FINAL TECHNICAL REPORT

Contract NAS8-33731

By

J. E. Rush

Principal Investigator

Prepared for

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812
I. INTRODUCTION

This report covers research on acoustic levitation, air-jet levitation, and heat transfer from molten samples. Although the work on these topics was not completely sequential, they are separated in this report for clarity. The performance period was May 8, 1980 to August 31, 1982.

The thrust of this research was toward obtaining a better understanding and improving the quality of containerless processing systems of interest to Marshall Space Flight Center (MSFC). Such systems have application to the study and processing of materials in situations in which contact with a container must be avoided, and have potential application in both ground-based and orbiting laboratories. An overview of the general subject of containerless processing is given by Naumann and Herring (1). Typical applications of the systems studied here are in the development and study of glasses from materials which normally crystallize upon cooling.

In addition to the reports which have resulted from this work and which are identified in the following sections, the PI gave presentations on this work at the Space Sciences Laboratory, MSFC; a
Materials Research Society Symposium (see Appendix B); the University of the South Sigma Xi Chapter; the University of Alabama (Tuscaloosa); and The University of Alabama in Huntsville.

II. ACOUSTIC LEVITATION

A. Background

A single-axis acoustic levitator has been under study and development for several years under NASA sponsorship, the primary contractor being Intersonics, Inc. The work of this report has been primarily in support of, and in addition to, that of Intersonics. The Principal Investigator (PI) is indebted to Dr. Roy Whymark and Dr. Charles Rey of Intersonics for their cooperation in sharing information during the course of this research.

The acoustic levitator utilizes a high-intensity standing-wave sound field to levitate or position small (1 cm or less) objects. The basic device was designed by St. Clair (2) in 1940 for a completely different purpose. Current levitator designs are available in reports from Intersonics, Inc. The basic theory for employing high-intensity sound sources for levitation was worked out by King (3) in 1934, based on earlier work by Rayleigh.

The PI had done work related to the levitator prior to the award of this contract, first as a NASA/ASEE Summer Faculty Fellow at MSFC (1979) and later as a physicist on a related contract. The fellowship involved experimental work relative to the causes of problems in spot-heating samples in the levitator at 1 g, and is summarized in a final report (4) and in the proceedings of a symposium (5). Earlier work on the levitator was reported by Oran, et al. (6), and by Whymark (7).
The work following the fellowship was directed at obtaining a better theoretical understanding of the failure of spot heating. It turned out that quite a bit of related research had been done in other contexts, primarily involving the effect of acoustic fields on heat transfer from cylinders. Most of the pertinent references are contained in a review article by Richardson (8). The significant effect noted by these authors is a dramatic increase in heat transfer from cylinders in air when placed in an acoustic field of 140 db or greater.

While several explanations for the enhanced heat transfer rate have been proposed and the definitive answer has not been given, it is clear (9) that the acoustic field causes the laminar flow of gravitational convection to become turbulent, and this turbulence is correlated with the drastically increased rate of heat transfer. It was this turbulence which caused spot-heated samples to be lost from the 170 db acoustic field in the laboratory. The turbulence, however, is not manifest in acoustic fields with unheated samples. The fact that neither heating nor high-intensity sound, independently, produce significant turbulence, while the combination does, is not particularly surprising because of the nonlinearity of the equations at the intensities employed for levitation. Indeed, the nonlinearity is basic to the levitation process (see the equation for the mean sound-pressure level derived by King, Ref. 3, p. 215).

A summary of the published results on heat transfer in acoustic fields, including a brief analysis of each paper, is available from the PI. Also available are 16 mm motion-picture films showing the behavior of samples at 1 g, including the effect of heating, and plots of dc
(time averaged) pressure, ac pressure, and temperature in the sound field, with and without spot-heated samples.

B. Further Studies of Heat Transfer in Acoustic Fields

Because no definitive conclusions about the mechanisms for heat transfer in acoustic fields had been reached in the published literature, and because the parameter space involved in our levitation work had not been explored by other authors, it was considered advisable to carry out further studies of the phenomenon. The apparatus consisted of a very small cylindrical solenoid of Kanthal wire coated with a nonconducting material to form a 1.0 cm long cylinder with a diameter of 0.25 cm. The cylinder (several similar ones were used) was attached to nichrome wire and rigidly supported in the sound field at a velocity antinode in the standing waves.

Detailed measurements were made of the heat transfer rate in thermal equilibrium for cylinder temperatures up to 700°C and sound pressure levels (SPL) up to 170 db. Higher SPL values were available from the apparatus but could not be accurately measured. An estimated SPL of 178 db was observed.

Further details of these measurements, results, and comparison with published papers, are contained in an interim report which is made Appendix A of this final report. Note that Appendix A has a self-contained list of references.

C. Analysis of Behavior of Sample in Acoustic Field at Low G

At the request of R. Naumann of MSFC, the PI assisted in the analysis of results of a single-axis levitator experiment on the
SPAR-VIII flight (Space Processing Applications Rocket). The experiment was No. 74-42/2, flown on November 18, 1980. A preliminary analysis had already been carried out by C. Rey of Intersonics, Inc. The additional analysis was done by the PI and by C. F. Schafer of MSFC, with the assistance of R. L. Holland of MSFC. The results are presented in a NASA Technical Memorandum (10).

In addition to the experiment analysis and recommendations for improvement in future SPAR flights, the memorandum contains unpublished results on the radial properties of the sound field in an acoustic levitator, an extension of the work of King (3), done by the PI. Experimental work on acoustic streaming in the sound field of the levitator was also carried out by the PI in collaboration with C. F. Schafer and with W. K. Stephens of UAH. This work was not completed because of more pressing matters, but preliminary results, including photographs and 16 mm film, are available from the PI.

Copies of the NASA Technical Memorandum are available from the PI, as well as from the customary sources.

D. Other Work on Acoustic Levitation

In connection with the work discussed in Sections A-C, several related measurements were made, some of which are described in Appendix A.

A frequency analysis of the acoustic driver and the standing-wave field was made to facilitate an understanding of the nature of the field, and the results are in Appendix A. A study was also made of the lateral force on a small sphere, in connection with the analysis of
SPAR-VIII. The results of this study are contained in laboratory notebooks available from the PI.

III. AIR-JET LEVITATION

A. Background

The technical monitor for the initial work on acoustic levitation was W. A. Oran of MSFC. Following Dr. Oran's assignment to NASA Headquarters in September 1980, the technical monitor has been Dr. E. C. Ethridge of MSFC. All work by the PI on the air-jet levitator was done under Dr. Ethridge. The work was initially begun at the request of Dr. R. J. Naumann of MSFC.

A constricted-tube gas-flow levitator was developed at MSFC by Berge, Oran, and Theiss (11). The device consists of a quartz tube with a constriction introduced by melting and stretching (tubes were prepared by R. Smith, glassblower at UAH); a source of compressed air or gas; and a furnace for heating the air before it passes through the tube. By means of this device, Berge, et al., were able to levitate a spherical sample with the tube upright or inverted, and to heat the sample to temperatures greater than 1300°C.

The only results in the open literature directly related to this device were by Schmidt and Springer (12), involving a sphere in a diffuser, and they are not particularly useful for the air-jet (or gas-flow) levitator. There is, however, an extensive literature on spheres in tubes (see references in Appendix B).

