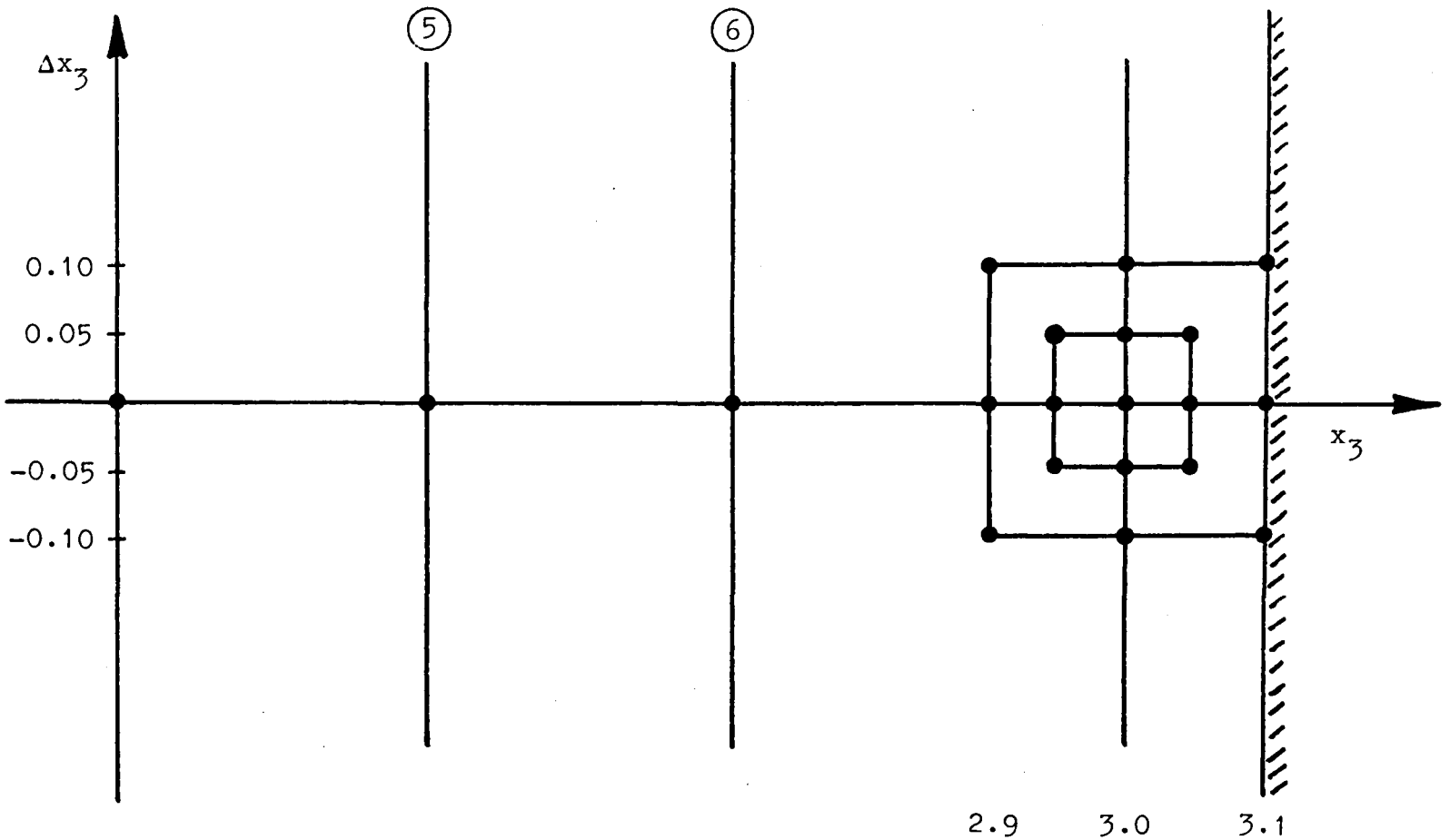

Figure 16. Interpolating table used to determine the value of F_2 at time step n for Example 2.

Controller	Overshoot	Settling Time
Linear (PID)	0.02	10.8
Noninterpolating		
1-dimensional integrating table	0.03	3.35
2-dimensional integrating table	0.05	3.35
Interpolating		
1-dimensional integrating table	0.04	3.05
2-dimensional integrating table	0.05	3.05

The minimum time to reach the set point with $F \equiv 5$ is approximately 2.75.

Table 3. Summary of results for Example 2.

Thus, depending upon the controller, the sampling period may have to be increased to accomodate these extra calculations. Also, multiplication and division are required for the interpolating controller. Thus, when this strategy is implemented, the computational costs will be greater than those for the noninterpolating scheme.

Again the integrating table of Figure 16 was projected onto the x_3 axis to investigate the effectiveness of the two-dimensional strategy. For this case linear interpolation was used to determine F_2 reducing the dimension of the parameter space to 37. The best design produced an overshoot of 0.04 and a settling time of, again, 3.05. For this problem, therefore, the two-dimensional strategy to determine F_2 was not any more effective than the one-dimensional strategy. We will see in the next example, however, that this is not always true. A summary of results for Example 2 is given in Table 3.

Example 3

Consider a class of problems for which it is desired to control an operating point which is near an unstable equilibrium point. One particular problem in this class is the control of the idle speed of an engine operating near the stall speed. Static curves for load and input torque versus engine speed for such a problem are shown schematically in Figure 17. For a given input curve in

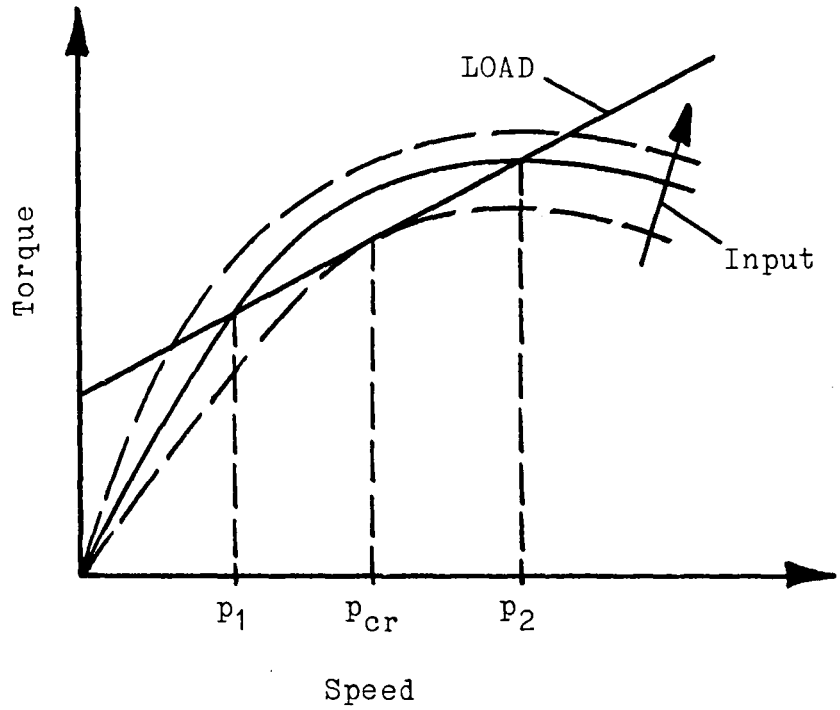

Figure 17. Schematic of the static curves for load and input torque versus engine speed for Example 3.

