ROTOR DYNAMIC SIMULATION AND SYSTEM IDENTIFICATION METHODS FOR APPLICATION TO VACUUM WHIRL DATA

A. Berman
N. Giansante
W. G. Flannelly

KAMAN AEROSPACE CORPORATION
Bloomfield, Connecticut 06002

CONTRACT NAS 1 13710
September 1980
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>SYMBOLS</td>
<td>vi</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>EQUATIONS OF MOTION</td>
<td>3</td>
</tr>
<tr>
<td>Rotor Equations</td>
<td>3</td>
</tr>
<tr>
<td>Addition of Hub Motions</td>
<td>8</td>
</tr>
<tr>
<td>Final Blade Equations of Motion</td>
<td>11</td>
</tr>
<tr>
<td>Hub Equations</td>
<td>18</td>
</tr>
<tr>
<td>Method of Solution</td>
<td>24</td>
</tr>
<tr>
<td>Program Features - V22</td>
<td>29</td>
</tr>
<tr>
<td>System Identification</td>
<td>31</td>
</tr>
<tr>
<td>Theoretical Background</td>
<td>31</td>
</tr>
<tr>
<td>Rotor Blade Application</td>
<td>32</td>
</tr>
<tr>
<td>Mass Constraints</td>
<td>35</td>
</tr>
<tr>
<td>Rotational Speed Effects</td>
<td>36</td>
</tr>
<tr>
<td>Mode Changes</td>
<td>36</td>
</tr>
<tr>
<td>Program Features - ROTS1</td>
<td>37</td>
</tr>
<tr>
<td>Method Applications</td>
<td>39</td>
</tr>
<tr>
<td>Simulation Data</td>
<td>39</td>
</tr>
<tr>
<td>Simulation Computations</td>
<td>39</td>
</tr>
<tr>
<td>System Identification</td>
<td>46</td>
</tr>
<tr>
<td>Conclusions and Recommendations</td>
<td>60</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>REFERENCES</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPENDIX A. - DEFINITIONS OF INTEGRALS</td>
<td>62</td>
</tr>
<tr>
<td>APPENDIX B. - USERS GUIDE.</td>
<td>63</td>
</tr>
<tr>
<td>APPENDIX C. - PROGRAM LISTINGS.</td>
<td>72</td>
</tr>
<tr>
<td>APPENDIX D. - NORMAL MODES AND NATURAL FREQUENCIES OBTAINED FROM VACUUM WHIRL DATA.</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>149</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Blade Coordinate System.</td>
</tr>
<tr>
<td>2</td>
<td>Point on Blade Referenced to Non-Rotating Hub Coordinate System.</td>
</tr>
<tr>
<td>3</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 1st OP Cantilever = 10.19 Rad/Sec.</td>
</tr>
<tr>
<td>4</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 1st IP and 2nd OP Frequencies = 54.55, 74.20 Rad/Sec.</td>
</tr>
<tr>
<td>5</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 0$. 3rd OP Frequency = 222 Rad/Sec</td>
</tr>
<tr>
<td>6</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 1st OP Frequency = 25.25 Rad/Sec.</td>
</tr>
<tr>
<td>7</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. An Apparent Highly Damped Response in Vicinity of 1st IP Frequency</td>
</tr>
<tr>
<td>8</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 2nd OP Frequency = 86.25 Rad/Sec.</td>
</tr>
<tr>
<td>9</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$ Rad/Sec. 1st OP Frequency = 30.49 Rad/Sec.</td>
</tr>
<tr>
<td>10</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$ Rad/Sec. Apparent Highly Damped Response in Vicinity of 1st IP Frequency</td>
</tr>
<tr>
<td>11</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$ Rad/Sec. 2nd OP Frequency = 95.52 Rad/Sec.</td>
</tr>
<tr>
<td>12</td>
<td>Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$ Rad/Sec. 3rd OP Frequency = 243.3 Rad/Sec.</td>
</tr>
<tr>
<td>13</td>
<td>Campbell Diagram Illustrating Natural Frequencies Obtained During Simulated Frequency Sweep.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------</td>
</tr>
<tr>
<td>14</td>
<td>In-Plane Mode Shape for All Frequencies</td>
</tr>
<tr>
<td>15</td>
<td>Torsional Mode Shape for All Frequencies</td>
</tr>
<tr>
<td>16</td>
<td>Out-of-Plane Shapes From 1st OP Coupled Modes</td>
</tr>
<tr>
<td>17</td>
<td>Out-of-Plane Shapes From 1st IP Coupled Modes</td>
</tr>
<tr>
<td>18</td>
<td>Out-of-Plane Shapes From 2nd OP Coupled Modes</td>
</tr>
<tr>
<td>19</td>
<td>Out-of-Plane Shapes From 3rd OP Coupled Modes</td>
</tr>
<tr>
<td>D-1</td>
<td>A Diagram of Acceleration Mobility Peak Frequencies</td>
</tr>
<tr>
<td>D-2</td>
<td>Bending Moment Modes</td>
</tr>
<tr>
<td>thru D-19</td>
<td></td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>44</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>56</td>
</tr>
<tr>
<td>10</td>
<td>57</td>
</tr>
<tr>
<td>11</td>
<td>59</td>
</tr>
</tbody>
</table>

**LIST OF TABLES**

1. BLADE PROPERTIES
2. IN-PLANE MODES
3. OUT-OF-PLANE MODES
4. TORSION MODE
5. THE BLADE INERTIAL, DAMPING, STIFFNESS MATRICES, AND INVERSE OF THE INERTIAL MATRIX AT $\Omega = 25$ RAD/SEC (SEE EQ. 36, 37)
6. HUB MATRICES (SEE EQ. 36, 37)
7. CANTILEVER NORMAL MODES
8. EIGHT STATION LUMPED MASS MODEL
9. SAMPLE PARAMETER IDENTIFICATION OUTPUT
10. SUMMARY OF MASS IDENTIFICATION RESULTS
11. MODE CHANGES REQUIRED FOR ORTHOGONALITY
SYMBOLS*

A \quad \text{blade cross-sectional area, coefficient matrix}

B_1^*, B_2^*, C_1, C_1^* \quad \text{blade cross-sectional integrals (see Ref. 3)}

BF \quad \text{vector of applied forces to blade, defined after Equation (34)}

BIN, BDAM, BSPR \quad \text{matrices in hub equations, defined after Equation (34)}

BIRI, BIRID, BIRIO, BIRIDH, BIRIIH \quad \text{matrices defined after Equation (34)}

C_{H_x}, C_{H_y}, C_{\alpha_x}, C_{\alpha_y} \quad \text{effective hub damping coefficients}

CIB \quad \text{blade coordinate transformation matrix, defined after Equation (34)}

COIR, COIH, CODR, CODH, COR \quad \text{blade equation matrices, defined after Equation (34)}

DYYI, DYYII, DZZII, etc \quad \text{definite integrals defined in Appendix A}

E \quad \text{Young's modulus}

E_l \quad = e_A E A K_A^2 - E B_2^2*

E_v \quad \text{effective in-plane stiffness} = E I_z' - (E I_z' - E I_y') \theta^2

E_w \quad \text{effective out-of-plane stiffness} = E I_y + (E I_z' - E I_y') \theta^2

E_{\phi} \quad \text{effective torsional stiffness} = G J - K_A^2 E A \theta^2

\quad \text{+ } E B_1^* \theta^2 + K_A^2 \omega^2 \tau_{1'}

e \quad \text{mass centroid offset from elastic axis, positive when centroid is forward}

e_A \quad \text{area centroid offset from elastic axis, positive when centroid is forward}

* Most symbols relating to blade parameters are consistent with the notation of Reference 3.
SYMBOLS (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{Hx}, F_{Hy}, F_{Hz}, F_{ax}, F_{ay}$</td>
<td>applied forces and moments at hub</td>
</tr>
<tr>
<td>$F_{NL}$</td>
<td>vector of nonlinear terms, defined after Equation (34)</td>
</tr>
<tr>
<td>$F_{R}$</td>
<td>vector of steady forces due to offsets, defined after Equation (34)</td>
</tr>
<tr>
<td>$G$</td>
<td>shear modulus</td>
</tr>
<tr>
<td>$g_y, g_w, g_{\Phi}$</td>
<td>blade inplane, out of plane, torsion damping, force/unit length/unit velocity</td>
</tr>
<tr>
<td>$HC, HF, HK$</td>
<td>hub damping, force, and stiffness matrices, defined after Equation (34)</td>
</tr>
<tr>
<td>$I$</td>
<td>as used in EI, appropriate area moment of inertia</td>
</tr>
<tr>
<td>$IB$</td>
<td>index referring to a particular blade of the rotor</td>
</tr>
<tr>
<td>$I_{y'}, I_{z'}$</td>
<td>blade section moments of inertia from $y'$ and $z'$ axes</td>
</tr>
<tr>
<td>$I_{ax}, I_{ay}$</td>
<td>effective moments of inertia of hub</td>
</tr>
<tr>
<td>$K_A$</td>
<td>area radius of gyration of blade cross-section</td>
</tr>
<tr>
<td>$K_{m1}, K_{m2}$</td>
<td>mass radius of gyration of blade cross-section, polar, from chord, from axis through c.g. perpendicular to chord.</td>
</tr>
<tr>
<td>$K_{Hx}, K_{Hy}$</td>
<td>effective stiffness of hub</td>
</tr>
<tr>
<td>$L_u, L_v, L_w$</td>
<td>components of applied forces to blade in $u, v, w$ coordinate system.</td>
</tr>
<tr>
<td>$m$</td>
<td>blade mass per unit length</td>
</tr>
<tr>
<td>$m_{Hx}, m_{Hy}$</td>
<td>effective hub masses</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>vector of elements of mass matrix</td>
</tr>
<tr>
<td>$\bar{M}_A$</td>
<td>vector of elements of approximate mass matrix</td>
</tr>
</tbody>
</table>
SYMBOLS (Continued)

NB  
number of blades

NY, NZ, NP  
number of in-plane, out-of-plane, torsion modes, respectively

NT  
total number of modes used = NY + NZ + NP

NX  
number of blade stations

\( \bar{r} \)  
right-hand side vector

R  
value of x at blade tip, blade radius

RIOC  
inverse of blade inertial coefficient matrix, COIR

SIB  
blade coordinate transformation matrix, defined after Equation (34)

t  
time

T  
tension, also kinetic energy

TM  
hub inertial matrix, defined after Equation (34)

u, v, w  
elastic displacements in radial, in-plane, and out-of-plane directions

\( \bar{v}, \bar{w}, \phi \)  
vector components of coupled blade normal modes, \( \psi \)

\( w_i \)  
weighting factor on i-th variable

W  
weighting matrix

x  
blade station, measured from hub

x, y, z  
blade displacement from undeformed blade coordinates

\( x_H, y_H, z_H \)  
coordinates of hub in inertial reference system, Figure 2

\( x_R, y_R, z_R \)  
non-rotating blade coordinates with origin at hub, Figure 2

\( y_i, z_i, \phi_i \)  
generalized coordinates, amplitudes of i-th in-plane, out-of-plane, and torsion modes in Galerkin method, functions of time only

\( v_i, z_i, \phi_i \)  
modal functions used in Galerkin method, function of x only
### SYMBOLS (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_I, Z_I, P_I$</td>
<td>integrals defined in Appendix A</td>
</tr>
<tr>
<td>$Y_z$</td>
<td>vector of blade generalized coordinates</td>
</tr>
<tr>
<td>$\alpha_x, \alpha_y$</td>
<td>pitch and roll angles of hub</td>
</tr>
<tr>
<td>$\beta_{pc}$</td>
<td>precone angle</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>$EI_{z}^{'}, EI_{y}^{'}, e_A^2EA$</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>$K_{m_2}^2 - K_{m_1}^2$</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>vector of changes in elements of mass matrix</td>
</tr>
<tr>
<td>$\eta$</td>
<td>blade section coordinate</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>built-in twist</td>
</tr>
<tr>
<td>$\xi$</td>
<td>dummy variable for blade station</td>
</tr>
<tr>
<td>$\tau$</td>
<td>centrifugal tension integral = $\int x m \xi d\xi$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>elastic twist about elastic axis</td>
</tr>
<tr>
<td>$\bar{\phi}$</td>
<td>vector torsional component of coupled blade normal mode</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>generalized coordinate, amplitude of $i$-th torsion mode in Galerkin method</td>
</tr>
<tr>
<td>$\Phi_i$</td>
<td>$i$-th torsional mode used in Galerkin method</td>
</tr>
<tr>
<td>$\psi$</td>
<td>blade azimuth</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>vector of coupled blade normal modes</td>
</tr>
<tr>
<td>$\omega$</td>
<td>blade natural frequency</td>
</tr>
<tr>
<td>$\omega_f$</td>
<td>frequency of forcing function</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>blade rotational speed</td>
</tr>
</tbody>
</table>
SYMBOLS (Continued)

\( J \)
for simplicity, often used to indicate \( \int_{-\infty}^{\infty} f(x) \, dx \)

\( (\cdot) \)
\( \frac{\partial}{\partial t} (\cdot) \)

\( (\cdot)' \)
\( \frac{\partial}{\partial x} (\cdot) \)
INTRODUCTION

The analysis of rotor dynamic and aeroelastic phenomena and the resulting capability to control and modify undesirable characteristics requires an understanding of the dynamics and aerodynamics of the rotor blade. Much of the theoretical and experimental research efforts have centered on the aerodynamic aspects of the problem. Of the recent work done in the field of rotor dynamics, most has been directed toward particular phenomena using idealized blade models. Little effort has been devoted to the development of methods of analyzing the dynamic characteristics of actual rotors.

The ability to analyze and predict the dynamic characteristics of a rotor blade has rarely been adequately tested. Non-rotating tests and rotating tests in the atmosphere omit the extreme structural operating conditions associated with the large centrifugal forces or involve significant aero-dynamic effects which cannot be analytically removed. One attempt (Reference 1) to test an idealized rotor model in a vacuum chamber resulted in the conclusion that the state-of-the-art of rotor dynamic analysis was not adequate for even a simple solid homogeneous uniform blade with a rectangular cross-section.

There are reasons why there are considerable uncertainties in the mathematical modeling of a rotor blade. In addition to the extreme centrifugal field effects, the major problem lies in the representation of the blade section properties. The state-of-the-art methods (for example, Reference 3) apply to blades with homogeneous sections. In actuality, a typical rotor blade will contain many of the following features: a tapered, twisted hollow spar; bonded thin skinned pockets with ribs or a honeycomb filler; leading edge balance weights; a bonded anti-icing boot; inboard stiffeners; multiple hinges; root cutout. The analytic determination of "effective stiffness", "elastic axis", and "structural damping coefficient" are, at best, intuitive approximations.

The vacuum chamber rotor testing planned at Langley Research Center offers a unique opportunity to significantly advance the state-of-the-art of rotor analytic modeling and rotor dynamic analysis. The purpose of the work presented in this report is to develop tools to augment the aforementioned testing program. Two specific computer programs have been developed. The V22 program has been developed to simulate the tests, including all the necessary special characteristics such as hub forcing, and independent rotational and forcing frequencies, including the non-rotating condition. In addition, the program was designed to be used as a research tool and emphasizes operational flexibility and ease of data input and solution controls.
The other program, ROTSI, is an attempt to use measured data to help identify better approximations to the mass and offset parameters of the rotor blade. The method is an adaptation of the method of incomplete models which has been used with success for other related structural problems.

The analytical developments necessary to implement these tools are derived and discussed in this report. The programs, operators guides, descriptions of special features, and illustrative computational results are also presented.

The major part of this work was completed in 1977, prior to the actual vacuum chamber tests. After the testing was performed an analysis of this data was carried out and is reported in Appendix D.

The contract research effort which has led to the results in this report was financially supported by the Structures Laboratory, USARTL (AVRADCOM).
EQUATIONS OF MOTION

A comprehensive development of the equations of motion of a rotor blade was first published by Houbolt and Brooks (Reference 2) in 1958. The equations were reformulated by Hodges and Dowell (Reference 3). Their major contributions were the improved generality, including nonlinear terms, and the independent verification of the earlier work. There being no need to rederive these equations again, the rotor equations used in this study were based on those given in Reference 3.

The addition of hub degrees of freedom necessitated the development of the additional terms in the blade equations and the development of the equations of motion of the hub itself which includes the effects of the blades.

The development of the equations of motion of the blades and hub, the application of the Galerkin method, the method of solution, and some of the major features of the program implementing these solutions is presented in the following sections.

ROTOR EQUATIONS

As suggested in Reference 3, the tension, $T$, and the longitudinal deflection, $u$, shall be eliminated from the equations. Using the nomenclature as shown in Figure 1 and considering $\theta$ and $\phi$ to be small with $\phi$ ignored compared to $\theta$ in the nonlinear terms, the equation for the tension in the blade becomes: (Equation 62 of Reference 3)

$$ T = EA\{u^1 + \frac{v^1}{2} + \frac{w^1}{2} + K_A^2 \theta^1 \phi^1 - \epsilon \{v^0 + w^0 \theta^0\} \} \quad (1) $$

Integrating with respect to $x$ and solving for $u$ yields:

$$ u = \int_0^x u' d\xi = \int_0^x \frac{T}{EA} - K_A^2 \theta^0 \phi^1 + \epsilon \{v^0 + \theta^0 \phi^0\} d\xi - \int_0^x \left( \frac{v^1}{2} + \frac{w^1}{2} \right) d\xi $$

with boundary condition $u(0) = 0$ \quad (2)

From Reference 3 the equation (Equation 61a) for the elastic displacement in the $x$ direction is:

$$ T' = -L_u - m(\omega^2 x + 2\omega^0 \dot{x}) \quad (3) $$
Integrating Equation (3) and using $L_u = 0$, $T(R) = 0$ and $\tau = \int \bar{m} \epsilon d\xi$, the resulting equation is:

$$T = \Omega^2 \tau + 2 \Omega \int \bar{m} \epsilon d\xi$$  (4)

Equation (3) and (4) and an expression for $u'$ developed from Equation (1) are substituted into the Equations (6ib), (6ic), (6ld) of Reference 3, the equations for the in-plane, out-of-plane, and torsion become (where third and higher order terms have been neglected):

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi'' - e_A \Omega^2 \tau'' - \Omega^2 \tau'' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi'' - e_A \Omega^2 \tau'' - \Omega^2 \tau'' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$

$$\{E_v'' - 2 \Omega e_A \int \bar{m} \epsilon d\xi + \Delta E\theta'' - E_{1*} \theta'' + E_{1*} \phi' - e_A \Omega^2 \tau' - \Omega^2 \tau' + \Omega^2 m\epsilon v'\}
- \Omega^2 m\epsilon + m\epsilon v - 2 \Delta m e\epsilon' + 2 \Delta m e\epsilon - \Omega^2 \bar{m} \epsilon d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi + \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' - \Omega^2 m\epsilon \epsilon'' d\xi'' = L_v + m\epsilon^2 e$$
These equations contain spatial derivatives of physical parameters which would be difficult to evaluate numerically. Integrating each equation twice between the limits \( x \) to \( R \) will eliminate this problem. Using the variable \( x \) as the lower limit is the more convenient because of the boundary conditions at the tip of the blade. For example, consider the double integration of functions \( f''(x) \) and \( f'(x) \) as follows:

\[
\int \int f''(x) \, dx \, dx = f'(R)(R-x) - f(R) + f(x)
\]

and

\[
\int \int f'(x) \, dx \, dx = f(R)(R-x) - \int f(x) \, dx
\]

Following the Galenkin (Ritz) procedure, arbitrary functions for the blade elastic displacements are substituted into the previous equations as follows:

\[
v(x_1t) = \sum_i y_i(t)Y_i(x) \equiv \sum_i y_i Y_i
\]

\[
w(x_1t) = \sum_j z_j(t)Z_j(x) \equiv \sum_j z_j Z_j
\]

\[
\phi(x_1t) = \sum_k \phi_k(t)\phi_k(x) \equiv \sum_k \phi_k \phi_k
\]

where \( Y_i(x), Z_j(x), \phi_k(x) \) are modal functions which satisfy the boundary conditions and \( y_i(t), z_j(t), \phi_k(t) \) are time dependent generalized co-ordinates. The modal functions are completely general and are not restricted to normal mode shapes.

In the following equations the short-hand notation \( \int = \int \left( \right) \, dx \) is used for simplicity.
\[\sum y_i (\mathcal{E} \mathcal{M} Y_i + 4 \pi^2 \mathcal{E} \mathcal{M}^2 \mathcal{E} \mathcal{M}^x \mathcal{E} \mathcal{M} Y_i) + 2 \pi \gamma_i [\mathcal{E} \mathcal{M} \gamma^* \mathcal{E} \mathcal{M} Y_i] + \frac{1}{e A R e A e Y_i} \mathcal{E} \mathcal{M} Y_i \mathcal{E} \mathcal{M} Y_i \]

\[= \mathcal{E} \mathcal{M} \left( L_v - m \Omega^2 e \right) - \Omega^2 (\mathcal{E} \mathcal{M} e - e_A \tau - R(\mathcal{E} \mathcal{M} e) R(R - x))\]  

\[\sum_{i} \left( 2 \pi \gamma_i [\mathcal{E} \mathcal{M} Y_i + \mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Y_i - e_A \mathcal{E} \mathcal{M} Y_i - (R - x)(\mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Y_i)] + y_i (\Delta \mathcal{E} \mathcal{M} Y_i) \right) \]

\[= \mathcal{E} \mathcal{M} \left( L_w - \Omega^2 e p_{m \mathcal{E} \mathcal{M} m} \right) - n^2 (\mathcal{E} \mathcal{M} e - e_A \tau - R(\mathcal{E} \mathcal{M} e) R(R - x))\]  

\[\sum_{j} \left( 2 \pi \gamma_j [\mathcal{E} \mathcal{M} Z_j + \mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j - e_A \mathcal{E} \mathcal{M} Z_j - (R - x)(\mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j)] + y_j (\Delta \mathcal{E} \mathcal{M} Z_j) \right) \]

\[= \mathcal{E} \mathcal{M} \left( L_w - \Omega^2 e p_{m \mathcal{E} \mathcal{M} m} \right) - n^2 (\mathcal{E} \mathcal{M} e - e_A \tau - R(\mathcal{E} \mathcal{M} e) R(R - x))\]  

\[\sum_{j} \left( 2 \pi \gamma_j [\mathcal{E} \mathcal{M} Z_j + \mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j - e_A \mathcal{E} \mathcal{M} Z_j - (R - x)(\mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j)] + y_j (\Delta \mathcal{E} \mathcal{M} Z_j) \right) \]

\[= \mathcal{E} \mathcal{M} \left( L_w - \Omega^2 e p_{m \mathcal{E} \mathcal{M} m} \right) - n^2 (\mathcal{E} \mathcal{M} e - e_A \tau - R(\mathcal{E} \mathcal{M} e) R(R - x))\]  

\[\sum_{j} \left( 2 \pi \gamma_j [\mathcal{E} \mathcal{M} Z_j + \mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j - e_A \mathcal{E} \mathcal{M} Z_j - (R - x)(\mathcal{E} \mathcal{M} \mathcal{E} \mathcal{M} Z_j)] + y_j (\Delta \mathcal{E} \mathcal{M} Z_j) \right) \]

\[= \mathcal{E} \mathcal{M} \left( L_w - \Omega^2 e p_{m \mathcal{E} \mathcal{M} m} \right) - n^2 (\mathcal{E} \mathcal{M} e - e_A \tau - R(\mathcal{E} \mathcal{M} e) R(R - x))\]  

7
ADDITION OF HUB MOTIONS

In this section the linear effects of the hub degrees of freedom are evaluated and will be combined with the blade equations.

The coordinate of a point on a blade in the nonrotating hub system, as shown in Figure 2, can be defined in terms of $r$, the undeformed reference line along the blade span as follows (including the major linear terms).

\[ x_R = r \cos \psi - (v + \eta \cos(\theta + \phi)) \sin \psi \]
\[ y_R = r \sin \psi + [(v + \eta \cos(\theta + \phi)) \cos \psi \]
\[ z_R = r\beta_{pc} + \omega + \eta \sin(\theta + \phi) \quad (11) \]

Assuming small angles for $\theta$ and $\phi$ in Equations (11), including hub displacements and angular motions $\alpha_x$ and $\alpha_y$ about the respective axes, the linear expression for the inertial coordinates for a point on the blade become:

\[ x = x_H + r \cos \psi - (\eta + v) \sin \psi + (r\beta_{pc} + \eta\theta)\alpha_y \]
\[ y = y_H + r \sin \psi + (\eta + v) \cos \psi - (r\beta_{pc} + \eta\theta)\alpha_x \]
\[ z = z_H + r\beta_{pc} + \eta(\theta + \phi) + \dot{\omega} + (r \sin \psi + \eta \cos \psi)\alpha_x \]
\[ - (r \cos \psi - \eta \sin \psi)\alpha_y \quad (12) \]

Accelerations of the inertial coordinates are derived from Equation (12) and are used in the formulation of the hub equations, below:

\[ \ddot{x} = \ddot{x}_H - \Omega^2(r \cos \psi - \eta \sin \psi) - \dot{v} \sin \psi - 2\Omega \dot{v} \cos \psi + \Omega^2 v \sin \psi \]
\[ + \eta \phi \sin \psi + 2\eta \Omega \dot{\phi} \cos \psi + (r\beta_{pc} + \eta\theta)\ddot{\alpha_y} \quad (13) \]
Figure 2. Point on Blade Referenced to Non-Rotating Hub Coordinate System
\[
\ddot{y} = \dddot{y}_H - \Omega^2(r \sin \psi + \eta \cos \psi) + \ddot{v} \cos \psi - 2\Omega \dot{v} \sin \psi - \Omega^2 v \cos \psi
\]

\[- \eta \dot{\phi} \cos \psi + 2\eta \dot{\phi} \sin \psi - (r_{\theta \phi} + \eta \dot{\phi}) \ddot{\alpha}_x \]

\[
\ddot{z} = \dddot{z}_H + \ddot{w} + \Omega^2(r \sin \psi + \eta \cos \psi) \alpha_x + 2\Omega(r \cos \psi - \eta \sin \psi) \ddot{\alpha}_x
\]

\[+ (r \sin \psi + \eta \cos \psi) \ddot{\alpha}_x + \Omega^2(r \cos \psi - \eta \sin \psi) \ddot{\alpha}_y
\]

\[+ 2\Omega(r \sin \psi + \eta \cos \psi) \ddot{\alpha}_y - (r \cos \psi - \eta \sin \psi) \ddot{\alpha}_y \] (15)

Applying LaGrange's equation, the additional terms in the equations for the elastic displacements \(v, w, \phi\) due to hub motions become:

\[v \text{ Equation}\]
\[- x_H \sin \psi \sin \theta + y_H \cos \psi \sin \theta + (\ddot{\alpha}_x \cos \psi + \ddot{\alpha}_y \sin \psi) (\beta_{\theta \phi} \dddot{x} \sin \theta + \dddot{x} \cos \theta) \]

\[+ (\ddot{x}_H \cos \psi - \dot{y}_H \sin \psi)(\beta_{\theta \phi} \dddot{y} + \dddot{y} \sin \theta + \dddot{y} \cos \theta) + \dddot{z}_H \sin \theta \cos \phi + \dddot{z}_H \cos \theta \sin \phi \]

\[= \dddot{z}_H \sin \theta \cos \phi + \dddot{z}_H \cos \theta \sin \phi \]

\[w \text{ Equation}\]
\[- (\ddot{\alpha}_x \cos \psi + \ddot{\alpha}_x \sin \psi) \sin \theta \cos \phi + \dddot{x}_H \cos \theta \sin \phi \sin \phi \cos \theta + \dddot{x}_H \cos \phi \sin \sin \theta \cos \phi \]

\[- (\ddot{\alpha}_x \cos \psi + \ddot{\alpha}_x \sin \psi) \sin \theta \cos \phi + \dddot{x}_H \cos \theta \sin \phi \sin \theta \cos \phi \]

\[= \dddot{x}_H \sin \theta \cos \phi - \dddot{x}_H \cos \theta \sin \phi \]

\[\phi \text{ Equation}\]
\[- \Omega^2 (x_{\sin \psi} - y_{\cos \psi} + \dot{y}_H \sin \psi) \sin \theta + \Omega (x_{H \cos \psi} - y_{H \sin \psi}) \sin \theta / \cos \phi + \dddot{z}_H \cos \theta \sin \phi \]

\[+ (\ddot{\alpha}_x - \Omega^2 \alpha_x + 2\Omega \ddot{\alpha}_y)(\sin \theta \cos \phi - \cos \theta \sin \phi) - (\ddot{\alpha}_y - \Omega^2 \alpha_y)
\]

\[- 2\Omega \ddot{\alpha}_x (\cos \theta \sin \phi - \sin \theta \cos \phi) + \Omega \ddot{\alpha}_x (\cos \theta \sin \phi - \sin \theta \cos \phi) \]

\[- \Omega (\ddot{\alpha}_x \sin \psi - \ddot{\alpha}_y \cos \psi)(\beta_{\theta \phi} \dddot{x} \sin \theta + \dddot{x} \cos \theta) \]

\[+ \frac{R}{x} \int \left( \right) d\xi \]

where \(f = f(x) \)
FINAL BLADE EQUATIONS OF MOTION

Combining the respective equations given in (8)-(10) and (16)-(18) yields the equations of motion for the elastic displacements \( v, w \) and \( \phi \).

