ROTOR DYNAMIC SIMULATION AND SYSTEM IDENTIFICATION METHODS FOR APPLICATION TO VACUUM WHIRL DATA

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<td>$E_l$</td>
<td>$= e_A EAK_A^2 - EB_2^*$</td>
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<td>$E_v$</td>
<td>effective in-plane stiffness $= EI_z' - (EI_z' - EI_y')\theta^2 - e_A^2EA$</td>
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<td>$E_w$</td>
<td>effective out-of-plane stiffness $= EI_y' + (EI_z' - EI_y')\theta^2 - e_A^2EA\theta$</td>
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<td>$E_\phi$</td>
<td>effective torsional stiffness $= GJ - K_A^4E_A\Omega^2 + EB_1\theta^2 + K_A^2\Omega^2 \tau_1^\phi$</td>
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<td>e</td>
<td>mass centroid offset from elastic axis, positive when centroid is forward</td>
</tr>
<tr>
<td>$e_A$</td>
<td>area centroid offset from elastic axis, positive when centroid is forward</td>
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*Most symbols relating to blade parameters are consistent with the notation of Reference 3.*
SYMBOLS (Continued)

\[ F_{Hx}, F_{Hy}, F_{Hz}, F_{\alpha x}, F_{\alpha y} \]

applied forces and moments at hub

\[ \text{FNL} \]

vector of nonlinear terms, defined after Equation (34)

\[ \text{FR} \]

vector of steady forces due to offsets, defined after Equation (34)

\[ G \]

shear modulus

\[ g_y, g_w, g_\phi \]

blade inplane, out of plane, torsion damping, force/unit length/unit velocity

\[ HC, HF, HK \]

hub damping, force, and stiffness matrices, defined after Equation (34)

\[ I \]

as used in EI, appropriate area moment of inertia

\[ IB \]

index referring to a particular blade of the rotor

\[ I_{y'}, I_{z'} \]

blade section moments of inertia from \( y' \) and \( z' \) axes

\[ I_{\alpha x}, I_{\alpha y} \]

effective moments of inertia of hub

\[ K_A \]

area radius of gyration of blade cross-section

\[ K_m, K_{m1}, K_{m2} \]

mass radius of gyration of blade cross-section, polar, from chord, from axis through c.g. perpendicular to chord.

\[ K_{Hx}, K_{Hy}, \text{etc} \]

effective stiffness of hub

\[ L_u, L_v, L_w \]

components of applied forces to blade in \( u, v, w \) coordinate system.

\[ m \]

blade mass per unit length

\[ m_{Hx}, m_{Hy}, \text{etc} \]

effective hub masses

\[ \bar{M} \]

vector of elements of mass matrix

\[ \bar{M}_A \]

vector of elements of approximate mass matrix
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<td>NB</td>
</tr>
<tr>
<td>number of blades</td>
</tr>
<tr>
<td>NY,NZ,NP</td>
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<tr>
<td>number of in-plane, out-of-plane, torsion modes, respectively</td>
</tr>
<tr>
<td>NT</td>
</tr>
<tr>
<td>total number of modes used = NY + NZ + NP</td>
</tr>
<tr>
<td>NX</td>
</tr>
<tr>
<td>number of blade stations</td>
</tr>
<tr>
<td>( \bar{r} )</td>
</tr>
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<td>right-hand side vector</td>
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<tr>
<td>R</td>
</tr>
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<td>value of ( x ) at blade tip, blade radius</td>
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<td>RIOC</td>
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<td>inverse of blade inertial coefficient matrix, ( COIR )</td>
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<td>blade coordinate transformation matrix, defined after Equation (34)</td>
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<tr>
<td>( t )</td>
</tr>
<tr>
<td>time</td>
</tr>
<tr>
<td>( T )</td>
</tr>
<tr>
<td>tension, also kinetic energy</td>
</tr>
<tr>
<td>( TM )</td>
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<td>hub inertial matrix, defined after Equation (34)</td>
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<td>( u,v,w )</td>
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<td>( w_i )</td>
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<td>weighting factor on ( i )-th variable</td>
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<td>( y_i,z_i,\phi_i )</td>
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SYMBOLS (Continued)

\( \mathbf{YI, ZI, PI} \)  
integrals defined in Appendix A

\( Y_z \)  
vector of blade generalized coordinates

\( \alpha_x, \alpha_y \)  
pitch and roll angles of hub

\( \beta_{pc} \)  
precone angle

\( \Delta E \)  
\( E_{I_z'} - E_{I_y'} - e_A^2 E_A \)

\( \Delta K \)  
\( K_{m_2}^2 - K_{m_1}^2 \)

\( \Delta m \)  
vector of changes in elements of mass matrix

\( \eta \)  
blade section coordinate

\( \theta \)  
built-in twist

\( \xi \)  
dummy variable for blade station

\( \tau \)  
centrifugal tension integral = \( \int_0^R \bar{m}\xi d\xi \)

\( \phi \)  
elastic twist about elastic axis

\( \phi \)  
vector torsional component of coupled blade normal mode

\( \phi_i \)  
generalized coordinate, amplitude of i-th torsion mode in Galerkin method

\( \phi_i \)  
i-th torsional mode used in Galerkin method

\( \psi \)  
blade azimuth

\( \psi \)  
vector of coupled blade normal modes

\( \omega \)  
blade natural frequency

\( \omega_f \)  
frequency of forcing function

\( \Omega \)  
blade rotational speed
SYMBOLS (Continued)

\[ \int \quad \text{for simplicity, often used to indicate} \int_{-R}^{R} \, dx \]

\( ^{(*)} \quad \frac{\partial}{\partial t} ( ) \)

\( (\cdot)' \quad \frac{\partial}{\partial x} ( ) \)
INTRODUCTION

The analysis of rotor dynamic and aeroelastic phenomena and the resulting capability to control and modify undesirable characteristics requires an understanding of the dynamics and aerodynamics of the rotor blade. Much of the theoretical and experimental research efforts have centered on the aerodynamic aspects of the problem. Of the recent work done in the field of rotor dynamics, most has been directed toward particular phenomena using idealized blade models. Little effort has been devoted to the development of methods of analyzing the dynamic characteristics of actual rotors.

The ability to analyze and predict the dynamic characteristics of a rotor blade has rarely been adequately tested. Non-rotating tests and rotating tests in the atmosphere omit the extreme structural operating conditions associated with the large centrifugal forces or involve significant aerodynamic effects which cannot be analytically removed. One attempt (Reference 1) to test an idealized rotor model in a vacuum chamber resulted in the conclusion that the state-of-the-art of rotor dynamic analysis was not adequate for even a simple solid homogeneous uniform blade with a rectangular cross-section.

There are reasons why there are considerable uncertainties in the mathematical modeling of a rotor blade. In addition to the extreme centrifugal field effects, the major problem lies in the representation of the blade section properties. The state-of-the-art methods (for example, Reference 3) apply to blades with homogeneous sections. In actuality, a typical rotor blade will contain many of the following features: a tapered, twisted hollow spar; bonded thin skinned pockets with ribs or a honeycomb filler; leading edge balance weights; a bonded anti-icing boot; inboard stiffeners; multiple hinges; root cutout. The analytic determination of "effective stiffness", "elastic axis", and "structural damping coefficient" are, at best, intuitive approximations.

The vacuum chamber rotor testing planned at Langley Research Center offers a unique opportunity to significantly advance the state-of-the-art of rotor analytic modeling and rotor dynamic analysis. The purpose of the work presented in this report is to develop tools to augment the aforementioned testing program. Two specific computer programs have been developed. The V22 program has been developed to simulate the tests, including all the necessary special characteristics such as hub forcing, and independent rotational and forcing frequencies, including the non-rotating condition. In addition, the program was designed to be used as a research tool and emphasizes operational flexibility and ease of data input and solution controls.
The other program, ROTSI, is an attempt to use measured data to help identify better approximations to the mass and offset parameters of the rotor blade. The method is an adaptation of the method of incomplete models which has been used with success for other related structural problems.

The analytical developments necessary to implement these tools are derived and discussed in this report. The programs, operators guides, descriptions of special features, and illustrative computational results are also presented.

The major part of this work was completed in 1977, prior to the actual vacuum chamber tests. After the testing was performed an analysis of this data was carried out and is reported in Appendix D.

The contract research effort which has led to the results in this report was financially supported by the Structures Laboratory, USARTL (AVRADCOM).
A comprehensive development of the equations of motion of a rotor blade was first published by Houbolt and Brooks (Reference 2) in 1958. The equations were reformulated by Hodges and Dowel (Reference 3). Their major contributions were the improved generality, including nonlinear terms, and the independent verification of the earlier work. There being no need to rederive these equations again, the rotor equations used in this study were based on those given in Reference 3.

The addition of hub degrees of freedom necessitated the development of the additional terms in the blade equations and the development of the equations of motion of the hub itself which includes the effects of the blades.

The development of the equations of motion of the blades and hub, the application of the Galerkin method, the method of solution, and some of the major features of the program implementing these solutions is presented in the following sections.

**ROTOR EQUATIONS**

As suggested in Reference 3, the tension, $T$, and the longitudinal deflection, $u$, shall be eliminated from the equations. Using the nomenclature as shown in Figure 1 and considering $\theta$ and $\phi$ to be small with $\phi$ ignored compared to $\theta$ in the nonlinear terms, the equation for the tension in the blade becomes: (Equation 62 of Reference 3)

$$T = EA\left(u' + \frac{v'^2}{2} + \frac{w'^2}{2} + K_A^2 \theta^2 \phi - e^2_1 (v'' + w'' \theta)\right)$$  \hspace{1cm} (1)

Integrating with respect to $x$ and solving for $u$ yields:

$$u = \int_0^x u' d\xi = \int_0^x \frac{T}{EA} - K_A^2 \theta^2 \phi + e^2_1 (v'' + \theta + \theta'') d\xi = \int_0^x \left(\frac{v'^2}{2} + \frac{w'^2}{2}\right) d\xi$$

with boundary condition $u(0) = 0$  \hspace{1cm} (2)

From Reference 3 the equation (Equation 61a) for the elastic displacement in the $x$ direction is:

$$T'' = - L_u - m(\ddot{\omega}^2 x + 2\dot{\omega} \dot{x})$$  \hspace{1cm} (3)
Integrating Equation (3) and using \( L_u = 0 \), \( T(R) = 0 \) and \( \tau \equiv \int m \dd{\xi} \), the resulting equation is:

\[
T = \Omega^2 \tau + 2 \Omega \int m v \dd{\xi}
\]  (4)

Equation (3) and (4) and an expression for \( u' \) developed from Equation (1) are substituted into the Equations (61b), (61c), (61d) of Reference 3, the equations for the in-plane, out-of-plane, and torsion become (where third and higher order terms have been neglected):

\[
\begin{align*}
\{E' v'' - 2 \Omega e_A \int m v d\xi + \Delta E \theta'' - EC_1 \times \phi'' + E_1 \theta' \phi' - e_A^2 \Omega^2 \tau'' - \Omega^2 \tau v'' + \Omega^2 m x v'' \} & \\
- \Omega^2 m v + m \dd{v} - 2 \Omega m v' + 2 \Omega m v'' + \Omega^2 \int m v d\xi v'' - 2 \Omega m \beta pc \dd{w} - 2 \Omega m e \dd{v}
\end{align*}
\] (5)

\[
\begin{align*}
\{\Delta E \theta'' - 2 \Omega e_A \int m v d\xi + e_w'' + EC_1 \times \phi'' + E_1 \theta' \phi' + e_A^2 \Omega^2 \tau \phi - \Omega^2 e_A \tau \theta'' \} & \\
+ 2 \Omega m \beta pc \dd{w} - \Omega^2 \tau w'' + \Omega^2 m x w' + \dd{w} + 2 \Omega m \dd{v}' - 2 \Omega \int m v d\xi w'' + m e \dd{\theta}
\end{align*}
\] (6)

\[
\begin{align*}
\{E_1 \theta' v'' + 2 \Omega K_A^2 \theta' \int m v d\xi + E_1 \theta' w'' + E_1 \phi' + \Omega^2 K_A^2 \theta' t' \} & \\
- \Omega^2 m x \dd{v}' + \Omega^2 m \dd{v} - m \dd{\theta} + \Omega^2 e_A \tau w'' + \Omega^2 m x w' + m \dd{w} + \Omega^2 m \dd{\phi}
\end{align*}
\] (7)
These equations contain spatial derivatives of physical parameters which would be difficult to evaluate numerically. Integrating each equation twice between the limits x to R will eliminate this problem. Using the variable x as the lower limit is the more convenient because of the boundary conditions at the tip of the blade. For example, consider the double integration of functions \( f''(x) \) and \( f'(x) \) as follows:

\[
\int \int f''(x) \, dx \, dx = f'(R)(R - x) - f(R) + f(x)
\]

and

\[
\int \int f'(x) \, dx \, dx = f(R)(R - x) - \int f(x) \, dx
\]

Following the Galenkin (Ritz) procedure, arbitrary functions for the blade elastic displacements are substituted into the previous equations as follows:

\[
v(x_1, t) = \sum_{i} y_i(t) Y_i(x) = \sum_{i} y_i Y_i
\]

\[
w(x_1, t) = \sum_{j} z_j(t) Z_j(x) = \sum_{j} z_j Z_j
\]

\[
\phi(x_1, t) = \sum_{k} \phi_k(t) \phi_k(x) = \sum_{k} \phi_k \phi_k
\]

where \( Y_i(x), Z_j(x), \phi_k(x) \) are modal functions which satisfy the boundary conditions and \( y_i(t), z_j(t), \phi_k(t) \) are time dependent generalized co-ordinates. The modal functions are completely general and are not restricted to normal mode shapes.

In the following equations the short-hand notation \( \int \int \) is used for simplicity.
\[
\sum \{\gamma_i (\sum \text{mY}_i + 4z^2 \sum \text{mY}_i^x, \left( e_{A_1} \text{mY}_i \right) + 2\gamma_i [\sum \text{meY}_i' - \text{meY}_i'] \\
- e_{A_1} \text{mY}_i, \text{meY}_i, - (R - x)(\text{meY}_i)_R \} \} \} + \gamma_i [E_{Y_i}' - \Omega^2 (\sum \text{meY}_i')] \\
- \sum \{2\gamma_i [\sum \text{mZ}_i + \sum \text{mZ}_i^x, e_{A_1} \text{Z}_i' - \text{meZ}_i'] + \beta_{pc} \sum \text{mZ}_i] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k (- \sum \text{meZ}_i', \text{meZ}_i') + \Omega^2 (\sum \text{meZ}_i', \sum \text{meZ}_i') + \text{f}_k \}
\]

\[
+ \sum \{ \text{f}_i + \sum \text{mY}_i' - \sum \text{mY}_i', \sum \text{mY}_i'' + \sum \text{mY}_i'' \}
\]

\[
= \sum \{ (L_v - \gamma_{v}^2 e) - \Omega^2 (\sum \text{meY}_i' - e_{A_1} R(\text{meY}_i)_R (R - x)) \}
\]

\[
\sum \{2\gamma_i [\beta_{pc} \sum \text{mY}_i + \sum \text{meY}_i' - e_{A_1} \sum \text{mY}_i' - (R - x)(\text{meY}_i)_R \} \} + \gamma_i (\Delta \text{eY}_i'') \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i, - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \text{f}_k [EC_1 \Phi_k' + E_{Y_i}' \Phi_k' + \Omega^2 (\sum \text{meZ}_i', \sum \text{meZ}_i') + \text{f}_k \}
\]

\[
+ \sum \{ (L_v - \gamma_{v}^2 e_{A_1} \sum \text{mY}_i' - \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' - \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i') \}
\]

\[
= \sum \{ (L_v - \gamma_{v}^2 e_{A_1} \sum \text{mY}_i' - \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i') \}
\]

\[
\sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \text{f}_i + \sum \text{mY}_i' - \sum \text{mY}_i', \sum \text{mY}_i'' + \sum \text{mY}_i'' \}
\]

\[
= \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
- \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' \phi_0 \}
\]

\[
\sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
- \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' \phi_0 \}
\]

\[
\sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
- \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' \phi_0 \}
\]

\[
\sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
- \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' \phi_0 \}
\]

\[
\sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
+ \sum \{ \text{f}_k (- \sum \text{meZ}_i', - 2\gamma_i [\sum \text{mK}_i^2 \text{meZ}_i'] + \Phi_k \}
\]

\[
+ \sum \{ \gamma_i [E_{Y_i}' + \sum \Phi_k' + \Omega^2 (\sum \text{mK}_i^2 \Phi_k') = \sum \{ \gamma_i (\sum \text{meY}_i' + \gamma_i [E_{Y_i}' - \sum \text{meY}_i') + \Omega^2 (\sum \text{meY}_i' + \sum \text{meY}_i')] \}
\]

\[
- \gamma_{v}^2 \beta_{pc} \sum \text{mY}_i' \phi_0 \}
\]
ADDITION OF HUB MOTIONS

In this section the linear effects of the hub degrees of freedom are evaluated and will be combined with the blade equations.

The coordinate of a point on a blade in the nonrotating hub system, as shown in Figure 2, can be defined in terms of $r$, the undeformed reference line along the blade span as follows (including the major linear terms).

\[ x_R = r \cos \psi - [v + \eta \cos(\theta + \phi)] \sin \psi \]
\[ y_R = r \sin \psi + [(v + \eta \cos(\theta + \phi))] \cos \psi \]
\[ z_R = r \beta_{pc} + w + \eta \sin(\theta + \phi) \tag{11} \]

Assuming small angles for $\theta$ and $\phi$ in Equations (11), including hub displacements and angular motions $\alpha_x$ and $\alpha_y$ about the respective axes, the linear expression for the inertial coordinates for a point on the blade become:

\[ x = x_H + r \cos \psi - (\eta + v) \sin \psi + (r \beta_{pc} + \eta \theta) \alpha_y \]
\[ y = y_H + r \sin \psi + (\eta + v) \cos \psi - (r \beta_{pc} + \eta \theta) \alpha_x \]
\[ z = z_H + r \beta_{pc} + \eta(\theta + \phi) + \ddot{w} + (r \sin \psi + \eta \cos \psi) \alpha_x \]
\[ - (r \cos \psi - \eta \sin \psi) \alpha_y \tag{12} \]

Accelerations of the inertial coordinates are derived from Equation (12) and are used in the formulation of the hub equations, below:

\[ \ddot{x} = \ddot{x}_H - \Omega^2 (r \cos \psi - \eta \sin \psi) - \dot{v} \sin \psi - 2\Omega \dot{v} \cos \psi + \Omega^2 \dot{v} \sin \psi \]
\[ + \eta \phi \dot{\sin} \psi + 2\eta \Omega \phi \cos \psi + (r \beta_{pc} + \eta \theta) \alpha_y \tag{13} \]
Figure 2. Point on Blade Referenced to Non-Rotating Hub Coordinate System
\[ \ddot{y} = \ddot{y}_H - \Omega^2(r \sin \psi + \eta \cos \psi) + \ddot{v} \cos \psi - 2\Omega \dot{v} \sin \psi - \Omega^2 \dot{v} \cos \psi \]
\[ - \eta \phi \cos \psi + 2\eta \Omega \phi \sin \psi - (r_{pc} + \eta \phi) \ddot{a}_x \]  
(14)

\[ \ddot{z} = \ddot{z}_H + \ddot{w} + \ddot{\phi} - \Omega^2(r \sin \psi + \eta \cos \psi) \alpha_x + 2\Omega(r \cos \psi - \eta \sin \psi) \alpha_x \]
\[ + (r \sin \psi + \eta \cos \psi) \ddot{a}_x + \Omega^2(r \cos \psi - \eta \sin \psi) \alpha_y \]
\[ + 2\Omega(r \sin \psi + \eta \cos \psi) \ddot{a}_y - (r \cos \psi - \eta \sin \psi) \ddot{a}_y \]  
(15)

Applying LaGrange's equation, the additional terms in the equations for the elastic displacements \( v, w, \phi \) due to hub motions become:

\[ v \text{ Equation} \]
\[ - \ddot{x}_H \sin \psi \dddot{m} + \ddot{y}_H \cos \psi \dddot{m} + (\ddot{a}_x \cos \psi + \ddot{a}_y \sin \psi) (3 \Omega \dddot{x}_m + \dddot{m} \phi) \]  
(16)

\[ w \text{ Equation} \]
\[ \ddot{z}_H \dddot{m} + (\ddot{a}_x - \Omega^2 \alpha_x + 2\Omega \ddot{a}_y)(\sin \psi \dddot{m} + \cos \psi \dddot{m}) \]
\[ - (\ddot{a}_y - \Omega^2 \alpha_y - 2\Omega \ddot{a}_x)(\cos \psi \dddot{m} - \sin \psi \dddot{m}) \]  
(17)

\[ \phi \text{ Equation} \]
\[ [(\ddot{x}_H \sin \psi - \ddot{y}_H \cos \psi) + \Omega(\ddot{x}_H \cos \psi - \ddot{y}_H \sin \psi)] \dddot{m} \theta + \dddot{z}_H \dddot{m} \phi \]
\[ + (\ddot{a}_x - \Omega^2 \alpha_x + 2\Omega \ddot{a}_y)(\sin \psi \dddot{m} + \cos \psi \dddot{m} \kappa^2_{m_2} ) - (\ddot{a}_y - \Omega^2 \alpha_y \]
\[ - 2\Omega \ddot{a}_x \cos \psi \dddot{m} - \sin \psi \dddot{m} \kappa^2_{m_2} ) + [(\ddot{a}_x \cos \psi + \ddot{a}_y \sin \psi) \]
\[ - \Omega (\ddot{a}_x \sin \psi - \ddot{a}_y \cos \psi) (3 \Omega \dddot{x}_m \phi + \dddot{m} \kappa^2_{m_2} \theta) \]  
(18)

where \( f \equiv \int \frac{f(x)}{x} \, dx \)
FINAL BLADE EQUATIONS OF MOTION

Combining the respective equations given in (8)-(10) and (16)-(18) yields the equations of motion for the elastic displacements \(v, w\) and \(\phi\).

\(v\) Equation

\[
\sum \{ \dot{y}_i \left[ \sum_{i} \dot{Y}_i \right] + 4 \Omega^2 \sum_{i} \dot{Y}_i \} = \frac{1}{EA} \sum_{i} \dot{Y}_i + 2 \Omega^2 \sum_{i} \dot{Y}_i \left[ \sum_{i} \dot{Y}_i \right] - \sum_{i} \dot{Y}_i - e_{A} \sum_{i} \dot{Y}_i + \sum_{i} \dot{Y}_i
\]

\[- (R - x)(\dot{Y}_i)_R] + y_i \left[ E_{i} \dot{Y}_i \dot{Y}_i - \Omega^2 \left( \sum_{i} \dot{Y}_i \right) ^2 \sum_{i} \sum_{i} \dot{Y}_i \right] + \dot{z}_j \left( \Delta \Theta \dot{Z}_j \right)
\]

\[+ \sum \{ 2 \Omega^2 \sum_{j} \dot{Z}_j \left[ \sum_{i} \dot{Y}_i \right] - \sum_{i} \dot{Y}_i \dot{Y}_i \} \]

\[- \sum \{ \phi_k \dot{Y}_i \dot{Y}_i + 2 \Omega^2 \sum_{k} \dot{Z}_j \left[ \sum_{i} \dot{Y}_i \right] ^2 \dot{Y}_i \dot{Y}_i \} \}

\[\dot{Y}_i \cos \psi \sum_{i} \dot{Y}_i + \dot{x} \cos \psi \sum_{i} \dot{Y}_i + \dot{y} \sin \psi \sum_{i} \dot{Y}_i \]

\[+ 2 \Omega \sum_{i} \dot{Y}_i \dot{Y}_i - \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]  

\[- \Omega \sum_{i} \dot{Y}_i \dot{Y}_i + \Omega^2 \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]

\[= \Omega \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \sum_{i} \dot{Y}_i \dot{Y}_i \]
\[ + 2\alpha_2 y (\sin \psi_{1\text{mx}} + \cos \psi_{1\text{me}}) + \Omega^2 \alpha_y (\cos \psi_{1\text{mx}} - \sin \psi_{1\text{me}}) \]

\[ + 2\Omega (\psi_{1\text{mvw}}' - \psi_{1\text{mvw}}) = \int L_w - \Omega^2 \beta_p \int \psi_{1\text{mx}} - \psi_{1\text{mex}} + \Omega^2 [e_A^{2\theta} \theta] \]

\[ + R(\psi_{1\text{me}})_{R(R-x)} \]

(20)

**Equation**

\[ \Sigma \{ - \ddot{Y}_i \int \psi_{1\text{me}} Y_i + 2\Omega y_i [\int (K_{\psi_{1\text{me}}}) + y_i [\int E_1 \theta_{\psi_i} Y_i - EC_1 \theta_{\psi_i} Y_i] \]

\[ + \Omega^2 (\int \psi_{1\text{mex}} Y_i' + \int \psi_{1\text{me}} Y_i)) + \int \ddot{Z}_j \int \psi_{1\text{mex}} Z_j + z_j [\int E_1 \theta_{\psi_j} Z_j] \]

\[ + EC_1 \psi_{1\text{mex}} - \Omega^2 (\int \psi_{1\text{mex}} Z_j') + \int \ddot{Z}_j \int \psi_{1\text{mex}} Z_j' + \int \ddot{Z}_j \int \psi_{1\text{mex}} Z_j'' \}

\[ + \int E_{\Phi_k} \psi_{1\text{mex}} + \Omega^2 (\int \psi_{1\text{mex}} \Phi_k) + \int \ddot{X}_i \int \psi_{1\text{mex}} \]

\[ - \ddot{Y}_i \cos \psi_{1\text{me}} - \Omega^2 \sin \psi_{1\text{me}} + \ddot{Z}_i \int \psi_{1\text{mex}} + \ddot{X}_i [\sin \psi_{1\text{mex}} + \cos \psi_{1\text{mex}}] \]

(21)

The Galerkin (Ritz) method of effecting approximate solutions of differential equations applied to the previous equations requires a set of averaging integrals. Equations (19) - (21) for \(v, w\) and \(\phi\) are multiplied by \(Y_i, Z_j, \phi_k\), respectively, where \(i = 1, NY; j = 1, NZ\) and \(k = 1, NP\) and each resulting equation is integrated from 0 to \(R\). This procedure yields NT equations (NT = NY + NZ + NP) which may be solved for the generalized coordinates.
\[\sum_{J=1}^{NY} \{ \dot{\gamma}_J \left[ \text{DYYII}(I,J,1) + 4\omega^2 \text{DYSI}(I,J,1) \right] + 2\omega \dot{\gamma}_J \left[ \text{DYSI}(I,J,2) - \text{DYYII}(I,J,5) \right] - \text{DYF}(I,J,2) + \text{DYII}(I,J,2) - \text{DYF}(I,J,1) \} + y_J \left[ \text{DYF}(I,J,3) \right] - \omega^2 \left( \text{DYYII}(I,J,7) - \text{DYYII}(I,J,4) + \text{DYYII}(I,J,1) \right) \}

+ \sum_{J=1}^{NZ} \{ 2\omega \dot{z}_J \left[ \text{DYSI}(I,J,3) - \text{DYZII}(I,J,5) - \beta_{pc} \text{DYZII}(I,J,1) \right] + z_J \text{DYF}(I,J,4) \}

+ \sum_{J=1}^{NP} \{ -\phi_J \text{DYZPII}(I,J,3) - 2\omega \phi_J \text{DYSI}(I,J,4) + \phi_J \text{DYF}(I,J,5) \}

- \ddot{x}_H \sin \psi \text{DYMII}(I,1) + \ddot{y}_H \cos \psi \text{DYMII}(I,1) + \ddot{a}_x \cos \psi \text{[DYMII}(I,5) + \beta_{pc} \text{DYMII}(I,2) \}

+ \ddot{a}_y \sin \psi \text{[DYMII}(I,5) + \beta_{pc} \text{DYMII}(I,2) \}

+ 2\omega \left[ \gamma_I \int \int (v'v' + x'x') - \gamma_I \int \int (v''v'' + x''x'') \right] \}

= \sum_{J=1}^{NY} \{ 2\omega \dot{y}_J \left[ \text{DZYI}(I,J,3) - \text{DZF}(I,J,1) + \beta_{pc} \text{DZYII}(I,J,1) \right] + y_J \text{DZF}(I,J,2) \}

+ \sum_{J=1}^{NZ} \{ \dot{z}_J \text{DZZII}(I,J,1) + z_J \left[ \text{DZF}(I,J,3) - \omega^2 \text{DZII}(I,J,6) - \text{DZII}(I,J,3) \right] \}

+ \sum_{J=1}^{NP} \{ \phi_J \text{DZPII}(I,J,1) + \phi_J \left[ \text{DZF}(I,J,4) + \omega^2 \text{DZPI}(I,J,2) - \text{DZF}(I,J,6) \right] \}

\text{where } I = 1 \text{ to } NY; \text{ thus, there is one equation for each in-plane mode. Similarly, for the } w \text{ equation:}

\[\sum_{J=1}^{NY} \left( 2\omega \dot{y}_J \left[ \text{DZYI}(I,J,3) - \text{DZF}(I,J,1) + \beta_{pc} \text{DZYII}(I,J,1) \right] + y_J \text{DZF}(I,J,2) \right) \]

\[+ \sum_{J=1}^{NZ} \left( \dot{z}_J \text{DZZII}(I,J,1) + z_J \left[ \text{DZF}(I,J,3) - \omega^2 \text{DZII}(I,J,6) - \text{DZII}(I,J,3) \right] \right) \]

\[+ \sum_{J=1}^{NP} \left( \phi_J \text{DZPII}(I,J,1) + \phi_J \left[ \text{DZF}(I,J,4) + \omega^2 \text{DZPI}(I,J,2) - \text{DZF}(I,J,6) \right] \right) \]
$ + \dot{z}_H DZMI (I, 1) + \dot{\alpha}_x [\sin \psi DZMI (I, 2) + \cos \psi DZMI (I, 3)]$

$ + 2 \Omega \dot{\alpha}_x [\cos \psi DZMI (I, 2) - \sin \psi DZMI (I, 3)] - \Omega^2 \alpha_x \sin \psi DZMI (I, 2)$

$ + \cos \psi DZMI (I, 3)] - \dot{\alpha}_y [\cos \psi DZMI (I, 2) - \sin \psi DZMI (I, 3)]$

$ + 2 \Omega \dot{\alpha}_y [\sin \psi DZMI (I, 2) + \cos \psi DZMI (I, 3)] + \Omega^2 \alpha_y [\cos \psi DZMI (I, 2)$

$ - \sin \psi DZMI (I, 3)] + 2 \Omega \int Z_1 \int \tilde{m} v \tilde{w} - \int Z_1 \int (w^a \int \tilde{m} v) \} = \int Z_1 \int \tilde{w}$

$ - \Omega^2 [DZMI (I, 6) - DZF (I, 1, 5) + \beta_{pc} DZMI (I, 2)]$  (23)

where I = 1 to NZ; following the same procedure, w, the $\phi$ equation is

$$\Sigma_{J=1}^{NY} \{ - \dot{y}_j DPYII (I, J, 3) + 2 \dot{\alpha} y_j DPSI (I, J, 5) + y_j [DPYI (I, J, 9) - DPFI (I, J, 1)$$

$$ + \Omega^2 (DPYII (I, J, 8) - DPYII (I, J, 6) + DPYII (I, J, 3))] \}$

$$\Sigma_{J=1}^{NZ} \{ \dot{z}_j DPZII (I, J, 2) + z_j [DPZI (I, J, 8) + DPFI (I, J, 3) - \Omega^2 (DPZII (I, J, 7)$$

$$ - DPZII (I, J, 4)] \}$

$$\Sigma_{J=1}^{NP} \{ \phi_j DPPII (I, J, 4) + \phi_j [DPF (I, J, 3) + DPPI (I, J, 6) + \Omega^2 (DPPII (I, J, 5)$$

$$ + DPPI (I, J, 7))] \} + \{ \dot{x}_H \sin \psi + \dot{\alpha}_x \cos \psi - \dot{y}_H \cos \psi - \Omega \dot{y}_H \sin \psi$$

$$ + \dot{z}_H DPMII (I, 3) + \dot{\alpha}_x [\sin \psi DPMII (I, 4) + \cos \psi DPMII (I, 7) + DPMII (I, 8)]$$

$$ + \beta_{pc} DPMII (I, 6)] + 2 \dot{\alpha}_x [\cos \psi DPMII (I, 4) - \sin \psi (DPMII (I, 7) + \frac{1}{2} DPMII (I, 8)$$

$$ + \frac{1}{2} \beta_{pc} DPMII (I, 6)] - \Omega^2 \alpha_x [\sin \psi DPMII (I, 4) + \cos \psi DPMII (I, 7)]$$
+ \ddot{\alpha}_y [- \cos \psi \text{DPMII}(I,4) + \sin \psi \text{(DPMII}(I,7) + \text{DPMII}(I,8) + \beta_{pc} \text{DPMII}(I,6))] \\
+ 2\alpha_y \ddot{y} [\sin \psi \text{DPMII}(I,4) + \cos \psi \text{(DPMII}(I,7) + \frac{1}{2} \text{DPMII}(I,8)] \\
+ \frac{1}{2} \beta_{pc} \text{DPMII}(I,6)] + \Omega^2 \alpha_y [\cos \psi \text{DPMII}(I,4) - \sin \psi \text{DPMII}(I,7)] \\

\begin{equation}
\frac{R}{\phi} = \int_{\phi_1}^{\phi_2} \frac{\phi \int_{\phi}^{\phi_2} - \Omega^2 \text{DPMII}(I,9) + \beta_{pc} \text{DPMII}(I,4) + \text{DPMI}(I,10)}{\phi_1}
\end{equation}

where I = 1 to NP. The coefficients shown in Equations (22), (23) and (24) are defined in Appendix A.

