NASA Technical Memorandum 84544

OPTIMIZATION OF COMPOSITE STRUCTURES

(NASA-TM-84544) OPTIMIZATION OF COMPOSITE STRUCTURES (NASA) 17 p HC A02/MF A01 CSCL 20K
N83-10447
Unclas
G3/39 38300

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AUGUST 1982

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INTRODUCTION

Composite materials greatly expand the options for obtaining efficient structural designs. Because of the large number of design options, the task of finding the optimum configuration for a composite structure is more difficult than for the corresponding metal structure. The opportunity to obtain superior designs together with the difficulty of selecting among the many options is making automated structural sizing--or structural optimization--an increasingly popular design tool for composite structures.

Three excellent reviews of structural optimization have appeared recently: Schmit (1981), Vanderplaats (1982), and Lev (1981). These three papers describe the history of and current work in structural optimization, particularly in the U.S., and discuss future applications. Another excellent review, Haftka (1981), which has been presented but is as yet unpublished, considers structural optimization with aeroelastic design requirements.

This paper provides a brief introduction to optimization and describes its application to composite structures. Two early approaches to systematic structural design are described. Then, basic concepts and definitions for modern optimization procedures are presented and contrasted with the earlier approaches. (A much more complete and detailed survey of the development of structural optimization is contained in the references cited above.) Several design studies illustrate factors that must be considered when using optimization techniques to design composite structures. One important factor is that composite structures can be tailored very well to meet a given set of design requirements, but the resulting structure may be very sensitive to off-design conditions--that is, conditions not considered in the original set of design requirements. Another factor is that optimized structures may be sensitive to imperfections. The design studies in this paper consider the effect of material strength, buckling, thermal loads, and geometric imperfections. All calculations were performed with the computer program PASCO (Anderson and Stroud, 1979; and Stroud, Greene, and Anderson, 1981). PASCO contains the computer program VIPASA (Wittrick and Williams, 1974) for the buckling analysis of stiffened panels and the computer program CONMIN (Vanderplaats and Moses, 1973) as the optimizer.

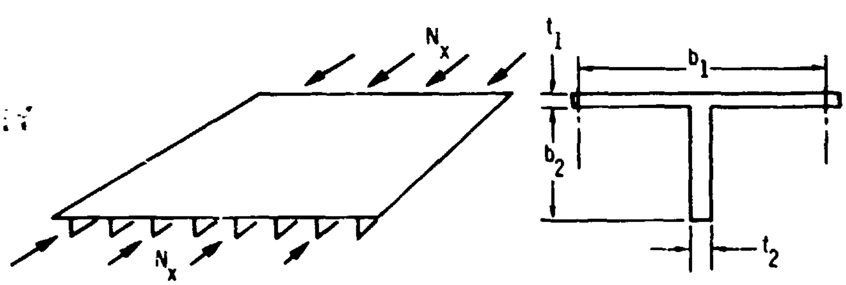
SYSTEMATIC STRUCTURAL DESIGN METHODS

An early systematic method for adjusting structural design variables is the fully-stressed design approach using a stress-ratio sizing algorithm. The method is based on the assumption that the lightest design is obtained if each structural member is stressed to an allowable stress in at least one of several loading conditions. With this method, thicknesses are changed using the formula

$$t_{i+1} = \frac{\sigma_i}{\sigma_a} t_i \tag{1}$$

where t_{i+1} is the thickness of a structural member for sizing iteration i+1, σ_a is the allowable stress, and t_i and σ_i are the thickness of, and stress in, the structural member for sizing iteration i. (Here, the word "stress" includes any representative measure of the stress state.) After all members are resized, the structure is reanalyzed, internal stresses are recalculated, and the thicknesses are updated using the above formula. This is an excellent approach if changes in the design variables cause only small changes in the load in each member (that is, if the structure is close to being statically determinate), and if a stress limitation is the only design requirement.

Another early design approach is based on the concept of simultaneous failure modes. It is similar, in some ways, to the fully-stressed design approach in that it is assumed the lightest design is obtained when two or more modes of failure occur simultaneously. It is also assumed that the failure modes that are active at the optimum (lightest) design are known in advance. Consider, for example, the way this procedure would be used to design a metal blade-stiffened panel having the cross section shown in Fig. 1. There are four design variables. Rules-of-thumb based on considerable experience are first used to establish proportions,



(a) BLADE-STIFFENED PANEL SUBJECTED TO COMPRESSIVE LOAD

(b) STIFFENER DETAIL SHOWING DESIGN VARIABLES b_1, b_2, t_1, t_2

Fig. 1. Metal blade-stiffened panel with four design variables.

such as plate width-to-thickness ratios. Establishing the proportions eliminates two of the design variables. The two remaining design variables are then calculated by setting the overall buckling load and the local buckling load equal to the applied load. This approach results in two equations in the two unknown design variables. The success of the method hinges on the experience and insight of the engineer who sets the proportions and identifies the resulting buckling modes. For metal structures having conventional configurations, insight has been gained through many tests. Limiting the proportions accomplishes two goals: It reduces the number of design variables, and it prevents failure modes that are difficult to analyze. This simplified design approach is, therefore, compatible with a simplified analysis capability.

