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A PRELIMINARY STUDY OF
CRACK INITIATION AND GROWTH AT
STRESS CONCENTRATION SITES

By

D. S. Dawicke
J. P. Gallagher
G. A. Hartman
A. M. Rajendran

University of Dayton
Research Institute

September 1982

Prepared For:
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio
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ABSTRACT

A literature search was conducted to examine current crack initiation and propagation models for notches. The Dowling crack initiation model and the El Haddad et al. crack propagation model were chosen for additional study. Existing data was used to make a preliminary evaluation of the crack propagation model. The results indicate that for the crack sizes in the test, the elastic parameter K gave good correlation for the crack growth rate data. Additional testing, directed specifically toward the problem of small cracks initiating and propagating from notches is necessary to make a full evaluation of these initiation and propagation models.
FOREWORD

This First Interim Technical Report is submitted in accordance with the NASA Grant NAG 3-246. It summarizes the work accomplished during the period 1 January 1982 through 31 August 1982.

The work is being performed for the National Aeronautics and Space Administration. The work is being performed by the Aerospace Mechanics Department of the University of Dayton Research Institute, with Mr. George Hartman and Dr. J. P. Gallagher acting as co-Principal Investigator. Technical support is being provided by Dr. A. M. Rajendran and Mr. D. S. Dawicke.
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CRACK GROWTH RATE TESTS FOR INDUSTRIAL GRADE BAR COPPER TESTED AT A STRESS RATIO (R) OF 0.1
SECTION I
INTRODUCTION

The growth of cracks can be divided into the two categories of initiation and propagation. In a notched structure, the initiation and subsequently the propagation of small cracks are governed by the stress field created by the notch. After the crack grows beyond the influence of the notch, crack propagation behavior is governed by the nominal stress in the structure.

A literature search was performed to examine the state-of-the-art crack propagation and initiation models. Thirteen recently published studies [1-13] provide examples of crack initiation and propagation models. The most promising of these models are: the El Haddad et al. crack propagation model [1], the elastic-plastic J-integral based crack propagation model described by Ratwani et al. [2], and the Dowling crack initiation cut-off model based on linear elastic fracture mechanics [3,4,5]. The work to date has included a close examination of the promising crack propagation models, specifically by testing their ability to describe existing data.
SECTION 2
LITERATURE REVIEW

A literature review was conducted to determine the existing approaches taken to solve the problem of cracks initiating and propagating from notches. This review was presented at NASA-Lewis on 16 July 1982. The remainder of this section summarizes the important aspects of the crack initiation and propagation models that have been chosen for additional study.

2.1 STRAIN LIFE INITIATION MODEL

A crack initiation model developed by Dowling [3,4,5] predicts life to initiation for notch geometries using smooth specimen strain life data. The severity of the notch is taken into account by elevating the applied stress by the elastic stress concentration factor ($k_t$). The equivalent strain amplitude for notched geometry is obtained from the cyclic stress-strain curve of the material.

Dowling found this method to be effective in predicting crack initiation life if the initiation crack length remained within the influence of the notch. Dowling defined an upper bound on the initiation crack length which ensured the crack was within the influence of the notch. This upper bound was defined to be the intersection of what he termed the short crack stress-intensity factor ($K_s$) and the long crack stress-intensity factor ($K_l$) solutions described as a function of crack length. The short crack stress-intensity factor was defined as a function of the stress concentration factor ($k_t$) of the notch and the crack length measured from the edge of the notch ($\ell$),

$$K_s = 1.12S k_t \sqrt{\pi \ell}$$  \hfill (1)

where 1.12 is the free surface correction factor for an edge crack. The long crack stress-intensity factor assumed the notch...
has no effect on the crack behavior,

\[ K_A = S \sqrt{\pi a} \]  

where \( a \) is the sum of the crack length (\( \lambda \)) and radius (\( r \)) of the notch; \( S \) in Equations 1 and 2 is the applied stress.

The long crack and short crack stress-intensity factor solutions for a diametrically cracked 1.0 inch diameter hole in a wide plate are shown in Figure 1. Analytically, the crack length (\( \lambda_m \)) associated with the intersection of the long crack and short crack stress-intensity factors can be expressed in terms of notch radius (\( r \)) and stress concentration factor of the notch.

\[ \lambda_m = \frac{r}{(1.12 k_e)^2 - 1} \]  

The Dowling crack initiation cut-off parameter (\( \lambda_m \)) given by Equation 3 is shown in Figure 2 as a function of notch severity. As the notch severity increases the initiation crack length decreases, i.e., the notch behaves like a crack. It can also be noted that the initiation crack length is on the order of 10 to 20% of the notch radius.