Equations (22), (23) and (24) may be written in partitioned matrix form as shown on the following pages.

In order to include a simple structural damping representation, terms of the form $g_v$, $g_w$, $g_\phi \phi'$ were added to Equations (5), (6), (7) resulting in the integrals $DYD$, $DZD$, $DPD$ which appear in the following pages and are defined in Appendix A.
\[
\begin{align*}
\begin{bmatrix}
\text{DYZI}(I,J,1) & \text{DYZI}(I,J,3) & \text{DYZI}(I,J,4) \\
\text{DYZI}(I,J,5) & \text{DYZI}(I,J,3) & \text{DYZI}(I,J,4) \\
-2\phi & 0 & 0 \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\text{DYII}(I,J,1) & 0 & -\text{DYII}(I,J,3) \\
0 & \text{DZII}(I,J,1) & \text{DZII}(I,J,1) \\
-\text{DYZII}(I,J,3) & \text{DZII}(I,J,2) & \text{DPZII}(I,J,4) \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
-\sin[\text{YMII}(I,1)] & \cos[\text{YMII}(I,1)] & 0 & \cos[\text{YMII}(I,5)] & \sin[\text{YMII}(I,5)] & \beta_{\text{YMII}(I,2)} & \beta_{\text{YMII}(I,2)} \\
0 & \sin[\text{DZMI}(I,1)] & -\cos[\text{DZMI}(I,2)] & -\cos[\text{DZMI}(I,2)] & \sin[\text{DZMI}(I,3)] & \sin[\text{DZMI}(I,3)] \\
\sin[\text{DPMI}(I,3)] & \cos[\text{DPMI}(I,3)] & \text{DPMI}(I,3) & \text{DPMI}(I,3) & \sin[\text{DPMI}(I,4)] & \cos[\text{DPMI}(I,4)] \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\text{DYSI}(I,J,2) & 0 \\
-\text{DYSI}(I,J,2) & \text{DYSI}(I,J,2) \\
+\text{DYF}(I,J,1) & -\beta_{\text{DYF}(I,J,1)} \\
\text{DYZI}(I,J,3) & 0 \\
+\beta_{\text{DYZI}(I,J,1)} & -\beta_{\text{DYZI}(I,J,1)} \\
\text{DYZI}(I,J,5) & 0 \\
\end{bmatrix}
\end{align*}
\]
\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & \cos\psi_DZMII(I,2) & \sin\psi_DZMII(I,2) \\
0 & \sin\psi_DZMII(I,3) & \cos\psi_DZMII(I,3) & 0 \\
-2\alpha & -\sin\psi_DPMII(I,4) & \sin\psi_DPMII(I,4) & 0 \\
\frac{1}{2}\cos\psi_DPMII(I,3) & -\frac{1}{2}\sin\psi_DPMII(I,3) & 0 & +\frac{1}{2}DPMII(I,8) +\frac{1}{2}DPMII(I,8) \\
& & +\frac{1}{2}DPMII(I,6) & +\frac{1}{2}DPMII(I,6)
\end{bmatrix}
\]
HUB EQUATIONS
Terms In Hub Equations Due to Blade Motions

The kinetic energy of a rotor blade may be expressed as follows:

\[ T = \frac{1}{2} \int_0^R \left( x^2 + y^2 + z^2 \right) dm \]

Assuming a spring-mass-damper model of the hub in each of the three orthogonal directions, and torsional models with respect to the body axes, the hub equations of motion including blade effects are:
Substituting the expressions for the accelerations of the inertial coordinates from Equations (13)-(15), performing the integration with respect to chord and blade span and assuming two of more symmetrical blades, the previous equations become:

\[
\begin{align*}
\dot{x}_H &= \frac{m_H}{\beta_1} + C_{Hx} + K_{Hx} \frac{\dot{x}_H}{\beta_1} + \sum b=1 \int m \ddot{x} \xi = F_{Hx} \\
\dot{y}_H &= \frac{m_H}{\beta_1} + C_{Hy} + K_{Hy} \frac{\dot{y}_H}{\beta_1} + \sum b=1 \int m \ddot{y} \xi = F_{Hy} \\
\dot{z}_H &= \frac{m_H}{\beta_1} + C_{Hz} + K_{Hz} \frac{\dot{z}_H}{\beta_1} + \sum b=1 \int m \ddot{z} \xi = F_{Hz} \\
I_{\alpha_x} &= \frac{\dot{\alpha}_x}{\beta_1} + C_{\alpha_x} + K_{\alpha_x} \frac{\dot{\alpha}_x}{\beta_1} + \sum b=1 \int m \ddot{\alpha} \xi = F_{\alpha_x} \\
I_{\alpha_y} &= \frac{\dot{\alpha}_y}{\beta_1} + C_{\alpha_y} + K_{\alpha_y} \frac{\dot{\alpha}_y}{\beta_1} + \sum b=1 \int m \ddot{\alpha} \xi = F_{\alpha_y}
\end{align*}
\]

Substituting the expressions for the accelerations of the inertial coordinates from Equations (13)-(15), performing the integration with respect to chord and blade span and assuming two of more symmetrical blades, the previous equations become:

\[
\begin{align*}
x_H &= m_H \frac{\ddot{x}_H}{\beta_1} + C_{Hx} \frac{\dot{x}_H}{\beta_1} + K_{Hx} \frac{\ddot{x}_H}{\beta_1} + \sum \int m \ddot{x} \xi \\
&\quad + 2 \int m v \sin \psi + \int m e \theta \phi \sin \psi + 2 \int m e \theta \phi \cos \psi + (\beta_p \int m \ddot{\alpha} \xi) = F_{Hx}
\end{align*}
\]
Equation II

\[ y_H \] Equation

\[
m_H \ddot{y}_H + C_H \dot{y}_H + K_H y_H + NB[M(1,1)\ddot{y}_H] + \sum_{IB=1}^{NB} \left\{ R \sum_{o} m v \cos \psi - 2\Omega \sum_{o} m v \sin \psi \right\} \]

\[
= F_{H_y} \] (27)

Equation III

\[ z_H \] Equation

\[
m_H \ddot{z}_H + C_H \dot{z}_H + K_H z_H + NB[M(1,1)\ddot{z}_H] + \sum_{IB=1}^{NB} \left\{ R \sum_{o} m w + R \sum_{o} m e \phi \right\} = F_{H_z} \] (28)

\[ \alpha_x \] Equation

\[
I_{\alpha_x} \ddot{\alpha}_x + C_{\alpha_x} \dot{\alpha}_x + K_{\alpha_x} \alpha_x - NB[\beta_{pc} M(1,2) + M(1,5)]\ddot{y}_H + \sum_{IB=1}^{NB} \left\{ R \sum_{o} \right\}

\[
+ \sum_{o} m e \phi \cos \psi + 2\Omega (\beta_{pc} \sum_{o} m x v + \sum_{o} m e \phi) \sin \psi + \Omega^2 (\beta_{pc} \sum_{o} m x v

\[
+ \sum_{o} m e \phi \cos \psi + (\beta_{pc} \sum_{o} m x e \phi + \sum_{o} m e \phi) \cos \psi - 2\Omega (\beta_{pc} \sum_{o}

\[
+ \sum_{o} m e \phi \cos \psi + (\beta_{pc} \sum_{o} m x^2 + 2\beta_{pc} \sum_{o} m x e \phi + \sum_{o} m e \phi) \dot{\alpha}_x

\[
+ \sum_{o} \sin \psi \sum_{o} m w + \cos \psi \sum_{o} m e \phi + \sum_{o} \cos \psi \sum_{o} m e \phi - \Omega^2 (\sum_{o} m x^2

\[
+ 2 \sin \psi \cos \psi \sum_{o} m x + \sum_{o} \cos \psi \sum_{o} m e \phi) \dot{\alpha}_x + \sum_{o} \sum_{o} \sin \psi \cos \psi \sum_{o} m x^2 - (\sum_{o} m x^2

\[
- \cos \psi \sum_{o} m x - \sin \psi \cos \psi \sum_{o} m e \phi \dot{\alpha}_x + (\sum_{o} \sum_{o} \sin \psi \cos \psi \sum_{o} m x^2 + 2 \sin \psi \cos \psi \sum_{o} m x

\[
}\]
$$+ \cos^2 \psi \sum_k R_k \left( \sum_k R_k \right) + \sum_k \left( \sin^2 \psi \right) \sum_k R_k \left( \sum_k R_k \right)$$

$$- \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) + \sum_k \left( \cos^2 \psi \right) \sum_k R_k \left( \sum_k R_k \right)$$

$$- \Omega^2 \left( \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) - \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) \right) = F_{\alpha_y} \quad (29)$$

$$\alpha_y \quad \text{Equation}$$

$$I_{\alpha_y} \ddot{\alpha}_y + C_{\alpha_y} \dot{\alpha}_y + K_{\alpha_y} \alpha_y + \sum_{IB=1}^{NB} \left( \beta_{pc} \sum_k R_k \left( \sum_k R_k \right) \right) \ddot{x}_H + \sum_k \left( \beta_{pc} \sum_k R_k \left( \sum_k R_k \right) \right) \dot{x}_H + \sum_k \left( \beta_{pc} \sum_k R_k \left( \sum_k R_k \right) \right) \ddot{x}_H$$

$$+ \sum_k \left( \beta_{pc} \sum_k R_k \left( \sum_k R_k \right) \right) \dot{x}_H + \sum_k \left( \beta_{pc} \sum_k R_k \left( \sum_k R_k \right) \right) \ddot{x}_H$$

$$- \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) + \sum_k \left( \cos^2 \psi \right) \sum_k R_k \left( \sum_k R_k \right)$$

$$- \Omega^2 \left( \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) - \sin^2 \psi \sum_k R_k \left( \sum_k R_k \right) \right) = F_{\alpha_y} \quad (30)$$

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Considering only the hub translational equations of motion and following a similar procedure as applied to the blade equations arbitrary functions for the elastic displacements are substituted into Equations (26)-(28) yielding:

\[ \begin{align*}
\dot{x}_H & \dot{y}_H \dot{z}_H = F_{Hx} \\
\dot{x}_H & = m_H \ddot{x}_H + C_H \dot{x}_H + K_H x_H + NB[M(1,1)\ddot{x}_H] + \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB \\
& + \sum_{IB=1}^{NB} \sum_{J=1}^{NP} \sin\psi \sum_{J=1}^{NP} \phi_{J,IB} + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB \\
& + \sum_{IB=1}^{NB} \sum_{J=1}^{NY} \sin\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos\psi \sum_{J=1}^{NY} \phi_{J,IB} \\
& + NB[\beta_{pc} M(1,2) + M(1,5)]\ddot{x}_H = F_{Hx} \\
\dot{y}_H & = m_H \ddot{y}_H + C_H \dot{y}_H + K_H y_H + NB[M(1,1)\ddot{y}_H] + \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB \\
& - \sum_{IB=1}^{NB} \sum_{IB=1}^{NP} \cos\psi \sum_{J=1}^{NP} \phi_{J,IB} - 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB \\
& - \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos\psi \sum_{J=1}^{NY} Y(I,J,1)\dot{y}_J,IB + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin\psi \sum_{J=1}^{NY} \phi_{J,IB} \\
& + NB[\beta_{pc} M(1,2) + M(1,5)]\ddot{y}_H = F_{Hy} \\
\dot{z}_H & = m_H \ddot{z}_H + C_H \dot{z}_H + K_H z_H + NB[M(1,1)\ddot{z}_H] + \sum_{IB=1}^{NB} \sum_{IB=1}^{NZ} \sin\psi \sum_{J=1}^{NZ} Z(I,J,1)\dot{z}_J,IB \\
& + \sum_{IB=1}^{NB} \sum_{IB=1}^{NP} \sin\psi \sum_{J=1}^{NP} \phi_{J,IB} = F_{Hz} \\
\end{align*} \]
Equations (31)-(33) may be solved for the hub accelerations and written in matrix form:

\[
\begin{bmatrix}
    m_{Hx} + NB \cdot MI(1,1) & 0 & 0 \\
    0 & m_{Hy} + NB \cdot MI(1,1) & 0 \\
    0 & 0 & m_{Hz} + NB \cdot MI(1,1)
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_H \\
    \ddot{y}_H \\
    \ddot{z}_H
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \sin\psi_{IB} & 0 & 0 \\
    0 & \cos\psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \ddot{y}_J \\
    \ddot{z}_J \\
    \dot{\phi}_J
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \cos\psi_{IB} & 0 & 0 \\
    0 & \sin\psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \ddot{y}_J \\
    \ddot{z}_J \\
    \dot{\phi}_J
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \sin\psi_{IB} & 0 & 0 \\
    0 & \cos\psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \ddot{y}_J \\
    \ddot{z}_J \\
    \dot{\phi}_J
\end{bmatrix}
\]

\[
\begin{bmatrix}
    C_{Hx} & 0 & 0 \\
    0 & C_{Hy} & 0 \\
    0 & 0 & C_{Hz}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_H \\
    \dot{y}_H \\
    \dot{z}_H
\end{bmatrix}
\]

\[
\begin{bmatrix}
    K_{Hx} & 0 & 0 \\
    0 & K_{Hy} & 0 \\
    0 & 0 & K_{Hz}
\end{bmatrix}
\begin{bmatrix}
    x_H \\
    y_H \\
    z_H
\end{bmatrix}
\]

\[
\begin{bmatrix}
    F_{HX} \\
    F_{HY} \\
    F_{HZ}
\end{bmatrix}
\]

\[
(34)
\]
METHOD OF SOLUTION

The coefficient matrices of Equation (25) with the hub angular motions $\alpha_x$ and $\alpha_y$ omitted may be defined thusly:

$$[\text{COIR}] = \begin{bmatrix}
DYYII(I,J,1) + 4\Omega^2 DYSI(I,J,1) & 0 & -DYPII(I,J,3) \\
0 & DZZII(I,J,1) & DZPII(I,J,1) \\
-DYPII(I,J,3) & DPZII(I,J,2) & DPII(I,J,4)
\end{bmatrix}$$

$$[\text{COIH}][\text{SIB}] = \begin{bmatrix}
-sin\psi DYMII(I,J,1) & cos\psi DYMII(I,J,1) & 0 \\
0 & 0 & DZMII(I,J,1) \\
sin\psi DPMII(I,J,3) & -cos\psi DPMII(I,J,3) & DPMII(I,J,3)
\end{bmatrix}$$

$$[\text{CODR}] = \begin{bmatrix}
D\Omega + 2\Omega \{-DYYI(I,J,2) & 2\Omega\{DYSIS(I,J,3) & -2\Omega DYSIS(I,J,4) \\
-DYYII(I,J,5) & -DYZII(I,J,5) \\
+D\Omega\{I,J,1\}-D\Omega\{I,J,2\} & -\beta pc D\Omega\{I,J,1\} & \\
+\beta pc D\Omega\{I,J,1\} & 2\Omega \{D\Omega\{I,J,1\}-D\Omega\{I,J,3\}-D\Omega\{I,J,1\}\} & 0 & 0 \\
2\Omega \{\Omega\{I,J,5\} & 0 & 0
\end{bmatrix}$$

$$[\text{CODH}][\text{CIB}] = -\Omega \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
cos\psi DPMII(I,J,3) & -sin\psi DPMII(I,J,3) & 0
\end{bmatrix}$$
\[ [\text{COR}] = \begin{bmatrix}
\text{DYF}(I,J,3) & \text{DZF}(I,J,3) & \text{DZF}(I,J,4) + \\
-\Omega^2 \{\text{DYYII}(I,J,7)\} & +\Omega^2 \{\text{DZZII}(I,J,3)\} & +\Omega^2 \{\text{DPII}(I,J,2)\} \\
-\text{DYYII}(I,J,4) & -\text{DZZII}(I,J,6) & -\text{DZF}(I,J,4) \\
+\text{DYYII}(I,J,1) & & \\
\end{bmatrix} \]

\[ \{\text{FR}\} = -\Omega^2 \begin{bmatrix}
-\text{DYMII}(I,J,3) + \text{DYMII}(I,J,4) - \text{DYF}(I,J,6) \\
\text{DZMI}(I,J,6) - \text{DZF}(I,J,5) + \beta_{pc} \text{DZMI}(I,J,2) \\
\text{DPII}(I,J,9) + \beta_{pc} \text{DPII}(I,J,4) \\
\end{bmatrix} \]

\[ \{\text{BF}\} = \begin{bmatrix}
\text{DYALII} \\
\text{DZALII} \\
\text{DPALII} \\
\end{bmatrix} \]

\[ \{\text{FNL}\} = 2\Omega \begin{bmatrix}
\text{R} \cdot \text{R} \cdot \text{R} \cdot \text{R} \cdot \text{R} \cdot \text{R} \cdot \text{R} \cdot \text{R} \cdot \text{X} \cdot \text{X} \\
\text{X} \cdot \text{X} \cdot \text{X} \cdot \text{X} \cdot \text{X} \cdot \text{O} \\
\text{O} \cdot \text{X} \cdot \text{X} \cdot \text{O} \cdot \text{X}
\end{bmatrix} \]
Similarly, the coefficients in the hub equations of motion, Equation (27) may be defined as:

\[
[SIB] = \begin{bmatrix}
\sin \psi_{IB} & 0 & 0 \\
cos \psi_{IB} & 1 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
[CIB] = \begin{bmatrix}
cos \psi_{IB} & 0 & 0 \\
0 & \sin \psi_{IB} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
[RIOC] = [COIR]^{-1}
\]

\[
\{y\}_{z_p} = \begin{bmatrix} y \\ z \\ \phi \end{bmatrix}, \quad \{x_H\} = \begin{bmatrix} x_H \\ y_H \\ z_H \end{bmatrix}
\]

Similarly, the coefficients in the hub equations of motion, Equation (27) may be defined as:

\[
[TM] = \begin{bmatrix}
m_Hx^{+NB\cdot MI(1,1)} & 0 & 0 \\
0 & m_Hy^{+NB\cdot MI(1,1)} & 0 \\
0 & 0 & m_Hz^{+NB\cdot MI(1,1)}
\end{bmatrix}
\]
$$[\text{BIN}] = \begin{bmatrix}
Y_{I(1, J, 1)} & 0 & -P_{I(1, J, 3)} \\
-Y_{I(1, J, 1)} & 0 & P_{I(1, J, 3)} \\
0 & -Z_{I(1, J, 1)} & -P_{I(1, J, 1)}
\end{bmatrix}$$

$$[\text{BDAM}] = 2\Omega 
\begin{bmatrix}
Y_{I(1, J, 1)} & 0 & -P_{I(1, J, 3)} \\
Y_{I(1, J, 1)} & 0 & -P_{I(1, J, 3)} \\
0 & 0 & 0
\end{bmatrix}$$

$$[\text{BSPR}] = \Omega^2 
\begin{bmatrix}
-Y_{I(1, J, 1)} & 0 & 0 \\
Y_{I(1, J, 1)} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$[\text{HC}] = -
\begin{bmatrix}
C_{H_x} & 0 & 0 \\
0 & C_{H_y} & 0 \\
0 & 0 & C_{H_z}
\end{bmatrix}$$

$$[\text{HK}] = - 
\begin{bmatrix}
K_{H_x} & 0 & 0 \\
0 & K_{H_y} & 0 \\
0 & 0 & K_{H_z}
\end{bmatrix}$$

27
\[
\{HF\} = \begin{bmatrix}
F_{Hx} \\
F_{Hy} \\
F_{Hz}
\end{bmatrix}
\]

\[
[BIRI] = [BIN][RIOC]
\]
\[
[BIRIO] = [BIRI][CODR]
\]
\[
[BIRIO] = [BIRI][COR] + [BSPR]
\]
\[
[BIRIDH] = [BIRI][CODH]
\]
\[
[BIRIIH] = [BIRI][COIH]
\]

Using the previous definitions, and assuming a sinusoidal forcing function, Equation (25) may be written as:

\[
\dddot{\{Y_P\}}_I = \left((RIOC)([CODR]\{\ddot{Y}_P\}_I + [COR]\{Y_P\}_I + \{FR\}_I + \{BF\}sin\omega_ft\right)_I
\]

\[
+ \{FNL\}_I + [COIH][SIB]_{I}{\ddot{x}_H}_I^I + [CODH][CIB]_{I}{\ddot{x}_H}_I^I
\]

(35)

Equation (34) for the hub accelerations is written as:

\[
[TM](\dddot{x}_H) = \sum_{IB=1}^{NB} [SIB]_{IB}[BIN](\dddot{Y}_P)_{IB} + \sum_{IB=1}^{NB} [CIB]_{IB}[BDAMP](\dddot{Y}_P)_{IB}
\]

\[
+ \sum_{IB=1}^{NB} [SIB][BSPR](Y_P)_{IB} + [HC](\dddot{x}_H) + [HK]_{IB}{\ddot{x}_H}_I^I + \{HF\}_{IB}
\]

(36)

28
Solving for the blade accelerations from Equation (35) and substituting the result into Equation (36) removes the inertial coupling in the system and allows solution of the hub accelerations directly.

\[
\{\ddot{x}_H\} = \left( [TM] - \sum_{IB=1}^{NB} [SIB]_{IB}[BIRIH][SIB]_{IB} \right)^{-1} \left( [HC] + \sum_{IB=1}^{NB} [SIB]_{IB}[BIRIDH][CIB]\{\dot{x}_H\} + [HK]\{x_H\} + \{F_T\} \right)
\]

\[
+ \sum_{IB=1}^{NB} \left( ([SIB]_{IB}[BIRID] + [CIB]_{IB}[BDAM])\{\ddot{y}_{ZP}\}_{IB} + [SIB]_{IB}[BIRI]\{FR\}_{IB} + \{BF\}\sin\omega_F t \right)
\]

\[+ \{FN}\}_{IB})
\]

Solution of Equation (37) is effected by use of a fourth order Runge-Kutta timewise integration technique. Once the hub responses are obtained for a particular time increment, Equation (35) is solved for the blade motions. These blade motions are, in turn, substituted into Equation (37) to yield the hub responses for the subsequent time increments. This procedure is continued until the total time interval of interest is reached.

PROGRAM FEATURES - V22

The V22 program, developed to implement the solutions of the equations developed above, was designed to achieve the flexibility and ease of use necessary to make it a useful research tool. The details of the necessary and optional inputs are described in Appendix B. Some of the major features of the program are outlined in this section.

1. General input - The input data, in most cases, may be input in any order. Certain data is optional as input and need not be entered unless used. In running successive cases, only changed data need be input.

2. No. of Blades - One to four blades may be specified. With a hub, a minimum of two is required.
3. Modal input - The method of solution (Galerkin's method) uses separate in-plane, out-of-plane, and torsion "modes" as generalized degrees of freedom. They need not be normal modes (and thus need not be changed for changes in parameters and rotor speed). The equations contain the modal displacement as well as the first and second derivatives. Only the second derivative and the root slope of each mode is required as input. The program integrates and normalizes each mode to a value of unit displacement at the tip. Modes which are representative of the expected normal mode shapes are suggested.

4. Frequencies - Rotational and forcing frequencies are input independently. A frequency sweep may be simulated with a single card for each discrete frequency. \( \Omega = 0 \) is allowed.

5. Hub data - The hub is represented by a single degree of freedom spring, mass, damper in each direction. These parameters may be easily changed with forcing frequency to simulate actual hub impedances. Optionally 0, 1, 2 or 3 directions of motion are allowed. Sinusoidal forcing in any of these directions may be specified.

6. Blade forces - Optional forces may be applied at any blade station. An optional 1- cos type excitation for a specified fraction of one revolution is available.

7. Floquet option - If this option is selected, the program automatically produces a Floquet transition matrix by performing one (force) cycle for each initial condition. A further option ignores the steady effects due to such quantities as twist and precone.

8. Periodic solution - A periodic solution is obtained through the Floquet matrix which allows the solution for the initial conditions which will result in periodicity.

9. Nonlinear options - All, in-plane only, or no nonlinear effects may be optionally included in the solution.

10. Solution controls - The integration procedure used includes error checks and automatically selects appropriate sized integration increments. The user specifies quantities such as the number of cycles, error bound, variable to be tested for error, initial condition (unless periodic solution is specified).
SYSTEM IDENTIFICATION

The mass parameters of any continuous structure are not amenable to direct verification. An operational rotor blade is subjected to very large centrifugal forces and undergoes a highly coupled motion which includes deformation of the elastic axis in and out of the plane of rotation and torsional deformations about this axis. Under these conditions, the adequacy of the mass parameters which are based on a fictitious homogeneous section are in some doubt. While there is no way of directly measuring these parameters, the relationship between them and the normal modes, which are at least conceptionally measurable, are well understood.

The method of incomplete models (References 4 and 5), which addresses the problem, has been adapted to the specific set of rotor blade parameters. This formulation determines the minimum changes required in the intuitively derived set of mass parameters to make them compatible with the measured modes. There are other related developments and features of the implementation program which will yield valuable information regarding the adequacy of the analytical model. These are derived and discussed in this section.

THEORETICAL BACKGROUND

Consider a discrete element dynamic model of a continuous structure. One part of this model is a mass matrix, \( M \). If \( \psi_k \) is a vector representing the k-th normal mode, there exists a necessary orthogonality relationship as follows:

\[
\psi_k^T M \psi_n = 0 \quad k \neq n \tag{38}
\]

If the modal vectors are considered to be known, and the masses unknown, this equation can be rewritten as a set of linear equations:

\[
A\bar{M} = 0 \tag{39}
\]

where \( A \) is a matrix whose elements are products of the elements of the modal vectors, and \( \bar{M} \) is a vector made up of the unknown elements of the mass matrix. There will be one equation for each unique pair of modes and one unknown for each of the elements of \( \bar{M} \). The problem is formulated so that the symmetrical off-diagonal elements in the (symmetrical) mass matrix appear only once in the mass vector, \( \bar{M} \).
Since the scalar product $\psi_k^T M \psi_n$ is identical to $\psi_n^T M \psi_k$, there will be $NM(NM-1)/2$ equations, where $NM$ is the number of modes. If $N$ is the number of coordinates, the number of unknowns may be between $N$ and $N(N+1)/2$ where the first corresponds to a pure diagonal matrix and the upper limit corresponds to a fully populated mass matrix. As discussed in References 4 and 5 it is usual and desirable to have many more unknowns than equations. There are, thus, an infinite number of solutions which will satisfy Equation (39).

It is, of course, desired to obtain that solution which is the most representative of the actual structure. This objective may be achieved by finding, of those mass matrices which satisfies Equation (39), and (38), that which is closest to an analytically derived model of the structure. That is to say, determine the smallest possible changes in the analytical mass matrix necessary to orthogonalize the measured modes. This may be done as follows. Let $\bar{M}_A$ be a vector which is made up of the elements of the analytical (or approximate) mass matrix and then write $\bar{M} = \bar{M}_A + \bar{M}$, where $\bar{M}$ represents the required changes in $\bar{M}_A$. Substituting into Equation (39) yields:

$$A\bar{M} = -A\bar{M}_A$$  \hspace{1cm} (40)

As discussed in Reference 5, the use of the matrix pseudoinverse yields a solution which has the minimum sum of the squares of the individual elements, i.e., $\bar{M}^T \bar{M} = \text{min}$. This solution may be written:

$$\bar{M}_{\text{min}} = -A^T (AA^T)^{-1} A\bar{M}_A$$  \hspace{1cm} (41)

The application to the specific rotor blade problem is given below, where certain other more detailed considerations of minimization and other constraints are discussed.

**ROTOR BLADE APPLICATION**

The normal modes of a rotor blade are conveniently expressed in terms of the in-plane, out-of-plane, and torsional components as follows:

$$\psi_k = \begin{bmatrix} \bar{v} \\ \bar{w} \\ -\phi \end{bmatrix}$$
where $\ddot{v}$, $\ddot{w}$, and $\ddot{\phi}$ are vectors, each having $NX$ elements, when $NX$ is the number of blade stations used in the analysis and test.

The mass matrix, as can be seen from the acceleration terms of Equations (5), (6), and (7) may be conveniently partitioned, where each of the partitions is a diagonal matrix of order $NX$. The rotor blade form of Equation (38) then may be written:

$$
\begin{bmatrix}
\dddot{v}^T & \ddot{w}^T & \dddot{\phi}^T
\end{bmatrix}_k
\begin{bmatrix}
m_i \\
0 \\
-(me\theta)_i
\end{bmatrix}
= 0 
\quad k \neq n
$$

The elements of these diagonal partitions ($i = 1, 2, ... NX$) represent a "lumped mass" (rather than a "distributed mass") formulation of the problem, which is inherent in the matrix representation.

Treating the modal displacements as knowns and the mass parameters as unknowns, the analogy of Equation (39) becomes:

$$
\begin{bmatrix}
v_k & w_k & \phi_k \\
+\dot{v}_k & +\dot{w}_k & +\dot{\phi}_k \\
-\ddot{v}_k & -\ddot{w}_k & -\ddot{\phi}_k
\end{bmatrix}
\begin{bmatrix}
\ddot{m} \\
\ddot{me} \\
\ddot{me\theta} \\
\ddot{mk^2}
\end{bmatrix}
= 0
$$

where, typically, $v_{ki}$ represents the in-plane displacement of mode $k$ at station $i$. Each partition of the matrix $A$ has $NM(NM-1)/2$ rows (one for each pair of modes, $k < n$) and $NX$ columns, one for each station ($i = 1, 2, ... NX$). This, there are $NM(NM-1)/2$ equations and $4 \cdot NX$ unknowns (in vector $M$).
As above, let \( \bar{M} = \bar{M}_A + \bar{M} \), then Equation (43) is:

\[
\Delta \bar{M} = -A\bar{M}_A
\]  

(44)

This equation may be solved for minimum \( \Delta \bar{M} \) as in Equation (41). However, if there are significant differences in size between elements of \( M_A \) it would not be appropriate to simply minimize the sum of the squares of the magnitudes of the changes. This procedure could result in excessively large percentage changes in the very small elements, even though these same changes would be quite small compared to the larger elements.

It is possible, through a simple modification in the method to minimize the sum of the squares of the percentage changes, which is a more reasonable criteria. In addition, it is also possible to allow the analyst to indicate a level of confidence in each element, so that items with higher confidence will tend to change least. The result is a solution which has a weighted sum of squares of the elements at a minimum.

Let the \( i \)-th element of \( \bar{M}_A \) be designated \( (\bar{M}_A)_i \) and the corresponding assigned weighting factor (confidence level) be \( w_i \). Form a diagonal matrix \( W \) such that \( W_{ii} = w_i/(\bar{M}_A)_i \). Then the elements of \( W \Delta \bar{M} \) are

\[
(W \Delta \bar{M})_i = w_i \Delta \bar{M}_i / (\bar{M}_A)_i
\]

which is the function that should be minimized. This is achieved by making \( W \Delta \bar{M} \) the unknown in Equation (44) by inserting \( I = W^{-1}W \) as follows:

\[
AW^{-1} W \Delta \bar{M} = -A\bar{M}_A
\]  

(45)

Then, as above:

\[
(W \Delta \bar{M})_{\text{min}} = -W^{-1}A^T(\Delta \bar{M}^2A^T)^{-1}A\bar{M}_A
\]

and

\[
\bar{M} = \bar{M}_A - W^{-2}A^T(\Delta \bar{M}^2A^T)^{-1}A\bar{M}_A
\]  

(46)

such that:

\[
\Delta \bar{M}^T W^2 \Delta \bar{M} = \text{min}
\]
MASS CONSTRAINTS

Since the number of equations is generally much less than the number of unknowns, it is possible to add equations to Equation (43) which will impose constraints on the mass parameters. In the method as implemented, five optional constraints are available. These each maintain the following mass characteristics at the same value they have in $\bar{R}_A$. These constraints refer to: total mass, radial static moment (cg), chordwise static moment (cg), flapping moment of inertia, and feathering moment of inertia. These five constraints result in the following equations added to Equation (43):

$$
\begin{align*}
1, 1, 1 & \quad 0 \quad 0 \quad 0 \\
x_1, x_2, x_3 \ldots & \quad 0 \quad 0 \quad 0 \\
0 & \quad 1, 1, 1 \ldots \quad 0 \quad 0 \\
x_1^2, x_2^2 \ldots & \quad 0 \quad 0 \quad 0 \\
0 & \quad 0 \quad 0 \quad 1, 1, 1 \ldots \\
\end{align*}
$$

Thus it is possible to find the necessary changes in the mass matrix to make the modes orthogonal, such that the weighted sum of squares of the percentage changes is a minimum and the specified mass characteristics remain invariant.

$$
\bar{M} = \bar{M}_A - W^{-2}A^T(A W^{-2}A^T)^{-1}(A \bar{M}_A - \bar{r})
$$

where $\bar{r}$ is the right-hand side vector of Equation (43) augmented by that of Equation (47).
ROTATIONAL SPEED EFFECTS

The mass matrix discussed above is independent of the blade rotational speed, $\Omega$. The natural frequencies and the mode shapes, however, do change as the rotational speed is changed. The analysis, as presented, is valid for any single $\Omega$ including the nonrotating condition, $\Omega = 0$.

The fact that the modes change with $\Omega$ provides an opportunity for obtaining additional information within a fixed range of forcing frequencies over that available for a conventional nonrotating structure. If several modes are measured at each of several values of $\Omega$, the same mass matrix must make the modes at any one $\Omega$ orthogonal.

Thus, the method above has been modified to accept modes at different values of $\Omega$ and to set up an equation for each pair of modes at each $\Omega$. For example, if the first three modes were identified at three $\Omega$'s, there would be nine equations which would provide information about the mass matrix.