Although both the fully-stressed design approach and the simultaneous failure mode approach are still being used for structural design, two other design approaches offer much greater potential for future structural design applications. These two approaches are optimality criteria methods and nonlinear mathematical programming techniques. During the 1970's there was considerable discussion regarding the relative merits of optimality criteria and nonlinear mathematical programming. Where applicable, optimality criteria methods produced an improved design with few iterations; nonlinear mathematical programming methods were rigorous and had broad generality but required more iterations. During this same time period, optimality criteria methods were becoming more rigorous. By the late 1970's, simularities between the two methods were being noted (Fleury, 1980). To some researchers, the differences between optimality criteria and nonlinear mathematical programming, once clear and distinct, are now beginning to blur. Some of the most up-to-date developments in both optimality criteria and nonlinear mathematical programming are discussed in the proceedings of a recent conference on optimum structural design (Gallagher and colleagues, 1981). The present paper focuses on design of composite structures using nonlinear mathematical programming.

BASIC CONCEPTS AND DEFINITIONS

A formal statement of the structural design problem is as follows: Find the values of the design variables $\overline{X}$ that:

(1) Minimize an objective function $W(\overline{X})$

(2) Satisfy a set of inequality design requirements $\overline{G}(\overline{X}) > 0$

(3) Satisfy lower and upper bounds $\overline{XL} < \overline{X} < \overline{XU}$

where a bar over a quantity indicates a vector. The design variables for filamentary composite structures can include ply orientation angles and ply thicknesses. These extra design variables allow composite structures to be more highly tailored than metal structures. The objective function in structural optimization is usually the weight. The design requirements $G > 0$ are referred to as behavioral constraints. (In many optimization codes the inequality is reversed, so that design requirements are written as $G(X) < 0$.) The lower and upper bounds on $\overline{X}$ are referred to as side constraints.

The constraints $\overline{G}(\overline{X}) > 0$ are mathematical expressions that must be satisfied in order to limit the structural behavior or prevent some failure mode. For example, if a stress σ in a structure is to be no more than a specified allowable stress σ_a, then this design requirement can be written as

$$1 - \frac{\sigma}{\sigma_a} > 0 \qquad (2)$$

The stress σ depends on the design variables and, given the loading system, can be calculated. For this case the allowable stress does not depend upon the design variables. The constraint G can be written as

$$G = 1 - \frac{\sigma(\overline{X})}{\sigma_a} \qquad (3)$$

In the more general case, the response quantity (represented by σ in eq. (3)) and the allowable response quantity (represented by σ_a in eq. (3)) are both functions of the design variables. In addition to stress, inequality design requirements are often placed on deflections, buckling loads, vibration frequencies, flutter speeds, and stiffnesses.

A major advancement in structural design technology occured when it was recognized that structural design is primarily an inequality design problem defined by the formal statement above and when computational procedures evolved that provided for the inequality constraints. A large portion of the credit for this advancement goes to Lucien Schmit (1960, 1981). It is now recognized that it is neither proper nor necessary to prescribe in advance which of many inequality constraints are to be critical at the optimum design.

A general and powerful discipline for solving the inequality design problem is nonlinear mathematical programming, which is often referred to as simply nonlinear programming. Nonlinear programming is a branch of the broader mathematical discipline denoted operations research that deals with the general problem of optimality. The word "nonlinear" is used because the objective function and constraints can be general nonlinear functions of the design variables. The nonlinear programming approach for solving optimization problems is to search for values of the design variables $\overline{X}$ while monitoring both the value of the objective function W and the values of the inequality constraints $\overline{G}$. The search is an iterative process that begins by assuming an initial design $\overline{X}_0$. A move direction $\overline{S}$ is then generated in design variable space. Steps are taken in that move direction and the best design in that direction is identified. The mathematical statement of the above search is

$$\overline{X}_1 = \overline{X}_0 + \alpha\overline{S} \qquad (4)$$

where α is the distance traveled in the $\overline{S}$ direction. The process of generating a move direction and calculating the distance traveled in that direction continues until the search converges to the set of design variables that minimized the objective function and satisfied the inequality constraints.

There are many approaches for calculating $\overline{S}$ and α, and there are many ways to account for inequality constraints (Fox, 1971; Kirsch, 1981; and Gallagher and colleagues, 1981). However, in all these nonlinear programming approaches, both the objective function and the constraints (or approximations to the objective function and/or constraints) are monitored during the iterative search and are used to calculate both $\overline{S}$ and α . The minimum value of the objective function (the best design) is not assumed to occur when a preselected set of constraints is critical--which is the fundamental assumption used in the fully-stressed and simultaneous failure mode approaches discussed earlier.