the family of input curves (solid line) the unstable and stable equilibrium points are p_1 and p_2 respectively. While we would like p_1 and p_2 to be close to each other to lower the idle speed we do not want the engine to stall when subjected to an increase in load. This increased load could be the sudden operation of the air conditioner. In this case the input would have to be increased (in the direction of the arrow) to avoid a stall. For a given load curve the input can be decreased (shown by the lower dashed curve) until p_1 and p_2 converge to a single point. This point, p_{cr}, represents the critical speed for which the engine can not physically run slower.

The dynamics associated with the static curves of Figure 17 are taken as first order for this example. The equations of motion are

$$\dot{x}_1 = 1(c_1 - x_1)$$
$$\dot{x}_2 = 2(u - x_2)$$
$$\dot{x}_3 = x_1 - \text{LOAD}$$

where (assuming a compatible set of units)

x_1 = actual torque (input)

x_2 = actual input

x_3 = engine speed

$c_1 = (x_2/100)(0.055x_3 - 0.46\text{x}10^{-4}(x_3)^2)$

u = desired input from controller

The schematic of the control system shown in Figure 10 is

applicable to this problem with the output x_3 replacing x_1 and the input from the controller, u, replacing VEL. The initial conditions are such that the engine is idling at steady state and $x_3 = 520$ for a load given by

$$LOAD = 50 + (0.1)x_3$$

when the load increases to

$$LOAD = 100 + (0.1)x_3$$

simulating perhaps someone turning on the air conditioner. It is desired that the controller maintain the idle speed while taking the following criteria into consideration. First, the engine should not stall. Thus, the first criterion is binary; 0 for no stall and 1 for stall. Second, the controller should minimize the 'integral of the error squared'. For an error defined as

$$e_n = R - x_3(nT)$$

with $R = 520$ and $T = 0.4$ this performance index is given by

$$ESQ_n = A \sum_{i=1}^{n} (e_i)^2 \, T$$

where A is a scaling factor. It should be noted that the set point of 520 is fairly close to the critical speed of 482.

Again, a linear PID controller was designed as in the previous examples resulting in an 'error squared' of 26.2

(with Stall = 0). The equations of motion were integrated numerically with a time step of 0.05 for a duration of 30.

A noninterpolating controller was considered next. Figures 18 and 19 show the structure of the tables to determine F_1 and F_2 defined in Example 2 where $\Delta e = (e_n - e_{n-1})/T$. The circled numbers indicate locations of boundaries which are allowed to vary. For this particular controller the dimension of the parameter space is 38. The best design gave an 'error squared' of 21.6 (21% better than the linear controller).

When the table of Figure 19 was projected onto the 'e' axis (reducing the dimension of the parameter space by 4) so that information regarding the pseudoderivative of the error was neglected in determining the value of F_2 the performance was degraded. The best design for this case produced an 'error squared' of 22.6. This is still better than that achieved by the linear controller but not as good as using the two-dimensional integrating table.

Finally, an interpolating controller was investigated and is shown in Figures 20 and 21. The basic structure is the same as that shown in Figures 18 and 19. Here, there are 58 controller parameters. The best design gave an 'error squared' of 21.3 (23% better than the linear controller).

Again, when the Δe information was disregarded in the F_2 determination the performance index was degraded to 22.6.

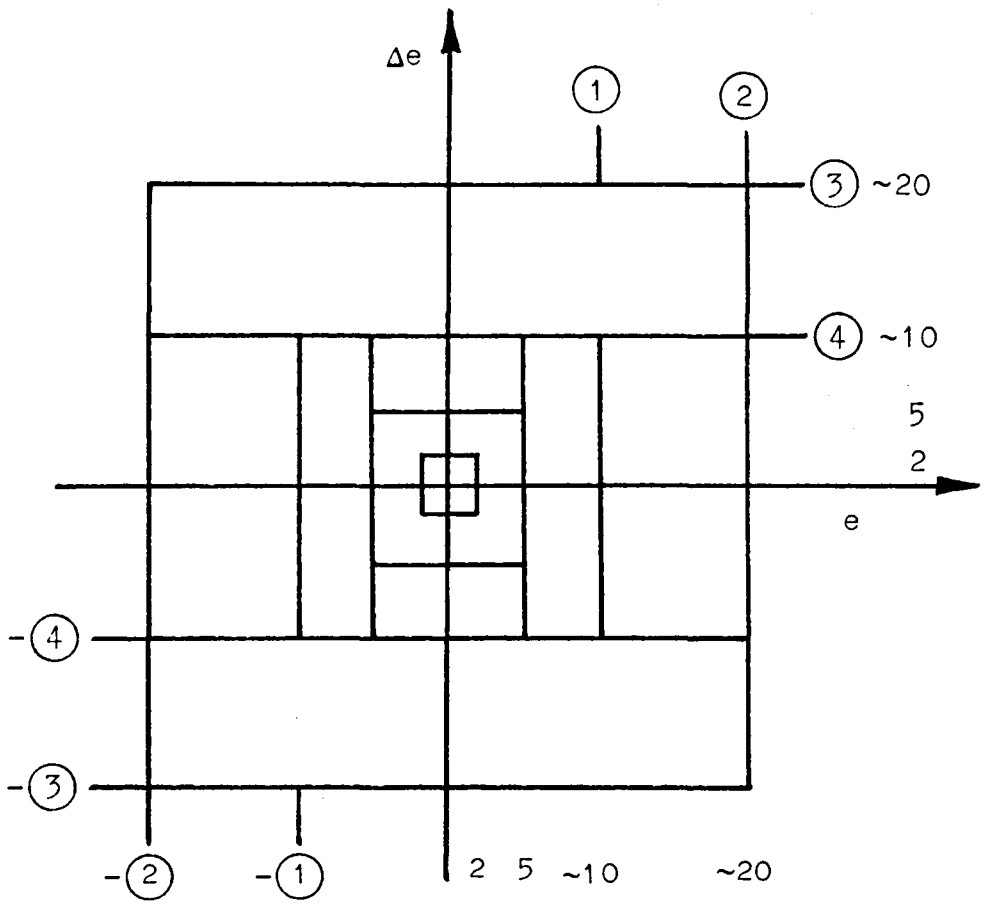

Figure 18. Noninterpolating table used to determine the value of F_1 at time step n for Example 3.