MODE CHANGES

The measured data, even if exact, is not sufficient to uniquely identify an analytical model and thus intuitive decisions are required of the user of this method. Some of these decisions have been described above. In addition to finding the necessary mass model changes, consideration should be given to the unavoidable errors in the measured modes. It is of interest to determine the minimum changes that would be required in the modes to achieve orthogonality using the analytical mass matrix. Methods of this general type have been suggested in the literature from time to time (References 6, 7, and 8). The method developed and implemented in this study uses techniques very similar to those for the mass identification, above.

If the modes are placed in order of decreasing confidence (usually in order of increasing natural frequency), the method assumes the first is correct, changes the second to make it orthogonal to the first, then changes the third to make it orthogonal to the first and the corrected second mode, and similarly for all higher modes. The changes are the minimum sum of squares of the percentage changes of each element as discussed above.

The first equation may be written:

$$\psi_1^T M(\psi_2 + \Delta\psi_2) = 0$$

or

$$A\Delta\psi_2 = -A\psi_2$$  \hspace{1cm} (49)
where \( A = \psi_1^T M \) is a \( 1 \times 3 \cdot M \times N \) matrix. The next equation then is:

\[
A \Delta \psi_3 = - \Delta \psi_3
\]

(50)

where:

\[
A = \begin{bmatrix}
\psi_1^T \\
\psi_2^T + \Delta \psi_2^T
\end{bmatrix}
\]

\( M \) and \( A \) is a \( 2 \times 3 \cdot M \times N \) matrix.

The equations for \( \Delta \psi_3 \) results in an \( A \) matrix of order \( M-1 \times 3 \cdot M \times N \). The procedure used for solving these equations is the same as that described above without any weighting function, \( w \), assigned to the individual elements.

**PROGRAM FEATURES - ROTS1**

This program has been designed to provide maximum flexibility as a research tool. The theoretical basis has been described in the previous paragraphs. The Users Guide with detailed input instructions is in Appendix B. This section will briefly outline several of the major features and capabilities of the program.

1. Normalization - the modes may be normalized so the diagonal elements of the generalized mass matrix are unity.

2. Add modes - after a computation is completed, additional modes may be added and further operations may be performed.

3. Rotational speed - modes of more than one rotational speed may be included (for mass identification) and the proper pairing takes place automatically.

4. Random errors - modes may be polluted with random errors with specified random or bias errors for sensitivity analyses.

5. Modal changes - necessary mode changes as described above with constant mass matrix may be determined.

6. Limited mode changes - modes may be changed as above but with limits specified for each mode. Truncation or scaling options are available.

7. Mass changes - weighted minimum mass changes may be obtained as described above.
8. Invariant stations - the mass parameters at selected stations may be held invariant.

9. Invariant parameters - mass, static moments, moments of inertia may optionally be maintained invariant during mass identification.

10. Sequential operations - the various options may be executed sequentially, for example, one may first change all the modes up to some specified percentages and then finish the correction by modifying the mass matrix.
METHOD APPLICATIONS

The two programs were continually checked for validity and reasonableness during their development. All features were at least qualitatively verified. The programs were then used to approximately simulate the tests to be carried out in the vacuum chamber at the Langley Research Center. These applications are described below.

SIMULATION DATA

The system simulated consisted of two blades and a hub with a vertical degree of freedom. The system was excited by a vertical force at the hub.

Each blade was represented by 17 stations. The parameters are shown in Table 1 which is taken from an actual computer run. The units are all in the lb-in-sec system.

Tables 2, 3, and 4 show the modes used as generalized degrees of freedom. These modes were developed from an approximate cantilever eigenvalue analysis. The one in-plane, three out-of-plane, and one torsional mode represent all the modes expected to have natural frequencies below 12/rev at \( \Omega = 25 \) rad/sec. The tables illustrate the second and first derivative and the displacements after normalization.

The hub was arbitrarily represented by a mass of .6 lb-sec\(^2\)/in and a spring rate of 20,000 lb/in. This implies a rigid rotor vertical natural frequency of 111. rad/sec or 4.44/rev at \( \Omega = 25 \) rad/sec.

Tables 5 and 6 give the blade and hub matrices as described in the section on Method of Solution and Equations (36) and (37).

SIMULATION COMPUTATIONS

Simulated frequency sweeps were carried out at \( \Omega = 0, 20, \) and 25 rad/sec. The Floquet option was used to obtain precise periodic responses to sinusoidal excitation at the hub. The objective of the simulated test was to locate the frequencies at which hub vertical antiresonances occur. At this frequency, cantilever conditions exist and since damping is light the displacement will be a good approximation to the coupled cantilever normal modes of the blades. Since discrete frequency inputs are required, a coarse sweep was first carried out, followed by necessary points at small frequency intervals to identify the point of zero hub displacement.
### Table 1. Blade Properties

<table>
<thead>
<tr>
<th>X</th>
<th>M</th>
<th>E</th>
<th>SMALL EA</th>
<th>KML</th>
<th>KM2</th>
<th>KA</th>
<th>THETA PRIME</th>
</tr>
</thead>
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<td>7.40E-01</td>
<td>4.89E-01</td>
<td>4.95E-01</td>
</tr>
</tbody>
</table>

### Notes
- **X**: Blade number
- **M**: Blade mass (kg)
- **E**: Blade equivalent modulus of elasticity (GPa)
- ** SMALL EA**: Small EA (GPa)
- **KML**: KML (GPa)
- **KM2**: KM2 (GPa)
- **KA**: KA (GPa)
- **THETA PRIME**: Blade angle (rad)

### Additional Values
- **EI OP**: EI OP (Nm²)
- **EI IP**: EI IP (Nm²)
- **GJ**: GJ (Nm²)
- **EA**: EA (Nm²)
- **EB1 E2**: EB1 E2 (Nm²)
- **EC1**: EC1 (Nm²)

### Conversion Factors
- 1 GPa = 1,000,000,000 Pa
- 1 Nm² = 1,000,000 Nm²
# Table 2: In-Plane Modes

## Second Derivatives

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<th>Mode</th>
<th>1st Deriv (Normalized)</th>
<th>1st Mode Shape</th>
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</thead>
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### Table 3. Out-of-Plane Modes

**Io = 4**  
**OUT-OF-PLANE MODES**

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# Table 6. Hub Matrices (See Eq. 36, 37)

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Note that since the responses are the steady-state periodic responses to
\( \sin \omega_f t \) forcing, the response at \( \omega_f t = 90^\circ \) is the "real" or in-phase
component and the response at \( \omega_f t = 0^\circ \) is the "imaginary" or out-of-phase
component. Figures 3-12 illustrate the hub responses in the vicinity of
the antiresonant frequencies. In most cases, the imaginary component is
too small to be observed and is not plotted. These figures also illus-
trate the system natural frequencies.

At each antiresonant frequency the amplitudes of the generalized co-
ordinates were determined and normalized on the largest component. These
represent cantilever coupled modes and are summarized in Table 7. A
Campbell diagram displaying these frequencies is given in Figure 13.

The actual mode shapes in each of the three directions are shown in
Figures 14-19. Figures 14 and 15 are the in-plane and torsion component
shapes. Since only one of each was used as a degree of freedom in the
simulation, these shapes are the same for all the coupled normal modes
obtained. The magnitudes are given in Table 7. The out-of-plane bending
was represented by three modes and different combinations appear for each
normal mode. Figures 16-19 illustrate these shapes for all the modes
referenced in Table 7. The amplitude of these normalized modes is the
sum of the \( z_1, z_2, z_3 \) components given in the table. The small but
noticeable effect of rotor speed is illustrated in these figures.

**SYSTEM IDENTIFICATION**

In order to test and illustrate the ROTSI methods and program, the data
obtained in the simulation runs, above, was treated as if it were actual
test data. The analytical model was first intuitively reduced to an
eight station lumped mass model as shown on Table 8.

Several combinations of these modes were used for mass identification.
A sample output is shown in Table 9 where the original parameter, the
modified parameter and the percentage changes are given. Table 10
summarizes the sample analyses that were carried out showing mean
absolute percent changes of the four parameters: \( m, e, \theta, K_m \). The
results are not satisfactory as shown. In addition to these cases, other
combinations of modes at different rotational speeds have yielded
very large percentage change requirements.

Since similar analyses on other structures using as many as ten modes
and 150 unknowns have been successfully carried out, the large changes
required for all but the simplest combinations is surprising. However, there are two significant considerations which may shed some light on
this problem.
Figure 3. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 1st OP Canti-lever = 10.19 Rad/Sec

Figure 4. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 1st IP and 2nd OP Frequencies = 54.55, 74.20 Rad/Sec

Figure 5. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 3rd OP Frequency = 222 Rad/Sec
Figure 6. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 1st OP Frequency = 25.25 Rad/Sec

Figure 7. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. An Apparent Highly Damped Response in Vicinity of 1st IP Frequency

Figure 8. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 2nd OP Frequency = 86.25 Rad/Sec
Figure 9. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 1st OP Frequency = 30.49 Rad/Sec

Figure 10. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. Apparent Highly Damped Response in Vicinity of 1st IP Frequency

Figure 11. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 2nd OP Frequency = 95.52 Rad/Sec

Figure 12. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 3rd OP Frequency = 243.3 Rad/Sec
### TABLE 7. CANTILEVER NORMAL MODES

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Figure 13. Campbell Diagram Illustrating Natural Frequencies Obtained During Simulated Frequency Sweep
Figure 14. In-Plane Mode Shape for All Frequencies

Figure 15. Torsional Mode Shape for All Frequencies
Figure 16. Out-of-Plane Shapes From 1st OP Coupled Modes

Figure 17. Out-of-Plane Shapes From 1st IP Coupled Modes
Figure 18. Out-of-Plane Shapes From 2nd OP Coupled Modes

Figure 19. Out-of-Plane Shapes From 3rd OP Coupled Modes
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<td>NEW M</td>
<td>PCT</td>
<td>ORIG E</td>
<td>NEW E</td>
<td>PCT</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>----------</td>
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<th>PCT</th>
<th>ORIG KM</th>
<th>NEW KM</th>
<th>PCT</th>
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### TABLE 10. SUMMARY OF MASS IDENTIFICATION RESULTS

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<th>Mean Change</th>
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<td>2</td>
<td>.7</td>
<td>.3</td>
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<tr>
<td>1a</td>
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<td>1.5</td>
<td>.6</td>
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<tr>
<td>2</td>
<td>x x x x</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>very large changes</td>
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</tr>
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<td>x x x</td>
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<tr>
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<tr>
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<td>mode 3 apparently inconsistent</td>
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<tr>
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<td>3 Equations</td>
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</table>

57
(1) Only five generalized coordinates (modes) were used in the simulation. The torsional mode participated only slightly in any of the normal modes, thus there are essentially only four degrees of freedom in the problem. Whenever the number of equations approaches four, the necessary changes can be expected to become large. This situation, of course, will not exist in a real test and, thus, it is expected that the analysis of actual test data may be considerably more successful. It is possible to use the simulation program using up to 11 degrees of freedom and it is expected that the results of such an analysis would be considerably improved.

(2) No case where data from two rotor speeds was used was successful. It is apparent, from Figures 14-19, that the predicted changes in mode shape with rotor speed is quite small. Thus, the equations resulting from the same modes at different speeds will be nearly identical and result in a nearly singular matrix. In the simulation program, as used in this report, the same modes were used as generalized coordinates for all rotor speeds, thus accentuating this condition. Whether the use of actual test data will improve this situation is uncertain since it is well known that the mode shapes change only slightly with rotor speed.

It is also noted that any combination which included the third mode at \( \Omega = 0 \) yielded poor results. No particular reason is seen for this effect, except that the second and third modes contain highly coupled in and out-of-plane responses. Since the in-plane and first out-of-plane mode are quite similar, there may be some analytical problems in orthogonalizing those modes with the analytical model used.

As an illustration of the mode change analysis, keeping the mass matrix invariant, the three modes at \( \Omega = 25 \) rad/sec. were processed. The required changes are quite small and the results are shown in Table 11.
TABLE 11. MODE CHANGES REQUIRED FOR ORTHOGONALITY

\[ \Omega = 25 \text{ rad/sec} \]

Percentage Changes

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<th>Mode 3</th>
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<td>w</td>
<td>v</td>
<td>w</td>
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Two separate analytical methods have been developed. They both have been used as a basis for computer programs. The two programs are expected to be useful research tools for evaluating rotor dynamic analytical models in conjunction with the vacuum chamber whirl tests to be conducted at the Langley Research Center.

The first program allows the analyst to attempt to model these tests and to observe the agreement between analysis and experiment. The analytical model includes the important dynamic features of the test, such as hub degrees of freedom, non-uniform parameters, stiffness coupling between out-of-plane and in-plane motion, and the ability to simulate forcing frequency sweeps independent of rotor speed. The program has been designed to allow convenient changes in parameters, number of degrees of freedom, types of nonlinearities, periodic or transient solutions. The effects of parameters in blade responses, natural frequencies, and normal modes may be easily studied.

The second program, which is an adaptation of methods previously applied to nonrotating structures, makes use of observed blade normal modes to correct the mass and inertial coupling terms used in the analytical model. Other options allow the analyst to study the possibility of inaccurate modal measurements and combinations of modal and mass parameter changes. In addition, a feature which produces controlled random variations in the measured modes allows for a study of sensitivities of these results to inaccuracies in the observed data. The method also has the capability of making use of modes measured at more than one rotational speed.

Both programs have been extensively tested for validity and sample computations have been presented in this report. The second program which performs a class of system identification analyses, was tested using results obtained from the simulation program. The capability to handle more than a few modes or modes at more than one rotational frequency has not been demonstrated. The lack of adequate success is believed to be due to the relatively small number of generalized degrees of freedom used in the simulation program. Since other related applications of this technique have been significantly more successful, it is anticipated that the analysis of actual test data or the use of simulations having a larger number of participating modes will yield useful results.
The simulation program has the capability to use eleven blade generalized degrees of freedom. This limit is purely due to the dimensioning limitations and simple program modifications can increase this limit to any desired value. The simulation carried out used five modes as degrees of freedom. The lower frequency responses obtained are believed to be quite valid and this validity only becomes weaker as frequency ranges are reached which in reality include participation of modes which were not included in the analysis.

The following recommendations are made for useful continuation of this research.

(1) Develop an analytical model, which is a better intuitive representation of the actual rotor system to be tested.

(2) Simulate specific test conditions and make direct comparisons with actual test responses. If obvious apparent discrepancies exist, make rational intuitive changes in the analytical parameters whenever such changes can be justified by consideration of the physical characteristics of the rotor.

(3) Use actual measured normal modes in both the nonrotating and rotating conditions to correct the mass and inertial coupling parameters and to study the sensitivities to measurement errors. Use these results to evaluate the possibility of obtaining significant information from non-rotating tests alone. Evaluate the use of this method to improve the analyst's capability to derive a more satisfactory model from the physical characteristics of the blades prior to any testing.

(4) Use the simulation program for conditions and blades other than those tested to study the effects of blade and hub parameters on natural frequencies, blade and rotor responses and stability.

(5) Because the simulation program is a convenient, flexible and adaptable program, it is strongly recommended that further developments of this program to include aerodynamics, controls and a more comprehensive fuselage representation be considered.
REFERENCES


# APPENDIX A

## DEFINITIONS OF INTEGRALS

Mass Integrals (Sta. No., Coefficient No.)

\[ \int_{x}^{R} f(x) \, dx \]

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<th>MII(I,2)</th>
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<td>( \int me )</td>
<td>( \int )</td>
<td>( \int mex )</td>
<td>( \int )</td>
<td>( \int me \theta )</td>
<td>( \int )</td>
<td>( \int mex \theta )</td>
<td>( \int )</td>
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<td>( \int )</td>
<td>( \int mk_{m} \theta )</td>
<td>( \int )</td>
<td>( \int mK_{m} \theta )</td>
<td>( \int )</td>
<td></td>
</tr>
</tbody>
</table>

\( I = 1 \) to number of blade stations
\[ \mathcal{J} \equiv \int_{0}^{R} \mathcal{F}(x) \, dx \]

\begin{align*}
\mathcal{Y}(I, J, 1) &= \int_{m} \mathcal{M}_{Y} \, dx \\
\mathcal{Y}(I, J, 2) &= \int_{m} \mathcal{E}_{Y} \, dx \\
\mathcal{Y}(I, J, 3) &= \int_{m} \mathcal{E}_{Y}^{2} \, dx \\
\mathcal{Y}(I, J, 4) &= \int_{m} \mathcal{E}_{Y}^{3} \, dx \\
\mathcal{Y}(I, J, 5) &= \int_{m} \mathcal{E}_{Y}^{4} \, dx \\
\mathcal{Y}(I, J, 6) &= \int_{m} \mathcal{E}_{Y}^{5} \, dx \\
\mathcal{Y}(I, J, 7) &= \int_{m} \mathcal{E}_{Y}^{6} \, dx \\
\mathcal{Y}(I, J, 8) &= \int_{m} \mathcal{E}_{Y}^{7} \, dx \\
\mathcal{Y}(I, J, 9) &= \int_{m} \mathcal{E}_{Y}^{8} \, dx \\
\mathcal{Y}(I, J, 10) &= \int_{m} \mathcal{E}_{Y}^{9} \, dx
\end{align*}

\[ \mathcal{Y}(I, J, 1) = \int_{m} \mathcal{Y}(I, J, 1) \]
\[ \mathcal{Y}(I, J, 2) = \int_{m} \mathcal{Y}(I, J, 2) \]
\[ \mathcal{Y}(I, J, 3) = \int_{m} \mathcal{Y}(I, J, 3) \]
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\[ \mathcal{Y}(I, J, 10) = \int_{m} \mathcal{Y}(I, J, 10) \]

\[ I = 1 \text{ to number of blade stations} \]
\[ J = 1 \text{ to number of in-plane modes} \]
Z Integrals (Sta. No., Mode No., Coefficient No.)

\[ \int_{x}^{R} f(\ )dx \]

\[ Z_{I(I,J,1)} = \int m_{Z_{j}} \]

\[ Z_{I(I,J,2)} = \int me_{Z_{j}} \]

\[ Z_{I(I,J,3)} = \int mx_{Z_{j}} \]

\[ Z_{I(I,J,4)} = \int mex_{Z_{j}} \]

\[ Z_{I(I,J,5)} = \int me\theta_{Z_{j}} \]

\[ Z_{I(I,J,6)} = \int \tau_{Z_{j}} \]

\[ Z_{I(I,J,7)} = \int e_{A}\tau_{Z_{j}} \]

\[ Z_{I(I,J,8)} = \int e_{L}\theta_{Z_{j}} \]

\[ Z_{I(I,J,9)} = \int e_{A}\theta_{Z_{j}} \]

I = 1 to number of blade stations

J = 1 to number of out-of-plane modes
\[ \phi \text{ Integrals (Sta. No., Mode No., Coefficient No.)} \]

\[ R \int_{x}^{\infty} f(x) \, dx \]

\[ \begin{align*}
\Pi(I,J,1) &= \int \Phi_J \\
\Pi(I,J,2) &= \int \Phi_J \\
\Pi(I,J,3) &= \int \Phi_J \\
\Pi(I,J,4) &= \int K_m^2 \Phi_J \\
\Pi(I,J,5) &= \int \Delta K \Phi_J \\
\Pi(I,J,6) &= \int e_{\phi}^J \\
\Pi(I,J,7) &= \int K_A^2 \phi_J' \\
\Pi(I,J,8) &= \int K_A^2 \theta_{\phi J}' \, dx \\
\end{align*} \]

\[ \Pi(I,J,1) = \int \Pi(I,J,1) \]

\[ \Pi(I,J,2) = \int \Pi(I,J,2) \]

\[ \Pi(I,J,3) = \int \Pi(I,J,3) \]

\[ \Pi(I,J,4) = \int \Pi(I,J,4) \]

\[ \Pi(I,J,5) = \int \Pi(I,J,5) \]

\[ \Pi(I,J,6) = \int \Pi(I,J,6) \]

\[ \Pi(I,J,7) = \int \Pi(I,J,7) \]

\[ \Pi(I,J,8) = \int \Pi(I,J,8) \]

\[ I = 1 \text{ to number of blade stations} \]

\[ J = 1 \text{ to number of torsional modes} \]
Special Integrals (Sta. No., Mode No., Coefficient No.)

\[ \int \int f(x,y) \, dx \, dy \]

\( S_{I,J,1} = \int \int \frac{1}{EA} Y_{I,J,1} \, dx \, dy \)

\( S_{I,J,2} = \int_{Z} Y_{I,J,10} \, dy \)

\( S_{I,J,3} = \int_{Z} Z_{I,J,9} \, dy \)

\( S_{I,J,4} = \int_{Z} P_{I,J,8} \, dy \)

\( S_{I,J,5} = \int_{K}^{R} 2 \gamma Y_{I,J,1} \, dx \)
v Equation Integrals

\[ J \equiv \int_{0}^{R} (\ ) dx \]

\[
\begin{align*}
DYYI(K,J,2) & = \int_{0}^{R} YI(I,J,2) \\
DYYII(K,J,1) & = \int_{0}^{R} YII(I,J,1) \\
DYYII(K,J,4) & = \int_{0}^{R} YII(I,J,4) \\
DYYII(K,J,5) & = \int_{0}^{R} YII(I,J,5) \\
DYYII(K,J,7) & = \int_{0}^{R} YII(I,J,7) \\
DYZII(K,J,1) & = \int_{0}^{R} ZII(I,J,1) \\
DYZII(K,J,5) & = \int_{0}^{R} ZII(I,J,5) \\
DYPII(K,J,3) & = \int_{0}^{R} PII(I,J,3) \\
DYMII(K,4) & = \int_{0}^{R} MII(I,4) \\
DYMII(K,1) & = \int_{0}^{R} MII(I,1) \\
DYMII(K,2) & = \int_{0}^{R} MII(I,2) \\
DYMII(K,3) & = \int_{0}^{R} MII(I,3)
\end{align*}
\]
\begin{align*}
\text{DYMII}(K,5) &= \int Y_K \text{MII}(I,5) \\
\text{DYSI}(K,J,i) &= \int Y_K \text{SI}(I,J,i) \quad i = 1 \text{ to } 4 \\
\text{DYF}(K,J,1) &= \int Y_K (R - x)(\text{me} Y_J)_R \\
\text{DYF}(K,J,2) &= \int Y_K e_A Y(I,J,1) \\
\text{DYF}(K,J,3) &= \int Y_K \text{Ev} Y'_J \\
\text{DYF}(K,J,4) &= \int Y_K \text{ E} Z'_J \\
\text{DYF}(K,J,5) &= \int Y_K \text{(E}_{1*} \text{P}_J + \text{E}_1 \text{P'}_J) \\
\text{DYF}(K,J,6) &= \int Y_K \text{(e} \tau + (\text{me})_R R(R - x) \\
\text{DYD}(K,J) &= \int Y_K \int X Y_J \\
\text{DYALII}(K) &= \int Y_K \int Y V \\
\end{align*}

\text{K, J} = 1 \text{ to number of (1-P, 0-P or torsion) modes}
\[ f = \int_{0}^{R} \cdot \cdot \cdot \, dx \]

\[
\begin{align*}
DZYI(K, J, 3) &= \int_{0}^{R} YI(I, J, 3) \\
DZPI(K, J, 2) &= \int_{0}^{R} PI(I, J, 2) \\
DZZII(K, J, 1) &= \int_{0}^{R} ZII(I, J, 1) \\
DZZII(K, J, 3) &= \int_{0}^{R} ZII(I, J, 3) \\
DZZII(K, J, 6) &= \int_{0}^{R} ZII(I, J, 6) \\
DZYII(K, J, 1) &= \int_{0}^{R} YII(I, J, 1) \\
DZPII(K, J, 1) &= \int_{0}^{R} PII(I, J, 1) \\
DZMI(K, 6) &= \int_{0}^{R} MI(I, 6) \\
DZMII(K, i) &= \int_{0}^{R} MII(I, i) \quad i = 1 \text{ to } 3 \\
DZI(K, J, 1) &= \int_{0}^{R} [(R - x)(me\theta Y_J)_R + e_A\theta Y_I(I, J, 1)] \\
DZF(K, J, 2) &= \int_{0}^{R} AZE\theta Y_J^2 \\
DZF(K, J, 3) &= \int_{0}^{R} EwZ_J^3 \\
DZF(K, J, 4) &= \int_{0}^{R} [EC_1^* P_J^2 + E_1^* \theta^* P_J^2] \\
DZF(K, J, 5) &= \int_{0}^{R} R(R - x)(me\theta)_R - e_A\theta^2 \\
DZF(K, J, 6) &= \int_{0}^{R} e_A P_J^2 - R(R - x)(meP_J)_R \\
DZD(K, J) &= \int_{0}^{R} F\int_{0}^{R} F\int_{0}^{R} F \\
DZALII(K) &= \int_{0}^{R} F\int_{0}^{R} F \\
K, J &= 1 \text{ to number of corresponding modes} 
\end{align*}
\]
\[ \phi \text{ Equation Integrals} \]

\[ f \equiv \int_{0}^{R} (\ ) dx \]

\[ \begin{align*}
DPYI(K, J, 9) & = \int_{0}^{R} \phi^K YI(I, J, 9) \\
DPYII(K, J, 3) & = \int_{0}^{R} \phi^K YII(I, J, 3) \\
DPYII(K, J, 6) & = \int_{0}^{R} \phi^K YII(I, J, 6) \\
DPYII(K, J, 8) & = \int_{0}^{R} \phi^K YYII(I, J, 8) \\
DPZI(K, J, 8) & = \int_{0}^{R} \phi^K ZI(I, J, 8) \\
DPZII(K, J, 2) & = \int_{0}^{R} \phi^K ZII(I, J, 2) \\
DPZII(K, J, 4) & = \int_{0}^{R} \phi^K ZII(I, J, 4) \\
DPZII(K, J, 7) & = \int_{0}^{R} \phi^K ZII(I, J, 7) \\
DPPI(K, J, 6) & = \int_{0}^{R} \phi^K PI(I, J, 6) \\
DPPI(K, J, 7) & = \int_{0}^{R} \phi^K PI(I, J, 7) \\
DPPII(K, J, 4) & = \int_{0}^{R} \phi^K PII(K, J, 4) \\
DPPII(K, J, 5) & = \int_{0}^{R} \phi^K PII(I, J, 5) \\
DPMII(K, i) & = \int_{0}^{R} \phi^K MI(I, i) \quad i = 3, 4; \ 6 \text{ to } 10 \\
DPSI(K, J, 1) & = \int_{0}^{R} \phi^K SI(I, J, 5) \\
DPF(K, J, 1) & = \int_{0}^{R} \phi^K EC_1^* Y_J \\
DPF(K, J, 2) & = \int_{0}^{R} \phi^K EC_1^* Y_J \\
DPF(K, J, 3) & = \int_{0}^{R} \phi^K EC_1^* Z_J \\
DPD(K, J) & \equiv \int\int\int \phi^K J \phi^J \phi^K \phi^K \\
DPALII(K) & \equiv \int\int\int \phi^K \phi^J \\
\end{align*} \]

K, J = 1 to number of appropriate modes
APPENDIX B

USERS GUIDE

V22

DYNAMIC ROTOR SIMULATION PROGRAM

First card of each case is HEADING CARD (see next page for description and exceptions).

All other data may be entered in any order (data blocks must maintain order within block). Data not entered (after 1st case) retains previous values (if any). All data is self identified by value of IO punched in col 1,2 of card on first card of block.

INPUT SUMMARY

<table>
<thead>
<tr>
<th>IO</th>
<th>Type of Data</th>
<th>No. of Cards</th>
<th>Required?</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Blade Properties</td>
<td>Block</td>
<td>Yes (Must precede IO = 3,4 or 5,13)</td>
</tr>
<tr>
<td>02</td>
<td>Blade Data</td>
<td>1</td>
<td>No (Default to 0's)</td>
</tr>
<tr>
<td>03</td>
<td>Modes: In-Plane (Y)</td>
<td>Block</td>
<td>No (At least one of 3,4,5 required)</td>
</tr>
<tr>
<td>04</td>
<td>Out-of-Plane (Z)</td>
<td>Block</td>
<td>No</td>
</tr>
<tr>
<td>05</td>
<td>Torsion (P)</td>
<td>Block</td>
<td>No</td>
</tr>
<tr>
<td>06</td>
<td>Frequencies ($\Omega$, $\omega_p$)</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>07</td>
<td>Hub Data, X,M,C,K,F</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>08</td>
<td>Y</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>09</td>
<td>Z</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Applied Forces, Blades</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Special Controls - Nonlin, Floquet</td>
<td>1</td>
<td>No (Default to Nonlinear)</td>
</tr>
<tr>
<td>18</td>
<td>Solution Controls</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Special IO Cancel</td>
<td>1</td>
<td>No</td>
</tr>
</tbody>
</table>

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HEADING CARD

<table>
<thead>
<tr>
<th>Col</th>
<th>IC</th>
<th>#0</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IC1</td>
<td></td>
<td>Ends run (same as IEND = 3, see below)</td>
</tr>
<tr>
<td>2</td>
<td>IC2</td>
<td>#0</td>
<td>All input printed (else only new data printed)</td>
</tr>
<tr>
<td>3</td>
<td>IC3</td>
<td>#0</td>
<td>Prints definite integrals</td>
</tr>
<tr>
<td>4</td>
<td>IC4</td>
<td>#0</td>
<td>Prints coefficient matrices</td>
</tr>
<tr>
<td>5</td>
<td>IC5</td>
<td>#0</td>
<td>Writes data on tape (see below)</td>
</tr>
<tr>
<td>6-80</td>
<td></td>
<td></td>
<td>Arbitrary heading</td>
</tr>
</tbody>
</table>

The heading card is the first card of the first case and the first card of each following case unless the preceding case ended with IEND = 2 (see below).

GENERAL INPUT

I0 in col 1,2 of 1st card only of each block.

IEND in col 80 of single card - see details of each block input.

- IEND = 1 end of data, followed by HEADING and new data
- = 2 same as 1 but omit HEADING card from next case
- = 3 ends run at completion of case

No special ending required for block data input

All data has following format. Real and integer input may be mixed.

I2, F8.0, 6F10.0, F9.0, I1

Do not use col 1 or 2 except for I0 (on first card of block)

Do not use col 80 except to end case

TAPE DATA (IC5 #0)

Uses FORTRAN unit 9. Data records are as follows \( \psi \) (in degrees, not limited to 360), tip in-plane deflection, tip out-of-plane deflection, tip torsional deflection, \( x_H, y_H, z_H \). Blade 1 only
**IO = 1**  
**BLADE PROPERTIES REQUIRED**

Must precede \( IO = 3,4,5,13 \)

\( IO \) on first card only, col 1,2 blank on all succeeding cards

2 cards per station (order 1,2,1,2...)

20 stations max

**IEND** (if used) on last card 1.

Definitions consistent with TN D-7818

<table>
<thead>
<tr>
<th>Word</th>
<th>Card 1</th>
<th>Card 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 X</td>
<td>sta (ascending sequence)</td>
<td>EOP - ( EI_{y'} ) (( EI ) out of chord plane)</td>
</tr>
<tr>
<td>2 M</td>
<td>mass/unit length</td>
<td>EIP - ( EI_{z'} ) (( EI ) for bending in chord plane)</td>
</tr>
<tr>
<td>3 E</td>
<td>e</td>
<td>GJ</td>
</tr>
<tr>
<td>4 SEA</td>
<td>e_A</td>
<td>EA - (if 0 then ( \frac{1}{EA} ) is set to 0)</td>
</tr>
<tr>
<td>5 Km1</td>
<td>( k_{m1} )</td>
<td>EB1 - EB1*</td>
</tr>
<tr>
<td>6 Km2</td>
<td>( k_{m2} )</td>
<td>EB2 - EB2*</td>
</tr>
<tr>
<td>7 KA</td>
<td>( k_A )</td>
<td>EC - EC1</td>
</tr>
<tr>
<td>8 THP</td>
<td>0° built in pitch - rad/ unit length</td>
<td>ECS - EC1*</td>
</tr>
</tbody>
</table>

**IO = 2**  
**BLADE DATA OPTIONAL** (Default to 0)

<table>
<thead>
<tr>
<th>Word</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 NB</td>
<td>nb of blades 4 max (Default to 1 if no hub DOF) (Default to 2 if hub DOF included)</td>
</tr>
<tr>
<td>2 THO</td>
<td>( \theta_0 ) angle at ( x(1) ) - radians</td>
</tr>
<tr>
<td>3 BPC</td>
<td>( \beta_{PC} ) - pre-cone - radians</td>
</tr>
<tr>
<td>4 GV</td>
<td>blade damping, 1-P appropriate units, viscous</td>
</tr>
<tr>
<td>5 GW</td>
<td>blade damping, 0-P appropriate units, viscous</td>
</tr>
<tr>
<td>6 GP</td>
<td>blade damping, torsion appropriate units, viscous</td>
</tr>
</tbody>
</table>
Each mode has one set of input - second derivative at each station followed by the first derivative at station 1 (slope and deflection are obtained by integration and normalized to unit deflection at tip).

Input - 8 elements per card - as many cards as necessary (3 max), all functions start on new card.

IO on first ( )" card - all other col 1,2 blank
IEND (if used) on 1st ( )" card of last mode

Order of input:
1st mode: ( )" x_1 ( )" x_2 ( )" x_3 ....
( )" x_9 ....

new card ( )' x_1 word 1 only, slope at station 1 (normally = 0)
new card ( )x_1 word 1 only, deflection at station 1 (normally = 0)
next mode ( )" x_1 ( )" x_2 ....
new card x_1 x_2 ....
   etc

IO = 6 FREQUENCIES REQUIRED

Word
1 OMEG - \( \Omega \) - rotor speed, rad/sec
2 OMF - \( \omega_f \) - forcing frequency, rad/sec
Impedance in each direction may be represented as spring-mass-damper at frequency $\omega_f$. Data omitted implies infinite impedance. If any hub data is input - at least two blades required.