As discussed in detail by Vanderplaats (1982), a computer program for structural optimization contains three key ingredients: (1) an analysis program, (2) an optimization program, and (3) an interface program through which the analyzer and optimizer communicate. The analysis program calculates values of the structural response quantities associated with the failure modes that are to be prevented. It then takes these response quantities and calculates values of the constraints $\overline{G}$. The analysis program also calculates the value of the objective function W. In addition, almost all optimization programs require the analysis program to calculate the derivatives of both the constraints and the objective function with respect to the design variables. This derivative information is calculated either analytically or using finite difference approximations. The optimization program contains the logic that uses the values of the constraints $\overline{G}$, the objective function W, and the derivatives of $\overline{G}$ and W to search design variable space for the optimum design. Several general purpose nonlinear programming optimizers are available. For relatively simple problems the optimizer can be coupled directly to the analysis program. Each time the optimizer requests information about the objective function or constraints, the analysis program carries out a new analysis. However, for optimization problems having analyses that are not simple (for example, any analysis using a large finite element model), the large number of analyses generally required by the optimizer makes it impractical to couple the optimizer directly to the analysis program. For these problems, an interface program[1] can be used to generate and provide approximate values of the constraints. Periodically, the interface program calls the analysis program and updates the approximations. For the most part, interface programs are not standard, off-the-shelf programs. They are written by the user to match his optimization problem. The computer program PASCO, which was used to generate all the results in this paper, uses an interface program approach.

At this point in the paper it is appropriate to review the topics that have been presented and to introduce the topics that complete the paper. The first part of the paper has focused on structural optimization--two early approaches to systematic design and some modern ideas. A researcher familiar with structural analysis but unfamiliar with structural optimization may now know, in general, what he should do in order to use optimization. The remainder of this paper discusses some of the characteristics of optimized structures, without regard to the approach used to carry out the optimization. Optimized structures--in particular, optimized composite structures--can be sensitive to off-design conditions and to imperfections. Knowledge of these potential dangers can help guide a researcher in his optimization studies.

SENSITIVITY OF OPTIMIZED COMPOSITE STRUCTURES TO OFF-DESIGN CONDITIONS AND IMPERFECTIONS

In all structures--metal as well as composite--there are several levels of design variables. At the detailed level, design variables can define plate element widths and thicknesses that make up a stiffener in a stiffened panel. At another level, design variables may define the arrangement of these panels in a wing structure. A third level design variable may consider the overall configuration, such as wing

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[1] An interface program may merely serve as a control program for calling various subroutines and for manipulating data to ensure data compatibility between these subroutines. Also, since most current structural analysis programs do not calculate derivatives of response quantities with respect to structural parameters, an interface program can be used to help calculate derivatives. However, as used here, an interface program is a program that requests information from the analysis program, generates approximate values of $\overline{G}$, W, $\overline{\nabla G}$, and $\overline{\nabla W}$, and supplies these approximate values to the optimizer. The symbol ∇ denotes the gradient vector operator.

span and aspect ratio. Composite materials provide additional design variables at the detailed level. These design variables define fiber orientation and the thickness of material at that orientation. If the physical properties of a metal and composite material are equivalent, the additional design variables provided by the composite material should lead to composite designs that are superior to metal designs. Some of the properties of graphite-epoxy, the composite material considered in this paper, are superior to those of aluminum. For example, the density of graphite-epoxy is about half that of aluminum, and its modulus of elasticity in the fiber direction is about twice that of aluminum. On the other hand, the modulus of elasticity transverse to the fiber direction is low, and there is evidence that slight damage--such as low speed impact damage--can cause a substantial reduction in strength (Starnes, Rhodes, and Williams, 1979).

Although it is true that the additional design variables afforded by composite materials provide an opportunity to obtain superior designs, it is also true that the designer should be aware of possible problems that can arise with these superior designs. These problems can arise because a structure that is tailored very well to meet a specific set of design conditions can fail at relatively low load levels for some other load condition. Also, optimized structures tend to have multiple modes of failure occurring simultaneously and can be sensitive to imperfections.

Example 1, Composite Laminate with Buckling and Material Strength Constraints

As a simple illustrative example, consider a laminate made of a graphite-epoxy material having the material properties given in Table 1. Subscripts 1 and 2 indicate the fiber direction and transverse to the fiber direction, respectively. The laminate is balanced and symmetric with plies oriented at $\pm\theta$ with respect to the x-direction as shown in Fig. 2. There are many plies so that bending-twisting coupling can be neglected. The loading is N_x = 175 kN/m (1000 lbf/in.). This example has two objectives:

(1) To show how the laminate thickness required to support the loading varies with ply orientation angle θ for both a buckling failure criterion and a material strength failure criterion

(2) To show that a laminate designed to prevent failure by buckling differs substantially from a laminate designed to prevent failure by material strength limitations--here, a strain criterion

