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TORQUES ON THE GYRO IN THE GYRO RELATIVITY EXPERIMENT

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LIST OF SYMBOLS

$\vec{T}$	Torque vector
$\vec{i}, \vec{j}, \vec{k}$	Orthogonal unit vectors in rotor axes
$\hat{F}_x, \hat{F}_y, \hat{F}_z$	Orthogonal unit vectors in electrode axes
V_i	Voltage on ith electrode (i = x, y, or z)
θ_o, ϕ_o	Polar and azimuthal angles of rotor spin vector in electrode axes
θ'	Angle between rotor spin vector and integration point of electrodes
d_o	Nominal rotor electrode gap
Δd	Variation in rotor electrode gap
α, β, γ	Direction cosines of integration point on electrodes
$\alpha_o, \beta_o, \gamma_o$	Direction cosines of rotor spin vector
r_o	Nominal rotor radius
ϵ_o	Permittivity constant
$r(\theta')$	Rotor shape as function of θ'
θ_1	Electrode half angle
M_i	$\frac{\epsilon_o r_o^2}{2 d_o^2} (V_{i+}^2 - V_{i-}^2)$ (+ and - refer to opposite electrodes)
	$= \frac{m}{\pi \sin^2 \theta_1} f_i$
P_i	$\frac{\epsilon_o r_o^2}{2 d_o^2} (V_{i+}^2 + V_{i-}^2) = \frac{m}{2\pi \sin^2 \theta_1} \left[h_i + \frac{f_i^2}{h_i} \right]$
h_i	Preload along ith axis
f_i	Acceleration along ith axis
Δ	Difference in preloads for different axes
t	Miscentering
ψ	Misalignment angle
m	Mass of rotor

TECHNICAL MEMORANDUM

TORQUES ON THE GYRO IN THE GYRO RELATIVITY EXPERIMENT

I. INTRODUCTION

The purpose of the Gyro Relativity Experiment is to determine if the orientation of inertial frames is affected by the motion of nearby matter. General relativity predicts that local inertial frames will precess due to the relative motion of the Earth as a whole (the geodetic precession) and due to the rotation of the Earth (the motional precession). The magnitude of these two effects in a low-Earth orbit are about 7 and 0.05 arc sec/yr, respectively. It has been shown [1] that all viable metric theories which fall into the PPN formalism predict the same geodetic and motional precession as general relativity to about the 1 percent level. This report addresses the question whether the Newtonian drifts on the gyro as presently conceived in the Gyro Relativity Experiment can be reduced to a level such that the geodetic and motional precessions of general relativity can be detected.

Section II is an extension of earlier works on suspension torques [2]. A simplified expression is derived for the torque on the gyro for the orientation which is planned for the experiment. If the harmonic content of the rotor shape is known, our expression allows a simple hand calculation of the torques. This is an improvement over the more complicated general formula for torque given in Reference 2. This simplified expression allows a computation of the roll-averaged torque for all the harmonics. Some numerical results are given for some typical state-of-the-art rotors. Also, the effect of scratches on the rotor coating is considered.

The remainder of the report is an analysis of the other torques on the gyro which are present. This is a parallel analysis to that of Everitt [3]. The results are not much different, but the analysis was done independently, and many of the details of the derivations are supplied. Section III gives an analysis of torques due to gas drag from first principles with all the steps in the calculation supplied. Section IV gives an independent analysis of torques due to rotor charging, based on a result of Reference 2. Section V gives a more general treatment of mass unbalance torques than in previous work. Section VI considers gravity-gradient torques in an inclined orbit and exhibits a new expression for suspension torques caused by gravity gradients in an inclined orbit after roll and orbit averaging. Section VII gives a general summary of previous work on magnetic torques. Section VIII considers cosmic ray impacts and corrects a slight error in Reference 3. Torques caused by brownian motion, jitter, photon bombardment, patch effect, and dirt particles are not discussed here, since they are small, and we have nothing to add to Reference 3.

II. ELECTRICAL TORQUES

The torques on the gyro caused by the suspension voltages have been analyzed in detail in Reference 2. In this section, we give a simplified expression for the torques valid for the planned configuration of the spin axis relative to the electrodes

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in the experiment. We also calculate the effects of roll averaging on the complete torque expression including all the odd harmonics.

The planned orientation of the spin axis of the gyro in the experiment is midway between two adjacent electrodes. That is, the direction cosines of the spin axis in the notation of Reference 2 are

$$\alpha_o = 0 \quad , \quad \beta_o = \gamma_o = 1/\sqrt{2} \quad .$$

Under these conditions the complete primary torque expression from pages 26 and 27 of Reference 2 can be greatly simplified. Defining as before the three integrals

$$I_1\,(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta d\theta \left[\alpha \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_2\,(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta d\theta \left[\beta \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_3\,(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta d\theta \left[\gamma \frac{dr(\theta')/d\theta'}{\sin\theta'}\right] \quad ,$$

where the terms are defined in the list of symbols and in Reference 2, we can, by manipulating the limits of integration in the ϕ integral, prove the following identities.

$$I_2\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = 0$$

$$I_2\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) = 0$$

$$I_1\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0$$

$$I_1\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 0$$

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$$I_2\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = -I_1\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$I_2\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right) = I_1\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$I_2\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -I_1\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$I_2\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -I_1\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$$

$$I_3\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = I_3\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$I_3\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right) = I_3\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$I_2\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = I_2\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$I_1\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = -I_1\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$I_3\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) = I_3\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right) .$$

Substituting these identities into the torque equations on pages 26 and 27 of Reference 2, we obtain upon using the expressions for acceleration f_i and preload h_i

$$T_x = \frac{m}{2\sqrt{2}\pi\sin^2\theta_1}\left\{\left[(f_y - f_z)\right]a + \frac{1}{2}\left[(h_y - h_z) + \left(\frac{f_y^2}{h_y} - \frac{f_z^2}{h_z}\right)\right]b\right\}$$

$$T_y = -T_z = -\frac{m}{\sqrt{2}\pi\sin^2\theta_1}\left[f_x c\right] , \qquad (1)$$

