General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

ON THE THERMAL STABILITY OF CORONAL LOOP PLASMA

S. K. Antiochos, A. G. Emslie and E. C. Shoub

Institute for Plasma Research

Stanford University

and

C.-H. An

Center for Astrophysics and Space Science

UCSD

ON THE THERMAL STABILITY OF CORONAL LOOP PLASMA

S. K. Antiochos, A. G. Emslie and E. C. Shoub

Institute for Plasma Research

Stanford University

and

C.-H. An

Center for Astrophysics and Space Science

UCSD

Abstract

We consider the stability to thermal perturbation of static models of coronal loops. The effects of cool, radiatively stable material at the loop base are included. The linear stability turns out to be sensitive only to the boundary conditions assumed on the velocity at the loop base. The question of the appropriate boundary conditions is discussed, and we conclude that the free-surface condition (the pressure perturbation vanishes), rather than the rigid-wall (the velocity vanishes), is relevant to the solar case. We find that the static models are thermally unstable, with a growth time of the order of the coronal cooling time. The physical implications of these results for the solar corona and transition-region are discussed.

I. Introduction

The linear theory for the thermal stability of static coronal-loop models has been investigated by a number of authors, but with differing conclusions. In his original work, Antiochos (1979) concluded that the static models were thermally unstable. Similar results were obtained by Hood and Priest (1980). However, Chiuderi, Einaudi, and Torricelli-Ciamponi (1981), Craig and McClymont (1981), and McClymont and Craig (1981a,b,c) have found that the models are either stable or that the growth rates for instability are too small to be physically significant. The reason for this difference in the results of the two sets of authors is in their treatment of the base of the loop models. Antiochos (1979) and Hood and Priest (1980) have not included cool material, $T \lesssim 10^5$ K, in their model, for which the form of radiative loss curve (e.g., Raymond, Cox, and Smith 1976) favors linear stability (Field 1965). In addition, Antiochos has considered only perturbations with a vanishing first-order heat flux at the base. Chiuderi, Einaudi, and Torricelli-Ciamponi; and Craig and McClymont argue that the growth rates for instability are very sensitive to these assumptions so that the models can be effectively stabilized by either including some chromospheric material or by changing the boundary conditions on the temperature perturbation; in particular, the models can be metastable (zero growth rate) if the temperature perturbation, instead of the heat-flux perturbation, is assumed to vanish at the base.

It is clear, therefore, that the proper treatment of the base is critical for determining to what extent the models are either stable or unstable. In the next section we discuss the question of the boundary conditions in detail.

II. Perturbation Equations and Boundary Conditions

From the form of the perturbation equations, e.g., Antiochos (1979), Field (1965), it is evident that there are four independent spatial derivatives in the problem and, hence, only four boundary conditions can be specified. This is to be expected since the full nonlinear equations also require four spatial boundary conditions for a unique solution (e.g., Richtmyer and Morton 1967). Hence, at each end of the loop two conditions must be specified. One condition generally describes the thermal properties of the base; for example, the loop base may act as a thermal bath so that the temperature perturbation vanishes there, $T_{1b} = 0$; or it may act as a thermal insulator so that the perturbation heat flux vanishes there, $F_{1b} = 0$. The other condition describes the inertial properties of the base; for example, it may act as a rigid wall so that the velocity perturbation vanishes, $v_{1b} = 0$, or a free surface, so that the pressure perturbation, $P_{1b} = 0$. Depending on the physical situation, either of the inertial conditions may be assumed, but clearly not both. Assuming both conditions is physically inconsistent; it implies that the loop base is both rigid and free. Also, it is mathematically incorrect since, along with the temperature condition, it is equivalent to imposing six independent boundary conditions on a problem that admits only four.