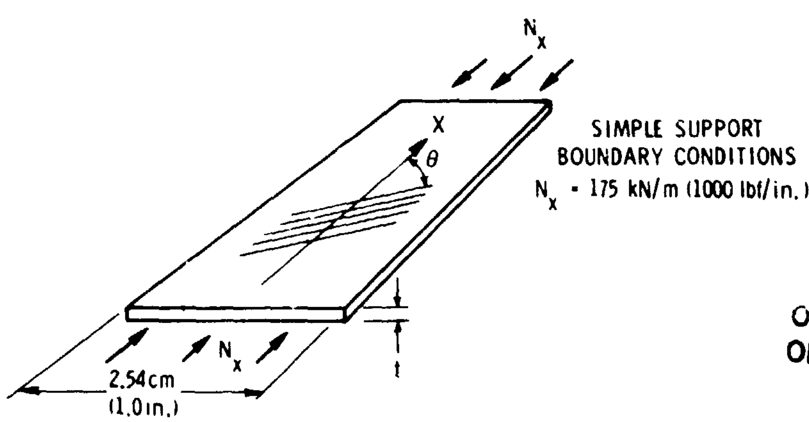

Fig. 2. Laminate loading and configuration, example 1.

These two objectives highlight the difference between an orthotropic composite material with a ply angle design variable and an isotropic material, such as a metal. For buckling calculations, the laminate is taken to be 2.54 cm (1.0 in.) wide and very long in the x-direction. Boundary conditions are simple support. For material strength calculations, limitations are placed on the strain ε_1 in the fiber direction, the strain ε_2 transverse to the fiber direction, and the shear strain γ_{12}. The limits are $|\varepsilon_1| < .004$, $|\varepsilon_2| < .004$, and $|\gamma_{12}| < .01$.

The laminate thickness required to support the load is presented in Fig. 3 as a function of the ply orientation angle. The vertical scale indicates the required thickness; the horizontal scale indicates the ply angle θ . The solid curve is for buckling; the dashed curve is for material strength. For buckling, the lightest laminate has $\theta = 45^o$. For material strength, the lightest laminate has $\theta = 0^o$. The notations "Governed by ε_1", etc., on the dashed curve indicate the portions of the curve for which the corresponding strain components govern the design. The strain ε_1 is critical for small values of θ, ε_2 is critical for $19^o < \theta < 28^o$, and γ_{12} is critical for $\theta > 28^o$. Since the required thickness varies with the ply angle, this example shows that a ply angle design variable can be useful for reducing structural weight. However, this example also points out that the best ply angle for one failure mode can be a poor ply angle for another failure mode.

The rationale for the design approach based on simultaneous failure modes is also illustrated in Fig. 3. The lightest design satisfying all design requirements is at the intersection of two constraint curves. In general, the lightest metal plate satisfying these design requirements would not have simultaneous failure modes unless the plate width were a design variable.[2] With the additional design

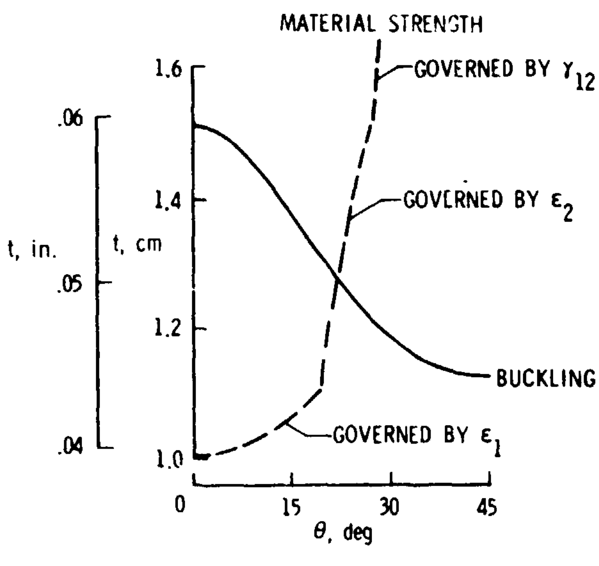

Fig. 3. Laminate thickness required to support given load as a function of ply orientation angle. Failure criteria are buckling and material strength. Load is N_x = 175 kN/m (1000 lbf/in.), example 1.

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[2] Differences between composites and metals can be illustrated with an even simpler example. A composite laminate designed by material strength for an N_x loading has a very low load-carrying capability when an N_y loading is added. A metal plate has no comparable reduction in load-carrying ability for this off-design loading.

variables provided by composite materials, it is reasonable to assume that simultaneous failure modes occur more frequently for optimized composite structures than optimized metal structures.

Example 2, Composite Stiffened Panel with Temperature Effects

This second example also uses a ply angle design variable and illustrates the sensitivity of an optimized composite structure to off-design conditions. Consider the minimum-weight graphite-epoxy blade-stiffened panel designed to support a compressive load of N_x = 525 kN/m (3000 lbf/in.) with a uniform temperature change of -111 K (-200°F). The failure criterion was buckling. (The strain limitations used in example 1 were also used in this example, but, since the strains were lower than the limiting strains, the strain limits did not influence the design.) The temperature change is measured with respect to the temperature for zero residual stress (curing temperature)--generally an elevated temperature. Assume that adjacent structure prevents the panel from undergoing temperature-induced deformation. That is, the panel remains flat.