**v Equation**

\[
\Sigma \{ y_i [f \int m_i \rho \frac{1}{EA} m Y_i] + 2 \Omega^2 [f \int m_i \epsilon_A Y_i] - e_A^m Y_i + f m Y_i \\
- (R - x)(m e Y_i)_R] + y_i \{ E_v Y_i \} - \Omega^2 (f \int m Y_i + f m x Y_i)_i + f m Y_i \} \\
+ \Sigma \{ 2 \Omega^2 [f \int m_i e_A \hat{z}_j'' - f \int m e \hat{z}_j'' - \beta_{pc} f \int m \hat{z}_j] + z_j (\Delta \Theta Z_j) \} \\
- \Sigma \{ \hat{z}_k f m \Theta_\phi \hat{z}_k + 2 \Omega^2 [f \int m_k \hat{z}_k' \phi_\phi + \phi_\phi (E_C_\phi \phi_\phi'' - E_\phi \phi_\phi') \} - \hat{z}_H \sin \psi f m \\
\hat{y}_H \cos \psi f m + \hat{\alpha}_x \cos \psi (\beta_{pc} f m x + f m \Theta_\theta) + \hat{\alpha}_y \sin \psi (\beta_{pc} f m x + f m \Theta_\theta) \\
+ 2 \Omega [f \int m \phi'' + f m \phi'' + f \int m \phi'' + f m \phi''] = f \int \phi' + \Omega^2 f m e \\
- \Omega^2 f m e + \Omega^2 [e_A \phi + (m e R (R - x))] \] (19)

**w Equation**

\[
\Sigma \{ 2 \Omega^2 [\beta_{pc} f m Y_i + f m \Theta Y_i - E_A \Theta m Y_i - (R - x)(m e Y_i)_R] + y_i \Delta \Theta Y_i'' \} \\
+ \Sigma \{ Z_j f \int m Z_j + z_j [E_v Z_j - \Omega^2 (f \int m Z_j + f m x Z_j')] \} + \Sigma \{ \hat{z}_k f m \Theta_\phi \hat{z}_k + \phi_\phi [E_C_\phi \phi'' + E_\phi \phi_\phi'' + \Omega^2 (f m x \phi_k - e_A \phi_k - R (R - x)(m \phi_k)_R)] \} \\
+ \hat{z}_H f m + \hat{\alpha}_x (\sin \psi f m x + \cos \psi f m e) + 2 \hat{\alpha}_x (\cos \psi f m x - \sin \psi f m e) \\
- \hat{y}_H (\sin \psi f m x + \cos \psi f m e - \hat{\alpha}_y (\cos \psi f m x - \sin \psi f m e) \]
The Galerkin (Ritz) method of effecting approximate solutions of differential equations applied to the previous equations requires a set of averaging integrals. Equations (19) - (21) for \( v, w \) and \( \phi \) are multiplied by \( y_i, z_j, \) and \( \phi_k \), respectively, where \( i = 1, NY; j = 1, NZ \) and \( k = 1, NP \) and each resulting equation is integrated from 0 to \( R \). This procedure yields NT equations (NT = NY + NZ + NP) which may be solved for the generalized coordinates.
\[
\sum_{J=1}^{NY} \{ \ddot{\psi}_J [DYYII(I,J,1) + 4\Omega^2DYSI(I,J,1)] + 2\Omega \dot{\psi}_J [DYSI(I,J,2) - DYYII(I,J,5)] \\
- DYF(I,J,2) + DYYI(I,J,2) - DYF(I,J,1)] + \gamma_J [DYF(I,J,3)] \\
- \Omega^2 (DYYII(I,J,7) - DYYII(I,J,4) + DYYII(I,J,1)) \}
\]

\[
\sum_{J=1}^{NZ} \{ 2\Omega \dot{\psi}_J [DYSI(I,J,3) - DYZII(I,J,5) - \beta_{pc} DYZII(I,J,1)] + z_J [DYF(I,J,4)] \}
\]

\[
\sum_{J=1}^{NP} \{ -\phi_J DYPII(I,J,3) - 2\Omega \dot{\psi}_J [DYSI(I,J,4) + \phi_J DYF(I,J,5)] \}
\]

\[
= \ddot{\chi}_H \sin \psi [DYMII(I,1) + \ddot{\chi}_H \cos \psi [DYMII(I,1) + \ddot{\alpha}_x \cos \psi [DYMII(I,5)] \\
+ \beta_{pc} DYMII(I,2)] + \ddot{\alpha}_y \sin \psi [DYMII(I,5) + \beta_{pc} DYMII(I,2)] \\
+ 2\Omega \left\{ \int Y_I \int mv' \right\} \int Y_I \int \left( v''m' \right) \right\} \int Y_I \int \left( \dot{v}'v' \right) - \int Y_I \int \left( \dot{w}'w' \right) \}
\]

\[
\sum_{I=1}^{NY} \sum_{J=1}^{NP} \{ 2\Omega \dot{\psi}_J [DZYI(I,J,3) - DZF(I,J,1) + \beta_{pc} DZYII(I,J,1)] + \gamma_J [DZF(I,J,2)] \}
\]

\[
\sum_{J=1}^{NZ} \{ 2\dot{\psi}_J [DZII(I,J,1)] + z_J [DZF(I,J,3) - \Omega^2 DZII(I,J,6) - DZII(I,J,3)] \}
\]

\[
\sum_{J=1}^{NP} \{ \phi_J DZPII(I,J,1) + \phi_J [DZF(I,J,4) + \Omega^2 (DZPI(I,J,2) - DZF(I,J,6))] \}
\]

where I = 1 to NY; thus, there is one equation for each in-plane mode. Similarly, for the w equation:

\[
\sum_{J=1}^{NY} \{ 2\dot{\psi}_J [DZYI(I,J,3) - DZF(I,J,1) + \beta_{pc} DZYII(I,J,1)] + \gamma_J [DZF(I,J,2)] \}
\]

\[
\sum_{J=1}^{NZ} \{ 2\dot{\psi}_J [DZYII(I,J,1)] + z_J [DZF(I,J,3) - \Omega^2 DZYII(I,J,6) - DZYII(I,J,3)] \}
\]

\[
\sum_{J=1}^{NP} \{ \phi_J DZPII(I,J,1) + \phi_J [DZF(I,J,4) + \Omega^2 (DZPI(I,J,2) - DZF(I,J,6))] \}
\]
\[ + \ddot{z}_H \text{DZMII}(I,1) + \ddot{\alpha}_x [\sin \psi \text{DZMII}(I,2) + \cos \psi \text{DZMII}(I,3)] \]
\[ + 2\Omega \ddot{\alpha}_x [\cos \psi \text{DZMII}(I,2) - \sin \psi \text{DZMII}(I,3)] - \Omega^2 \dot{\alpha}_x [\sin \psi \text{DZMII}(I,2) \]
\[ + \cos \psi \text{DZMII}(I,3)] - \ddot{\alpha}_y [\cos \psi \text{DZMII}(I,2) - \sin \psi \text{DZMII}(I,3)] \]
\[ + 2\Omega \ddot{\alpha}_y [\sin \psi \text{DZMII}(I,2) + \cos \psi \text{DZMII}(I,3)] + \Omega^2 \alpha_y [\cos \psi \text{DZMII}(I,2) \]
\[ - \sin \psi \text{DZMII}(I,3)] + 2\Omega \{ \int Z_I \int \text{m} \text{w}' - \int Z_I \int (w'' \text{m}\dot{\text{w}}) \} = \int Z_I \int \text{w} \dot{\text{w}} \]
\[ - \Omega^2 [\text{DZMII}(I,6) - \text{DF}(I,1,5) + \beta_{pc} \text{DZMII}(I,2)] \] (23)

where \( I = 1 \) to \( NZ \); following the same procedure, \( w \), the \( \phi \) equation is

\[ \sum_{J=1}^{NY} \{ - y_J \text{DPYII}(I,J,3) + 2z_J \text{DPYII}(I,J,9) - \text{DPYII}(I,J,1) \] 
\[ + \Omega^2 (\text{DPYII}(I,J,8) - \text{DPYII}(I,J,6) + \text{DPYII}(I,J,3)) \} \]

\[ + \sum_{J=1}^{NZ} \{ \dot{z}_J \text{DPZII}(I,J,2) + z_J [\text{DPZII}(I,J,8) + \text{DPF}(I,J,3) - \Omega^2 (\text{DPZII}(I,J,7) \]
\[ - \text{DPZII}(I,J,4)) \} \}

\[ + \sum_{J=1}^{NP} \{ \phi_J \text{DPPII}(I,J,4) + \phi_J [\text{DPF}(I,J,3) + \text{DPPI}(I,J,6) + \Omega^2 (\text{DPPII}(I,J,5) \]
\[ + \text{DPPI}(I,J,7)) \} + \{ \dot{x}_H \sin \psi + \Omega \dot{x}_H \cos \psi - \dot{y}_H \cos \psi - \Omega y_H \sin \psi \]
\[ + \ddot{z}_H \} \text{DPMII}(I,3) + \ddot{\alpha}_x [\sin \psi \text{DPMII}(I,4) + \cos \psi \text{DPMII}(I,7) + \text{DPMII}(I,8) \]
\[ + \beta_{pc} \text{DPMII}(I,6)] + 2\Omega \ddot{\alpha}_x [\cos \psi \text{DPMII}(I,4) - \sin \psi \text{DPMII}(I,7) + \frac{1}{2} \text{DPMII}(I,8) \]
\[ + \frac{1}{2} \beta_{pc} \text{DPMII}(I,6) ] - \Omega^2 \alpha_x [\sin \psi \text{DPMII}(I,4) + \cos \psi \text{DPMII}(I,7) \]

14
\[ + \ddot{\alpha}_y \left[ - \cos \psi \text{DPMII}(I,4) + \sin \psi (\text{DPMII}(I,7) + \text{DPMII}(I,8) + \beta_{pc} \text{DPMII}(I,6)) \right] \]

\[ + 2\omega_y \left[ \sin \psi \text{DPMII}(I,4) + \cos \psi (\text{DPMII}(I,7) + \frac{1}{2} \text{DPMII}(I,8) \right] \]

\[ + \frac{1}{2} \beta_{pc} \text{DPMII}(I,6) \right] + \omega^2 \alpha_y \left[ \cos \psi \text{DPMII}(I,4) - \sin \psi \text{DPMII}(I,7) \right] \]

\[ = \int_0^R \left[ \phi_I \int_\phi^M - \omega^2 [\text{DPMII}(I,9) + \beta_{pc} \text{DPMII}(I,4) + \text{DPMII}(I,10)] \right] \tag{24} \]

where \( I = 1 \) to \( NP \). The coefficients shown in Equations (22), (23) and (24) are defined in Appendix A.

Equations (22), (23) and (24) may be written in partitioned matrix form as shown on the following pages.

In order to include a simple structural damping representation, terms of the form \( g_v \phi \), \( g_w \phi \), \( g_\phi \psi \) were added to Equations (5), (6), (7) resulting in the integrals \( \text{DYD} \), \( \text{DZD} \), \( \text{DPD} \) which appear in the following pages and are defined in Appendix A.
\[\begin{array}{cccc}
DYYII(I,J,1) & +4\Omega^2DYSI(I,J,1) & 0 & -DYPII(I,J,3) \\
0 & 0 & DZZII(I,J,1) & DZPII(I,J,1) \\
- DYPII(I,J,3) & DPZII(I,J,2) & 0 & \sin DZMI I (I ,2) \\
\end{array}\]

\[\begin{array}{cccc}
0 & 0 & 0 & DZMII(I,1) \\
\sin DPMII(I,3) & \cos DPMII(I,3) & DPMII(I,3) & 0 \\
\sin DPMII(I,3) & \cos DPMII(I,3) & DPMII(I,3) & 0 \\
\end{array}\]

\[\begin{array}{cccc}
\sin DPMII(I,3) & \cos DPMII(I,3) & DPMII(I,3) & 0 \\
- \sin DPMII(I,4) & \cos DPMII(I,4) & DPMII(I,4) & - \cos DPMII(I,4) \\
- \sin DPMII(I,4) & \cos DPMII(I,4) & DPMII(I,4) & - \cos DPMII(I,4) \\
\end{array}\]

\[\begin{array}{cccc}
-DYSI(I,J,3)-DZII(I,J,5) & DYSI(I,J,3)-DZII(I,J,5) & -DYSI(I,J,4) \\
-DYF(I,J,2)-DYYI(I,J,2) & -R_DYZII(I,J,1) \\
+DZII(I,J,1) & +R_DZFI I (I ,1) & + \beta_DZFI I (I ,1) \\
\end{array}\]

\[\begin{array}{cccc}
DPSI(I,J,5) & 0 & 0 & 0 \\
0 & 0 & 0 & DPD \\
0 & 0 & 0 & \phi_J \\
\end{array}\]

\[\begin{array}{c}
\frac{\dot{y}_J}{y_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{z}_J}{z_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\frac{\dot{\phi}_J}{\phi_J} \\
\end{array}\]
HUB EQUATIONS

Terms In Hub Equations Due to Blade Motions

The kinetic energy of a rotor blade may be expressed as follows:

\[ T = \frac{1}{2} \int_0^R (x'^2 + y'^2 + z'^2) \, dm \]

Assuming a spring-mass-damper model of the hub in each of the three orthogonal directions, and torsional models with respect to the body axes, the hub equations of motion including blade effects are:

\[
\begin{align*}
R \int \mathcal{J}_1 \int L_v - \Omega^2 \left[ DYMII(I,3) + DYMII(I,4) - DWF(I,1,6) \right] \\
+ R \int \mathcal{J}_2 \int L_w - \Omega^2 \left[ DZMI(I,6) - DZF(I,1,5) + \beta_{pc} DZMI(I,2) \right] \\
+ R \int \mathcal{J}_3 \int M_\phi - \Omega^2 \left[ DPMMI(I,9) + \beta_{pc} DPMMI(I,4) \right]
\end{align*}
\]
Substituting the expressions for the accelerations of the inertial coordinates from Equations (13)-(15), performing the integration with respect to chord and blade span and assuming two of more symmetrical blades, the previous equations become:

\[ m_H \ddot{x}_H + C_H \dot{x}_H + K_H x_H + \sum b=1 \frac{m \ddot{x}_H}{R} = F_{Hx} \]

\[ m_H \ddot{y}_H + C_H \dot{y}_H + K_H y_H + \sum b=1 \frac{m \ddot{y}_H}{R} = F_{Hy} \]

\[ m_H \ddot{z}_H + C_H \dot{z}_H + K_H z_H + \sum b=1 \frac{m \ddot{z}_H}{R} = F_{Hz} \]

\[ I_{\alpha_x} \ddot{\alpha}_x + C_{\alpha_x} \dot{\alpha}_x + K_{\alpha_x} \alpha_x + \sum b=1 \frac{m \ddot{\alpha}_x}{R} = F_{\alpha_x} \]

\[ I_{\alpha_y} \ddot{\alpha}_y + C_{\alpha_y} \dot{\alpha}_y + K_{\alpha_y} \alpha_y + \sum b=1 \frac{m \ddot{\alpha}_y}{R} = F_{\alpha_y} \]

\[ x_H \text{ Equation} \]

\[ m_H \ddot{x}_H + C_H \dot{x}_H + K_H x_H + \text{NB MI}(1,1) \ddot{x}_H + \sum b=1 \frac{- m \ddotsin\phi - 2\Omega m \ddottcos\phi}{R} \]

\[ \left. \begin{array}{c}
\int_0^{\beta_p \text{ MI}(1,2) + \text{MI}(1,5)} \frac{m v \sin \phi + m \theta \phi \sin \psi + 2\Omega m \theta \phi \cos \psi + (\beta_p \text{ MI}(1,2) + \text{MI}(1,5)} \alpha_y \right} \\
\end{array} \right\} \]

\[ = F_{Hx} \]
\[
\begin{align*}
\gamma_H \text{ Equation} \\
\frac{m_H}{y_H} \ddot{y}_H + C_H \dot{y}_H + K_H y_H + NB[M(1,1)y_H] + \sum_{IB=1}^{NB} \left\{ \int_0^R m_i \dot{w}_i \cos\psi - 2\Omega \int_0^R m_i \dot{v}_i \sin\psi \right\} \\
- \Omega^2 \int_0^R m_i \dot{w}_i \cos\psi - \int_0^R m_i \dot{w}_i \dot{\phi} \cos\psi + 2\Omega \int_0^R m_i \dot{w}_i \dot{\phi} \sin\psi - (\beta_{pc} M(1,2) + M(1,5))\alpha_x \} \\
= F_{Hy} \tag{27}
\end{align*}
\]

\[
\begin{align*}
z_H \text{ Equation} \\
\frac{m_H}{z_H} \ddot{z}_H + C_H \dot{z}_H + K_H z_H + NB[M(1,1)z_H] + \sum_{IB=1}^{NB} \left\{ \int_0^R m_i \dot{w}_i + \int_0^R m_i \dot{\phi} \right\} = F_{Hz} \tag{28}
\end{align*}
\]

\[
\begin{align*}
\alpha_x \text{ Equation} \\
\frac{1}{I_{\alpha_x}} \ddot{\alpha}_x + C_{\alpha_x} \dot{\alpha}_x + K_{\alpha_x} \alpha_x - NB[\beta_{pc} M(1,2) + M(1,5)]\ddot{y}_H + \sum_{IB=1}^{NB} \left\{ - (\beta_{pc} \int_0^R m_i \dot{w}_i \right\} \\
+ \int_0^R m_i \dot{w}_i \dot{\phi} \cos\psi + 2\Omega (\beta_{pc} \int_0^R m_i \dot{w}_i \dot{\phi} \sin\psi + \Omega^2 (\beta_{pc} \int_0^R m_i \dot{w}_i \dot{\phi} \\
+ \int_0^R m_i \dot{w}_i \dot{\phi} \cos\psi + (\beta_{pc} \int_0^R m_i \dot{w}_i \dot{\phi} \sin\psi + 2\Omega (\beta_{pc} \int_0^R m_i \dot{w}_i \dot{\phi} \\
+ \int_0^R m_i \dot{w}_i \dot{\phi} \cos\psi + (\beta_{pc} \int_0^R m_i \dot{w}_i \dot{\phi} \sin\psi + \Omega^2 (\sin^2 \psi \int_0^R m_i \dot{w}_i \\
+ 2\sin\psi \cos\psi \int_0^R m_i \dot{w}_i \dot{\phi} \alpha_x + \Omega [\sin\psi \cos\psi \int_0^R m_i \dot{w}_i - (\sin^2 \psi \\
- \cos^2 \psi \int_0^R m_i \dot{w}_i - \sin\psi \cos\psi \int_0^R m_i \dot{w}_i \dot{\phi} \alpha_x + (\sin^2 \psi \int_0^R m_i \dot{w}_i + 2\sin\psi \cos\psi \int_0^R m_i \dot{w}_i \dot{\phi} \\
\end{align*}
\]
\[
\begin{align*}
\alpha_y & \equiv \frac{d^2\alpha_y}{dt^2} \\
& = R_a \left( \omega^2 \psi' \mu^2 \frac{d^2}{dt^2} \alpha_x + \Omega^2 (\sin \psi \cos \psi \mu x^2 - (\sin^2 \psi - \cos^2 \psi) \mu \text{mex} \right) \\
& - \sin \psi \cos \psi \mu^2 \alpha_y + 2\Omega (\sin^2 \psi \mu x^2 + 2\sin \psi \cos \psi \mu \text{mex} + \cos^2 \psi \mu^2 \alpha_y) \\
& - \Omega^2 (\sin \psi \cos \psi \mu x^2 - (\sin^2 \psi - \cos^2 \psi) \mu \text{mex} - \sin \psi \cos \psi \mu^2 \alpha_y) = F_{\alpha_y}
\end{align*}
\]
Considering only the hub translational equations of motion and following a similar procedure as applied to the blade equations, arbitrary functions for the elastic displacements are substituted into Equations (26)-(28) yielding:

\[ \dot{x}_H \] Equation

\[
m_H \ddot{x}_H + C_H \dot{x}_H + K_H x_H + NB[M(1,1)\ddot{x}_H] - \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} y_I(1,J,1)\ddot{y}_J,IB
\]

\[
+C \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin \psi I(I,J,1)\ddot{y}_J,IB
\]

\[ + \Omega^2 \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin \psi I(I,J,1)\ddot{y}_J,IB + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos \psi I(I,J,1)\ddot{y}_J,IB
\]

\[ + \Omega^2 \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin \psi I(I,J,1)\ddot{y}_J,IB + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos \psi I(I,J,1)\ddot{y}_J,IB
\]

\[ + NB[B_{pc} M(1,2) + M(1,5)] = F_H \]

\[ y_H \] Equation

\[
m_H \ddot{y}_H + C_H \dot{y}_H + K_H y_H + NB[M(1,1)\ddot{y}_H] + \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} y_I(1,J,1)\ddot{y}_J,IB
\]

\[ - \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos \psi I(I,J,1)\ddot{y}_J,IB - 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin \psi I(I,J,1)\ddot{y}_J,IB
\]

\[ - \Omega^2 \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \cos \psi I(I,J,1)\ddot{y}_J,IB + 2\Omega \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} \sin \psi I(I,J,1)\ddot{y}_J,IB
\]

\[ + NB[B_{pc} M(1,2) + M(1,5)] = F_H \]

\[ z_H \] Equation

\[
m_H \ddot{z}_H + C_H \dot{z}_H + K_H z_H + NB[M(1,1)\ddot{z}_H] + \sum_{IB=1}^{NB} \sum_{IB=1}^{NZ} z_I(1,J,1)\ddot{z}_J,IB
\]

\[ + \sum_{IB=1}^{NB} \sum_{IB=1}^{NY} I(I,J,1)\ddot{z}_J,IB = F_H \]
Equations (31)-(33) may be solved for the hub accelerations and written in matrix form:

\[
\begin{bmatrix}
    m_H + NB \cdot MI(1,1) & 0 & 0 \\
    0 & m_H + NB \cdot MI(1,1) & 0 \\
    0 & 0 & m_H + NB \cdot MI(1,1)
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_H \\
    \ddot{y}_H \\
    \ddot{z}_H
\end{bmatrix}
\]

\[
\sum_{IB=1}^{NB} \begin{bmatrix}
    \sin \psi_{IB} & 0 & 0 \\
    0 & \cos \psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    YI(1,J,1) & 0 & -PI(1,J,3) \\
    -YI(1,J,1) & 0 & PI(1,J,3) \\
    0 & -ZI(1,J,1) & -PI(1,J,3)
\end{bmatrix}
\begin{bmatrix}
    \dot{y}_J \\
    \dot{z}_J \\
    \dot{\phi}_J
\end{bmatrix}
\]

\[
\sum_{IB=1}^{NB} 2\Omega \begin{bmatrix}
    \cos \psi_{IB} & 0 & 0 \\
    0 & \sin \psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    YI(1,J,1) & 0 & -PI(1,J,3) \\
    YI(1,J,1) & 0 & -PI(1,J,3) \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    \ddot{y}_J \\
    \ddot{z}_J \\
    \ddot{\phi}_J
\end{bmatrix}
\]

\[
\sum_{IB=1}^{NB} \Omega^2 \begin{bmatrix}
    \sin \psi_{IB} & 0 & 0 \\
    0 & \cos \psi_{IB} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    -YI(1,J,1) & 0 & 0 \\
    YI(1,J,1) & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    \dot{y}_J \\
    \dot{z}_J \\
    \dot{\phi}_J
\end{bmatrix}
\]

\[
\begin{bmatrix}
    C_{Hx} & 0 & 0 \\
    0 & C_{Hy} & 0 \\
    0 & 0 & C_{Hz}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_H \\
    \dot{y}_H \\
    \dot{z}_H
\end{bmatrix}
- \begin{bmatrix}
    K_{Hx} & 0 & 0 \\
    0 & K_{Hy} & 0 \\
    0 & 0 & K_{Hz}
\end{bmatrix}
\begin{bmatrix}
    x_H \\
    y_H \\
    z_H
\end{bmatrix}
+ \begin{bmatrix}
    F_{Hx} \\
    F_{Hy} \\
    F_{Hz}
\end{bmatrix}
\]
METHOD OF SOLUTION

The coefficient matrices of Equation (25) with the hub angular motions \( \alpha_x \) and \( \alpha_y \) omitted may be defined thusly:

\[
[\text{COIR}] = \begin{bmatrix}
DYYII(I, J, 1) + 4\Omega^2 \text{DYSI}(I, J, 1) & 0 & -DZII(I, J, 3) \\
0 & DZIII(I, J, 1) & DZPII(I, J, 1) \\
-DZII(I, J, 3) & DPZII(I, J, 2) & DPPII(I, J, 4)
\end{bmatrix}
\]

\[
[\text{COIH}[\text{SIB}]] = \begin{bmatrix}
\sin \psi \text{DYMII}(I, 1) & \cos \psi \text{DYMII}(I, 1) & 0 \\
0 & 0 & DZMII(I, 1) \\
\sin \psi \text{DPMII}(I, 3) & -\cos \psi \text{DPMII}(I, 3) & \text{DPMII}(I, 3)
\end{bmatrix}
\]

\[
[\text{CODR}] = \begin{bmatrix}
DYD + 2\Omega \{ -DYYI(I, J, 2) \\
-\text{DYYII}(I, J, 5) \\
+\text{DYF}(I, J, 1) - \text{DYF}(I, J, 2) \\
+\text{DYSI}(I, J, 2) \}
\end{bmatrix}
\]

\[
[\text{CODH}[\text{CIB}]] = -\Omega \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\cos \psi \text{DPMII}(I, 3) & -\sin \psi \text{DPMII}(I, 3) & 0
\end{bmatrix}
\]
\[
\begin{array}{c|c|c}
\text{DYF}(I,J,3) & \text{DYF}(I,J,4) & \text{DYF}(I,J,5) \\
-\Omega^2 \{\text{DYII}(I,J,7)\} & -\Omega^2 \{\text{DYII}(I,J,4)\} & +\Omega^2 \{\text{DYII}(I,J,1)\} \\
+\Omega^2 \{\text{DYII}(I,J,4)\} & +\Omega^2 \{\text{DYII}(I,J,2)\} & -\Omega^2 \{\text{DYII}(I,J,1)\} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{DZF}(I,J,3) & \text{DZF}(I,J,4) & \text{DZF}(I,J,5) \\
+\Omega^2 \{\text{DZZII}(I,J,3)\} & +\Omega^2 \{\text{DZZII}(I,J,4)\} & -\Omega^2 \{\text{DZZII}(I,J,5)\} \\
-\Omega^2 \{\text{DZZII}(I,J,6)\} & -\Omega^2 \{\text{DZZII}(I,J,7)\} & +\Omega^2 \{\text{DZZII}(I,J,8)\} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{DPYI}(I,J,9) & \text{DPZI}(I,J,8) & \text{DPF}(I,J,3) + \text{DPI}(I,J,6) \\
-\text{DPF}(I,J,1) & +\text{DPF}(I,J,2) & +\Omega^2 \{\text{DPI}(I,J,5)\} \\
+\Omega^2 \{\text{DPI}(I,J,3)\} & +\Omega^2 \{\text{DPI}(I,J,4)\} & +\Omega^2 \{\text{DPI}(I,J,7)\} \\
\end{array}
\]
Similarly, the coefficients in the hub equations of motion, Equation (27) may be defined as:

\[
[SIB] = \begin{bmatrix}
\sin\psi_{IB} & 0 & 0 \\
0 & \cos\psi_{IB} & 1 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
[CIB] = \begin{bmatrix}
\cos\psi_{IB} & 0 & 0 \\
0 & \sin\psi_{IB} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
[RIOC] = [COIR]^{-1}
\]

\[
\{Y_{ZP}\} = \begin{bmatrix}
y \\
z \\
\phi
\end{bmatrix} \quad \{x_H\} = \begin{bmatrix}
x_H \\
y_H \\
z_H
\end{bmatrix}
\]

Similarly, the coefficients in the hub equations of motion, Equation (27) may be defined as:

\[
[TM] = \begin{bmatrix}
m_{Hx} + m_{H} \cdot NI(1,1) & 0 & 0 \\
0 & m_{Hy} + m_{H} \cdot NI(1,1) & 0 \\
0 & 0 & m_{Hz} + m_{H} \cdot NI(1,1)
\end{bmatrix}
\]
\[
[BIN] = \begin{bmatrix}
Y_I(1, J, 1) & 0 & -PI(1, J, 1, 3) \\
-Y_I(1, J, 1) & 0 & PI(1, J, 1, 3) \\
0 & -ZI(1, J, 1) & -PI(1, J, 1)
\end{bmatrix}
\]

\[
[BDAM] = 2\Omega
\]

\[
[BSPR] = \Omega^2
\]

\[
[HC] = -\begin{bmatrix}
C_{Hx} & 0 & 0 \\
0 & C_{Hy} & 0 \\
0 & 0 & C_{Hz}
\end{bmatrix}
\]

\[
[HK] = -\begin{bmatrix}
K_{Hx} & 0 & 0 \\
0 & K_{Hy} & 0 \\
0 & 0 & K_{Hz}
\end{bmatrix}
\]

27
\[
\{HF\} = \begin{bmatrix}
F_{HX} \\
F_{HY} \\
F_{HZ}
\end{bmatrix}
\]

\([BIRI] = [BIN][RIOC]\]

\([BIRID] = [BIRI][CODR]\]

\([BIRIO] = [BIRI][COR] + [BSPR]\]

\([BIRIDH] = [BIRI][CODH]\]

\([BIRIH] = [BIRI][COIH]\]

Using the previous definitions, and assuming a sinusoidal forcing function, Equation (25) may be written as:

\[
\ddot{Y}_{p IB} = ([RIOC][CODR]\{\dot{Y}_{zp IB}\} + [COR]\{Y_{zp IB}\} + \{FR\}_{IB} + \{BF\} \cdot \sin \omega_F t + \{FNL\}_{IB} + [COIH][SIB]_{IB}\{x_{h IB}\} + [CODH][CIB]_{IB}\{\dot{x}_{H IB}\})
\]

Equation (34) for the hub accelerations is written as:

\[
[TM]\{\ddot{x}_H\} = \sum_{IB=1}^{NB} [SIB]_{IB}[BIN]\{\ddot{Y}_{zp IB}\} + \sum_{IB=1}^{NB} [CIB]_{IB}[BDAMP]\{\dot{Y}_{zp IB}\} + \sum_{IB=1}^{NB} [SIB][BSPR]\{Y_{zp IB}\} + [HC]\{\dot{x}_{H IB}\} + [HK]\{x_{H IB}\} + \{HF\}_{IB}
\]
Solving for the blade accelerations from Equation (35) and substituting the result into Equation (36) removes the inertial coupling in the system and allows solution of the hub accelerations directly.

\[
\{\ddot{x}_H\} = ([\mathbf{T}_M] - \sum_{IB=1}^{NB} [\mathbf{SIB}]_{IB}[\mathbf{BIRIH}][\mathbf{SIB}]_{IB})^{-1} ([\mathbf{HC}]

+ \sum_{IB=1}^{NB} [\mathbf{SIB}]_{IB}[\mathbf{BIRIDH}][\mathbf{CIB}]\{\ddot{x}_H\} + [\mathbf{HK}]\{x_H\} + \{H_F\}

+ \sum_{IB=1}^{NB} (([\mathbf{SIB}]_{IB}[\mathbf{BIRID}] + [\mathbf{CIB}]_{IB}[\mathbf{BDAM}]\{\dot{Y}_{ZP}\}_{IB}

+ [\mathbf{SIB}]_{IB}[\mathbf{BIRIO}]\{Y_{ZP}\}_{IB} + [\mathbf{SIB}]_{IB}[\mathbf{BIRI}](\{FR\}_{IB} + \{BF\sin\omega_Ft

+ \{FNL\}_{IB})))
\]  

(37)

Solution of Equation (37) is effected by use of a fourth order Runge-Kutta timewise integration technique. Once the hub responses are obtained for a particular time increment, Equation (35) is solved for the blade motions. These blade motions are, in turn, substituted into Equation (37) to yield the hub responses for the subsequent time increments. This procedure is continued until the total time interval of interest is reached.