However, previous authors (Antiochos 1979; Chiuderi, Einaudi, and Torricelli-Ciamponi 1980; and Craig and McClymont 1981) have, in fact, imposed both inertial boundary conditions on the perturbation. In their case, they were able to find solutions to the overconstrained problem because they considered only the restricted class of models in which gravity is neglected; and the loop area and the coronal heating do not have any spatial variation. Under these simplifications, the static equations are autonomous, and the equilibrium models are symmetric about the loop apex. Hence, the solutions to the first-order equations, i.e., the normal modes of the loop, are either purely symmetric or antisymmetric about the apex. But, for the simplified system with no gravity, the only possible antisymmetric solution to the force equation is that P_1 vanishes identically. Hence, all the antisymmetric solutions, irrespective of their boundary conditions, will also trivially satisfy the free-surface conditions; in particular, all antisymmetric modes for the rigid-wall conditions will satisfy the overconstrained problem.

Previous authors have used this result to exclude the symmetric modes as valid solutions. The argument used is that the antisymmetric modes are more physical because all the perturbed quantities, T_1, v_1, and P_1 vanish at the boundary. But it is evident from the discussion above that this argument is spurious since, in general, no mode can satisfy all these conditions. For a realistic equilibrium model that has no special symmetry properties, the modes will be neither symmetric nor

antisymmetric. Instead, there will be two distinct sets of modes corresponding to the two distinct physical conditions of either rigid walls or free surfaces, and no mode will satisfy both. Hence, within each set all the modes are equally acceptable, or unacceptable.

The question remains as to which set of boundary conditions best represents the physical situation in a solar loop. Ideally, one would like to place the loop boundary sufficiently deep down in the atmosphere that conditions there do not affect the stability properties of the corona and transition region. This can be done for the thermal boundary conditions by placing the loop base at a sufficiently low temperature. Since both conduction and radiation decrease rapidly for $T < 6 \times 10^4$ K, the thermal time scale at the base can be made to be very long compared to the time scales in the corona or transition region simply by including sufficiently cool material at the base of the static models. We expect that in this case the growth rates and the eigenfunctions will be insensitive to the thermal boundary conditions.

However, it is <u>not</u> possible to select the base point so that the growth rates and eigenfunctions are independent of the inertial boundary conditions. The reason for this is that the sound travel time in the corona, chromosphere and even photosphere is very rapid compared to typical thermal time scales, such as the coronal cooling time. Hence, we expect that the stability properties of the static models will, in general, be sensitive to the inertial boundary conditions assumed for the perturbations; we show below that this is indeed the case.

We are left, therefore, with having to decide which is the proper inertial boundary condition at the loop base. We believe that the correct condition is the free surface: $P_{1b} = 0$, and that the rigid-wall condition is not applicable to solar loops. Assuming the rigid-wall condition implies that at some level in the solar atmosphere a true physical boundary exists across which no mass transfer can occur. One may think of a rigid wall as representing material of infinite inertia and, hence, immobile. For example, in the loop models, the magnetic field is assumed to provide infinite inertia to motions perpendicular to the field direction, thereby justifying a one-dimensional loop geometry. However, in our model the plasma itself has essentially no inertia since that is the assumption implicit in dropping the acceleration terms. Therefore, by assuming rigid-wall conditions at the base, we are postulating that there is a distinct physical difference between the plasma in the loop and the plasma below, i.e., the inertial properties of the solar atmosphere must change abruptly at some level. To our knowledge, there exists no such level.

It is important to note that such an inertial change is not equivalent to abrupt density or pressure changes, which do occur in the atmosphere. In particular, the chromosphere may be thought of as a narrow interface between the low density and pressure corona, and the high density and pressure photosphere. Due to the density difference, the velocities in the photosphere will tend to be very small compared to those in the corona. For example, the velocities in the photosphere beneath a coronal hole

are negligible compared to solar wind speeds. But the important quantity is the mass flux, and this is the same in both sections. Also, due to the pressure ratio, the position of the top of the photosphere will be negligibly affected by pressure changes in the corona, but this does not mean that the top of the photosphere acts as a rigid wall with respect to the corona. On the contrary, material moves freely from the photosphere to the corona and vice versa. If the top of the photosphere acted as a true rigid wall, the mass flux would necessarily vanish there; and there would be no steady-state flows such as the solar wind or siphon flow into sunspots.