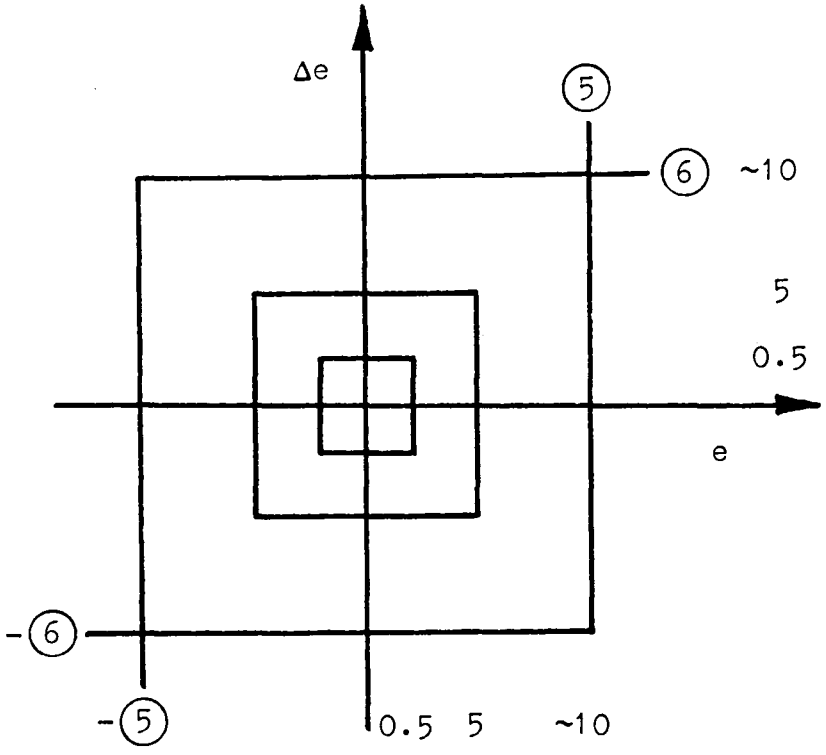

Figure 19. Noninterpolating table used to determine the value of F_2 at time step n for Example 3.

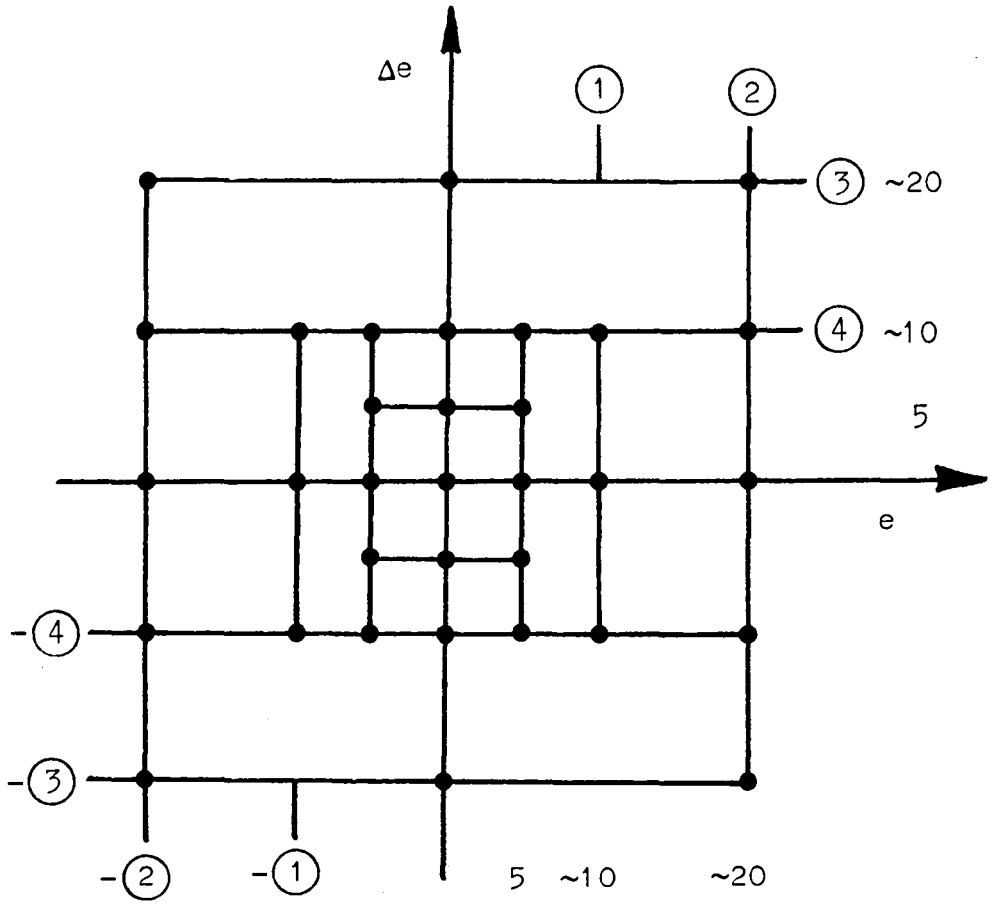

Figure 20. Interpolating table used to determine the value of F_1 at time step n for Example 3.

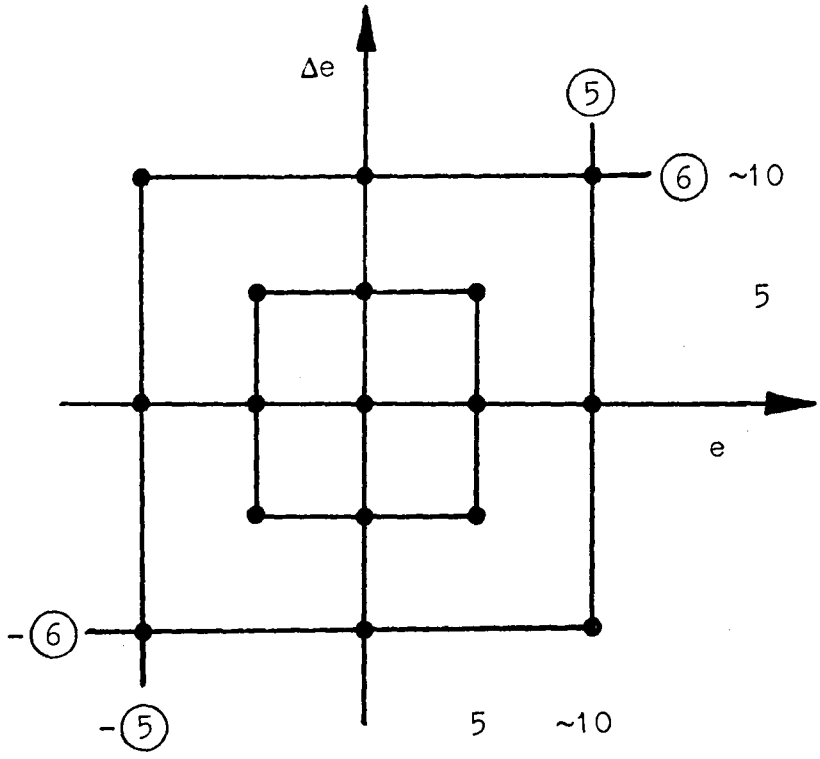

Figure 21. Interpolating table used to determine the value of F_2 at time step n for Example 3.