PROGRAM FEATURES - V22

The V22 program, developed to implement the solutions of the equations developed above, was designed to achieve the flexibility and ease of use necessary to make it a useful research tool. The details of the necessary and optional inputs are described in Appendix B. Some of the major features of the program are outlined in this section.

1. General input - The input data, in most cases, may be input in any order. Certain data is optional as input and need not be entered unless used. In running successive cases, only changed data need be input.

2. No. of Blades - One to four blades may be specified. With a hub, a minimum of two is required.
3. Modal input - The method of solution (Galerkin's method) uses separate in-plane, out-of-plane, and torsion "modes" as generalized degrees of freedom. They need not be normal modes (and thus need not be changed for changes in parameters and rotor speed). The equations contain the modal displacement as well as the first and second derivatives. Only the second derivative and the root slope of each mode is required as input. The program integrates and normalizes each mode to a value of unit displacement at the tip. Modes which are representative of the expected normal mode shapes are suggested.

4. Frequencies - Rotational and forcing frequencies are input independently. A frequency sweep may be simulated with a single card for each discrete frequency. \( \Omega = 0 \) is allowed.

5. Hub data - The hub is represented by a single degree of freedom spring, mass, damper in each direction. These parameters may be easily changed with forcing frequency to simulate actual hub impedances. Optionally 0, 1, 2 or 3 directions of motion are allowed. Sinusoidal forcing in any of these directions may be specified.

6. Blade forces - Optional forces may be applied at any blade station. An optional \( 1 - \cos \) type excitation for a specified fraction of one revolution is available.

7. Floquet option - If this option is selected, the program automatically produces a Floquet transition matrix by performing one (force) cycle for each initial condition. A further option ignores the steady effects due to such quantities as twist and precone.

8. Periodic solution - A periodic solution is obtained through the Floquet matrix which allows the solution for the initial conditions which will result in periodicity.

9. Nonlinear options - All, in-plane only, or no nonlinear effects may be optionally included in the solution.

10. Solution controls - The integration procedure used includes error checks and automatically selects appropriate sized integration increments. The user specifies quantities such as the number of cycles, error bound, variable to be tested for error, initial condition (unless periodic solution is specified).
SYSTEM IDENTIFICATION

The mass parameters of any continuous structure are not amenable to direct verification. An operational rotor blade is subjected to very large centrifugal forces and undergoes a highly coupled motion which includes deformation of the elastic axis in and out of the plane of rotation and torsional deformations about this axis. Under these conditions, the adequacy of the mass parameters which are based on a fictitious homogeneous section are in some doubt. While there is no way of directly measuring these parameters, the relationship between them and the normal modes, which are at least conceptionally measurable, are well understood.

The method of incomplete models (References 4 and 5), which addresses the problem, has been adapted to the specific set of rotor blade parameters. This formulation determines the minimum changes required in the intuitively derived set of mass parameters to make them compatible with the measured modes. There are other related developments and features of the implementation program which will yield valuable information regarding the adequacy of the analytical model. These are derived and discussed in this section.

THEORETICAL BACKGROUND

Consider a discrete element dynamic model of a continuous structure. One part of this model is a mass matrix, \( M \). If \( \psi_k \) is a vector representing the \( k \)-th normal mode, there exists a necessary orthogonality relationship as follows:

\[
\psi_k^T M \psi_n = 0 \quad k \neq n
\]  

(38)

If the modal vectors are considered to be known, and the masses unknown, this equation can be rewritten as a set of linear equations:

\[
A \hat{M} = 0
\]  

(39)

where \( A \) is a matrix whose elements are products of the elements of the modal vectors, and \( \hat{M} \) is a vector made up of the unknown elements of the mass matrix. There will be one equation for each unique pair of modes and one unknown for each of the elements of \( \hat{M} \). The problem is formulated so that the symmetrical off-diagonal elements in the (symmetrical) mass matrix appear only once in the mass vector, \( \hat{M} \).
Since the scalar product $\psi_k^T M \psi_n$ is identical to $\psi_n^T M \psi_k$, there will be $NM(NM-1)/2$ equations, where $NM$ is the number of modes. If $N$ is the number of coordinates, the number of unknowns may be between $N$ and $N(N+1)/2$ where the first corresponds to a pure diagonal matrix and the upper limit corresponds to a fully populated mass matrix. As discussed in References 4 and 5 it is usual and desirable to have many more unknowns than equations. There are, thus, an infinite number of solutions which will satisfy Equation (39).

It is, of course, desired to obtain that solution which is the most representative of the actual structure. This objective may be achieved by finding, of those mass matrices which satisfies Equation (39), and (38), that which is closest to an analytically derived model of the structure. That is to say, determine the smallest possible changes in the analytical mass matrix necessary to orthogonalize the measured modes. This may be done as follows. Let $\bar{M}_A$ be a vector which is made up of the elements of the analytical (or approximate) mass matrix and then write $\bar{M} = \bar{M}_A + \Delta \bar{M}$, where $\Delta \bar{M}$ represents the required changes in $\bar{M}_A$. Substituting into Equation (39) yields:

$$A \Delta \bar{M} = - A \bar{M}_A$$

As discussed in Reference 5, the use of the matrix pseudoinverse yields a solution which has the minimum sum of the squares of the individual elements, i.e., $\Delta \bar{M}^T \Delta \bar{M} = \text{min}$. This solution may be written:

$$\Delta \bar{M}_\text{min} = - A^T (AA^T)^{-1} A \bar{M}_A$$

The application to the specific rotor blade problem is given below, where certain other more detailed considerations of minimization and other constraints are discussed.

**ROTOR BLADE APPLICATION**

The normal modes of a rotor blade are conveniently expressed in terms of the in-plane, out-of-plane, and torsional components as follows:

$$\psi_k = \begin{bmatrix} \bar{v} \\ \bar{w} \\ - \phi \end{bmatrix}_k$$

32
where \( \vec{v}, \vec{w}, \) and \( \phi \) are vectors, each having \( NX \) elements, when \( NX \) is the number of blade stations used in the analysis and test.

The mass matrix, as can be seen from the acceleration terms of Equations (5), (6), and (7) may be conveniently partitioned, where each of the partitions is a diagonal matrix of order \( NX \). The rotor blade form of Equation (38) then may be written:

\[
\begin{bmatrix}
\vec{v}^T \\
\vec{w}^T \\
\phi^T
\end{bmatrix}_k
\begin{bmatrix}
m_i & 0 & -(m\epsilon_i) \\
0 & m_i & (m\epsilon_i) \\
-(m\epsilon_i) & (m\epsilon_i) & (m\epsilon_i)^2
\end{bmatrix}
\begin{bmatrix}
\vec{v} \\
\vec{w} \\
\phi
\end{bmatrix}_n = 0 \quad k \neq n
\]

(42)

The elements of these diagonal partitions \( (i = 1, 2, \ldots NX) \) represent a "lumped mass" (rather than a "distributed mass") formulation of the problem, which is inherent in the matrix representation.

Treating the modal displacements as knowns and the mass parameters as unknowns, the analogy of Equation (39) becomes:

\[
\begin{bmatrix}
v_k \phi_i & w_k \phi_i & -v_k \phi_i & k_i \phi_i \\
+w_k \phi_i & +w_i \phi_k & -v_n \phi_k & k_i \phi_i \\

\end{bmatrix}
\begin{bmatrix}
m \\
-m\epsilon \\
m n\epsilon \\
-mk^2
\end{bmatrix} = 0
\]

(43)

where, typically, \( v_{ki} \) represents the in-plane displacement of mode \( k \) at station \( i \). Each partition of the matrix \( A \) has \( NM(NM-1)/2 \) rows (one for each pair of modes, \( k < n \)) and \( NX \) columns, one for each station \( (i = 1, 2, \ldots NX) \). This, there are \( NM(NM-1)/2 \) equations and \( 4 \cdot NX \) unknowns (in vector \( \mathbb{M} \)).
As above, let $\overline{M} = \overline{M}_A + \overline{\Delta M}$, then Equation (43) is:

$$A\overline{\Delta M} = -A\overline{M}_A$$  \hspace{1cm} (44)

This equation may be solved for minimum $\overline{\Delta M}$ as in Equation (41). However, if there are significant differences in size between elements of $M_A$ it would not be appropriate to simply minimize the sum of the squares of the magnitudes of the changes. This procedure could result in excessively large percentage changes in the very small elements, even though these same changes would be quite small compared to the larger elements.

It is possible, through a simple modification in the method to minimize the sum of the squares of the percentage changes, which is a more reasonable criteria. In addition, it is also possible to allow the analyst to indicate a level of confidence in each element, so that items with higher confidence will tend to change least. The result is a solution which has a weighted sum of squares of the elements at a minimum.

Let the $i$-th element of $\overline{M}_A$ be designated $\overline{(M)_i}$ and the corresponding assigned weighting factor (confidence level) be $w_i$. Form a diagonal matrix $W$ such that $W_{ii} = w_i/(\overline{(M)_i})$. Then the elements of $W\overline{\Delta M}$ are

$$(W\overline{\Delta M})_i = w_i(\overline{\Delta M})_i/\overline{(M)_i}$$

which is the function that should be minimized. This is achieved by making $W\overline{\Delta M}$ the unknown in Equation (44) by inserting $I = W^{-1}W$ as follows:

$$AW^{-1}W\overline{\Delta M} = -A\overline{M}_A$$  \hspace{1cm} (45)

Then, as above:

$$(W\overline{\Delta M})_{\text{min}} = -W^{-1}A^T(AW^{-2}A^T)^{-1}A\overline{M}_A$$

and

$$\overline{M} = \overline{M}_A - W^{-2}A^T(AW^{-2}A^T)^{-1}A\overline{M}_A$$  \hspace{1cm} (46)

such that:

$$\overline{\Delta M}^T W^2 \overline{\Delta M} = \text{min}$$
MASS CONSTRAINTS

Since the number of equations is generally much less than the number of unknowns, it is possible to add equations to Equation (43) which will impose constraints on the mass parameters. In the method as implemented, five optional constraints are available. These each maintain the following mass characteristics at the same value they have in \( \bar{M}_A \). These constraints refer to: total mass, radial static moment (cg), chordwise static moment (cg), flapping moment of inertia, and feathering moment of inertia. These five constraints result in the following equations added to Equation (43):

\[
\begin{bmatrix}
1,1,1... \\
x_1,x_2,x_3... \\
0 \\
x_1^2,x_2^2... \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
m \\
\theta
\end{bmatrix}
= \begin{bmatrix}
\Sigma m_{A_i} \\
\Sigma X_i m_{A_i} \\
\Sigma (m_e) A_i \\
\Sigma (\theta m_e)^2 A_i \\
\Sigma (\theta m_k)^2 A_i
\end{bmatrix}
\] (47)

The solution then becomes:

\[
\bar{M} = \bar{M}_A - W^{-2} A^T (A W^{-2} A^T)^{-1} (A \bar{M}_A - \bar{F})
\] (48)

where \( \bar{F} \) is the right-hand side vector of Equation (43) augmented by that of Equation (47).

Thus it is possible to find the necessary changes in the mass matrix to make the modes orthogonal, such that the weighted sum of squares of the percentage changes is a minimum and the specified mass characteristics remain invariant.
ROTATIONAL SPEED EFFECTS

The mass matrix discussed above is independent of the blade rotational speed, $\Omega$. The natural frequencies and the mode shapes, however, do change as the rotational speed is changed. The analysis, as presented, is valid for any single $\Omega$ including the nonrotating condition, $\Omega = 0$.

The fact that the modes change with $\Omega$ provides an opportunity for obtaining additional information within a fixed range of forcing frequencies over that available for a conventional nonrotating structure. If several modes are measured at each of several values of $\Omega$, the same mass matrix must make the modes at any one $\Omega$ orthogonal.

Thus, the method above has been modified to accept modes at different values of $\Omega$ and to set up an equation for each pair of modes at each $\Omega$. For example, if the first three modes were identified at three $\Omega$'s, there would be nine equations which would provide information about the mass matrix.

MODE CHANGES

The measured data, even if exact, is not sufficient to uniquely identify an analytical model and thus intuitive decisions are required of the user of this method. Some of these decisions have been described above. In addition to finding the necessary mass model changes, consideration should be given to the unavoidable errors in the measured modes. It is of interest to determine the minimum changes that would be required in the modes to achieve orthogonality using the analytical mass matrix. Methods of this general type have been suggested in the literature from time to time (References 6, 7, and 8). The method developed and implemented in this study uses techniques very similar to those for the mass identification, above.

If the modes are placed in order of decreasing confidence (usually in order of increasing natural frequency), the method assumes the first is correct, changes the second to make it orthogonal to the first, then changes the third to make it orthogonal to the first and the corrected second mode, and similarly for all higher modes. The changes are the minimum sum of squares of the percentage changes of each element as discussed above.

The first equation may be written:

$$\psi_1^T M (\psi_2 + \Delta \psi_2) = 0$$

or

$$A \Delta \psi_2 = -A \psi_2$$

(49)

36
where $A = \psi_1^TM$ is a $1 \times 3\cdot NX$ matrix. The next equation then is:

$$A\Delta\psi_3 = -A\psi_3$$  \hspace{1cm} (50)

where:

$$A = \begin{bmatrix} \psi_1^T \\ M \psi_2^T + \Delta\psi_2^T \end{bmatrix}$$

and $A$ is a $2 \times 3\cdot NX$ matrix.

The equations for $\Delta\psi_M$ results in an $A$ matrix of order $M-1 \times 3\cdot NX$. The procedure used for solving these equations is the same as that described above without any weighting function, $w$, assigned to the individual elements.

**PROGRAM FEATURES - ROTSI**

This program has been designed to provide maximum flexibility as a research tool. The theoretical basis has been described in the previous paragraphs. The Users Guide with detailed input instructions is in Appendix B. This section will briefly outline several of the major features and capabilities of the program.

1. **Normalization** - the modes may be normalized so the diagonal elements of the generalized mass matrix are unity.

2. **Add modes** - after a computation is completed, additional modes may be added and further operations may be performed.

3. **Rotational speed** - modes of more than one rotational speed may be included (for mass identification) and the proper pairing takes place automatically.

4. **Random errors** - modes may be polluted with random errors with specified random or bias errors for sensitivity analyses.

5. **Modal changes** - necessary mode changes as described above with constant mass matrix may be determined.

6. **Limited mode changes** - modes may be changed as above but with limits specified for each mode. Truncation or scaling options are available.

7. **Mass changes** - weighted minimum mass changes may be obtained as described above.
8. Invariant stations - the mass parameters at selected stations may be held invariant.

9. Invariant parameters - mass, static moments, moments of inertia may optionally be maintained invariant during mass identification.

10. Sequential operations - the various options may be executed sequentially, for example, one may first change all the modes up to some specified percentages and then finish the correction by modifying the mass matrix.
METHOD APPLICATIONS

The two programs were continually checked for validity and reasonableness during their development. All features were at least qualitatively verified. The programs were then used to approximately simulate the tests to be carried out in the vacuum chamber at the Langley Research Center. These applications are described below.

SIMULATION DATA

The system simulated consisted of two blades and a hub with a vertical degree of freedom. The system was excited by a vertical force at the hub.

Each blade was represented by 17 stations. The parameters are shown in Table 1 which is taken from an actual computer run. The units are all in the lb-in-sec system.

Tables 2, 3, and 4 show the modes used as generalized degrees of freedom. These modes were developed from an approximate cantilever eigenvalue analysis. The one in-plane, three out-of-plane, and one torsional mode represent all the modes expected to have natural frequencies below 12/rev at $\Omega = 25$ rad/sec. The tables illustrate the second and first derivative and the displacements after normalization.

The hub was arbitrarily represented by a mass of 0.6 lb-sec$^2$/in and a spring rate of 20,000 lb/in. This implies a rigid rotor vertical natural frequency of 111. rad/sec or 4.44/rev at $\Omega = 25$ rad/sec.

Tables 5 and 6 give the blade and hub matrices as described in the section on Method of Solution and Equations (36) and (37).

SIMULATION COMPUTATIONS

Simulated frequency sweeps were carried out at $\Omega = 0$, 20, and 25 rad/sec. The Floquet option was used to obtain precise periodic responses to sinusoidal excitation at the hub. The objective of the simulated test was to locate the frequencies at which hub vertical antiresonances occur. At this frequency, cantilever conditions exist and since damping is light the displacement will be a good approximation to the coupled cantilever normal modes of the blades. Since discrete frequency inputs are required, a coarse sweep was first carried out, followed by necessary points at small frequency intervals to identify the point of zero hub displacement.
<table>
<thead>
<tr>
<th>X</th>
<th>M</th>
<th>E</th>
<th>SMALL EA</th>
<th>KM1</th>
<th>KM2</th>
<th>KA</th>
<th>THETA PRIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>6.470E-03</td>
<td>-2.925E 00</td>
<td>-2.850E 00</td>
<td>1.265E 00</td>
<td>5.557E 00</td>
<td>5.699E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>1.0</td>
<td>5.900E-03</td>
<td>-2.275E 00</td>
<td>-2.163E 00</td>
<td>1.281E 00</td>
<td>5.819E 00</td>
<td>5.958E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>2.0</td>
<td>4.450E-03</td>
<td>-1.407E 00</td>
<td>-1.269E 00</td>
<td>1.219E 00</td>
<td>6.075E 00</td>
<td>6.147E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>3.0</td>
<td>3.190E-03</td>
<td>-1.030E 00</td>
<td>-8.750E-01</td>
<td>1.057E 00</td>
<td>5.820E 00</td>
<td>5.915E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>4.0</td>
<td>2.320E-03</td>
<td>-9.250E-01</td>
<td>-7.250E-01</td>
<td>9.133E-01</td>
<td>6.018E 00</td>
<td>6.087E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>5.0</td>
<td>1.720E-03</td>
<td>-1.100E 00</td>
<td>-8.500E-01</td>
<td>8.010E-01</td>
<td>6.475E 00</td>
<td>6.524E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>6.0</td>
<td>1.650E-03</td>
<td>-9.050E-01</td>
<td>-5.630E-01</td>
<td>8.160E-01</td>
<td>6.260E 00</td>
<td>6.313E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>7.0</td>
<td>1.490E-03</td>
<td>-7.900E-01</td>
<td>-3.750E-01</td>
<td>8.090E-01</td>
<td>6.186E 00</td>
<td>6.239E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>8.0</td>
<td>1.340E-03</td>
<td>-7.000E-01</td>
<td>-1.500E-01</td>
<td>7.910E-01</td>
<td>6.082E 00</td>
<td>6.133E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>9.0</td>
<td>1.300E-03</td>
<td>-4.130E-01</td>
<td>2.300E-01</td>
<td>8.000E-01</td>
<td>5.785E 00</td>
<td>5.845E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>10</td>
<td>1.420E-03</td>
<td>-3.250E-01</td>
<td>7.000E-01</td>
<td>7.840E-01</td>
<td>5.394E 00</td>
<td>5.451E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>11</td>
<td>1.550E-03</td>
<td>1.050E 00</td>
<td>7.640E-01</td>
<td>5.066E 00</td>
<td>5.123E 00</td>
<td>5.180E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>12</td>
<td>1.540E-03</td>
<td>1.037E 00</td>
<td>7.670E-01</td>
<td>4.937E 00</td>
<td>4.996E 00</td>
<td>5.054E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>13</td>
<td>1.540E-03</td>
<td>1.025E 00</td>
<td>7.670E-01</td>
<td>4.914E 00</td>
<td>4.972E 00</td>
<td>5.030E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>14</td>
<td>1.590E-03</td>
<td>1.087E 00</td>
<td>7.560E-01</td>
<td>4.899E 00</td>
<td>4.956E 00</td>
<td>5.012E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>15</td>
<td>1.620E-03</td>
<td>1.162E 00</td>
<td>7.480E-01</td>
<td>4.899E 00</td>
<td>4.956E 00</td>
<td>5.012E 00</td>
<td>-4.850E-04</td>
</tr>
<tr>
<td>16</td>
<td>1.680E-03</td>
<td>1.162E 00</td>
<td>7.480E-01</td>
<td>4.899E 00</td>
<td>4.956E 00</td>
<td>5.012E 00</td>
<td>-4.850E-04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E1 OP</th>
<th>E1 IP</th>
<th>GJ</th>
<th>EA</th>
<th>EBl</th>
<th>EB2</th>
<th>ECI</th>
<th>ECI*</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.500E 08</td>
<td>9.000E 09</td>
<td>2.400E-08</td>
<td>2.440E-08</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.000E 08</td>
<td>8.250E 09</td>
<td>1.850E-08</td>
<td>2.275E-08</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.530E 08</td>
<td>6.030E 09</td>
<td>1.550E-08</td>
<td>1.649E-08</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.280E 08</td>
<td>4.000E 09</td>
<td>9.500E 07</td>
<td>1.144E 08</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9.250E 07</td>
<td>3.250E 09</td>
<td>4.700E 07</td>
<td>8.310E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7.000E 07</td>
<td>2.650E 09</td>
<td>3.350E 07</td>
<td>6.060E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.000E 07</td>
<td>2.350E 09</td>
<td>3.300E 07</td>
<td>5.750E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4.000E 07</td>
<td>2.040E 09</td>
<td>3.120E 07</td>
<td>5.250E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.500E 07</td>
<td>1.720E 09</td>
<td>3.500E 07</td>
<td>4.750E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>1.460E 09</td>
<td>3.150E 07</td>
<td>4.630E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>1.250E 09</td>
<td>3.300E 07</td>
<td>4.580E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>1.070E 09</td>
<td>3.450E 07</td>
<td>4.630E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>9.700E 08</td>
<td>3.200E 07</td>
<td>4.600E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>1.000E 09</td>
<td>3.400E 07</td>
<td>4.600E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.000E 07</td>
<td>1.000E 09</td>
<td>3.300E 07</td>
<td>4.750E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.130E 07</td>
<td>1.000E 09</td>
<td>3.300E 07</td>
<td>4.900E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.250E 07</td>
<td>1.010E 09</td>
<td>3.300E 07</td>
<td>4.900E 07</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
### Table 2. In-Plane Modes

#### Second Derivatives

<table>
<thead>
<tr>
<th>Mode</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.504E-05</td>
</tr>
<tr>
<td>2</td>
<td>1.580E-05</td>
</tr>
<tr>
<td>3</td>
<td>2.114E-05</td>
</tr>
<tr>
<td>4</td>
<td>3.034E-05</td>
</tr>
<tr>
<td>5</td>
<td>3.301E-05</td>
</tr>
<tr>
<td>6</td>
<td>3.518E-05</td>
</tr>
<tr>
<td>7</td>
<td>3.393E-05</td>
</tr>
<tr>
<td>8</td>
<td>3.268E-05</td>
</tr>
<tr>
<td>9</td>
<td>3.196E-05</td>
</tr>
<tr>
<td>10</td>
<td>2.942E-05</td>
</tr>
<tr>
<td>11</td>
<td>2.566E-05</td>
</tr>
<tr>
<td>12</td>
<td>2.089E-05</td>
</tr>
<tr>
<td>13</td>
<td>1.454E-05</td>
</tr>
<tr>
<td>14</td>
<td>7.446E-06</td>
</tr>
<tr>
<td>15</td>
<td>2.708E-06</td>
</tr>
<tr>
<td>16</td>
<td>2.557E-07</td>
</tr>
<tr>
<td>17</td>
<td>0.0</td>
</tr>
</tbody>
</table>

#### First Deriv (Normalized)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.234E-04</td>
</tr>
<tr>
<td>3</td>
<td>1.972E-04</td>
</tr>
<tr>
<td>4</td>
<td>4.032E-04</td>
</tr>
<tr>
<td>5</td>
<td>1.037E-03</td>
</tr>
<tr>
<td>6</td>
<td>1.719E-03</td>
</tr>
<tr>
<td>7</td>
<td>2.310E-03</td>
</tr>
<tr>
<td>8</td>
<td>3.076E-03</td>
</tr>
<tr>
<td>9</td>
<td>3.720E-03</td>
</tr>
<tr>
<td>10</td>
<td>4.332E-03</td>
</tr>
<tr>
<td>11</td>
<td>4.883E-03</td>
</tr>
<tr>
<td>12</td>
<td>5.348E-03</td>
</tr>
<tr>
<td>13</td>
<td>5.703E-03</td>
</tr>
<tr>
<td>14</td>
<td>5.922E-03</td>
</tr>
<tr>
<td>15</td>
<td>6.024E-03</td>
</tr>
<tr>
<td>16</td>
<td>6.054E-03</td>
</tr>
<tr>
<td>17</td>
<td>6.055E-03</td>
</tr>
</tbody>
</table>