The PI was asked to predict the behavior of samples in the levitator in a low-g environment.
B. Behavior of Samples in Air-Jet Levitator in Low G

The experimental parameters involved in understanding the behavior of the air-jet levitator are:

(a) flow rate of air or gas
(b) pressures in tube
(c) relative diameters of tube and constriction of sample
(d) angle and shape of constriction
(e) net force on sample at different positions in tube
(f) viscosity of air or gas

In order to predict the behavior of samples at low g, a study was made in which (b) and (e) were measured as (a), (c), and (d) were varied. The effect of (f) can be calculated from the Reynolds number.

The results of these measurements were presented at the Materials Research Society Symposium, "Materials Processing in the Reduced Gravity Environment of Space," Boston, November 16-18, 1981, with W. K. Stephens of UAH and E. C. Ethridge of MSFC. They are given in Appendix B, which is a copy of the paper which was presented. The conclusion is that the levitator should work as a positioning device at low g, but that care must be taken to regulate flow rates so that the molten sample never touches the tube. The development of the device for low g will thus require a careful study of flow rates vs. temperature, in a low gravity environment, to maintain sample stability.

C. Behavior of Samples in the Laboratory

Studies were also made of sample behavior in the laboratory at 1 g, and some of the results are contained in Appendix B. It is possible to
do a theoretical analysis of the air-jet levitator for laminar flow with a precisely defined geometry, and such an analysis would be quite similar to flow-meter theory. However, the flow is not laminar in the actual levitation experiments, and the academic interest of such an analysis hardly seems appropriate in the context of developing working devices.

The furnace arrangement built by Berge, Cran, and Theiss was redesigned and rebuilt by the PI with the help of R. Eakes of UAH, and is available for continued work at 1 g.

IV. HEAT TRANSFER FROM A MOLTEN SAMPLE

A. Background

The basic problem involved in this part of the project was to develop a computer model for the temperature distribution, as a function of position and time, in a small spherical drop of material which is cooling by conduction (in the material) and radiation. The material is neither opaque nor fully transparent. This problem is related to the processing of samples in drop tubes, on stings, etc.

The problem described above has never been solved. However, there are several simpler, related problems that have been solved approximately. The subject is discussed generally in a review article by Viskanta and Anderson (13). The basic problems of radiative transfer are presented by Chandrasekhar (14).

Some steady-state problems for which approximate solutions are available are

1) a gray spherical shell with no conduction (15-17);
2) a special nongray spherical shell with no conduction (18);
3) a gray spherical shell with radiation and conduction (19, 20);
4) conduction with no radiation (21).

Time-dependent problems which have been considered are
5) cooling of a spherical gray gas with no conduction (22, 23);
6) conduction in a sphere with no radiation (24).

B. Theoretical Model

In order to simplify use of the word processor for this report, we modify the conventional notation somewhat. Some pertinent variables in the problem are

- $u$ - energy density
- $q$ - heat flux vector
- $t$ - time
- $D$ - density
- $c$ - specific heat
- $T$ - temperature (absolute)
- $f$ - frequency of radiation
- $k$ - thermal conductivity
- $K$ - linear radiative absorptivity (depends on $f$)
- $n$ - index of refraction (depends on $f$)

The energy conservation equation is then

$$\frac{du}{dt} = - \text{div } q$$

where, for this section, $d$ implies partial differentiation. Then from

$$q = - k \text{ grad } T + F$$
and \( v = DcT \)

we get

\[ Dc \frac{dT}{dt} = \text{div} (k \text{grad} T) - \text{div} F \]

which, with spherical symmetry, becomes

\[ Dc \frac{dT}{dt} = r^{-2} \frac{d}{dr} (kr^2 \frac{dT}{dr} - r^2 F) \]

where \( F \) is the radial component of \( F \) and \( r \) is the radial coordinate.

The problem of the distribution of radiant energy for a given temperature distribution was solved by Chandrasekhar (14). The term \( \text{div} F \) can be represented as

\[ \text{div} F = \int_{0}^{\infty} K(f) [4\pi n^2(f) I(f) - G(f)] df \]

where \( I(f) \) is the blackbody distribution function and \( G(f) \) is the contribution from absorption in each small volume of material. The expression for \( \text{div} F \) in terms of temperature is given by Viskanta and Lall (22) in terms of exponential integral functions.

For the simplest case, we assume that \( c, k, K, \) and \( n \) are constant. Then we can change to dimensionless variables

\[
\begin{align*}
x &= \frac{r}{r_0} \\
w &= xT/T_0 \\
t^* &= t/(Dc/KB T_0^3) \\
N &= kK/B T_0^3 \\
s &= t^* N/(Kr_0^2),
\end{align*}
\]

where \( r_0 \) is the spherical boundary, \( T_0 \) is the initial temperature, and \( B \) is the Stefan-Boltzmann constant. Then we get
\[ \frac{du}{ds} = \frac{d^2u}{dx^2} - H \]

where \( H \) is an integral representing the flux, which depends on position and temperature.

If we define

\[ y = Kr \]

so that \( x = y/y_o \)

we find

\[ H = y(4E' - G') \]

where \( E' = (T/T_o)^4 \)

and \( G' = G/BT_o^4 \)

with \( G \) the integral over frequency of \( G(f) \). Viskanta and Lall get for \( G \),

\[ G = \left( \frac{2}{y} \right) \int_0^{y_o} E'[E_1(|y-y'|)] - E_1(y + y')] y'dy' \]

where \( E_1 \) is the exponential integral function.

The basic equation is thus an integro-differential equation of the parabolic type. We have explored a variety of possibilities for
approximate solutions to this equation, and finally settled on a direct computer approach using finite differences and numerical integration.

The most significant approximations which have been made are taking $k$ independent of $T$ and taking $K$ independent of $f$. Relaxing the first restriction would make the calculation slightly more complicated, but the equation is already strongly nonlinear because of the flux integral, which is not small, so it would be a minor complication. Relaxing the second restriction is quite a serious matter.

Whether or not the second approximation is reasonable depends on the shape of the absorption curve for a given material in the vicinity of the blackbody peak. The position of the peak follows from Wien's displacement law

$$\lambda_{\text{max}} T = 2898 \ \mu\text{m-K}$$

for wavelength $\lambda$. For the temperature range 2000 K to 3000 K, this gives $\lambda_{\text{max}}$ from 1.5 to 1 microns. We have as yet no data on $K$ for liquid alumina, but for solid alumina the transmittance is essentially uniform from 0.2 to 5 microns. To this extent the approximation of constant $K$ for alumina ($\text{Al}_2\text{O}_3$) is a good one. For each additional substance of interest, the approximation must be reconsidered.