<table>
<thead>
<tr>
<th>Word</th>
<th>HM</th>
<th>HC</th>
<th>HK</th>
<th>HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x$</td>
<td>$y$</td>
<td>$z$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mass</td>
</tr>
<tr>
<td>2</td>
<td>$x$</td>
<td>$y$</td>
<td>$z$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Damping Coeff</td>
</tr>
<tr>
<td>3</td>
<td>$x$</td>
<td>$y$</td>
<td>$z$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Spring Rate</td>
</tr>
<tr>
<td>4</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Force - multiplied by $\sin \omega_f t$ (or by 1 if $\omega_f = 0$)</td>
</tr>
</tbody>
</table>
Load may be applied at any one station, but in three directions. Amplitudes are multiplied by $\sin \omega_f t$ (or by 1 if $\omega_f = 0$). Forces may be applied to one or all blades. $\omega_f t$ always refers to blade 1, however, producing "umbrella mode" forcing. (See IO = 7, 8, 9 for hub forcing).

<table>
<thead>
<tr>
<th>Word</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NXF</td>
</tr>
<tr>
<td>2</td>
<td>AFY</td>
</tr>
<tr>
<td>3</td>
<td>AFZ</td>
</tr>
<tr>
<td>4</td>
<td>AFP</td>
</tr>
<tr>
<td>5</td>
<td>NBF</td>
</tr>
<tr>
<td>6</td>
<td>PER</td>
</tr>
</tbody>
</table>

Note: If in-plane hub degrees of freedom are used (IO = 7 or 8) AFY or NBF must = 0.
IO = 17  SPECIAL CONTROLS - NONLIN, FLOQUET OPTIONAL (Default to nonlinear, no floquet)

FLOQUET OPTION: Produces Floquet transition matrix using force cycle ($\omega_f$) unless $\omega_f = 0$ then rotor cycle is used. Note that if in-plane hub D-0-F are used equation contains terms periodic in $\Omega t$. If a force is applied then the boundary conditions for a (linear) periodic solution are determined and solution is executed for number of cycles specified in IO = 18. This overrides any other initial condition(s).

A maximum at 15 degrees of freedom are allowed for this option (30 variables including velocities).

Word

1  NLIN  =  0  All nonlinear terms included  
     =  1  In-plane nonlinear terms only  
     =  2  Linear terms only  

2  NFLOQ =  1  Floquet option (see discussion just above)  
     =  2  Same as 1, but steady effects of offsets and twists and precone are ignored.

IO = 18  SOLUTION CONTROLS REQUIRED

Errors and initial conditions are limited to one variable.

Word

1  CYCLES  Number of force* cycles for solution to run  
2  HINIT  Number of integration intervals per cycle  
3  ERROR  Error bound (appropriate units), see IYE  
4  IYE  Index of variables tested for ERROR**  
5  CIC  Initial condition (appropriate units), see IYIC  
6  IYIC  Index of variable for initial condition  
7  BERR  Upper limit (abs) of variable (IYE) which stops run. If = 0 no limit

* Force cycle is used unless $\omega_f = 0$ (IO = 06), then rotor cycle is used.

** See section on variable numbers following.
IO = 21    SPECIAL IO CANCEL    OPTIONAL

For cases after the first, IO's previously used may be cancelled. When this option is used all coefficients are recalculated and IC2 is set to 1 (see HEADING CARD) to insure data printout. There is no necessity to cancel when data is replaced.

Word

1-8   IO's to be cancelled (0's ignored)
VARIABLE NUMBERS

In I018 the variables are referred to by numbers. These numbers are as follows:

<table>
<thead>
<tr>
<th>I</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_H )</td>
</tr>
<tr>
<td>2</td>
<td>( x_H )</td>
</tr>
<tr>
<td>3</td>
<td>( y_H )</td>
</tr>
<tr>
<td>4</td>
<td>( y_H )</td>
</tr>
<tr>
<td>5</td>
<td>( z_H )</td>
</tr>
<tr>
<td>6</td>
<td>( z_H )</td>
</tr>
</tbody>
</table>

\[ I = 9 + 2 \text{ NM(IB-1)} \]

\[ I = 9 + 2 \text{IB} \text{ NM} \]

BLADE 1

\[ \text{IB} \text{ = blade number} \]

\[ \text{NM} \text{ = no. of modes} \]

\[ \text{last y} \]

\[ \text{last z} \]

\[ \text{last } \phi \]

BLADE 2

\[ \text{etc.} \]

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ERROR MESSAGES

Certain errors terminate the run. Others are warnings with correction as indicated below. All error numbers refer to a Fortran statement number in vicinity of error. (All are in INPU except for the 5000 series which occur in SOL).

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>REASON</th>
<th>TERMINATE</th>
<th>NUMBER</th>
<th>REASON</th>
<th>TERMINATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Inactive IO</td>
<td>Yes</td>
<td>510</td>
<td>I013, NYF &lt; 0 CR</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>Yes</td>
<td></td>
<td>I013, All forces 0</td>
<td>Yes</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>Yes</td>
<td>511</td>
<td>I013, NB &lt; NBF &lt; 0</td>
<td>No, NBF*</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>Yes</td>
<td>512</td>
<td>Sets NBM to 6</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>Invalid IO</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>202</td>
<td>More than one input</td>
<td>No, I0*</td>
<td>1100</td>
<td>I018, Error &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>of same IO, last one</td>
<td></td>
<td>1105</td>
<td>I018, IYIC &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>used</td>
<td></td>
<td>1106</td>
<td>I018, IYIC &gt; NDIM</td>
<td>Yes</td>
</tr>
<tr>
<td>203</td>
<td>I021, Attempt to cancel</td>
<td></td>
<td>1107</td>
<td>I018, IYE &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>invalid I$</td>
<td></td>
<td>1108</td>
<td>I018, IYE &gt; NDIM</td>
<td>Yes</td>
</tr>
<tr>
<td>215</td>
<td>I01, Stations out of seq</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>216</td>
<td>I01, Too many stations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>262</td>
<td>I03, Too many Y modes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>264</td>
<td>I04, Too many Z modes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>266</td>
<td>I05, Too many P modes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>No I0 = 1</td>
<td>Yes</td>
<td>5010</td>
<td>Too many D-0-F</td>
<td>Yes</td>
</tr>
<tr>
<td>501</td>
<td>No I0 = 3,4 or 5</td>
<td>Yes</td>
<td>502</td>
<td>IHLF = 11</td>
<td>Yes</td>
</tr>
<tr>
<td>502</td>
<td>No I0 = 6</td>
<td>Yes</td>
<td>506</td>
<td>IHLF = 12</td>
<td>Yes</td>
</tr>
<tr>
<td>506</td>
<td>I02 NB &gt; 4, set to 4</td>
<td>No, NB*</td>
<td>507</td>
<td>IHLF = 13</td>
<td>Yes</td>
</tr>
<tr>
<td>507</td>
<td>I02 NB &lt; 1, set to 1 or 2</td>
<td>No, 1*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>509</td>
<td>I0 = 18 Missing</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>510</td>
<td>In-plane hub with</td>
<td>No, NBF*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* This quantity is printed with warning.
**INPUT**

<table>
<thead>
<tr>
<th>COL</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) HEADING</td>
</tr>
<tr>
<td>1-10</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5-80</td>
</tr>
</tbody>
</table>

**MASS DATA - ONE CARD PER BLADE STATION**, 20 MAX

1-10 | Q(II) STATION |
| 11 | *(SEE NOTE) WM |
| 12-20 | M - LUMPED MASS |
| 21 | *(SEE NOTE) WE |
| 22-30 | E - CG OFFSET FROM EA WHEN CG FORWARD |
| 31 | *(SEE NOTE) WT |
| 32-40 | TH - PITCH ANGLE - RAD |
| 41 | *(SEE NOTE) WK |
| 42-50 | KM - RADIUS OF GYRATION IN TORSION |

* 1ST COL OF EACH WORD CONTAINS WEIGHTING FACTOR FROM 1-9 (0=1) HIGHER VALUE INDICATES GREATER CONFIDENCE

SEE 101 = 3 - WD1

**CONTROL CARD - MODES**

1-10 | CALV - MULTIPLIES I-P MODE DEFL - (0=1) |
| 11-20 | CALW - MULTIPLIES D-P MODE DEFL (0=1) |
| 21-30 | CALP - MULTIPLIES TOR MODE DEFL (0=1) |
| 31-40 | IHO - ROOT PITCH ANGLE - RAD |
| ADDS TO TH - (TH NOT CHANGED) |
(4) MODES — STATIONS CORRESPOND TO MASS DATA

<table>
<thead>
<tr>
<th>EACH MODE</th>
<th>1-10</th>
<th>FREQ NATURAL, RAD/SEC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11-20</td>
<td>OMEG ROTATIONAL, RAD/SEC</td>
</tr>
<tr>
<td>21-30</td>
<td>IF NE.0 TEMPORARILY REPLACES CALV</td>
<td></td>
</tr>
<tr>
<td>31-60</td>
<td>IF NE.0 TEMPORARILY REPLACES CALW</td>
<td></td>
</tr>
<tr>
<td>41-50</td>
<td>IF NE.0 TEMPORARILY REPLACES CALP</td>
<td></td>
</tr>
</tbody>
</table>

NEXT CDS V I-P DISPLACEMENTS, 8F10. UP TO 3 CARDS
NEXT CDS W O-P START ON NEW CD
NEXT CDS P TOR

FOLLOW BY NEXT MODE — 8 MODES MAX AT ONE OMEG

*** 30 EQS MAX (NOT INCL INVARIANCES) ***
END WITH BLANK CARD.

(5) OPERATION CODES COL 1,2 IO1,IO2

<table>
<thead>
<tr>
<th>COL 1</th>
<th>IO1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MODIFY MODES WITH RANDOM ERRORS — MODES REPLACED</td>
</tr>
<tr>
<td></td>
<td>WD1 PERCENT RANDOM + OR — RECTANGULAR DIST</td>
</tr>
<tr>
<td></td>
<td>WD2 PERCENT BIAS</td>
</tr>
<tr>
<td></td>
<td>WD3 INTEGER SEED TO START RANDOM SEQUENCE</td>
</tr>
<tr>
<td>***</td>
<td>FOLLOW BY NEXT OPERATION CARD. (5) ***</td>
</tr>
</tbody>
</table>

2 SOLVE FOR MINIMUM MODAL CHANGES — MASS MATRIX UNCHANGED

ALL MODES MUST BE AT SAME OMEGA — 8 MAX
FIRST MODE UNCHANGED, LAST MODE WILL CHANGE MOST MINIMUM SUM PERCENT CHANGES USED
WEIGHTING FACTORS NOT USED IN THIS OPTION

| WD1.EQ.0 | NO LIMIT ON CHANGES |
| WD1.EQ.1 | LIMIT CHANGES — SCALE OPTION |
| WD2-8    | MAX PCT CHANGE ALLOWED IN EACH MODE |
|          | CHANGES ARE SCALLED SO MAX CHANGE I.E., MAXIMUM |
|          | 0 INDICATES NO LIMIT |
| WD1.EQ.2 | LIMIT CHANGES — TRUNCATE OPTION |
| WD2-8    | SAME AS FOR SCALE OPTION EXCEPT THAT ONLY |
|          | CHANGES WHICH EXCEED LIMITS ARE TRUNCATED |
|          | OTHER CHANGES ARE NOT MODIFIED |

83
3  INCOMP MODEL MASS CHANGES

WD1.EQ.1  WEIGHTING FACTORS ALL SET TO 1 (TEMP)
WD1.EQ.2  STAS WITH INVARIANT PARAM. READ 51(A)

THE FOLLOWING CONTROLS CAUSE THE CORRESPONDING
PROPERTIES TO REMAIN INVARIANT - IF NE 0.

<table>
<thead>
<tr>
<th>COL 20</th>
<th>TOTAL MASS</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>RADIAL CG</td>
<td>M*X</td>
</tr>
<tr>
<td>40</td>
<td>CHORDWISE CG</td>
<td>M*E</td>
</tr>
<tr>
<td>50</td>
<td>FLAPPING MOM OF INERT</td>
<td>M*X**2</td>
</tr>
<tr>
<td>60</td>
<td>FEATHERING MOM OF INERT</td>
<td>M<em>K</em>M**2</td>
</tr>
</tbody>
</table>

0  ABOVE OPERATIONS DO NOT DISTURB ORIGINAL DATA
1  ABOVE OPERATIONS REPLACE ORIGINAL DATA IN PREPARATION
   FOR SEQUENTIAL OPERATIONS

(5A)  USED ONLY FOR INVAR STAS.  SEE 3, ABOVE, WD1 = 2

COL1 = NO OF STATIONS (8 MAX)
WD1, WD2, ..., STATION NUMBERS, NO ZEROES

NEXT HEADING CARD.

*******************************************************************************
*******************************************************************************
APPENDIX C
PROGRAM LISTINGS

C REAL M,KMI,KPZ,KA
C LOGICAL DM
C COMMON FOR INPUT
C COMMON/INDAT/X(20),E(20),SM(20),KM1(20),KM2(20),KAI(20),
1 TH(20),EP(20),EA(20),EB(20),ECS(20),EIP(20),
2 THG,BC,YPP(20,3),LPP(20,5),ZPP(20,3),PPP(20,3),PP(20,3),
3 OMEG,OMF,EC(20),NY,NZ,NP,NP,OMEGS,OMFS,DIM,NMAX,NLIN
4,NB,HX,MY,HCZ,HCX,HCY,HCZ,HKY,HKZ,NX,NFLQQ
5,HNET,ERROR,ICYC,BERR,CYCLE+,NXF,AFY,AFP,NBF
6 *R,GV,GW,HE(3),PER
C COMMON COEFFICIENT MATRICES
C COMMON/COEF/COI(11,11),COII(11,11),CDOI(11,11),CDOI(11,11),
1 COII(11,11),CCOI(11,11),F(11),DF(11),FLN(11),CIR(11,12),
2 CIR(11,11),CCR(11,11),FR(11),RLOC(11,12),BF(11)
3 ,BIR(3,11),BDAM(3,11),BSOR(3,11),COH(11,3),COOH(11,3),BIRI(3,11)
4 ,BIRIO(3,11),BIRIOI(3,11),BIRIOH(3,3),HFI(3),TH(3,3),BIRITH(3,3)
5 ,HC(3,3),HK(3,3)
C COMMON FOR HEADING, CONTROL DATA
C COMMON/HED/IC1,IC2,IC3,IC4,HEAD(19),IPAGE,INPUT(20),IEND,LINE,IC5
C COMMON DIMENSION DATA
C COMMON/DIM/NINPUT,NSTA,NMODE,NMODEP,NMODEP,NMODE,NM1,NDIM,NLABLE
C COMMON BASIC DERIVED DATA
C COMMON/DER/TH(20),EV(20),EH(20),EP(20),Y(20,3),Z(20,5),P(20,3)
C COMMON VARIABLES AND SOLUTION CONTROLS
C COMMON/VAR/YVAR(98),DYVAR(98),PRMT(6),LY(98)
C DIMENSIONALIZATION

NINPUT = 20
NSTA = 20
NYMODE = 3
NZMODE = 5
NPMODE = 3
NMODE = 11
NM1 = NMODE + 1
NDIM = 98
NDIM = 98
DO 10 I = 1,NINPUT
FSI = I,0E+51
10 INPUT(I) = 0
ICASE = 0
IEND = 0
20 CALL INPUT (ICASE)

LINE = 100
CALL SOL(PRMT,YVAR,DYVAR,HLF,LY)
IF (ICASE.EQ.0) WRITE(9) (FSI,I=1,7)
100 IF (IEND.EQ.3) CALL EXIT

GO TO 20
END
FUNCTION DINT (DUMP,DUMPP,X,NX)

C DUMP IS INTEGRAL OF DUMPP
REAL DUMPI ,DUMPP(L)
CALL INT (DUMP,DUMPP,0,X,NX,1)
DINT=DUMPP(NX)
RETURN
END

FUNCTION DINTL (A,B,I1 ,N,X,NA,NX,DUMP,DUMPP)
REAL A(NA,1) ,B(NA,1) ,X(1) ,DUMP(1) ,DUMPP(1)
DO 10 I=1,NX
10 DUMPP(I) = A(I,1)*B(I,N)
CALL INT (DUMP,DUMPP,0,X,NX,1)
DINT1=DUMPP(NX)
RETURN
END

FUNCTION DINT2 (A,B,I1 ,I2,N,NB,X,NA,NX,DUMP,DUMPP)
REAL A(NA,1) ,B(NA,NB,1) ,X(1) ,DUMP(1) ,DUMPP(1)
DO 10 I=1,NX
10 DUMPP(I) = A(I,1) * B(I,I2,N)
CALL INT (DUMP,DUMPP,0,X,NX,1)
DINT2=DUMPP(NX)
RETURN
END

SUBROUTINE ERR(N,I)
C I = 0, TERMINATES RUN
I NE 0 WARNING ONLY, PRINTS I
PRINT 10,N
10 FORMAT(//10X,17H*** ERROR NUMBER ,15,5H *** )
IF (I,NE,0) GOTO 20
CALL EXIT
20 FORMAT(30,I)
30 FORMAT (20X,20H*** WARNING ONLY *** ,15/) 
RETURN
END
**SUBROUTINE FCT**

**C**

**NOTE**

**DIM** NOT USED INCLUDED FOR COMPATIBILITY ONLY

**C**

**MULTI BLADES, 3 DDF HUB, NON-LIN CORiolIS FORCES**

**DIMENSION** YVAR(1), DERY(1)

**LOGICAL** LY(1)

**REAL** HM(KML), HKM2, HK, KA

**REAL** DUMP(20), DUNPP(20), VDM(20)

**REAL** VD(20), VDP(20), VP(20), VPP(20), WDI(20), WDP(20), WP(20), WPP(20)

**COMMON** /INDAT/ X(20), M(20), E(20), SE(20), KM(20), KM2(20), KA(20)

**1** THI(20), EEP(20), GJ(20), ETA(20), EB(20), EB2(20), EDS(20), EI(20), EIP(20)

**2** THO, BPC, YPP(20, 3), YP(20, 3), ZZP(20, 5), ZP(20, 5), PPP(20, 3), PP(20, 3)

**3** OME(OMF, ECI(20)), NY, N, N, N, N, N, CMEGS, CMFS, IDIM, NMAX, NLIN

**4** NB, HMX, HMY, HNC, HCY, HCE, HXK, HCY, HKZ, NX, NFDLQ

**5** HBI(3, 3), HUB(1, 1), BERR, YPP(20), YDI(20), Y(20, 3), Z(20, 5), P(20, 3)

**6** RY, Y, GW, Y, Y, C, PER

**COMMON** /COSF/CNL(I1, I1), DCD(11, 11), CDD(11, 11), CD(11, 11), CD(11, 11)

**1** C(11, 11), CDD(11, 11), F(11, 11), FNL(11, 11), CDR(11, 11),

**2** CDR(11, 11), CCR(11, 11), FR(11, 11), RGC(11, 12), BF(11, 12)

**3** BNI(N, 11), BDM(3, 11), BPR(3, 11), COH(11, 3), CD(11, 3), BII(3, 11)

**4** BIRH(3, 11), BIR(3, 11), BIRR(3, 11), HI(3, 3), HI(3, 3), HI(3, 3), HI(3, 3)

**5** H(3, 3), H(3, 3), H(3, 3), H(3, 3)

**COMMON** /HED/I1, IC1, IC2, IC3, ICG, HEAD(19), IPAGE, INPUT(20), INEND, INEL, IC5

**COMMON** /DIM/NINPUT, NSTA, NMODE, NMODE, NMODE, NMODE, N1, N1, 1, NBLADE

**COMMON** /DER/TH(20), EV(20), EV(20), EP(20), Y(20, 3), Z(20, 5), P(20, 3)

**NOTE**

**DO** NCT, **DO** NOT USE COMMON/YVAR/

**LOGICAL** LHub

**INTEGER** ICol(4), IRow(4)

**REAL** XH(13), XH(3), XH(3), XH(3), XH(3), FHI(11, 4)

**REAL** YB(11), YBD(11), HUB(3, 4), HUBC(3, 4), HUBV(3, 11), HUBD(3, 11),

**1** HUBF(3, 11), HUB(3, 11), HUBV(3, 11), COS(4), PSI(4), RHS(11, 3), FHB(11),

**2** HVIB(4), VDBD(11),

**LHub** = .FALSE.,

**IF** (LY(1), OR, LY(3), OR, LY(5)) **LHub** = .TRUE.,

**SOFT** = **SIN**(OMF*T),

**IF** (OMF < 0.0) **SOFT** = 1.0,

**IF** (.NOT. LHub) GC TO 45

**PSI** = **AMOD** (**TPMEG**, 6.28319)

**DPSI** = 6.28319/**FLOAT**(NB)

**SIN** = **SIN**(PSI(1))

**COS** = **COS**(PSI(1))

**DO** 10 I = 1, 2, **NB**

**PSI**(IB) = PSI**(IB - 1) + **DPSI**

**IF** (PSI**(IB) > 6.28319) **PSI**(IB) = PSI**(IB) - 6.28319

**SIN**(IB) = **SIN**(PSI**(IB))

**10** **COS**(IB) = **COS**(PSI**(IB))

**DO** 20 I = 1, 3

**DO** 20 J = 1, 3

**HUB**(I, J) = TM(I, J)

**20** **HUB**(I, J) = **HUB**(I, J)

**DO** 30 I = 1, **NB**

**HUB**(1, 1) = **HUB**(1, 1) - **SIN**(IB)****2** * **BIRH**(1, 1)

**HUB**(1, 2) = **HUB**(1, 2) - **SIN**(IB)****2** * **COS**(IB) * **BIRH**(1, 2)

**87**
HUB1(2,1) = HUB1(2,1) - SIN(B(1)) * COS(B(2)) * BIR1BH(2,1)  
HUB1(1,3) = HUB1(1,3) - SIN(B(3)) * BIR1BH(1,3)  
HUB1(3,1) = HUB1(3,1) - SIN(B(1)) * BIR1BH(3,1)  
HUB1(2,3) = HUB1(2,3) - COS(B(3)) * BIR1BH(2,3)  
HUB1(3,2) = HUB1(3,2) - COS(B(3)) * BIR1BH(3,2)  
HUB1(2,2) = HUB1(2,2) - COS(B(2)) * BIR1BH(2,2)  
HUB1(3,3) = HUB1(3,3) - BIR1BH(3,3)  
HUB1(1,1) = HUB1(1,1) + SIN(B(1)) * COS(B(1)) * BIR1BH(1,1)  
HUB1(1,2) = HUB1(1,2) + SIN(B(1)) * BIR1BH(1,2)  
HUB1(2,1) = HUB1(2,1) + COS(B(1)) * BIR1BH(2,1)  
HUB1(1,3) = HUB1(1,3) + SIN(B(3)) * BIR1BH(1,3)  
HUB1(3,1) = HUB1(3,1) + COS(B(1)) * BIR1BH(3,1)  
HUB1(2,3) = HUB1(2,3) + SIN(B(3)) * BIR1BH(2,3)  
HUB1(3,2) = HUB1(3,2) + COS(B(3)) * BIR1BH(3,2)  

30 HUBC(3,3) = HUBC(3,3) + BIR1BH(3,3)

XHD(1) = YVAR(1)  
XH(1) = YVARI(2)  
XHD(2) = YVAR(3)  
XH(2) = YVAR(4)  
XHD(3) = YVAR(5)  
XH(3) = YVAR(6)

DO 40 I = 1, 3

40 RHS(I) = HF(I) * SOFT

CALL MXV(RHS, HUBC, XHD, 3, 3, 1)

CALL MXV(RHS, HUBC, XH, 3, 3, 1)

50 YBJ = YVAR(I)

IF(NOT.LHUB) GO TO 62

DO 60 J = 1, NM

HUBB(1, J) = SIN(B(1)) * BIR1BH(1, J) + COS(B(1)) * BDM(1, J)

HUBB(2, J) = COS(B(2)) * BIR1BH(2, J) + SIN(B(1)) * BDM(2, J)

HUBB(3, J) = BIR1BH(3, J) + ECAM(3, J)

HUBB(1, J) = SIN(B(1)) * BIR1BH(1, J)

HUBB(2, J) = COS(B(2)) * BIR1BH(2, J)

HUBB(3, J) = BIR1BH(3, J)

HUBBF(1, J) = SIN(B(1)) * BIR1BH(1, J)

HUBBF(2, J) = COS(B(2)) * BIR1BH(2, J)

60 HUBBF(3, J) = BIR1BH(3, J)

62 DO 65 I = 1, NM

65 FNL(I) = 0

NCN LINEAR TERMS

IF(NLIN.EQ.2) GO TO 160

IF(NY.EQ.0) GO TO 160

CALL SUMODE(VD, YDB, YN, NSTA, NX, NY)

CALL SUMODE(VDP, YDB, YP, NSTA, NX, NY)

CALL SUMODE(VPP, YB, YPP, NSTA, NX, NY)

CALL SUMODE(VP, YB, YP, NSTA, NX, NY)

DO 70 I = 1, NX

WD(I) = 0

88
WP(I) = 0
WP(I) = 0
70 WP(I) = 0
IF(NZ.EQ.0) GO TO 85
DO 80 I=1,NZ
DUMP(I) = YB(NY+I)
80 DUMP(I) = YB(NY+I)
CALL SUMODE(WP,DUMP,ZP,NSTA,NX,NZ)
CALL SUMODE(WP,DUMP,ZP,NSTA,NX,NZ)
CALL SUMODE(WP,DUMP,ZP,NSTA,NX,NZ)
CALL SUMODE(WP,DUMP,ZP,NSTA,NX,NZ)
85 DO 90 I=1,NX
90 DUMP(I) = VD(I)*WP(I)
CALL INT(DUMP,DUMP1,0,X,NX,1)
DO 95 I=1,NX
95 DUMP(I) = M(I)*VD(I)
CALL INT(VD,M,DUMP1,0,X,NX,2)
DO 100 I=1,NX
100 DUMP(I) = M(I)*VD(I)
CALL INT(VD,M,DUMP1,0,X,NX,2)
CALL INT(DUMP,DUMP1,0,X,NX,2)
DO 120 J=1,NY
120 DUMP(J) = Y(J,I)*DUMP(I)
CALL INT(VD,Y,DUMP1,0,X,NX,2)
DO 130 I=1,NX
130 DUMP(I) = WPP(I)*VOM(I) - M(I)*VC(I)*WP(I)
CALL INT(DUMP,DUMP1,0,X,NX,2)
CALL INT(DUMP,DUMP1,0,X,NX,2)
DO 140 J=1,NZ
140 DUMP(J) = Z(I,J)*DUMP(I)
140 FNL(NY+J) = DCAT(DUMP,DUMP1,X,NX)*2.*OMEG
150 CONTINUE
160 DO 170 I=1,NX
170 FIB(I,IB) = FIB(I)
C  BLADE FORCING
   IF(INPUT(I),EQ.0) GO TO 190
   IF(NLF.NE.0 .AND .IB.NE.NAF) GO TO 190
   DO 180 I=1,NW
   180 FIB(I) = FIB(I) + BF(I)*SOFT
   GO TO 180
   175 CONST = PSI(IB)/PER
   IF(CONST.GE.0.28319) GC TO 180
   FIB(I) = FIB(I) + BF(I)*(1.0 - COS(CCAST))
   180 FIB(I,IB) = FIB(I)
   IF(NOT.LHUB) GC TO 200
   CALL MXV(RHS,HUBEV,YB,3,NM,3,1)
   CALL MXV(RHS,HUBEF,YB,3,NM,3,1)
   CALL MXV(RHS,HUBEF,FB,3,NM,3,1)
200 CONTINUE
     IF (.NOT. LHUB) GO TO 300
     CALL INVRS(HUBI,3,HINV,HUBC,IRCH,ICOL,3,4)
     CALL MXV(HHOD,HINV,RHS,3,3,3,0)
     C NOTE THAT ALL 3 HUB MOTIONS COMPUTED, THEY ARE IGNORED IF NOT
     IF (LY(1)) 00001640
     IDERY(1) = XHDD(1) 00001650
     IF (LY(2)) 00001660
     IDERY(2) = YVAR(1) 00001670
     IF (LY(3)) 00001680
     IDERY(3) = XHDD(2) 00001690
     IF (LY(4)) 00001700
     IDERY(4) = YVAR(2) 00001710
     IF (LY(5)) 00001720
     IDERY(5) = XHDD(3) 00001730
     IF (LY(6)) 00001740
     IDERY(6) = YVAR(5) 00001750
     C BLADES
     300 DO 360 IB=1,NB
          I = 10 + NM*(IB-1) #2
          DO 310 J = 1, NM
          I = I + 1
          YDB(J) = YVAR(I) 00001760
          I = I + 1
     310 YB(J) = YVAR(I)
          DO 320 I = 1, NM
     320 RHS(I) = FIB(I,1, IB)
          CALL MXV(RHS,CODR,YDB,NM,NM,NM,AMODE,1)
          CALL MXV(RHS,COR,YB,AM,NM,NM,AMODE,1)
          IF (.NOT. LHUB) GO TO 350
          DUMP(2) = COSB(I) * DERY(1)
          DUMP(3) = DERY(6)
          CALL MXV(RHS,CCIH,DUMP,AM,3,AMODE,1)
          DUMP(1) = COSB(I) * XHDD(1)
          DUMP(2) = SINB(I) * XHDD(2)
          DUMP(3) = XHDD(3)
          CALL MXV(RHS,CSCD,HUMP,AM,3,AMODE,1)
     350 CALL MXV(YDB,ROI,C,RHS,NM,NM,AMODE,0)
          I = 10 + NM*(IB-1) #2
          DO 360 J = 1, NM
          I = I + 1
          DERY(I) = YDB(J)
          I = I + 1
     360 DERY(I) = YVAR(I-1)
          RETURN
     END
SUBROUTINE HEAD1
COMMON/HED/IC1,IC2,IC3,IC4,HEAD(19),IPAGE,INPUT(20),IEND,LINe,IC5
IPAGE=IPAGE+1
PRINT 100,IC1,IC2,IC3,IC4,IC5,HEAD,IPAGE,(I,I=1,20),INPUT
100 FORMAT (1H1,9X,13HV22 11/12/76 /10X,15H--- --------- **,  
1 "19(5H ****)/ 8X,5I2,14X,19A4,3X,4HPAGE,15/
2 10X,10(5H***)2013/50X,10INPUT = ,2013) 
RETURN
END
9 (D(2192),DZALL(1)),(C(2197),DPAL(1))
EQUIVALENCE (D(2201),DYD(1)),(D(2210),DZD(1)),(D(2235),DPD(1))

C
INITIALIZATION

C
IHEADING

50 IF(IEND.EQ.2) GO TO 52
READ 9000,IC1,IC2,IC3,IC4,IC5,HEAD
9000 FORMAT (5F18.4,E3)
IF(IC1.NE.0) CALL EXIT
52 ICASE = ICASE+1
IPAGE = 0
C INPUT(I) = 0, NEVER USED = 1, USED = 2, MODIFIED OR NEW
DO 100 I=1,NINPUT
IF (INPUT(I).EQ.0) GO TO 100
IF (INPUT(I).EQ.6) GO TO 100
IF (INPUT(I).EQ.0) GO TO 100
INPUT(I) = 1
C CLEAR TO CLEAN UP OUTPUT OF INTEGRALS
DO 90 I=1,2243
90 D(I)=0.
IF (INPUT(I).EQ.0) OLDCP = 1.
IF (INPUT(I).EQ.6) OLDCM = OMEG
OLDCMS = GLDCM*OLDCM
IF (IC5.EQ.0) GO TO 201
I=7
WRITE (9)
GO TO 201
C GENERAL INPUT
C
C 200 IF (IEND.NE.0) GO TO 500
201 READ 9010,IO,DUM,IEND
9010 FORMAT (2F8.0,6F15.0,F9.0,I1)
IF(IO.NE.21) GO TO 202
CALL HEADIN
PRINT 9011
9011 FORMAT('//20X*28HFOLLOWING ID'S ARE CANCELLED //')
DO 203 J=1,8
I=DUM(J)
IF (I.EQ.0) GC TO 203
PRINT 9012,I
9012 FORMAT(3X,I10)
IF (I.LT.0.OR.I.GT.NINPUT) CALL ERR (203,0)
INPUT(I) = 0
C NOTE INPUT(1) SET TO 2 TO INSURE THAT ALL COEFS ARE RE CALCULATE
C
C C
C INPUT(I)=2
I2=1
GO TO 200
C
202 IF(IO.GT.NINPUT,CR.IO.GT.1) CALL ERR(200,0)
IF (INPUT(IO).EQ.2) CALL ERR (202,IO)
INPUT (IO) = 2
GO TO (210,220,230,230,230,230,270,320,330,340,10,11,12)
CALL ERR(10,0)
CALL ERR(11,0)
CALL ERR(12,0)
CALL ERR(14,0)
CALL ERR(15,0)
CALL ERR(16,0)
CALL ERR(19,0)
CALL ERR(20,0)

C       IO=1 BLOCK PROPERTIES

210 I = 1
215 X(I) = DUM(1)
M(I) = DUM(2)
E(I) = DUM(3)
SEA(I) = DUM(4)
KMI(I) = DUM(5)
KM2(I) = DUM(6)
KA(I) = DUM(7)
THPI(I) = DUM(8)
READ 9010, IO, DUM
EOPI(I) = DUM(1)
EIP(I) = DUM(2)
GJ(I) = DUM(3)
EA(I) = DUM(4)
EBI(I) = DUM(5)
EB2(I) = DUM(6)
EC(I) = DUM(7)
ECII(I) = DUM(8)
R=X(I)
 IF(IEND,NE.0) GO TO 500
READ 9010, IO, DUM, IEND
IF(IO,NE.0) GO TO 202
IF(DUM(I).LT.X(I)) CALL ERR(215,0)
I = I+1
NX = I
IF(NX.GT.NSTA) CALL ERR(216,0)
GO TO 215