#### Mode Shapes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>4.934E-04</td>
</tr>
<tr>
<td>3</td>
<td>1.135E-03</td>
</tr>
<tr>
<td>4</td>
<td>3.536E-03</td>
</tr>
<tr>
<td>5</td>
<td>1.793E-02</td>
</tr>
<tr>
<td>6</td>
<td>4.595E-02</td>
</tr>
<tr>
<td>7</td>
<td>8.677E-02</td>
</tr>
<tr>
<td>8</td>
<td>1.416E-01</td>
</tr>
<tr>
<td>9</td>
<td>2.096E-01</td>
</tr>
<tr>
<td>10</td>
<td>2.901E-01</td>
</tr>
<tr>
<td>11</td>
<td>3.823E-01</td>
</tr>
<tr>
<td>12</td>
<td>4.846E-01</td>
</tr>
<tr>
<td>13</td>
<td>5.951E-01</td>
</tr>
<tr>
<td>14</td>
<td>7.113E-01</td>
</tr>
<tr>
<td>15</td>
<td>8.308E-01</td>
</tr>
<tr>
<td>16</td>
<td>9.516E-01</td>
</tr>
<tr>
<td>17</td>
<td>1.000E+00</td>
</tr>
</tbody>
</table>
### Table 3. Out-of-Plane Modes

$\text{iO} = 4$  \hspace{1cm} \text{Out-of-Plane Modes}

#### Second Derivatives

<table>
<thead>
<tr>
<th>(i)</th>
<th>1.402E-05</th>
<th>-4.294E-05</th>
<th>1.909E-04</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.792E-05</td>
<td>-4.191E-05</td>
<td>1.780E-04</td>
</tr>
<tr>
<td>3</td>
<td>2.657E-05</td>
<td>-6.097E-05</td>
<td>2.582E-04</td>
</tr>
<tr>
<td>4</td>
<td>4.590E-05</td>
<td>-1.012E-04</td>
<td>3.900E-04</td>
</tr>
<tr>
<td>5</td>
<td>4.056E-05</td>
<td>-7.579E-05</td>
<td>1.832E-04</td>
</tr>
<tr>
<td>6</td>
<td>3.784E-05</td>
<td>-4.479E-05</td>
<td>7.925E-05</td>
</tr>
<tr>
<td>7</td>
<td>4.318E-05</td>
<td>-2.610E-06</td>
<td>4.891E-05</td>
</tr>
<tr>
<td>8</td>
<td>2.808E-05</td>
<td>6.047E-05</td>
<td>5.184E-04</td>
</tr>
<tr>
<td>9</td>
<td>2.124E-05</td>
<td>1.152E-04</td>
<td>5.069E-04</td>
</tr>
<tr>
<td>10</td>
<td>1.691E-05</td>
<td>1.713E-04</td>
<td>1.989E-04</td>
</tr>
<tr>
<td>11</td>
<td>1.188E-05</td>
<td>1.906E-04</td>
<td>2.975E-04</td>
</tr>
<tr>
<td>12</td>
<td>8.304E-06</td>
<td>1.672E-04</td>
<td>7.154E-04</td>
</tr>
<tr>
<td>13</td>
<td>5.926E-06</td>
<td>1.575E-04</td>
<td>9.073E-04</td>
</tr>
<tr>
<td>14</td>
<td>3.170E-06</td>
<td>1.074E-04</td>
<td>7.839E-04</td>
</tr>
<tr>
<td>15</td>
<td>1.278E-06</td>
<td>4.914E-05</td>
<td>4.176E-04</td>
</tr>
<tr>
<td>16</td>
<td>1.137E-07</td>
<td>4.592E-06</td>
<td>4.200E-05</td>
</tr>
<tr>
<td>17</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

#### (C) First Deriv (Normalized)

<table>
<thead>
<tr>
<th>(i)</th>
<th>0.0</th>
<th>0.0</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.437E-04</td>
<td>-3.394E-04</td>
<td>1.475E-03</td>
</tr>
<tr>
<td>3</td>
<td>2.327E-04</td>
<td>-5.452E-04</td>
<td>2.337E-03</td>
</tr>
<tr>
<td>4</td>
<td>5.226E-04</td>
<td>-1.194E-03</td>
<td>4.908E-03</td>
</tr>
<tr>
<td>5</td>
<td>1.387E-03</td>
<td>-2.963E-03</td>
<td>1.064E-02</td>
</tr>
<tr>
<td>6</td>
<td>2.171E-03</td>
<td>-4.169E-03</td>
<td>1.168E-02</td>
</tr>
<tr>
<td>7</td>
<td>2.981E-03</td>
<td>-4.594E-03</td>
<td>5.996E-03</td>
</tr>
<tr>
<td>8</td>
<td>3.694E-03</td>
<td>-3.980E-03</td>
<td>4.710E-03</td>
</tr>
<tr>
<td>9</td>
<td>4.187E-03</td>
<td>-2.203E-03</td>
<td>1.599E-02</td>
</tr>
<tr>
<td>10</td>
<td>4.560E-03</td>
<td>-6.215E-03</td>
<td>2.265E-02</td>
</tr>
<tr>
<td>11</td>
<td>4.857E-03</td>
<td>-6.279E-03</td>
<td>2.187E-02</td>
</tr>
<tr>
<td>12</td>
<td>5.050E-03</td>
<td>-8.054E-03</td>
<td>1.174E-02</td>
</tr>
<tr>
<td>13</td>
<td>5.197E-03</td>
<td>-1.150E-02</td>
<td>4.491E-03</td>
</tr>
<tr>
<td>14</td>
<td>5.284E-03</td>
<td>-1.415E-02</td>
<td>2.140E-02</td>
</tr>
<tr>
<td>15</td>
<td>5.328E-03</td>
<td>-1.571E-02</td>
<td>3.342E-02</td>
</tr>
<tr>
<td>16</td>
<td>5.342E-03</td>
<td>-1.629E-02</td>
<td>3.801E-02</td>
</tr>
<tr>
<td>17</td>
<td>5.343E-03</td>
<td>-1.627E-02</td>
<td>3.818E-02</td>
</tr>
</tbody>
</table>

#### (C) Mode Shapes

<table>
<thead>
<tr>
<th>(i)</th>
<th>0.0</th>
<th>0.0</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.749E-04</td>
<td>-1.358E-03</td>
<td>5.901E-03</td>
</tr>
<tr>
<td>3</td>
<td>1.328E-03</td>
<td>-3.127E-03</td>
<td>1.353E-02</td>
</tr>
<tr>
<td>4</td>
<td>4.346E-03</td>
<td>-1.008E-02</td>
<td>4.251E-02</td>
</tr>
<tr>
<td>5</td>
<td>2.346E-02</td>
<td>-5.165E-02</td>
<td>1.980E-01</td>
</tr>
<tr>
<td>6</td>
<td>5.903E-02</td>
<td>-1.230E-01</td>
<td>4.212E-01</td>
</tr>
<tr>
<td>7</td>
<td>1.106E-01</td>
<td>-2.106E-01</td>
<td>5.979E-01</td>
</tr>
<tr>
<td>8</td>
<td>1.773E-01</td>
<td>-2.961E-01</td>
<td>6.108E-01</td>
</tr>
<tr>
<td>9</td>
<td>2.561E-01</td>
<td>-3.577E-01</td>
<td>4.078E-01</td>
</tr>
<tr>
<td>10</td>
<td>3.437E-01</td>
<td>-3.732E-01</td>
<td>2.534E-02</td>
</tr>
<tr>
<td>11</td>
<td>4.379E-01</td>
<td>-3.238E-01</td>
<td>4.188E-01</td>
</tr>
<tr>
<td>12</td>
<td>5.371E-01</td>
<td>-2.004E-01</td>
<td>7.538E-01</td>
</tr>
<tr>
<td>13</td>
<td>6.396E-01</td>
<td>-4.882E-03</td>
<td>8.263E-01</td>
</tr>
<tr>
<td>14</td>
<td>7.444E-01</td>
<td>2.516E-01</td>
<td>5.673E-01</td>
</tr>
<tr>
<td>15</td>
<td>8.506E-01</td>
<td>5.503E-01</td>
<td>1.910E-02</td>
</tr>
<tr>
<td>16</td>
<td>9.573E-01</td>
<td>8.599E-01</td>
<td>6.952E-01</td>
</tr>
<tr>
<td>17</td>
<td>1.000E 00</td>
<td>1.000E 00</td>
<td>1.000E 00</td>
</tr>
</tbody>
</table>
### Table 4. Torsion Mode

| Mode | Second Derivatives
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.207E-04</td>
</tr>
<tr>
<td>2</td>
<td>1.207E-04</td>
</tr>
<tr>
<td>3</td>
<td>1.207E-04</td>
</tr>
<tr>
<td>4</td>
<td>1.207E-04</td>
</tr>
<tr>
<td>5</td>
<td>1.207E-04</td>
</tr>
<tr>
<td>6</td>
<td>1.063E-05</td>
</tr>
<tr>
<td>7</td>
<td>-2.343E-05</td>
</tr>
<tr>
<td>8</td>
<td>-2.749E-05</td>
</tr>
<tr>
<td>9</td>
<td>-3.226E-05</td>
</tr>
<tr>
<td>10</td>
<td>-3.299E-05</td>
</tr>
<tr>
<td>11</td>
<td>-3.299E-05</td>
</tr>
<tr>
<td>12</td>
<td>-3.299E-05</td>
</tr>
<tr>
<td>13</td>
<td>-3.299E-05</td>
</tr>
<tr>
<td>14</td>
<td>-3.299E-05</td>
</tr>
<tr>
<td>15</td>
<td>-3.642E-05</td>
</tr>
<tr>
<td>16</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### (c) First Deriv (Normalized)

<table>
<thead>
<tr>
<th>Mode</th>
<th>First Deriv (Normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>9.659E-04</td>
</tr>
<tr>
<td>3</td>
<td>1.449E-03</td>
</tr>
<tr>
<td>4</td>
<td>2.415E-03</td>
</tr>
<tr>
<td>5</td>
<td>4.829E-03</td>
</tr>
<tr>
<td>6</td>
<td>6.414E-03</td>
</tr>
<tr>
<td>7</td>
<td>8.178E-03</td>
</tr>
<tr>
<td>8</td>
<td>1.036E-03</td>
</tr>
<tr>
<td>9</td>
<td>1.362E-03</td>
</tr>
<tr>
<td>10</td>
<td>1.765E-03</td>
</tr>
<tr>
<td>11</td>
<td>2.112E-03</td>
</tr>
<tr>
<td>12</td>
<td>2.492E-03</td>
</tr>
<tr>
<td>13</td>
<td>2.792E-03</td>
</tr>
<tr>
<td>14</td>
<td>2.123E-03</td>
</tr>
<tr>
<td>15</td>
<td>1.472E-03</td>
</tr>
<tr>
<td>16</td>
<td>5.783E-04</td>
</tr>
<tr>
<td>17</td>
<td>5.526E-04</td>
</tr>
</tbody>
</table>
TABLE 5. THE BLADE INERTIAL, DAMPING, STIFFNESS MATRICES, AND INVERSE OF THE INERTIAL MATRIX AT $\Omega = 25$ RAD/SEC (SEE EQ. 36, 37)

<table>
<thead>
<tr>
<th>COIR</th>
<th>3.842E 02</th>
<th>0.0</th>
<th>0.0</th>
<th>0.0</th>
<th>5.527E 01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.659E 02</td>
<td>2.379E 02</td>
<td>-2.544E 01</td>
<td>5.633E 02</td>
<td></td>
</tr>
<tr>
<td>-0.0</td>
<td>-4.712E 02</td>
<td>-1.848E 02</td>
<td>9.288E 01</td>
<td>-5.558E 02</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>6.047E 02</td>
<td>1.278E 02</td>
<td>-1.883E 02</td>
<td>6.554E 02</td>
<td></td>
</tr>
<tr>
<td>1.128E 02</td>
<td>1.005E 03</td>
<td>5.848E 02</td>
<td>-8.914E 01</td>
<td>3.080E 04</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CODR</th>
<th>-6.604E 02</th>
<th>-1.493E 01</th>
<th>-3.420E 01</th>
<th>-2.353E 01</th>
<th>-4.104E 02</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.850E 01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.819E 01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-2.737E 01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6.686E 02</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COR</th>
<th>-1.803E 06</th>
<th>6.378E 04</th>
<th>8.455E 05</th>
<th>3.021E 06</th>
<th>8.414E 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.294E 05</td>
<td>-4.525E 05</td>
<td>-1.113E 06</td>
<td>-1.739E 06</td>
<td>-4.470E 06</td>
<td></td>
</tr>
<tr>
<td>-9.931E 04</td>
<td>4.995E 05</td>
<td>5.408E 05</td>
<td>-1.227E 06</td>
<td>2.074E 06</td>
<td></td>
</tr>
<tr>
<td>5.645E 04</td>
<td>-5.593E 05</td>
<td>3.336E 05</td>
<td>3.681E 06</td>
<td>-5.062E 05</td>
<td></td>
</tr>
<tr>
<td>-9.821E 03</td>
<td>-9.167E 05</td>
<td>-1.632E 06</td>
<td>-4.679E 05</td>
<td>-1.179E 09</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RIOC</th>
<th>2.604E 03</th>
<th>-1.773E 06</th>
<th>-2.421E 05</th>
<th>-9.396E 06</th>
<th>-4.879E 06</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.526E 06</td>
<td>1.408E 02</td>
<td>2.540E 02</td>
<td>1.064E 02</td>
<td>-2.563E 05</td>
<td></td>
</tr>
<tr>
<td>-8.327E 06</td>
<td>-2.001E 02</td>
<td>-4.458E 02</td>
<td>-1.928E 02</td>
<td>-2.836E 05</td>
<td></td>
</tr>
<tr>
<td>-4.841E 06</td>
<td>3.168E 02</td>
<td>5.188E 02</td>
<td>1.599E 02</td>
<td>1.648E 05</td>
<td></td>
</tr>
<tr>
<td>-9.959E 06</td>
<td>1.233E 05</td>
<td>1.683E 04</td>
<td>6.531E 05</td>
<td>3.391E 05</td>
<td></td>
</tr>
</tbody>
</table>

44
### Table 6. Hub Matrices (see Eq. 36, 37)

<table>
<thead>
<tr>
<th>BIRIIH</th>
<th>1.915E-01</th>
<th>-1.915E-01</th>
<th>-8.149E-08</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.915E-01</td>
<td>1.915E-01</td>
<td>8.149E-08</td>
</tr>
<tr>
<td></td>
<td>1.313E-04</td>
<td>-1.313E-04</td>
<td>3.109E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIRID</th>
<th>2.465E-01</th>
<th>-5.578E-03</th>
<th>-1.277E-02</th>
<th>-8.790E-03</th>
<th>-1.533E-01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.465E-01</td>
<td>-5.578E-03</td>
<td>1.277E-02</td>
<td>8.790E-03</td>
<td>1.533E-01</td>
</tr>
<tr>
<td></td>
<td>5.712E-02</td>
<td>-4.236E-07</td>
<td>9.700E-07</td>
<td>6.675E-07</td>
<td>1.164E-05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIRIO</th>
<th>-7.632E-02</th>
<th>2.381E-01</th>
<th>3.156E-02</th>
<th>1.128E-03</th>
<th>2.748E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.632E-02</td>
<td>-2.381E-01</td>
<td>-3.156E-02</td>
<td>-1.128E-03</td>
<td>-2.748E-02</td>
</tr>
<tr>
<td></td>
<td>-2.440E-01</td>
<td>-1.329E-02</td>
<td>-2.770E-02</td>
<td>-3.819E-03</td>
<td>3.364E-03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIRIDH</th>
<th>6.286E-03</th>
<th>-6.286E-03</th>
<th>0.0</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-6.286E-03</td>
<td>6.286E-03</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>2.920E-03</td>
<td>-2.920E-03</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIRI</th>
<th>3.736E-04</th>
<th>-7.553E-08</th>
<th>-1.032E-06</th>
<th>-4.002E-07</th>
<th>-2.078E-07</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.736E-04</td>
<td>7.553E-08</td>
<td>1.032E-06</td>
<td>4.002E-07</td>
<td>2.078E-07</td>
</tr>
<tr>
<td></td>
<td>-2.837E-08</td>
<td>-2.920E-03</td>
<td>5.237E-03</td>
<td>-2.087E-03</td>
<td>-9.654E-08</td>
</tr>
</tbody>
</table>
Note that since the responses are the steady-state periodic responses to \( \sin \omega f t \) forcing, the response at \( \omega f t = 90^\circ \) is the "real" or in-phase component and the response at \( \omega f t = 0^\circ \) is the "imaginary" or out-of-phase component. Figures 3-12 illustrate the hub responses in the vicinity of the antiresonant frequencies. In most cases, the imaginary component is too small to be observed and is not plotted. These figures also illustrate the system natural frequencies.

At each antiresonant frequency the amplitudes of the generalized coordinates were determined and normalized on the largest component. These represent cantilever coupled modes and are summarized in Table 7. A Campbell diagram displaying these frequencies is given in Figure 13.

The actual mode shapes in each of the three directions are shown in Figures 14-19. Figures 14 and 15 are the in-plane and torsion component shapes. Since only one of each was used as a degree of freedom in the simulation, these shapes are the same for all the coupled normal modes obtained. The magnitudes are given in Table 7. The out-of-plane bending was represented by three modes and different combinations appear for each normal mode. Figures 16-19 illustrate these shapes for all the modes referenced in Table 7. The amplitude of these normalized modes is the sum of the \( z_1, z_2, z_3 \) components given in the table. The small but noticeable effect of rotor speed is illustrated in these figures.

SYSTEM IDENTIFICATION

In order to test and illustrate the ROTSI methods and program, the data obtained in the simulation runs, above, was treated as if it were actual test data. The analytical model was first intuitively reduced to an eight station lumped mass model as shown on Table 8.

Several combinations of these modes were used for mass identification. A sample output is shown in Table 9 where the original parameter, the modified parameter and the percentage changes are given. Table 10 summarizes the sample analyses that were carried out showing mean absolute percent changes of the four parameters: \( m, e, \theta, K_m \). The results are not satisfactory as shown. In addition to these cases, other combinations of modes at different rotational speeds have yielded very large percentage change requirements.

Since similar analyses on other structures using as many as ten modes and 150 unknowns have been successfully carried out, the large changes required for all but the simplest combinations is surprising. However, there are two significant considerations which may shed some light on this problem.
Figure 6. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 1st OP Frequency $= 25.25$ Rad/Sec

Figure 7. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. An Apparent Highly Damped Response in Vicinity of 1st IP Frequency

Figure 8. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 20$ Rad/Sec. 2nd OP Frequency $= 86.25$ Rad/Sec
Figure 9. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 1st OP Frequency = 30.49 Rad/Sec

Figure 10. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. Apparent Highly Damped Response in Vicinity of 1st IP Frequency

Figure 11. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 2nd OP Frequency = 95.52 Rad/Sec

Figure 12. Hub Vertical Deflection vs Forcing Frequency, $\Omega = 25$. 3rd OP Frequency = 243.3 Rad/Sec
<table>
<thead>
<tr>
<th>Type</th>
<th>( \Omega ) (Rad/Sec)</th>
<th>( \omega )</th>
<th>( \gamma )</th>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( z_3 )</th>
<th>( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st OP</td>
<td>0</td>
<td>10.19</td>
<td>.0655</td>
<td>1.0</td>
<td>.0868</td>
<td>-.0100</td>
<td>.00097</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>25.25</td>
<td>.0408</td>
<td>1.0</td>
<td>.0020</td>
<td>-.0013</td>
<td>.00049</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>30.49</td>
<td>.0354</td>
<td>1.0</td>
<td>-.0198</td>
<td>.0013</td>
<td>.00037</td>
</tr>
<tr>
<td>1st IP</td>
<td>0</td>
<td>54.55</td>
<td>1.0</td>
<td>-.3393</td>
<td>.8503</td>
<td>-.0537</td>
<td>.000801</td>
</tr>
<tr>
<td>2nd OP</td>
<td>0</td>
<td>74.20</td>
<td>-1.928</td>
<td>-.3015</td>
<td>1.0</td>
<td>-.0561</td>
<td>.000348</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>86.25</td>
<td>-.6268</td>
<td>-.2863</td>
<td>1.0</td>
<td>-.0448</td>
<td>.000845</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>95.52</td>
<td>-.4180</td>
<td>-.2839</td>
<td>1.0</td>
<td>-.0379</td>
<td>.00104</td>
</tr>
<tr>
<td>3rd OP</td>
<td>0</td>
<td>222.0</td>
<td>.1569</td>
<td>.3240</td>
<td>.4024</td>
<td>1.0</td>
<td>.003650</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>243.3</td>
<td>-.131</td>
<td>.287</td>
<td>-.359</td>
<td>1.0</td>
<td>.000756</td>
</tr>
</tbody>
</table>
Figure 13. Campbell Diagram Illustrating Natural Frequencies Obtained During Simulated Frequency Sweep
Figure 14. In-Plane Mode Shape for All Frequencies

Figure 15. Torsional Mode Shape for All Frequencies
Figure 16. Out-of-Plane Shapes From 1st OP Coupled Modes

Figure 17. Out-of-Plane Shapes From 1st IP Coupled Modes
Figure 18. Out-of-Plane Shapes From 2nd OP Coupled Modes

Figure 19. Out-of-Plane Shapes From 3rd OP Coupled Modes
<table>
<thead>
<tr>
<th>I</th>
<th>STA</th>
<th>W</th>
<th>M</th>
<th>W</th>
<th>E</th>
<th>W</th>
<th>TH</th>
<th>W</th>
<th>KM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.000</td>
<td>1.</td>
<td>1.445E-01</td>
<td>-1.</td>
<td>010E 00</td>
<td>-2.</td>
<td>430E-02</td>
<td>1.</td>
<td>6.310E 00</td>
</tr>
<tr>
<td>2</td>
<td>110.000</td>
<td>1.</td>
<td>7.300E-02</td>
<td>-7.</td>
<td>450E-01</td>
<td>-5.</td>
<td>340E-02</td>
<td>1.</td>
<td>6.190E 00</td>
</tr>
<tr>
<td>3</td>
<td>150.000</td>
<td>1.</td>
<td>5.540E-02</td>
<td>-4.</td>
<td>400E-02</td>
<td>-7.</td>
<td>280E-02</td>
<td>1.</td>
<td>5.650E 00</td>
</tr>
<tr>
<td>4</td>
<td>190.000</td>
<td>1.</td>
<td>4.580E-02</td>
<td>1.</td>
<td>000E 00</td>
<td>-9.</td>
<td>220E-02</td>
<td>1.</td>
<td>5.060E 00</td>
</tr>
<tr>
<td>5</td>
<td>210.000</td>
<td>1.</td>
<td>3.080E-02</td>
<td>1.</td>
<td>030E 00</td>
<td>-1.</td>
<td>020E-01</td>
<td>1.</td>
<td>5.010E 00</td>
</tr>
<tr>
<td>6</td>
<td>230.000</td>
<td>1.</td>
<td>3.130E-02</td>
<td>1.</td>
<td>060E 00</td>
<td>-1.</td>
<td>120E-01</td>
<td>1.</td>
<td>5.000E 00</td>
</tr>
<tr>
<td>7</td>
<td>250.000</td>
<td>1.</td>
<td>3.500E-02</td>
<td>1.</td>
<td>130E 00</td>
<td>-1.</td>
<td>210E-01</td>
<td>1.</td>
<td>4.960E 00</td>
</tr>
<tr>
<td>8</td>
<td>268.000</td>
<td>1.</td>
<td>1.000E-02</td>
<td>1.</td>
<td>160E 00</td>
<td>-1.</td>
<td>300E-01</td>
<td>1.</td>
<td>4.960E 00</td>
</tr>
</tbody>
</table>
### Table 9. Sample Parameter Identification Output

<table>
<thead>
<tr>
<th></th>
<th>ORIG M</th>
<th>NEW M</th>
<th>PCT</th>
<th>ORIG E</th>
<th>NEW E</th>
<th>PCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.445E-01</td>
<td>1.443E-01</td>
<td>-0.2</td>
<td>1.010E-00</td>
<td>1.012E-00</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>7.300E-02</td>
<td>7.191E-02</td>
<td>-1.5</td>
<td>7.450E-01</td>
<td>7.562E-01</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>5.540E-02</td>
<td>5.415E-02</td>
<td>-2.3</td>
<td>4.400E-02</td>
<td>4.502E-02</td>
<td>2.3</td>
</tr>
<tr>
<td>4</td>
<td>4.580E-02</td>
<td>4.511E-02</td>
<td>-1.5</td>
<td>1.000E-00</td>
<td>1.015E-00</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>3.080E-02</td>
<td>3.072E-02</td>
<td>-0.3</td>
<td>1.030E-00</td>
<td>1.033E-00</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>3.130E-02</td>
<td>3.160E-02</td>
<td>1.0</td>
<td>1.060E-00</td>
<td>1.050E-00</td>
<td>-1.0</td>
</tr>
<tr>
<td>7</td>
<td>3.500E-02</td>
<td>3.605E-02</td>
<td>3.0</td>
<td>1.130E-00</td>
<td>1.097E-00</td>
<td>-2.9</td>
</tr>
<tr>
<td>8</td>
<td>1.000E-02</td>
<td>1.015E-02</td>
<td>1.5</td>
<td>1.160E-00</td>
<td>1.143E-00</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>ORIG TH</th>
<th>NEW TH</th>
<th>PCT</th>
<th>ORIG KM</th>
<th>NEW KM</th>
<th>PCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.430E-02</td>
<td>-2.430E-02</td>
<td>0.0</td>
<td>6.315E-00</td>
<td>6.315E-00</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>-5.340E-02</td>
<td>-5.340E-02</td>
<td>0.0</td>
<td>6.190E-00</td>
<td>6.237E-00</td>
<td>-0.8</td>
</tr>
<tr>
<td>3</td>
<td>-7.280E-02</td>
<td>-7.280E-02</td>
<td>0.0</td>
<td>5.650E-00</td>
<td>5.715E-00</td>
<td>1.1</td>
</tr>
<tr>
<td>4</td>
<td>-9.220E-02</td>
<td>-9.220E-02</td>
<td>0.0</td>
<td>5.060E-00</td>
<td>5.098E-00</td>
<td>-0.8</td>
</tr>
<tr>
<td>5</td>
<td>-1.020E-01</td>
<td>-1.020E-01</td>
<td>0.0</td>
<td>5.010E-00</td>
<td>5.017E-00</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>-1.120E-01</td>
<td>-1.120E-01</td>
<td>0.0</td>
<td>5.000E-00</td>
<td>4.976E-00</td>
<td>-0.5</td>
</tr>
<tr>
<td>7</td>
<td>-1.210E-01</td>
<td>-1.210E-01</td>
<td>0.0</td>
<td>4.960E-00</td>
<td>4.887E-00</td>
<td>-1.5</td>
</tr>
<tr>
<td>8</td>
<td>-1.300E-01</td>
<td>-1.300E-01</td>
<td>0.0</td>
<td>4.960E-00</td>
<td>4.924E-00</td>
<td>-0.7</td>
</tr>
</tbody>
</table>
### TABLE 10. SUMMARY OF MASS IDENTIFICATION RESULTS

**Input Modes**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>( \Omega = 0 )</th>
<th>20 Rad/Sec</th>
<th>Maximum Change (%)</th>
<th>Mean Change (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x x</td>
<td>1 2 3 4 1 2 1 2 3</td>
<td>0.7</td>
<td>0.3</td>
<td>1 Eq., 24 unknowns</td>
</tr>
<tr>
<td>1a</td>
<td>x x</td>
<td></td>
<td>1.5</td>
<td>0.6</td>
<td>5 mass constraints, 6 Equations</td>
</tr>
<tr>
<td>2</td>
<td>x x x x</td>
<td></td>
<td>-</td>
<td>-</td>
<td>very large changes</td>
</tr>
<tr>
<td>3</td>
<td>x x x</td>
<td></td>
<td>25.5</td>
<td>9.0</td>
<td>3 Equations</td>
</tr>
<tr>
<td>3a</td>
<td>x x x</td>
<td></td>
<td>26.4</td>
<td>9.0</td>
<td>mass const, 4 Equations</td>
</tr>
<tr>
<td>3b</td>
<td>x x x</td>
<td></td>
<td>24.7</td>
<td>9.2</td>
<td>5 mass constraints, 8 Equations</td>
</tr>
<tr>
<td>4</td>
<td>x x x x x</td>
<td></td>
<td>379.0</td>
<td>65.0</td>
<td>mode 3 apparently inconsistent</td>
</tr>
<tr>
<td>5</td>
<td>x x</td>
<td></td>
<td>1.2</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>x x</td>
<td></td>
<td>3.0</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>x x x x</td>
<td></td>
<td>13.6</td>
<td>3.8</td>
<td>3 Equations</td>
</tr>
<tr>
<td>7a</td>
<td>x x x</td>
<td></td>
<td>250.0</td>
<td>43.0</td>
<td>5 mass constraints, 8 Equations</td>
</tr>
<tr>
<td>8</td>
<td>x x x x x</td>
<td></td>
<td>307.0</td>
<td>45.0</td>
<td>2 Equations</td>
</tr>
<tr>
<td>9</td>
<td>x x</td>
<td></td>
<td>412.0</td>
<td>51.0</td>
<td>3 Equations</td>
</tr>
</tbody>
</table>
(1) Only five generalized coordinates (modes) were used in the simulation. The torsional mode participated only slightly in any of the normal modes, thus there are essentially only four degrees of freedom in the problem. Whenever the number of equations approaches four, the necessary changes can be expected to become large. This situation, of course, will not exist in a real test and, thus, it is expected that the analysis of actual test data may be considerably more successful. It is possible to use the simulation program using up to 11 degrees of freedom and it is expected that the results of such an analysis would be considerably improved.