The panel and loading are defined in Fig. 4. The length of the panel is fixed at 0.76 m (30 in.); the stiffener spacing is fixed at 0.13 m (5 in.). The skin consists of ±θ plies; the attachment flange and blade stiffener consist of 0° plies surrounded by ±45° plies. The design variables are the skin ply angle θ, the five thicknesses t_1 - t_5, the width b_1 of the attachment flange, and the depth b_2 of the blade. The buckling boundary conditions are simple support. The optimum design for the given design condition has a skin ply angle of ±66.2°.

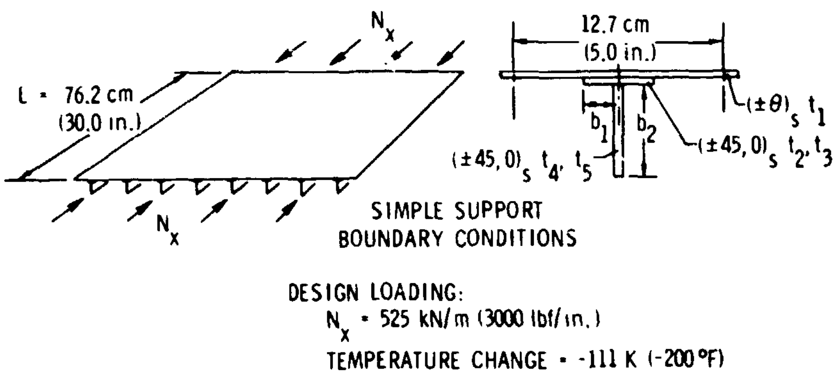

Fig. 4. Overall configuration, design variables, and loading for example 2, graphite-epoxy blade-stiffened panel with temperature effects.

The buckling load of the optimum panel design was calculated for various changes in temperature. The results of these buckling analyses are given by the solid curve in Fig. 5. The vertical scale represents the ratio of the buckling load to the design load. The horizontal scale represents the change in temperature from the temperature for zero residual stress. The filled circular symbol indicates the design condition. The solid curve shows that when the temperature is increased by only 18 K (32°F) above the design temperature, the buckling load of the optimized panel falls to only 20% of the design load. This behavior is caused by the fact that the coefficient of thermal expansion of a laminate is a function of ply orientation. Since the skin ply angle is variable, the various design variables can adjust themselves so that only a small portion of the design load is reacted by the skin at the design temperature. The skin is, therefore, very thin--0.01 cm (0.004 in.). In fact, at the design temperature and load, the skin is slightly in

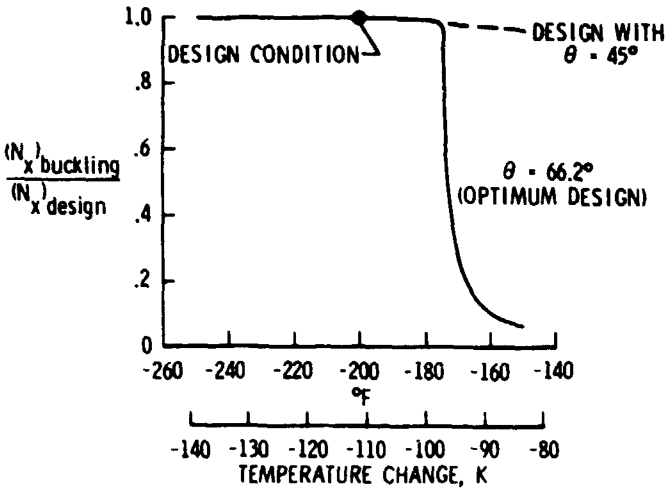

Fig. 5. Buckling load as a function of change in temperature for graphite-epoxy panel designed for temperature change of -111 K (-200°F), example 2.

tension. The temperature must be increased 13 K (24°F) before the skin is in compression. A small additional increase in temperature causes the buckling load of the panel to fall off rapidly. The dashed line in Fig. 5 is for a similar panel for which the ply angle of the skin is fixed at ±45°. In this case, temperature (in the range considered) has little effect on buckling load. For a metal panel, a uniform temperature change causes no differential expansion or thermal stress and, therefore, has no effect on the buckling load.

The substantial reduction in the buckling load shown in Fig. 5 does not necessarily mean that the load-carrying ability of the panel is similarly reduced. If buckled skins are acceptable, the panel may be useful for much higher loadings. Designing a panel to carry load with a buckled skin requires a nonlinear analysis, and such a design capability is not considered in this paper.

Example 3, Stiffened Panels with Effects of Imperfections

The third example illustrates that optimized structural panels tend to have multiple modes of failure occuring simultaneously. In this example, there are several buckling modes that are critical at the same design loading. This third example also illustrates the extent to which geometric imperfections can affect the buckling load of an optimized panel, whether the panel is made of graphite-epoxy or aluminum.

A graphite-epoxy blade-stiffened panel and an aluminum blade-stiffened panel having the same configuration and overall dimensions as the second example, Fig. 4, were designed to support a compressive load of N_x = 525 kN/m (3000 lbf/in.). Both panels were assumed to be perfectly straight, and temperature effects were not considered. The skin ply angle θ for the graphite-epoxy panel was set at 45°. The material properties of the graphite-epoxy are given in Table 1; material properties of the aluminum are given in Table 2.