From an analysis of energy transfer at the spherical surface, it would appear that the appropriate boundary condition is

$$\frac{dT}{dr} = 0 \text{ at } r_o.$$ 

However, this gives
\[ \frac{dw}{dx} = \frac{T}{T_0} \text{ at } r_0, \]

which is impracticable. A relatively minor modification is

\[ \frac{dw}{dx} = 0 \text{ at } r_0 \]

which gives

\[ \frac{dT}{dr} = -\frac{T}{r_0} \text{ at } r_0. \]

Starting in thermal equilibrium at \( t = 0 \), we also expect

\[ T = T_0 \text{ for } r \text{ between } 0 \text{ and } r_0, \]

but this gives a null solution, so we have chosen

\[ T = T_0 \exp(-aT) \]

for a near zero. We also have

\[ w = 0 \text{ for } x = 0, \text{ all } t. \]

The difference equation is given, e.g., by Ames (24). The computer program (in Fortran V) is given in Appendix C. We have incorporated the necessary condition for convergence (see Ref. 24, p. 323).

The general result of the computer calculation is what one would expect. The temperature at any point decreases with time. At a given
time, the $T$ vs. $r$ curve looks roughly like a decreasing exponential function.

For sample numerical results, we used the following values:

- melting point of $\text{Al}_2\text{O}_3 = 2323$ K
- $T_0 = 2500$ K
- density $D = 4.8 \text{ g/cm}^3$ (Ref. 25)
- heat capacity $c = 197 \text{ J/mole} \cdot \text{K}$ (Ref. 25)
- thermal conductivity $k = 0.065 \text{ W/cm} \cdot \text{K}$ (Ref. 26)
- optical absorptivity $K = 0.2/\text{cm}$ (Ref. 27)

We find

$$N = 0.13$$

$$t^* = 1.5 \times 10^{-5} t$$

We divided $x$ into 50 equal segments. To satisfy the convergence requirement, this means that a good value for an increment of $s$ is $10^{-4}$ or less. Then we get

$$t = (140 \text{ sec/cm}^2) r_o^2$$

For $r_o = 1 \text{ mm}$, incrementing $t$ 50 times, we get a total time of 72 sec, which should be quite adequate.

Since the computer program involves dimensionless variables, a few runs for typical values of $y_o$ are sufficient to cover all cases of interest with different substances. The results have been made available to the technical monitor at MSFC.
The PI is continuing to search for improved experimental data on substances of interest to MSFC and to study ways of expanding the applicability of the model. Any additional results will be made available to appropriate personnel at MSFC. It is intended that the results of this work be published.
REFERENCES


APPENDIX A

Interim Report on Contract NAS8-33731

J. E. Rush

This is a much-expanded version of the first quarterly report on subject contract, covering work for the period June 2 - August 7 (no work done on the contract for May 8 - June 1). All the work involved the two acoustic-levitation systems at SSL/MSFC and included:

(a) instrument analysis

(b) experimental study of heat transfer from cylinders and spheres, and

(c) calculations of relevant physical quantities and comparison with published data.

Detailed data are contained in my laboratory notebook No. 2.

A. Instrument Analysis

Studies were done of the stability of the Intersonics (Model 15-1A) feedback amplifier and driver system (St. Clair generator). It was determined that a cooling fan directed at the amplifier could reduce the downward power drift of ~4 db in 8 min to ~0.5 db, and reduce the rapid fluctuations from ± 0.3 db to ± 0.1 db.

During the course of these studies, an instability in the driver reflector system was noted. This consists of a rapid fluctuation of ~4 db when the system is tuned to resonance, at ~178 db. It occurs with a concave driver and large flat reflector or a flat driver and concave reflector. It was eliminated by use of a flat driver and flat reflector or a concave driver and small flat reflector. Studies with oscilloscope and wave analyzer showed that the instability involves alternate coupling and decoupling to the first and second harmonics of the fundamental vibrating frequency (~15 k Hz).
Because the B & K microphones were registering 178 to 180 db for the system in resonance, and they are inaccurate beyond 170 db, an effort was made to obtain an alternate source for measurement of high sound pressure levels (SPL). With the help of John Theiss of MSFC, I obtained a strain-gauge pressure transducer, a sensor amplifier module (SAM-1) to accompany it, an oscillograph to record the signals, and a deadweight tester and 1 kHz standard source to calibrate it. The transducer was rated from 0 to 5 psi (184 db) and 0 to 40 kHz.

When the dc signal was calibrated with the deadweight tester, the transducer agreed well with the microphones at 1 kHz and 160 db, using the standard source. Unfortunately, it gave readings about 4 db higher than the microphones when placed in the field of the acoustic system, at SPL values from 155 to 180 db. The signal from the transducer was also quite different from the microphones as seen on the oscilloscope and the wave analyzer. Since the B & K microphones are known to be accurate below 170 db, I was forced to discard the strain-gauge transducer as a reliable measuring instrument for the acoustic system, and be left with no accurate measuring device for SPL values greater than 170 db. Because of instabilities at higher SPL values, this limitation may not be a practical problem.

During the above calibration process, I did wave analysis studies of the drivers with no reflector and clearly observed the sawtooth effect predicted for high SPL values.

B. Experimental Study of Heat Transfer

As a continuation of studies begun in the summer of 1979 and continued during the period September 1979 to May 1980, I constructed several heating elements and measured their heat transfer characteristics
in the acoustic field. The configurations were as given in Table 1.

<table>
<thead>
<tr>
<th>Shape</th>
<th>No. Turns</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere (approx.)</td>
<td>5</td>
<td>0.4 cm</td>
<td>0.4 cm</td>
</tr>
<tr>
<td>cylinder</td>
<td>20</td>
<td>1.0 cm</td>
<td>0.3 cm</td>
</tr>
<tr>
<td>cylinder</td>
<td>20</td>
<td>1.0 cm</td>
<td>0.25 cm</td>
</tr>
</tbody>
</table>

In each case the coil was wound with B & S # 28 Nichrome wire and spot-welded to leads of B & S # 20 Kanthal wire. The coil was dipped in Sauereisen resistor cement, shaped, and allowed to dry with an iron-constantan thermocouple, made of 0.004 in. wire, imbedded approximately in the center. The leads were then inserted into a rigid ceramic holder which was taped to a rod which was mounted on a ring stand. Power was provided by a very steady (± 0.003 A) d.c. power supply. The thermocouple leads were connected to an Omega 2168A digital thermometer which showed integer degrees.

The initial heat-transfer measurements were made in a sound field with SPL recorded at ~ 178 db by B & K 1/8" microphone (see notes about SPL measurement in Part A), using the Intersonics Model 15-1A systems. A large flat aluminum reflector was placed about 3.0 cm from the edge of a concave aluminum driver. The reflector was then raised slightly (~ 2 mm) so that the system was just far enough from resonance to avoid the large pressure fluctuations described in Part A. Several sets of data were taken with the sphere and the second cylinder, the thermocouple in the first cylinder having failed. Measurements were
also made of heat transfer from the sphere with no sound field, with no pieces of apparatus near enough to influence convective heat transfer. In the sound field, the coil was placed at the location of the first minimum below the highest maximum below the reflector, putting it approximately halfway between driver and reflector. This location should give maximum velocity of the a.c. field and maximum cooling. (This fact had been checked in earlier measurements. See laboratory notebook No. 1.)

In order to determine the effect of radiation and conduction of heat through the leads, the emissivity of the sphere was measured with an IRCON radiometer calibrated to a standard blackbody source; and thermocouples were attached at each end of a Kanthal lead with Sauer­eisen to determine the temperature gradient for various temperatures of the sphere. (These connections and measurements were independent of the convection measurements.)