(2) No case where data from two rotor speeds was used was successful. It is apparent, from Figures 14-19, that the predicted changes in mode shape with rotor speed is quite small. Thus, the equations resulting from the same modes at different speeds will be nearly identical and result in a nearly singular matrix. In the simulation program, as used in this report, the same modes were used as generalized coordinates for all rotor speeds, thus accentuating this condition. Whether the use of actual test data will improve this situation is uncertain since it is well known that the mode shapes change only slightly with rotor speed.

It is also noted that any combination which included the third mode at \( \Omega = 0 \) yielded poor results. No particular reason is seen for this effect, except that the second and third modes contain highly coupled in and out-of-plane responses. Since the in-plane and first out-of-plane mode are quite similar, there may be some analytical problems in orthogonalizing those modes with the analytical model used.

As an illustration of the mode change analysis, keeping the mass matrix invariant, the three modes at \( \Omega = 25 \text{ rad/sec.} \) were processed. The required changes are quite small and the results are shown in Table 11.
### TABLE 11. MODE CHANGES REQUIRED FOR ORTHOGONALITY

\( \Omega = 25 \text{ rad/sec} \)

Percentage Changes

<table>
<thead>
<tr>
<th>Sta</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( v )</td>
<td>( w )</td>
</tr>
<tr>
<td>1</td>
<td>No change</td>
<td>0</td>
<td>-.15</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>-.01</td>
<td>-1.45</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>-.02</td>
<td>-2.18</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>-.04</td>
<td>-1.42</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>-.04</td>
<td>-.22</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>-.06</td>
<td>1.01</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>-.09</td>
<td>3.01</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>-.03</td>
<td>1.46</td>
</tr>
</tbody>
</table>
CONCLUSIONS AND RECOMMENDATIONS

Two separate analytical methods have been developed. They both have been used as a basis for computer programs. The two programs are expected to be useful research tools for evaluating rotor dynamic analytical models in conjunction with the vacuum chamber whirl tests to be conducted at the Langley Research Center.

The first program allows the analyst to attempt to model these tests and to observe the agreement between analysis and experiment. The analytical model includes the important dynamic features of the test, such as hub degrees of freedom, non-uniform parameters, stiffness coupling between out-of-plane and in-plane motion, and the ability to simulate forcing frequency sweeps independent of rotor speed. The program has been designed to allow convenient changes in parameters, number of degrees of freedom, types of nonlinearities, periodic or transient solutions. The effects of parameters in blade responses, natural frequencies, and normal modes may be easily studied.

The second program, which is an adaptation of methods previously applied to nonrotating structures, makes use of observed blade normal modes to correct the mass and inertial coupling terms used in the analytical model. Other options allow the analyst to study the possibility of inaccurate modal measurements and combinations of modal and mass parameter changes. In addition, a feature which produces controlled random variations in the measured modes allows for a study of sensitivities of these results to inaccuracies in the observed data. The method also has the capability of making use of modes measured at more than one rotational speed.

Both programs have been extensively tested for validity and sample computations have been presented in this report. The second program which performs a class of system identification analyses, was tested using results obtained from the simulation program. The capability to handle more than a few modes or modes at more than one rotational frequency has not been demonstrated. The lack of adequate success is believed to be due to the relatively small number of generalized degrees of freedom used in the simulation program. Since other related applications of this technique have been significantly more successful, it is anticipated that the analysis of actual test data or the use of simulations having a larger number of participating modes will yield useful results.
The simulation program has the capability to use eleven blade generalized degrees of freedom. This limit is purely due to the dimensioning limitations and simple program modifications can increase this limit to any desired value. The simulation carried out used five modes as degrees of freedom. The lower frequency responses obtained are believed to be quite valid and this validity only becomes weaker as frequency ranges are reached which in reality include participation of modes which were not included in the analysis.

The following recommendations are made for useful continuation of this research.

1. Develop an analytical model, which is a better intuitive representation of the actual rotor system to be tested.

2. Simulate specific test conditions and make direct comparisons with actual test responses. If obvious apparent discrepancies exist, make rational intuitive changes in the analytical parameters whenever such changes can be justified by consideration of the physical characteristics of the rotor.

3. Use actual measured normal modes in both the nonrotating and rotating conditions to correct the mass and inertial coupling parameters and to study the sensitivities to measurement errors. Use these results to evaluate the possibility of obtaining significant information from non-rotating tests alone. Evaluate the use of this method to improve the analyst's capability to derive a more satisfactory model from the physical characteristics of the blades prior to any testing.

4. Use the simulation program for conditions and blades other than those tested to study the effects of blade and hub parameters on natural frequencies, blade and rotor responses and stability.

5. Because the simulation program is a convenient, flexible and adaptable program, it is strongly recommended that further developments of this program to include aerodynamics, controls and a more comprehensive fuselage representation be considered.
REFERENCES


APPENDIX A
DEFINITIONS OF INTEGRALS

Mass Integrals (Sta. No., Coefficient No.)

\[
\int_{x}^{R} f(x) \, dx
\]

\[MI(I,1) = \int m\]
\[MI(I,2) = \int mx\]
\[MI(I,3) = \int me\]
\[MI(I,4) = \int mex\]
\[MI(I,5) = \int me\theta\]
\[MI(I,6) = \int mex\theta\]
\[MI(I,7) = \int \frac{mK}{m_2}^2\]
\[MI(I,8) = \int \frac{mK}{m_2} \theta\]
\[MI(I,9) = \int mK\theta\]
\[MI(I,10) = \int \frac{K^2}{A^2} \tau\theta'\]

\[I = 1 \text{ to number of blade stations}\]
Y Integrals (Sta. No., Mode No., Coefficient No.)

\[
\int_s^R \int_x J \, dx
\]

\[
Y_I(I,J,1) = \int m_j
\]
\[
Y_I(I,J,2) = \int m_e y_j
\]
\[
Y_I(I,J,3) = \int m_e \theta y_j
\]
\[
Y_I(I,J,4) = \int m_x y_j
\]
\[
Y_I(I,J,5) = \int m_e \theta y_j
\]
\[
Y_I(I,J,6) = \int m_e x \theta y_j
\]
\[
Y_I(I,J,7) = \int y_j
\]
\[
Y_I(I,J,8) = \int e^{-A \theta} y_j
\]
\[
Y_I(I,J,9) = \int e^{-A \theta} y_j
\]
\[
Y_I(I,J,10) = \int_{0}^{x} e^{-A \theta} y_j \, dx
\]

\[
Y_{II}(I,J,1) = \int y_I(I,J,1)
\]
\[
Y_{II}(I,J,2) = \int y_I(I,J,2)
\]
\[
Y_{II}(I,J,3) = \int y_I(I,J,3)
\]
\[
Y_{II}(I,J,4) = \int y_I(I,J,4)
\]
\[
Y_{II}(I,J,5) = \int y_I(I,J,5)
\]
\[
Y_{II}(I,J,6) = \int y_I(I,J,6)
\]
\[
Y_{II}(I,J,7) = \int y_I(I,J,7)
\]
\[
Y_{II}(I,J,8) = \int y_I(I,J,8)
\]
\[
Y_{II}(I,J,9) = \int y_I(I,J,9)
\]

\[I = 1 \text{ to number of blade stations}\]

\[J = 1 \text{ to number of in-plane modes}\]
Z Integrals (Sta. No., Mode No., Coefficient No.)

\[
\int_{x}^{R} f(x) \, dx
\]

\[
Z_{I}(I,J,1) = \int mZ_{J}
\]

\[
Z_{I}(I,J,2) = \int meZ_{J}
\]

\[
Z_{I}(I,J,3) = \int mxZ_{J}^{i}
\]

\[
Z_{I}(I,J,4) = \int mexZ_{J}^{i}
\]

\[
Z_{I}(I,J,5) = \int me\theta Z_{J}^{i}
\]

\[
Z_{I}(I,J,6) = \int \tau Z_{J}^{u}
\]

\[
Z_{I}(I,J,7) = \int e_{A} \tau Z_{J}^{u}
\]

\[
Z_{I}(I,J,8) = \int e_{L} \theta \theta' Z_{J}^{u}
\]

\[
Z_{I}(I,J,9) = \int e_{A} \theta Z_{J}
\]

\[
Z_{II}(I,J,1) = \int Z_{I}(I,J,1)
\]

\[
Z_{II}(I,J,2) = \int Z_{I}(I,J,2)
\]

\[
Z_{II}(I,J,3) = \int Z_{I}(I,J,3)
\]

\[
Z_{II}(I,J,4) = \int Z_{I}(I,J,4)
\]

\[
Z_{II}(I,J,5) = \int Z_{I}(I,J,5)
\]

\[
Z_{II}(I,J,6) = \int Z_{I}(I,J,6)
\]

\[
Z_{II}(I,J,7) = \int Z_{I}(I,J,7)
\]

\[
Z_{II}(I,J,8) = \int Z_{I}(I,J,8)
\]

\[
Z_{II}(I,J,9) = \int Z_{I}(I,J,9)
\]

I = 1 to number of blade stations

J = 1 to number of out-of-plane modes
\( \phi \) Integrals (Sta. No., Mode No., Coefficient No.)

\[
\int R \int x \frac{dx}{x}
\]

\[
\begin{align*}
\text{PI}(I,J,1) &= \int \text{m}e\phi_J \\
\text{PI}(I,J,2) &= \int \text{m}e\phi_J \\
\text{PI}(I,J,3) &= \int \text{m}e\phi_J \\
\text{PI}(I,J,4) &= \int \text{m}K_m^2\phi_J \\
\text{PI}(I,J,5) &= \int \text{m}K\phi_J \\
\text{PI}(I,J,6) &= \int \text{E}^\phi_J \\
\text{PI}(I,J,7) &= \int K_A^2 \phi_J \\
\text{PI}(I,J,8) &= \int K_A^2 \phi_J dx
\end{align*}
\]

I = 1 to number of blade stations

J = 1 to number of torsional modes
Special Integrals (Sta. No., Mode No., Coefficient No.)

\[ \int \int f(x) \, dx \, dx \]

\[ SI(I,J,1) = \int_0^R \frac{1}{EA} YI(I,J,1) \, dx \]

\[ SI(I,J,2) = \int mYI(I,J,10) \]

\[ SI(I,J,3) = \int mZI(I,J,9) \]

\[ SI(I,J,4) = \int mPI(I,J,8) \]

\[ SI(I,J,5) = \int K A^2 \theta YI(I,J,1) \, dx \]
Equation Integrals

\[ \int_0^{\infty} \int ( ) \, dx \]

\[
\begin{align*}
\text{DYYI}(K,J,2) &= \int_K^Y Y_{I,J,2} \\
\text{DYYII}(K,J,1) &= \int_K^Y Y_{I,J,1} \\
\text{DYYII}(K,J,4) &= \int_K^Y Y_{I,J,4} \\
\text{DYYII}(K,J,5) &= \int_K^Y Y_{I,J,5} \\
\text{DYYII}(K,J,7) &= \int_K^Y Y_{I,J,7} \\
\text{DYZII}(K,J,1) &= \int_K^Y Z_{I,J,1} \\
\text{DYZII}(K,J,5) &= \int_K^Y Z_{I,J,5} \\
\text{DYPII}(K,J,3) &= \int_K^Y P_{I,J,3} \\
\text{DYMI}(K,4) &= \int_K^Y M_{I,4} \\
\text{DYMII}(K,1) &= \int_K^Y M_{I,1} \\
\text{DYMII}(K,2) &= \int_K^Y M_{I,2} \\
\text{DYMII}(K,3) &= \int_K^Y M_{I,3}
\end{align*}
\]
<table>
<thead>
<tr>
<th>Expression</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{DYMII}(K,5)$</td>
<td>$\int Y_K \text{MII}(I,5)$</td>
</tr>
<tr>
<td>$\text{DYSI}(K,J,i)$</td>
<td>$\int Y_K \text{SI}(I,J,i) \quad i = 1 \to 4$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,1)$</td>
<td>$\int Y_K (R - x)(m\theta Y_J)_R$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,2)$</td>
<td>$\int Y_K e_A Y_I(I,J,1)$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,3)$</td>
<td>$\int Y_K E_v Y_J^\prime$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,4)$</td>
<td>$\int Y_K E Z_J^\prime$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,5)$</td>
<td>$\int Y_K (E_1 \phi P_J^\prime + E_1 \phi P_J')$</td>
</tr>
<tr>
<td>$\text{DYF}(K,J,6)$</td>
<td>$\int Y_K (e \tau + (m)_{R}(R - x))$</td>
</tr>
<tr>
<td>$\text{DYD}(K,J)$</td>
<td>$g \int Y_J \int \int Y_J$</td>
</tr>
<tr>
<td>$\text{DYALII}(K)$</td>
<td>$\int Y_K \int \int L_V$</td>
</tr>
</tbody>
</table>

$K, J = 1 \to \text{number of (1-P, 0-P or torsion) modes}$
\[ \int_{0}^{R} f(x) \, dx = \int_{0}^{R} f(x) \, dx \]

\[
\begin{align*}
DZYI(K, J, 1) &= \int Z_K YI(I, J, 3) \\
DZPI(K, J, 2) &= \int Z_K PI(I, J, 2) \\
DZII(K, J, 1) &= \int Z_K ZI(I, J, 1) \\
DZII(K, J, 3) &= \int Z_K ZI(I, J, 3) \\
DZII(K, J, 6) &= \int Z_K ZI(I, J, 6) \\
DZYII(K, J, 1) &= \int Z_K YII(I, J, 1) \\
DZPII(K, J, 1) &= \int Z_K PII(I, J, 1) \\
DZMI(K, 6) &= \int Z_K MI(I, 6) \\
DZMII(K, i) &= \int Z_K MII(I, i) \quad i = 1 \text{ to } 3 \\
DZI(K, J, 1) &= \int Z_K [(R - x) (m \theta Y_j)_R + e_A \theta YI(I, J, 1)] \\
DZF(K, J, 2) &= \int Z_K \Delta \theta Y_j \\
DZF(K, J, 3) &= \int Z_K \varepsilon \theta Z_j^\nu \\
DZF(K, J, 4) &= \int Z_K [\varepsilon_1 P' P'' J + E_1 \theta Y_{j}'] \\
DZF(K, J, 5) &= \int Z_K [R(R - x) (m \theta)_R - e_A \theta] \\
DZF(K, J, 6) &= \int Z_K [e_A \theta P_j - R(R - x) (m \theta P)_R] \\
DZD(K, J) &= \int Z_K [l_{W} Z_{K} / \int Z_{J}^{x} x / R / R / R] \\
DZALII(K) &= \int Z_K [l_{W} Z_{K} / \int Z_{L}^{x} x / R / R] \\
\end{align*}
\]

\[ K, J = 1 \text{ to number of corresponding modes} \]
\[ \phi \text{ Equation Integrals} \]

\[ \int_R \phi_K \text{ } dx \]

\[ D_{\text{PI}}(K,J,9) = \int_{\text{phi}_K} \text{ } Y(I,J,9) \]

\[ D_{\text{PI}}(K,J,3) = \int_{\text{phi}_K} \text{ } Y(I,J,3) \]

\[ D_{\text{PI}}(K,J,6) = \int_{\text{phi}_K} \text{ } Y(I,J,6) \]

\[ D_{\text{PI}}(K,J,8) = \int_{\text{phi}_K} \text{ } Y(Y(I,J,8) \]

\[ D_{\text{PZI}}(K,J,8) = \int_{\text{phi}_K} \text{ } Z(I,J,8) \]

\[ D_{\text{PZII}}(K,J,2) = \int_{\text{phi}_K} \text{ } Z(I,J,2) \]

\[ D_{\text{PZII}}(K,J,4) = \int_{\text{phi}_K} \text{ } Z(I,J,4) \]

\[ D_{\text{PZII}}(K,J,7) = \int_{\text{phi}_K} \text{ } Z(I,J,7) \]

\[ D_{\text{PPI}}(K,J,6) = \int_{\text{phi}_K} \text{ } P(I,J,6) \]

\[ D_{\text{PPI}}(K,J,7) = \int_{\text{phi}_K} \text{ } P(I,J,7) \]

\[ D_{\text{PPII}}(K,J,4) = \int_{\text{phi}_K} \text{ } P(I,J,4) \]

\[ D_{\text{PPII}}(K,J,5) = \int_{\text{phi}_K} \text{ } P(I,J,5) \]

\[ D_{\text{PMII}}(K,i) = \int_{\text{phi}_K} \text{ } M(I,i) \quad i = 3, 4; \text{ } 6 \text{ to } 10 \]

\[ D_{\text{PSI}}(K,J,1) = \int_{\text{phi}_K} \text{ } S(I,J,5) \]

\[ D_{\text{PF}}(K,J,1) = \int_{\text{phi}_K} \text{ } E_{1} \ * \ Y_{j} \]

\[ D_{\text{PF}}(K,J,2) = \int_{\text{phi}_K} \text{ } E_{1} \ * \ Y_{j}'' \]

\[ D_{\text{PF}}(K,J,3) = \int_{\text{phi}_K} \text{ } E_{1} \ * \ Z_{j}'' \]

\[ D_{\text{PD}}(K,J) = g_{\phi} \int_{\phi_K} \text{ } \int_{\phi_K} \text{ } \int_{\phi_K} \]

\[ D_{\text{PALLI}}(K) = \int_{\phi} \text{ } \int_{\phi} \text{ } \int_{\phi} \]

\[ K, J = 1 \text{ to number of appropriate modes} \]
First card of each case is HEADING CARD (see next page for description and exceptions).

All other data may be entered in any order (data blocks must maintain order within block). Data not entered (after 1st case) retains previous values (if any). All data is self identified by value of IO punched in col 1,2 of card on first card of block.

### INPUT SUMMARY

<table>
<thead>
<tr>
<th>IO</th>
<th>Type of Data</th>
<th>No. of Cards</th>
<th>Required?</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Blade Properties</td>
<td>Block</td>
<td>Yes (Must precede IO = 3,4 or 5,13)</td>
</tr>
<tr>
<td>02</td>
<td>Blade Data</td>
<td>1</td>
<td>No (Default to 0's)</td>
</tr>
<tr>
<td>03</td>
<td>Modes: In-Plane (Y)</td>
<td>Block</td>
<td>No (At least one of 3,4,5 required)</td>
</tr>
<tr>
<td>04</td>
<td>Out-of-Plane (Z)</td>
<td>Block</td>
<td>No</td>
</tr>
<tr>
<td>05</td>
<td>Torsion (P)</td>
<td>Block</td>
<td>No</td>
</tr>
<tr>
<td>06</td>
<td>Frequencies (Ω, ω_p)</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>07</td>
<td>Hub Data, X,M,C,K,F</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>08</td>
<td>Y</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>09</td>
<td>Z</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Applied Forces, Blades</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Special Controls - Nonlin, Floquet</td>
<td>1</td>
<td>No (Default to Nonlinear)</td>
</tr>
<tr>
<td>18</td>
<td>Solution Controls</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Special IO Cancel</td>
<td>1</td>
<td>No</td>
</tr>
</tbody>
</table>
HEADING CARD

Col 1  IC1  #0  Ends run (same as IEND = 3, see below)
2     IC2  #0  All input printed (else only new data printed)
3     IC3  #0  Prints definite integrals
4     IC4  #0  Prints coefficient matrices
5     IC5  #0  Writes data on tape (see below)
6-80  Arbitrary heading

The heading card is the first card of the first case and the first card of each following case unless the preceding case ended with IEND = 2 (see below)

GENERAL INPUT

10 in col 1,2 of 1st card only of each block.
IEND in col 80 of single card - see details of each block input.

IEND = 1  end of data, followed by HEADING and new data
          = 2  same as 1 but omit HEADING card from next case
          = 3  ends run at completion of case

No special ending required for block data input
All data has following format. Real and integer input may be mixed.

  I2, F8.0, 6F10.0, F9.0, I1

Do not use col 1 or 2 except for I0 (on first card of block)
Do not use col 80 except to end case

TAPE DATA (IC5 #0)

Uses FORTRAN unit 9. Data records are as follows ψ (in degrees, not limited to 360), tip in-plane deflection, tip out-of-plane deflection, tip torsional deflection, x_H, y_H, z_H. Blade 1 only
**IO = 1**  \textbf{BLADE PROPERTIES REQUIRED}  

Must precede IO = 3, 4, 5, 13  
IO on first card only, col 1, 2 blank on all succeeding cards  
2 cards per station (order 1, 2, 1, 2...  
20 stations max  
IEND (if used) on last card 1.  
Definitions consistent with TN D-7818

<table>
<thead>
<tr>
<th>Word</th>
<th>Card 1</th>
<th>Card 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X - sta (ascending sequence)</td>
<td>EOP - EI_{y'} (EI out of chord plane)</td>
</tr>
<tr>
<td>2</td>
<td>M - mass/unit length</td>
<td>EIP - EI_{z'} (EI for bending in chord plane)</td>
</tr>
<tr>
<td>3</td>
<td>E - e</td>
<td>GJ</td>
</tr>
<tr>
<td>4</td>
<td>SEA - e_A</td>
<td>EA - (if 0 then ( \frac{1}{EA} ) is set to 0)</td>
</tr>
<tr>
<td>5</td>
<td>KmA - k_A</td>
<td>EB1 - EB1*</td>
</tr>
<tr>
<td>6</td>
<td>KM2 - k_m2</td>
<td>EB2 - EB2*</td>
</tr>
<tr>
<td>7</td>
<td>KA - k_A</td>
<td>EC - EC1</td>
</tr>
<tr>
<td>8</td>
<td>THP - 0' built in pitch - rad/ unit length</td>
<td>ECS - EC1*</td>
</tr>
</tbody>
</table>

**IO = 2**  \textbf{BLADE DATA OPTIONAL} (Default to 0)

<table>
<thead>
<tr>
<th>Word</th>
<th></th>
</tr>
</thead>
</table>
| 1    | NB - no of blades 4 max (Default to 1 if no hub DOF  
\( \text{Default to 2 if hub DOF included} \) |
| 2    | THO - \( \theta_0 \) angle at x(1) - radians |
| 3    | BPC - \( \beta_{PC} \) - pre-cone - radians |
| 4    | GV - blade damping, l-P appropriate units, viscous |
| 5    | GW - blade damping, O-P appropriate units, viscous |
| 6    | GP - blade damping, torsion appropriate units, viscous |
IO = 3 MODES IN-PLANE (Max 3 modes)

IO = 4 MODES OUT-OF-PLANE (Max 5 modes)

IO = 5 MODES TORSION (Max 3 modes)

Each mode has one set of input - second derivative at each station followed by the first derivative at station 1 (slope and deflection are obtained by integration and normalized to unit deflection at tip)

Input - 8 elements per card - as many cards as necessary (3 max), all functions start on new card

IO on first (.)" card - all other col 1,2 blank
IEND (if used) on 1st (.)" card of last mode

Order of input:
1st mode: ( )" x₁ ( )" x₂ ( )" x₃ . . .
( )" x₉ . . .
new card ( )" x₁ word 1 only, slope at station 1 (normally = 0)
new card ( ) x₁ word 1 only, deflection at station 1 (normally = 0)
next mode ( )" x₁ ( )" x₂ . . .
new card x₁ x₂ . . . etc

IO = 6 FREQUENCIES REQUIRED

Word
1 OMEG - ω - rotor speed, rad/sec
2 OMF - ωₕ - forcing frequency, rad/sec

75
\( IO = 7 \) HUB DATA, \( X \) OPTIONAL
\( IO = 8 \) HUB DATA, \( Y \) OPTIONAL
\( IO = 9 \) HUB DATA, \( Z \) OPTIONAL

Impedance in each direction may be represented as spring-mass-damper at frequency \( \omega_f \). Data omitted implies infinite impedance. If any hub data is input - at least two blades required.

Word

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HM</td>
<td>( x ) Mass ( y ) ( z )</td>
</tr>
<tr>
<td>2</td>
<td>HC</td>
<td>( x ) Damping Coeff ( y ) ( z )</td>
</tr>
<tr>
<td>3</td>
<td>HK</td>
<td>( x ) Spring Rate ( y ) ( z )</td>
</tr>
<tr>
<td>4</td>
<td>HF</td>
<td>(1) Force - multiplied by ( \sin \omega_f t ) (or by 1 if ( \omega_f = 0 )) (2) (3)</td>
</tr>
</tbody>
</table>
$\text{IO = 13 APPLIED FORCES, BLADES OPTIONAL}$

Load may be applied at any one station, but in three directions. Amplitudes are multiplied by $\sin \omega_f t$ (or by 1 if $\omega_f = 0$). Forces may be applied to one or all blades. $\omega_f t$ always refers to blade 1, however, producing "umbrella mode" forcing. (See IO = 7, 8, 9 for hub forcing).

**Word**

1. N XF Station index number (see IO = 1)
2. AF Y Amplitude in y direction
3. AF Z
4. AF P
5. N BF Blade number to which force is applied - 0 applies forces to all blades simultaneously. If $> NB$, NBF is set to 0.
6. P ER Period as fraction of $360^\circ (1 - \cos)$ force is applied from $\psi = 0$ to $\psi = \text{PER} \times 2$. OMF (IO = 6) is ignored. Integration interval must be selected with core (IO = 18).

**Note:** If in-plane hub degrees of freedom are used (IO = 7 or 8) AFY or NBF must $= 0$. 

77
IO = 17  SPECIAL CONTROLS - NONLIN, FLOQUET OPTIONAL (Default to nonlinear, no floquet)

FLOQUET OPTION: Produces Floquet transition matrix using force cycle (\(\omega_f\)) unless \(\omega_f = 0\) then rotor cycle is used. Note that if in-plane hub D-O-F are used equation contains terms periodic in \(\omega t\). If a force is applied then the boundary conditions for a (linear) periodic solution are determined and solution is executed for number of cycles specified in IO = 18. This overrides any other initial condition(s).

A maximum at 15 degrees of freedom are allowed for this option (30 variables including velocities).

Word

1 NLIN = 0 All nonlinear terms included
   = 1 In-plane nonlinear terms only
   = 2 Linear terms only
2 NFLOQ = 1 Floquet option (see discussion just above)
   = 2 Same as 1, but steady effects of offsets and twists and precone are ignored.

IO = 18  SOLUTION CONTROLS REQUIRED

Errors and initial conditions are limited to one variable.

Word

1 CYCLES Number of force* cycles for solution to run
2 HINIT Number of integration intervals per cycle
3 ERROR Error bound (appropriate units), see IYE
4 IYE Index of variables tested for ERROR**
5 CIC Initial condition (appropriate units), see IYIC
6 IYIC Index of variable for initial condition
7 BERR Upper limit (abs) of variable (IYE) which stops run.
   If = 0 no limit

* Force cycle is used unless \(\omega_f = 0\) (IO = 06), then rotor cycle is used.

** See section on variable numbers following.
IO = 21  SPECIAL IO CANCEL  OPTIONAL

For cases after the first, IO's previously used may be cancelled. When 
this option is used all coefficients are recalculated and IC2 is set to 
1 (see HEADING CARD) to insure data printout. There is no necessity to 
cancel when data is replaced.

Word

1-8  IO's to be cancelled (0's ignored)
In I018 the variables are referred to by numbers. These numbers are as follows:

<table>
<thead>
<tr>
<th>I</th>
<th>Variable</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_H)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(x_H)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(y_H)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(y_H)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(z_H)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(z_H)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(\dot{y}_1)</td>
<td>Blade 1 (I = 9 + 2 \text{NM}(IB-1))</td>
</tr>
<tr>
<td>12</td>
<td>(y_1)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(y_2)</td>
<td>NM = no. of modes</td>
</tr>
<tr>
<td></td>
<td>(\vdots)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>last (y)</td>
<td>IB = blade number</td>
</tr>
<tr>
<td></td>
<td>(z_1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(z_1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\vdots)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>last (z)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\phi_1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\phi_1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\vdots)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>last (\phi)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\dot{y}_1)</td>
<td>Blade 2</td>
</tr>
<tr>
<td></td>
<td>(y_1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

80
ERROR MESSAGES

Certain errors terminate the run. Others are warnings with correction as indicated below. All error numbers refer to a Fortran statement number in vicinity of error. (All are in INPU except for the 5000 series which occur in SOL).