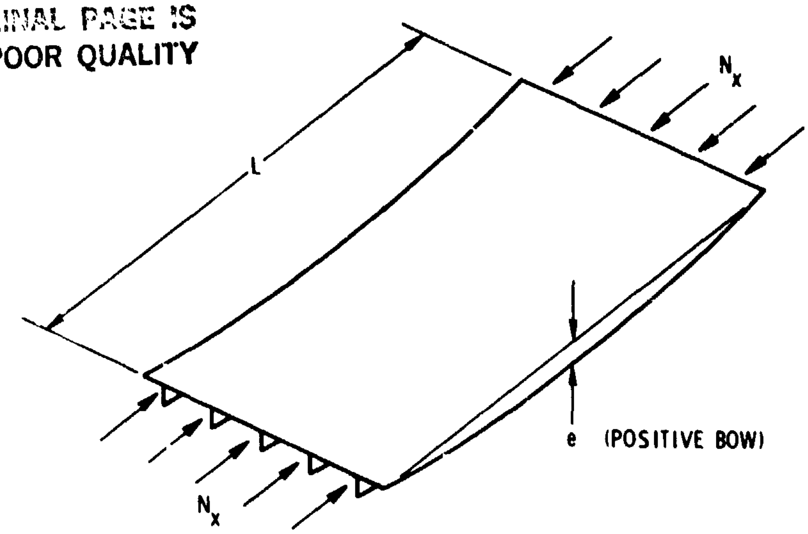

Fig. 6. Panel with initial bow, example 3.

The final design configuration for each panel was then analyzed assuming that it had an overall bow-type imperfection with magnitude e as shown in Fig. 6. A beam-column approach (Anderson and Stroud, 1979) is used in PASCO to calculate the bending stress caused by the bow. Buckling loads for both panels, with and without an imperfection, are shown in Figs. 7 and 8 as a function of the buckling half-wavelength λ . The buckling half-wavelength λ is measured down the length of the panel (in the stiffener direction). The values of λ considered are $\lambda = L/m$ where L is the panel length and m is an integer.

Consider, first, the results for the graphite-epoxy panel presented in Fig. 7. The vertical scale is the ratio of the buckling load to the design load. The horizontal scale is the ratio of the buckling half-wavelength to panel length. Calculated results are shown by the symbols. At the design load, the straight graphite-epoxy panel buckles at half-wavelengths of $\lambda = L$, L/4, and L/12 (filled circular symbols), which illustrates the multiple simultaneous failure modes mentioned earlier. These values of λ are denoted critical wavelengths. At $\lambda = L$, there is a second eigenvalue 6% above the design load. The filled triangular and square symbols in Fig. 7 indicate the lowest buckling loads for the panel with bows of $e/L = \pm 0.003$.

Consider, second, the results for the aluminum panel presented in Fig. 8. At the design load, the straight panel buckles at half-wavelengths of $\lambda = L$, L/4, L/7, and L/16 (filled circular symbols), which, again, illustrates multiple simultaneous failure modes. For all half-wavelengths from L/3 to L/9 the buckling loads are less than 1% above the design load. At $\lambda = L$, the aluminum panel has a second eigenvalue 3% above the design load. The filled triangular and square symbols in Fig. 8 indicate the lowest buckling loads for the panel with bows of $e/L = \pm 0.003$.

Buckle mode shapes for each critical wavelength for the straight graphite-epoxy panel are shown in Fig. 9. The second buckling mode for $\lambda = L$ is also shown. These mode shapes show the deformation of the panel cross section. Mode shapes down the length of the panel are sinusoidal with half-wavelength λ . The mode shapes for the straight aluminum panel are similar to those shown in Fig. 9. For the aluminum panel, the mode shape for $\lambda = L/7$ is a combination of the L/4 and L/16 modes, which, in turn, correspond to the L/4 and L/12 modes for the graphite-epoxy panel.

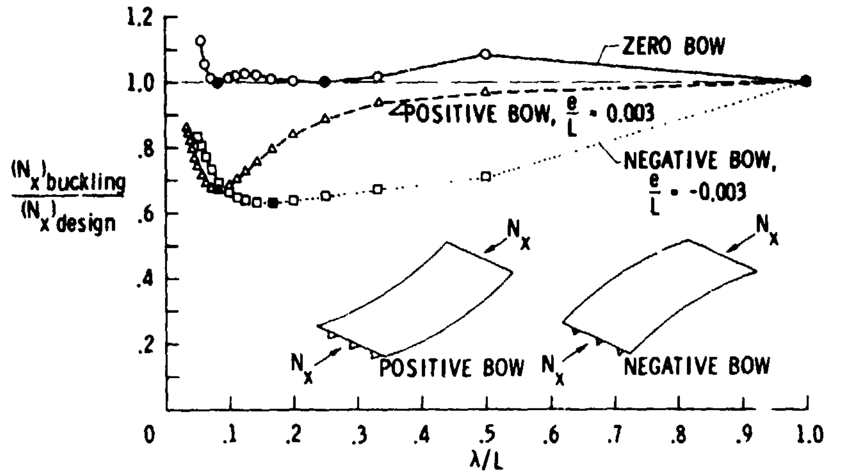

Fig. 7. Ratio of buckling load to design load as a function of buckling half-wavelength for graphite-epoxy blade-stiffened panel designed for zero bow, example 3.