Dowling also suggested that linear elastic fracture mechanics parameters could be utilized beyond the initiation cut-off.

2.2 CRACK PROPAGATION MODELS

2.2.1 Elastic J-Integral (K)

The linear elastic fracture mechanics (LEFM) parameter \( K \), the stress-intensity factor, has been found to be an effective basis for life analysis procedures when extensive plasticity does not occur. Localized cyclic plasticity around the root of a notch has been observed to accelerate the growth rate of cracks propagating from notches. El Haddad et al. [1], Leis et al. [11,12], and Newman [13] have shown that the LEFM parameter \( K \), correlates the crack growth rate data in a manner which would lead to unconservative life predictions for small cracks propagating from plastically deforming notches under fully reversed loading.
Figure 1. The Long and Short Crack Stress-Intensity Factor Solutions for a 1 inch Diameter Cracked Hole.

Crack length measured from edge of notch.

\[
\frac{K}{\sigma} = 1.12 \sqrt{\frac{a}{R}}
\]
Figure 2. Effect of Notch Severity on Dowling's Crack Initiation Cutoff Parameter.
The advantage of using the LEFM parameter $K$, is that it is a material independent property of a specific geometry. Handbook solutions exist for many simple geometries and a series of relatively inexpensive linear finite element runs will generate the solution for more complex geometries. One of the more direct methods for calculating the stress-intensity factor via finite element analysis involves using the path independent line integral derived by Rice [14], i.e., the $J$-integral, which is given by

$$ J = \int (Wdy - T \frac{\partial u}{\partial x} dx) $$

(4)

In the elastic case, the stress-intensity factor is proportional to the square root of the $J$-integral according to the formula,

$$ K = \sqrt{JE} $$

(5)

where $E$ is the elastic modulus. The LEFM parameter $K$ provides a starting point in the life analysis of cracks propagating from notches due to the availability of the solution and its acceptance in making linear elastic life predictions.

2.2.2 Elastic-Plastic $J$-Integral

Ratwani et al. [2] suggested that the behavior of cracks propagating from plastically deforming notches could be described using the $J$-integral calculated to account for the plastic behavior. The $J$-integral proposed for this correlation is calculated with Equation 4 based on finite element results for elastic-plastic material properties. Actually, once Ratwani et al. calculated the value of the $J$-integral under elastic-plastic conditions for a given loading and crack length, they converted $J$ to a quasi-stress intensity factor ($K_p$) using Equation 5, i.e.,

$$ K_p = \sqrt{JE} $$

(6)

which was then used for calculating crack growth behavior in the notch region.
2.2.3 Strain Based Stress-Intensity Factor

El Haddad et al. [1] proposed a different type of plasticity enhanced stress-intensity factor to account for the behavior of cracks propagating from notches. This approach is based upon a strain concentration factor ($k_E'$), which is the ratio of local strain ($e$) as a function of distance ($x$) from the notch to the nominal gross strain ($e$).

\[ k_E' = k_E'(x) = \frac{e(x)}{e} \]  

(7)

The stress-intensity factor ($K_{EH}$) proposed by El Haddad et al. is given by

\[ K_{EH} = S k_E' \sqrt{\pi \lambda} \]  

(8)

where $\lambda$ is the crack length measured from the notch root and $k_E'$ is evaluated for $x=\lambda$ (See Equation 7).

The strain concentration factor is evaluated by a nonlinear finite element solution of the uncracked geometry. The El Haddad et al. stress-intensity factor is equivalent to the LEFM $K$ for different levels of load when the material at the notch root does not experience plastic deformation. For stress levels which result in plastic deformation at the notch root, the strain concentration factor will elevate the stress-intensity factor to account for the accelerated growth rate of the crack. For crack lengths outside the plastic zone of the notch, the stress-intensity factor approaches that of the LEFM long crack stress-intensity factor. El Haddad et al. presented results which indicate this method correlates the crack growth rate data for cracks propagating from several types of notches under several loading conditions.
SECTION 3
APPLICATION OF EXISTING DATA TO CRACK PROPAGATION MODELS

A small data base of crack growth rate data [15] and a series of finite element results [16] for industrial grade buss bar copper were available for use in analyzing the crack propagation models. Fatigue crack growth rate data had been collected using 4.0 inch wide plates with center cracks (CCP) and with radially cracked holes (RCH). Specimen types and test stress levels are given in Table 1. Tensile tests and incremental step tests were conducted to obtain the monotonic and cyclic stress-strain curves for material modeling. The finite element based study was conducted using the MAGNA code and the material model was obtained from the tensile test results.