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where

$$a = \left[I_3\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) + I_3\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\right]$$

$$-\left[I_2\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) - I_2\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\right]$$

$$b = \left[I_3\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) - I_3\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)\right]$$

$$-\left[I_2\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) + I_2\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\right]$$

$$c = I_3\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) ,$$

and the gyro spin axis is midway between the y and z axes and perpendicular to the x-axis. This expression is an improvement over previous expressions because the dependence on f_i and h_i is explicit and all the harmonics are included in the a, b, and c expressions. The roll-averaged torque can then be computed directly from equation (1). Equation (1) has been checked with our previous computer program described in Reference 2 and agreement has been obtained. For a given rotor, we need only compute a, b, and c, and we have the complete torque.

If we expand $r(\theta')$ in a fourier cosine series, the coefficients of the a_n's can be computed numerically (as in Table 1). These numbers have been checked analytically for the second, third, fourth, and fifth harmonic and found to be accurate to six figures, verifying the accuracy of our numerical integration scheme. We notice that even harmonics only contribute to b, and odd harmonics only to a and c. Also, the coefficients do not decrease much as the order of the harmonics increases, verifying earlier work. So if we have the harmonic coefficients a_n for a given ball, we can use Table 1 to compute the complete torque. In practice, the a_n's will alternate in sign as will the X_n, Y_n, and Z_n, so a lot of cancelation will take place. The result of the summation for a, b, and c will then not be significantly greater than any one term in the series except perhaps in pathological cases. So we can get a good idea of the magnitude of a, b, and c by just looking at the general size of each term in the series. This gives us a way to quickly estimate the drifts for any given ball.

To illustrate this, consider the three rotor profiles in Reference 2, Figures 1, 2, and 3. The values of a, b, and c for these rotors are given in Table 2; we see that the general level of the harmonics is reflected in a, b, and c. b is dominated

TABLE 1. COEFFICIENTS OF a_n (θ_1 = 30 deg)

Harmonic No.	Y_i	Harmonic No.	X_i	Z_i
2	-3.84765	3	-2.94524	1.76715
4	0.96191	5	3.68156	-1.47262
6	2.52502	7	-0.27918	0.25771
8	-0.81161	9	1.30465	0.82835
10	2.78992	11	1.50071	-0.96181
12	-0.73694	13	-0.97497	0.20939
14	0.68016	15	1.45899	0.62342
16	0.02978	17	-1.69550	-0.77133
18	-1.46040	19	0.11291	0.17999
20	0.57729			

$$r(\theta') = \sum_{n=2}^{20} a_n \cos n\theta'$$

$$a = \sum_{n=2}^{20} X_n a_n$$

$$b = \sum_{n=2}^{20} Y_n a_n$$

$$c = \sum_{n=2}^{20} Z_n a_n$$

Harmonic No.	$X_i + 2 Z_i$
3	0.58906
5	0.73632
7	0.23624
9	2.96135
11	-0.42291
13	-0.55619
15	2.70573
17	-3.23816
19	0.47289

TABLE 2. ROTOR TORQUE COEFFICIENTS (μin.)

	a	b	c
Case I	0.018	4.3	-0.078
Case II	0.207	14.5	-0.106
Case III	1.065	10.1	-0.337

by the large second harmonic, while a and c are dependent on the general level of the odd harmonics. Case III shows that higher harmonics can be fairly significant. However, the random nature of both coefficients in the series for a, b, and c improves the situation. Table 3 shows the harmonic coefficients for our sample rotors from Reference 2.

TABLE 3. HARMONIC COEFFICIENTS a_n (μin.)

Harmonic No.	Case I	Case II	Case III
2	-1.0974	-3.6691	-2.5278
3	-0.0797	-0.0958	0.0217
4	0.0578	0.1878	-0.0921
5	-0.0420	-0.0328	0.2589
6	0.0656	0.0400	0.2019
7	-0.1091	-0.0445	-0.0820
8	0.0897	-0.0704	-0.0999
9	-0.0150	0.0052	0.0312
10	-0.0022	0.0366	-0.0263
11	-0.0550	-0.0055	0.0069
12	0.0158	0.0302	0.0756
13	-0.0215	0.0023	-0.0536
14	-0.0151	-0.0004	0.0150
15	-0.0122	0.0014	0.0489
16	0.0301	-0.0010	-0.0481
17	-0.0031	-0.0209	0.0123
18	0.0112	-0.0124	0.0203
19	-0.0092	-0.0032	-0.0136
20	0.0037	0.0035	0.0160

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A series of computer runs was undertaken for some general rotor orientations with the program described in Reference 2. These computations indicate that the general level of torque is given to good approximation by equation (1) for orientations near to that assumed in calculating equation (1). This means that the torque is not very sensitive to misalignment of the rotor spin axis in the experiment.

In the final gyro experiment, the spacecraft will be rolled about the spin axis of each of the gyros with a period of about 15 min. So we need to calculate the roll-averaged value of equation (1). Since the roll-averaged value of the h_y-h_z terms is obviously zero if h_y and h_z remain constant over the roll period, we need only consider the terms involving the f_i's. We first define a new set of axes such that the z' axis is parallel to the spin axis and the y' axis is perpendicular to it and the x = x' axis. In this coordinate system we have

$$f_y' = \frac{1}{\sqrt{2}} (f_y - f_z)$$

$$f_z' = \frac{1}{\sqrt{2}} (f_y + f_z)$$

$$T_x = \frac{m}{2\pi \sin^2 \theta_1} \left[f_y' \, a + \frac{f_y' f_z'}{\sqrt{2}\, h} b \right] = a' f_y'$$

$$T_y' = \frac{1}{\sqrt{2}} (T_y - T_z)$$

$$= - \frac{m}{\pi \sin^2 \theta_1} (f_x \, c) = c' \, f_x$$

$$T_z' = 0 \quad ,$$

where we have assumed $h_y = h_z = h$.