C       IO=2 BLOCK CATA

220 NB = DUM(1)
TH0=DUM(2)
BPC=DUM(3)
GV = DUM(4)
GW = DUM(5)
GP = DUM(6)
GO TO 200

C

IO = 3,4,5 MODES
230 IF(INPUT(I),EQ.0) CALL ERR(230,0)
J = 0
235 J = J+1
DO 240 I=1,8
240 DUMPP(I) = DUM(I)
IF (NX.LE.8) GO TO 250
READ 9020, (DUMPP(I),I=1,NX)
9020 FORMAT (7F10.0,F9.0)
250 READ 9020, SC
C INTEGRATE AND NORMALIZE MODES
CALL INT(DUMP,DUMPP,SC,X,NX,1)
CALL INT(DUMPP,DUMP,0,X,NX,1)
CONST=DUMPP(NX)
IF(CONST.EQ.0) CCNST=1.0
DO 260 I=1,NX
IF (10.4) 252,254,256
252 YPP(I,J) = DUMPP(I)/CONST
YP(I,J) = DUMP(I)/CONST
Y(I,J) = DUMPP(I)/CONST
GO TO 260
254 ZPP(I,J) = DUMPP(I)/CONST
ZP(I,J) = DUMP(I)/CONST
Z(I,J) = DUMPP(I)/CONST
GO TO 260
256 PPI(I,J) = DUMPP(I)/CONST
PPI(I,J) = DUMP(I)/CONST
P(I,J) = DUMPP(I)/CONST
260 CONTINUE
IF (IEND.NE.0) GC TO 261
READ 9010 IENDT
IF (I1.EQ.0) GC TO 235
261 IF (I10.4) 262,264,266
262 NY = J
IF (NY.GT.NYMODE) CALL ERR(262,0)
GO TO 267
264 NZ = J
IF (NZ.GT.NZMODE) CALL ERR(264,0)
GO TO 267
266 NP = J
IF (NP.GT.NPMODE) CALL ERR(266,0)
267 IF (I10.4) GO TO 500
IEND = IENDT
IO = 11
GO TO 202
C IO = 6 FREQUENCIES
270 QMF = DUM(1)
QMFW = DUM(2)
GO TO 200
C IO = 17 NON LINEAR CONTROLS
280 NFLOQ = DUM(1)
GO TO 200
C IO = 18 SOLUTION CONTROLS
300 CYCLES = DUM(1)
HINIT = DUM(2)
ERROR = DUM(3)
IYE = DUM(4)
CIC = DUM(5)
IYIC = DUM(6)
BERR = DUM(7)
GO TO 200
C IO = 7 HUBX
320 HMX = DUM(1)
HCX = DUM(2)
HKX = DUM(3)
HE(1) = DUM(4)
GO TO 200

C 10 = 8 HUB Y
330 HMY = DUM(1)
HCY = DUM(2)
HE(2) = DUM(4)
GO TO 200

C 10 = 9 HUB Z
340 HMZ = DUM(1)
HCZ = DUM(2)
HE(2) = DUM(4)
GO TO 200

C 10 = 13 BLADE FORCE
350 NXF = DUM(1)
AFY = DUM(2)
AFZ = DUM(3)
AFP = DUM(4)
NBF = DUM(5)
PER = DUM(6)
GO TO 200

C 500 IF(Input(1).EQ.0) CALL ERR(500,0)
IF(Input(2).NE.0) GO TO 501
NB = 1
THO = 0
BPC = 0
GV = 0
GW = 0
GP = 0

501 IF(Input(3).EQ.0) NY = 0
IF(Input(4).EQ.0) NZ = 0
IF(Input(5).EQ.0) NP = 0
NM = NY + NZ + NP
IF(NM.EQ.0) CALL ERR(501,0)
NMAX = NM
IF(NP.GT.NMAX) NMAX = NP
IF(NY.GT.NMAX) NMAX = NY
IF(Input(6).EQ.0) CALL ERR(502,0)
IF(NB.EQ.1 .AND. (Input(7).NE.0 .OR. Input(8).NE.0 .OR. Input(9).NE.0)) CALL ERR(503,0)
 IF(NB.EQ.2 .AND. (Input(7).NE.0 .OR. Input(8).NE.0 .OR. Input(9).NE.0)) CALL ERR(504,0)
1
NB = 2
IF(NB.GT.NBLADE) CALL ERR(506,0)
IF(NB.GT.NBLADE) NB = NBLADE
IF(NB.LE.1) CALL ERR(507,1)
IF(NB.LE.1) NB = 1
IF(NB.EQ.1 .AND. (Input(7).NE.0 .OR. Input(8).NE.0 .OR. Input(9).NE.0)) CALL ERR(508,0)
1
NB = 2

96
IF (INPUT(7) .NE. 0) GO TO 502
HMX = 0
HCX = 0
H#X = 0
HE(II) = 0
HM = 0
HCY = 0
HKY = 0
HE(2) = 0
502 IF (INPUT(8) .NE. 0) GO TO 503
HM = 0
HCZ = 0
HKZ = 0
HE(3) = 0
503 IF (INPUT(9) .NE. 0) GO TO 504
HM = 0
HCZ = 0
HKZ = 0
HE(3) = 0
504 OMRAT = OMG/OMRT
OMRAT = OMRAT .OR. OMRAT
IF (INPUT(11) .NE. 0 .AND. (NXF .. GT .. NBRF .OR. NBRF .LE. 0)) CALL ERR(510,0)
IF (INPUT(11) .NE. 0 .AND. (AFY .. EQ .. 0 .AND. AFZ .. EQ .. 0 .AND. AFY .. EQ .. 0))
1 CALL ERR(510,0)
IF (INPUT(11) .NE. 0 .AND. NBF .GT. NBF) CALL ERR(512, NBF)
IF (INPUT(11) .NE. 0 .AND. NBF .GT. NBF) NBF = 0
IF (INPUT(11) .NE. 0 .AND. NBF .LT. 0) CALL ERR(512, NBF)
IF (INPUT(11) .NE. 0 .AND. NBF .LT. 0) NBF = 0
IF (INPUT(17) .EQ. 0) NLIN = 0
IF (INPUT(17) .EQ. 0) NFCC = 0
IF (INPUT(18) .EQ. 0) CALL ERR(509, 0)
C ADD BLACE LOADS TO HUE
DO 508 I = 1, 3
508 HF(II) = HE(II)
IF (INPUT(13) .EQ. 0) GO TO 509
IF (AFZ .. EQ .. 0) GO TO 506
IF (INPUT(9) .EQ. 0) GO TO 506
CONST = AFZ
IF (NBF .. EQ .. 0) CCNST = NB .. CONST
HF(III) = HF(III) + CONST
C COMPUTE COEFFICIENTS, ETC.
509 CALL INTO(TH, THP, TH0, X, NX, 1)
DO 510 I = 1, NX
DUMMY1 = SEAI I * 2 * EA(I)
DUMMY2 = EIPI(I) - EOPI(I)
EV(I) = EIP(I) - DUMMY2 * TH(I) ** 2 - DUMMY1
DELE(I) = DUMMY2 - DUMMY1
EOE(I) = SEA(I) * EA(I) * KA(I) ** 2 - EB2(I)
EWI(I) = EOP(I) + DUMMY2 * TH(I) ** 2 - DUMMY1 * TH(I)
EPI(I) = GJ(I) - (KA(I) ** 4 * EA(I) * EB1(I)) * THP(I) ** 2
DEK(I) = KM2(I) ** 2 - KM1(I) ** 2
510 KM1(I) = KM2(I) ** 2 + KM1(I) ** 2
C FORM MASS INTEGRALS

97
C RECOMPUTE ALL COEFS UNLESS ONLY IC = 6 OR GE 17 ARE CHANGED
L CALC=.TRUE.
600 DO 601 I=1,16
IF (I.EQ.6) GO TO 601
IF (INPUT(I).EQ.2) GO TO 1075
601 CONTINUE
L CALC=.FALSE.
IF (INPUT(6).EQ.2) GO TO 1075
GO TO 1100
C FORM INTEGRANDS
602 DO 610 I=1,NX
MI(I,1) = M(I)
MI(I,2) = M(I)*X(I)
MI(I,3) = M(I)*E(I)
MI(I,4) = MI(I,3)*X(I)
MI(I,5) = MI(I,3)*TH(I)
MI(I,6) = MI(I,3)*X(I)
MI(I,7) = MI(I,3)*TH(I)
MI(I,8) = MI(I,3)*TH(I)
610 MI(I,9) = M(I)*DEL(K(I))*TH(I)
DO 620 J=1,NX
DO 630 I=1,NX
CALL INT (DUMP, DUMPP, 0, X, NX, 2)
DO 630 I=1,NX
MI(I,J) = DUMP(I)
DO 635 J=1,NX
DO 640 I=1,NX
MI(I,10) = DUMPP(I)
DO 635 I=1,NX
640 MI(I,10) = DUMPP(I)
C FORM INTEGRALS
650 IF (INPUT(3).EQ.0) GO TO 700
C FORM INTEGRANTS
660 DO 670 I=1,16
DO 670 IM=1,9
Y(I,IM,1) = M(I)*Y(I,IM)
Y(I,IM,2) = Y(I,IM,1)*E(I)
Y(I,IM,3) = Y(I,IM,2)*TH(I)
Y(I,IM,4) = Y(I,IM,3)*Y(I,IM)
Y(I,IM,5) = MI(I,3)*Y(I,IM)
Y(I,IM,6) = Y(I,IM,5)*X(I)*TH(I)
Y(I,IM,7) = MI(I,3)*YP(I,IM)
Y(I,IM,8) = Y(I,IM,7)*SEA(I)*TH(I)
Y(I,IM,9) = YPP(I,IM)*EONE(I)*THP(I)
660 Y(I,IM,10) = YPP(I,IM)*SEA(I)
DO 670 J=1,9
DO 670 IM=1,9
DO 665 I=1,9
665 DUMPP(I) = Y(I,IM,J)
CALL INT (DUMP, DUMPP, 0, X, NX, 2)
CALL INT (DUMPP, DUMPP, 0, X, NX, 2)
DO 670 I = 1,NX
YN(I,IM,J) = DUMP(I)
670 YN(I,IM,J) = DUMPP(I)
DO 680 IM = 1,NX
DO 675 I = 1,NX
675 DUMPP(I) = YN(I,IM,10)
CALL INT (DUMP,DUMPP,0,X,NX,1)
DO 680 I = 1,NX
680 YN(I,IM,10) = DUMP(I)
DO 682 I = 1,NX
DO 682 IM = 1,NX
IF(EA(I).EQ.0) SI(I,IM,1) = 0
IF(EA(I).NE.0) SI(I,IM,1) = YN(I,IM,1)/EA(I)
SI(I,IM,2) = M(I)*YN(I,IM,10)
682 SI(I,IM,5) = KA(I)*S2*THP(I)*YN(I,IM,1)
DO 685 IM = 1,NX
DO 683 DUMPP(I) = SI(I,IM,1)
CALL INT(DUMP,DUMPP,0,X,NX,1)
DO 684 I = 1,NX
DO 684 IM = 1,NX
684 DUMPP(I) = DUMP(I)*M(I)
CALL INT(DUMPP,DUMPP,0,X,NX,2)
CALL INT(DUMPP,DUMPP,0,X,NX,2)
DO 685 I = 1,NX
685 SI(I,IM,1) = DUMP(I)
DO 690 IM = 1,NX
DO 686 I = 1,NX
686 DUMPP(I) = SI(I,IM,2)
CALL INT(DUMPP,DUMPP,0,X,NX,2)
CALL INT(DUMPP,DUMPP,0,X,NX,2)
DO 690 I = 1,NX
690 SI(I,IM,2) = DUMPP(I)
DO 695 IM = 1,NX
DO 692 I = 1,NX
692 DUMPP(I) = SI(I,IM,5)
CALL INT(DUMPP,DUMPP,0,X,NX,2)
DO 695 I = 1,NX
695 SI(I,IM,5) = DUMPP(I)
C FORM Z INTEGRALS
700 IF( INPUT(4).EQ.0 ) GO TO 750
DO 710 I = 1,NX
DO 710 JM = 1,NZ
ZI(I,JM,1) = M(I)*ZI(I,JM)
ZI(I,JM,2) = ZI(I,JM,1)*E(I)
ZI(I,JM,3) = M(I)*X(I)*ZP(I,JM)
ZI(I,JM,4) = ZI(I,JM,3)*E(I)
ZI(I,JM,5) = M(I)*IP(I,JM)*E(I)*TH(I)
ZI(I,JM,6) = M(I)*ZP(I,JM)
ZI(I,JM,7) = ZI(I,JM,6)*SEA(I)
ZI(I,JM,8) = ZPP(I,JM)*ECNE(I)*TH(I)*TH(I)
710 ZI(I,JM,9) = ZPP(I,JM)*SEA(I)*TH(I)
DO 720 J = 1,8
DO 720 JM = 1,NZ
DO 715 I = 1,NX
715 DUMPP(I) = ZI(I,JM)
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004380
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004390
DO 720 I = 1,NX  000004400
Z[I,J,M,J] = DUMPP(I)  000004410
720 Z[I,J,M,J] = DUMPP(I)  000004420
DO 730 JM = 1,NZ  000004430
DO 725 I = 1,NX  000004440
725 DUMPP(I) = Z[I,J,M,J]  000004450
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004460
DO 730 I = 1,NX  000004470
730 Z[I,J,M,J] = DUMPP(I)  000004480
DO 740 JM = 1,NZ  000004490
DO 735 I = 1,NX  000004500
735 DUMPP(I) = M(I)*Z[I,J,M,J]  000004510
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004520
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004530
DO 740 I = 1,NX  000004540
740 S[I,J,M,J] = DUMPP(I)  000004550
C FORM P INTEGRALS
750 IF(INPUT(5)=EQ.0) GO TO 800  000004560
DO 760 I=1,NX  000004570
DO 760 IM = 1,NP  000004580
PI(I,IM,1) = M(I)*E(I)*F(I,IM)  000004590
PI(I,IM,2) = PI(I,IM,1)*F(I)  000004600
PI(I,IM,3) = PI(I,IM,1)*TH(I)  000004610
PI(I,IM,4) = M(I)*TH(I)*P(I,IM)  000004620
PI(I,IM,5) = M(I)*DELK(I)*P(I,IM)  000004630
PI(I,IM,6) = E(I)*PP(I,IM)  000004640
PI(I,IM,7) = KA(I)**2*PP(I,1)*PP(I,IM)  000004650
760 PI(I,IM,8) = KA(I)**2*THP(I)*PP(I,IM)  000004660
DO 770 J = 1,T  000004670
DO 770 IM = 1,NP  000004680
DO 765 I = 1,NX  000004690
765 DUMPP(I) = PI(I,IM,J)  000004700
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004710
IF (J/GT.5 ) GO TO 766  000004720
CALL INT ( DUMP,DUMPP,0,X,NX,2)  000004730
766 DO 770 I = 1,NX  000004740
PI(I,IM,J) = DUMPP(I)  000004750
IF (J/GT.5 ) GO TO 770  000004760
PI(I,IM,J) = DUMPP(I)  000004770
770 CONTINUE  000004780
DO 780 IM = 1,NP  000004790
DO 775 I = 1,NX  000004800
775 DUMPP(I) = PI(I,IM,8)  000004810
CALL INT (DUMP,DUMPP,0,X,NX,1)  000004820
DO 780 I = 1,NX  000004830
780 PI(I,IM,8) = DUMPP(I)  000004840
DO 790 IM = 1,NP  000004850
DO 785 I = 1,NX  000004860
785 DUMPP(I) = M(I)*PI(I,IM,8)  000004870
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004880
CALL INT (DUMP,DUMPP,0,X,NX,2)  000004890
DO 790 I = 1,NX  000004900
790 SI(I,IM,4) = DUMPP(I)  000004910
100
DO 882 K=1,NX
882 DUMPP(K)=AFP*P(K,I)*ALII(K)
DPLII(I)=DINT(DUMP,DUMPP,X,NX)
884 IF(NY.EQ.0) GO TO 886
DO 885 J=1,NY
DPSII(I,J,1)=DINT2(P,SI,I,J,5,5,X,NSTA,NX,DUMP,DUMPP)
DO 885 K=1,N
DPSII(I,J,2)=DINT2(P,II,I,J,K,NMDED,X,NSTA,NX,DUMP,DUMPP)
885 DPPII(I,J,K)=DINT2(P,YII,I,J,K,NMDED,X,NSTA,NX,DUMP,DUMPP)
886 IF(NZ.EQ.0) GO TO 891
DO 892 J=1,NZ
DO 890 K=1,N
DPIII(I,J,K)=DINT2(P,II,I,J,K,NMDED,X,NSTA,NX,DUMP,DUMPP)
890 IF (K(IP,5) .GT. 5) GO TO 895
DPIII(I,J,K)=DINT2(P,II,I,J,K,NMDED,X,NSTA,NX,DUMP,DUMPP)
895 CONTINUE
896 DO 897 K=1,N
DPMI(I,K)=DINT1(P,MI,I,K,NSTA,NX,DUMP,DUMPP)
897 DPMII(I,K)=DINT1(P,MI,I,K,NSTA,NX,DUMP,DUMPP)
900 DPMI(I,10)=DINT1(P,MI,I,10,X,NSTA,NX,DUMP,DUMPP)
901 IF(NZ.EQ.0) GO TO 931
DO 910 J=1,NY
DO 910 K=1,NK
DO 902 I=1,NX
902 DUMPP(I)=Y(I,J)*[R-X(I)]*M(I,NX)*E(NX)*Y(NX,K)
DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
DO 904 I=1,NX
904 DUMPP(I)=Y(I,J)*EPA(I)*Y(I,K,1)
DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
DO 906 I=1,NX
906 DUMPP(I)=Y(I,J)*EIV(J)YPJ(I,K)
910 DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
IF(NZ.EQ.0) GO TO 916
DO 915 K=1,NZ
DO 912 I=1,NX
912 DUMPP(I)=Y(I,J)*DEDE(I)*TH(I)*ZPPI(I,K)
915 DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
916 IF(NZ.EQ.0) GO TO 925
DO 920 K=1,NP
DO 917 I=1,NX
917 DUMPP(I)=Y(I,J)*[-ECS(I)]*TH(I)*PPP(I,K)+EDEN(I)THI*PPP(I,K)
920 DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
925 DO 927 I=1,NX
927 DUMPP(I)=Y(I,J)*[SEADE[I,H[I,2]+R-[R-X(I)]*M(NX)*E(NX)]
930 DYPJ(J,K)=DINT(DUMP,DUMPP,X,NX)
931 IF(NZ.EQ.0) GO TO 961
DO 960 J=1,NZ
DO 960 K=1,NZ
DO 935 K=1,NZ
932 DUMPP(I) = Z(I,J)*(R-X(I))*M(NX)*E(NX)*H(I)*TH(NX)*Y(NX,K)

934 DUMPP(I) = Z(I,J)*DELE(I)*TH(I)*YPP(I,K)

936 DO 938 K=1,NX

937 DUMPP(I) = Z(I,J)*EWH(I)*YPP(I,K)

938 DZF(J,K,3) = DINT(DUMP,DUMPP,X,NX)

IF (NP.EQ.0) GO TO 946

939 DUMPP(I) = Z(I,J)*DELE(I)*TH(I)*YPP(I,K)

940 DUMPP(I) = Z(I,J)*EWH(I)*YPP(I,K)

942 DUMPP(I) = Z(I,J)*EWH(I)*YPP(I,K)

945 DZF(J,K,6) = DINT(DUMP,DUMPP,X,NX)

946 DO 950 J=1,NX

950 DUMPP(I) = Z(I,J)*EWH(I)*YPP(I,K)

951 IF (NP.EQ.0) GO TO 991

960 DZF(J,K,5) = DINT(DUMP,DUMPP,X,NX)

961 IF (NP.EQ.0) GO TO 991

964 DUMPP(I) = P(I,J)*EWH(I)*YPP(I,K)

965 IF (NZ.EQ.0) GO TO 971

970 DPF(J,K,2) = DINT(DUMP,DUMPP,X,NX)

971 IF (NZ.EQ.0) GO TO 990

980 DPF(J,K,3) = DINT(DUMP,DUMPP,X,NX)

990 CONTINUE

C DUMPING DEFINITE INTEGRALS

DO 999 IF(NY.EQ.0,OR.GW.EQ.0) GO TO 999

992 YZPI(K) = Y(K,J)

DO 999 IF(NY.EQ.0,OR.GW.EQ.0) GO TO 999

DO 998 J=1,NY

DO 997 J=1,NX

DO 996 K=1,NX

103
996 YZPI(K)=Z(K,J)
    CALL INT(DUMP, YZPI, 0, X, NX, 2)
    CALL INT(YZPI, DUMP, 0, X, NX, 2)
    DO 998 I=1,NZ
    DO 997 K=1,NX
997 DUMP(K)=YZPI(K)*Z(K,I)
    CALL INT(DUMP, DUMP, X, NX, 2, GP)
    IF(NP.EQ.0) GO TO 1010
    DO 1002 J=1,NP
    DO 1000 K=1,NX
1000 YZPI(K)=P(K,J)
    CALL INT(DUMP, YZPI, 0, X, NX, 2)
    CALL INT(YZPI, DUMP, 0, X, NX, 2)
    DO 1002 I=1,NP
    DO 1000 K=1,NX
1001 DUMP(K)=YZPI(K)*P(K,I)
    CALL INT(DUMP, DUMP, X, NX, 2, GP)
    IF(NP.EQ.0) GO TO 1010
1010 II=0
    IF(NY.EQ.0) GO TO 1031
    DO 1030 I=1,NY
    JJ=0
    II=II+1
    DO 1015 J=1,NY
    JJ=JJ+1
    COI(II,JJ)=DYII(I,J,I)
    DCOII(I,JJ)=4*DYII(I,J,1)
    COO(I,JJ)=-DYYI(I,J,5)-DYYII(I,J,7)
7  COO(I,JJ)=-DYYI(I,J,4)
1  COO(I,JJ)=DYYII(I,J,1)
    COII(JJ)=-DYYII(I,J,3)
1015 COII(JJ)=DYII(I,J,7)-DYII(I,J,4)+DYII(I,J,1)
1016 IF(NZ.EQ.0) GO TO 1021
    DO 1020 J=1,NZ
    JJ=JJ+1
    CCII(I,JJ)=0
    DCII(I,JJ)=0
    COII(I,JJ)=0
    DCII(I,JJ)=-2*(DYII(I,J,3)-DYII(I,J,5)-BPC*DYYII(I,J,1))
    COII(I,JJ)=-DYYII(I,J,4)
1020 COII(JJ)=0
1021 IF(NP.EQ.0) GO TO 1026
    DO 1025 J=1,NP
    JJ=JJ+1
    CCII(I,JJ)=-DYYII(I,J,3)
    DCII(I,JJ)=0
    COII(I,JJ)=0
    DCII(I,JJ)=2*DYII(I,J,4)
    COII(I,JJ)=-DYYII(I,J,5)
1025 COII(JJ)=0
1026 IF(NP.EQ.0) GO TO 1026
    BFII=0
    IF(INPUT(131).NE.0.AND.AFY.TE.0) BFII=1
1030 CONTINUE
1031 IF(NZ.EQ.0) GO TO 1051

104
DO 1050 I = 1, NZ
    JJ = 0
    II = II+1
    IF(NY.EQ.0) GO TO 1036
    DO 1035 J = 1, NY
    JJ = JJ+1
    GO1035
    COII(I,JJ) = 0
    DOII(I,JJ) = 0
    DDII(I,JJ) = 0
    DDII(I,JJ) = -2*(DZYI(I,J,3)-DZF(I,J,1)+BPC*DZYII(I,J,1))
    CO(I,JJ) = -DZF(I,J,2)
    GO1039
1035  DOCl(I,JJ) = 0
1036  DO 1040 J = 1, NZ
    JJ = JJ+1
    COII(I,JJ) = DZII(I,J,J)
    DOII(I,JJ) = 0
    DDII(I,JJ) = 0
    DDII(I,JJ) = -2*OPSI(I*J,II)+DPF(I*J,II)
    GO1104
1040  DOCl(I,JJ) = -DZF(I,J,3)
1041  IF(NP.EQ.0) GO TO 1046
    DO 1045 J = 1, NP
    JJ = JJ+1
    COII(I,JJ) = DZPI(I,J,1)
    DOII(I,JJ) = 0
    DDII(I,JJ) = 0
    DDII(I,JJ) = -DZF(I,J,4)
    GO1105
1045  DOCl(I,JJ) = -DZPI(I,J,2)+DZF(I,J,6)
1046  DFII(I) = -(DZMI(I,6)-DZF(I,1,5)+BPC*DZMII(I,1,2))
    BFII(I) = 0
    IF(INPUT(I3).NE.0.AND.AFZ.NE.0) BFII(I)=DZALIII(I)
    CONTINUE
1050  IF(NP.EQ.0) GO TO 1075
    DO 1070 I = 1, NP
    JJ = 0
    II = II+1
    IF(NY.EQ.0) GO TO 1056
    DO 1055 J = 1, NY
    JJ = JJ+1
    COII(I,JJ) = -DPYII(I,J,3)
    DOII(I,JJ) = 0
    DDII(I,JJ) = 0
    DDII(I,JJ) = -2*DPSS(I,J,1)
    CO(I,JJ) = -DPYI(I,J,9)+DPF(I,J,1)
    GO1105
1055  COII(I,JJ) = -DPYII(I,J,8)-DPYII(I,J,6)+DPYII(I,J,3)
1056  IF(NL.EQ.0) GO TO 1061
    DO 1060 J = 1, NZ
    JJ = JJ+1
    COII(I,JJ) = DPZII(I,J,2)
    DOII(I,JJ) = 0
    DDII(I,JJ) = 0
    DDII(I,JJ) = 0
    COII(I,JJ) = -(DPZII(I,J,8)+DPF(I,J,2))
1060  COII(I,JJ) = DPZII(I,J,7)-DPZII(I,J,4)
1061 DO 1065 J = 1, NP
   JJ = JJ + 1
   COI(I, JJ) = DPPI(I, J, 4)
   DCOI(I, JJ) = 0
   COD(I, JJ) = -DPD(I, J)
   DCD(I, JJ) = 0
   COI(I, J) = -DF(I, J, 3) - DPPI(I, J, 6)
1065 DCOI(I, J) = -(DPPI(I, J, 5) + DPP(I, J, 7))
   BF(I) = 0
   IF(1 .LT. 13) .NE. 0 .AND. AFP .NE. 0) BF(I) = DPAI(I)
   DF(I) = -(DPM(I, J, 9) + PC + DPM(I, I, 4))
1070 CONTINUE
C     SUM WITH OMEGAS
1075 OMEGS = OMEG * OMEG
   QMFS = CMF * QMF
   DO 1080 I = 1, NM
   DO 1081 J = 1, NM
   COIR(I, J) = COI(I, J) + OMEGS * OMEG(I, J)
   DCOIR(I, J) = COD(I, J) + OMEGS * DCD(I, J)
1081 COR(I, J) = COI(I, J) + OMEGS * DCOI(I, J)
   C     NOTE F IS EVALUATED IF FCT
1080 FR(I) = OMEGS * OF(I)
   CALL INVRS(COIR, NM, RICC, WORK, IRC, ICOL, NMODE, NMI)
   C     INVERT COIR
   CALL INVRS(COIR, NM, RICC, WORK, IRC, ICOL, NMODE, NMI)
   C     HUB EFFECTS WITH OLD OMEG TO BE RATIOED LATER
   IF(1 .LT. 13) .LT. 0 .AND. INPUT(9) .EQ. 0 .AND. INPUT(9) .EQ. 0) GO TO 1100
   IF(.NOT. LCALC) GO TO 1090
   JJ = 0
   IF(NY .EQ. 0) GO TO 1087
   DO 1081 J = 1, NY
   JJ = JJ + 1
   CONST = Y(1, J, 1)
   BIN1(J, JJ) = CONST
   BIN2(J, JJ) = -CONST
   BIN3(J, JJ) = 0
   BDAM(1, JJ) = CONST * 2 * CLDOM
   BDAM(2, JJ) = CONST * 2 * CLDOM
   BDAM(3, JJ) = 0
   BSPR(1, JJ) = -COAST * OLCCMS
   BSPR(2, JJ) = COAST * OLCCMS
   BSPR(3, JJ) = 0
   COIH(JJ, 1) = DYM(I, J, 1)
   COIH(JJ, 2) = -DYM(I, J, 1)
   COIH(JJ, 3) = 0
   DO 1081 I = 1, 3
1081 COIH(JJ, I) = 0
1087 IF(NZ .EQ. 0) GO TO 1083
   DO 1082 J = 1, NZ
   JJ = JJ + 1
   BIN1(J, JJ) = 0
   BIN2(J, JJ) = 0
   BIN3(J, JJ) = -ZI(1, J, 1)
   BDAM(1, JJ) = 0
   BDAM(2, JJ) = 0
   BDAM(3, JJ) = 0
BSPR(1, JJ) = 0
BSPR(2, JJ) = 0
BSPR(3, JJ) = 0
COIH(JJ1) = 0
COIH(JJ2) = 0
COIH(JJ3) = -CZII(JJ)
DO 1082 I=1,3
1082 CDH(JJ, J) = 0
1083 IF (NP.EQ.0) GO TO 1085
DO 1084 J=1,NP
JJ = JJ + 1
CONST = PI(1, J, JJ)
BIN(1, JJ) = -CONST
BIN(2, JJ) = CONST
BIN(3, JJ) = -PI(1, J, JJ)
BDAM(1, JJ) = -CONST*2*CLDOM
BDAM(2, JJ) = -CONST*2*CLDOM
BDAM(3, JJ) = 0
BSPR(1, JJ) = 0
BSPR(2, JJ) = 0
BSPR(3, JJ) = 0
CONST = OMEGA(I1(J, JJ))
COIH(JJ1) = -CONST
COIH(JJ2) = 0
COIH(JJ3) = -CONST
DO 1084 J=1,3
1084 CDH(JJ, J) = 0
1085 DO 1086 I=1,3
1086 TM(1, J) = 0
TM(1, 1) = HMX + NB*MII(1, 1)
TM(2, 2) = HMY + NB*MII(1, 1)
TM(3, 3) = HMZ + NB*MII(1, 1)
HC(1, J) = -HCX
HC(2, J) = -HCY
HC(3, J) = -HCZ
HK(1, J) = 0
HK(2, J) = -HKX
HK(3, J) = -HKX
C INCLUDE OMEGA IN HUB EFFECTS USES RATIOS
C
C 1050 DO 1091 I=1,3
1091 BDAM(I, J) = BDAM(I, J)*CMRAT
BSPR(I, J) = BSPR(I, J)*CMRATS
C 1051 CDH(J, J) = CDH(J, J)*CMRAT
C
CALL MXM(BIRIH,BIRI,TCCOH,3,NM,3,3,NMODE)  00008780
CALL MXM(BIRIH,BIRI,TCCIH,3,NM,3,3,NMODE)  00008790

C  SOLUTION CONTROLS
1100  PRMT(1)=0  00008800
OM=OMF  00008810
IF(OM.EQ.0)OM=OMEG  00008820
PRMT(2)=6.28319*CYCLES/OM  00008830
PRMT(3)=6.28319/HINIT/OM  00008840
PRMT(4)=ERROR  00008850
IF(ERROR.LE.0)CALL ERR(1100,0)  00008860
PRMT(6)=BRR  00008870
DO 1105 I=1,NDIM  00008880
YVAR(I)=0  00008890
LY(I)=.FALSE.  00008900
IF((IYIC.LE.0)CALL ERR(1105,0)  00008910
IF((IYIC.GT.NDIM)CALL ERR (1106,0)  00008920
YVAR(IYIC)=CIC  00008930
IF((IYIC.GT.NDIM)CALL ERR (1107,0)  00008940
DERY(IY)=1.0  00008950
IF(IINPUT(7).NE.0)LY(1)=.TRUE.  00008960
IF(IINPUT(7).NE.0)LY(2)=.TRUE.  00008970
IF(IINPUT(8).NE.0)LY(3)=.TRUE.  00008980
IF(IINPUT(8).NE.0)LY(4)=.TRUE.  00008990
IF(IINPUT(9).NE.0)LY(5)=.TRUE.  00009000
IF(IINPUT(9).NE.0)LY(6)=.TRUE.  00009010
1200 IDIM = 10+2*NDIM  00009020
DO 1205 I=1,IDIM  00009030
IF(IYIC.GT.NDIM)CALL ERR (1200,0)  00009040
IF(YVAR(IYIC).EQ.FALSE)CALL ERR (1205,0)  00009050
1205 LY(I)=.TRUE.  00009060
1200 I=1,NDIM  00009070

C  OUTPUT
C
C  OUTPUT
C
C  OUTPUT
C  2000 CALL HEADIN
C  IF (IINPUT(1).NE.2.AND.IINPUT(2).NE.2.AND.IC2.EQ.0) GO TO 2050  00009100
C
C
C
C  2010 PRINT 9070,1X(I),M(I),E(I),SEA(I),KMI(I),KM2(I),KA(I),THP(I)  00009230
C
C  2020 PRINT 9070,1X(I),E(I),IP,ECP(I),ECP(I),ECP(I),ECP(I)  00009320
C