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>REASON</th>
<th>TERMINATE</th>
<th>NUMBER</th>
<th>REASON</th>
<th>TERMINATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Inactive IO</td>
<td>Yes</td>
<td>510</td>
<td>I013, NYF &lt; 0 CR</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td>&quot;</td>
<td>Yes</td>
<td>511</td>
<td>I013, All forces 0</td>
<td>Yes</td>
</tr>
<tr>
<td>14</td>
<td>&quot;</td>
<td>Yes</td>
<td>512</td>
<td>I013, NB &lt; NBF &lt; 0</td>
<td>No, Sets NBM to NBF*</td>
</tr>
<tr>
<td>15</td>
<td>&quot;</td>
<td>Yes</td>
<td>1100</td>
<td>I018, Error &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>16</td>
<td>&quot;</td>
<td>Yes</td>
<td>1105</td>
<td>I018, IYIC &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>19</td>
<td>&quot;</td>
<td>Yes</td>
<td>1106</td>
<td>I018, IYIC &gt; NDIM</td>
<td>Yes</td>
</tr>
<tr>
<td>20</td>
<td>&quot;</td>
<td>Yes</td>
<td>1107</td>
<td>I018, IYE &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>200</td>
<td>Invalid IO</td>
<td>Yes</td>
<td>1108</td>
<td>I018, IYE &gt; NDIM</td>
<td>Yes</td>
</tr>
<tr>
<td>202</td>
<td>More than one input of same IO, last one used</td>
<td>No, IO*</td>
<td>1100</td>
<td>I018, Error &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>203</td>
<td>I021, Attempt to cancel invalid I$</td>
<td>Yes</td>
<td>1105</td>
<td>I018, IYIC &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>215</td>
<td>I01, Stations out of seq</td>
<td>Yes</td>
<td>1106</td>
<td>I018, IYIC &gt; NDIM</td>
<td>Yes</td>
</tr>
<tr>
<td>216</td>
<td>I01, Too many stations</td>
<td>Yes</td>
<td>1107</td>
<td>I018, IYE &lt; 0</td>
<td>Yes</td>
</tr>
<tr>
<td>262</td>
<td>I03, Too many Y modes</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>264</td>
<td>I04, Too many Z modes</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>266</td>
<td>I05, Too many P modes</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>No IO = 1</td>
<td>Yes</td>
<td>5010</td>
<td>Too many D-O-F</td>
<td>Yes</td>
</tr>
<tr>
<td>501</td>
<td>No IO = 3,4 or 5</td>
<td>Yes</td>
<td>5010</td>
<td>Too many D-O-F</td>
<td>Yes</td>
</tr>
<tr>
<td>502</td>
<td>No IO = 6</td>
<td>Yes</td>
<td>5030</td>
<td>IHLF = 11</td>
<td>Yes</td>
</tr>
<tr>
<td>506</td>
<td>I02 NB &gt; 4, set to 4</td>
<td>No, NB*</td>
<td>5031</td>
<td>IHLF = 12</td>
<td>Yes</td>
</tr>
<tr>
<td>507</td>
<td>I02 NB &lt; 1, set to 1 or 2 (2 if HUB DOF)</td>
<td>No, 1*</td>
<td>5032</td>
<td>IHLF = 13</td>
<td>Yes</td>
</tr>
<tr>
<td>509</td>
<td>I0 = 18 Missing</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>510</td>
<td>In-plane hub with AFY<em>OR</em>NBF<em>NE</em>0</td>
<td>No, NBF*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* This quantity is printed with warning.
INPUT

(1) HEADING
1 IC1 .EQ 0 - FIRST OR NORM RUN - ALL INPUT
2 REPLACE MODES - INPUT 3,4,5
3 ADD MODES - INPUT 4,5
8 NEW OP CODE ONLY - INPUT 5
9 END OF RUN - LAST CARD OF RUN

2 IC2 .EQ.1 PRINTS ORTHO CHECKS
2 AND NORMALIZES MODES
NOTE - MODES ARE REPLACED
AFTER INPUT

3 IC3 .NE.0 PRINTS EQS FOR MASS IDENT

4 IC4 .NE.0 RESTORES INPUT MODES, IF IC1 .EQ. 8

5-80 ARBITRARY HEADING HEAD[19]

(2) MASS DATA - ONE CARD PER BLADE STATION, 20 MAX

1-10 (11) STATION
11 * (SEE NOTE) WM
12-20 M - LUMPED MASS
21-30 E - CG OFFSET FROM EA * WHEN CG FORWARD
31 * (SEE NOTE) WT
32-40 TH - PITCH ANGLE - RAD
41 * (SEE NOTE) WK
42-50 KM RADIUS OF GYRATION IN TORSION

* 1ST COL OF EACH CARD CONTAINS WEIGHTING FACTOR
FROM 1-9 (0=1) HIGHER VALUE INDICATES GREATER CONFIDENCE
SEE 101 - WD1

END WITH BLANK CARD

(3) CONTROL CARD - MODES

1-10 CALV - MULTIPLIES I-P MODE DEPL - (0=1)
11-20 CALW - MULTIPLIES D-P MODE DEPL (0=1)
21-30 CALP - MULTIPLIES TOR MODE DEPL (0=1)
31-40 THO - ROOT PITCH ANGLE - RAD
ADD TO TH - (TH NOT CHANGED)
### MODES — STATIONS CORRESPOND TO MASS DATA

<table>
<thead>
<tr>
<th>EACH MODE</th>
<th>FREQ</th>
<th>NATURAL</th>
<th>RAD/SEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-20</td>
<td>OMEG</td>
<td>ROTATIONAL</td>
<td>RAD/SEC</td>
</tr>
<tr>
<td>21-30</td>
<td>IF. NE. 0 TEMPORARILY REPLACES CALV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31-40</td>
<td>IF. NE. 0 TEMPORARILY REPLACES CALW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41-50</td>
<td>IF. NE. 0 TEMPORARILY REPLACES CALP</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NEXT CDS</th>
<th>V</th>
<th>I-P</th>
<th>DISPLACEMENTS, 8F10.</th>
<th>UP TO 3 CARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEXT CDS</td>
<td>W</td>
<td>O-P</td>
<td>START ON NEW CD</td>
<td></td>
</tr>
<tr>
<td>NEXT CDS</td>
<td>P</td>
<td>TOR</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FOLLOW BY NEXT MODE — 8 MODES MAX AT ONE OMEG

*** 30 Eqs MAX (NOT INCL INVARIANCES) ***

END WITH BLANK CARD.

### OPERATION CODES

<table>
<thead>
<tr>
<th>COL 1, 2</th>
<th>IO1, IO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>COL-1</td>
<td>IO1</td>
</tr>
</tbody>
</table>

1. MODIFY MODES WITH RANDOM ERRORS — MODES REPLACED
   - WD1 PERCENT RANDOM + OR — RECTANGULAR DIST
   - WD2 PERCENT BIAS
   - WD3 INTEGER SEED TO START RANDOM SEQUENCE

*** FOLLOW BY NEXT OPERATION CARD (5) ***

2. SOLVE FOR MINIMUM MODAL CHANGES — MASS MATRIX UNCHANGED
   - ALL MODES MUST BE AT SAME OMEGA — 8 MAX
   - FIRST MODE UNCHANGED, LAST MODE WILL CHANGE MOST
   - MINIMUM SUM PERCENT CHANGES USED
   - WEIGHTING FACTORS NOT USED IN THIS OPTION

   - WD1.EQ.0 NO LIMIT ON CHANGES
   - WD1.EQ.1 LIMIT CHANGES — SCALE OPTION
     - WD2-B MAX PCT CHANGE ALLOWED IN EACH MODE
     - CHANGES ARE SCALED SO MAX CHANGExLE MAXIMUM
     - 0 INDICATES NO LIMIT

   - WD1.EQ.2 LIMIT CHANGES — TRUNCATE OPTION
     - WD2-B SAME AS FOR SCALE OPTION EXCEPT THAT ONLY
     - CHANGES WHICH EXCEED LIMITS ARE TRUNCATED
     - OTHER CHANGES ARE NOT MODIFIED
### INCOMP Model Mass Changes

1. **WD1.EQ.1**WEIGHTING FACTORS ALL SET TO 1 (TEMP)
2. **WD1.EQ.2**STAS WITH INVARIANT PARAM. READ-54A

The following controls cause the corresponding properties to remain invariant if not 0.

<table>
<thead>
<tr>
<th>COL 20</th>
<th>TOTAL MASS</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>RADIAL CG</td>
<td>M*X</td>
</tr>
<tr>
<td>40</td>
<td>CHORDWISE CG</td>
<td>M*E</td>
</tr>
<tr>
<td>50</td>
<td>FLAPPING MOM OF INERT</td>
<td>M*X**2</td>
</tr>
<tr>
<td>60</td>
<td>FEATHERING MOM OF INERT</td>
<td>M*Y**2</td>
</tr>
</tbody>
</table>

**Col-2-162**

0 ABOVE OPERATIONS DO NOT DISTURB ORIGINAL DATA

1 ABOVE OPERATIONS REPLACE ORIGINAL DATA IN PREPARATION FOR SEQUENTIAL OPERATIONS

*(SA)* USED ONLY FOR INVAR STAS. SEE 3, ABOVE, WD1 = 2

**COL1** = NO OF STATIONS (8 MAX)

WD1, WD2, ..., STATION NUMBERS, NO ZEROS

Next Heading Card.

*-------------------------------------------------------------------------------------------------------------------------------------*

---

84
APPENDIX C
PROGRAM LISTINGS

C REAL M,KMI,KP2,KAL
C LOGICAL LY
C COMMON FOR INPUT
C COMMON/INDAT/X(20),M(20),E(20),SE(20),KM1(20),KM2(20),KA(20),
1 THP(20),EOP(20),GJ(20),EA(20),EB1(20),EB2(20),ECS(20),EIP(20),
2 THQ,BPC,YPP(20,3),ZP(20,5),ZP(20,5),PP(20,3),PP(20,3),
3 OMEG,OMF,EC(20),NY,NZ,NP,NP,OMGS,OMFS,IDIM,NMAX,NLIN
4,NB,MD,MY,HCX,HCY,HCZ,HCX,HCY,HCZ,HCX,HCY,HCZ,HCX,HCY,HCZ
5,HI,NET,ERROR,IVEC,IYIC,BERR,CYCLES,NXF,AFY,AFR,NBF
6,K,CL,WE,PAR,HE(3),PER
C COMMON COEFFICIENT MATRICES
C COMMON/COEF/CG(11,11),CDO(11,11),CDO(11,11),CDO(11,11),
1 CO(11,11),CDO(11,11),F(11),DF(11),FN(11),CGIR(11,12),
2 COOR(11,11),CGR(11,11),FR(11),RIOC(11,12),BF(11)
1, BIRD(3,11),BDAM(3,11),BSPR(3,11),COIM(11,3),CDOH(11,3),BIRI(3,11)
4,BIRD(3,11),BIRD(3,11),BIRD(3,11),BIRD(3,11),BF(3),BF(3),BF(3)
5,HC(3,3),HK(3,3)
C COMMON FOR HEADING, CONTROL DATA
C COMMON/HED/IC1,IC2,IC3,IC4,HEAD(19),IPAGE,INPUT(20),IEND,LINE,IC5
C COMMON DIMENSION DATA
C COMMON/DIM/NINPUT,NSTA,NUMODE,NMODE,NMODE,NMODE,NM1,NDIM,NBLADE
C COMMON BASIC DERIVED DATA
C COMMON/DER/TH(20),EV(20),EW(20),EP(20),W(20,3),Z(20,5),P(20,3)
C COMMON VARIABLES AND SOLUTION CONTROLS
C COMMON/VAR/YVAR(98),DERY(98),PRMT(6),LY(98)
C DIMENSIONALIZATION
NINPUT = 20
NSTA = 20
NUMODE = 3
NMODE = 5
NMODE = 11
NM1 = NMODE+1
NDIM = 98
NDIM \* NINP = 1.0E+51
10 INPUT(I)=0
ICASE=0
IEND = 0
20 CALL INPU (ICASE)
LINE = 100
CALL SOL(PRMT,YVAR,DERY,ILF,LY)
IF(ICYC,NE,0) WRITE(9) (FSI,I=1,7)
100 IF (IEND,NE,3) CALL EXIT
GO TO 20
END
FUNCTION DINT (DUMP, DUMPP, X, NX)

C
DUMP IS INTEGRAL OF DUMPP
REAL DUMPI, DUMPP(1)
CALL INT (DUMP, DUMPP, 0, X, NX, 1)
DINT = DUMPP(NX)
RETURN
END

FUNCTION DINTL (A, B, I1, N, X, NA, NX, DUMP, DUMPP)
REAL A(NA, 1), B(NA, 1), X(1), DUMPP(1), DUMPI
DO 10 I = 1, NX
10 DUMPP(I) = A(I, I1) * B(I, N)
CALL INT (DUMP, DUMPP, 0, X, NX, 1)
DINTL = DUMPP(NX)
RETURN
END

FUNCTION DINT2 (A, B, I1, I2, N, NB, X, NA, NX, DUMP, DUMPP)
REAL A(NA, 1), B(NB, 1), X(1), DUMPP(1), DUMPI
DO 10 I = 1, NX
10 DUMPP(I) = A(I, I1) * B(I, I2, N)
CALL INT (DUMP, DUMPP, 0, X, NX, 1)
DINT2 = DUMPP(NX)
RETURN
END

SUBROUTINE ERR(N, I)
C I = 0, TERMINATES RUN
I NE 0 WARNING ONLY, PRINTS I
PRINT 10, N
10 FORMAT (13X, 17H*** ERROR NUMBER, IS, 5H *** )
IF (I .NE. 0) GOTO 20
CALL EXIT
20 FORMAT (30X)
30 FORMAT (20X, 20H*** WARNING ONLY *** , IS)
SUBROUTINE FCTYVAR,DERY,LY,INDIM) 00000010
  NOTE INDIM NOT USED INCLUDED FOR COMPATIBILITY ONLY 00000020
  MULTI BLADES, 3 DOF HUB, NON-LIN CORIOLIS FORCES 00000030
DIMENSION YVAR(1),DERY(1) 00000040
LOGICAL LY(1) 00000050
REAL H,KML,KM2,KA 00000060
REAL DUMPI(20),DUMPI(20),VW(i(20),VPP(20),WDP(20),WP(20),WPP(20)) 00000070
COMMON/INDAT/X(20),Y(20),Z(20),E(20),SEY(20),VM(20),KM(20),V2(20),V3(20),V4(20), 00000080
  KAT(20) 00000090
1 THP(20),EOP(20),GJ(20),EAT(20),EBI(20),EB2(20),ECS(20),EIP(20), 00000100
2 TH0,EPC,YPP(20),ZP(20,5),ZIP(20,5),PPP(20,3),PP(20,3),PP(20,3) 00000110
3 OMEG,OMF,EC(20),NY,NZ,NP,NK,CMEGS,CMEFS,IDIM,NMAX,NLINC 00000120
4,NS,HX,HY,Hz,HX,HX,HK,HKY,HKZ,HH,NN,NE,LQ 00000130
5*INIT*ERROR*LY*IC1*LY*IC2*IC3**IC4*HEADI19*IPAGE*INPUT*20*5*IE 00000140
  COMMON/COEF/COI(120),DCDI(120),DCD1(120),DCD1(120),DCD1(120),DCD1(120) 00000150
  COMMON/HEO/IC(120),IC(120),IC(120),IC(120),IC(120),IC(120) 00000160
  COMMON/DIM/NINPUT,NSTA,NMODE,NMODE,NMODE,NMODE,NM5,NDLM,NDIM,NDL, 00000170
  COMMON/DER/THI(20),EV(20),EV(20),EP(20),Y(20,3),Z(20,5),P(20,3) 00000180
  COMMON/DER/THI(20),EV(20),EV(20),EP(20),Y(20,3),Z(20,5),P(20,3) 00000190
  COMMON/DER/THI(20),EV(20),EV(20),EP(20),Y(20,3),Z(20,5),P(20,3) 00000200
C NOTE DO NOT USE COMMON/VAR/ 00000210
  *LOGICAL LHUB 00000220
  INTEGER ICOL(4),IRW(4) 00000230
  REAL XHD(3),XH(3),XHDD(3),FIB(11,4) 00000240
  REAL YB(11),YDB(11),HUB(1),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3), 00000250
  HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3) 00000260
  HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3),HUB1(3) 00000270
  1 HUBF(1),HUBF(1),HUBF(1),HUBF(1),HUBF(1),HUBF(1),HUBF(1),HUBF(1) 00000280
  2 HINV(1),HDB(1) 00000290
LHUB=.FALSE. 00000300
IF(LY(I).OR.LY(J).OR.LY(5)) LHUB=.TRUE. 00000310
SOFT = SIN(OMF*T) 00000320
IF(LHUB.EQ.FALSE) SOFT = 1. 00000330
IF(LY(I).NOT.LHUB)IC TO 45 00000340
PSI(1) = AMOD(T*OMF6.28319) 00000350
DPSI=6.28319/FLOAT(NB) 00000360
-SIN(1)=SIN(PSI(1)) 00000370
COSB(1)=COS(PSI(1)) 00000380
DO 10 IB=2,NB 00000390
PSI(18)=PSI(18-1)+DPSI 00000400
IF(PSSI(18)>6.28319) PSI(18)=PSI(18)-6.28319 00000410
SINB(18)=SIN(PSSI(18)) 00000420
10 COSB(18)=COS(PSSI(18)) 00000430
DO 20 IB=1,NB 00000440
DO 20 IB=1,NB 00000450
20 HUBC(1,1)=HUBC(1,1) 00000460
DO 30 IB=1,NB 00000470
HUB(1,1) = HUB1(1,1)-SINB(18)*2*BIRIT(1,1) 00000480
HUB(1,2) = HUB1(1,2)-SINB(18)*COSB(18)*BIRIT(1,2) 00000490
WP(I)=0
WDP(I)=0
70 WP(I)=0
    IF(NZ.EQ.0) GO TO 85
    DO 80 I=1,NZ
    DUMP(I)= YB(NY+I)
80 DUMP(I)= YB(NY+I)
    CALL SUMODE (WD,DUMPP,Z,NSTA,NX,NZ)
    CALL SUMODE (WDP,DUMPP,ZP,NSTA,NX,NZ)
    CALL SUMODE (WPP,DUMPP,ZPP,NSTA,NX,NZ)
    CALL SUMODE (W,DUMPP,ZP,NSTA,NX,NZ)
    DO 90 I=1,NX
    DUMP(I)= YB(1+NY+I)
70 DUMP(I)= YB(NY+I)
    CALL SUMODE (WDP,DUMPP,Z,NSTA,NX,NZ)
    CALL SUMODE (WDP,DUMPP,ZP,NSTA,NX,NZ)
    CALL SUMODE (WPP,DUMPP,ZP,NSTA,NX,NZ)
    CALL SUMODE (W,DUMPP,ZP,NSTA,NX,NZ)
    DO 90 I=1,NX
     DUMP(I)= YB(1+NY+I)
    CALL SUMODE (WDP,DUMPP,ZP,NSTA,NX,NZ)
    CALL SUMODE (WPP,DUMPP,ZP,NSTA,NX,NZ)
    CALL SUMODE (W,DUMPP,ZP,NSTA,NX,NZ)
    DO 90 I=1,NX
80 DUMP(I)= YB(NY+I)
    CALL INT(DUMP,DUMPP,0,X,NX,1)
    DO 95 I=1,NX
85 DUMP(I)= YB(NY+I)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 100 I=1,NX
90 DUMP(I)= YB(NY+I)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 120 J=1,NY
    DO 110 I=1,NX
100 DUMP(I)= YB(NY+I)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 120 J=1,NY
110 DUMP(I)= YB(NY+I)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 120 J=1,NY
120 FNL(J)= INT(DUMP,DUMPP,0,X,NX,2)*2.0*OMEG
    IF(NLIN.EQ.1) GO TO 150
    DO 130 J=1,NY
130 DUMP(I)= WPP(I)*DUMPP(I)- VC(I)*W(I)*W(I)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 140 J=1,NY
140 FNL(NY+J)= INT(DUMP,DUMPP,0,X,NX,2)*2.0*OMEG
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    CALL INT(DUMP,DUMPP,0,X,NX,2)
    DO 140 J=1,NY
150 CONTINUE
160 DO 170 I=1,NX
170 FB(I)=FR(I)+FNL(I)
    C  BLADE FORCING
    IF(INPUT(13),EQ.0) GO TO 190
    IF(NBF.NE.0.AND.IB.NE.ABF) GO TO 190
    DO 180 I=1,NX
180 FB(I)=FB(I)+BF(I)*SFTH
    DO 180 I=1,NX
    GO TO 180
190 IF(LHUB) GO TO 200
    CALL MXV(RHSHUB,YDB,3,NM,3,1)
    CALL MXV(RHSHUB,YB,3,NM,3,1)
    CALL MXV(RHSHUB,YB,3,NM,3,1)
    GO TO 180
175 CONST=PSI(IB)/PER
    IF(CONST.GE.4.2839) GC TO 180
    FB(I)=FB(I)+BF(I)*(1.0-COS(PI*CAST))
180 FB(I)=FB(I)
180 FB(I)=FB(I)
190 IF(LHUB) GO TO 200
    CALL MXV(RHSHUB,YDB,3,NM,3,1)
    CALL MXV(RHSHUB,YB,3,NM,3,1)
    CALL MXV(RHSHUB,YB,3,NM,3,1)
200 CONTINUE
IF (.NOT. LHUB) GO TO 300
CALL INVRS (HUB1, 3, HINV, HUBC, IRCW, ICOL, 3, 4)
CALL MXV (XHDD, HINV, RHS, 3, 3, 3, 0)
C NOTE THAT ALL 3 HUB MOTIONS COMPUTED, THEY ARE IGNORED IF NOT
IF (LY(1))
1DERY(1) = XHDD(1)
IF (LY(2))
1DERY(2) = YVAR(1)
IF (LY(3))
1DERY(3) = XHDD(2)
IF (LY(4))
1DERY(4) = YVAR(3)
IF (LY(5))
1DERY(5) = XHDD(3)
IF (LY(6))
1DERY(6) = YVAR(5)
C ELAVES
300 DO 360 IB = 1, NB
I = 10 + NM*(IB - 1)*2
DO 310 J = 1, NM
I = I + 1
YDB(J) = YVAR(I)
I = I + 1
310 YB(J) = YVAR(I)
DO 320 I = 1, NM
320 RHS(I) = FIB(I, IB)
CALL MXV (RHS, CODR, YDB, NM, NM, NMCDE, 1)
CALL MXV (RHS, CCR, YB, NM, NM, NMCDE, 1)
IF (.NOT. LHUB) GO TO 350
DUMP(2) = COSB(1) * DERY(3)
DUMP(3) = DERY(5)
CALL MXV (RHS, CCIH, DUMP, NM, 3, NMCDE, 1)
DUMP(1) = COSB(1) * XHDD(1)
DUMP(2) = SINB(1) * XHDD(2)
DUMP(3) = XHDD(3)
CALL MXV (RHS, CODH, DUMP, NM, 3, NMCDE, 1)
350 CALL MXV (YDB, RIOC, RHS, NM, NM, NMCDE, 0)
I = 10 + NM*(IB - 1)*2
DO 360 J = 1, NM
I = I + 1
DERY(I) = YDB(J)
I = I + 1
360 DERY(I) = YVAR(I - 1)
RETURN
END
SUBROUTINE HEAD1
COMMON/HED/IC1, IC2, IC3, IC4, HEAD, IPAGE, INPUT(20), IEND, LINE, IC5
IPAGE = IPAGE + 1
PRINT 100, IC1, IC2, IC3, IC4, IC5, HEAD, IPAGE, (I, I = 1, 20), INPUT
100 FORMAT (1H1, 9X, 13HV22, 11/12/76, 10X, 15H---- **, 19(5H ****), 8X, 5I2, 14X, 19A4, 3X, 4MPAGE, I5/
1 10X, 10(5H ****), 2013/50X, 10HINPUT = , 2013) RETURN END
SUBROUTINE INPUT (ICASE)  
V-22

REAL M,KM1,KM2,KA
LOGICALLY,LAG,LNCALC
INTEGERIRD(12),ICOL(12)
COMMON FOR INPUT
COMM/INDAT/X(20),M(20),E(20),SEAL(20),KM1(20),KM2(20),KA(20),
1 THP(20),EP(20),GJ(20),EA(20),EB1(20),EB2(20),EGS(20),EIP(20),
2 THP(20),E(20),GJ(20),EA(20),EB1(20),EB2(20),EGS(20),EIP(20),
3 OMEQ,OMF,EC(20),NY,NZ,NP,NM,CMEGS,GMFS,IDIM,MAX,NLDO
4,5NB,HMX,HMY,HMX,HCX,HCE,HCZ+KK,HKY,HKE,NX,NLDO
5 &HINIT,ERROR,1,MC1,CIC,ICY,BCR,CYCL,1,CFASE,AFZ,AFZ+NB
6,CONS,CGFA,CGFA,HE(3),PER

COMMON COEFFICIENT MATRICES
COMM/COEF/COI(11,11),DCO(11,11),DCD(11,11),DCD(11,11),
1 COI(11,11),DCO(11,11),F(11),DF(11),FNL(11),G(I1R(11,12),
2 CODR(11,11),ARCIR(11,11),RFR(11),RDCR(11,12),RF(11,12)
3 &BIN(3,11),DIF(3,11),SPR(3,11),MG1H(11,3),GODH(11,3),BIR(3,11)
4,BIR(3,11),BIR(3,11),BIR(3,11),BIR(3,11),BIR(3,11),
5 &HC(3,3),HK(3,3)

COMMON FOR HEADING, CONTROL DATA
COMM/HED/IC1,IC2,IC3,IC4,HEAD(19),IPAGE,INPUT(20),IEND,LINE,IC5
COMM DIMENSION DATA
COMM/DIM/NINPUT,NSTA,NMODE,NZMODE,NMODE,NM1,NDIM,NBLADE
COMM BASIC DERIVED DATA
COMM/DER/TH(20),EV(20),EH(20),EIP(20),Y(20),Z(20),5,P(20,3)
COMM VARIABLES AND SOLUTION CONTROLS
COMM/VAR/YYVAR(98),DERY(98),PRMT(6),LY(98)
REAL DUM(18),DUMP(20),CUMP(20),KOMP(11,12),DUMP(20)
REAL TELE(20),EDATE(20),CELK(20),KM(20)
REAL K(20),K1(20),KII(20,9),YII(20,3,9),ZII(20,5,9),
1 ZII(20,5,9),PI(20,3,8),QII(20,3,7),SII(20,5,4)
REAL DYY(5,3,0),DYY(3,3,9),DYY(3,3,5,8),DYY(3,3,7)
1 DYY(3,3,8),DYY(3,3,9),DYY(3,3,9),DYY(3,3,9)
2 DYY(1,1,0),DYY(1,1,0),DYY(1,1,0),DYY(1,1,0)
3 DYY(1,1,0),DYY(1,1,0),DYY(1,1,0),DYY(1,1,0)
4 DYY(1,1,0),DYY(1,1,0),DYY(1,1,0),DYY(1,1,0)
5 DYY(1,1,0),DYY(1,1,0),DYY(1,1,0)
6 &ALL(20),DALL(20),DALL(20),DALL(20)
REAL ZYII(20),ZDII(20),ZDII(20),ZDII(20)
REAL DI(2243)
EQUVALENCE(D(1),DYY(1),D(1)),(D(91),DYY(1),D(1)72),DYY(1)
1,3(D(92),DYY(1)),(D(935),DYY(1)),(D(9415),DYY(1)),
2(D(445),DYY(1)),(D(447),DYY(1)),(D(6622),DYY(1)),
3(D(9173),DYY(1)),(D(9175),DYY(1)),(D(9177),DYY(1)),
4(D(1182),DZY(3,1),DZY(1),DZY(5)),
5(D(1223),DZY(3,1),DZY(1)),(D(1583),DZY(3,1)),
6(D(1203),DZY(1),D(1277),DZY(1)),
7(D(1877),DZY(1),D(1877)),
8(D(1994),DZY(1),D(2144),DZY(1))
92
INITIALIZATION

C

EQUIVALENCE (D(2201),DYO(1)),(D(2210),DZO(1)),(D(2235),DPD(1))

C

HEADING

IF(IEND.NE.2) GO TO 52
READ 9000,IC1,IC2,IC3,IC4,IC5,HEAD

9000 FORMAT (511*18A4,A31)

IF(IC1.NE.0) CALL EXIT

52 ICASE = ICASE+1

IPAGE = 0

INPUT(1) = 0, NEVER USED = 1, USED = 2, MODIFIED OR NEW

DO 100 1=1,NINPUT

IF (INPUT(1).EQ.0) GO TO 100

INPUT(1) = 1

100 CONTINUE

C CLEAR TO CLEAN UP OUTPUT OF INTEGRALS

DO 90 1=1,2243

90 D(I) = 0.

IF(INPUT(6).EQ.0) OLDCP = 1.