The current to the coils was measured with a Hewlett Packard digital ammeter for all values \( \leq 2.00 \, \text{A} \), and the voltage across the coil was estimated by sliding probes connected to a digital voltmeter along the Kanthal leads and extrapolating to the limiting value at the points of attachment of the coil. Current values greater than 2.00 A were gotten by setting the ammeter on the power supply to a corrected value estimated from lower current readings with the digital ammeter. (Only a zero-point correction was needed).

Following the measurements at \(~ 178\) db, measurements were made at 170, 160, 150, 140, and 130 db. Those at 170 db were made with the system configuration noted above, with the reflector adjusted for lower SPL. The remainder were made on the second amplifier-driven system, using an ALTEC 9440 A, 800 watt amplifier with oscillator and magnet
circuits constructed at MSFC. The second system was used to allow stability at lower power. To arrive at 130 db, the driving cylinder had to be firmly clamped. At 130, 140, and 150 db, the signal read by the microphone was an almost pure sine wave at 15.2 to 15.5 k Hz (depending on driving cylinder). At 160 db it was slightly distended toward a sawtooth. At 170 and 178 db it was clearly distorted. All SPL readings were taken with a 1/8" B & K microphone connected to a B & K Type 2607 measuring amplifier. The microphone was attached to a cathetometer so that it could be easily moved in and out of the sound field. All SPL readings were made at the maximum SPL point, and the microphone was removed before temperature readings were taken. All readings were rms pressure $P_{\text{rms}}$. The shape of the signal was determined by feeding the output of the B & K measuring amplifier into an oscilloscope.

Comparison with a good sine wave was easily made by inserting a 22.5 k Hz low-pass filter (included in the B & K unit) or by passing the signal through a wave analyzer and looking at the output of the wave analyzer on the oscilloscope. A dual trace unit on the oscilloscope was used for easy comparison.

C. Heat-Transfer Calculations

Most of the calculations were done using an HP 9835 A desktop computer located in SSL Room 215, by means of five programs - labeled SPL, QTRANS, QCOR, QMEAS, and QCAL, which I wrote and which are stored on an HP cassette. The programming language is BASIC and the programs can be easily read and interpreted by anyone familiar with FORTRAN.
1. Free Convection

For free convection, Yuge (1) has obtained an empirical formula for spheres,

\[ \text{Nu} = 2 + 0.392 \text{Gr}^{0.25}, \quad 1 < \text{Gr} < 10^5 \]

where \( \text{Nu} \) is the Nusselt number and \( \text{Gr} \) is the Grashof number. His defining equations are

\[ \text{Nu} = \frac{hD}{k_m} \]

and \[ \text{Gr} = gD^3 \frac{\Delta t}{\nu_m^2} T_a. \]

Here \( D \) is the diameter, \( h \) the heat-transfer coefficient, \( k \) the thermal conductivity of air, \( g \) the acceleration due to gravity, \( \Delta t \) the difference between surface and ambient temperature, \( \nu \) the kinematic viscosity of air, and \( T_a \) the absolute ambient temperature. The subscript \( m \) indicates the value at the mean between surface and ambient temperatures (the film temperature) and \( h \) is defined by

\[ h = \frac{Q}{A \Delta t} \]

where \( Q \) is the heat-transfer rate and \( A \) the surface area \((= \pi D^2)\).

A complete list of symbols with definitions is given at the end of this report. All Grashof numbers for my data were between 20 and 200, thus falling within Yuge's limits.

The free-convection measurements which I made agree with Yuge's formula to within 16% at all temperatures up to 550°C, even though his data only went to 70°C. In the comparison, the
radiation and conduction losses were subtracted from the measured power input to the coil. The conduction loss was obtained from

\[ Q_C = 2 k \omega A \omega \frac{\Delta t \omega}{L \omega} \]

where \( A \omega \) is the cross-sectional area of the Kanthal wire, \( k \omega \) the thermal conductivity, and \( \Delta t \omega \) the temperature difference over the length \( L \omega \). This does not allow for convective cooling of lead wires, and so overestimates the convection loss of the sphere. This fact coincides with the data, which show convection losses progressively larger than those calculated from Yuge's formula as the temperature is increased. At 550°C, the calculated radiation and conduction losses accounted for 19% and 18%, respectively, of the measured power input to the coil.

2. Convection in the Sound Field

While there do not appear to be any published data on heat transfer from spheres in an acoustic field, there are several published papers dealing with heat transfer from cylinders, either in a sound field (2-4, 6-12) or vibrated (5,7). For sufficiently long wavelengths, there should be no distinction between the effects of sound and vibration (5). Ford and Peebles (7) determined that the effects were indistinguishable for \( \lambda/D \geq 12 \), where \( \lambda \) is the effective wavelength and \( D \) is the cylinder diameter. The \( \lambda/D \) value for my data is 9, so some distortion might be expected when comparing them with vibration data. There are other data available in reports, which are referenced in the published papers, but it does not appear that they would add much to what has been published.
The nature of the published data is summarized in Table 1. From the table, we see that there are empirical formulae given in References 3, 5, and 9. The highest frequency is 5 k Hz, the largest SPL value is 150 db, and the highest temperature is 130°C. Thus, the data themselves are not directly useful for 15 k Hz sound fields at 170 db and temperatures up to 700°C. Nevertheless we can compare our data with the empirical formulae, parametrize our data, extrapolate curves, etc., and consider what inferences might be appropriate for understanding the physical processes involved.

The empirical formulae in Refs. 3, 5, and 9 involve the following dimensionless numbers: the Grashof Number Gr, the Prandtl number Pr, the vibration Mach number Ma, the vibration Reynolds number Rev, and the Nusselt number Nu. The definitions used in the papers are

\[ \text{Gr} = g D^3 \beta A t \frac{\Delta t}{V_m^2} \]

where \( \beta \) is the volume coefficient of thermal expansion (for an ideal gas, \( \beta = \frac{1}{T} \));

\[ \text{Pr} = \frac{\mu m}{k_m} \]

where \( \mu_m \) is the viscosity and \( C_p \) is the specific heat at constant pressure;

\[ \text{Ma} = \frac{V_s}{C} \]