III. Growth Rates and Eigenfunctions

A. Free-Surface Modes

In this letter we calculate the growth rates and eigenfunctions only for the simplified problem that has been discussed by previous authors; more realistic models will be considered in a subsequent paper. In addition, we consider only the so-called thermally isolated static model, in which the heat flux is assumed to vanish at the loop base (Vesecky, Antiochos and Underwood 1979). However, we investigate in detail the effects of adding cool material to the coronal loop by including a model chromosphere, and we include in the analyses the first symmetric mode, which we have omitted previously. For comparison we calculate the growth rates for both the free-surface and the rigid-wall-boundary conditions; although as discussed above, we believe that the free-surface condition is the physically relevant one.

ORIGINAL PAGE IS
OF POOR QUALITY

Under the assumptions above, and neglecting the acceleration terms (Field 1965), the first-order equations can be written in Lagrangian form as:

$$\xi'' + Q(x)\,\xi + \lambda\, T_0^{-3/2}\,\xi = P_1\,(.4\lambda T_0 + G(x))/P_0 \tag{1}$$

where:

$$\xi = T_o^{3/2}\, T_1 \quad , \tag{2}$$

$$Q(x) = (n_o\Lambda + T_o\, \partial/\partial T_o\, (\varepsilon/n_o - n_o\Lambda)$$

$$- n_o\, \partial/\partial n_o\, (\varepsilon/n_o))/(10^{-6}\, n_o T_o^{7/2}) \quad , \tag{3}$$

$$G(x) = (\varepsilon/n_o - n_o\, \partial/\partial\, n_o\, (\varepsilon/n_o))/(10^{-6} n_o T_o) \quad , \tag{4}$$

$$\lambda = -\,10^7\, k^2\, \nu/P_o\ . \tag{5}$$

In (1) - (5) P is the total plasma pressure; n is the electron density; Λ is the radiative loss coefficient (e.g., Raymond, Cox and Smith 1976); ε is the coronal heating function; the coefficient of thermal conduction is that of Spitzer (1962); the subscript "o" refers to the static model; k is Boltzmann's constant; ν is the growth rate, i.e. all first-order variables are assumed to vary as $e^{\nu t}$; and the prime indicates differentiation with respect to the independent variable x, which corresponds to the total number of electrons initially in the loop and measured from the top (see Antiochos 1976 or McClymont and Craig 1981b).

Equation (1) must now be solved for the various boundary conditions. The thermal boundary conditions are straightforward to implement: if the temperature perturbation is assumed to

vanish at the base, then $\xi_b = 0$; if the heat flux perturbation vanishes, then $(\xi')_b = 0$. This is the main advantage to using a Lagrangian formulation. In Eulerian coordinates the boundary condition of a vanishing perturbation heat flux has an integral form in general; whereas it has the simple differential form above in Lagrangian coordinates. Note that there is a physical difference between the two formulations. In the Eulerian case we are setting the boundary at a particular point in the loop, whereas in the Lagrangian case we are setting the boundary at a particular plasma particle. (Of course, both formulations are identical in the case of rigid-wall conditions). Since we will show below that the stability properties are insensitive to the particular position of the boundary, as long as it is somewhere in the chromosphere, we may as well use the Lagrangian formulation for mathematical convenience.

The free-surface condition is also straightforward. Since P_1 is a constant for the case of no gravity, the free-surface condition implies that the righthand side of (1) vanishes. Hence, we are left with a standard Sturm-Liouville problem as in Antiochos (1979), which we solve in exactly the same manner as before. The rigid-wall condition, on the other hand, is more difficult to implement. The simplest procedure is to express it as an integral constraint, equivalent to Equation (20) in Antiochos (1979). Letting the position of the base points be $\pm X$, ($\pm L$ in Eulerian coordinates), we find that the perturbed pressure, P_1, can be determined in terms of the eigenfunctions:

$$P_1 = (k/L) \int_{-X}^{X} \xi T_o^{-3/2} \, dx \tag{6}$$

Therefore, for the rigid-wall condition, we must solve the inhomogeneous problem (1), subject to the constraint (6) (cf. McClymont and Craig 1981c). Note that this is not a standard Sturm-Liouville problem; hence, none of the standard theorems (e.g., Morse and Feshbach 1953) on the behavior of the eigenfunctions and eigenvalues need apply. Since for this problem there are two parameters, λ and P_1, that must be determined, we do not use a "shooting" technique as in the homogeneous case to obtain solutions. Instead, we consider (1) and (6) as defining a nonlinear problem for λ and ξ/P_1 and use basically Newton-Raphson iteration. Convergence was rapid, $\lesssim$ 20 iterations, for all the cases we ran.

There are two functions in the problem that must be specified: the radiative loss coefficient, $\Lambda(T)$, and the energy input per particle, $\varepsilon(n,T)/n$. For the former we use a smooth analytic approximation to the curve derived by Raymond, Cox and Smith (1976). The stability properties of the models are not sensitive to the exact form of $\Lambda(T)$ as long as this form is such that it implies radiative instability above $\sim 10^5$ K and stability below $\sim 10^5$ K.

Since the mechanism for coronal heating is not known, the form of the energy input ε is essentially arbitrary. In our previous work we found that the stability properties are also insensitive to the exact form for ε, at least, for the case where this form is a power law dependence on n and T. Hence, we

simply assume that in the corona and transition region, i.e., $T \gtrsim 6 \times 10^4$K, the energy input per particle is constant: ε/n = const. However, for the lower temperatures we use a different form for the energy input so that there will be a significant mass of material at "chromospheric" temperature, $T < 6 \times 10^4$ K. The simplest procedure for obtaining a thick chromosphere in the static models is to adjust the energy input rate at low temperatures so that it becomes almost equal to the radiation loss rate (e.g., Craig, Robb and Rollo 1981). Therefore, we use the following form for the energy input:

$$\varepsilon/n = C_o \tanh(\chi(T)) + n\Lambda\,(1 - \tanh(\chi(T))) \qquad (7)$$

where

$$\chi(T) = C_1\,(T/T_{ob} - 1 + \delta)^m \qquad (8)$$

and

C_o, C_1, T_{ob}, m and δ are constants. Equations (7) and (8) imply that for $\delta \ll 1$: $\varepsilon/n \to C_o$ for $T > T_{ob}$ and that $\varepsilon/n \to n\Lambda$ for $T \simeq T_{ob}$. The amount of material at T_{ob} becomes arbitrarily large for arbitrarily small δ; in particular, one finds a chromospheric size scale $\propto 1/\delta$ for the value, m = 3, of the exponent in Equation (8). Of course, this is a highly unrealistic model for the true solar chromosphere. However, it permits us to investigate conveniently the effect of adding cool, radiatively stable material to the loop base.

We have analyzed the stability properties of a wide range of static models. In Figure 1 we plot the growth rates of the three lowest free-surface modes for a series of static models. These

modes are the first symmetric modes, $T'_{1,top} = 0$, with either $T_{1,base} = 0$ or $T'_{1,base} = 0$, and the first antisymmetric mode, $T_{1,top} = 0$, with $T'_{1,base} = 0$. The static models all have almost identical coronal properties, i.e., at the loop apex the temperature, $T_{cor} = 2.3 \times 10^6$ K, the density, $n_{cor} = 5.5 \times 10^9$ cm^{-3}, and the half-length, $L = 1.7 \times 10^9$ cm. The models were generated by fixing: the energy input rate into the corona, $C_o = 10^{-12}$ ergs/sec/particle [Equation (7)]; the chromospheric temperature, $T_{ob} = 3 \times 10^4$ K; and the exponent $m = 3$. We vary either the base temperature (T_b), or the mass of the base ($T_b = T_{ob}$ and δ varies).