We can now average over the roll by considering a transformation from the rotating spacecraft coordinate system X', Y', Z', to an inertially-fixed coordinate system X", Y", Z"

$$X'' = X \cos \omega t - Y' \sin \omega t$$

$$Y'' = X \sin \omega t + Y' \cos \omega t$$

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$$Z'' = Z'$$

where ω is the rotation rate. Then we have

$$T_{x''} = T_x \cos \omega t - T_{y'} \sin \omega t$$

$$T_{y''} = T_x \sin \omega t + T_{y'} \cos \omega t$$

substituting in the inverse transformation

$$T_{x'} = a' \left[- f_{x''} \sin \omega t + f_{y''} \cos \omega t \right]$$

$$T_{y'} = c' \left[f_{x''} \cos \omega t + f_{y''} \sin \omega t \right]$$

and averaging, we find

$$\langle T_{x''} \rangle = \frac{a' - c'}{2} f_{y''}$$

$$\langle T_{y''} \rangle = \frac{c' - a'}{2} f_{x''}$$

which relates the roll-averaged torques to the accelerations in the inertially-fixed system. The roll-averaged drift will then be

$$\Omega = \frac{5}{2\pi} \left[\frac{a+2c+bf_\ell/\sqrt{2}\, h}{r_o} \right] \frac{f_{tr}}{v} \quad , \tag{2}$$

for $\theta_1 = 30$ deg where v is the peripheral velocity of the ball, f_{tr} is the magnitude of the acceleration perpendicular to the spin axis, and $f_\ell = f_{z'}$ is the longitudinal acceleration. Thus, if we know a, b, and c for a given rotor, we can calculate Ω for any acceleration level. The values of Ω for the three rotors considered in Reference 2 are given in Table 4 for several values of f_ℓ/h. Drifts scale as f_{tr}. So we see for the balls consider, we are at about the 0.1 mas/yr level for $f_{tr} = 10^{-10}$ g and correspondingly higher values for larger f_{tr}. For example, $f_{tr} = 10^{-8}$ g, a level which may be attainable without drag-free control, gives drifts of the order 10 mas/yr. Notice that the roll averaging does not reduce the odd harmonic torques much as can be seen from Tables 1 and 2, where there is not much cancelation in the a + 2c terms except for the third and fifth harmonic.

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TABLE 4. ROLL-AVERAGED DRIFTS (MAS/YR)
($f_{tr} = 10^{-10}$ g v = 2400 cm/sec)

f_ℓ/h	Case I	Case II	Case III
10^{-3}	0.037	0.001	0.110
10^{-2}	0.029	0.026	0.128
10^{-1}	0.032	0.282	0.306

For rotor misalignments and $h_y \neq h_z$, we will get imperfect roll averaging of the even harmonics, and this effect will be proportional to the misalignment angle. The exact magnitude of this effect for all the harmonics has not been calculated, but for misalignment angles of $\sim 10^{-4}$ rad, the effects are negligible.

In the process of depositing the niobium film on the gyro rotor, it is possible that small scratches on the film may occur. To estimate the effect of these scratches on the torque, we consider the following situations. Suppose the ball has a line along a great circle in which the ball radius changed by Δr in a distance Δt. This would represent one side of a scratch. Then along that scratch, we have

$$\frac{dr(\theta')}{d\theta'} \sim \frac{\Delta r(r_o)}{\Delta t}$$

the solid angle covered by the scratch would be

$$\Delta\Omega \sim \frac{(\Delta t)(2\theta_1 r_o)}{r_o^2} .$$

Hence the contributions to I_1, I_2, and I_3 can be estimated as

$$I \sim \frac{dr(\theta')}{d\theta'} \frac{\Delta\Omega}{\sin\theta_{min}}$$

$$\sim \frac{2\theta_1 \Delta r}{\sin\theta_{min}} \sim 4\,\Delta r$$

where θ_{min} is the minimum angle between the gyro spin axis and an electrode (θ_{min} = 15 deg). For a 100-μin. scratch, this could be a large effect, except for the following points. The other side of the scratch will contribute an equal and opposite torque if the scratch is symmetrical. Furthermore, $r(\theta')$ is the average shape of the rotor over a spin cycle so that, unless the scratch happens to be perpendicular to the spin axis, the contribution to the average shape is

$$\Delta r' = \int_0^{2\pi} \Delta r(\theta,\phi)d\phi \sim \Delta r\left(\frac{\Delta t}{r_o}\right) ,$$

so the effective Δr is reduced by a factor $(\Delta t/r_o)$ due to spin averaging of the shape. For $\Delta t \sim \Delta r \sim 100$ μin., the contribution to I becomes

$$I \sim 4\Delta r' \sim 4 \cdot 10^{-8} \ \mu\text{in.} \quad ,$$

which is considerably less than the contributions to I_1, I_2, and I_3, due to large-scale ball asphericity. Thus, we conclude that sufficiently thin or shallow scratches are not a problem, except under very improbably circumstances.

III. TORQUES DUE TO GAS DRAG

The residual gas in the gyro housing will cause rundown torques on the rotor if the pressure is uniform and drifts of the gyro if the pressure is nonuniform. We calculate these torques in this section. Since the mean-free path of the gas molecules is many orders of magnitude larger than the dimensions of the rotor-housing gap and the pressure is very low, we can use the kinetic theory of gases for an ideal gas. We calculate the number of molecules impinging on the rotor per unit time and then assume that each molecule of mass m transfers a momentum mV to the ball, where V is the peripheral velocity of the ball at the point where the molecule hits the ball. Integrating over the ball then gives the total torque.