IF(INPUT(6).NE.0) OLDCP = OMEG

OLDCM = GLDCM*OLDCM

IF(IC5.EQ.0) GO TO 201

I = 7

WRITE (9) I

GO TO 201

C

C

C

GENERAL INPUT

C

C

C

200 IF (IEND.NE.0) GO TO 500

201 READ 9010,IO,DUM,IEND

9010 FORMAT (12,F8.0,6F15.0,F9-09)

IF(IO.NE.21) GO TO 202

CALL HEADD

PRINT 9011

9011 FORMAT(//20X,28BFOLLOWING IO'S ARE CANCELLED /)

DO 203 J=1,8

I = DUM(J)

IF(I.EQ.0) GC TO 203

PRINT 9012,I

9012 FORMAT(30X,I10)

IF(I.LT.0.OR.I.GT.NINPUT) CALL ERR (203,0)

INPUT(I) = 0

203 CONTINUE

C NOTE INPUT(1) SET TO 2 TO INSURE THAT ALL COEFS ARE RE CALCULATED

INPUT(1) = 2

IC2 = 1

GO TO 200

202 IF(IO.GT.NINPUT.OR.IO.LT.1) CALL ERR(200,0)

IF(INPUT(IO).EQ.2) CALL ERR (202,IO)

INPUT(IO) = 2

GO TO (210,220,230,2230,2270,320,330,340,10,11,12)
1 350,14,15,16,280,300,19,20),10
10 CALL ERR(10,0)
11 CALL ERR(11,0)
12 CALL ERR (12,0)
13 CALL ERR(13,0)
14 CALL ERR(14,0)
15 CALL ERR(15,0)
16 CALL ERR(16,0)
17 CALL ERR(17,0)
18 CALL ERR(18,0)
19 CALL ERR(19,0)
20 CALL ERR(20,0)
C 10=1 BLADE PROPERTIES
210 I = 1
215 X(I) = DUM(1)
M(I) = DUM(2)
E(I) = DUM(3)
SEA(I) = DUM(4)
KML(I) = DUM(5)
KM2(I) = DUM(6)
KA(I) = DUM(7)
THP(I) = DUM(8)
READ 9010,IO,DUM
EOPI(I) = DUM(1)
EIP(I) = DUM(2)
GJ(I) = DUM(3)
EA(I) = DUM(4)
EBL(I) = DUM(5)
EB2(I) = DUM(6)
EC(I) = DUM(7)
EC2(I) = DUM(8)
R= X(I)
IF(INEND,NE.,0) GO TO 500
READ 9010,IO,DUM,INEND
IF(IO,NE.,0) GO TO 202
IF(DUM(1),LT, X(I)) CALL ERR(215,0)
I = I+1
NX = I
IF(NX.GT.NSTA) CALL ERR (216,0)
GO TO 215
C 10=2 BLADE CATA
220 NB = DUM(1)
TH0=DUM(2)
BPC=DUM(3)
GV=DUM(4)
GW = DUM(5)
GP = DUM(6)
GO TO 200
C 10 = 3,4,5 MOCDES
230 IF(INPUT(1),EQ,0) CALL ERR (230,0)
J = 0
235 J = J+1
DO 240 I=1,8
240 DUMPP(I) = DUM(1)
IF (NX.LE.8) GO TO 250
READ 9020, (DUMPP(I),I=9,NX)
9020 FORMAT (7FL0,0,F9.0)
250 READ 9020, SC
C INTEGRATE AND NORMALIZE MODES
CALL INT (DUMP, DUMPP, SC, X, NX, 1)
CALL INT (DUMPPP, DUMP, 0, X, NX, 1)
CONST = DUMPPP (NX)
IF (CONST.EQ.0) CCNST = 1.0
DO 260 I = 1, NX
IF (10-4) 252, 254, 256
252 YPP (I, J) = DUMPP (I) /CONST
YP (I, J) = DUMPP (I) /CONST
256 YPP (I, J) = DUMPP (I) /CONST
P(I, J) = DUMPPP (I) /CONST
GO TO 260
254 ZPP (I, J) = DUMPP (I) /CONST
ZP (I, J) = DUMPP (I) /CONST
GO TO 260
260 CONTINUE
IF (IEND .NE. 0) GC TO 261
READ 9010, II, DUM, IENDT
IF (11 .EQ. 0) GC TO 235
261 IF (10-4) 262, 264, 266
262 NY = J
IF (NY.GT. NYMODE) CALL ERR (262, 0)
GO TO 267
264 NZ = J
IF (NZ.GT. NZMODE) CALL ERR (264, 0)
GO TO 267
266 NP = J
IF (NP.GT. NPMODE) CALL ERR (266, 0)
267 IF (IEND .NE. 0) GC TO 500
IEND = IENDT
IO = II
GO TO 202
C IO = 6 FREQUENCIES
270 QMEG = DUM (1)
QMF = DUM (2)
GO TO 200
C IO = 17 NON LINEAR CONTROLS
280 NLIN = DUM (1)
NFLOQ = DUM (2)
GO TO 200
C IO = 18 SOLUTION CONTROLS
300 CYCLES = DUM (1)
HINIT = DUM (2)
ERROR = DUM (3)
IYE = DUM (4)
CIC = DUM (5)
IYIC = DUM (6)
BERR = DUM (7)
GO TO 200
C IO = 7 HUBX
320 HMX = DUM (1)
HCX = DUM (2)
HKX = DUM(3)  00002180
HE(1) = DUM(4)  00002190
GO TO 200  00002200
C
IO = 8  HUB Y  00002210
330 HMY = DUM(1)  00002220
HCY = DUM(2)  00002230
HE(2) = DUM(4)  00002240
GO TO 200  00002250
C
IO = 9  HUB Z  00002260
340 HMZ = DUM(1)  00002270
HCZ = DUM(2)  00002280
HE(3) = DUM(4)  00002290
GO TO 200  00002300
C
IO = 13  BLADE FORCE  00002310
350 NXF = DUM(1)  00002320
AFY = DUM(2)  00002330
AFZ = DUM(3)  00002340
AFP = DUM(4)  00002350
NBF = DUM(5)  00002360
PER = DUM(6)  00002370
GO TO 200  00002380
C
500 IF(INPUT(1).EQ.0) CALL ERR(500,0)  00002480
IF(INPUT(2).NE.0) GO TO 501  00002490
NB=1  00002500
THO=0  00002510
BPC=0  00002520
GV=0  00002530
GW=0  00002540
GP=0  00002550
501 IF(INPUT(3).EQ.0) NY=0  00002560
IF(INPUT(4).EQ.0) NZ=0  00002570
IF(INPUT(5).EQ.0) NP=0  00002580
NM=NY+NZ+NP  00002590
IF(NM.EQ.0) CALL ERR(501,0)  00002600
NMAX = NM  00002610
IF(NP.GT.NMAX) NMAX = NP  00002620
IF(NY.GT.NMAX) NMAX = NY  00002630
IF(INPUT(6).EQ.0) CALL ERR(502,0)  00002640
IF(NB.EQ.1 .AND. (INPUT(7).NE.0 .OR. INPUT(8).NE.0 .OR. INPUT(9).NE.0))  00002650
1 = NB=2  00002660
IF(NB.GT.NBLADE) CALL ERR(506, NB)  00002670
IF(NB.GT.NBLADE) NB=NBLADE  00002680
IF(NB.LT.1) CALL ERR(507,1)  00002690
IF(NB.LT.1) NB = 1  00002700
IF(NB.EQ.1 .AND. (INPUT(7).NE.0 .OR. INPUT(8).NE.0 .OR. INPUT(9).NE.0))  00002710
1  00002720
IF(INPUT(7).NE.0) GO TO 502
HMX=0
HCX=0
HEX=0
HE(II)=0
HCY=0
HKY=0
HE(3)=0
502 IF(INPUT(8).NE.0) GO TO 503
HM=0
HC=0
HE(II)=0
HCY=0
HKY=0
HE(3)=0
503 IF(INPUT(9).NE.0) GO TO 504
HM=0
HC=0
HE(II)=0
HCY=0
HKY=0
HE(3)=0
504 OMRAT = CMG/OLDCM
OMRAT=OMRAT*OMRAT
IF(INPUT(13).NE.0.AND.(NXF.GT.NX.OR.NXF.LE.0)) CALL ERR1(510,0)
IF(INPUT(13).NE.0.AND.(AFY.EQ.0.AND.AFZ.EQ.0.AND.AFP.EQ.0))
1 CALL ERR (511,0)
IF(INPUT(13).NE.0.AND.NBF.GT.NE) CALL ERR1(512,NBF)
IF(INPUT(13).NE.0.AND.NBF.NE.0) NBF = 0
IF(INPUT(13).NE.0.AND.NBF.LT.0 ) CALL ERR1(512,NBF)
IF(INPUT(9).NE.0) NBF=0
IF(INPUT(17).EQ.0) NBF=0
IF(INPUT(17).EQ.0) NBF=0
IF(INPUT(18).EQ.0) CALL ERR(509,0)
ADD BLACE LOADS TO HUE
DO 508 I=1,3
508 HF(I)=HE(I)
IF(INPUT(13).EQ.0) GO TO 509
IF(AFZ.EQ.0) GO TO 506
IF(INPUT(9).EQ.0) GO TO 506
CONST=AFZ
IF(NBF.EQ.0) CONST=NB*CONST
HF(3)=HF(3)+CONST
506 IF(AFZ.EQ.0) GO TO 509
IF(INPUT(7).EQ.0.AND.INPUT(8).EQ.0.OR.NBF.EQ.0) GO TO 509
CALL ERR(510,NBF)
NBF=0
C COMPUTE COEFFICIENTS, ETC.
C 00003140
C 00003150
509 CALL INTH(TH,THP,TH0,X,NX,1)
DO 510 I=1,NX
DUMMY1 = SEA(I)*E2*EA(I)
DUMMY2 = EIP(I)-EOP(I)
EV(I) = EIP(I)-DUMMY2*TH(I)**2-DUMMY1
DELE(I) = DUMMY2-DUMMY1
EONE(I) = SEA(I)*EA(I)*KA(I)**2-EBZ(I)
EW(I) = EOP(I)+DUMMY2*TH(I)**2-DUMMY1*TH(I)
EP(I) = GJ(I)-(KA(I)**4*EA(I)-EB1(I))*THP(I)**2
DELK(I) = KMZ(I)**2-KM1(I)**2
510 KM1(I) = KM1(I)**2+KM1(I)**2
C FORM MASS INTEGRALS
C 00003270
97
C RECOMPUTE ALL COEFS UNLESS ONLY IC = 6 OR GE 17 ARE CHANGED
LCALC=.TRUE.
600 DO 601 I=1,16
   IF(I.EQ.6) GO TO 601
   IF(INPUT(I).EQ.2) GO TC 602
601 CONTINUE
   LCALC=.FALSE.
   IF(INPUT(6).EQ.2) GO TC 1075
   GO TO 1100
C FORM INTEGRANDS
602 DO 610 I = 1,NX
   MI(1,1) = M(I)
   MI(1,2) = MI(1)*X(I)
   MI(1,3) = MI(1)*E(I)
   MI(1,4) = MI(1,3)*X(I)
   MI(1,5) = MI(1,3)*TH(I)
   MI(1,6) = MI(1,5)*X(I)
   MI(1,7) = MI(1)*K(2(I))*2
   MI(1,8) = MI(1,7)*TH(I)
610 MI(1,9) = MI(1)*DELK(I)*TH(I)
   DO 620 J = 1,9
   DO 620 I = 1,NX
   MI(I,J) = DUMP(I)
620 DUMP(I) = MI(I,J)
   CALL INT (DUMP,DUMPP,O,X,NX,2)
   CALL INT (DUMPP,CUMPP,O,X,NX,2)
   DO 630 I = 1,NX
   MI(I,J) = DUMP(I)
630 MI(I,J) = DUMPP(I)
   CALL INT (DUMP,DUMPP,O,X,NX,2)
   CALL INT (DUMPP,CUMPP,O,X,NX,2)
640 MI(I,J) = DUMP(I)
C FORM Y INTEGRALS
650 IF(INPUT(3).EQ.0) GO TC 700
   CALL INT (DUMP,DUMPP,O,X,NX,2)
C FORM INTEGRANDS
660 YI(I,IM,1) = Y(I,IM)*Y(I,IM)
   YI(I,IM,2) = YI(I,IM,1)*E(I)
   YI(I,IM,3) = YI(I,IM,2)*TH(I)
   YI(I,IM,4) = YI(I,IM,3)*Y(I,IM)
   YI(I,IM,5) = MI(I,1)*X(I)*YP(I,IM)
   YI(I,IM,6) = MI(I,1)*X(I)*TH(I)
   YI(I,IM,7) = MI(I,2)*YP(I,IM)
   YI(I,IM,8) = MI(I,1)*TH(I)
   YI(I,IM,9) = YPP(I,IM)*EONE(I)*TH(I)
670 YI(I,IM,10) = YPP(I,IM)*EONE(I)
   DO 670 J = 1,9
   DO 670 IM = 1,NY
   YI(I,IM) = Y(I,IM)
   DO 665 I = 1,NX
   DUMP(I) = YI(I,IM,J)
   CALL INT (DUMP,DUMPP,O,X,NX,2)
   CALL INT (DUMP,DUMPP,O,X,NX,2)
DO 670 I = 1,NX
  YI(I,IM,J) = DUMPP(I)
670 YII(I,IM,J) = DUMPP(I)
DO 680 IM = 1,NX
DO 675 I = 1,NX
675 DUMPP(I) = YI(I,IM,10)
  CALL INT (DUMP,DUMPP,0,X,NX,1)
DO 680 I = 1,NX
680 YI(I,IM,10) = DUMPP(I)
DO 682 I = 1,NX
  IF(EA(I).EQ.0) SI(I,IM,1) = 0
  IF(EA(I).NE.0) SI(I,IM,1) = YII(I,IM,1)/EA(I)
SI(I,IM,2) = M(I)*YII(I,IM,10)
682 SI(I,IM,5) = KA(I)**2*THP(I)**YII(I,IM,1)
DO 685 IM = 1,NX
DO 680 I = 1,NX
DO 683 DUMPP(I) = SI(I,IM,1)
  CALL INT (DUMP,DUMPP,0,X,NX,1)
DO 684 I = 1,NX
684 DUMPP(I) = DUMP(I)*M(I)
  CALL INT (DUMP,DUMPP,0,X,NX,2)
  CALL INT (DUMP,DUMPP,0,X,NX,2)
DO 685 IM = 1,NX
DO 680 I = 1,NX
DO 685 SI(I,IM,1) = DUMPP(I)
DO 690 IM = 1,NX
DO 686 I = 1,NX
686 DUMPP(I) = SI(I,IM,2)
  CALL INT (DUMP,DUMPP,0,X,NX,2)
  CALL INT (DUMP,DUMPP,0,X,NX,2)
DO 690 I = 1,NX
690 SI(I,IM,2) = DUMPP(I)
DO 695 IM = 1,NX
DO 692 I = 1,NX
692 DUMPP(I) = SI(I,IM,5)
  CALL INT (DUMP,DUMPP,0,X,NX,2)
DO 695 I = 1,NX
695 SI(I,IM,5) = DUMPP(I)
C FORM Z INTEGRALS
700 IF(INPUT(4).EQ.0) GO TO 750
DO 710 I = 1,NX
710 ZI(I,JM,1) = M(I)**Z(I,JM)
ZI(I,JM,2) = ZI(I,JM,1)**E(I)
ZI(I,JM,3) = M(I)**X(I)**ZP(I,JM)
ZI(I,JM,4) = ZI(I,JM,3)**E(I)
ZI(I,JM,5) = M(I)**ZP(I,JM)*E(I)**THP(I)
ZI(I,JM,6) = M(I)**ZP(I,JM)
ZI(I,JM,7) = ZI(I,JM,6)**E(I)
ZI(I,JM,8) = ZP(I,JM)**E(I)**THP(I)
710 ZI(I,JM,9) = ZP(I,JM)**E(I)**THP(I)
DO 720 J = 1,8
DO 720 JM = 1,NZ
DO 715 I = 1,NX
715 DUMPP(I) = ZI(I,JM,J)
99
CALL INT (DUMP,DUMPP,0,X,NX,2) 0000043 80
CALL INT (DUMPP,DUMPP,0,X,NX,2) 0000043 90
DO 720 I = 1,NX 0000044 00
ZI(I,JM,J) = DUMPP(I) 000004410
720 ZI(I,JM,J) = DUMPP(I) 000004420
DO 730 JM = 1,NZ 000004430
DO 725 I = 1,NX 000004440
725 DUMPP(I) = ZI(I,JM,9) 000004450
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004460
DO 730 I = 1,NX 000004470
730 ZI(I,JM,9) = DUMPP(I) 000004480
DO 740 JM = 1,NZ 000004490
DO 735 I = 1,NX 000004500
735 DUMPP(I) = M(I)*ZI(I,JM,9) 000004510
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004520
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004530
DO 740 I = 1,NX 000004540
740 SI(I,JM,3) = DUMPP(I) 000004550
C FORM P INTEGRALS
750 IF (INPUT(5).EQ.0) GO TO 800 000004560
DO 760 I = 1,NX 000004570
DO 760 IM = 1,NP 000004580
PI(I,IM,1) = M(I)*E(I)*F(I,IM) 000004590
PI(I,IM,2) = PI(I,IM,1)*X(I) 000004600
PI(I,IM,3) = PI(I,IM,1)*TH(I) 000004610
PI(I,IM,4) = M(I)*DELK(I)*PI(I,IM) 000004620
PI(I,IM,5) = M(I)*DELK(I)*PI(I,IM) 000004630
PI(I,IM,6) = EP(I)*PP(I,IM) 000004640
PI(I,IM,7) = KA(I)**2*PI(I,2)*PI(I,IM) 000004650
760 PI(I,IM,8) = KA(I)**2*THP(I)*PI(I,IM) 000004660
DO 770 J = 1,7 000004670
DO 770 IM = 1,NP 000004680
DO 765 I = 1,NX 000004690
765 DUMPP(I) = PI(I,IM,J) 000004700
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004710
IF (J.GT.5) GO TO 766 000004720
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004730
766 DO 770 I = 1,NX 000004740
PI(I,IM,J) = DUMPP(I) 000004750
IF (J.GT.5) GO TO 770 000004760
PI(I,IM,J) = DUMPP(I) 000004770
770 CONTINUE 000004780
DO 780 IM = 1,NP 000004790
DO 775 I = 1,NX 000004800
775 DUMPP(I) = PI(I,IM,8) 000004810
CALL INT (DUMP,DUMPP,0,X,NX,1) 000004820
DO 780 I = 1,NX 000004830
780 PI(I,IM,8) = DUMPP(I) 000004840
DO 790 IM = 1,NP 000004850
DO 785 I = 1,NX 000004860
785 DUMPP(I) = M(I)*PI(I,IM,8) 000004870
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004880
CALL INT (DUMP,DUMPP,0,X,NX,2) 000004890
DO 790 I = 1,NX 000004900
790 SI(I,IM,4) = DUMPP(I) 000004910
700
DEFINITE INTEGRALS

BLADE FORCE INTEGRALS

800 IF(INPUT(13).EQ.0) GO TO 810
DO 802 I=1,NX
802 ALII(I)=MAX(0.0,X(NXF)-X(I))
810 IF(NY.EQ.0) GO TO 851
DO 850 I = 1,NY
IF(INPUT(13).EQ.0.OR.AFY.EQ.0) GO TO 824
DO 815 K=1,NX
815 DUMP(K)=AFY*Y(K,I)*ALII(K)
DYALII(I)=DINT(DUMP,DUMPP,X,AX)
824 DO 825 J = 1,NY
DYSI(I,J,1) = DINT2(Y,SI,I,J,1,5,X,NSTA,NX,DUMP,DUMPP)
DYSI(I,J,2) = DINT2(Y,SI,I,J,2,5,X,NSTA,NX,DUMP,DUMPP)
DO 825 K = 1,9
DYII(I,J,K) = DINT2(Y,YII,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
825 IF (NZ.EQ.0) GC TO 832
DO 830 J = 1,NZ
DYSI(I,J,3) = DINT2(Y,SI,I,J,3,5,X,NSTA,NX,DUMP,DUMPP)
DO 830 K = 1,8
830 DYII(I,J,K) = DINT2(Y,YII,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
832 IF (NP.EQ.0) GC TO 836
DO 835 J = 1,NP
DYSI(I,J,4) = DINT2(Y,SI,I,J,4,5,X,NSTA,NX,DUMP,DUMPP)
835 DYPII(I,J,3) = DINT2(Y,PII,I,J,3,NYMODE,X,NSTA,NX,DUMP,DUMPP)
836 DO 845 K = 1,9
DYII(I,K) = DINT1(Y,MI,I,K,X,NSTA,NX,DUMP,DUMPP)
845 DYII(I,K) = DINT1(Y,MI,I,K,X,NSTA,NX,DUMP,DUMPP)
850 DYII(I,J) = DINT1(Y,MI,I,J,10,X,NSTA,NX,DUMP,DUMPP)
851 IF (NZ.EQ.0) GC TO 881
DO 880 I = 1,NZ
IF(INPUT(13).EQ.0.OR.AFZ.EQ.0) GO TO 854
DO 852 K=1,NX
852 DUMPP(K)=AFZ*Z(K,I)*ALII(K)
DZALII(I)=DINT(DUMP,DUMPP,X,AX)
854 IF(NY.EQ.0) GO TO 856
DO 855 J = 1,NY
855 DZSI(I,J,K) = DINT2(Z,SI,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
855 DZII(I,J,K) = DINT2(Z,YII,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
856 DO 860 J = 1,NZ
860 DZZII(I,J,K) = DINT2(Z,ZII,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
IF (NP.EQ.0) GO TO 866
DO 865 J = 1,NP
DZPII(I,J,2) = DINT2(Z,PI,I,J,2,NYMODE,X,NSTA,NX,DUMP,DUMPP)
865 DZPII(I,J,1) = DINT2(Z,PI,I,J,1,NYMODE,X,NSTA,NX,DUMP,DUMPP)
866 DO 870 K = 1,9
DZMI(I,K) = DINT1(Z,MI,I,K,X,NSTA,NX,DUMP,DUMPP)
870 DZMI(I,K) = DINT1(Z,MI,I,K,X,NSTA,NX,DUMP,DUMPP)
880 DZMI(I,10) = DINT1(Z,MI,I,10,X,NSTA,NX,DUMP,DUMPP)
881 IF (NP.EQ.0) GC TO 901
DO 900 I = 1,NP
IF(INPUT(13).EQ.0.OR.AFP.EQ.0) GO TO 884
00004930
00004940
00004950
00004960
00004970
00004980
00004990
00005000
00005010
00005020
00005030
00005040
00005050
00005060
00005070
00005080
00005090
00005100
00005110
00005120
00005130
00005140
00005150
00005160
00005170
00005180
00005190
00005200
00005210
00005220
00005230
00005240
00005250
00005260
00005270
00005280
00005290
00005300
00005310
00005320
00005330
00005340
00005350
00005360
00005370
00005380
00005390
00005400
00005410
00005420
00005430
00005440
00005450
00005460
00005470
DO 882 K=1,NX
882 DUMPP(K)=AFP*P(K,1)*ALII(K)
DPALII(I)=INT(DUMP,DUMPP,X,NX)
884 IF(NY.EQ.0) GO TO 886
DO 885 J = 1,NY
885 K = 1,N9
DPYII(I,J,K) = INT2(P,Y1,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
885 DYNII(I,J,K) = INT2(P,Y1,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
886 IF(NZ.EQ.0) GO TO 891
DO 892 J = 1,NZ
890 K = 1,N9
DPYIII(I,K,J) = INT2(P,Y11,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
892 DYNIII(I,K,J) = INT2(P,Y11,I,J,K,NYMODE,X,NSTA,NX,DUMP,DUMPP)
895 CONTINUE
896 DO 897 K = 1,N9
DPMY(I,K) = INTI(P,Y1,I,K,NSTA,NX,DUMP,DUMPP)
897 DPMII(I,K) = INTI(P,Y11,I,K,NSTA,NX,DUMP,DUMPP)
900 DPM(I,10) = INTI(P,Y1,I,10,X,NSTA,NX,DUMP,DUMPP)
901 IF(NY.EQ.0) GO TO 931
DO 910 J = 1,NY
909 K = 1,N9
DO 902 I = 1,NX
902 DUMPP(I) = Y(I,J)*R-X(I)=MX)*NY)*Y(NX,K)
DYF(J,K,1) = INT(DUMP,DUMPP,X,NX)
DO 904 I = 1,NX
904 DUMPP(I) = Y(I,J)*EA(I)*Y1(I,K,1)
DYF(J,K,1) = INT(DUMP,DUMPP,X,NX)
DO 906 I = 1,NX
906 DUMPP(I) = Y(I,J)*EV(I)*PPI(K,1)
910 DYF(J,K,3) = INT(DUMP,DUMPP,X,NX)
IF(NZ.EQ.0) GO TO 916
DO 912 K = 1,NZ
912 DUMPP(I) = Y(I,J)*DELE(I)*TH(I)*ZPP(I,K)
915 DYF(J,K,4) = INT(DUMP,DUMPP,X,NX)
916 IF(NP.EQ.0) GO TO 929
DO 920 K = 1,NP
920 DUMPP(I) = Y(I,J)*TH(I)*PP(I,K)+EDNE(I)*TP(I)*PP(I,K)
920 DYF(J,K,5) = INT(DUMP,DUMPP,X,NX)
925 DO 927 J = 1,NX
927 DUMPP(I) = Y(I,J)*SEAI(I)*MI(I,2)+R*(R-X(I))*MX)*NX)*Y(NX)
930 DYF(J,1,6) = INT(DUMP,DUMPP,X,NX)
931 IF(NZ.EQ.0) GO TO 961
DO 960 J = 1,NZ
IF(NY.EQ.0) GO TO 936
DO 935 K = 1,NY
DO 932 I = 1,NX