where \( V_s \) is the velocity of acoustic streaming, arbitrarily chosen to be given by \( V_s = a \omega \), \( a \) being the maximum ...
<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Frequency Range (Hz)</th>
<th>'SPL Range (db)</th>
<th>λ/2D Range</th>
<th>Temperature Range (°C)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Hor. St. Wave</td>
<td>120</td>
<td>110 - 117</td>
<td>300</td>
<td>49 - 52</td>
<td>Temperature vs. angle</td>
</tr>
<tr>
<td>3</td>
<td>Hor. St. Wave</td>
<td>1100 - 4900</td>
<td>0 - 151</td>
<td>2 - 8</td>
<td>20 - 130</td>
<td>Pictures; graphs; formulae</td>
</tr>
<tr>
<td>4</td>
<td>Hor. Tr. Wave</td>
<td>1000 - 500</td>
<td>0 - 148</td>
<td>2 - 9</td>
<td>100</td>
<td>Pictures; Δh vs. SPL</td>
</tr>
<tr>
<td>5</td>
<td>Vert. Vibrations</td>
<td>54 - 225</td>
<td>0 - 150</td>
<td>35 - 145</td>
<td>35 - 125</td>
<td>Graphs, formulae</td>
</tr>
<tr>
<td>6</td>
<td>Hor. St. Wave</td>
<td>1500</td>
<td>0 - 146</td>
<td>6</td>
<td>115 - 165</td>
<td>Local effects</td>
</tr>
<tr>
<td>7</td>
<td>Hor. St. Wave &amp; Vib.</td>
<td>100</td>
<td>136 - 150</td>
<td>78</td>
<td>50</td>
<td>Compare vibration &amp; sound effects</td>
</tr>
<tr>
<td>8</td>
<td>Hor. St. Wave</td>
<td>680 - 1090</td>
<td>130 - 140</td>
<td>8 - 13</td>
<td>30 - 50</td>
<td>Local effects</td>
</tr>
<tr>
<td>9</td>
<td>Hor. St. Wave</td>
<td>645 - 672</td>
<td>131 - 148</td>
<td>13 - 14</td>
<td>85 - 93</td>
<td>L vs. Δt; formulae</td>
</tr>
<tr>
<td>10</td>
<td>Vert. St. Wave</td>
<td>710 - 1470</td>
<td>125 - 140</td>
<td>1.2 - 2.5</td>
<td></td>
<td>Local effects</td>
</tr>
<tr>
<td>11</td>
<td>Hor. &amp; Vert. St. Waves</td>
<td>710 - 1470</td>
<td>125 - 140</td>
<td>1.2 - 2.5</td>
<td></td>
<td>Local effects</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>20 - 40</td>
<td>20 - 40</td>
<td></td>
<td></td>
<td>Local effects</td>
</tr>
<tr>
<td>This Report</td>
<td>Vertical St. Waves</td>
<td>15,000</td>
<td>0 - 180</td>
<td>4.5</td>
<td>20 - 700</td>
<td></td>
</tr>
</tbody>
</table>
sound-particle amplitude of oscillation and \( \omega \) the angular frequency;

\[
\text{Re}_v = \frac{V_s D}{\nu m} \quad \text{(Ref. 5)}
\]

\[
\text{Re}_s = \frac{V_s' D}{\nu m} \quad \text{(Ref. 9)}
\]

with \( V_s' = (2 \pi a \omega)^2 \) \( \phi \), \( \phi \) being determined empirically at each temperature; and

\[
\text{Nu} = \frac{h}{A \Delta T}
\]

as before.

There are two possible ways to define \( \Delta t \). Since the sound field heats the coil even with no current supplied to it, the temperature \( t_0 \) at \( Q = 0 \) will be higher than the ambient temperature \( t_a \). The difference is about 5°C for SPL = 170 and 12°C for SPL = 180 db. The Grashof number would normally be defined so that \( Gr = 0 \) when \( Q = 0 \). On the other hand, the forced air flow, by acoustic streaming, involves air at ambient temperature (outside the standing-wave sound field), so the Reynolds number would normally be defined for a mean temperature relative to ambient.

The difference between \( t_0 \) and \( t_a \) at 170 - 180 db is not trivial when one compares data plots using each one. If \( h \) is linear in one plot, it will not be linear in the other, even within experimental error.

There is also an ambiguity in the definition of the Grashof number. Ford and Kaye (3,5) state that it is evaluated at the mean film temperature, presumably implying \( \beta = \beta_m = 1/T_m \). Yuge
uses $\delta = 1/T_a$. For $\Delta t < 100^\circ C$ it made little difference, but Gr peaks at $\Delta t \approx 250^\circ C$ using Yuge's definition and at a lower temperature with a lower value using the definition of Ford and Kaye.

The empirical formulae given in Ref. 3 are:

$$h_o = 0.245 \left(\frac{\Delta t}{D}\right)^{1/4}$$

$$h_v = [b_v \left(\frac{\Delta t}{D}^3\right)^m / D] \left[\left(a f\right)^2 F\right]^n$$

$$\text{Nu}_o = 0.485 \left(\text{Gr Pr}\right)^{1/4}$$

$$\text{Nu}_v = B_v \left(\text{Gr Pr}\right)^{1/3} \left(\frac{a_2 F}{V}\right)^n$$

Here the subscript 0 implies $\text{SPL} = 0$, V implies $\text{SPL} \neq 0$; $h_o$ and $h_v$ are in Btu/ft$^2$ · hr · °F; $b_v$ and $B_v$ vary with frequency or, more appropriately, with $\lambda/2D$; $m$ and $n$ vary with $\lambda/2D$; $f = 2\pi \omega$; and $F$ is a geometrical factor included to compensate for the different values of the amplitude of over the surface of the cylinder, so it, too, varies with $\lambda/2D$. The values of $b_v$ and $B_v$ are tabulated and graphed, and the values of $F$ are graphed.

The formulae in Ref. 5 are:

$$h_o = 0.255 \left(\frac{\Delta t}{D}\right)^{1/4}$$

$$h_v = b_v \left(\frac{\Delta t}{D}\right)^{0.2} (a\ell)$$

$$\text{Nu}_o = 0.495 (\text{Gr Pr})^{1/4}$$

$$\text{Nu}_v = B_v (\text{Gr Pr})^{0.2} \text{Re}_v$$
with $h_0$ and $h_v$ also in Btu/ft$^2$·hr·°F. Thus the parameterization used by Ford and Kaye for horizontal (3) and vertical (5) standing waves is similar, except for a change from $Ma$ to $Re_s$. However, the number $n$ is about 1/3, which means that there is a change from $(af)^{2/3}$ to $(af)$ and $(Ma)^{2/3}$ to $Re_v$ in going from horizontal to vertical. Note that the equations for $h_0$ and $Nu_0$ correspond to identical situations in the two cases. In both cases, the $v$-subscript equations are for "fully developed vortex flows" which, according to the authors, means $SPL > 146$ db.

The parameterization of Ref. 9 is more complicated. Lee and Richardson (9) arrive at a formula:

$$\frac{Nu}{Gr^{1/4}} \left(1 + 1.61 \frac{Gr^{-1/4}}{}\right) = 0.372 \left[1 + A_1(Re_s')^{2/Gr}\right]^{1/4}$$

where $Re_s$ (discussed above) is not calculated directly, but is replaced by means of the relation

$$B_1 = (0.372)^4 A_1 \phi^2 \frac{d^2/v^2}{Gr}$$

with $B_1$ determined independently at each temperature. Note that the definition of $Re_s$ gives $Re_s \propto (af)^2$ while Ref. 5 has $Re_v \propto (af)$. Since the dependence of $Re_s$ on $(af)$ is never used, however, no comparison with Ref. 5 can be made. Lee and Richardson (9) claim to fit the data and Peebles (7) as well as their own, and demonstrate the fit with a graph. In Refs. 3 and 5, there is no demonstration of the degree to which the equations fit the data, except for $h_v$. Since the data are
shown on a graph of $\log h$ vs $\log \Delta t$, it would be possible to investigate the quality of fit for $N_u$ (which I expect is rather poor) but I have not done so.