Of those molecules which have an x component of momentum p_x, $(N/V)dA(p_x/m)$ of them will strike per second on an area dA, perpendicular to the x axis, N/V being the number per unit volume and $(p_x/m)dA$ being the volume of the prism, in front of dA, which contains all the molecules that will strike dA in a second. The average number of molecules striking dA per second is then obtained by integrating over the Maxwell distribution.

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$$(\#/\text{sec}) = \frac{N}{V}\, dA \, \frac{1}{(2\pi mkT)^{3/2}} \int_0^\infty e^{-p_x^2/2mkT} \left(\frac{p_x}{m}\right) dp_x$$

$$\int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \, e^{-\left[\frac{(p_y^2+p_z^2)}{2mkT}\right]}$$

$$= \frac{N}{V}\, dA \sqrt{\frac{kT}{2\pi m}} \quad ,$$

where k is Boltzmann's constant and T is temperature. Multiplying by mV and dividing by dA, the momentum transfer per unit area per unit time B is then given by

$$B = V\, P \sqrt{\frac{m}{2\pi kT}}$$

where P is the pressure, and we have used the ideal gas law. This agrees with Reference 4, equation (1.113) except for a factor of order 1. This factor accounts for the fact that the total momentum mV may not all be transferred to the ball, depending on the character of the surface of the ball. Our equation for B is clearly a worst-case calculation.

If we define a coordinate system by unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, where $\hat{z}$ defines the spin axis of the ball, then the momentum transfer will be in a direction perpendicular to the spin axis and tangent to the ball, that is, in a direction $\hat{\rho}$ given by

$$\hat{\rho} = \hat{x} \sin \phi - \hat{y} \cos \phi$$

where ϕ is the azimuth angle from the x-axis. The total torque is then

$$\vec{T} = \int (\vec{r} \times \hat{\rho}) B dS \quad , \tag{3}$$

where $\vec{r}$ is the radius vector to the surface element dS. Defining θ as the angle from the z axis and using $V = \omega r_o \sin \theta$, we find, assuming P is uniform,

$$T_z = -P \sqrt{\frac{m}{2\pi kT}}\, \omega r_o^4 \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \, [\sin^2\theta] = -\frac{4}{3} P \sqrt{\frac{2\pi m}{kT}}\, \omega r_o^4$$

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this produces an exponential rundown of the form

$$\omega = \omega_o e^{-t/\tau}$$

where

$$\tau = \frac{1}{5} \rho \frac{r_o}{P} \sqrt{\frac{2\pi kT}{m}}$$

in agreement with Reference 3, equation (7), where ρ and r_o are the density and radius of the ball. For $P = 10^{-9}$ torr and $T = 1.6K$, we find $\tau = 300$ yr so the ball speed will decrease by about 0.3 percent in a year. From equation (3), we find

$$T_x = \sqrt{\frac{m}{2\pi kT}}\, \omega r_o^4 \int_o^{2\pi} d\phi \int_o^{\pi} \sin\theta d\theta P\, [\sin\theta \cos\theta \cos\phi]$$

and a similar expression for T_y with $\cos\phi$ replaced by $\sin\phi$. If P is uniform, both these expressions vanish. However, if P varies in the housing, it can cause drift-producing torques. Letting $P = P_o + \Delta P$ and letting ΔP vary over an area A, the worst-case drift becomes

$$\Omega_p = \frac{15}{16\pi} \sqrt{\frac{m}{2\pi kT}} \frac{\Delta P A}{\rho r_o^3}$$

where we have replaced the bracketed quantity in the integrand by its maximum value, 1/2. This is similar to equation (126) of Reference 3. Letting $A = 1\ cm^2$, $\Delta P = 10^{-10}$ torr, and $T = 1.6K$, we find $\Omega_p \sim 1$ arc sec/yr. So we see that drifts caused by pressure variations can be quite large. However, they are averaged very well by spacecraft roll, since the pressure variations will roll with the spacecraft. Since the averaging effect goes as 1/# rolls and at a 15-min roll period there are about 10^4 rolls in a year, the drift is reduced to about 0.1 milliarcsec/yr which is similar to the number given in Reference 3, page 571. Again, we are relying strongly on spacecraft roll to reach the accuracy goal of the experiment. However, if the gyro housings are sealed during the experiment, the pressure uniformity may be considerably better than the 10 percent assumed here. The effects calculated here could be tested on the ground at higher pressures and extrapolated to the lower pressures, but they could not be checked at the lower pressures required for the experiment on the ground, since drifts from other sources are much larger. Reference 3 reports data in which the rundown was measured to be about 1 order of magnitude larger than that expected

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from the theory presented here. This extra rundown may have been due to the suspension system which can cause large rundown at high g-levels. This effect would be greatly reduced with a low-g suspension system.

IV. TORQUES DUE TO BALL CHARGING

The rotor in the gyro experiment will acquire some charge due to the charged-particle environment in space and possibly electrical breakdown during the initial levitation. In this section, we calculate the torque due to the electrical charge and put limits on its size.