Thus, for both the interpolating and noninterpolating controllers a two-dimensional integrating table performed better than a one-dimensional table. A summary of results for Example 3 is shown in Table 4.

Controller	Stall	'Error Squared'
Linear (PID)	0	26.2
Noninterpolating		
1-dimensional integrating table	0	22.6
2-dimensional integrating table	0	21.6
Interpolating		
1-dimensional integrating table	0	22.6
2-dimensional integrating table	0	21.3

Table 4. Summary of results for Example 3.

CHAPTER 5

DETAILS OF THE METHODOLOGY WHEN BOTH ADJUSTABLE AND NONADJUSTABLE PARAMETERS ARE PRESENT

When nonadjustable as well as adjustable parameters are present the qualitative nature of a system is drastically altered. In considering problems in the previous chapter each sample of the parameter space provided information as to the relative 'goodness' of the resulting solution. This is not true when nonadjustable parameters are present since the satisfaction or nonsatisfaction of design criteria is usually affected by these parameters. Thus, any one sample of the parameter space taken by itself is meaningless. We must then estimate the probability of obtaining desirable system response given the uncertainty in the nonadjustable parameters. The best design is the one which maximizes this probability.

In the context of a CELSS system the nonadjustable uncertain parameters are those system parameters which are inherently poorly defined. These include growth rate coefficients, light shading coefficients, and diffusion parameters to name just a few. The adjustable parameters are those associated with the CELSS control strategy.

These might include nutrient flow rates, light intensity, and soil moisture.

If the uncertainty in both the nonadjustable and adjustable parameters is considered from the beginning of the problem, it is clear that the number of samples of the parameter space required to specify a satisfactory design by the random sampling procedure illustrated in the previous chapter is impractical. Even if the number of samples taken from the nonadjustable parameter space for every sample taken from the adjustable parameter space is as low as 10, this means 10 times as many mappings will have to be performed as those in the previous chapter. This problem is solved as follows.

Our first task is to locate 'desirable' regions in the parameter space as quickly as possible. Therefore, initially set all nonadjustable parameters to representative values (e.g., mean values). These are now considered as certain parameters. The sampling procedure of the previous chapter is then used to obtain a design satisfying the given criteria under the assumption of perfect knowledge of the uncertain nonadjustable parameters. Again, the criteria can be made less restrictive initially to obtain a solution more readily with subsequent 'convergence' to a design satisfying the original criteria. If a binary criterion is used (such as survival) this sampling stops after the first design is found satisfying the criterion. At this point, since the

nonadjustable parameters were set to representative values, a point in a region of the parameter space representing desirable system response has been located. We must then deal with the fact that we do not have perfect knowledge of the nonadjustable parameters. In light of this, we reassign the a priori probability distributions to these parameters; considering them, again, as uncertain. The problem still exists, however, of how to best locate a design which maximizes the probability of obtaining satisfactory system response. Here, the measure of performance is the estimate of this probability. For each sample of the adjustable parameter space a number (say, initially taken as 10) of samples from the nonadjustable parameter space must be taken to obtain the estimate. Our goal is to locate the design producing the highest estimate of the probability of obtaining desirable system response while keeping the total number of samples (and subsequent mappings) to a minimum.

To illustrate this idea consider the two-dimensional parameter space shown in Figure 22. Here, s_1 is considered as an adjustable parameter while s_2 is considered as nonadjustable. For simplicity, assume that uniform probability distributions have been assigned to each parameter. The enclosed areas represent regions which produce desirable system response for any (s_1, s_2) pair within the regions. These areas as defined by the

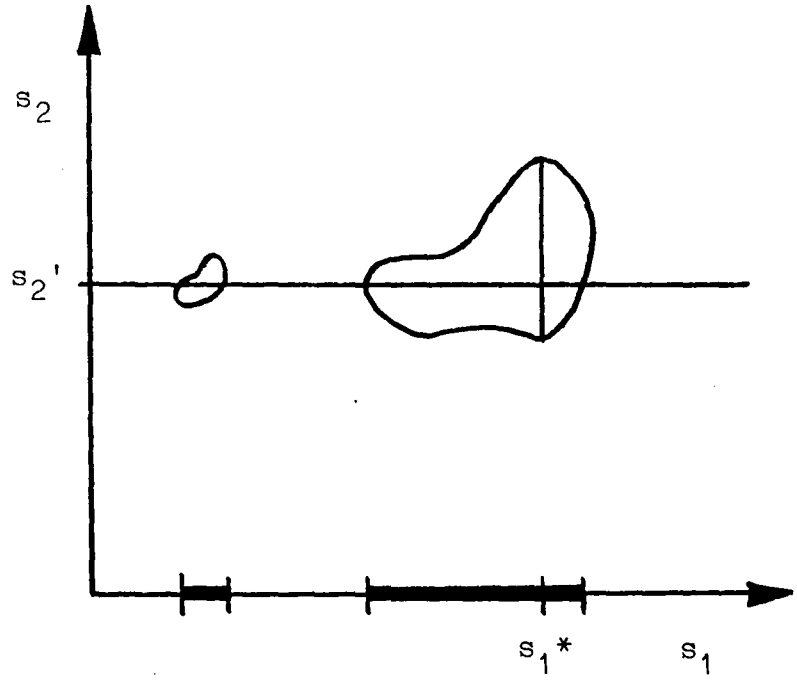

Figure 22. Example of a two-dimensional parameter space where s_1 is adjustable and s_2 is nonadjustable.

boundaries are, of course, unknown. Initially, s_2 is set to a representative value, s_2'. For this value, designs in the one-dimensional adjustable parameter space leading to desirable system response are shown by the heavy lines along the s_1 axis. Thus, if s_1 is chosen anywhere in these regions desirable system response will result given perfect knowledge of the nonadjustable parameter, s_2. During the first part of the design process the adjustable parameter space is sampled until a value for s_1 has been chosen within these regions. The uniform probability distribution is then reassigned to s_2 and a search strategy implemented to locate the desired solution, s_1*. As can be seen in Figure 22, this design maximizes the probability of obtaining desirable system response given the uncertainty in the nonadjustable parameter, s_2. This concept is easily extended to higher dimensional parameter spaces and to general probability distributions.