108
CALL HEADIN
PRINT 9090
9090 FORMAT (//X,29H(C)EW (C)EV (C)EP //)
DO 2030 I = 1,NX
2030 PRINT 9070,I,EW(I),EV(I),EP(I)
C
DO 2050 I = 3,4,5
2050 IF (INPUT(3) .NE. 2 .AND. IC2 .EQ. 0 OR. INPUT(3) .EQ. 0) GO TO 2075
CALL HEADIN
PRINT 9100
9100 FORMAT (//20X,23H I0 = 3 IN-PLANE MODES // 20X,18SECOND DERIVATIVES //)
1ATIVES //
DO 2055 I = 1,NX
2055 PRINT 9070,I,(YP(I,J),J=1,NY)
PRINT 9110
9110 FORMAT (//20X,28H(C) FIRST DERIV (NORMALIZED) //)
DO 2060 I = 1,NX
2060 PRINT 9070,I,(YP(I,J),J=1,NY)
CALL HEADIN
PRINT 9120
9120 FORMAT (//20X,15H(C) MCDE SHAPES//)
DO 2065 I = 1,NX
2065 PRINT 9070,I,(Y(I,J),J=1,NY)
2075 IF (INPUT(4) .NE. 2 .AND. IC2 .EQ. 0 OR. INPUT(4) .EQ. 0) GO TO 2100
CALL HEADIN
PRINT 9130
9130 FORMAT (//20X,27H I0 = 4 OJT-OF-PLANE MODES // 20X,18SECOND DERIVATIVES //)
1IVES //
DO 2080 I = 1,NX
2080 PRINT 9070,I,(ZPP(I,J),J=1,NZ)
PRINT 9110
9110 FORMAT (//20X,20H(C) ZPPALS //)
DO 2085 I = 1,NX
2085 PRINT 9070,I,(ZP(I,J),J=1,NZ)
CALL HEADIN
PRINT 9120
9120 FORMAT (//20X,15H(C) MCDE SHAPES//)
DO 2090 I = 1,NX
2090 PRINT 9070,I,(Z(I,J),J=1,NZ)
2100 IF (INPUT(5) .NE. 2 .AND. IC2 .EQ. 0 OR. INPUT(5) .EQ. 0) GO TO 2150
CALL HEADIN
PRINT 9140
9140 FORMAT ( //20X,22H I0 = 4 TCRSICN MODES // 20X,18SECOND DERIVATIVES //)
1ES //
DO 2105 I = 1,NX
2105 PRINT 9070,I,(PPP(I,J),J=1,NP)
PRINT 9110
9110 FORMAT (//20X,20H(C) PPALS //)
DO 2110 I = 1,NX
2110 PRINT 9070,I,(PP(I,J),J=1,NP)
CALL HEADIN
PRINT 9120
9120 FORMAT (//20X,15H(C) MCDE SHAPES//)
DO 2115 I = 1,NX
2115 PRINT 9070,I,(P(I,J),J=1,NP)
2150 IF (INPUT(6) .EQ. 0) GO TO 2500
C DEFINITE INTEGRALS
IF (INPUT(3) .EQ. 0 OR. (INPUT(3) .EQ. 1 .AND. IC2 .EQ. 0)) GO TO 2200
CALL HEADIN
PRINT 9150

109
9150 FORMAT (/20X,20H*** DYYI (I,J,N) *** //) 00009880
PRINT 9160,(I,I=1,10) 00009890
9160 FORMAT (1X,3i1H J17,9(12X,1I)) 00009900
DO 2160 I = 1,NY 00009910
DO 2160 J = 1,NY 00009920
2160 PRINT 9170,i,j,(DYYI(I,J,N),N=1,10) 00009930
9170 FORMAT (1X,12,1P10E12.3) 00009940
PRINT 9180 00009950
9180 FORMAT (/20X,21H*** DYYII (I,J,N) *** //) 00009960
PRINT 9160,(I,I=1,9) 00009970
DO 2170 I = 1,NY 00009980
DO 2170 J = 1,NY 00009990
2170 PRINT 9170,i,j,(DYYII(I,J,N),N=1,9) 00010000
IF(INPUT(4).EQ.0) GO TO 2176 00010010
PRINT 9190 00010020
9190 FORMAT (/20X21H*** DYZII (I,J,N) *** //) 00010030
PRINT 9160,(I,I=1,8) 00010040
DO 2175 I = 1,NY 00010050
DO 2175 J = 1,NZ 00010060
2175 PRINT 9170,i,j,(DYZII(I,J,N),N=1,8) 00010070
CALL HEADIN 00010080
2176 IF(INPUT(5).EQ.0) GO TO 2182 00010090
PRINT 9200 00010100
9200 FORMAT (/20X,21H*** DYPII (I,J,N) *** //) 00010110
PRINT 9160,(I,I=1,3) 00010120
DO 2180 I = 1,NY 00010130
DO 2180 J = 1,NP 00010140
2180 PRINT 9210 00010150
9210 FORMAT (/20X,20H*** DYSII (I,J,N) *** //) 00010160
PRINT 9160,(I,I=1,4) 00010170
DO 2185 I = 1,NY 00010180
DO 2185 J = 1,NMAX 00010190
2185 PRINT 9170,i,j,(DYSII(I,J,N),N=1,4) 00010200
PRINT 9220 00010210
9220 FORMAT (/20X,18H*** DYMII (I,N) *** //) 00010220
PRINT 9160,(I,I=1,10) 00010230
DO 2190 I = 1,NY 00010240
2190 PRINT 9070,i,(DYMII(1,N),N=1,10) 00010250
PRINT 9230 00010260
9230 FORMAT (/20X,19H*** DYMII (I,N) *** //) 00010270
PRINT 9160,(I,I=1,9) 00010280
DO 2195 I = 1,NY 00010290
2195 PRINT 9070,i,(DYMII(1,N),N=1,9) 00010300
2200 IF(INPUT(4).EQ.0.OR.(INPUT(4).EQ.1.AND.IC2.EQ.0)) GO TO 2250 00010310
CALL HEADIN 00010320
IF(INPUT(3).EQ.0) GO TO 2211 00010330
PRINT 9240 00010340
9240 FORMAT (/20X,20H*** DZYI (I,J,N) *** //) 00010350
PRINT 9160,(I,I=1,10) 00010360
DO 2205 I = 1,NZ 00010370
DO 2205 J = 1,NY 00010380
2205 PRINT 9170,i,j,(DZYI(I,J,N),N=1,10) 00010390
PRINT 9250 00010400
9250 FORMAT (/20X,21H*** DZYII (I,J,N) *** //) 00010410

PRINT 9160, (I, I=1,9)
DO 2210 I = 1,NZ
DO 2210 J = 1,NY
2210 PRINT 9260
2211 PRINT 9260
9260 FORMAT('/20X,21H*** DZZII (I,J,N) *** /*
PRINT 9160, (I, I=1,8)
DO 2215 I = 1,NZ
DO 2215 J = 1,NY
2215 PRINT 9170, (I,J) (DZZII(I,J,N),N=1,8)
CALL HEADIN
PRINT 9270
9270 FORMAT('/20X,21H*** DPYII (I,J,N) *** /*
PRINT 9160, (I, I=1,1)
DO 2225 I = 1,NZ
DO 2225 J = 1,NP
2225 PRINT 9170, (I,J) (DPYII(I,J,N),N=1,2)
PRINT 9280
9280 FORMAT('/20X,21H*** DPYII (I,J,N) *** /*
PRINT 9160, (I, I=1,1)
DO 2225 I = 1,NZ
DO 2225 J = 1,NP
2225 PRINT 9170, (I,J) (DPYII(I,J,N),N=1,1)
PRINT 9290
9290 FORMAT('/20X,18H*** DPYII (I,N) *** /*
PRINT 9160, (I, I=1,10)
DO 2230 I = 1,NZ
2230 PRINT 9070, (I,DPM(I,N),N=1,10)
PRINT 9300
9300 FORMAT('/20X,19H*** DPYII (I,N) *** /*
PRINT 9160, (I, I=1,9)
DO 2235 I = 1,NZ
2235 PRINT 9070, (I,DPM(I,N),N=1,9)
2250 IF (INPUT(5).EQ.0.OR.(INPUT(5).EQ.1.AND.-IC2 .EQ.0) GO TO 2300
CALL HEADIN
IF (INPUT(5).EQ.0) GO TC 2261
PRINT 9310
9310 FORMAT('/20X,20H*** DPYII (I,J,N) *** /*
PRINT 9160, (I, I=1,10)
DO 2255 I = 1,NP
DO 2255 J = 1,NY
2255 PRINT 9170, (I,J) (DPYII(I,J,N),N=1,10)
PRINT 9320
9320 FORMAT('/20X,21H*** DPYII (I,J,N) *** /*
PRINT 9160, (I, I=1,9)
DO 2260 I = 1,NP
DO 2260 J = 1,NY
2260 PRINT 9170, (I,J) (DPYII(I,J,N),N=1,9)
2261 IF (INPUT(4).EQ.0) GO TC 2271
PRINT 9330
9330 FORMAT('/20X,20H*** DPYII (I,J,N) *** /*
PRINT 9160, (I, I=1,9)
DO 2265 I = 1,NP
DO 2265 J = 1,NZ
111
2265 PRINT 9170, I, J, (CPZI(I, J, N), N=1, 9) 00010980
CALL HEADIN 00010990
PRINT 9340 00011000
9340 FORMAT(/20X,21H*** DPZII (I, J, N) *** //) 00011010
PRINT 9160, (I, I=1, 8) 00011020
DO 2270 I = 1, NP 00011030
DO 2270 J = 1, N 00011040
2270 PRINT 9170, I, J, (CPZI(I, J, N), N=1, 8) 00011050
2271 PRINT 9350 00011060
9350 FORMAT(/20X,20H*** DPFI (I, J, N) *** //) 00011070
PRINT 9160, (I, I=1, 8) 00011080
DO 2275 I = 1, NP 00011090
DO 2275 J = 1, N 00011100
2275 PRINT 9170, I, J, (CPPI(I, J, N), N=1, 8) 00011110
PRINT 9360 00011120
9360 FORMAT(/20X,21H*** DPPI (I, J, N) *** //) 00011130
PRINT 9160, (I, I=1, 7) 00011140
DO 2280 I = 1, NP 00011150
DO 2280 J = 1, NY 00011160
2280 PRINT 9170, I, J, (CPPI(I, J, N), N=1, 7) 00011170
CALL HEADIN 00011180
PRINT 9370 00011190
9370 FORMAT(/20X,20H*** DPSI (I, J, 1) *** //) 00011200
PRINT 9160, (I, I=1, 1) 00011210
DO 2285 I = 1, NP 00011220
DO 2285 J = 1, NY 00011230
2285 PRINT 9170, I, J, CPSI(I, J, 1) 00011240
PRINT 9380 00011250
9380 FORMAT(/20X,18H*** DPMI (I, N) *** //) 00011260
PRINT 9160, (I, I=1, 10) 00011270
DO 2290 I = 1, NP 00011280
2290 PRINT 9070, I, (CPPI(I, N), N=1, 10) 00011290
PRINT 9390 00011300
9390 FORMAT(/20X,19H*** DPPII (I, N) *** //) 00011310
PRINT 9160, (I, I=1, 9) 00011320
DO 2295 I = 1, NP 00011330
2295 PRINT 9070, I, (CPPI(I, N), N=1, 9) 00011340
2300 CALL HEADIN 00011350
IF INPUT(3).EQ.0 GO TO 2310 00011360
PRINT 9400 00011370
9400 FORMAT(/20X,19H*** DYF (I, J, N) *** //) 00011380
PRINT 9160, (I, I=1, 6) 00011390
DO 2305 I = 1, NY 00011400
DO 2305 J = 1, NMAX 00011410
2305 PRINT 9170, I, J, (DYF(I, J, N), N=1, 6) 00011420
2310 IF INPUT(4).EQ.0 GO TO 2320 00011430
PRINT 9410 00011440
9410 FORMAT(/20X,19H*** DZF (I, J, N) *** //) 00011450
PRINT 9160, (I, I=1, 6) 00011460
DO 2315 I = 1, NZ 00011470
DO 2315 J = 1, NMAX 00011480
2315 PRINT 9170, I, J, (DZF(I, J, N), N=1, 6) 00011490
2320 IF INPUT(5).EQ.0 GO TO 2330 00011500
PRINT 9420 00011510
9420 FORMAT(/20X,19H*** DPF (I, J, N) *** //) 00011520
PRINT 9160, (I, I=1, 3)
DO 2325 I = 1, NP
DO 2325 J = 1, NMAX
2325 IF (GVOE(I, J) .AND. GP .EQ. 1 .AND. GP .EQ. 0) GO TO 2500
CALL HEADIN
PRINT 9421
9421 FORMAT (/20X,21H*** CYD, OZD, DPD *** //)
PRINT 9160, (I, I=1, 5)
DO 2335 I = 1, NY
2335 PRINT 9070, (DYC(I, J), J=1, NY)
PRINT 9470
DO 2340 I = 1, NZ
2340 PRINT 9070, (OZD(I, J), J=1, NZ)
PRINT 9470
DO 2345 I = 1, NP
2345 PRINT 9070, (DPD(I, J), J=1, NP)
2500 IF (INPUT(I) .NE. 2 .AND. ICZ .EQ. 0) GO TO 2525
PRINT 9430, OMEG, CMF
9430 FORMAT (/20X,22H FORCING FREQ = 7F6.2, 17H //)
C COEFFICIENT MATRICES
2525 IF (IC4 .EQ. 0) GO TO 2600
CALL HEADIN
PRINT 9450
9450 FORMAT (/20X,31H*** CCIR, CCOR, COR, FR, BF *** //)
DO 2530 I = 1, NM
2530 PRINT 9460, (CCIR(I, J), J=1, NM)
9460 FORMAT (3X,1P111E11.3)
PRINT 9470
9470 FORMAT (/)
DO 2540 I = 1, NM
2540 PRINT 9460, (CCOR(I, J), J=1, NM)
PRINT 9470
DO 2550 I = 1, NM
2550 PRINT 9460, (COR(I, J), J=1, NM)
PRINT 9470
PRINT 9460, (FR(I), I=1, NP)
PRINT 9470
PRINT 9460, (BF(I), I=1, NM)
CALL HEADIN
PRINT 9480
9480 FORMAT (/20X,24H*** RICC = INV(COIR) *** //)
DO 2560 I = 1, NM
2560 PRINT 9460, (RICC(I, J), J=1, NP)
IF (INPUT(I) .EQ. 0 .AND. INPUT(8) .EQ. 1 .AND. INPUT(9) .EQ. 0) GO TO 2600
PRINT 9500
9500 FORMAT (/20X,20H*** BIRIH, BIRID *** //)
DO 2565 I = 1, 3
2565 PRINT 9460, (BIRIH(I, J), J=1, 3)
PRINT 9470
DO 2570 J = 1, 3
2570 PRINT 9460, (BIRID(I, J), J=1, NP)
CALL HEADIN
PRINT 9510
9510 FORMAT (//20X,37H*** BIRIC, BIRICH, BIRIT, TM, HC, HK, HF *** //))
DO 2575 I = 1, 3
2575 PRINT 9460, (BIRIC(I, J), J = 1, NM)
PRINT 9470
DO 2580 I = 1, 3
2580 PRINT 9460, (BIRIC(I, J), J = 1, 3)
PRINT 9470
DO 2585 I = 1, 3
2585 PRINT 9460, (BIRIC(I, J), J = 1, NM)
PRINT 9470
PRINT 9460, (TM(I, I), I = 1, 3)
PRINT 9470
PRINT 9460, (HC(I, I), I = 1, 3)
PRINT 9470
PRINT 9460, (HK(I, I), I = 1, 3)
PRINT 9470
PRINT 9460, HF
2600 IF (INPUT(7) /= NE.0) PRINT 9600, FMX, HCX, HKX, HF(1)
9600 FORMAT (//20X, 19HIO = 7) HUB DATA 10X, 16HMX, HCX, HKX, HF =
1 4F10.3)
IF (INPUT(8) /= NE.0) PRINT 9601, HMX, HCY, HKY, HF(2)
9601 FORMAT (//20X, 19HIO = 8) HUB DATA 10X, 16HMY, HCY, HKY, HF =
1 4F10.3)
IF (INPUT(9) /= NE.0) PRINT 9602, FMZ, HCZ, HKZ, HE(3)
9602 FORMAT (//20X, 19HIO = 9) HUB DATA 10X, 16HMZ, HCZ, HKZ, HF =
1 4F10.3)
3900 IF (INPUT(13) /= NE.0) PRINT 9740, X(NX), AFY, AFZ, AFP
9740 FORMAT (//20X, 27HIO = 13) STA, FY, FZ, FP = , 4F10.3)
IF (INPUT(13) /= NE.0 AND, NF.E.0) PRINT 9743, PER
IF (INPUT(13) /= NE.0 AND, NF.E.0) NBF = 0
IF (INPUT(13) /= NE.0 AND, NF.E.0) NBF = 0
IF (INPUT(13) /= NE.0 AND, NF.E.0) PRINT 9741, NBF
IF (INPUT(13) /= NE.0 AND, NF.E.0) PRINT 9742, NBF
9741 FORMAT (30X, 27HIO = 13) HUB NO.
9742 FORMAT (30X, 27HIO = 13) HUB NO.
9743 FORMAT (//20X, 16HIO = 1) HUB NO.
9744 FORMAT (//20X, 16HIO = 2) HUB NO.
9745 FORMAT (//20X, 16HIO = 3) HUB NO.
9746 FORMAT (//20X, 16HIO = 4) HUB NO.
9747 FORMAT (//20X, 16HIO = 5) HUB NO.
9748 FORMAT (//20X, 16HIO = 6) HUB NO.
9749 FORMAT (//20X, 16HIO = 7) HUB NO.
9750 FORMAT (//20X, 16HIO = 8) HUB NO.
9751 FORMAT (//20X, 16HIO = 9) HUB NO.
9752 FORMAT (30X, 10HALL BLADES )
9753 FORMAT (//20X, 16HIO = 10) HUB NO.
9754 FORMAT (//20X, 16HIO = 11) HUB NO.
9755 FORMAT (//20X, 27HIO = 1 P KAC-LINEARITIES ***)
9760 FORMAT (//20X, 27HIO = 2 ALL KAC-LINEARITIES ***)
9770 FORMAT (//20X, 25HIO = 3 NO CCXILOS TERMS ***)
9780 FORMAT (//20X, 43HIO = 4 AUTOMATIC FLOQUET TRANSITION MATRIX ***)
9785 FORMAT (//20X, 55HIO = 5 STEACY FORCES DUE TO STRUCTURAL EFFECTS IGNORED)
C
4000 IF (INPUT18) /= NE.2, AND, IC2.EQ.0) GO TO 5000
PRINT 9800, CYCLES, HINIT, ERROR, IFE, IYF, IYF, BERR
9800 FORMAT (//3X, 20HIO = 18) CYCLES = , F5.1, 4X, THHINIT = , F5.1,
1 4X, TERROR = , F6.3, 4X, 5H1YE = , I4, 4X, 5HCIC = , F5.2, 4X, 6HEIIC = , I4,
2 4X, 6HIBERR = , F6.2)
5000 RETURN
END
SUBROUTINE INT(A,B,A0,X,NX,ICONT)

C A(X) = INTEGRAL OF B(X) WITH BC = A0 AT X(1)
C X IS INDEPENDENT VARIABLE
C NX IS NUMBER OF STATIONS
C ICONT = 1 INTEGRAL FROM 0 TO X
C 2 INTEGRAL FROM X TO R (LAST X)
C
C TRAPEZOIDAL INTEGRATION

REAL A(I),B(I),X(I)
A(1)=A0
DO 10 I=2,NX
10 A(I)=A(I-1)+(B(I-1)+B(I))*(X(I)-X(I-1))/2
IF (ICONT.EQ.1) RETURN
C=A(NX)
DO 20 I=1,NX
20 A(I)=C-A(I)
RETURN
END
SUBROUTINE INYPS (B, N, A, D, IROW, ICOL, NRW, NCL)  
C A = INVERSE OF B (UNCISTURBED)  
C VARIABLE DIMENSIONS, NCL MUST BE AT LEAST ONE GREATER THAN N.  
C NRW MUST BE AT LEAST EQUAL TO N.  
C IROW, ICOL ARE VECTORS OF LENGTH NCL.  
REAL A(NRW, NCL), E(NRW, NCL), D(NRW, NCL)  
INTEGER IROW(NCL), ICOL(NCL)  
DO 1 I = 1, N  
DO 1 J = 1, N  
1 A(I, J) = B(I, J)  
M = N + 1  
DO 7 I = 1, M  
DO 7 J = 1, N  
7 ICOL(I) = I  
DO 20 K = 1, N  
AMAX = A(K, K)  
DO 10 I = K, N  
DO 10 J = K, N  
IF (ABS(A(I, J)) > AMAX) GO TO 1, K  
CONTINUE  
KI = ICOL(K)  
ICOL(K) = ICOL(K)  
ICOL(K) = ICOL(K)  
IROW(K) = IROW(K)  
IROW(K) = IROW(K)  
IF(AMAX) 11, 12, 11  
10 CONTINUE  
11 PRINT 13  
12 FORMAT (* SOLUITION OF MATRIX NOT POSSIBLE*)  
GO TO 100  
13 GO TO 14  
14 A(I, J) = E  
DO 15 I = 1, N  
E = A(I, K)  
A(I, K) = A(I, J)  
15 A(I, J) = E  
DO 16 I = 1, N  
IF(I - K) 18, 17, 18  
17 A(I, M) = 1.  
GO TO 16  
18 A(I, M) = 0.  
16 CONTINUE  
PVT = A(K, K)  
DO 8 J = 1, M  
A(K, J) = A(K, J) / PVT  
DO 19 I = 1, N  
IF(I - K) 21, 19, 21  
116
21 AMULT = A(I,K)
   DO 22 J = 1, N
22 A(I,J) = A(I,J) - AMULT*A(K,J)
   DO 19 I = 1, N
   CONTINUE
19 CONTINUE
   DO 20 I = 1, N
20 A(I,K) = A(I,K)
   DO 25 I = 1, N
   DO 24 L = 1, N
   IF (IRCW(I,J*L) .EQ. 24) CONTINUE
24 CONTINUE
   DO 23 J = 1, N
23 D(L,J) = A(I,J)
   DO 26 J = 1, N
   DO 28 L = 1, N
   IF (ICOL(J)L) .EQ. 28) CONTINUE
28 CONTINUE
   DO 29 I = 1, N
29 A(I,L) = D(I,J)
26 A(I,L) = D(I,J)
100 RETURN
   END
SUBROUTINE MXM(A,B,C,M,N,K,NA,AB,NC)
C
C MATRIX MULT A(M,N)=B(K,K)*C(K,M)
C
DIMENSION A(NA,1),B(NB,1),C(AC,1)
DO 20 I=1,N
DO 20 J=1,M
A(I,J)=0
DO 20 L=1,K
20 A(I,J)=A(I,J)+B(I,L)*C(L,J)
RETURN
END

SUBROUTINE MXV(A,B,C,M,N,NDIM,ICONT)
C
C MATRIX TIMES VECTOR A(M)=B(M,N)+C(N)
C
DIMENSION A(I),B(NDIM,1),C(I)
DO 10 I=1,M
IF (ICONT.EQ.0) A(I)=0
DO 10 J=1,N
10 A(I)=A(I)+B(I,J)*C(J)
RETURN
END
SUBROUTINE OUTP(TV, YV, DERY, IHLF, DIM, PRMT, LY)
REAL H, KM, KP2, KA
LOGICAL LY(1)
REAL DATA (61, DATAT(3))
DIMENSION YV(I), DATAT(I), PRMT(I)

COMMON/INDAT/X(20), M(20), E(20), SHE(20), KM(20), KP2(20), KA(20),
THP(20), YPP(20, 3), ZPP(20, 5), ZP(20, 5), PPP(20, 3), PPP(20, 3),
OMG, OMF, ECI(20), NY, NZ, NP, NFLO, CMEGS, OMFS, DIM, NMAX, NLIN

*NB, H, MY, HZ, HX, HY, HKY, HKZ, NX, NFLOQ

COMMON/HED/IC1, IC2, IC3, IC4, HEAC(I), IPA, INP(20), IEND, LINE, IC5

IF (NFLOQ .NE. 0) RETURN
CYCF = T*OMF/628319
CYCR = T*OMF/628319
CYCR = CYCR
NCRY = CYCF
NCYCR = NCYCR
DEGF = (CYCF - FLOAT(NCYCF)) * 360
DEGR = (CYCR - FLOAT(NCYCR)) * 360
LINE = LINE + NMAX + NE + 1
IF (NB .GT. 1) LINE = LINE + NB
IF (NMAX .GT. 1) LINE = LINE + N
IF (LY(1) .LT. L) LINE = LINE + 1
IF (LINE .GT. 56) LINE = 10
IF (LINE .GT. 10) GC TO 50
CALL HEADIN
PRINT 1000
1000 FORMAT (/11H TIME OMF OMEGA I Y(I) DOT Y(I) DOT PHII DOT PHII DOT)
1 / 26H SEC CY DEG CY DEG )
50 DO 110 IB = 1, KB
III = 2*NM*(IB - 1) + 9
DATAT(I) = 0
DATAT(2) = 0
DATAT(3) = 0
DO 100 I = 1, NMAX
DO 90 J = 1, 6
90 DATA(J) = 0
IF (NY .LT. 1) GC TO 91
II = III + 2*I
DATA(1) = YV(II)
DATA(2) = YV(II+1)
DATA(3) = DATAT(1) + DATA(2)
91 IF (NZ .LT. 1) GO TO 92
II = III + 2*(I + NY) + NZ
DATA (3) = DATA (3) + YV (II)
DATA (4) = DATA (4) + YV (II+1)
DATA(2) = DATA(2) + DATA(1)
92 IF (NP .LT. 1) GO TO 93
II = III + 2*(I + NY + NZ)
119
DATA (5) = YV(11)
DATA (6) = YV(11+1)
DATA(3) = DATAT(3)+DATA(6)
93 IF (I.GT.1) GO TO 95
 IF (IB.EQ.1) GO TO 94
 IF (IB.EQ.1) PRINT 1004, T, NCYCF, DEGF, NCYCR, DEGR
 IF (IB.EQ.1) PRINT 1005, IB
 GO TO 95
1004 FORMAT (/1X,F6.3,2(I4,F6.1),10H *BLADE 1*)
1005 FORMAT (27X,7H *BLADE, I2,1H*)
94 PRINT 1010, T, NCYCF, DEGF, NCYCR, DEGR, I, DATA
1010 FORMAT (/1X,F6.3,2(I4,F6.1),13,3(1PE12.3, E13.3,8X))
 GO TO 100
95 PRINT 1020, I, DATA
1020 FORMAT (20X, I10, 3(1PE12.3, E13.3, 8X))
100 CONTINUE
 IF (NMAX.GT.1) PRINT 1021, DATAT
 IF (IC5.NE.0.AND.IB.EQ.1) WRITE (9) CYCR, DATAT, YV(2), YV(4), YV(6)
110 CONTINUE
 IF (LY(1), CR, LY(3), CR, LY(5)) PRINT 1025, (YV(L), L=1, 6)
1025 FORMAT (/4X, 26HUB XDOT, X, YDOT, Y, ZDOT, Z, 3(1PE12.3, E13.3,8X))
 IF (PRMT(6).EE0) GO TO 200
 IF (ABS(YV(1)).LT.PRMT(6)) GO TO 200
 PRINT 1030
1030 FORMAT (/24H *** LIMIT EXCEEDED *** */)
 PRMT(5) = 1
200 RETURN
END
SUBROUTINE SCLI(PRMT,YVAR,DERY,ILHF,LY)
INTEGER IRON(31),ICOL(31)
LOGICAL LY(I)
REAL PRMT(I),YVAR(I),DERY(I)
REAL AUX(898),BFTEMP(11),ERW(36),FLTM(30,31),FLTM(30,31),
WORK(30,31)
REAL HTEMP(3),FRTEMP(11)
COMMON/INDAT/X(201),M(20),E(20),S(20),K(20),K(20),KA(201),
THP(20),EOP(20),GJ(20),EA(20),EBL(20),EBL(20),ECS(20),EIP(20),
OMG(OMF)=E(20),NY,NZ,NP,NP,CMESG,OMFS,IDIM,NMAX,NL IN
4,HR,HPX,HMY,HMZ,HX,HY,HCZ+HX,HK,PK,YPP,PHZ,
COMMON/COEF/CGI
DO 10 I=IDIM
EQUIVALENCE(AUX(11 eWGRK41 I
COMMON/D
SUBROUTINE SCLY(PRMT,YVAR,DERY,ILHF,LY)
COMMON/INDAT/X(201),M(20),E(20),S(20),K(20),K(20),KA(201),
THP(20),EOP(20),GJ(20),EA(20),EBL(20),EBL(20),ECS(20),EIP(20),
OMG(OMF)=E(20),NY,NZ,NP,NP,CMESG,OMFS,IDIM,NMAX,NL IN
4,HR,HPX,HMY,HMZ,HX,HY,HCZ+HX,HK,PK,YPP,PHZ,
COMMON/COEF/CGI
DO 10 I=IDIM
EQUIVALENCE(AUX(11 eWGRK41 I
IF(I,G,T,E8) GO TO 101
DO 30 J=1,IDIM
DERY(J)=ERW(J)
30 YVAR(I)=0
CALL RKGSV(PSMT,YVAR,DERY,IDIM,HLF,AUX,LY)
IF(HLF.EQ.11) CALL ERR (5030,0)
IF(HLF.EQ.12) CALL ERR (5031,0)
J=0
DO 50 J=1,IDIM
IF(.NOT.(LY(J))) GC TO 50
J=J+1
50 CONTINUE
GO TO 101
010 CONTINUE
PRINT 1010,II,(FLTM(I,J),JJ=1,NVAR)
1010 FORMAT(1X,13,P10,E12,3/(4X,10,E12,3))
100 CONTINUE
GO TO 109
109 CONTINUE
IF(ID1=II-NM2)
ID1=ID1-1
ICD1=II
DO 108 J8=2,NB
DO 108 J=1,NM2
JJ=ID11+(J8-1)*NM2+J
JREF=ID11+J
IF(ID11.EQ.0) GO TO 103
DO 102 I=1,ID11
102 CONTINUE
108 CONTINUE
DO 107 J8=1,NB
IREF=ID01-1
IF(J8.EQ.J8)IREF=ID11
DO 107 I=1,NM2
II=ID11+(J8-1)*NM2+I
107 CONTINUE
108 CONTINUE
GO TO 110
110 YVAR(I)=0
DO 115 J=1,NL
DERY(J)=ERW(J)
115 CALL RKGSV(PSMT,YVAR,DERY,IDIM,HLF,AUX,LY)
110 CONTINUE
DO 130 I=1,NVAR
130 CONTINUE
CONTINUE
CALL INVRS(FLTM,AVAR,FLTMI,WRK,IROW,ICOL,30,31) 00001080
II=0 00001090
DO 140 I=1,IDIM 00001100
IF(.NOT.LY(I)) GC TO 140 00001110
II=II+1 00001120
YVAR(I)=YVAR(I) 00001130
140 CONTINUE 00001140
PRINT 1020,(YVAR(I),I=1,NVAR) 00001150
1020 FORMAT (130X,19HPARTICULAR SOLUTION /4X,1P10E12.3) 00001160
CALL MXV(DERY,FLTMI,YVAR,AVAR,NVAR,30,0) 00001170
II=0 00001180
DO 150 I=1,IDIM 00001190
YVAR(I)=0. 00001200
IF(.NOT.LY(I)) GC TO 150 00001210
II=II+1 00001220
YVAR(I)=DERY(I) 00001230
150 CONTINUE 00001240
DO 160 I=1,IDIM 00001250
160 DERY(I)=ERW(I) 00001260
PRMT(2)=PRMT2 00001270
NFLQ=NLFLOQ 00001280
CALL RKGSV(PRMT,YVAR,DERY,IDIM,ILHF,AJX,LY) 00001300
NFLQ=NFLT 00001310
IF(NFLQ*NE.2) RETURN 00001320
DO 170 I=1,II 00001330
170 BF(I)=BFTEMP(I) 00001340
RETURN 00001350
END 00001360
SUBROUTINE RKGSV(PRMT, Y, DERY, ACIM, IHLF, AUX, LY)

SUBROUTINE RKGSV

MODIFIED TO INCLUDE OPTIONAL COMPUTATION OF EACH Y(I)

FCT, OUTP REMOVED FROM ARG LIST, THUS NO EXTERNAL SMT REQ

PURPOSE

TO SOLVE A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL

EQUATIONS WITH GIVEN INITIAL VALUES.

USAGE

CALL RKGSV (PRMT, Y, DERY, NDIM, IHLF, FCT, OUTP, AUX, LY)

PARAMETERS FCT and OUTP REQUIRE AN EXTERNAL STATEMENT.

DESCRIPTION OF PARAMETERS

PRMT - AN INPUT AND OUTPUT VECTOR WITH DIMENSION GREATER

OR EQUAL TO 5, WHICH SPECIFIES THE PARAMETERS OF

THE INTERVAL AND ACCURACY AND WHICH SERVES FOR

COMMUNICATION BETWEEN OUTPUT SUBROUTINE (FURNISHED)

BY THE USER) AND SUBROUTINE RKGS, EXCEPT PRMT(5)

THE COMPONENTS ARE NOT DESTROYED BY SUBROUTINE

RKGS AND THEY ARE

PRMT(1) - LOWER BOUND OF THE INTERVAL (INPUT),

PRMT(2) - UPPER BOUND OF THE INTERVAL (INPUT),

PRMT(3) - INITIAL INCREMENT OF THE INDEPENDENT VARIABLE

(INPUT),

PRMT(4) - UPPER ERROR BOUND (INPUT). IF ABSOLUTE ERROR IS

GREATER THAN PRMT(4), INCREMENT GETS HALVED.