From Reference 2, the torque on the gyro is given by the expression

$$\vec{T} = \frac{\varepsilon_o r_o^2}{2d_o^2} \sum V_i^2 \iint \left(1 - 2\frac{\Delta d}{d_o}\right) [\hat{F}_x (\beta\gamma_o - \gamma\beta_o) + \hat{F}_y (\gamma\alpha_o - \alpha\gamma_o)$$

$$+ \hat{F}_z (\alpha\beta_o - \beta\alpha_o)] \frac{(dr(\theta')/d\theta')}{\sin\theta'} \sin\theta d\theta d\phi \quad , \tag{4}$$

where the terms are defined in the list of symbols and in Reference 2. For a charged rotor, we can then take all the V_i's to be given by

$$V_i = \frac{Q}{4\pi\varepsilon_o} \frac{d_o}{r_o^2} \tag{5}$$

where Q is the total charge on the ball. This follows from Gauss' law if we assume that the charge is distributed uniformly to first approximation. Then, if σ is the surface charge per unit area, the radial electric field is $E_r = \sigma/\varepsilon_o = Q/4\pi\varepsilon_o r_o^2$, following the construction of a Gaussian surface as in elementary physics. Since the rotor-electrode gap is much smaller than the rotor radius, we can assume a parallel plate capacitor and $V_i = E_r d_o$ from which equation (5) follows. This equation also follows for the situation of two spherical conductors with the inner one charged, which our situation here only approximates.

We can now use equation (4) to evaluate the torque. If we neglect the $\Delta d/d_o$ terms and consider the case $\beta_o = \gamma_o = 1/\sqrt{2}$, we can see from the equations on pages 26 and 27 of Reference 2 that $\vec{T} = 0$. This follows from the identities in Section II. If we are not precisely at this symmetry point (which is the planned configuration for the experiment), then it is not obvious from the general expression that $\vec{T} = 0$, but expansion in terms of harmonics indicates that it is, since only differences in voltages occur. This means that the torques will come from the $\Delta d/d_o$ terms in equation (4). Since these terms are very complex to evaluate (Reference 2, Section VIII), we will

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only estimate their order of magnitude. Taking, for example, the contribution from the z+ electrode, we find

$$\vec{T}_{z+} = \frac{Q^2}{32\pi^2 \varepsilon_o r_o^2} \left[\frac{\hat{F}_x}{\sqrt{2}} (-J_2 + J_3) + \frac{\hat{F}_y}{\sqrt{2}} (+J_1) + \frac{\hat{F}_z}{\sqrt{2}} (-J_1) \right]$$

where

$$J_1 = \int_0^{\theta_1} \sin\theta d\theta \int_0^{2\pi} d\phi \left(\frac{\Delta d}{d_o}\right) \alpha \frac{dr(\theta')/d\theta'}{\sin\theta'}$$

$$J_2 = \int_0^{\theta_1} \sin\theta d\theta \int_0^{2\pi} d\phi \left(\frac{\Delta d}{d_o}\right) \beta \frac{dr(\theta')/d\theta'}{\sin\theta'}$$

$$J_3 = \int_0^{\theta_1} \sin\theta d\theta \int_0^{2\pi} d\phi \left(\frac{\Delta d}{d_o}\right) \gamma \frac{dr(\theta')/d\theta'}{\sin\theta'}$$

$$\cos\theta' = \frac{1}{\sqrt{2}} [\sin\theta \sin\phi + \cos\theta] \quad .$$

Similar terms appear for the other five electrodes. An estimate of the order of magnitude of these terms can be obtained by factoring out $(\Delta d/d_o)$ and taking only the second harmonic for $r(\theta')$. For example,

$$J_3 \sim \frac{\Delta d}{d_o} 4a_2 \int_0^{\theta_1} \sin\theta d\theta \int_0^{2\pi} d\phi \frac{[\sin\theta \sin\phi + \cos\theta]}{\sqrt{2}} \cos\theta$$

$$= \frac{\Delta d}{d_o} \frac{8\pi a_2}{\sqrt{2}} \left[\frac{1 - \cos^3\theta_1}{3} \right] \quad .$$

For $\theta_1 = 30$ deg, then just for the J_3 term on the z+ electrode, we get

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$$T_{z+} \sim \frac{Q^2}{8\epsilon_o r_o^2} \; a_2 \left(\frac{\Delta d}{d_o}\right) \; \frac{(0.117)}{\pi} \quad .$$

Multiplying by a conservative factor of 3 for the J_1 and J_2 contribution and a factor of 6 for the six electrodes, we find

$$T \sim \frac{Q^2}{4\epsilon_o r_o^2 \pi} \left(\frac{\Delta d}{d_o}\right) \; a_2 \quad .$$

This is a very conservative number since a lot of cancelation will take place if the original expression for $\vec{T}$ were calculated exactly. The higher harmonics will contribute to the torque also, but we know from Reference 2 that a_2 is the dominant contributor to these calculations. This gives a drift rate

$$\dot{\Omega} = \frac{15}{32\pi^2} \; \frac{Q^2}{\rho\epsilon_o r_o^6} \left(\frac{a_2}{V}\right) \left(\frac{\Delta d}{d_o}\right) \quad ,$$

which is similar to that of Reference 3, equation (120) [equation (120) requires a factor of ρr_o in the denominator to be dimensionally correct]. Now, assuming a_2 = 2 μin., $\Delta d/d_o \sim 1/100$, V = 2400 cm/sec, we have as a limit on Q in order to obtain 0.3 milliarsec/yr drift

$$Q = 4 \times 10^7 \text{ electrons}$$

which is the same order of magnitude as in Reference 3, page 561. We know, however, that the $\Delta d/d_o$ terms are averaged by roll so that assuming the standard reduction of 10^{-4} due to roll averaging, we get a limit of

$$Q = 4 \times 10^9 \text{ electrons}$$

which is again similar to Reference 3.

Whether this level of charge can be maintained is an open question. Reference 3 gives some data which indicate that for the DISCOS satellite, the charge level was kept acceptably low. Reference 3 gives a discussion of this and a method to measure the charge buildup.