Since the 'desirable' regions in the parameter space are often disjoint a global-local adaptive random search strategy is used in this thesis. The first part of the search technique follows that given in [8]. The algorithm must first determine if it should search 'near' to or 'far away' from the point found above by suppressing the uncertainty in the nonadjustable parameters. For uniform a priori distributions on the adjustable parameters this is accomplished by assigning a standard range to these distributions and centering them about the above point.

Different ranges are then constructed by multiplying the standard range by various numbers (e.g., 1,2,3). The regions in the parameter space defined by these ranges are each sampled a given number of times (say 20). The region producing the highest estimate or performance measure is then sampled a number of times (say 40). So, by sampling 3 ranges 20 times each and then sampling the 'best' range 40 times, (3X20 + 40)10 = 1000 mappings have been performed thus far. (Remember that each sample of the adjustable parameter space requires a number (taken here as 10) of samples of the nonadjustable parameter space to obtain an estimate of the performance measure.) The uniform distributions are then centered about the best solution found. This procedure is repeated until the standard (smallest) region produces the best solution. The final desired solution is, therefore, 'close' to the current center of the adjustable parameter space.

The algorithm then 'converges' to the 'best' solution as was done in Chapter 4. The ranges are halved and centered about the current best solution found and a given number of samples are taken. The only difference here is that to obtain a more accurate estimate of the probability of producing desirable system response the number of samples taken from the nonadjustable parameter space for every sample taken from the adjustable parameter space (previously 10) is increased (perhaps by 25%) each time the ranges are halved. In the beginning of the search the

estimate is usually small and increases as the procedure continues. Hence, the sample size starts out small and is increased with the need for more accurate information. This is done to keep the total number of mappings to a minimum while still obtaining a desirable solution. Finally, for the 'best' design, the nonadjustable parameter space is sampled a given number of times to achieve the desired statistical properties to be used with the confidence limits for the binomial distribution.

So, the procedure is as follows.

1. Center the adjustable parameter space about the solution found assuming certainty in the nonadjustable parameters.

2. Sample a given number of times from each of the regions in the adjustable parameter space established from multiples of a standard range. This is done to determine if the algorithm should search 'near' to or 'far away' from the current center of the parameter space.

3. Sample the region containing the 'best' solution obtained in Step 2 a given number of times. Then, center the adjustable parameter space about the 'best' solution found thus far.

4. If the 'best' region is defined by the standard range (smallest), continue to Step 5; otherwise, return to Step 2.

5. 'Converge' to the 'best' design by performing a number of samples, reducing and centering the adjustable

parameter space, and increasing the number of samples from the nonadjustable parameter space for every sample of the adjustable parameter space. Step 5 is terminated when sampling the current adjustable parameter space does not produce a design (and associated probability estimate) which is 'better' than that previously found.

6. Obtain desired statistical properties by performing an additional number of samples of the nonadjustable parameter space using the final design from the adjustable parameter space.

A schematic of this adaptive random search is shown in Figure 23. When the a priori probability distributions are not taken as uniform the standard deviation (or variance) is used to define the search ranges and is illustrated in [8].

During the second part of the design procedure the performance measure is the estimate of the probability of obtaining acceptable system response. Since the number of samples from the nonadjustable parameter space is finite, this performance index is only an estimate of the real but unknown probability. Thus, the confidence limits for the binomial distribution are used for statistical inference. (The binomial distribution is utilized due to the basic binary classification scheme of either obtaining or not obtaining the behavior.) These confidence limits are shown in Figures 24 and 25 for the 95% and 99% confidence levels, respectively. The charts in these figures have

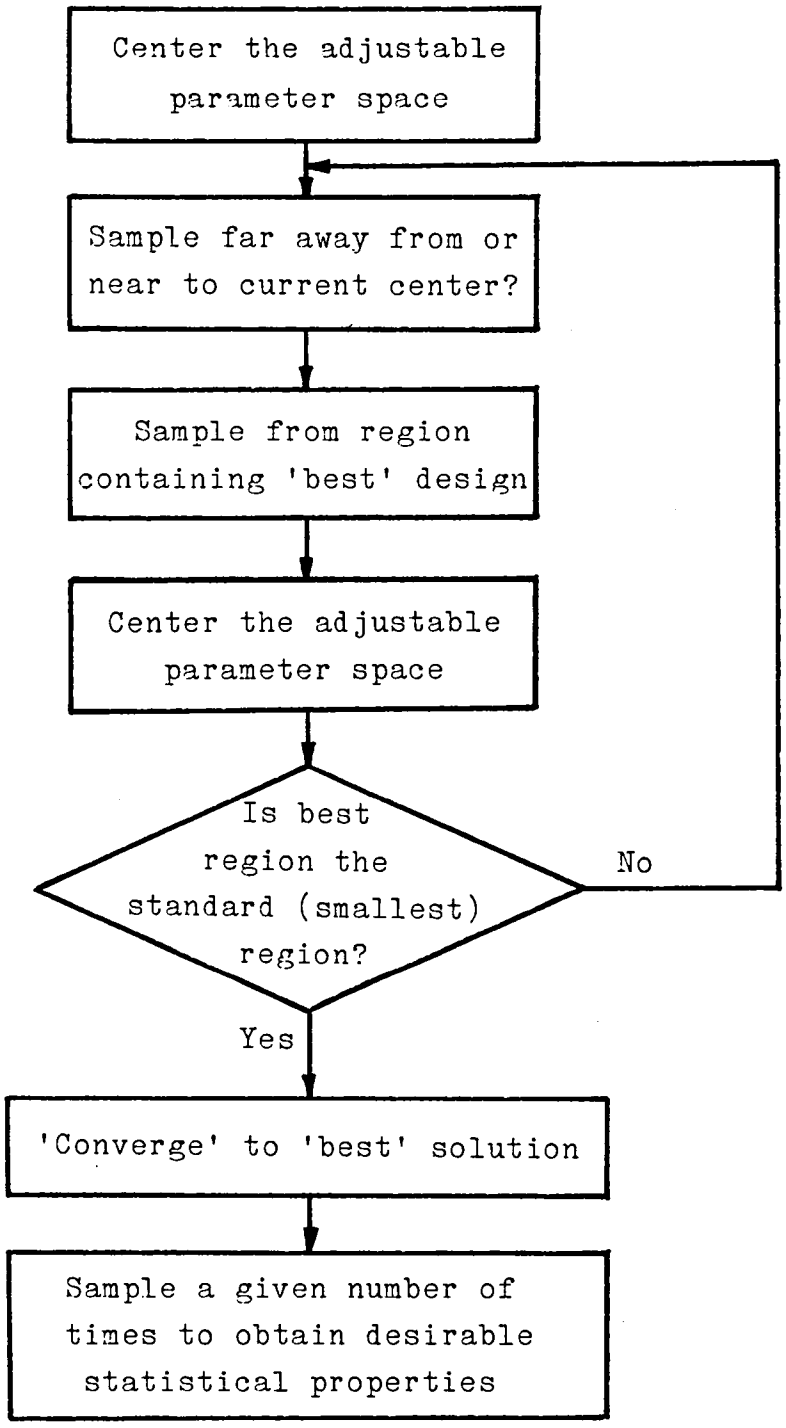

Figure 23. Schematic of the adaptive random search strategy.