The growth rates in Figure 1 are expressed in terms of the conductive cooling time of the coronal plasma, i.e., we plot $\lambda = \nu\tau_c$, where τ_c is defined as:

$$\tau_c = \frac{5/2\ P_o\ L^2}{10^{-6}\ T_{cor}^{7/2}} \tag{9}$$

For the particular values of the loop parameters above, $_c = 1.4 \times 10^3$ sec; however, is insensitive to the value of $_c$. Hence, Figure 1 should be valid for any static model whose parameters lie within the observed range of solar values. Note that for all the free-surface modes shown in Figure 1, λ is negative, indicating <u>instability</u>.

There are several interesting features of Figure 1. First, it is evident that the growth rates of the modes with thermal boundary condition, $T_{1,base} = 0$, are insensitive to the value of the base temperature or to the mass of chromospheric material. In particular, the first symmetric mode has $\lambda \simeq -.65$; the first

antisymmetric mode can be shown analytically to have $\lambda \equiv 0$ (Craig and McClymont 1981). The modes with a vanishing-perturbation heat flux, on the other hand, are sensitive to the value of the base temperature, in agreement with our previous results (Antiochos 1979); but only if the amount of chromospheric material is very small, $\lesssim .1\%$ of the total loop mass. For models with a significant amount of cool material, the growth rates are independent of the thermal boundary conditions or of the amount of base material.

We conclude, therefore, that the static models of coronal loops are linearly unstable, at least for the class of models considered here. The fastest growing mode in all cases is the lowest order one; hence, the instability is a global one involving the whole coronal loop. This can be seen in Figure 2, where we have plotted the eigenfunction $T_1(x)$ for the case with the largest chromosphere, ~ 20% of the total loop mass. Only the mode with $T_{1,base} = 0$ is shown; however, the mode with $T'_{1,base} = 0$ is indistinguishable from it except very near the base. The modes exhibit a significant amplitude only in the coronal and transition region part of the loop. This amplitude peaks at approximately the temperature where the radiative loss rate $n^2 \Lambda$, is maximum ~ 8×10^4 K. Below this temperature the amplitude falls off exponentially. Although the peak occurs in the transition region, the coronal contribution to the "energy" in the mode, i.e., $\int T_1^2 \, dx$, is approximately equal to the transition region contribution. This emphasizes the point that the instability is a global one and not restricted solely to the

transition region.

B. Rigid-Wall Modes

In Figure 1 λ is plotted for the first symmetric rigid-wall modes and for each of the two thermal boundary conditions. (The antisymmetric modes are identical to those of the free-surface case as discussed above.) It is evident that the values of λ are very different from the free-surface case. First, they are all positive, indicating stability, except for the cases with very little chromosphere, < .1%, and $T_{1b}' = 0$. For these cases the value of λ is almost identical to the corresponding free-surface modes. However, for the physically relevant cases with some chromospheric mass, the values for λ are positive. Note also that for models with significant amounts of chromospheric mass, > 10%, the values of λ decrease, indicating a tendency toward instability as the chromospheric mass increases.

The situation can be clarified by inspection of the eigenfunction T_1 (x) for the lowest symmetric mode (Figure 2). As in the free-surface case, the mode peaks at $T \sim 8 \times 10^4$ K, but note that it has a zero crossing, whereas the lowest free-surface mode can have none. The amplitude in the chromosphere is large; it dominates the contribution to the "energy" in the mode. As the chromospheric mass increases, this amplitude increases and becomes more constant. Hence the symmetric rigid-wall modes are primarily chromospheric perturbations rather than coronal perturbations as in the free-surface case. Since our model for the chromosphere is highly artificial, the physical significance of the rigid-wall modes is questionable.

Discussion

The key result of this paper is that the static models of coronal loops are thermally unstable, even if the models contain arbitrary amounts of chromospheric material. This disagrees with the conclusions of several authors (Habbal and Rosner 1979, Chiuderi, Einaudi and Torricelli-Ciamponi 1981, Craig and McClymont 1981 and McClymont and Craig 1981a,b,c), but agrees with the conclusions of Hood and Priest (1980) and Wragg and Priest (1982). The growth rate of the instability is of the order of the cooling time of the coronal plasma, independent of the particular parameter of the model. We can identify two possible physical manifestations of this instability. One is in the observations of velocities in the transition region (e.g., Lites et al. 1976). Since the amplitude of the unstable mode peaks in the transition region, one might expect the largest velocities to be generated there. Another possible manifestation of thermal instability is in observation of cool condensations in the corona; particularly, quiescent prominences. Our results imply that coronal loops are naturally unstable; and, hence, condensations may form spontaneously in the corona.