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V. MASS UNBALANCE TORQUES

If the center of mass is displaced from the center of support of the rotor, then there will be a torque of the form

$$\hat{T} = (\Delta r)\, \hat{S} \times \vec{F} \quad ,$$

where $\hat{S}$ is a unit vector in the direction of the gyro spin vector and $\vec{F}$ is the external force. Δr is defined as

$$\Delta r = \hat{S} \cdot \frac{\int \vec{r}\, \rho(\vec{r}) dV}{\int \rho(\vec{r}) dV} \quad ,$$

where the volume integral of the density is taken over the actual mass distribution. The origin is taken as the center of support of the rotor. For a spherical coordinate system with θ defined with respect to $\hat{S}$, the integral can be written

$$m\Delta r = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{r(\theta,\phi)} r^3 dr\ \rho(r,\theta,\phi) \cos\theta \quad ,$$

where $r(\theta,\phi)$ is the actual shape of the rotor and m its mass. We wish to calculate the effect of nonuniformities of ρ and $r(\theta,\phi)$ from this integral. They will lead to worst-case drifts of the form

$$\Omega = \frac{5}{2} \left(\frac{\Delta r}{r_o}\right) \frac{f}{V} \quad .$$

We can expand ρ in spherical harmonics [5]

$$\rho(r,\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \rho_{\ell m}(r)\, Y_{\ell m}(\theta,\phi) \quad .$$

If we now consider only density nonuniformity in a spherical ball, then, since $P_1(\cos\theta) = \sqrt{4\pi/3}\, Y_{10}(\theta,\phi) = \cos\theta$ and using the orthogonality properties of $Y_{\ell m}$

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$$m\Delta r = \sqrt{\frac{4\pi}{3}} \int_0^{r_o} r^3 dr \; \rho_{10}(r)$$

where

$$\rho_{10}(r) = \sqrt{\frac{3}{4\pi}} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \; \rho(r,\theta,\phi) \cos\theta \quad .$$

Given the actual density profile, then this integral could be evaluated. Notice that only the lowest spherical harmonic coefficient contributes to the torque. If we now assume $\rho_{10}(r)$ is independent of r, then, for a given drift rate Ω_o, this requires

$$\frac{\rho_{10}}{\rho} < \frac{8}{5} \sqrt{\frac{4\pi}{3}} \frac{V}{f} \Omega_o = 6 \times 10^{-6}$$

for V = 2400 cm/sec, f = 10^{-10} g, and Ω_o = 0.5 milliarcsec/yr. This is a condition on ρ_{10} not on the higher $\rho_{\ell m}$. If density variations were due only to $P_1(\cos\theta)$, then

$$\Delta\rho = \rho(\theta=0) - \rho(\theta=\pi) = \sqrt{\frac{3}{\pi}} \; \rho_{10} \quad ,$$

but this is an overly conservative criteria and much larger peak-to-valley excursions in density can occur, provided they can be characterized by spherical harmonics other than Y_{10}.

If we had a bubble at (R_o, θ_o, ϕ_o), then

$$m' r = \Delta m \; R_o \cos\theta_o$$

and for the above parameters

$$\frac{\Delta m}{m} = \frac{r_o}{R_o} \frac{1}{\cos\theta_o} (7.6 \times 10^{-7})$$

or the bubble has radius

$$r \sim 0.2 \text{ mm} \quad .$$

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Now consider a nonspherical ball of uniform density and radius

$$r(\theta,\phi) = r_o + \Delta r(\theta,\phi) \quad .$$

If we choose the origin at the center of the ball for $\Delta r = 0$, then the r_o term contributes nothing and

$$m\Delta r = \rho_c r_o^3 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \, \Delta r(\theta,\phi) \cos\theta \quad ,$$

where we are considering Δr as due to a coating of density ρ_c. Expanding Δr in spherical harmonics and using the orthogonality relations again gives

$$\Delta r(\theta,\phi) = \sum_{\ell=o}^{\infty} \sum_{m=-\ell}^{\infty} a_{\ell m} Y_{\ell m}$$

$$m\Delta r = \sqrt{\frac{4\pi}{3}} \, \rho_c r_o^3 a_{10}$$

where

$$a_{10} = \sqrt{\frac{3}{4\pi}} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \, \Delta r(\theta,\phi) \cos\theta$$

or for a given drift rate

$$\frac{a_{10}}{r_o} < \frac{4}{5} \sqrt{\frac{\pi}{3}} \frac{V}{f} \frac{\rho_r}{\rho_c} \Omega_o \sim 4 \times 10^{-7}$$

for $\rho_c/\rho_r \sim 4$ where ρ_r is the rotor density and the previous values of V, f, and Ω_o. If $\Delta r(\theta,\phi)$ is known, we can compute a_{10} from the above integral and check the condition on a_{10}. If $\Delta r(\theta,\phi)$ were all due to $P_1(\cos\theta)$, then

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$$\Delta r(\theta=0) - \Delta r(\theta=\pi) = \sqrt{\frac{3}{\pi}}\, a_{10}$$

but again this is over pessimistic. In particular, for ball-coating procedures in which selected, equally-spaced sites are coated, the coating shape would be largely due to the higher $Y_{\ell m}$ which do not contribute to Δr. So much larger density variations could be tolerated than are evident from the condition on a_{10}. A more quantitative statement requires an actual coating profile.

Finally, we consider a scratch of depth $\Delta r'$ and width $d = r_o \Delta\theta$ at $\theta=\theta_o$. Then

$$m\Delta r = 2\pi\rho_c r_o^{\,2} \sin\theta_o \cos\theta_o \,(\Delta r' d)$$

which requires for previous values of $\Omega_o f, V$

$$\frac{\Delta r' d}{r_o^{\,2}} < \frac{4}{15}\,\frac{\rho_r}{\rho_c \sin\theta_o \cos\theta_o}\,(1.9 \times 10^{-6}) \quad .$$

For $\Delta r' = d = 100$ μin., this seems to be satisfied for $\rho_c/\rho_r \sim 4$. So from the point of view of mass unbalance, we can tolerate fairly large scratches on the rotor.