The data of this report are tabulated in my laboratory notebook No. 2, and represented in Figure 1, which is a plot of $\log h$ vs. $\log \Delta t$ for the data of Ford and Kaye (3,5) is shown, and we can compare data in this range for SPL = 130, 140, and 150 db. (Note: the data at 130 db are so close to those at 140 db that they are not plotted). The range of Grashof numbers in Refs. 3 and 5 is $1 \times 10^4$ to $5 \times 10^4$, while my range is 40 to 115; the range of Reynolds numbers is 0 to 3500 for Ref. 5, 0 to 300 for my data (up to 150 db), but typical ratios $Re_v/(Gr)^{1/2}$ are of the same order of magnitude.

A distinct difference is that at higher values of $\Delta t$ the Grashof number peaks, then decreases.

An analysis of ordinary forced and mixed convection indicates (13) that one could expect to get

$$Nu = f(Gr, Pr, Re_v)$$

with $Gr$ and $Pr$ appearing as a product. Consequently, I calculated $Gr$, $Pr$, and $Re_v$ for my data. In the calculation, I chose Yuge's definition of $Gr$ ($\beta = 1/T_a$) and defined $\Delta t$ relative to $t_a$. In $Re_v$, $v_s = (\alpha \omega)$ is defined at $t_a$. By extrapolating to corresponding values of $\Delta t$ at different SPL, I plotted $\log Nu$ vs. $\log Re_v$ at constant $(Gr Pr)$, as shown in Fig. 2. Although $(Gr Pr)$ is double valued, the curves below and above the peaks are clearly separated by different values.
of $\text{Nu}$. Note that the data are very close to straight lines, and that the slopes are all close to the same value. For $\Delta t \leq 150^\circ \text{C}$, the slopes are about 0.56. For $\Delta t \geq 300^\circ \text{C}$, they are about 0.52. Fig. 3 shows a plot of $(\text{Gr Pr})$ is $\Delta t$ and $\text{Re}_v$ is $\Delta t$. The dependence on $\text{Re}_v$ may be compared to the data for ordinary forced convection (See Ref. 13, p. 242). For $\text{Re}$ from 0 to 400,000, the data are well fit by

$$\text{Nu} = 0.43 + C (\text{Re})^m \quad \text{with} \quad C = 0.48, \quad m = 0.5$$

for $0 < \text{Re} < 4,000$ and $C = 0.174, \quad m = 0.618$ for $4,000 < \text{Re} < 40,000$. It is not possible to plot $\text{Nu} \text{ vs. } (\text{Gr Pr})$ at constant $\text{Re}_v$, because there are no common values of $\text{Re}_v$. Consequently, I plotted $(\text{Nu}/\text{Re}_v^{0.52}) \text{ vs. } (\text{Gr Pr})$ and got the graph shown in Fig. 4. The low temperature data would obviously agree better if I used $\text{Re}_v^{0.56}$; nevertheless, the trend is clear. For $\Delta t \leq 100^\circ \text{C}$ we get

$$\text{Nu} \approx 0.03 \sqrt[0.52]{\text{Re}_v} \sqrt{\text{Gr Pr}}$$

while for $\Delta t \geq 200^\circ \text{C}$ we have

$$\text{Nu} \approx 54 \sqrt[0.52]{\text{Re}_v} / (\text{Gr Pr}).$$

Between $100^\circ \text{C}$ and $200^\circ \text{C}$ there is obviously some sort of transition.

D. Further Work

The figures included in this report contain no direct indication of agreement or disagreement with the published formulae.
However, since the low temperature dependence of $\text{Nu}$ is clearly

$$\text{Nu} = (\text{Gr Pr})^{0.45} (\text{Re}_v)^{0.56}$$

there is no need to look for a comparison with the formulae of Ref. 5 where $\text{Nu} \propto \text{Re}_v$. There is also little hope for $\text{Nu} \propto \text{Ma}^{2/3}$ as in Ref. 3, but this should be checked, as should the complicated formula in Ref. 9. It would also be worthwhile to investigate a parameterization like

$$\text{Nu} = (\text{Const.}) + (\text{Const.}) (\text{GrPr})^m (\text{Re}_v)^n$$

by analogy with the good data on forced convection in Ref. 13. All of this can be accomplished fairly easily by computerizing the data and using a minimizing program which I have available.

A good check would be accomplished by taking additional data at 145, 155, and 165 db. The data at 178 db are suspect because we don't have a good measurement of SPL, but it would be worthwhile using the low-pass filter on the B & K amplifier and using the resulting SPL value as the correct one, just to see how the results would agree with data at 140, 150, and 160 db. The 170 db data are also suspect for purposes of comparison because the field is not close to a sine wave and we are assuming a sine wave in our parameterization.

Once the matters above have been investigated or accomplished, I plan to write a paper for publication (authors: Rush and Dean) in J. Acoust. Soc. Am. or J. Heat Transfer. I suggest the former, because they are more likely to be interested in heat transfer at high frequencies. I will, of course, try
to get a theoretical basis for the change in $\text{Nu} \ vs. \ (Gr \ Pr)$ at 150° C, and for the differences between our results and others, before submitting the paper.

Following the above work, which should take about one month or perhaps during the process of the above) I plan to investigate the stability of objects in the acoustic system at room temperature, introducing specific perturbations such as air currents and reflector changes, and quantifying the observations as much as possible.


Notation

a = amplitude of sound vibration
A = surface area
A_\omega = cross-sectional area
c = speed of sound (at t_a)
C_p = specific heat of air at constant pressure
D = diameter of cylinder
f = frequency (sound or vibration)
g = acceleration due to gravity
Gr = Grashof number = g D^3 \beta \Delta \tau / \nu_m
h = heat transfer coefficient = Q / A \Delta t
k = thermal conductivity
L = length of segment of lead wire
M_a = vibration mach number = V_s / C
Nu = Nusselt number = hD / k_m
Pr = Prandtl number = \mu_m C_p / k_m
P_{rms} = root-mean-square sound pressure
Q = convective heat transfer rate
Re_s = streaming Reynolds number = V'_s D / \nu_m
Re_v = vibration Reynolds number = V'_s D / \nu_m
SPL = sound pressure level in db (re 2 X 10^{-4} dyne/cm^2)
T = absolute temperature, K
t = temperature, °C
V_s = velocity amplitude = a\omega
V'_s = streaming speed = \sqrt{(af)^2 \phi} (Ref. 9)
\beta = volume coefficient of thermal expansion = 1/T for ideal gas
\[ \Delta t = \text{temperature difference} \]
\[ \lambda = \text{wavelength} \]
\[ \mu = \text{viscosity} \]
\[ \nu = \text{kinematic viscosity} = \frac{\mu}{\rho} \]
\[ \varphi = \text{function of } f, D, \text{ and } \nu \text{ such that } (af)^2 \varphi \text{ is a characteristic streaming velocity (Ref. 9)} \]
\[ \rho = \text{density of air} \]
\[ \omega = \text{angular frequency} = 2 \pi f \]

**Subscripts**

\( o \) implies \( Q = o \) (this report) or \( \text{SPL} = o \) (Refs. 3,5)

\( m \) implies mean film temperature, average of surface temperature and ambient temperature

\( a \) implies ambient temperature

\( \omega \) refers to Kanthal wire leads to heating coil
Fig. 1

Log-log

Data from Figs. 67-69.