Of course, in order to investigate these possibilities, the nonlinear evolution of the instability must be calculated. Clearly, the instability must not saturate at a low level if it is to produce sizable velocities in the transition region or cool H-alpha condensations in the corona. As discussed in our previous paper (Antiochos 1979), we believe that the instability will result in a nonlinear oscillation of the loop about its

static state. The period of the oscillation should be related to the growth rate of the instability, 10^3 sec. for typical loop parameters. The amplitude of the oscillation, which must be calculated by nonlinear simulation, will determine the physical importance of the instability.

Numerical simulations of coronal loops have been performed by several authors. These simulations have not exhibited any significant evidence for thermal instability. However, in all cases rigid-wall boundary conditions were used, and it is clear from our results here that these conditions will tend to stabilize the models, at least linearly. In addition, numerical effects may be providing unphysical stability to the models. We intend to consider, in detail, the nonlinear stability of the static loop models in a forthcoming paper.

Acknowledgments

This work was supported by NASA grants NGL 05-020-272 and NAGW-92, and ONR contract N00014-75-C-0673.

References

Antiochos, S. K. 1976, Stanford University Institute for Plasma Research Report No. 679.

Antiochos, S. K. 1979, Ap. J. Lett., 232, L125.

Chiuderi, C., Einaudi, G. and Torricelli-Ciamponi, G. 1981, Astron. Ap., 97, 27.

Craig, I. J. D. and McClymont, A. N. 1981, Nature 294 333.

Craig, I. J. D, Robb, T. D. and Rollo, M. D. 1982, Sol. Phys. (in press).

Field, G. B. 1965, Ap. J., 142, 531.

Habbal S. R. and Rosner, R. 1979, Ap. J., 234, 1113.

Hood, A. W. and Priest, E. R. 1980 Astron. Ap., 87, 126.

Lites, B. W., Bruner, E. C, Chipman E. G, Shine, R.A., Rottman, G. J., White, O. R. and Athay, R. G. 1976, Ap. J. Lett., 210, L111.

McClymont, A. N. and Craig, I. J. D. 1981a, Univ. of Waikato Math. Res. Report No. 96.

_____ 1981b, Univ. of Waikato Math. Res. Report No. 99.

_____ 1981c, Univ. of Waikato Math. Res. Report No. 100.

Morse, P. M. and Feshbach, H. 1953, Methods of Theoretical Physics (New York: McGraw-Hill), ch. 6.

Oran, E.S., Mariska, J. T. and Boris, J. P. 1982, Ap. J., 254, 349.

Peres, G., Rosner, R., Serio, S. and Vaiana, G. S. 1982, Ap. J., 252, 791.

Raymond, J. C., Cox D. P. and Smith, B. W. 1976, Ap. J., 204, 290.

Richtmyer, R. D. and Morton, K. W. 1967, Difference Methods for Initial-Value Problems, (2nd ed., New York: Interscience Publishers).

Spitzer L. 1962, Physics of Fully Ionized Gases (New York: Wiley Interscience).

Vesecky J. F., Antiochos, S. K. and Underwood, J. H. 1979, Ap. J., 233, 987.

Wragg, M. A. and Priest, E. R. 1982, Astron. Ap. (in press).

Figure Captions

Figure 1

Absolute value of λ for a series of static models with decreasing base temperature and/or increasing chromospheric mass. The results for five modes are shown. Solid lines refer to free-surface modes, dashed lines refer to rigid-wall modes.

Figure 2

The eigenfunction T_1 for the lowest symmetric free-surface (solid line) and rigid-wall (dashed line) modes. They are plotted as a function of log (r) where r is defined as: $r \equiv \int_0^x n_0(x)\, x\, dx$.