VI. GRAVITY GRADIENTS IN AN INCLINED ORBIT

In an inclined orbit, the spin axis of the gyros and spacecraft will not be in the orbit plane. This introduces extra terms in the gravity-gradient expressions not considered in Reference 2, Section IX. These terms are discussed in this section.

If the orbit is in the y-z plane and the spacecraft and gyro spin axis make an angle θ with this plane, then using the expression for the gravity-gradient forces in Reference 2, Section IX, the gravity-gradient forces are seen to be

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \frac{GM}{R^3} \begin{pmatrix} -\rho_o \cos\omega t \cos\theta - \rho_z \sin\theta \\ (3\cos^2\Omega t - 1)\,\rho_o \cos\omega t + 3\cos\Omega t \sin\Omega t\,(-\rho_o \sin\omega t \sin\theta + \rho_z \cos\theta) \\ (3\cos\Omega t \sin\Omega t)\,\rho_o \cos\omega t + (3\sin^2\Omega t - 1)(-\rho_o \sin\omega t \sin\theta + \rho_z \cos\theta) \end{pmatrix},$$

where

ρ_o = displacement of gyro perpendicular to spin axis of spacecraft

ρ_z = displacement of gyro parallel to spin axis of spacecraft

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ω = spacecraft roll frequency

Ω = orbit angular frequency

R = orbit radius .

If we now average this over time in the manner of Reference 2, Section IX, we find

$$\begin{pmatrix} \langle f_x \rangle \\ \langle f_y \rangle \\ \langle f_z \rangle \end{pmatrix} = \frac{GM}{R^3} \begin{pmatrix} -\rho_z \sin\theta \\ o \\ +\frac{1}{2}\rho_z \cos\theta \end{pmatrix} .$$

Transforming this to a coordinate system with the z' axis coinciding with the spacecraft spin axis, we find that the component of force perpendicular to the gyro spin axis is

$$\langle f_{tr} \rangle = -\frac{3GM}{2R^3} \rho_z \sin\theta \cos\theta \quad ,$$

and the force along the spin axis is

$$\langle f_\ell \rangle = \frac{GM}{R^3} \rho_z \left(\frac{3}{2}\cos^2\theta - 1 \right) \quad .$$

For $\rho_z \sim 10$ cm, $\langle f_{tr} \rangle \sim 10^{-8}$ g, this will produce torques on the gyros of order 10 mas/yr from Table 3 with our sample gyros. This is 2 orders of magnitude larger than gravity-gradient effects in a polar orbit, indicating that a polar orbit is a much better choice for the experiment. The only other possibilities would be to (1) minimize ρ_z in the "four square" configuration or (2) use the known time dependence of θ which can be calculated from the known orbit regression rate to separate the gravity-gradient drifts from the relativity drifts. Considering the effort involved in this experiment, neither of these alternatives seems reasonable. So, it would seem that these gravity-gradient considerations make a polar orbit imperative.

Notice also that the product $f_\ell \, f_{tr}/h$ in equation (2) can be fairly large and does not average to zero for the gravity gradients. For $h \sim 10^{-6}$ g, this term is comparable to the other terms in equation (2), so it must be included in the analysis if modeling is attempted for the torques in an inclined orbit. The exact form of this term after orbital and roll averaging equation (1) in the manner of Sections I and IX, Reference 2, is

$$\langle T_1 \rangle = \frac{-b}{16\sqrt{2}\,h} \; \frac{m}{2\pi \sin^2\theta_1} \left(\frac{GM}{R^3}\,\rho_o^{\,2}\right) \left(\cos^4\theta - \sin^3\theta\cos\theta + 3\sin\theta\cos\theta\right)$$

$$\langle T_{11} \rangle = \frac{b}{4\sqrt{2}\,h} \; \frac{m}{2\pi \sin^2\theta_1} \left(\frac{GM}{R^3}\right) \left[-\rho_z^{\,2}\left(3 - \frac{27}{4}\cos^2\theta\right)\sin\theta\cos\theta \right.$$

$$\left. + \frac{\rho_o^{\,2}}{2}\left(-\frac{1}{2}\cos^2\theta - \frac{3}{2}\sin^2\theta\cos^2\theta + \sin\theta\cos^3\theta\right)\right] ,$$

where T_1 means the component parallel to the orbit plane and T_{11} means the component perpendicular to the orbit plane. ρ_o is the displacement of the gyro from the spin axis. For $h \sim 10^{-6}$ g, the $\rho_z^{\,2}$ term leads to drifts of order 10 mas/yr for $\rho_z \sim 10$ cm. Thus the "in line" configuration produces unacceptably large gravity-gradient drifts from the f^2/h terms in an inclined orbit. However, in this case, going to the "four square" configuration will not help, since then we pick up the $\rho_o^{\,2}$ terms which are also of order 10 mas/yr. In both cases, these terms could probably be modeled and removed in the data reduction. Again, the polar orbit would avoid this complicated procedure.

VII. MAGNETIC TORQUES

The rotating gyro in the gyro experiment will produce the London moment field which will then interact with its surroundings, including the readout loop, the electrodes, and any superconducting shields present. This interaction will produce back-reaction torques which have been analyzed in great detail primarily by Holdeman [6,7,8] and Ebner and Sung [9]. A brief discussion and summary of these calculations is given here.

The basic equations governing superconductivity are the London equations [10]. These equations predict that magnetic field is excluded from a nonrotating superconductor, except for a very thin region near the surface. Hence, as a good approximation, one can assume that the appropriate boundary conditions are that the normal component of the magnetic field $\vec{B}$ is zero at the surface of the superconductor. The magnetostatic form of Maxwell's equations in a vacuum is

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{B} = 0$$

so that one can define a potential function ψ such that

$$\nabla^2 \psi = 0$$

and this, together with the above-mentioned boundary conditions on $\vec{B}$, constitute a boundary value problem very similar to traditional electrostatic boundary value problems. Solving the problem then allows one to calculate torque on the gyro from the integral of the expression for the Lorentz force on the surface currents in the rotor.