ORIGINAL PAGE IS OF POOR QUALITY

\( h \) vs. \( \Delta t \)

Insert field data and key data.
Overlay with Fig. 2 at edge, so that lines meet.

At relative to $t = 24^\circ$.
Fig. 3
Gr Pr w. at & Rev w. at
Data from pp. 67-68

Gr Pr

Rev
\[ \frac{Nu}{(Re)^{0.588}} \text{ vs. } Gr \text{ for Pr}[\text{upper}] \]

Data from pg. 72-73

Nu vs. Re at constant Gr Pr)

\[ Nu = 0.5 \log (Gr \cdot Pr) \text{ vs. } (Re) \text{ for } \text{lower} \]

ORIGINAl PAGE IS
OF POOR QUALITY
APPENDIX B

PROPERTIES OF A CONSTRICTED-TUBE AIR-FLOW LEVITATOR

J. E. RUSH,* W. K. STEPHENS,* AND E. C. ETHRIDGE**
*Department of Physics, University of Alabama in Huntsville, Huntsville, Alabama, USA
**Space Sciences Lab, Marshall Space Flight Center, Alabama, USA

ABSTRACT

A constricted-tube gas flow levitator first developed by Berge, Oran, and Theiss shows promise both as a space-positioning device and as a levitator for ground-based work. We present results of laboratory studies which were designed to predict the behavior of the device in a low-g environment.

INTRODUCTION

There is much interest in levitation techniques which can be used in ground based research to study a number of phenomena. The effective levitation of liquid nonconductors remains an elusive goal in materials science. A constricted-tube gas flow levitator has been developed at the Marshall Space Flight Center by Berge, Oran, and Theiss [1]. Its advantages are that it is a simple levitator which is essentially orientation and gravity independent, will operate over a broad temperature range, and can be used to process both conducting and nonconducting materials. Solid spherical samples of a number of different densities at 1200°C have been successfully levitated. We have continued to study the properties of such levitators as a possible solution to this goal and as a possible levitator for low g and report here on our work to date (Sept., 1981).

The levitator consists of a constricted (quartz) tube fed at one end by a source of heated air or gas. A spherical sample is positioned by the air stream on the downstream side of the constriction, where it can be melted and resolidified without touching the tube. The primary source of heat for the sample is the air itself, although secondary sources are also being investigated. The air is heated by being passed through a furnace, by being blown past a torch, or both.

The behavior of spheres in flowing fluid in a lowg environment has been studied by many investigators. The earliest work was on spheres in free flow [2], where the drag coefficient vs. Reynolds number Re was determined. Also investigated was the pressure distribution around the sphere and the behavior of the fluid as a function of Re. A more complex situation is flow past spheres in vertical, slightly tapered tubes forming the basis of flowmeter technology. Quite a bit of work has also been done on spheres in straight tubes and pipes, both experimentally [3] and theoretically [4]. However, the only published work on spheres in diffusers which we have found is by Schmidt and Springer [5].

In a straight, vertical tube one can maintain an unstable equilibrium in the vertical direction by balancing the gravitational force with a drag force. A relatively stable equilibrium exists laterally because of a Bernoulli effect due to the radial variation in fluid velocity, coupled with a weaker Magnus force due to rotation of the sphere [6]. With a slightly tapered tube, as in a flowmeter, one can easily produce stable equilibrium, but levitation is gravity-dependent. The effect of a constriction is to supply an upstream...
(Bernoulli) force which can give stable equilibrium with or without gravity.

The purpose of the work presented here is to study the properties of the levitator in order to predict its behavior in a gravity-free environment and at elevated temperatures.

EXPERIMENTAL PROCEDURE

We define the inner diameter of the tube as $D_1$, the constriction diameter as $D_2$, and the diameter of the spherical sample as $d$. The equilibrium position of the sample in $g$ and the stable pressure were measured at each end of the tube as a function of $D_2/D_1$, $d/D_1$, shape of the constriction, flowrate through the tube, and shape of the diffuser. We also studied the net axial force on samples suspended by a wire from an electrobalance as a function of position of sample and of the variables given above. Thus we were able to map the force in the neighborhood of the expected low-$g$ equilibrium position.

Since we are ultimately interested in levitating and melting samples at elevated temperatures we want to know how the levitation forces are affected by temperature changes. For a given tube and sample size, with the system in mechanical and thermal equilibrium, the net forces on a sample at any given point in the tube should depend only on the value of $Re$. By varying the fluid flow rate we can vary $Re$ thereby modeling the effects of varying temperature on $Re$. Thus temperature effects can be studied by varying the flow rate.

The tubes used in these studies had an inner diameter $D_1$ of approximately 0.65 cm with ratios $D_1/D_1$ of $1/4$ to $1/2$. The ratios $d/D_1$ ranged from $1/2$ to $7/8$. For the shape of the constriction, we found that a simple but adequate quantitative measure was the approximate angle $\alpha$ of the inside surface with respect to the tube axis near the equilibrium position of the sample. The values of $\alpha$ ranged from $7^\circ$ to $18^\circ$.

Each end of the constricted tube was attached to copper tubing by means of a plastic heat-shrinkable tubing. Fittings were mounted on each copper tube for flexible-tubing connections to pressure gauges and to a flowmeter on the upstream side. Gauge pressures were measured with and without samples inserted for the range of flow rates used (3 to 17 liters per minute). At the lower flow rates we also measured differential pressures. We also measured the variation of pressure with sample position for suspended samples. The pressure at the downstream end was never significantly different from atmospheric pressure ($\Delta P < 0.01$ psi). The precise position of the sample was read with a cathetometer.

The force measurements were made using a Cahn electrobalance. A fine wire was suspended from the electrobalance, passing through the tube, and attached to the spherical sample by means of a small bit of epoxy. In these measurements the sample material was not important, so we used assorted steel ball bearings.

In each case the suspended samples were balanced before the airflow was begun, so that the forces measured were due entirely to the airflow. The suspended samples, of course, could not rotate. Rotation would probably occur for most freely-positioned samples, but this effect would not greatly modify the restoring force ($6$) and would in fact improve stability.

EXPERIMENTAL RESULTS

In Table I we show some basic data for five constricted tubes which were used in these studies.

ORIGINAL PAGE IS OF POOR QUALITY
TABLE I
Basic parameters of constricted tubes

<table>
<thead>
<tr>
<th>Tube Number</th>
<th>Inner Diameter D1 (cm)</th>
<th>Constricted Diameter D2 (cm)</th>
<th>Ratio D2/D1</th>
<th>Angle α</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.650</td>
<td>0.256</td>
<td>0.39</td>
<td>13°</td>
</tr>
<tr>
<td>2</td>
<td>0.650</td>
<td>0.350</td>
<td>0.54</td>
<td>9.5°</td>
</tr>
<tr>
<td>3</td>
<td>0.650</td>
<td>0.167</td>
<td>0.26</td>
<td>7°</td>
</tr>
<tr>
<td>4</td>
<td>0.650</td>
<td>0.157</td>
<td>0.24</td>
<td>18°</td>
</tr>
<tr>
<td>5</td>
<td>0.676</td>
<td>0.157</td>
<td>0.23</td>
<td>17.5°</td>
</tr>
</tbody>
</table>

A plot of the force, F, on a sphere in tube #5 is shown in Figure 1. Since the sphere is balanced, the force is due entirely to the fluid flow. This force is plotted against displacement of the sphere along the axis of the tube where the zero point of displacement corresponds to the point where the sphere touches the constriction and blocks the flow. The sphere used for Figure 1 had a diameter of 0.475 cm (d/D1=0.23) and the flow rate was 13.2 liters per minute, corresponding to an average velocity of 612 cm/s at diameter D1 (upstream).