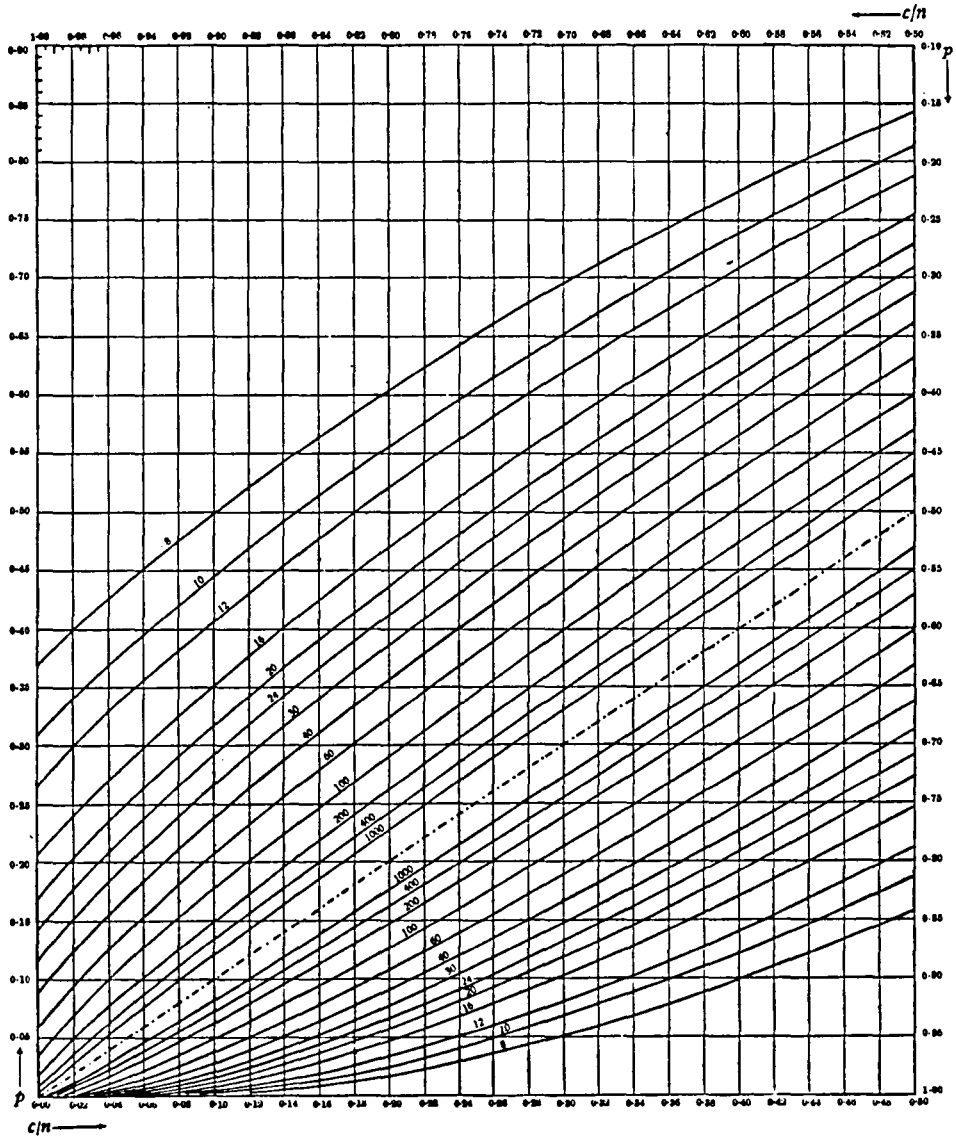

Figure 24. Chart providing confidence limits for p in binomial sampling given a sample fraction, c/n, where the confidence coefficient is equal to 0.95.