Using this type of approximation the following magnetic torques have been analyzed.

1) Torques due to external magnetic fields on the gyro. These torques have been analyzed both with the full London equations and with the approximation mentioned above. The results are similar and are understandable as the traditional expression for the torque on a magnetic dipole in a magnetic field where the magnetic dipole moment is given by the London moment.

2) Torques due to the readout ring. These torques have been analyzed with and without a spherical shield. The result is understandable as the torque due to the interaction of two magnetic dipoles, one with the London moment and one with the magnetic moment of the readout loop.

3) Torque due to flux coupling. The orientation of the magnetic moment effects the flux linked by the readout loop and thus the energy of the system. This energy is angle dependent and thus leads to a torque.

4) Torques due to residual trapped flux in the shield or the electrodes.

5) Torques due to a flat section in an otherwise spherical shield.

6) Torques due to holes in a spherical shield.

7) Dissipation torques from electrodes and shields.

All these torques are found to be negligible provided (1) magnetic field, currents, and alignments are kept to levels which are already required for other reasons in the experiment; and (2) roll averaging is relied on. These calculations rely primarily on simple magnetostatics, so one can have a great deal of confidence in them.

Holdeman [6] has given a simple example to show how a superconducting shield can torque the gyro. Consider a plane shield a distance d from the gyro. Then, one can use the method of images familiar from electrostatics to insure that the boundary conditions are satisfied at the shield. The torque will then be given by

$$\dot{N} = \dot{M} \cdot \left[\frac{3\,\hat{n}\,(\vec{M}'\cdot\hat{n}) - \vec{M}'}{(2d)^3} \right]$$

$$= \dot{M} \cdot \dot{B}$$

where $\dot{B}$ is the field of the image magnetic dipole M' and $\hat{n}$ is a unit vector connecting the image dipole with the real dipole, both with magnetic moment equal to the London moment. For d $\sim$ 4 cm and ω = 200 Hz, we find drift rates of order

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$$\Omega \sim (15\ \Omega_o) \sin 2\ \alpha$$

where Ω_o = 1 mas/yr and α is the angle between $\hat{n}$ and $\hat{M}'$. The superconducting shields, by excluding magnetic fields, can produce relatively large drifts, and symmetry must be relied on (in this case $\alpha \sim 0$ or $\alpha \sim \pi/2$).

While a spherical shield will produce no torque on the gyro, a cylindrical shield will, and one can get a general idea of the order of magnitude from the plane calculation. So if cylindrical shields are used, care must be taken to preserve the symmetry of the situation. This means the gyro must be on the axis of the cylinder and either parallel or perpendicualr to it. Otherwise the torques could be unacceptably large. The calculation of torque due to a cylindrical shield has not been carried out and would require extensive analytical and numerical work.

VIII. COSMIC RAY IMPACTS

The energy loss of a minimum ionizing proton per unit length dE/dx is about 2 meV/gm/cm^2 when traversing material due to electromagnetic interactions. Its momentum loss ΔP is approximately $1/c\ dE/dx\ \Delta x$ for a relativistic particle where Δx is the path length. If all this momentum loss were transferred to the gyro a deflection of the spin axis $\Delta\theta$ would result of order

$$\Delta\theta \sim \frac{r_o \Delta P}{I\omega}$$

$$\sim \frac{r_o^2 \frac{dE}{dx}}{I\omega c}$$

$$= 7.5 \times 10^{-21} \text{ rad} \quad .$$

For heavy cosmic rays this result is multiplied by z^2 where z is the atomic number. If N particles were incident on the gyro from random directions, then we have a random walk process and the total deflection would be multiplied by $\sqrt{N}$. For protons the flux f_p of particles is about 2/cm^2/sec so for Δt = 1 year

$$N = (\pi r_o^2 f_p \Delta t) = 6.8 \times 10^8$$

and

$$\Omega = \frac{\Delta\theta\sqrt{N}}{\Delta t} = 4 \times 10^{-8} \text{ mas/yr} \quad .$$

This is a factor of 10^3 smaller than that of Reference 3 Section L3(f) due to a dropping of a factor of ω in the denominator of the equation in that section. For heavy nuclei, Ω is increased by a factor of $Z^2\sqrt{f_z/f_p}$ where f_z is the flux of particles of atomic number z. For iron this can result in about a two order of magnitude increase in Ω. For nuclear interactions, if n secondaries are produced then Ω is modified by a factor $n\sqrt{f_s/f_p}$ where f_s is the flux that produces showers and this can result in an increase in Ω of about an order of magnitude. So we see that cosmic rays are not a problem. Solar flares and particles trapped in the radiation belts are also not significant since the number of such particles which reach the gyro in a year is not much larger than the number of cosmic rays impacting the rotor. Any directional effect would destroy the $\sqrt{N}$ dependance of Ω, but this is unlikely and would only increase Ω by about 4 orders of magnitude.

IX. CONCLUSION

The conclusion was reached that for the current gyro concept of the Gyro Relativity Experiment the Newtonian drifts can be reduced to levels well below the relativistic effects to be measured provided the conditions on rotor sphericity, mass unbalance, and charge and magnetic field, residual gas pressure, and alignment described in this report are met. These requirements, though difficult to meet, seem to be feasible. The most challenging conditions appear to be coating uniformity to meet the mass unbalance criteria and residual gas pressure. Also, a sufficiently low g environment is crucial for the experiment. Such a low g environment is only available in space and is the reason for the dramatic improvement of gyro performance possible in the Gyro Relativity Experiment.

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