![Graph of Fluid Force vs. Displacement for Balanced Sphere](image-url)

**Fig. 1.** Fluid force vs. displacement for balanced sphere.
Since we want stable equilibrium, we are looking for a force vs. displacement curve with a large negative slope and a broad range of positive/negative values to the left/right of zero. We also want the maximum and minimum values of the force to be reasonably large in comparison to typical sample weights. For a sample of the size used in Figure 1, which has a good d/D₁ ratio for stability, the weight for the steel sample is 440 mg. Thus the negative value for the F (10³ mg) is quite acceptable. Since the sphere is easily levitated at this flow rate, the positive value is acceptable also. The least desirable feature of Figure 1 is the small value of the equilibrium displacement, but this can be adjusted by varying the shape of the tube.

In Table II we summarize the results with the various tubes in terms of varying diameters of samples. In each case the flow rate chosen for comparison was based on the minimum value of the force for the range of flow rates used.

**CONCLUSIONS**

From the results given above, we conclude that the constricted-tube levitator can be used successfully as a positioning device for solid spherical samples at low g.

For operation in thermal equilibrium at high temperatures, we note that the important fluid parameter is the kinematic viscosity. If air is heated from room temperature to, say 1200°C, the kinematic viscosity increases by a factor of 14. To maintain a given value of the Reynolds number, the flow rate would...
have to be increased by the same factor, for a specific geometry of tube and sample. Thus, to maintain stable equilibrium, one increases the flow rate as the air or other gas is heated. The feasibility of this process has already been demonstrated in 1 g by Berge, Oran, and Theiss.

Fig. 2. Position of freely levitated steel sphere vs. flow rate at 1 g.

The other stability problem which must be considered for processing of samples is the change in shape of the spherical sample as it melts. The solution to this problem involves selecting a shape for the constriction so that the solid sample does not spin too rapidly and does not contact the tube on melting. From the data of Tables I and II, one can see that by decreasing the constriction angle (changing from Tube 5 to Tube 3) one can maintain a sufficiently large force and significantly increase the separation between tube and sample. We are currently beginning a study of the stability of melted samples at 1 g.

REFERENCES


RADIAT, RADIAT
108/31/82-09:12(,0)
1. REAL *6. DE, HNDEI
2. REAL *6. DTA
3. EXTERNAL HNDEI
4. COMMON /AA1/, THETA(50,50), THETA0(50) , TAU(50)
5. COMMON INT/ PS1(50,50), PS10(50)
6. 1 , ST(50) TSTAR(50) TPRIME(50)
7. , X(50) , T(50) , U(50,50).
8. 1 , EI(100)
9. READ (5,191) NX, NT, M
10. 191 FORMAT (3I5)
11. READ (5,102) AN, TAUO, DELTAT, ALPHA, BETA
12. 102 FORMAT (5F10.0)
13. WRITE (6,151) AN, TAUO, DELTAT
14. 151 FORMAT (//1H5 = ,F5.2,4X,7HTAUO = ,F5.2,4X,10HDELTA T = ,E8.2//)
15. DELX = 1, //X
16. DELTAU = TAUO*DELX
17. R = DELTAT/DELX**2
18. NXPP = NX + 2
19. NXP = NX + 1
20. DO 201 I=1,NXPP
21. X(I) = (R-1)*DELX
22. TAU(I) = TAUO*X(I)
23. WRITE (6,193) TAU(I)
24. 193 FORMAT (60X,E8.2)
25. GAMMA = 0.1
26. V = - GAMMA*X(1)
27. U(1,1) = EXP(V)*X(1)
28. 201 CONTINUE
29. NTP = NT + 1
30. DO 211 J=1,NTP
31. T(J) = T(J-1)*DELTAT
32. TSTAR(J) = ALPHA*T(J)
33. TPRIME(J) = BETA*TSTAR(J)
34. 211 CONTINUE
35. NXD = 2, *NX + 4
36. DO 221 I=2,NXD
37. XA = (L-1)*DELX
38. TA = TAUO*XA
39. DTA = TA
40. DE = -XHDEI(2, D, IER).
41. IF (IER.NE.130) GO TO 310
42. WRITE (6,181)
43. 181 FORMAT (////5H ZERO)
44. 310 CONTINUE
45. IF (IER.NE.131) GO TO 311
46. WRITE (6,182)
47. 182 FORMAT (////SHOWERFLOW)
48. 311 CONTINUE
49. EI(I) = DE
50. WRITE (6,191) EI(I)
51. 191 FORMAT (30X,E8.2)
52. 221 CONTINUE
53. DO 241 J=1,NTP
54. DO 251 I=2,NXP
55. THETA(I,J) = U(I,J)/X(I)
56.  251 CONTINUE
57.  FN = 0.0
58.  DO 261 I=2,NXP
59.  DO 271 K=1,NXP
60.  T4 = THETA(K,J)*4
61.  KA = IABS(I-K).
62.  KB = I + K
63.  IF (KA+KB.EQ.0) GO TO 253
64.  ED(KA) = 0.0
65.  253 CONTINUE
66.  EDIF = ED(KA) - L1(KB)
67.  FCN = 2.*T4*EDIF
68.  IF (K.EQ.1) GO TO 252
69.  IF (K.EQ.NX) GO TO 252
70.  FCN = 3.*FCN
71.  252 CONTINUE
72.  FN = FN + FCN
73.  71 CONTINUE
74.  TG = 0.375*DELTAU*FN
75.  U(1,J+1) = 0.0
76.  U(NXP,J+1) = U(NX,J+1)
77.  T4 = THETA(I,J)*4
78.  PH1 = -4.*TAU(I)*TH4 + TG
79.  FLUX = TAUO/AN*PH1
80.  WRITE (6,192) FLUX
81.  192 FORMAT (50X,ES,2)
82.  UP = U(I+1,J)
83.  UM = U(I-1,J)
84.  UN = U(I,J)
85.  U(I,J+1) = R*(UP-UM) + (1.-2.*R)*UM + DELTA/U*FLUX
86.  261 CONTINUE
87.  U(1,J+1) = 3.*U(2,J+1) - 3.*U(3,J+1) + U(4,J+1)
88.  241 CONTINUE
89.  WRITE (6,160)
90.  160 FORMAT (L//L)
91.  WRITE (6,161)
92.  161 FORMAT (2X,1HX,10X,1HT,11X,1HU,8X,5HTHEHA/
93.  DO 301 J=1,HT
94.  DO 302 I=1,NX
95.  WRITE (6,162) X(I), T(J), U(I,J), THETA(I,J)
96.  162 FORMAT (F5.2,4X,E8.2,4X,E8.2,4X,E12.6)
97.  302 CONTINUE
98.  301 CONTINUE
99.  STOP
100. END