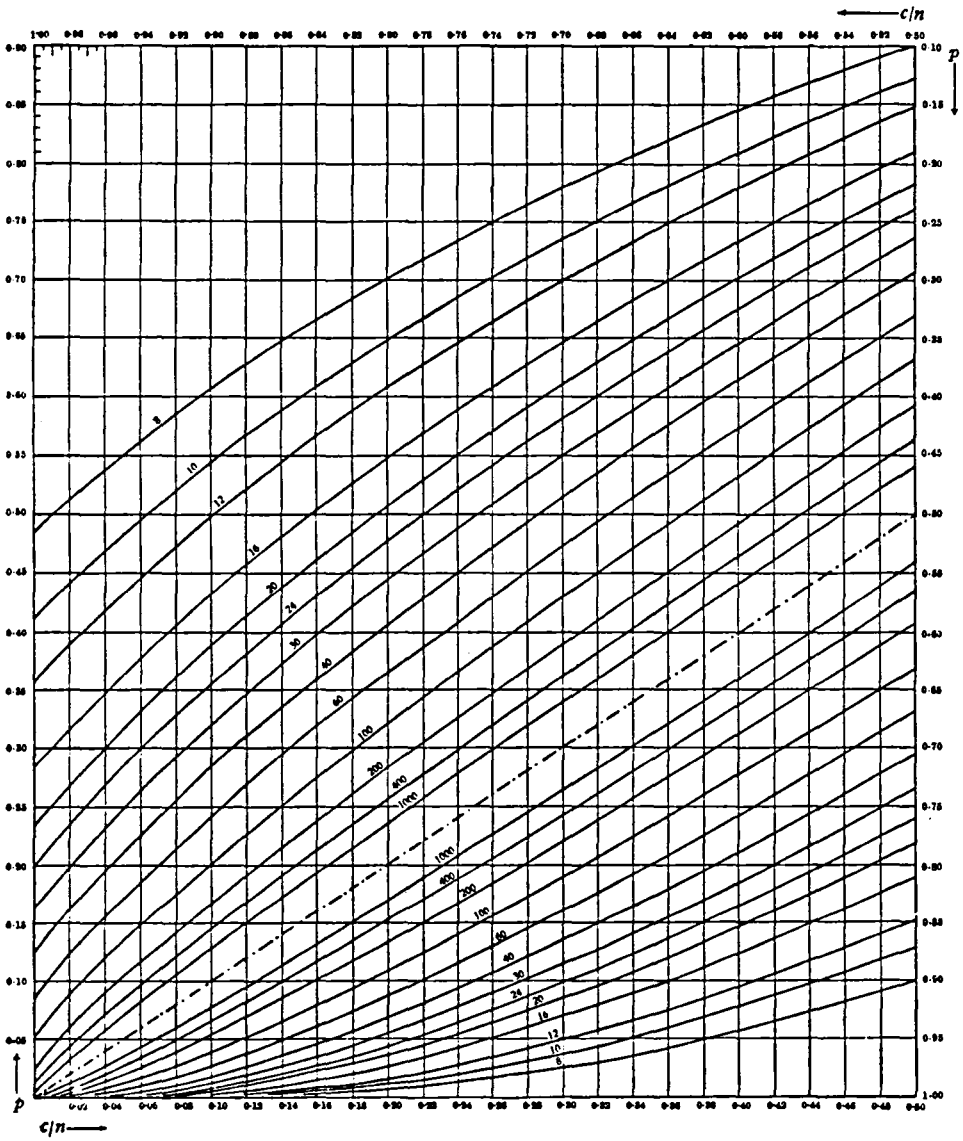

Figure 25. Chart providing confidence limits for p in binomial sampling given a sample fraction, c/n, where the confidence coefficient is equal to 0.99.

been reproduced, with permission, from the Biometrika Tables for Statisticians [9]. To illustrate the use of these charts consider an example for which the estimate of obtaining some behavior is 0.70 with a sample size of 100. The numbers printed along the upper and lower curves indicate the sample size, n. The probability estimate, c/n, is given along the abscissa. Then, using Figure 24 (with confidence coefficient equal to 0.95) we are 95% confident that the actual probability of obtaining the behavior, p, is such that

$$0.60 < p < 0.78$$

As the sample size increases the range of uncertainty of the probability decreases. If the above sample size was 400 rather than 100, then we would be 95% confident that

$$0.65 < p < 0.74$$

Of course, for many problems the design providing the highest estimate of obtaining desirable response may not be 'good enough'. In this case, better information about the parameters to which the satisfaction of the design criteria (the behavior) is sensitive must be obtained in order to provide a 'better' design. This is where the generalized sensitivity analysis discussed in Chapter 2 becomes extremely important.

Example 1 of Chapter 4 is continued where two of the system parameters are now considered to be uncertain.

Example 1 (continued)

The two parameters associated with damping and first order dynamics are now considered to be uncertain to 5% and 2% respectively. So, here

$$b = 1 \pm 0.05$$

$$c = 5 \pm 0.10$$

with all other parameters remaining unchanged. Thus, the adjustable parameter space is still 21-dimensional. However, the nonadjustable parameter space is now 2-dimensional, resulting in a total parameter space of dimension 23. When b and c were considered as certain the best controller design produced an overshoot which was less than or equal to 0.1 and a settling time of 3.70. With b and c considered as uncertain we wish to obtain a controller design which maximizes the probability that the overshoot will be less than or equal to 0.1 and the settling time will be less than or equal to 3.70. Initially, the center of the adjustable parameter space is the design obtained in the previous chapter. So, thus far, only about 500 mappings have been performed to locate this point in the parameter space.

The adaptive random search procedure was then implemented which produced a design giving an estimate of the probability of obtaining desirable system response of 0.73. 120 samples from the two-dimensional nonadjustable parameter space were taken for this design to achieve

satisfactory statistical properties. Approximately 3900 total mappings were performed to obtain this result; roughly 8 times that required for the first part of the procedure. Here, initially 20 samples from the nonadjustable parameter space were taken for each sample taken from the adjustable parameter space. This means that after the initial 500 samples of the adjustable parameter space used to obtain the best design while suppressing the uncertainty in b and c, less than 200 additional samples were used to locate the design maximizing the estimate. It is clear that if the uncertainty in the nonadjustable parameters was considered from the beginning of the problem an extremely large number of mappings would result making this search technique impractical.

With the sample size of 120 and the sample estimate (sample fraction) of 0.73 we can use the chart of Figure 24 to say that

$$0.64 < p < 0.81$$

with 95% confidence. These numbers were obtained by interpolating between the '100' and '200' sample size curves. Using the chart of Figure 25 we are 99% confident that

$$0.58 < p < 0.83$$

The final design producing this estimate (to three digits) is given in Table 5. The structure of the controller was presented in Chapter 4 and is shown in Figure 11.

Again, the above estimate of obtaining acceptable response may not be sufficiently high. The generalized sensitivity analysis must then be used in order to specify a design providing a higher estimate. For this example, the uniform distribution ranges were taken about the best design found above with the given uncertainty in the nonadjustable parameters, b and c. The distinction between the adjustable and nonadjustable parameter spaces is temporarily suspended while the parameter space as a whole is sampled a number (here, 400) of times. Each sample point results in a system response which either satisfies (passes) or does not satisfy (fails) the design criteria. For this example the number of passes was $m = 153$ and the number of fails was $n = 247$. The Kolmogorov-Smirnov two-sample test statistic, $d_{m,n}$, was then calculated for each of the 23 parameters. The satisfaction of the criteria was overwhelmingly sensitive to the damping parameter, b, with $d_{m,n} = 0.686$ for this parameter. From Table 1 (Chapter 2) we see that this value indicates that the 'pass' and 'fail' distributions for b separate at well above the 99% confidence level which gives $d_{m,n} > 0.168$ for separation. On the other hand, for the parameter c, we have $d_{m,n} = 0.109$ which

Boundary	Location	Region	VEL X 10
1	1.00	1	9.96
2	1.89	2	7.36
3	2.52	3	3.07
4	3.56	4	3.60
		5	-0.55
		6	0.26
		7	1.67
		8	0.82
		9	-0.69
		10	-1.29
		11	0.11
		12	-0.70
		13	1.57
		14	-2.32
		15	0.32
		16	-0.11
		17	-0.08

Table 5. Final design for the nonlinear controller of Example 1 with b unknown to $\pm$ 5%.

indicates that the 'pass' and 'fail' distributions do not separate even at the 80% confidence level where it is required that $d_{m,n} > 0.110$ for separation. Correlation coefficients were also calculated and found near zero indicating that for the given range of uncertainty the satisfaction of the design criteria was, indeed, insensitive to c. Therefore, to find a better design we can neglect c and focus attention on obtaining better information on the parameter b alone.

In this example we assume that this has been done narrowing the range of uncertainty on b to 2.5%. So, now we have

$$b = 1 \pm 0.025$$

where the uncertainty on c remains unchanged.