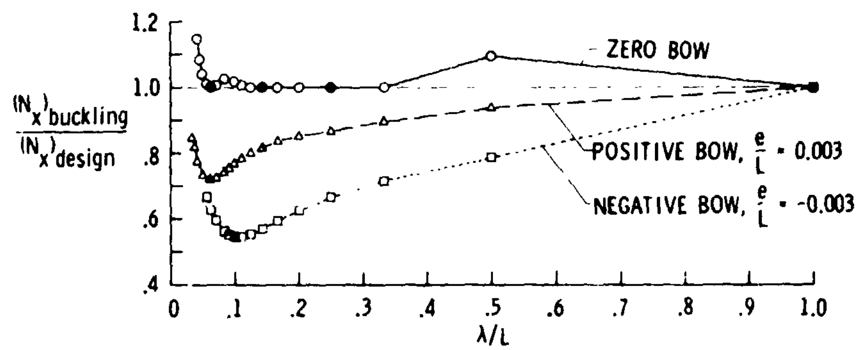

Fig. 8. Ratio of buckling load to design load as a function of buckling half-wavelength for aluminum blade-stiffened panel designed for zero bow, example 3.

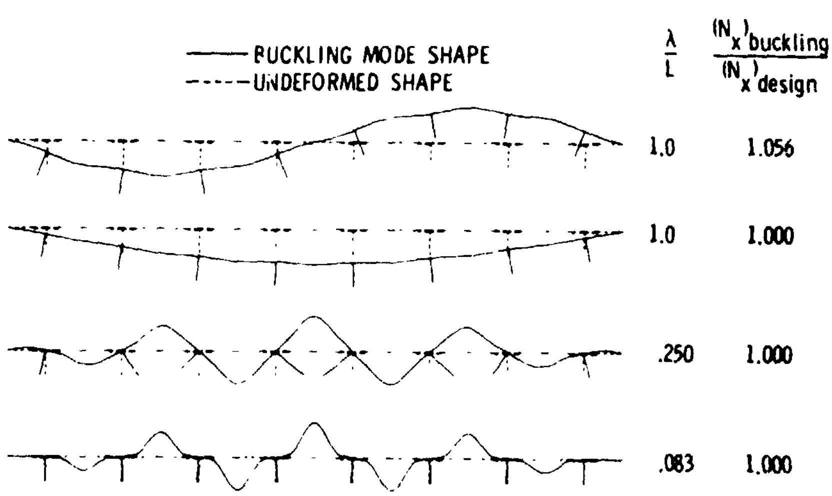

Fig. 9. Buckling mode shapes for straight graphite-epoxy blade-stiffened panel, example 3.

The lowest buckling loads of both panels are shown in Fig. 10 as a function of the size of the bow. The vertical scale in Fig. 10 represents the ratio of the buckling load of the panel with a bow-type imperfection to the buckling load of the perfect panel. (Recall that both panels were designed under the assumption of zero bow.) The horizontal scale gives the value e/L of the bow. The results show that slight imperfections can cause a substantial reduction in the buckling load and that aluminum panels can be affected to about the same extent as graphite-epoxy panels. A bow of e/L = 0.001 in a panel designed for zero bow can cause a 25% reduction in the buckling load.

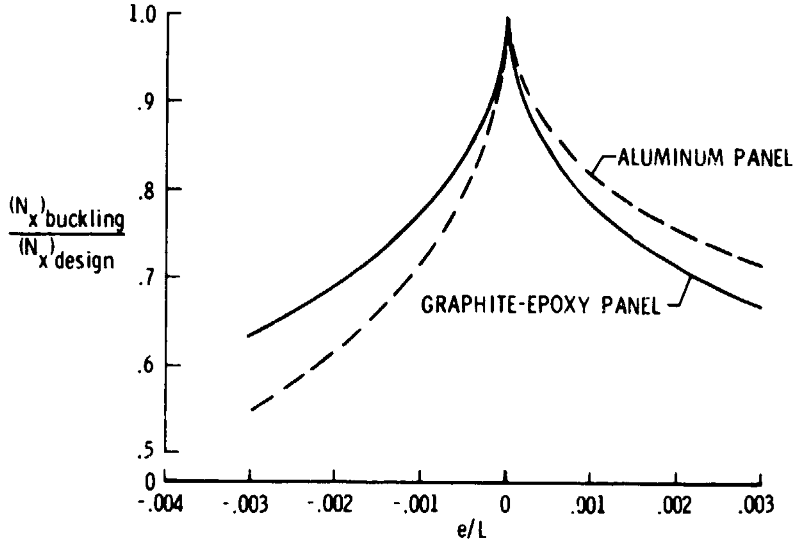

Fig. 10. Effect of bow-type imperfection on the buckling load of graphite-epoxy and aluminum blade-stiffened panels designed for zero bow, example 3.