IF INCREMENT IS LESS THAN PRMT(3) AND ABSOLUTE

ERROR LESS THAN PRMT(4)/50, INCREMENT GETS DOUBLED.

THE USER MAY CHANGE PRMT(4) BY MEANS OF HIS

OUTPUT SUBROUTINE.

PRMT(5) - AN INPUT PARAMETER. SUBROUTINE RKGS INITIALIZES

PRMT(5) = 0. IF THE USER WANTS TO TERMINATE

SUBROUTINE RKGS AT ANY OUTPUT POINT, HE HAS TO

CHANGE PRMT(5) TO NON-ZERO BY MEANS OF SUBROUTINE

OUTP. FURTHER COMPONENTS OF VECTOR PRMT ARE

FEASIBLE IF ITS DIMENSION IS DEFINED GREATER

 THAN 5. HOWEVER SUBROUTINE RKGS DOES NOT REQUIRE

AND CHANGES THEM. NEVERTHELESS THEY MAY BE USEFUL

FOR HANDLING RESULT VALUES TO THE MAIN PROGRAM

(CALLING RKGS) WHICH ARE OBTAINED BY SPECIAL

MANIPULATIONS WITH OUTPUT DATA IN SUBROUTINE OUTP.

Y - INPUT VECTOR OF INITIAL VALUES. (DESTROYED)

LATERON Y IS THE RESULTING VECTOR OF DEPENDENT

VARIABLES COMPUTED AT INTERMEDIATE POINTS X.

DERY - INPUT VECTOR OF ERROR WEIGHTS. (DESTROYED)

THE SUM OF ITS COMPONENTS MUST BE EQUAL TO 1.

LATERON DERY IS THE VECTOR OF DERIVATIVES, WHICH

BELONG TO FUNCTION VALUES Y AT A POINT X.

NDIM - AN INPUT VALUE, WHICH SPECIFIES THE NUMBER OF

EQUATIONS IN THE SYSTEM.

IHLF - AN OUTPUT VALUE, WHICH SPECIFIES THE NUMBER OF
BISECTIONS OF THE INITIAL INCREMENT. IF IHLF GETS
C 00000530
C GREATER THAN 10, SUBROUTINE RKGS RETURNS WITH
C 00000540
C ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM. ERROR
C 00000550
C MESSAGE IHLF=12 OR IHLF=13 APPEARS IN CASE
C 00000560
C PRMT(3)=0 OR IN CASE SIGN(PRMT(3))=NE.SIGN(PRMT(2))=00000570
C PRMT(1)) RESPECTIVELY. 00000580
C
FCT - THE NAME OF AN EXTERNAL SUBROUTINE USED. THIS
C 00000550
C SUBROUTINE COMPUTES THE RIGHT HAND SIDES DERY
C 00000560
C THE SYSTEM TO GIVEN VALUES X AND Y. ITS PARAMETER
C 00000570
C LIST MUST BE X,Y,DERY,LY SUBROUTINE FCT SHOULD
C 00000580
C NCT DESTROY X AND Y.
C 00000590
C OUTP - THE NAME OF AN EXTERNAL OUTPUT SUBROUTINE USED.
C 00000640
C ITS PARAMETER LIST MUST BE X,Y,DERY,IHLF,NDIM,PRMT,00000650
C 00000560
C NONE OF THESE PARAMETERS (EXCEPT, IF NECESSARY, 00000570
C PRMT(4),PRMT(5),....) SHOULD BE CHANGED BY
C 00000580
C SUBROUTINE OUTP. IF PRMT(5) IS CHANGED TO NON-ZERO, 00000590
C SUBROUTINE RKGS IS TERMINATED. 00000600
C 00000610
C AUX - AN AUXILIARY STORAGE ARRAY WITH 8 ROWS AND MDIM
C 00000620
C COLUMNS.
C 00000630
C LY - LOGICAL ARRAY, IF TRUE, CORRESPONDING Y(I)
C 00000640
C IS CALCULATED
C 00000650
C
REMARKS
C THE PROCEDURE TERMINATES AND RETURNS TO CALLING PROGRAM, IF
C 00000770
C (1) MORE THAN 10 BISECTIONS OF THE INITIAL INCREMENT ARE
C 00000780
C NECESSARY TO GET SATISFACTORY ACCURACY (ERROR MESSAGE
C 00000790
C IHLF=11), 00000800
C 00000700
C (2) INITIAL INCREMENT IS EQUAL TO 0 OR HAS WRONG SIGN
C 00000810
C (ERROR MESSAGES IHLF=12 OR IHLF=13), 00000820
C 00000710
C (3) THE WHOLE INTEGRATION INTERVAL IS WORKED THROUGH,
C 00000830
C 00000720
C (4) SUBROUTINE OUTP HAS CHANGED PRMT(5) TO NON-ZERO.
C 00000840
C 00000730
C 00000850
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C 00000860
C THE EXTERNAL SUBROUTINES FCT(X,Y,DERY) AND
C 00000870
C OUTP(X,Y,DERY,IHLF,NDIM,PRMT) MUST BE FURNISHED BY THE USER.
C 00000880
C 00000740
C METHOD
C EVALUATION IS DONE BY MEANS OF FOURTH ORDER RUNGE-KUTTA
C 00000900
C FORMULAE IN THE MODIFICATION DUE TO GILL. ACCURACY IS
C 00000910
C TESTED COMPARING THE RESULTS OF THE PROCEDURE WITH SINGLE
C 00000920
C AND DOUBLE INCREMENT.
C 00000930
C 00000940
C SUBROUTINE RKGS AUTOMATICALLY ADJUSTS THE INCREMENT DURING
C 00000950
C THE WHOLE COMPUTATION BY HALVING OR DOUBLING. IF MORE THAN
C 00000960
C 10 BISECTIONS OF THE INCREMENT ARE NECESSARY TO GET
C 00000970
C SATISFACTORY ACCURACY, THE SUBROUTINE RETURNS WITH
C 00000980
C ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM.
C 00000990
C TO GET FULL FLEXIBILITY IN OUTPUT, AN OUTPUT SUBROUTINE
C 00001000
C MUST BE FURNISHED BY THE USER.
C 00001010
C FOR REFERENCE, SEE
C 00001020
C RALSTON/ILF, MATHEMATICAL METHODS FOR DIGITAL COMPUTERS,
C 00001030
C WILEY, NEW YORK/LONDON, 1960, PP.110-120.
C 00001040
C 00001050
C 00001060
C 00001070
SUBROUTINE RKGSV(PRMT,Y,DERY,ACIP,IHLF,FCT,OUTP,AUX,LY)

DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)

LOGICAL LY(1)
DO 100 I=1,NCIM
100 AUX(8,1)=.06666667*DERY(1)
X=PRMT(1)
XEND=PRMT(2)
H=PRMT(3)
PRMT(5)=0.
CALL FCT(X,Y,DERY,LY,ACIM)

ERROR TEST
IF(H*(XEND-X))470,460,110

PREPARATIONS FOR RUNGE-KUTTA METHOD
110 A(1)=.5
A(2)=2.928932
A(3)=1.707107
A(4)=.1666667
B(1)=2.
B(2)=1.
B(3)=1.
B(4)=2.
C(1)=.5
C(2)=2.928932
C(3)=1.707107
C(4)=.5

PREPARATIONS OF FIRST RUNGE-KUTTA STEP
DO 120 I=1,NCIM
IF(.NOT.EY(I)) GC TO 120
AUX(I,1)=Y(I)
AUX(2,1)=DERY(I)
AUX(3,1)=0.
AUX(6,1)=0.
120 CONTINUE

CONTINUE
IR=0
H=H/H
IHLF=-1
ISTEP=0
IEND=0

START OF A RUNGE-KUTTA STEP
130 IF((X-H)=XEND)*H)160,150,140
140 H=XEND-X
150 IEND=1

RECORDING OF INITIAL VALUES OF THIS STEP
160 CALL GLTP(X,Y,DERY,IR,NDIM,PRMT,LY)
IF(PRMT(5))490,170,490
170 ITEST=0
180 ISTEP=ISTEP+1

126
C START OF INNERMGST RUNGE-KUTTA LOOP
J=1
190 AJ=A(J)
BJ=B(J)
CJ=C(J)
DO 200 I=1,NDIM
IF(.NOT.LY(I)) GC TO 200
R1=H*DERY(I)
R2=AJ*(R1-BJ*AUX(6,I))
Y(I)=Y(I)+R2
R2=R2*R2+R2
AUX(6,I)=AUX(6,I)+R2-CJ*R1
200 CONTINUE
IF(J-4)210,240,210
210 J=J+1
X=X+H
CALL FCT(X,Y,DERY,LY,NCIM)
GO TO 190
C END OF INNERMOST RUNGE-KUTTA LOOP
C TEST OF ACCURACY
IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
240 DO 250 I=1,NDIM
IF(LY(I))AUX(I,I) = Y(I)
250 CONTINUE
ISTEP=ISTEP+ISTEP-2
270 IHLF=IHLF+1
X=X-2
H=H/2
DO 280 I=1,NDIM
IF(.NOT.LY(I)) GC TO 280
Y(I)=AUX(I,I)
DERY(I)=AUX(2,I)
AUX(6,I)=AUX(3,I)
280 CONTINUE
GO TO 180
C IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
290 IMOD=ISTEP/2
IF(ISTEP-IMOD-IMOD)300,320,300
300 CALL FCT(X,Y,DERY,LY,ADIP)
DO 310 I=1,NDIM
IF(.NOT.LY(I)) GC TO 310
AUX(5,I)=Y(I)
AUX(7,I)=DERY(I)
310 CONTINUE
GO TO 180
C COMPUTATION OF TEST VALUE DELT
320 DELT=0.
DO 330 I=1,NCIM
IF(.NOT.LY(I)) GC TO 330
330 CONTINUE
DELT = DELT + AUX(8,1) * ABS(AUX(4,1) - Y(1))
330 CONTINUE
C IF (DELT - PRMT(4)) < 370, 370, 340
C ERROR IS TOO GREAT
340 IF (IHLF - 10) < 350, 450, 450
350 DO 360 I = 1, NDIM
C IF (LY(I)) AUX(4, I) = AUX(5, I)
360 CONTINUE
C ISTEP = ISTEP + ISTEP - 4
C X = X - H
C END = 0
GO TO 270
C RESULT VALUES ARE GOOD
370 CALL FCT(X, Y, DERY, LY, NDIM)
C DO 380 I = 1, NDIM
C IF (.NOT. LY(I)) GC TO 380
C AUX(1, I) = Y(I)
C AUX(2, I) = DERY(I)
C AUX(3, I) = AUX(6, I)
Y(I) = AUX(5, I)
C DERY(I) = AUX(7, I)
380 CONTINUE
C CALL OUTP(X - H, Y, DERY, IH LF, NDIM, PRMT, LY)
C IF (PRMT(5)) < 490, 390, 490
360 DO 400 I = 1, NDIM
C IF (.NOT. LY(I)) GC TO 400
Y(I) = AUX(1, I)
C DERY(I) = AUX(2, I)
400 CONTINUE
C IREC = IHLF
C IF (IERD1410, 410, 480
C C INCREMENT GETS DCUBLED
410 IHLF = IHLF - 1
C ISTEP = ISTEP / 2
C H = H / H
C IF (IHLF) < 420, 420
C IMOD = ISTEP / 2
C IF (IMOD = IMOD - IMOD) < 430, 430, 430
430 IF (DELT - 0.02 * PRMT(4)) < 440, 440, 130
440 IHLF = IHLF - 1
C ISTEP = ISTEP / 2
C H = H / H
GO TO 130
C RETURNS TO CALLING PROGRAM
450 IHLF = 11
C CALL FCT(X, Y, DERY, LY, NDIM)
GO TO 480
460 IHLF = 12
GO TO 480
470 IHLF = 13
480 CALL OUTP(X, Y, DERY, IHLF, NDIM, PRMT, LY)
490 RETURN
END
SUBROUTINE SLPCDE(A,Q,PHI,STA,AX,N)
C   MODAL SUMMATION  STA=DIMENSION  AX=NO OF STATIONS  N=NO OF MJD
C
C      A(I) = SUM(Q(J)*PHI(I,J))
C
REAL A(1),Q(1),PHI(STA,1)
DO 10 I=1,NX
A(I)=0.
DO 10 J=1,N
10  A(I)=A(I)+Q(J)*PHI(I,J)
RETURN
END
INPUT

--

(1) HEADING

1. IGL=EQ.0 -- FIRST OR NORMAL RUN -- ALL INPUT
   1. REPLACE MODES -- INPUT 3, 4, 5
   2. ADD MODES -- INPUT 4, 5
   8. NEW DP CODE ONLY -- INPUT 5
   9. END OF RUN -- LAST CARD OF RUN

2. IC2 .EQ.1 PRINTS ORTHO CHECKS
   AND NORMALIZES MODES
   NOTE MODES ARE REPLACED
   AFTER INPUT AND AFTER
   RANDOM ERRORS.

3. IC3 .NE.0 PRINTS EOS FOR MASS IDENT

4. IC4 .NE.0 RESTORES INPUT MODES IF IGL.EQ.8

5-80 ARBITRARY HEADING HEAD(19)

(2) MASS DATA -- ONE CARD PER BLADE STATION \(20\) MAX

1-10 \(x(i)\) -- STATION
11 \*(SEE NOTE) WM
12-20 \(M\) -- LUMPED MASS
21 \*(SEE NOTE) WE
22-30 \(E\) -- CG OFFSET FROM EA WHEN CG FORWARD
31 \*(SEE NOTE) WT
32-40 \(TH\) -- PITCH ANGLE -- RAD
41 \*(SEE NOTE) WK
42-50 \(KM\) -- RADIUS OF GYRATION IN TORSION

* 1ST COL OF EACH WORD CONTAINS WEIGHTING FACTOR
FROM 1-9 \(10=1\) HIGHER VALUE INDICATES GREATER CONFIDENCE
SEE 101 = 3 \(WDL\)

END WITH BLANK CARD

(3) CONTROL CARD -- MODES

1-10 CALV MULTIPLIES 1-P MODE DEFL \(0=1\)
11-20 CALW MULTIPLIES 0-P MODE DEFL \(0=1\)
21-30 CALP MULTIPLIES TOR MODE DEFL \(0=1\)
31-40 THQ -- ROOT PITCH ANGLE -- RAD
   ADDS TO TH -- (TH NOT CHANGED)
(4) MODES — STATIONS CORRESPOND TO MASS DATA

<table>
<thead>
<tr>
<th>EACH MODE</th>
<th>FREQ, RAD/SEC</th>
<th>OMEG, ROTATIONAL, RAD/SEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>11-20</td>
<td>21-30 IF .NE. 0 TEMPORARILY REPLACES CALV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31-40 IF .NE. 0 TEMPORARILY REPLACES CALW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>41-50 IF .NE. 0 TEMPORARILY REPLACES CALP</td>
</tr>
</tbody>
</table>

NEXT CDS V 1-P DISPLACEMENTS, 8F10.6 UP TO 3 CARDS
NEXT CDS W 0-P START EV NEH CD
NEXT CDS P TOR

FOLLOW BY NEXT MODE — 8 MODES MAX AT ONE OMEG
16 MODES MAX AT ALL OMEG

*** 30 EQS MAX (NOT INCLUDING INVARIANCES) ***

END WITH BLANK CARD

(5) OPERATION CODES COL 1,2 101,102

<table>
<thead>
<tr>
<th>COL 1</th>
<th>101</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MODIFY MODES WITH RANDOM ERRORS — MODES REPLACED</td>
</tr>
<tr>
<td></td>
<td>WD1 PERCENT RANDOM + OR - RECTANGULAR DIST</td>
</tr>
<tr>
<td></td>
<td>WD2 PERCENT BIAS</td>
</tr>
<tr>
<td></td>
<td>WD3 INTEGER SEED TO START RANDOM SEQUENCE</td>
</tr>
</tbody>
</table>

*** FOLLOW BY NEXT OPERATION CARD (5) ***

2 SOLVE FOR MINIMUM MODAL CHANGES — MASS MATRIX UNCHANGE

ALL MODES MUST BE AT SAME OMEGA — 8 MAX
FIRST MODE UNCHANGED, LAST MODE WILL CHANGE MOST
MINIMUM SUM PERCENT CHANGES USED
WEIGHTING FACTORS NOT USED IN THIS OPTION

WD1,EQ,0 NO LIMIT ON CHANGES
WD1,EQ,1 LIMIT CHANGES — SCALE OPTION
WD2,B MAX PCT CHANGE ALLOWED IN EACH MODE
CHANGES ARE SCALING 50 MAX CHANGE L.E. MAXIMUM
0 INDICATES NO LIMIT

WD1,EQ,2 LIMIT CHANGES — TRUNCATE OPTION
WD2,B SAME AS FOR SCALE OPTION EXCEPT THAT ONLY
CHANGES WHICH EXCEED LIMITS ARE TRUNCATED
OTHER CHANGES ARE NOT MODIFIED
### 3. INCOMP-MODEL-MASS CHANGES

WD1.EQ.1 WEIGHTING FACTORS ALL SET TO 1 (TEMP)
WD1.EQ.2-STATS WITH INVARIANT PARAMS-READ 5(A)

THE FOLLOWING CONTROLS CAUSE THE CORRESPONDING
PROPERTIES TO REMAIN INVARIANT-IF .NE. 0.

<table>
<thead>
<tr>
<th>COL 20</th>
<th>TOTAL MASS  M</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>RADIAL CG</td>
</tr>
<tr>
<td>40</td>
<td>CHORDWISE CG</td>
</tr>
<tr>
<td>50</td>
<td>FLAPPING MOM OF INERT  M**2</td>
</tr>
<tr>
<td>60</td>
<td>FEATHERING MOM OF INERT  M**2</td>
</tr>
</tbody>
</table>

### COL 2

0 ABOVE OPERATIONS DO NOT DISTURB ORIGINAL DATA

1 ABOVE OPERATIONS REPLACE ORIGINAL DATA IN PREPARATION FOR SEQUENTIAL OPERATIONS

(5A) USED ONLY FOR INVAR STAS. SEE 3.ABOVE, WD1 = 2

<table>
<thead>
<tr>
<th>COL 1</th>
<th>NO OF STATIONS (8 MAX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD1*WD2</td>
<td>STATION NUMBERS, NO ZEROS</td>
</tr>
</tbody>
</table>

NEXT HEADING CARD

<table>
<thead>
<tr>
<th>JJ1</th>
<th>INTEGER HEAD(19), IROW(46), ICOL(46)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JJ2</td>
<td>INTEGER IJEQ(40,2)</td>
</tr>
<tr>
<td>JJ3</td>
<td>INTEGER INI(0)</td>
</tr>
<tr>
<td>JJ4</td>
<td>REAL X(21), WM(20), M(21), W(20), E(21), T(20), TH(21), HK(20), KH(21),</td>
</tr>
<tr>
<td></td>
<td>OMEG(16), VEQ(16), V(16, 20), W(16, 20), P(16, 20), VEQ(21), V(21, 20),</td>
</tr>
<tr>
<td></td>
<td>E(21), T(20), TH(21), HK(20), KH(21),</td>
</tr>
<tr>
<td>JJ5</td>
<td>REAL DUM(8), V(16, 20), W(16, 20), P(16, 20), E(21), T(20), TH(21), HK(20),</td>
</tr>
<tr>
<td></td>
<td>A(60, 7), P(60), B(60, 7), C(7, 8), D(7, 8), WORK(7, 8)</td>
</tr>
<tr>
<td>JJ6</td>
<td>REAL WOR(60), V(60)</td>
</tr>
<tr>
<td>JJ7</td>
<td>REAL GSM(16), OCHECK(16, 16)</td>
</tr>
<tr>
<td>JJ8</td>
<td>REAL EQ(35, 80), MA(80), WA(80)</td>
</tr>
<tr>
<td>JJ9</td>
<td>REAL M(120), E2(120), TH2(20), HK(20), KH(20), ME(20), NET(20),</td>
</tr>
<tr>
<td></td>
<td>1 MK2(20), SM(5)</td>
</tr>
<tr>
<td>JJ10</td>
<td>REAL VSAV(16, 20), WSAV(16, 20), PSAV(16, 20)</td>
</tr>
<tr>
<td>JJ11</td>
<td>REAL VV(80), VM(80), AHAI(35, 36), AHAI(35, 36), DAA(35, 36)</td>
</tr>
<tr>
<td>JJ12</td>
<td>1 READ 1000, IC1, IC2, IC3, IC4, HEAD</td>
</tr>
<tr>
<td>JJ13</td>
<td>1000 FORMAT(411, 19A4)</td>
</tr>
<tr>
<td>JJ14</td>
<td>IF(I1. EQ. 9) CALL EXIT</td>
</tr>
<tr>
<td>JJ15</td>
<td>PRINT 1001, IC1, IC2, IC3, IC4, HEAD</td>
</tr>
<tr>
<td>JJ16</td>
<td>1001 FORMAT(1H1, 10X, 100(1H4))/</td>
</tr>
</tbody>
</table>

1/19/77 //10X,19A4 //10X,100(1H4) //
1006 FORMAT (/10X,31H*** ORIGINAL MODES RESTORED *** //)
GO TO 100
9 NX=0
DO 10 I=1,21
READ 1005,X(I),IM,M(I),IE,E(I),IT,TH(I),IK,KM(I)
1005-FORMAT(F10.0,4(I1,F9.0))
IF(M(I),EQ.0) GO TO 20
NX=NX+1
W(KM(I))=AMAX0(W(I),1)
WX(I)=AMAX0(WX(I),1)
WT(I)=AMAX0(WT(I),1)
W(KM(I))=AMAX0(W(KM(I)),1)
CALL ERR(100,0)
20 PRINT 1010,(I1,X(I),W(KM(I)),M(I),WE(I),E(I),WT(I),TH(I),IK)
1 I=1,NX)
1010 FORMAT (110X,90HI STA W M W E
1 W TH W KM /(10X,OP 12,F12.3,
2 4(OPEF0.0,1PE12.3)
25 READ 1015, CALV,CALW,CALP,THO
1015 FORMAT (8F10.0)
N2=2*NX
N3=3*NX
N4=4*NX
If(CALV.EQ.0) CALV = 1
IF(CALW.EQ.0) CALW = 1
IF(CALP.EQ.0) CALP = 1
PRINT-1016-THO
1016 FORMAT (/10X,32HROOT PITCH ANGLE (ADDS TO TH) = 1PE12.3/ 1H1,
1 10X,25HINPUT MODES (CARD IMAGES) //
NH=0
PRINT 1017,CALV,CALW,CALP,THO
1017 FORMAT (/10X,8F12.5)
29 IF(1C1.EQ.2) PRINT-1019
1019 FORMAT (/H1,10X,2THADDED MODES CARD IMAGES //)
30 READ 1015,F,F,OF,O,CV,CW,CP
PRINT-1017,F,O,OF,O,CV,CW,CP
IF(F.EQ.0 AND O.EQ.0) GO TO 70
NH=NH+1
FREQ(NH)=F
0NEG(NH)=O
READ 1015,(V(NH,I),I=1,NX)
PRINT-1018,(V(NH,I),I=1,NX)
1018 FORMAT (10X,8F12.5)
READ 1015,(W(NH,I),I=1,NX)
C. APPLY CALIBRATION

IF (CV .EQ. 0) CV = CALV
IF (CW .EQ. 0) CW = CALW
IF (CP .EQ. 0) CP = CALP
I = NM

DO 45 J = 1, NX
45 VI (I, J) = VI (I, J) * CV

DO 55 J = 1, NX
55 WI (I, J) = WI (I, J) * CW

DO 65 J = 1, NX
65 PI (I, J) = PI (I, J) * CP

GO TO 30

DO 85 I = 1, NX
85 VS(AV (J, I)) = V (J, I)

DO 86 J = 1, NM
86 WS(AV (J, I)) = W (J, I)

C. PRINT-MODES

PRINT 1020
1020 FORMAT (1H1//50X,31HINPUT MODE SHAPES (CAL APPLIED )
CALL - MODES -(X,V,W,P,OME, FREQ,NM,NX,16)

90 AM = 0
AM = 0
AMET = 0
AMK = 0

SM(2) = 0
SM(3) = 0
SM(4) = 0

M(J) = ME(J) + TH(I) * THO
MK(I) = M(I) * KM(I) * 2
AM = AM + M(I)
SM(2) = SM(2) + M(I) * X(I)
SM(3) = SM(3) + ME(I)
SM(4) = SM(4) + M(I) * X(I) * 2
AME = AM + ABS (ME(I))
AMET = AMET + ABS (MET(I))

-95 AMK = AMK + MK(I)

SM(1) = AM
SM(5) = AMK

AM = AM / NX
AME = AM / NX
AMET = AMET / NX

AMK = AMK / NX

IF (AM . EQ. 0) CALL ERR (95, 0)
IF (AMK . EQ. 0) CALL ERR (96, 0)
IF(C2.EQ.0) GO TO 130

PRINT 1031
1031 FORMAT (1H1//30X,25HINPUT ORTHOGONALITY CHECK //)
CALL ORTH(V,W,P,M,ME,MET,M,K,NM,NX,16,GMASS,OCHECK,16,IC2)
IF(1C2.EQ.2) PRINT 1032
1032 FORMAT(40X,42H*** MODES REPLACED BY NORMALIZED MODES *** //)
IF(C2.EQ.2) CALL PMODES(X,V,W,P,OMEG,FREQ,NM,NX,16)
IF(C2.EQ.2.AND.IM1.NE.1) IC3=1

READ 1035,IO1,IO2,DUM
1035 FORMAT(2I1,F6.0,7F10.0)
GO TO (110,200,500,130),IO1

C FOR IO1=1
C WDI=UNIFORMLY DISTRIBUTED RANDOM ERROR HAVING A
C +/- MAXIMUM OF PERCENT ON AMPLITUDE
C WDO=BIAS ERROR OF PCTB ON AMPLITUDE
C IZ IS USED IN CALCULATING AN INTEGER RANDOM NUMBER
C USED IN SUBROUTINE RANDU

110 WDI=DUM(1)/100.
WDO=DUM(2)/100.
IZ=DUM(3)
IX=IZ*2+1
CALL ERR(A(V,WDI,WDO,NM,NX,IX,16)
CALL ERR(A(W,WDI,WDO,NM,NX,16)
CALL ERR(A(P,WDI,WDO,NM,NX,16)
PRINT 2050, DUM(1), DUM(2), IZ
2050 FORMAT(1H1//30X,27H*** RANDOM ERROR OPTION ***)
1 21HRANDOM NO SEED=110)
IF(C2.NE.0) CALL ORTH(V,W,P,M,ME,MET,M,K,NM,NX,16,GMASS,OCHECK,16,
IC2)
130 CALL ERR(130,0)

C CORRECT MODES ONLY 101 = 2
C ORIGINAL MODES UNDISTURBED
C CORRECTED MODES IN V2, M2, P2
C CHECK FREQUENCIES, MODES

200 PRINT 1040
1040 FORMAT(1H1//30X,18HMODE CHANGE-OPTION-//30X,26HPERCENTAGE CHANGES-1
1V(W,P) //20X,16HMODE 1 UNCHANGED )
1041 IF(DUM(1).EQ.1) PRINT1043
1042 IF(DUM(1).EQ.2) PRINT1044
1043 FORMAT(20X,21HLIMIT OPTION - SCALED )
1044 FORMAT(20X,24HLIMIT OPTION - TRUNCATED )
IF (INM*G1 = 8) CALL ERR(200,0)

OM = OMEG(1)
DO 210 I = 2, NM
IF (OMEG(I)*OM*OM) CALL ERR(210, J)

210 CONTINUE
C CHANGED MODE IN V2*, 2, P2
C FIRST MODE-UNCHANGED

C FORM A WITH COLUMN A IS COMPRESSED

250 M1 = N
N = N + 1
DO 260 I = 1, NX
A(I, M1) = M(I) * V2(M1, I) - MET(I) * P2(M1, I)
A(NX + 1, M1) = M(I) * W2(M1, I) + ME(I) * P2(M1, I)

260 A(N2 + 1, M1) = MET(I) * V2(M1, I) + ME(I) * W2(M1, I) + MK(I) * P2(M1, I)

C FORM COMPRESSED MTH MODE

DO 270 I = 1, NX
PHI(I) = V(N, I)
PHI(NX + 1) = W(N, I)
270 PHI(N2 + 1) = P(N, I)

DO 280 I = 1, N3
DO 280 J = 1, M1
280 B(I, J) = PHI(I) * A(I, J)

C C = BTRAN * B (M X M1)

DO 290 I = 1, M1
DO 290 J = 1, M1
290 C(I, J) = B(I, J) * B(I, J)

C INVERT C INTO D
IF (M1, NE, 1) GO TO 300
D(I,J) = 1.0 / C(I, J)

GO TO 310

300 CALL INVRS (C, M1, D, WORK, IROW, ICOL, 7, 8)

C ATRAN * PHI

310 DO 320 I = 1, M1
WOR(I) = 0
320 DO 320 J = 1, N3
WOR(I) = WOR(I) + A(J, I) * PHI(J)
320 CALL MXV (WOR, 50, WOR, M1, 1, 1, 1, 7, 0)

C WOR = FRACTIONAL CHANGE IN EACH ELEMENT
C PRINT PERCENT CHANGES

EMAX = 0
DO 330 I = 1, N3
WOR(I) = WOR(I) * 100.
330 EMAMX = AMAX1 (EMAX, ABS(WOR(I)))
PRINT 1050, N, EMAX

1050 FORMAT (12X, 5H, MODE, 12, 10H, MAX CHANGE F8.1)

IF (DUM(1) .EQ. 0) GO TO 331
3198 IF(DUM(N).NE.0) PRINT 1051,DUM(N)
3199 1051 FORMAT(45X,18HMAX ALLOWED CHANGE F6.2)
3200 PRINT 1055,(WOR(I),I=1,NX)
3201 I1 = NX+1
3202 I3 = N2+1
3203 PRINT 1055,(WOR(I),I=1,N2)
3204 1055 FORMAT(420X,10F10.3)
3205 PRINT 1055,(WOR(I),I=1,N3)
3206 TEMP = .01
3207 IF(DUM(I).EQ.0) GO TO 335
3208 IF(DUM(N).EQ.0 OR EMAX.NE.DUM(N)) GO TO 335
3209 IF(DUM(I).EQ.2) GO TO 342
3210 TEMP = .01*DUM(N)/EMAX
3211 335 DO 340 I = 1,N3
3212 340 PHI(I) = PHI(I)*I1*TEMP*WOR(I)
3213 GO TO 349
3214 342 DO 345 I = 1,N3
3215 IF(WOR(I).GT.0) WOR(I) = AMIN(WOR(I),DJM(N))
3216 IF(WOR(I).LT.0) WOR(I) = AMAX(WOR(I),-DUM(N))
3217 345 PHI(I) = PHI(I)*I1*TEMP*WOR(I)
3218 349 DO 350 I = 1,NX
3219 V2(N+I) = PHI(I)
3220 W2(N+I) = PHI(NX+I)
3221 350 P2(N+I) = PHI(N+I)
3222 IF(N+LT,NM) GO TO 250
3223 355 PRINT 1060
3224 1060 FORMAT(1H1 // 30X,15HCORRECTED MOCES //)
3225 CALL PMODES((X,Y2,W2,P2,OMEG,FREQ,NM,NX,16))
3226 370 IF (IC2.EQ.0) GO TO 1
3227 PRINT 1061
3228 1061 FORMAT(1H1 // 30X,30HCORRECTED-ORTHOGONALITY CHECK //)
3229 CALL ORTH (V2,W2,P2,M,ME,PET,PK,NM,NX,16,GMASS,OCHECK,16,IC2)
3230 IF (IC2.EQ.2) CALL PHREMOS (X,Y2,W2,P2,OMEG,FREQ,NM,NX,16)
3231 IF (IC2.EQ.0) GO TO 1
3232 DO 380 I = 1,NX
3233 DO 380 J = 1,NM
3234 W1J,J = V2(J,J)
3235 W1J,J = W2(J,J)
3236 380 PHI(J,J) = P2(J,J)
3237 PRINT 1065
3238 1065 FORMAT(/10X,47HORIGINAL DATA REPLACED BY MODIFIED DATA ***
3239 1 //)
3239 GO TO 1
3240 C MASS ONLY SI IO1 = 3
3240 C ORIGINAL MOCIS PARAMETERS UNDISTURBED
3240 C CORRECTED VALUES IN M2, E2, F2, K2
3240 C SET UP EQUATION PAIRS
3241 500 NEQ = 0
3242 NSI = 0
3243 NM1 = NM-1
3244 DO 510 I = 1,NM1
3245 IF (OMEG(I).NE.OMEG(I)) GO TO 510
0262 520 DO 550 I = 1,NEQ
0263 550 JJ = JEQ(I,1)
0265 DD 550 J = 1,NX
0268 EQ(I+1,J) = -V[I,J]*P(JJ,J) + W(JJ,J) + P[I,J]
0269 DD 551 I = 1,NEQ
0270 551 W(V) = 0.
0272 IF(DUM(2),EQ=0) GO TO 553
0273 PRINT 2001,SM(1)
0274 2001 FORMAT (33X,36HTOTAL MASS INVARIANT AT 510.3 )
0275 NEQ = NEQ+1
0276 W(V) = SM(1)
0277 DD 552 I = 1,NX
0278 EQ(NEX+I) = 1.0
0279 EQ(NEX+NX+I) = 0.0
0280 EQ(NEX+NX+2+I) = 0.0
0281 EQ(NEX) = -0.0
0282 IF(DUM(3),EQ=0) GO TO 555
0283 TEMP = SM(2)/SM(1)
0284 PRINT 2002, TEMP
0285 2002 FORMAT (30X,36HRADIAL CG INVARIANT AT 510.2 )
0286 NEQ = NEQ+1
0287 W(V) = SM(2)
0288 DD 554 I = 1,NX
0289 EQ(NEX+I) = X(I)
0290 EQ(NEX+NX+I) = 0.0
0291 EQ(NEX+NX+2+I) = 0.0
0292 EQ(NEX+NX+2+I) = 0.0
0293 IF(DUM(4),EQ=0) GO TO 557
0294 TEMP = SM(3)/SM(1)
0295 PRINT 2003, TEMP
0296 2003 FORMAT (30X,36HCRCREWSE CG INVARIANT AT 510.4 )
0297 NEQ = NEQ+1
0298 W(V) = SM(3)
DO 556 I = 1, NX
EQ(NEQ, I) = 0.0
EQ(NEQ, NX+I) = 1.0
EQ(NEQ, N2+I) = 0.0
556 EQ(NEQ, N3+I) = 0.0
557 IF(DUM(5), EQ.0) GO TO 559
559 PRINT 2005, SM(4)
2004 FORMAT (30X, 34HFLAPPING MOM OF INERT INVARIANT AT FI1.2)
NEQ = NEQ + 1
558 W(VNEQ) = - SM(4)
DO 559 I = 1, NX
EQ(NEQ, I) = X(I) * Y
EQ(NEQ, NX+I) = 0.0
EQ(NEQ, N2+I) = 0.0
558 EQ(NEQ, N3+I) = 0.0
559 IF(DUM(6), EQ.0) GO TO 565
PRINT 2005, SM(5)
2005 FORMAT (30X, 36HFEATHERING MOM OF INERT INVARIANT AT FI0.4)
NEQ = NEQ + 1
556 W(VNEQ) = - SM(5)
DO 559 I = 1, NX
EQ(NEQ, I) = 0.0
EQ(NEQ, NX+I) = 0.0
EQ(NEQ, N2+I) = 0.0
560 EQ(NEQ, N3+I) = 1.0
565 N41 = N4
569 IF(DUM(1), EQ. 2.) N41 = N4 - NSI
PRINT 2006, NEQ, N41
2006 FORMAT (30X, 1HTOTAL EQUATIONS = I5, 18H, NO OF UNKNOWNS = I4/)
IF(IC3, EQ.0) GO TO 580
PRINT 2010
2010 FORMAT (30X, 35HEQUATION COEFFICIENTS FOR MASS SI/)
DO 570 J = 1, NEQ
PRINT 2020, (EQ(I, J), J = 1, N4)
2020 FORMAT (30X, 1IP10E12, 3)
FORM COMPRRESSED MA MATRIX
580 DO 590 I = 1, NX
MA(I) = M(I)
MA(NX+I) = ME(I)
MA(N2+I) = ME(T)
590 MA(N3+I) = MK(I)
FORM INVERSE, COMPRESSED PERCENTAGE WEIGHTED WEIGHTING FUNCTION
IF(DUM(1), EQ. 1.) GO TO 602
600 DO I = 1, NX
WA(I) = M(I) / WK(I)
WA(NX+I) = ME(I) / WE(I)
602 WA(N2+I) = M(T) / WT(I)
IF(ME(I), EQ.0) WA(N2+I) = AME / WT(I)
WA(N3+I) = MK(I) / WK(I)
0347        IF(MK(I),EQ,0) WA(N3+I)=AMK/WK(I)
0348        600 CONTINUE
0349        IF(N51,LE,0) GO TO 609
0350        DO 601 I=1,N51
0351        J=NIN(I)
0352        IF(J,LE,0,OR, J,GT,NX) CALL ERR(601,0)
0353        WA(J)=0
0354        WA(NX*J)=0
0355        WA(N2*J)=0
0356        601 WA(N3+J)=0
0357        GO TO 609
0358        602 DQ 605 I = 1,NX
0359        WA(I)=M(I)
0360        WA(NX*I)=ME(I)
0361        IF(ME(I),EQ,0) WA(NX+I)=AME
0362        WA(N2+I)=MET(I)
0363        IF(MET(I),EQ,0) WA(N2+I)=AMET
0364        WA(N3+I)=MK(I)
0365        IF(MK(I),EQ,0) WA(N3+I)=AMK
0366        605 CONTINUE