The adaptive random search procedure was again implemented with the result that the 'best' design produced an estimate of 0.99 with 120 samples. Using the chart of Figure 24 this means that

$$0.95 < p < 1$$

with 95% confidence. The chart of Figure 25 indicates that now we are 99% confident that

$$0.93 < p < 1$$

This is a great improvement over the previous design. The cost of this improvement was the time and effort spent in

obtaining better information on b. If, for some reason, better information could not have been obtained, the design providing the estimate of 0.73 would have been the final design. As the information concerning the uncertain parameters improves, the nonadjustable parameter space shrinks to a point so that for a given design the probability (and estimate) of obtaining acceptable response will be either 0 or 1. This case was considered in the previous chapter where all the parameters are adjustable.

The design producing the above estimate with b unknown to $\pm 2.5\%$ is given (to three digits) in Table 6.

Boundary	Location	Region	VEL X 10
1	1.00	1	10.00
2	1.89	2	7.32
3	2.51	3	3.08
4	3.55	4	3.54
		5	-0.58
		6	0.25
		7	1.64
		8	0.82
		9	-0.64
		10	-1.33
		11	0.09
		12	-0.72
		13	1.57
		14	-2.38
		15	0.32
		16	-0.13
		17	-0.09

Table 6. Final design for the nonlinear controller of Example 1 with b unknown to $\pm$ 2.5%.

CHAPTER 6

CONCLUDING REMARKS

A design methodology for nonlinear (more accurately, not necessarily linear) systems containing parameter uncertainty has been presented. Several fundamental concepts have been utilized in this methodology.

Fundamental to the design process is the mapping from the parameter space to the indices of performance. It is assumed that for any selection of parameter values for the system of interest performance measures can be obtained. For general nonlinear systems, however, the analytical form of this mapping is unknown. It is usually quite complicated and not one-to-one. Therefore, the parameter space must be sampled.

The idea of separating the parameter space into regions which produce a given system behavior and those which do not has also been incorporated into this methodology. The behavior could be the satisfaction of some design criteria or the occurrence of some qualitative system response.

A generalized sensitivity analysis is used to determine to what degree the behavior (or nonbehavior) is sensitive to the various parameters of the system. The

analysis is based on the degree to which the cumulative distributions for the behavior and not the behavior separate for each of the parameters under the behavioral classification.

The nonparametric Kolmogorov-Smirnov two-sample test statistic is the basis for the sensitivity ranking. Since this statistic is a function of the number of samples producing behaviors and nonbehaviors only, the results are independent of the number of system parameters. This statement must be qualified, however, since as the dimension of the parameter space increases, in general, the relative 'volume' producing desirable system response decreases. So, to obtain a given level of statistical confidence the number of samples may have to be increased. This will depend upon the sensitivity of the behavior to the parameters and through experience is seen not to be a major effect.

The parameter space is divided into the adjustable and nonadjustable parameter spaces. When the nonadjustable uncertain parameters are set to fixed values each sample of the parameter space results in a deterministic solution with subsequent satisfaction or nonsatisfaction of the design criteria. The problem in this case is one of locating regions in the parameter space producing desirable system response. Since these regions are usually disjoint, a random search technique is used. When the nonadjustable parameters are allowed to

vary, any one given sample of the parameter space is meaningless since the satisfaction of design criteria is generally sensitive to these parameters. The problem here, then, is one of obtaining a design in the adjustable parameter space which maximizes the probability of achieving satisfactory system response given the uncertainty in the nonadjustable parameters. The confidence limits for the binomial distribution are used to provide a measure of confidence in the probability estimate. If this probability is not high enough for a given problem, the generalized sensitivity analysis can be used to indicate for which parameters better information should be found. The design procedure is then repeated with this new information to obtain a higher probability estimate. To minimize the number of samples taken from the parameter space an adaptive random search technique is used in this part of the method.

The uncertainty in the nonadjustable parameters is suppressed during the first part of the design procedure. This is done to enable us to locate 'desirable' regions in the parameter space as quickly as possible. Each sample provides a measure of the relative 'goodness' of the resulting solution. Starting from a point in the adjustable parameter space satisfying the design criteria under perfect knowledge of the nonadjustable parameters, the uncertain nonadjustable parameters are then allowed to vary. Here, each sample of the adjustable parameter space

requires a number of samples from the nonadjustable parameter space to obtain an estimate of the probability of achieving satisfactory response. Thus, during this second part of the design procedure the number of mappings which must be performed is drastically increased over those required for the first part. If the uncertainty in the nonadjustable parameters was not suppressed during the first part, in general the design method would not be practical to implement.

As stated above, sampling of the parameter space and subsequent mapping to the indices of performance is vital to this design methodology. It is necessary, therefore, to perform a significant number of mappings during the design process. When all of the parameters are adjustable this number is on the order of several hundred. However, when nonadjustable uncertain parameters are present this number is on the order of several thousand. If the amount of time required to perform one mapping for a particular problem is 'large' relative to the facilities available and the desired amount of effort to be spent then the methodology described in this thesis may not be practical to implement. One thing to notice, however, is that the methodology requires no supervision from the designer. Once the a priori parameter distributions are set, algorithms can be written to carry out the rest of the design procedure. So, for a given problem, the designer must define the mapping (in many cases, by a set of

differential equations and associated performance indices), the parameter space (specified by the probability distributions), and the design criteria. The completion of the design can be automated.

These procedures were applied to nonlinear controller design. Several controller strategies were presented while illustrating this methodology. Many problems exist for which a nonlinear controller would 'out perform' a linear one. These nonlinear controllers can be used to control either linear or nonlinear systems. The difficulty arises in being able to design the nonlinear controller. It was shown that nonlinear controllers could be designed with the same effort as linear controllers. In fact, whether the control algorithm is linear or nonlinear is irrelevant when it is implemented using a microprocessor.

In the examples considered in this thesis interpolating controllers were shown to perform slightly better than noninterpolating controllers but at a cost of increased overhead. An example was given for which inclusion of the derivative of the state in the integrating table resulted in better controllers than those which did not use this information. However, in another example, the inclusion of this information made no difference in the performance of the controller. For what class of problems this is expected to occur is unknown at this time.

With the advent of the microprocessor the variety and complexity of nonlinear controllers is virtually limitless. The methodology presented in this thesis provides a means to design such controllers.

The problem not addressed by this thesis is the case where the system of interest contains uncertain functions of time of which band-limited white noise is an example. Here, even when all of the uncertain parameters are adjustable, the boundaries separating regions in the parameter space producing behaviors and nonbehaviors are no longer distinct. The transition from one region to another is now fuzzy. Thus, the binary classification must be replaced by a scheme which admits a region in which no behavior/nonbehavior classification is made. Once this problem has been solved, the class of problems for which sampling of the parameter space and statistical inference can be used for design purposes will be greatly increased.

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16. Abstract

This report discusses a design methodology capable of dealing with nonlinear systems, such as a controlled ecological life support system (CELSS), containing parameter uncertainty. The methodology was applied to the design of discrete-time nonlinear controllers. These nonlinear controllers can be used to control either linear or nonlinear systems. Several controller strategies were presented while illustrating this design procedure.

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