IMPLICATIONS

Of the above three examples, the first two examples illustrate that structures tailored for a specific load condition can perform poorly in an off-design condition. Since composite materials provide additional design variables for more refined tailoring, optimized composite structures can be especially susceptible to this problem. Numerous other studies have come to this same conclusion for specific structures and specific loading. For example, an aluminum wing optimized for strength may have a low flutter speed and vice versa (Stroud, Dexter, and Stein, 1971). In a study that considered design for global damage tolerance and simulated the damage by removing structural elements, optimized wing structures of both graphite-epoxy and aluminum showed substantial reductions in load-carrying ability when they were analyzed under the assumption of damage. In that study, composite structures were affected more severely by the assumed damage condition than were aluminum structures (Starnes and Haftka, 1980).

The third example illustrates multiple failure modes and the effects of geometric imperfections. The imperfection sensitivity of optimized structures has received considerable attention (for example Thompson and Hunt, 1973; Reis and Roorda, 1979; and Budiansky and Hutchinson, 1979). Structures that are optimized primarily on the basis of buckling generally have more than one buckling mode critical at the design load. It is these multiple simultaneous buckling modes that can cause the imperfection sensitivity. The mechanism is unstable mode interaction. The more modes, the greater the possibility of an unstable interaction. In example 3 above, the composite panel was designed by three buckling modes, and the aluminum panel was designed by four modes. In another study (Stroud, Agranoff, and Anderson, 1977), a hat-stiffened panel was designed by four buckling modes--$\lambda = L$, L/2, L/3, and L/20. In addition to showing that highly optimized panels are frequently designed by several buckling modes, example 3 illustrates that imperfections having a reasonable magnitude ($e/L = 0.003$) can cause a 45% reduction in the buckling load of highly optimized flat panels.

The solution to the problem of sensitivity of optimized structures to off-design load conditions and to imperfections is to use a multiple load condition approach in which various off-design load conditions and imperfections are accounted for in the design process. Some types of imperfections can only be accounted for with costly nonlinear analyses. Techniques for making the nonlinear problem more manageable are discussed by Rosen and Schmit (1981) and Almroth, Stern, and Bushnell (1981). Approximate analysis techniques are used by Rosen and Schmit (1981) in the design of truss structures having local and system imperfections. Almroth, Stern, and Bushnell (1981) describe a system of computer programs for designing stiffened panels including the effects of a random set of initial imperfections. In that paper, the goal is to obtain computational efficiency by using several levels of analysis. Once nonlinear analyses are used, design requirements that prevent buckling can be changed to design requirements that maintain a given post-buckling strength or limit the stress or displacements. Initial progress along these lines is described by Rosen and Schmit (1981), Almroth, Stern, and Bushnell (1981), and Dickson and Biggers (1980).

CONCLUDING REMARKS

This paper discusses several topics associated with structural optimization. These topics include early systematic design procedures, a modern procedure denoted nonlinear mathematical programming, and sensitivity of optimized structures to off-design conditions and imperfections. The focus of the paper is on optimization of composite structures.

The following factors must be considered when using optimization techniques to design composite structures:

(1) Structural design is primarily an inequality design problem. It is neither proper nor necessary to prescribe in advance which of many failure modes are to be critical at the optimum design.

(2) Optimized structures that are tailored to a specific set of design conditions (loads, temperatures, failure criteria, etc.) can perform poorly in an off-design condition--that is, a design condition not considered in the set of original design conditions.

(3) Optimized structures tend to have multiple modes of failure occurring simultaneously. For example, there may be several buckling modes that are critical at the same design loading. Such structures can be sensitive to imperfections.

(4) Compared with metal materials, composite materials provide additional design variables (ply orientation and ply thickness) for more refined tailoring and more extensive optimization. Optimized composite structures can, therefore, be especially susceptible to problems arising from off-design conditions and imperfections.

During the 1980's the combination of improved composite materials, better manufacturing techniques, and better analysis and design procedures will allow engineers to exploit more fully the potential of composite materials in structural design.

TABLE 1 Lamina Properties of Graphite-Epoxy Material Used in Calculations

Symbol	Value in SI Units	Value in U.S. Customary Units
E_1	131.0 GPa	19.0×10^6 psi
E_2	13.0 GPa	1.89×10^6 psi
G_{12}	6.41 GPa	$.93 \times 10^6$ psi
μ_1	.38	.38
α_1	$-.378 \times 10^{-6}$ 1/K	$-.21 \times 10^{-6}$ 1/°F
α_2	28.8×10^{-6} 1/K	16×10^{-6} 1/°F

TABLE 2 Properties of Aluminum Used in Example Calculations

Symbol	Value in SI Units	Value in U.S. Customary Units
E	68.9 GPa	10×10^6 psi
G	26.2 GPa	3.8×10^6 psi
μ	.33	.33

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