ORIGINAL PAGE IS
OF POOR QUALITY

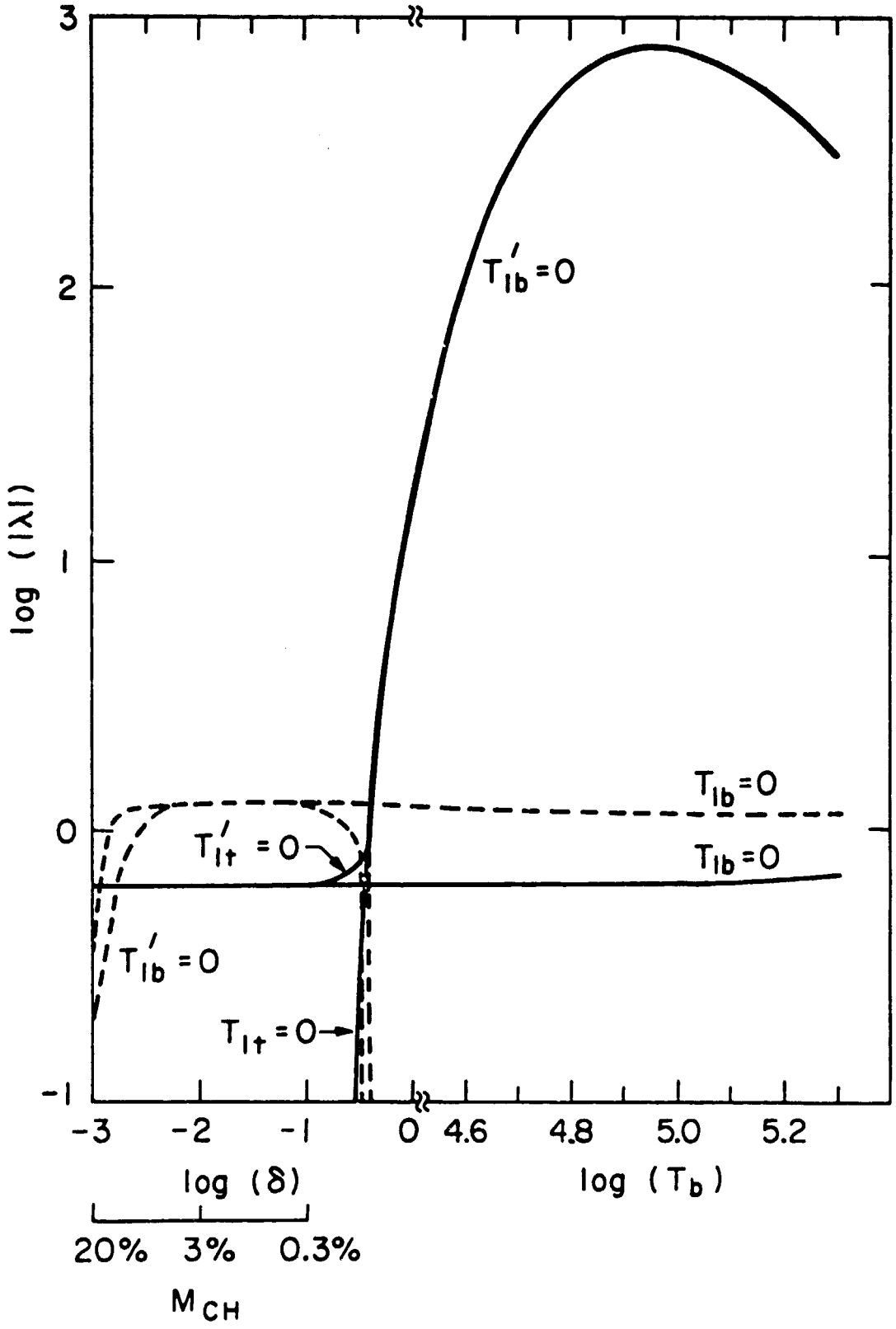

Figure 1