3.1 ELASTIC J-INTEGRAL \( (K) \)

A series of elastic finite element solutions were obtained for a crack propagating from the RCH specimen. The LEFM parameter \( K \), stress-intensity factor, for this geometry was calculated from the J-integral results of the elastic finite element solution. Figure 3 describes the finite element grids, crack tip elements, and a J-integral path for a 1 inch crack as measured from the edge of the 1 inch diameter hole in the 4 inch wide plate. The finite element solution for the stress-intensity factor is shown in Figure 4. The Bowie [17] solution for the stress-intensity factor of a crack propagating from a 1 inch diameter hole in an infinite plate has been added for comparison.

The difference between the Bowie solution and the finite element solution for \( K \) is due to the finite width of the plate.

The RCH crack growth rate data correlated with the finite element determined \( K \) are shown in Figure 5, along with the crack growth rate data for the CCP specimens. The fatigue crack growth rate obtained from these specimens were correlated with the analytical elastic \( K \) solution with a width correction factor used in ASTM standard E647 [18].
### TABLE 1

**CRACK GROWTH RATE TESTS FOR INDUSTRIAL GRADE BAR COPPER TESTED AT A STRESS RATIO (R) OF 0.1**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Specimen Type</th>
<th>Thickness (inch)</th>
<th>CRACK INTERVAL</th>
<th>Gross Stress (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a₀ (inch)</td>
<td>aₖ (inch)</td>
</tr>
<tr>
<td>1</td>
<td>CCP1</td>
<td>CCP*</td>
<td>0.50</td>
<td>0.261</td>
<td>0.688</td>
</tr>
<tr>
<td>2</td>
<td>CCP2</td>
<td>CCP</td>
<td>0.50</td>
<td>0.497</td>
<td>0.785</td>
</tr>
<tr>
<td>3</td>
<td>RCH1</td>
<td>RCH**</td>
<td>0.375</td>
<td>0.035</td>
<td>0.561</td>
</tr>
<tr>
<td>4</td>
<td>RCH2</td>
<td>RCH</td>
<td>0.375</td>
<td>0.034</td>
<td>0.725</td>
</tr>
<tr>
<td>5</td>
<td>RCH3</td>
<td>RCH</td>
<td>0.375</td>
<td>0.035</td>
<td>0.367</td>
</tr>
<tr>
<td>6</td>
<td>RCH4</td>
<td>RCH</td>
<td>0.375</td>
<td>0.062</td>
<td>0.396</td>
</tr>
</tbody>
</table>

* CCP is a center cracked panel with length, width, and notch length of 22.0, 4.0, and 0.5 inch, respectively.

** RCH is a radially cracked panel with a 1.0 inch diameter hole in a 22.0 inch long and 4.0 inch wide plate.
Figure 3. Finite Element Mesh Representation of a 4 inch Wide Plate with a Crack Propagating from One Side of a 1 inch Diameter Hole.
Figure 4. The Elastic Finite Element Solution and the Bowie Approximation.
Figure 5. Crack Growth Rate Data Correlated with the Elastic Stress-Intensity Factor for Industrial Grade Buss Bar Copper Subjected to a Fatigue Loading with Stress Ratio (R) of 0.1.
\[ K = S \sqrt{\pi a \sec \left( \frac{\pi a}{W} \right)} \]  

where \( a \) is the half crack length and \( W \) is the plate width.

For this material, the correlation with the finite element elastic \( K \) is very good. The accelerated crack growth rate for small cracks near the notch reported by others was not observed. The smallest crack lengths shown in Figure 5 are on the order of 0.1 inch which is small compared to the calculated size of the plastic zone shown in Figure 6.

A study by Newman [13] suggests that crack closure is the cause for the accelerated crack growth rate experienced by small cracks propagating from notches. The current data base seems to agree with this conclusion. The previously reported accelerated crack growth rate was seen in tests run with a stress ratio of -1 (fully reversed loading). The existing test results were for tests at a stress ratio of 0.1, thus crack closure may not be as important in the cracking process. We are planning further investigation in this area.

Newman reasoned that once closure has occurred, the load needed to open a short crack is less than that needed to open a long crack. This would result in a greater stress range in which damage is being done for the short crack, thus an increased crack growth rate.