C

0367        609 DQ 610 I=1,NEQ
0368        DO 610 J=1,NEQ
0369        AW(A(I,J))=0
0370        DO 610 L=1,N4
0371        610 AWA(I,J)=AWA(I,J)*EQ(I,L)*EQ(J,L)*WA(L)*W(L)

C

0372        IF(I3EQ,0) GO TO 612

0373        PRINT 2021
0374        2021 FORMAT (20H1 // 30X,2INH MATRIX TO BE INverted //)
0375        DO 611 I=1,NEQ
0376        611 PRINT 2020,AWA(I,J),J=1,NEQ
0377        612 CALL INVRS (AWA,NEQ,AWA1,OWA1,ICOL,35,36)
0378        IF(I3EQ,0) GO TO 615

0379        PRINT 2022
0380        2022 FORMAT (20H1 // 30X,14H INVERSE //)
0381        DD 614 I=1,NEQ
0382        614 PRINT 2020,AWA1(I,J),J=1,NEQ
0383        615 DD 618 I=1,NEQ
0384        DO 618 J=1,N4
0385        618 W(V(I))=EQ(I,J)+WA(J)*W(V(I))
0386        IF(I3EQ,0) GO TO 619

0387        PRINT 2023
0388        2023 FORMAT (20H1 // 30X,12N2L**MA (TRAN) //)
0389        PRINT 2020,N(V(I)),I=1,NEQ)
0390        619 DD 625 I=1,NEQ
0391        DM(I)=0
0392        DO 625 J=1,NEQ
0393        625 DM(I)=BM(I)*AWA1(I,J)*W(V(J))

C

FORM W(V)=EQ(T)*DM THEN DM=DELTA MASS
0394        DO 620 I=1,N4
0395        620 W(V(I))=0
0396        DO 620 J=1,NEQ
0397        620 W(V(I))=W(V(I)+EQ(J,I)*DM(J))
DO 630 I = 1, N4

630 DM(I) = -WV(I)*WA(I)**2

C FORM CORRECTED CHARACTERISTICS

DO 640 I = 1, NX

640 M2(I) = M(I)*DM(I)

ME2(I) = ME(I) + DM(NX*I)

E2(I) = ME2(I)/M2(I)

MET2(I) = MET(I) + DM(N2*I)

IF (ME2(I).EQ.0) GO TO 635

TH2(I) = MET2(I)/ME2(I) - TH0

GO TO 636

635 TH2(I) = TH(I)

636 MK2(I) = MK(I) + DM(N3*I)

637 TEMP = MK2(I)/M2(I)

IF (TEMP.GE.0) GO TO 639

KM2(I) = -SQRT(-TEMP)

GO TO 640

639 KM2(I) = SQRT(TEMP)

640 CONTINUE

C COMPUTE PCT CHANGES IN AWA

DO 650 I = 1, NX

650 AWA(I) = DM(I)/M(I) * 100.

AWA(I, 4) = (KM2(I) - KM(I))/KM(I) * 100.

IF (TH(I).EQ.0) GO TO 647

AWA(I, 3) = (TH2(I) - TH(I))/TH(I) * 100.

GO TO 648

647 AWA(I, 3) = 100.

IF (TH2(I).EQ.0) AWA(I, 3) = 0.

648 IF (E2(I).EQ.0) GO TO 649

649 AWA(I, 2) = 100.

IF (E2(I).EQ.0) AWA(I, 2) = 0.

650 CONTINUE

C PRINT CHANGED VALUES

PRINT 2030

2030 FORMAT (1H1/130H - ORIG M - NEW M - PCT - ORIG E -

1 NEW E PCT ORIG TH NEW TH PCT ORIG KN

2 NEW KN PCT)

DO 655 I = 1, NX

655 PRINT 2040, I, M(I), M2(I), AWA(I, 1), E(I), E2(I), AWA(I, 2),

TH(I), TH2(I), AWA(I, 3), KM(I), KM2(I), AWA(I, 4)

2040 FORMAT (13, 9.1, 3, 9.1, 3, 10F7.1)

C ORTH CHECK

IF (IC2.EQ.0) GO TO 1

1061 PRINT 1061

1032 PRINT 1032

1039 CALL PHOMDES (X, V, H, P, ONEG, FREQ, KM, NX, 16)

1040 IF (IC2.EQ.0) CALL PHOMDES (X, V, H, P, ONEG, FREQ, KM, NX, 16)

1041 DO 660 I = 1, NX

660 M(I) = M2(I)

1042 E(I) = E(I)

1043 TH(I) = TH2(I)

141
3445 $K_M(I) = K_M(2(I))$
3446 $N_E(I) = N_E(2(I))$
3447 $N_M(I) = N_M(2(I))$
3448 $N_K(I) = N_K(2(I))$
3449 PRINT 1065
3450 GO TO 1
3451 END
SUBROUTINE _RMODES_ (X, V, W, P, OMG, FREQ, NM, NX, NDIM)

REAL X(1), V(NDIM, 1), W(NDIM, 1), P(NDIM, 1), OMG(1), FREQ(1)

IM0 = 1

IM1 = MIN(NM, 3)

75 PRINT 1025, (OMG(I), I=IM0, IM1)
1025 FORMAT (/13X, 8HOMEGA = , F18.3, 2F39.3)

PRINT 1026, (FREQ(I), I=IM0, IM1)

1026 FORMAT (/13X, 7HFREQ = , F18.3, 2F39.3)

DO 80 I = 1, NX
80 PRINT 1030, I, X(I), (V(I,J), J = 1, IM0), (W(I,J), J = 1, IM1)

1030 FORMAT (1X, 12, 0P F10.3, 3(3X, 1P 3E12.3))

IF (IM1.GE.NM) GO TO 90

IM0 = IM0 + 3

IM1 = MIN(NM, IM1 + 3)

IF (IM0.EQ.4 .OR. IM0.EQ.10 .OR. IM0.EQ.16) GO TO 75

PRINT 1020

1020 FORMAT (1H1, 50X, 11HPRODUCE SHAPES /)

GO TO 75

90 RETURN

END

SUBROUTINE _ERR(N, I_

C I = 0, TERMINATES RUN I NE O WARNING ONLY, PRINTS I

PRINT 10, N
10 FORMAT (/10X, 17H***ERROR NUMBER = , I5, 5H ***

IF (I.NE.0) GOTO 20

CALL EXIT

20 PRINT -30, I

30 FORMAT (20X, 20H*** WARNING ONLY *** , I5//)

31 RETURN

END
SUBROUTINE CRTINV(V, W, M, ME, MET, MK, NM, NX, MDIM, GMASS, OCHECK, MCDIM, IP)

PERFORMS ORTHOGONALITY CHECK

GMASS ARE DIAGONAL ELEMENTS

OCHECK IS NORMALIZED BY DIVIDING ROW, COL BY SQRT

OF DIAGONAL

IP.NE.0 GMASS, OCHECK ARE PRINTED
IP.EQ.2 MODES ARE NORMALIZED (GEN MASS = 1.0)

REAL V(MDIM,1), W(MDIM,1), P(MDIM,1), ME(1), MET(1), MK(1), GMASS(1),
1 OCHECK(MCDIM,1), M(1)

DO 20 I = 1, NM
DO 20 J = 1, NM
OCHECK(I, J) = 0
DO 20 I = 1, NM
OCHECK(I, J) = OCHECK(I, J) + V(I, L) * M(L) * V(J, L) - P(I, L) * MET(L) * V(J, L)
1 + W(I, L) * M(L) * W(J, L) + P(I, L) * ME(L) * W(J, L) - V(I, L) * MET(L) * P(J, L)
2 + W(I, L) * ME(L) * P(J, L) + P(I, L) * MK(L) * P(J, L)

DO 30 I = 1, NM
GMASS(I) = OCHECK(I, I)

SQ = SQRT(GMASS(I))

IF(IP.NE.2) GO TO 29
DO 25 L = 1, NX
V(I, L) = V(I, L) / SQ
DO 25 L = 1, NX
W(I, L) = W(I, L) / SQ
DO 29 J = 1, NM
OCHECK(I, J) = OCHECK(I, J) / SQ

30 OCHECK(I, J) = OCHECK(I, J) / SQ
29 IF(IP.EQ.0) RETURN
PRINT 100, GMASS(I), I = 1, NM

100 FORMAT (2X, 1HDIAGONAL ELEMENTS OF ORTHO CHECK MATRIX /)
1 (10X, 1PBE14.3))

PRINT 200

200 FORMAT (1H/2X, 10HNORMALIZED ORTHO CHECK MATRIX /)

DO 30 I = 1, NM
40 PRINT 300, OCHECK(I, J), J = 1, NM
30 FORMAT (2X, 16F8.3)

IF(IP.EQ.2) PRINT 350
PRINT 350 FORMAT (1H1, 30X, 16HNORMALIZED MODES //)
RETURN
END
SUBROUTINE INVRSD (B, N, A, D, IROW, ICOL, NRW, NCL)
C A = INVERSE OF B  B UNDISTURBED
C VARIABLE DIMENSIONS  NCL MUST BE AT LEAST ONE GREATER THAN NRW
C NRW MUST BE AT LEAST EQUAL TO N
C IROW, ICOL ARE VECTORS OF LENGTH NCL

REAL A(NRW,NCL), B(NRW,NCL), D(NRW,NCL)
INTEGER IROW(NCL), ICOL(NCL)

DO 1 1=1,N
DO 1 J=1,N
1 A(I,J)=0 {I,J}
M=N+1
DO 7 1=1,N
7 IROW(I)=I
ICOL(I)=I
DO 12 10 K=1,N
AMAX=A(I,K)
12 DO 10 I=K,N
10 DO 10 J=K,N
IF (ABS(A(I,J))-ABS(AMAX)) LT 10,9,9
9 AMAX=A(I,J)
IC=I
JC=J
10 CONTINUE
K I=ICOL(K)
ICOL(K)=ICOL(IC)
ICOL(IC)=K
KI=IROW(K)
IROW(K)=IROW(JC)
IROW(JC)=K
IF (AMAX) LT 11,12,13
12 PRINT 13
13 FORMAT (* SOLUTION OF MATRIX NOT POSSIBLE *)
GO TO 100
DO 14 J=1,N
14 E=A(K,J)
A(K,J)=A(IC,J)
14 A(IC,J)=E
DO 15 I=1,N
15 E=A(I,K)
A(I,K)=A(I,JC)
A(I,JC)=E
DO 16 I=1,N
16 IF (I-K) LT 18,17,18
17 A(I,M)=1.
GO TO 16
18 A(I,M)=0.
19 CONTINUE
PVT=A(K,K)
DO 30 J=1,M
28 A(K,J)=A(K,J)/PVT
30 DO 20 I=1,N
31 IF (I-K) LT 121,19,21
21 AMUL=A(I,K)
30 DO 22 J=1,N
32

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22. \( A(I+J) = A(I+J) - A \times A(K+J) \)

19 CONTINUE

DO 20 I = 1, N

20. \( A(I+K) = A(I+K) \)

DO 25 I = 1, N

DO 24 L = 1, N

DO 28 I = 1, N

DO 28 L = 1, N

IF (IROW(I) - L) 24, 23, 24

CONTINUE

DO 25 J = 1, N

DO 26 J = 1, N

28 CONTINUE

DO 26 I = 1, N

29 CONTINUE

END
SUBROUTINE MXV(A,B,C,M,N,NDIM,ICNT)

C MATRIX TIMES VECTOR A(M)=B(M,N)*C(N) FOR ICNT = 0
   +A(M) FOR ICNT = 1

C

DIMENSION A(1), B(NDIM,1), C(1)

DO 10 I=1, M
   IF(ICNT.EQ.0) A(I)=0
   DO 10 J=1, N
      10 A(I)=A(I)+B(I,J)*C(J)
RETURN
END

SUBROUTINE ERRAL(A,PCT,PCTB,NJ,NM,IX,NDIM)

C A BIAS ERROR PCTB(RATIC) ON AMPLITUDE AND A UNIFORMLY DISTRIBUTED
C RANDOM ERROR HAVING A +/- MAXIMUM OF PCT(RATIO) ON AMPLITUDE

DIMENSION A(NDIM,1)

IF(PCT.NE.0) GO TO 110

100 IF(.PCTB.EQ.0) GO TO 130

110 DO 120 K=1, NM
   DO 120 I=1, NJ
      CALL RANDU(I1,1,1,YFL-1)
      IF(YFL.EQ.1) 120
      E=1+2*(YFL-.5)*PCTB
      120 A(K,I)=A(K,I)*E
RETURN
END
SUBROUTINE RANDU (IX, IY, YFL)

USAGE

CALL RANDU (IX, IY, YFL)

COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN
0-AND-1.0 AND RANDOM REAL INTEGERS BETWEEN 0-AND-2**31.

EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER AND
PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.

VARIABLES

IX = FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER NUMBER
WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY IX SHOULD BE
THE PREVIOUS VALUE OF IY COMPUTED BY THIS SUBROUTINE

IY = A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT ENTRY
TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS BETWEEN 0 AND 2**31

YFL = THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT, RANDOM
NUMBER IN THE RANGE 0-TO-1.0

0002  IY=IX*65539
0003  IF(IY) 100,110,110
0004  100 IY=IY+2147483647+1
0005  110 YFL=IY
0006  YFL=YFL*.4656613E-9
0007  RETURN
0008  END
APPENDIX D
NORMAL MODES AND NATURAL FREQUENCIES OBTAINED FROM VACUUM WHIRL DATA*

INTRODUCTION

The rotor was forced vertically along the axis of rotation with no other external forces. The natural frequencies of the symetric flapping modes with infinite hub impedance are the driving point antiresonant frequencies along the rotational axis. These frequencies were identified and a modal analysis done to determine the mode shapes using strain/hub acceleration transmissibility in the following manner.

Strain readings, calibrated in terms of bending moments, and hub vertical accelerations were recorded simultaneously on analog tape at the selected rotational speeds of 0, 5.24, 10.47 and 15.71 rod/sec. (0, 50, 100 and 150 RPM). The time domain hub acceleration signal was fed from the tape reader to the force input of a Fast Fourier Transform Digital Signal Analyzer, type Hewlett Packard 5420, while the time domain strain signal from the $j^{th}$ station along the blade was fed to the response input of the Digital Signal Analyzer for stations $j = 1$ to $j = 12$ at each of the rotor RPM settings. Over a narrow band of frequency covering each hub antiresonant frequency, determined approximately from broad band analysis in which the hub driving point antiresonant frequencies appear in the Fourier Transform in the form of natural frequencies, a Fourier Transform of $2^8$ frequency line was obtained for each strain/hub acceleration transfer function. The narrow band data were then analyzed for global properties.

The transmissibility residues for the 12 blade stations in a given mode were found to be complex, due to the nature of the transfer function, but complex normalization showed the bending moment modes to be real (classical). The deflection modes were obtained from the bending moment modes by simple double trapezoidal integration of the curvature from the root to the tip.

*The tests from which this data were obtained are described in Ref. 9
The Antiresonant Method. - It is obviously impossible to achieve infinite terminating impedance in practice but the modal effects of infinite terminating impedance along a single motion coordinate can be obtained quite accurately through antiresonance theory even though the terminating coordinate never reaches absolutely zero motion. It never reaches absolute zero because, and only because, in this case, the rotor dissipates energy to a sink. The nature of this energy dissipation, called "damping", is not known. If the rotor were undamped the vertical motion along the axis of rotation, the coordinate of sole external excitation, would be absolutely zero at the natural frequencies of the symmetric flapping modes of infinite hub impedance regardless of the actual hub impedance. The sum of the inertial forces of the undamped rotor acting vertically on the hub would, at these frequencies, be exactly equal to the sole excitation force acting vertically at the hub, regardless of its magnitude (within the linear range) or phasing to any base, in the steady state. This is the principle of the undamped vibration absorber of 1909; its notable early 19th century predecessor, the una corda or "soft" pedal on aftermath of the concert grand piano; the Thearle invention of the 1930 on which shaft and turbine balancing machines are based; the 1947 method of stabilization by Thor which made spin dry home washing machines practical and the many obvious helicopter applications along with the less obvious one recently in which a military helicopter initially had little pilot seat vibration at the expense of intolerable tail fatigue.

Mathematically, a damped antiresonance is merely a zero of zero magnitude. In the case at hand the single excitation along the axis of rotation is unknown (because the measured applied force in the rig is below the hub with an intervening unknown impedance) but as it is the same for hub vertical acceleration and blade bending moment the quotient of blade bending mobility and hub acceleration mobility involves cancellation of the pole roots leaving the denominator a polynomial whose roots are hub driving point zeros the undamped parts of which are the desired antiresonances. These can be determined from the Fourier Transform of the transfer function as will be shown below.
From elementary considerations of complex variable theory it is easily seen that the residues are without physical significance in themselves because the polynomial quotient has an arbitrary factor. For this reason one cannot use this procedure to obtain physically meaningful orthonormal modes. However, in normalizing on a station on the blade the arbitrary factor of the multiplying factor cancels, being the same for each station, and a valid bending-moment mode shape can be readily obtained. That is, the validity of the quotient of residues is maintained. This is precisely the same as ratioing the vectorial chords of the Nyquist plots of each blade station between given frequencies in the zero root range of the hub mobility to that of any given blade station.

Because the complex chordal vectors between given frequencies are parallel to the modal diameter of any transmissibility having the hub driving point product of roots of the zeros in the denominator and because the length of such chords are necessarily proportional to their associated diameters each it follows that the ratio of the complex chordal vectors is the same as that of the complex diametral vectors. In other words, if one were to transfer the Nyquist axes to an origin corresponding to the antiresonant frequency, do a bilinear transformation and ratio the distances of the resulting lines to the origin for any station to a given blade station one would find a canonical invariance of the polynomial in the poles and the frequency invariant factor for any given pole.
Finding the Natural Frequency. - Most often one will find three peaks in mobility associated with a mode, two in the real and one in the imaginary or vice versa. If the angle of a complex mode is near 45°, 135°, 225° or 315° there will be only two sharp peaks, one in the real and one in the imaginary.

The following is done for acceleration mobility. \( q \) refers to a frequency in the imaginary and \( p \) to a frequency in the real. The subscript \( x \) refers to an acceleration mobility maximum and \( m \) to an acceleration mobility minimum.

If the modal angle is in the range from about -40° to about +40° or narrower there will be a maximum in the acceleration imaginary and a minimum and maximum in the real.

\[
2 q_x^2 - \frac{p_x^2 + p_m^2}{2} = \Omega^2 [1 + g(2 \tan \phi/2 - \tan \phi)]
\]  
(D-1)

Let the natural frequency be approximated by

\[
\Omega^2 \approx 2 q_x^2 - \frac{p_x^2 + p_m^2}{2}
\]  
(D-2)

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( g = .02 )</th>
<th>( g = .05 )</th>
<th>( g = .10 )</th>
<th>( g = .20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°</td>
<td>0.22%</td>
<td>0.56%</td>
<td>1.11%</td>
<td>2.22%</td>
</tr>
<tr>
<td>30°</td>
<td>0.08%</td>
<td>0.21%</td>
<td>0.41%</td>
<td>0.83%</td>
</tr>
<tr>
<td>20°</td>
<td>0.02%</td>
<td>0.06%</td>
<td>0.11%</td>
<td>0.23%</td>
</tr>
<tr>
<td>10°</td>
<td>0.003%</td>
<td>0.006%</td>
<td>0.01%</td>
<td>0.03%</td>
</tr>
<tr>
<td>0°</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>-10°</td>
<td>0.003%</td>
<td>0.006%</td>
<td>0.01%</td>
<td>0.03%</td>
</tr>
<tr>
<td>-20°</td>
<td>0.02%</td>
<td>0.06%</td>
<td>0.11%</td>
<td>0.23%</td>
</tr>
<tr>
<td>-30°</td>
<td>0.08%</td>
<td>0.21%</td>
<td>0.41%</td>
<td>0.83%</td>
</tr>
<tr>
<td>-40°</td>
<td>0.22%</td>
<td>0.56%</td>
<td>1.11%</td>
<td>2.22%</td>
</tr>
</tbody>
</table>
If the modal angle is in the range from 50° to 130° one will observe a $p_m$, $q_x$ and $q_m$ with the identical errors over the range as given in Table D-I by adding 90° to the angle. Similarly for the other cases.

\[ \Omega^2 = 2p_x^2 - \frac{q_x^2 + q_m^2}{2} \]  \hspace{1cm} (D-3)

\[ \Omega^2 = 2q_m^2 - \frac{p_x^2 + p_m^2}{2} \]  \hspace{1cm} (D-4)

\[ \Omega^2 = 2q_x^2 - \frac{p_x^2 + p_m^2}{2} \]  \hspace{1cm} (D-5)

Equation D-2, D-3, D-4 and D-5 involve frequencies merely as twice the square of the single peak frequency less half the sum of the squares of the double peak frequencies.

![Diagram](image)

Figure D-1. A diagram of acceleration mobility peak frequencies.
Two Peaks Only - Natural Frequency. - If there is only one real and one imaginary peak associated with a mode, the modal angle must be near $45^\circ + n \times 90^\circ$ for $n = 0, 1, 2, 3$ as seen in Figure D-1.

For $n = 0$

$$q^2_x + p^2_m = \Omega^2 [2 + g (\tan \phi/2 - \cot \frac{\phi + \pi/2}{2})]$$  \hspace{1cm} (D-6)

Let the natural frequency be approximated by

$$\Omega^2 = \frac{q^2_x + p^2_m}{2}$$  \hspace{1cm} (D-7)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$g = .02$</th>
<th>$g = .05$</th>
<th>$g = .10$</th>
<th>$g = .20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \times 90^\circ + 35^\circ$</td>
<td>0.206%</td>
<td>0.516%</td>
<td>1.04%</td>
<td>2.096%</td>
</tr>
<tr>
<td>$n \times 90^\circ + 40^\circ$</td>
<td>0.102%</td>
<td>0.257%</td>
<td>0.514%</td>
<td>1.034%</td>
</tr>
<tr>
<td>$n \times 90^\circ + 45^\circ$</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$n \times 90^\circ + 50^\circ$</td>
<td>0.102%</td>
<td>0.257%</td>
<td>0.514%</td>
<td>1.034%</td>
</tr>
<tr>
<td>$n \times 90^\circ + 55^\circ$</td>
<td>0.206%</td>
<td>0.516%</td>
<td>1.04%</td>
<td>2.096%</td>
</tr>
</tbody>
</table>

The actual inherent error in natural frequency is about half those in Table D-II.

Local Spectrum Analysis of a Complex Mode Given the Natural Frequency

This procedure may be used over any portion of the modal arc. In an acceleration mobility Kennedy-Pancu plot let $N$ be the natural frequency and $f_1$ be any frequency on the modal arc selected by the operator. The chord from frequency $f_1$ at $N \sqrt{T - b}$ to frequency $f_2 = N \sqrt{T + b}$ over an arc of $180^\circ$ or less is perpendicular to a diameter through the natural frequency, $b$ is an arbitrary number less than unity. See Figure D-2.
The modal angle is $\phi$.

$$\frac{c/2}{D-h} = \tan \frac{\alpha}{2}$$  \hspace{1cm} (D-8)

For practical purposes (see the mensuration section of any standard engineering handbook)

$$\frac{c/2}{D-h} = \frac{2h}{c} = \tan \frac{\alpha}{2}$$  \hspace{1cm} (D-9)

and

$$c = D \sin \alpha.$$  \hspace{1cm} (D-10)

$$c = \sqrt{\left(\gamma_2^R - \gamma_1^R\right)^2 + \left(\gamma_2^I - \gamma_1^I\right)^2}$$  \hspace{1cm} (D-11)
\[ y_A^R = \left( y_2^R + y_1^R \right)/2, \quad y_A^I = \left( y_2^I + y_1^I \right)/2 \]  \hspace{1cm} (D-12) \\
\[ h = \sqrt{\left( y_N^R - y_A^R \right)^2 + \left( y_N^I - y_A^I \right)^2} \]  \hspace{1cm} (D-13) \\
\[ e_1 = \sqrt{\left( y_N^R - y_1^R \right)^2 + \left( y_N^I - y_1^I \right)^2} \]  \hspace{1cm} (D-14) \\
\[ e_2 = \sqrt{\left( y_N^R - y_2^R \right)^2 + \left( y_N^I - y_2^I \right)^2} \]  \hspace{1cm} (D-15) \\

If \( e_2/e_1 \leq 1.0 \) then \( N \) is not the natural frequency for points 1 and 2 on the modal arc. If \( e_2/e_1 < 1.0 \) then the natural frequency is less than \( N \), if \( e_2/e_1 > 1.0 \) then the natural frequency is greater than \( N \). 

\[ \frac{f_2^2}{N^2} = 1 + g \tan \frac{\alpha}{2} \]  \hspace{1cm} (D-16) \\
\[ \frac{f_1^2}{N^2} = 1 - g \tan \frac{\alpha}{2} \] \\
\[ \frac{f_2^2 - f_1^2}{N^2} = 2 g \tan \frac{\alpha}{2} \]  \hspace{1cm} (D-17) \\
\[ g = \frac{1}{2} \frac{f_2^2 - f_1^2}{N^2 \tan \alpha/2} \]
The natural frequencies determined from HP 5420 data using Equations D-2 through D-5 are shown in Table D-III in comparison to the natural frequencies found by NASA. The strain data for 100 RPM was quite noisy and was therefore not analyzed. Figures D-3 through D-11 show the bending moment normal modes and Figures D-12 through D-20 show the normalized deflection mode shapes.

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Frequency</th>
<th>Record Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Flapping</td>
<td>8.2 NASA</td>
<td>8.7 NASA</td>
</tr>
<tr>
<td></td>
<td>8.16 (1,1)</td>
<td>8.46 (1,37)</td>
</tr>
<tr>
<td></td>
<td>8.18 (2,41)</td>
<td>8.47 (3,19)</td>
</tr>
<tr>
<td>3rd Flapping</td>
<td>21.8 NASA</td>
<td>22.2 NASA</td>
</tr>
<tr>
<td></td>
<td>21.71 (1,10)</td>
<td>21.93 (2,1)</td>
</tr>
<tr>
<td></td>
<td>21.82 (3,1)</td>
<td>21.97 (3,26)</td>
</tr>
<tr>
<td></td>
<td>21.81 (2,48)</td>
<td>21.93 (1,46)</td>
</tr>
<tr>
<td>4th Flapping</td>
<td>41.2 NASA</td>
<td>42.0 NASA</td>
</tr>
<tr>
<td></td>
<td>41.66 (1,19)</td>
<td>41.92 (2,5)</td>
</tr>
<tr>
<td></td>
<td>41.73 (3,5)</td>
<td>41.99 (3,33)</td>
</tr>
<tr>
<td></td>
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<tr>
<td>1st Torsion</td>
<td>26.6 NASA</td>
<td>27.4 NASA</td>
</tr>
<tr>
<td></td>
<td>26.41 (1,28)</td>
<td>27.02 (2,14)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27.02 (3,40)</td>
</tr>
</tbody>
</table>
RECOMMENDATIONS

If this test were to be repeated it would be useful to measure strain on the hub near the center of rotation to provide the initial condition for integration of strains and it would be practical to calibrate in terms of the differential strains of the bending bridges, instead of bending moment, to eliminate the need for theoretical EI values in the integration.

In the photographic method of obtaining mode shapes the assumption is that the modes are uncoupled, that is, that the shaking excites only one mode. With that assumption, a promising method of obtaining rotating mode shapes is that pioneered by Hassal of the Royal Aircraft Establishment:

\[
\{q(R)\} = \Phi \{\phi(\varepsilon)\} + \{\varepsilon(R)\}
\]

where \(\varepsilon(R)\) is the vector of blade strains measured in rotation

\(\Phi\) is the matrix of nonrotating normalized normal translational modes

\(\phi(\varepsilon)\) is the matrix of nonrotating normalized normal strain modes

Normalization of the Left Hand Side at a natural frequency, given very light damping and widely separated natural frequencies, would be the rotating normal mode. \(\Phi\) and \(\phi(\varepsilon)\) are obtained in a nonrotating shake test after which the accelerometers are removed from the blade and have the same number of columns but not necessarily the same number of rows. The strains used need not be directly related to bending moments.

CONCLUSIONS

The rotating and nonrotating modes in flatwise bending for the cantilever condition were found to be real. The natural frequencies found in bending moment modal analysis agreed closely with those found by other methods.
3rd FLAPPY BENDING MOMENT MODE
150 RPM

BLADE STATION
Figure D-10

X/R
0

1.2
0.8
0.4
0.2
0.1
4th FLAPWISE BENDING MOMENT MODE
150 RPM

BLADE STATION
Figure D-11

\[ x/R \]

\[ .1 \quad .2 \quad .3 \quad .4 \quad .5 \quad .6 \quad .7 \quad .8 \quad .9 \quad 1.0 \]
Figure D-14

4th FLATWISE MODE SHAPE

0 RPM

X/R

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

0 0.4 0.8 1.2

0 -0.4 -0.8 -1.2
4th FLATWISE MODE SHAPE

50 RPM

Figure D-17
Figure D-20

4th FLATWISE MODE SHAPE

150 RPM
The work presented in this report was performed in order to develop methods of using rotor vacuum whirl data to improve the ability to model helicopter rotors. The work consisted of the following: (1) formulation of the equations of motion of elastic blades on a hub using a Galerkin method; (2) development of a general computer program for simulation of these equations; (3) study and implementation of a procedure for determining physical parameters based on measured data; (4) application of a method for computing the normal modes and natural frequencies based on test data.