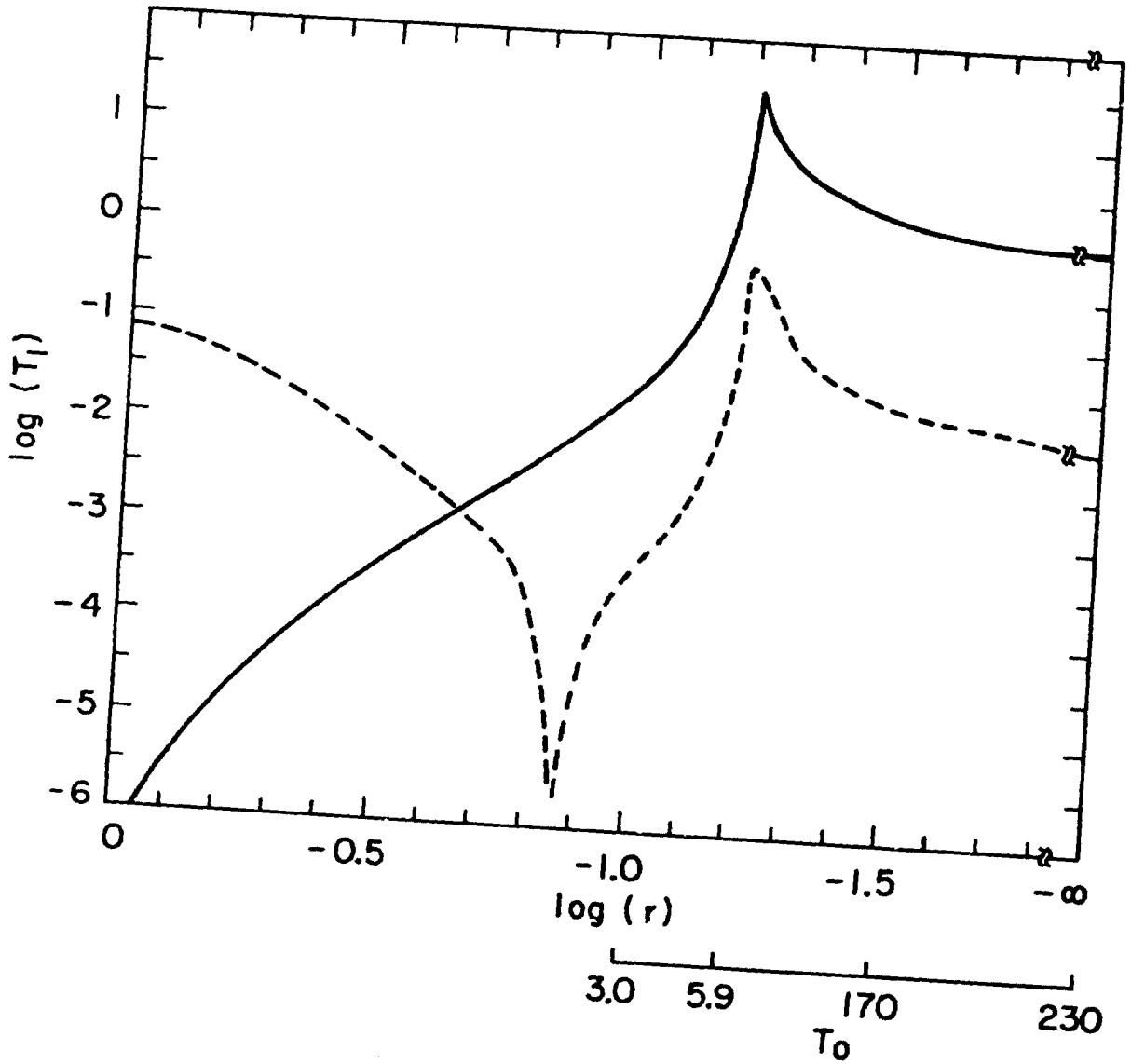

Figure 2

ORIGINAL PAGE IS
OF POOR QUALITY