3.2 ELASTIC-PLASTIC J-INTEGRAL

Nonlinear finite element based studies were also conducted for cracks propagating in the RCH specimens. Figure 7 summarizes the results of the J-integral calculations in terms of the ratio of "plastically" corrected stress-intensity factor \( K_p \) (given by Equation 6) to the elastic stress-intensity factor, for the two applied stress levels utilized during the test.
Figure 6. Extent of Plastic Zone Along Expected Path of Crack for a 1 inch diameter Hole in a 4 inch Wide Plate.
Figure 7. Extent of Plastic Enhancement of the Stress-Intensity Factor, as Calculated by the Finite Element Solution.
The "plastically" corrected stress-intensity factors based on Equation 6 were used to correlate the crack growth rate data from the RCH specimens as shown in Figure 8. Finite element solutions for the CCP specimens are not complete; thus, the crack growth rate data for the CCP specimens were not included in the figure.

3.3 STRAIN BASED STRESS-INTENSITY FACTOR

A nonlinear finite element solution for an uncracked RCH specimen was used as an initial estimate of the plastic strain near the notch. The results of this program were used to calculate the strain concentration factor \( k'_\varepsilon \). This factor was used in the plastically enhanced stress-intensity parameter \( K_{EH} \). \( K_{EH} \) was then used to correlate the crack growth rate data for the four RCH tests. The results of this correlation, along with the elastic long crack (CCP) data are shown in Figure 9.

The plastically enhanced stress-intensity factor, as described by El Haddad et al., failed to correlate the crack growth rate data from the RCH specimens with the long crack data. An additional test was conducted to determine the validity of the strains calculated by the finite element analysis. The test specimen consisted of an uncracked RCH specimen with strain gages attached to the inside of the notch. The specimen was then cycled at a stress level similar to that used in the fatigue tests. The experimental results indicate that the material at the notch root experienced a large decrease in plastic strain after the first cycle due to redistribution of stresses. The plastic strain amplitude stabilized after approximately the tenth cycle. The finite element solution agrees with the first cycle but differs substantially with the tenth and succeeding cycles, as shown in Figure 10. The strain concentration factor for the uncracked RCH specimen, as calculated by the finite element solution, does not give an accurate description of the strain along the expected crack path. A cyclic nonlinear finite element analysis is
Figure 8. Crack Growth Rate Data Correlated with the Plastically Enhanced Stress-Intensity Factor from the Finite Element Solution.
Figure 9. Fatigue Crack Growth Rate Behavior of the Copper Alloy Using the Stress-Intensity Factor Proposed by El Haddad and the Elastic Long Crack Data.
Figure 10. Extent of Redistribution of Stresses Experienced by the Copper and Finite Element Predictions of Strain.
planned and when the data is available it will be used to obtain a better estimate of the plastic strain for the El Haddard model.

3.4 SUMMARY OF EXPERIMENTAL RESULTS

The experimental results used thus far to evaluate the crack propagation models show that the elastic stress-intensity factor (K) correlates the crack growth rate data for the conditions tested. The elastic-plastic J-integral based (K_p) and the El Haddad et al. plastically enhanced stress-intensity factor (K_{EH}) are strongly dependent upon the material model used in the finite element analysis.

Additional testing is required to determine the conditions under which the elastic K fails to correlate the data. Tests are planned which will specifically address the problem of cracks propagating from notches. These tests will provide crack growth rate data for small cracks (less than 0.05 inch) near notches. Tests will be run under fully reversed loading and under a stress ratio of 0.1, to determine the effect of crack closure on the small crack growth rate.
SECTION 4
SUMMARY OF WORK ACCOMPLISHED AND WORK PLANNED FOR THE NEXT PERIOD

During the past six months, the work accomplished can be summarized as follows:

A. A literature review of current crack initiation and propagation model for notched structures was conducted.

B. A crack initiation model and several crack propagation models were chosen for further study.

C. Analysis, using existing crack growth rate data and finite element solutions, was performed to evaluate the crack propagation models.

D. The materials for additional testing has been acquired and the machining of the specimens has begun.

Between September 1 and 31 December 1982, the following work will be accomplished:

A. Baseline testing of the aluminum will be conducted.

B. Life predictions using the crack initiation and crack propagation models will be made.

C. Fatigue tests will be conducted on the aluminum.

D. The crack initiation and crack propagation models will be evaluated for the aluminum.

E. A plate of INCONEL 718 will be procured.
REFERENCES