932 DUMPP(I) = Z(I,J)*(R-X(I))*K(NX)*E(NX)*TH(NX)*Y(NX,K) 00006030
1  *SEA(I)*TH(I)*Y(I,K) 00006040
DZF(J,K,1) = DINT (DUMPP,DUMPP,X,NX) 00006050
DO 934 I = 1,NX 00006060
934 DUMPP(I) = Z(I,J)*SEAE(I)*TH(I)*YPP(I,K) 00006070
935 DZF(J,K,2) = DINT (DUMPP,DUMPP,X,NX) 00006080
936 DO 938 K = 1,NZ 00006090
937 DUMPP(I) = Z(I,J)*EWH(I)*YPP(I,K) 00006100
938 DZF(J,K,3) = DINT (DUMPP,DUMPP,X,NX) 00006120
IF (NP.EQ.0) GO TO 946 00006130
DO 945 K = 1,NP 00006140
DO 940 I = 1,NX 00006150
940 DUMPP(I) = Z(I,J)*ECS(I)*PPP(I,K)*EON(I)*TH(I)*TP(I)*PPP(I,K) 00006160
DZF(J,K,4) = DINT (DUMPP,DUMPP,X,NX) 00006170
DO 942 I = 1,NX 00006180
942 DUMPP(I) = Z(I,J)* 00006190
1 (SEA(I)*MI(I,2)*P(I,K)+X(NX)*X(X(NX)-X(I))*M(NX)*E(NX)*P(NX,K)) 00006200
945 DZF(J,K,6) = DINT (DUMPP,DUMPP,X,NX) 00006210
946 DO 950 I = 1,NX 00006220
950 DUMPP(I) = Z(I,J)* (SEA(I)*MI(I,2)*TH(I)*X(NX)*M(NX)*E(NX)*TH(NX) 00006230
1  *(R-X(I))) 00006240
960 DZF(J,K,5) = DINT (DUMPP,DUMPP,X,NX) 00006250
961 IF (NP.EQ.0) GO TO 991 00006260
DO 990 J = 1,NP 00006270
IF (NY.EQ.0) GO TO 965 00006280
DO 963 K = 1,NY 00006290
DO 962 I = 1,NX 00006300
962 DUMPP(I) = P(I,J)*ECS(I)*TH(I)*YPP(I,K) 00006310
963 DPFI(J,K,1) = DINT (DUMPP,DUMPP,X,NX) 00006320
965 IF (NZ.EQ.0) GO TO 971 00006330
DO 970 K = 1,NZ 00006340
970 DO 964 I = 1,NX 00006350
964 DUMPP(I) = P(I,J)*ECS(I)*ZPP(I,K) 00006360
970 DPFI(J,K,2) = DINT (DUMPP,DUMPP,X,NX) 00006370
971 IF (NZ.EQ.0) GO TO 990 00006380
DO 980 J = 1,NP 00006390
DO 975 I = 1,NX 00006400
975 DUMPP(I) = P(I,J)*ECS(I)*PPP(I,K) 00006410
980 DPFI(J,K,3) = DINT (DUMPP,DUMPP,X,NX) 00006420
990 CONTINUE 00006430
C Damping Definite Integrals 00006440
991 IF (NY.EQ.0 OR GW.EQ.0) GO TO 995 00006450
DO 994 J = 1,NY 00006460
DO 992 K = 1,NX 00006470
992 YZPI(K) = Y(K,J) 00006480
CALL INT(DUMPP,YZPI,0,X,NX,Z) 00006490
CALL INT(YZPI,DUMPP,0,X,NX,Z) 00006500
DO 994 I = 1,NY 00006510
DO 993 K = 1,NX 00006520
993 DUMPP(K) = YZPI(K)*Y(K,I) 00006530
994 DYP(I,J) = DINT (DUMPP,DUMPP,X,NX)*GV 00006540
995 IF (NZ.EQ.0 OR GW.EQ.0) GO TO 999 00006550
DO 998 J = 1,NZ 00006560
DO 996 K = 1,NX 00006570
996 YZPI(K)=Z(K,J)
   CALL INT(DUMP, YZPI, 0, X, NX, 2)
   CALL INT(YZPI, DUMP, 0, X, NX, 2)
DO 998 I=1,NZ
DO 997 K=1,NX
997 DUMP(K)=YZPI(K)*Z(K,J)
998 DZD1(I,J)=DINT(DUMP, DUMP, X, NX)*GW
999 IF(NP.EQ.0,CR,GP,EQ.0) GO TO 1010
DO 1002 J=1,NP
DO 1000 K=1,NX
1000 YZPI(K)=P(K,J)
   CALL INT(DUMP, YZPI, 0, X, NX, 2)
   CALL INT(YZPI, DUMP, 0, X, NX, 2)
DO 1002 I=1,NP
DO 1000 K=1,NX
1001 DUMP(K)=YZPI(K)*P(K,J)
1002 DPD(I,J)=DINT(DUMP, DUMP, X, NX)*GP
C FORM BLADE COEFFICIENT MATRICES
1010 II=0
IF(NY.EQ.0) GO TO 1031
DO 1030 I=1,NY
   JJ = 0
   II = II+1
DO 1015 J=1,NY
   JJ = JJ+1
   COI (II,JJ) = DYYII(I,J,1)
   DCOI (II,JJ) = 4*DSII(I,J,1)
   CCOI (II,JJ) = -DYII(I,J,1)
   DDCOI (II,JJ) = -2*(DSII(I,J,2)-DYYII(I,J,2)+DYYII(I,J,2))
1 = -DYII(I,J,1)
COI (II,JJ) = -DYII(I,J,3)
1015 DCOI (II,JJ) = DYYIII(I,J,7)-CYYIII(I,J,4)+DYYIII(I,J,1)
1016 IF(NZ.EQ.0) GO TO 1021
DO 1020 J=1,NZ
   JJ = JJ+1
   CCII (II,JJ) = 0
   DCII (II,JJ) = 0
   CCOII (II,JJ) = 0
   DDCOII (II,JJ) = -2*(DSII(I,J,3)-DYYIII(I,J,5)-BPC*DYII(I,J,1))
   CCII (II,JJ) = -DYII(I,J,4)
1020 DCCII (II,JJ) = 0
1021 IF(NP.EQ.0) GO TO 1026
DO 1025 J=1,NP
   JJ = JJ+1
   CCOII (II,JJ) = -DYIII(I,J,3)
   DCII (II,JJ) = 0
   CCOII (II,JJ) = 0
   DDCII (II,JJ) = 2*DSII(I,J,4)
1025 DCCII (II,JJ) = 0
1026 IF(NP.EQ.0) = DYYII(I,3)-CYYII(I,4)+DYII(I,1,6)
   BFIII = 0
IF(INPUT(13).NE.0.AND.AFY.NE.0) BFII = DYIIIII(I)
1030 CONTINUE
1031 IF(NZ.EQ.0) GO TO 1051

104
DO 1050 I = 1,NZ
   JJ = 0
   II = I+1
   IF(NY.EQ.0) GO TO 1036
   DO 1035 J = 1,NY
      JJ = JJ+1
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = -2*(DZYI(I,J,3)-DZF(I,J,1)+BPC*DZYII(I,J,1))
      CO(I,I,JJ) = -DZF(I,J,2)
   1035
   DO 1036 JJ = 0
      I1 = I1+1
      IF(NY.EQ.0) GO TO 1036
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = -2*(DZYI(I,J,3)-DZF(I,J,3))
      CO(I,I,JJ) = -DZF(I,J,2)
   1036
   DO 1040 J = 1,NZ
      JJ = JJ+1
      COI(II,JJ) = DZZII(I,J,1)
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = -2*(DZYII(I,J,3)-DZF(I,J,1))
      CO(I,I,JJ) = -DZF(I,J,2)
   1040
   DO 1041 JJ = 0
      I1 = I1+1
      IF(NY.EQ.0) GO TO 1041
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = -2*(DZYII(I,J,5)+BPC*DZMII(I,2))
      CO(I,I,JJ) = DZF(I,J,2)
   1041
   DO 1042 J = 1,NP
      JJ = JJ+1
      COI(II,JJ) = DZPI(I,J,1)
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = 0
      CO(I,I,JJ) = -DZF(I,J,2)
   1042
   DO 1043 J = 1,NY
      JJ = JJ+1
      COI(II,JJ) = -DZPII(I,J,3)
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = 0
      CO(I,I,JJ) = 0
   1043
   DO 1044 J = 1,NP
      JJ = JJ+1
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = 0
      CO(I,I,JJ) = 0
   1044
   DO 1045 J = 1,NP
      JJ = JJ+1
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = 0
      CO(I,I,JJ) = 0
   1045
   DO 1046 J = 1,NP
      JJ = JJ+1
      COI(II,JJ) = 0
      COO(II,JJ) = 0
      COD(II,JJ) = 0
      DCOOD(II,JJ) = 0
      CO(I,I,JJ) = 0
   1046
   CONTINUE
1061 DO 1065 J = 1, NP
   JJ = JJ + 1
   COI(I, J) = DPPII(I, J, 4)
   DCOI(I, J) = 0
   COD(I, J) = -DPD(I, J)
   DCOD(I, J) = 0
   CO(I, J) = -DPF(I, J, 3) - DPPII(I, J, 6)
1065 DCO(I, J) = -(DPPII(I, J, 5) + DPPII(I, J, 7))
   BF(I) = 0
   IF (INPUT(13) .NE. 0 .AND. AFP .NE. 0) BF(I) = DPALII(I)
   DF(I) = -(DPMII(I, 9) + BPC*DPMII(I, 1))
1070 CONTINUE
C SUM WITH OMEGAS
1075 OMEGS = OMEG*OMEG
   CMFS = CMF*OMF
   DO 1080 I = 1, NM
   DO 1076 J = 1, NM
      CONI(I, J) = 0
      CONI(I, J) = CONI(I, J) + OMEGS*DCII(I, J)
      CODR(I, J) = COD(I, J) + OMEG*DCOD(I, J)
1076 CON(I, J) = CON(I, J) + CMEGS*DCO(I, J)
C NOTE F IS EVALUATED IF FCT
1080 FR(I) = OMEGS*OF(I)
C INVERT COIR
   CALL INVRS (COIR, NM, RICC, WORK, IRW, ICOL, NMODE, NM1)
C HUB EFFECTS WITH OLD OMEG TO BE RATIOED LATER
   IF (INPUT(7) .EQ. 0 .AND. INPUT(8) .EQ. 0 .AND. INPUT(9) .EQ. 0) GO TO 1100
   IF (.NOT. LCALC) GO TO 1090
   JJ = 0
   IF (NY .EQ. 0) GO TO 1087
   DO 1081 J = 1, NY
      JJ = J + 1
      CONST = YI(J, J + 1)
      BINT(I, J) = CCNST
      BINT(I, J) = -CCNST
      BINT(1, J) = 0
      BDAM(1, J) = CCNST + 2.*CLDOM
      BDAM(2, J) = CCNST + 2.*CLDOM
      BDAM(3, J) = 0
      BSPR(1, J) = -COAST*OLCCMS
      BSPR(2, J) = COAST*OLCCMS
      BSPR(3, J) = 0
      COIH(JJ, 1) = DYMII(J, 1)
      COIH(JJ, 2) = -DYMII(J, 1)
      COIH(JJ, 3) = 0
   DO 1081 J = 1, 3
1081 COIH(JJ, I) = 0
1087 IF (NZ .EQ. 0) GO TO 1083
   DO 1082 J = 1, NZ
      JJ = J + 1
      BINT(I, J) = 0
      BINT(I, J) = 0
      BINT(I, J) = -ZI(I, J, 1)
      BDAM(1, J) = 0
      BDAM(2, J) = 0
      BDAM(3, J) = 0
1082 FR(I) = OMEGS*OF(I)
1083 CONTINUE
BSPR(1,JJ) = 0
BSPR(2,JJ) = 0
BSPR(3,JJ) = 0
COIH(JJ,1) = 0
COIH(JJ,2) = 0
COIH(JJ,3) = -COH(JJ,1)
DO 1082 I=1,3
1082 CDH(I,JJ) = 0
1083 IF (NP.EQ.0) GO TO 1085
DO 1084 J=1,NP
JJ = JJ + 1
CONST = PI(1,1,3)
BIN(1,JJ) = -CONS
BIN(2,JJ) = CONS
BIN(3,JJ) = PI(1,1,1)
BDAM(1,JJ) = -CONS**2*CLDOM
BDAM(2,JJ) = -CONS**2*CLDOM
BDAM(3,JJ) = 0
BSPR(1,JJ) = 0
BSPR(2,JJ) = 0
BSPR(3,JJ) = 0
CONST = DPMI(J,3)
COIH(JJ,1) = -CONS
COIH(JJ,2) = CONS
COIH(JJ,3) = -CONS
CDH(JJ,1) = -DPMI(J,3)*CLDOM
CDH(JJ,2) = DPMI(J,3)*CLDOM
1084 CDH(JJ,3) = 0
1085 DO 1086 I=1,3
DO 1086 J=1,3
HC(1,JJ) = 0
HK(1,JJ) = 0
1086 TM(1,J) = 0
TM(1,1) = HMX + NB*MI(1,1)
TM(2,1) = HMY + NB*MI(1,1)
TM(3,1) = HMS + NB*MI(1,1)
HC(1,1) = -HCX
HC(2,1) = -HCX
HC(3,1) = -HCX
HK(1,1) = -HKX
HK(2,1) = -HKX
HK(3,1) = -HKX
HC(1,1) = -HCX
HC(2,1) = -HCX
HC(3,1) = -HCX
HK(1,1) = -HKX
HK(2,1) = -HKX
HK(3,1) = -HKX
C INCLUDE OMEGA IN HUB EFFECTS USES RATIOS
1090 DO 1091 I=1,3
DO 1091 J=1,NW
BDAM(I,J) = BDAM(I,J)*CMRAT
BSPR(I,J) = BSPR(I,J)*CMRATS
1091 CDH(I,J) = CDH(I,J)*CMRAT
C NOTE NOTE - - - SPECIFIC FOR 3 HUB DOF
CALL MXMBI1, BI1, R1OC, 3,NF,NM,3,3,NMODE
CALL MXMBI1, BI1, R1OC, 3,NF,NM,3,3,NMODE
CALL MXMBI1, BI1, CDR, 3,NF,NM,3,3,NMODE
CALL MXMBI1, BI1, CDR, 3,NF,NM,3,3,NMODE
DO 1092 I=1,3
DO 1092 J=1,NM
1092 BRI0(I,J) = BRI0(I,J) + BSPR(I,J)
CALL MXM(BIRIH,BIRI,CDDH,3,NM,3,3,NMODE ) 00008780
CALL MXM(BIRIH,BIRI,CDDH,3,NM,3,3,NMODE ) 00008750
C SCLUTIC CONTROLS
1100 PRMT(I) = 0
OM=OMF
IF(OM.EQ.0)OM=OMEG
PRMT(2) =6.28319*CYCLES/OM
PRMT(3) =6.28319/HINIT /OM
PRMT(4) =ERROR
IF(ERROR.LE.0) CALL ERR(1100,0)
PRMT(6) = BERR
DO 1105 I = 1,NDIM
YVAR(I) = 0
LY(I) = .FALSE.
1105 DERY(I) = 0
IF(YIC.LE.0) CALL ERR(1105,0)
IF(YIC.GT.NDIM) CALL ERR(1105,0)
YVAR(I) = CIC
DERY(I) = 0
IF(IYIC.LE.0) CALL ERR(1105,0)
IF(IYIC.GT.NDIM) CALL ERR(1105,0)
DERY(IYIC) = 1.0
IF (INPUT(7).NE.0) LY(I) = .TRUE.
IF (INPUT(7).NE.0) LY(2) = .TRUE.
IF (INPUT(8).NE.0) LY(3) = .TRUE.
IF (INPUT(8).NE.0) LY(4) = .TRUE.
IF (INPUT(9).NE.0) LY(5) = .TRUE.
IF (INPUT(9).NE.0) LY(6) = .TRUE.
1200 DIM = 10+2*NM*NE
DO 1205 I=1,NDIM
LY(I) = .TRUE.
1205 CALL HEADIN
C OUTPUT OUTPUT OUTPUT
2000 CALL HEADIN
IF (INPUT(1).NE.2.AND. INPUT(2).NE.2.AND. IC2.EQ.0) GO TO 2050
C
IO = 1+2
PRINT 9060,NM,EPCTH,OM,GN,CP
9060 FORMAT (/10X,27H0 = 1.2
1 5X,9HEPREPONE = ,F6.3,5X,$\theta = \theta _0 ,F6.3,5X,$
2 15HDAMPING (V,\beta, P) ,1P3E11.3
3 ///10X,9BH M E
4 SMALL EA KMI K*2 KA \theta PRIME (G) \theta 00009220
5 ///
DO 2010 I = 1,AX
2010 PRINT 9070,I,X(I),M(I),E(I),SEA(I),KMI(I),KM2(I),KA(I),THP(I) ,
1 THP(I) 00009250
1 THP(I) 00009260
9070 FORMAT (1X,I3,1P10E12.3)
PRINT 9080
9080 FORMAT (/17X,89HE1 CP EA IP GJ EA
1 EB1* = EB2* EC1 EC1* //
DO 2020 I = 1,AX
2020 PRINT 9070,I,ECP(I),EIP(I),GJ(I),EA(I),EB1(I),EB2(I),EC(I),EC5(I) 00009320
CALL HEADIN
PRINT 9090
9090 FORMAT (//8X,29H(C)EW (C)EV (C)EP //)
DO 2030 I = 1,NX
2030 PRINT 9070,I,EX(I),EV(I),EP(I)
C 1Q = 3,4,5
2050 IF(INPUT(3).NE.2.AND.IC2.EQ.0.OR.INPUT(3).EQ.0) GO TO 2075
CALL HEADIN
PRINT 9100
9100 FORMAT (//20X,23HIO = 3 IN-PLANE MODES // 20X,18SECOND DERIV
1ATIVES //)
DO 2055 I = 1,NX
2055 PRINT 9070,I,(VPP(I,J),J=1,NY)
PRINT 9110
9110 FORMAT (//20X,28H(C) FIRST DERIV (NORMALIZED) //)
DO 2060 I = 1,NX
2060 PRINT 9070,I,(YP(I,J),J=1,NY)
CALL HEADIN
PRINT 9120
9120 FORMAT (//20X,15HIC MCDE SHAPES//)
DO 2065 I = 1,NX
2065 PRINT 9070,I,(Y(I,J),J=1,NY)
2075 IF(INPUT(4).NE.2.AND.IC2.EQ.0.OR.INPUT(4).EQ.0) GO TO 2100
CALL HEADIN
PRINT 9130
9130 FORMAT (//20X,27HIO = 4 OJT-OF-PLANE MODES// 20X,18SECOND DERIV
1ATIVES //)
DO 2080 I = 1,NX
2080 PRINT 9070,I,(ZPP(I,J),J=1,NZ)
PRINT 9110
9110 FORMAT (//20X,28H(C) SECOND DERIV
2 ATIVES //)
DO 2085 I = 1,NX
2085 PRINT 9070,I,(ZP(I,J),J=1,NZ)
CALL HEADIN
PRINT 9120
9120 FORMAT (//20X,15HIC SECOND DERIV
2 ATIVES //)
DO 2090 I = 1,NX
2090 PRINT 9070,I,(Z(I,J),J=1,NZ)
2100 IF(INPUT(5).NE.2.AND.IC2.EQ.0.OR.INPUT(5).EQ.0) GO TO 2150
CALL HEADIN
PRINT 9140
9140 FORMAT (//20X,22HIO = 5 TCRSICN MODES // 20X,18SECOND DERIVATIVES //)
DO 2105 I = 1,NX
2105 PRINT 9070,I,(PPP(I,J),J=1,NP)
PRINT 9110
9110 FORMAT (//20X,28H(C) DEFINITE INTEGRALS //)
DO 2110 I = 1,NX
2110 PRINT 9070,I,(PP(I,J),J=1,NP)
CALL HEADIN
PRINT 9120
2150 IF(INC3 .EQ.0) GO TO 2500
CALL HEADIN
PRINT 9150
9150 FORMAT (/20X,20H*** DYYI (I,J,N) *** //) 00009880
PRINT 9160,(I,I=1,10) 00009880
9160 FORMAT (1X,3HI J=1T17,9112 /)
DO 2160 I = 1,NY
DO 2160 J = 1,NY
2160 PRINT 9170,I,J,{DYYI(I,J,N),N=1,10)
9170 FORMAT (1X,I,12,P10E12.3)
PRINT 9180
9180 FORMAT(/20X,**DYYII (I,J,N) *** //)
DO 2160 I = 1,NY
DO 2160 J = 1,NY
IF{INPUT(4)=.EQ.0.) GO TO 2176
PRINT 9190
9190 FORMAT (/20X,**DYZII (I,J,N) *** //)
DO 2170 I = 1,NY
DO 2170 J = 1,NZ
2175 PRINT 9170,I,J,{DYZII(I,J,N),N=1,1)
CALL HEADIN
2176 IF{INPUT(5).EQ.0.) GO TO 2182
PRINT 9200
9200 FORMAT (/20X,**DYII (I,J,N) *** //)
DO 2180 I = 1,NY
DO 2180 J = 1,NP
2185 PRINT 9170,I,J,{DYII(I,J,N),N=1,3)
PRINT 9210
9210 FORMAT (/20X,**DYSII (I,J,N) *** //)
DO 2185 I = 1,NY
DO 2185 J = 1,NMAX
2185 PRINT 9170,I,J,{DYSII(I,J,N),N=1,4)
PRINT 9220
9220 FORMAT (/20X,**DWMII (I,N) *** //)
DO 2190 I = 1,NY
2150 PRINT 9070,I,{DYMII(I,N),N=1,10)
PRINT 9230
9230 FORMAT (/20X,**DMMII (I,N) *** //)
DO 2195 I = 1,NY
2155 PRINT 9070,I,{DMMII(I,N),N=1,9)
2200 IF{INPUT(4).EQ.0.OR.(INPUT(4).EQ.1.AND.IC2.EQ.0)) GO TO 2250
CALL HEADIN
IF{INPUT(3).EQ.0.) GO TO 2211
PRINT 9240
9240 FORMAT (/20X,**DZYII (I,J,N) *** //)
DO 2205 I = 1,NZ
DO 2205 J = 1,NY
2205 PRINT 9170,I,J,{DZYII(I,J,N),N=1,10)
PRINT 9250
9250 FORMAT (/20X,**DZII (I,J,N) *** //)
PRINT 9160, (I, I=1, 9) DO 2210 I = 1, NZ DO 2210 J = 1, NY 2210 PRINT 9260 9260 FORMAT (11X, 21H*** DZII (I, J, N) *** ) DO 2215 I = 1, NZ DO 2215 J = 1, NY 2215 PRINT 9170, (I, J, DZII (I, J, N), N=1, 9, 1) CALL HEADIN IF (INPUT(5) .EQ. 0) GO TO 2226 PRINT 9270 9270 FORMAT (11X, 21H*** DZII (I, J) *** ) DO 2220 I = 1, NZ DO 2220 J = 1, NP 2220 PRINT 9180, (I, J, DZII (I, J, N), N=1, 2) PRINT 9280 9280 FORMAT (11X, 21H*** DPYII (I, J, N) *** ) DO 2230 I = 1, NZ DO 2230 J = 1, NY 2230 PRINT 9070, (I, J, DZI (I, N), N=1, 10) PRINT 9300 9300 FORMAT (11X, 21H*** DPYII (I, N) *** ) DO 2235 I = 1, NZ DO 2235 J = 1, NY 2235 PRINT 9070, (I, J, DZI (I, N), N=1, 9) 2250 IF (INPUT(5).EQ.0.OR. (INPUT(5).EQ.1.AND..IC2 .EQ.0)) GO TO 2300 CALL HEADIN IF (INPUT(5).EQ.0) GO TO 2261 PRINT 9310 9310 FORMAT (11X, 21H*** DPYII (I, J, N) *** ) DO 2255 I = 1, NP DO 2255 J = 1, NY 2255 PRINT 9170, (I, J, DPYII (I, J, N), N=1, 10) PRINT 9320 9320 FORMAT (11X, 21H*** DPYII (I, J, N) *** ) DO 2260 I = 1, NP DO 2260 J = 1, NY 2260 PRINT 9170, (I, J, DPYII (I, J, N), N=1, 9) 2261 IF (INPUT(4).EQ.0) GO TO 2271 PRINT 9330 9330 FORMAT (11X, 21H*** DPYII (I, J, N) *** ) DO 2265 I = 1, NP DO 2265 J = 1, NY
2265 PRINT 9170, I, J, (CPZI(I, J, N), N=1,9)
    CALL HEADIN
7930 PRINT 9340
9340 FORMAT(/20X,21H*** DPZII (I, J, N) *** //)
    PRINT 9160, (I, I=1,8)
    DO 2270 J = 1, NP
       DO 2270 I = 1, NP
          CALL HEADIN
2270 PRINT 9170, I, J, (CPZI(I, J, N), N=1,8)
2271 PRINT 9350
9350 FORMAT(/20X,21H*** DPFI (I, J, N) *** //)
    PRINT 9160, (I, I=1,7)
    DO 2280 J = 1, NP
       DO 2280 I = 1, NP
          PRINT 9360
2280 PRINT 9170, I, J, (CPFI(I, J, N), N=1,7)
    CALL HEADIN
9370 FORMAT(/20X,20H*** DPFI (I, J, I) *** //)
    PRINT 9160, (I, I=1,1)
    DO 2285 I = 1, NP
       DO 2285 J = 1, NP
          PRINT 9380
2285 PRINT 9170, I, J, (CPFI(I, J, I), I=1,1)
    CALL HEADIN
9380 FORMAT(/20X,18H*** DPMII (I, H) *** //)
    PRINT 9160, (I, I=1,10)
    DO 2290 I = 1, NP
       DO 2290 J = 1, NP
          PRINT 9390
2290 PRINT 9070, I, (CPPI(I, I), I=1,10)
    CALL HEADIN
9390 FORMAT(/20X,19H*** DPMII (I, N) *** //)
    PRINT 9160, (I, I=1,9)
    DO 2295 I = 1, NP
       DO 2295 J = 1, NP
          PRINT 9400
2295 PRINT 9070, I, (CPPI(I, I), I=1,9)
2300 CALL HEADIN
    IF INPUT(3).EQ.0 GO TO 2310
7940 PRINT 9400
9400 FORMAT(/20X,19H*** DVF (I, J, N) *** //)
    PRINT 9160, (I, I=1,6)
    DO 2305 I = 1, NP
       DO 2305 J = 1, NMAX
          PRINT 9410
2305 PRINT 9170, I, J, (DVF(I, J, N), N=1,6)
2310 IF INPUT(4).EQ.0 GO TO 2320
7941 PRINT 9410
9410 FORMAT(/20X,19H*** DZF (I, J, N) *** //)
    PRINT 9160, (I, I=1,6)
    DO 2315 I = 1, NMAX
       DO 2315 J = 1, NMAX
          PRINT 9420
2315 PRINT 9170, I, J, (DZF(I, J, N), N=1,6)
2320 IF INPUT(5).EQ.0 GO TO 2330
    PRINT 9420
9420 FORMAT(/20X,19H*** DPFI (I, J, N) *** //)
PRINT 9160, (I, I=1,3)
DO 2325 I = 1, NP
DO 2325 J = 1, NMAX
2325 IF (GVP.EQ.0.AND.GK.EQ.0.AND.GP.EQ.0) GO TO 2500
CALL HEADIN
PRINT 9421
9421 FORMAT (120X, 21H *** CYD, DZD, DPD ***
PRINT 9160, (I, I=1,5)
DO 2335 I = NY
2335 PRINT 9070, (DYC(I, J), J=1, NY)
PRINT 9470
DO 2340 I = NZ
2340 PRINT 9070, (OZD(I, J), J=1, NZ)
PRINT 9470
DO 2345 I = NP
2345 PRINT 9070, (DPO(I, J), J=1, NP)
2500 IF (INPUT(6).NE.2.AND.IC2.EQ.0) GO TO 2525
PRINT 9430, OMEG, CMF
9430 FORMAT (120X, 22H *** RICCR ***
1 , 6.2, 12H ** RAD/SEC ***
C COEFFICIENT MATRICES
2525 IF (IC4.EQ.0) GO TO 2600
CALL HEADIN
PRINT 9450
9450 FORMAT (120X, 21H *** CCIR, COIR, COR, FR, BF ***
DO 2530 I = 1, NM
2530 PRINT 9460, (COIR(I, J), J=1, NM)
9460 FORMAT (3X, 1P11E11.3)
PRINT 9470
9470 FORMAT (//)
DO 2540 I = 1, NM
2540 PRINT 9460, (CODR(I, J), J=1, NM)
PRINT 9470
DO 2550 I = 1, NM
2550 PRINT 9460, (COR(I, J), J=1, NM)
PRINT 9470
PRINT 9460, (FR(I), I=1, NP)
PRINT 9470
PRINT 9460, (BF(I), I=1, NM)
CALL HEADIN
PRINT 9480
9480 FORMAT (120X, 24H *** RICC = INV(COIR) ***
DO 2560 I = 1, NM
2560 PRINT 9460, (RICR(I, J), J=1, NP)
IF (INPUT(7).EQ.0.AND.INPUT(8).EQ.0.AND.INPUT(9).EQ.0) GO TO 2600
PRINT 9500
9500 FORMAT (120X, 20H *** BIRIIH, BIRID ***
DO 2565 I = 1, 3
2565 PRINT 9460, (BIRIIH(I, J), J=1, 3)
PRINT 9470
DO 2570 I = 1, 3
2570 PRINT 9460, (BIRID(I, J), J=1, NM)
CALL HEADIN
PRINT 9510
113
9510 FORMAT(//20X,3TH*** BIRIC, BIRIC, BIRI, TM, HG, HK, HF *** //) 00012080
DO 2575 I = 1, 3 00012090
2575 PRINT 9460, (BIRI(I, J), J = 1, NM) 00012100
PRINT 9470 00012110
DO 2580 I = 1, 3 00012120
2580 PRINT 9460, (BIRIC(I, J), J = 1, 3) 00012130
PRINT 9470 00012140
DO 2585 I = 1, 3 00012150
2585 PRINT 9460, (BIRI(I, J), J = 1, NM) 00012160
PRINT 9470 00012170
PRINT 9460, (TM(I, I), I = 1, 3) 00012180
PRINT 9470 00012190
PRINT 9460, (HC(I, I), I = 1, 3) 00012200
PRINT 9470 00012210
PRINT 9460, (HK(I, I), I = 1, 3) 00012220
PRINT 9470 00012230
PRINT 9460, HF 00012240
2600 IF (INPUT(7) .NE. 0) PRINT 9600, FMX, HCX, HKX, HF(1) 00012250
9600 FORMAT(//20X,19H10 = 7, HUB DATA 10X, 16HMX, HCX, HKX, HF = 00012260
1 4F10.3) 00012270
IF (INPUT(8) .NE. 0) PRINT 9601, HMY, HCY, HKY, HF(2) 00012280
9601 FORMAT(//20X,19H10 = 8, HUB DATA 10X, 16HMY, HCY, HKY, HF = 00012290
1 4F10.3) 00012300
IF (INPUT(9) .NE. 0) PRINT 9602, FMZ, HCZ, HKZ, HE(3) 00012310
9602 FORMAT(//20X,19H10 = 9, HUB DATA 10X, 16HMZ, HCZ, HKZ, HF = 00012320
1 4F10.3) 00012330
3900 IF (INPUT(13) .NE. 0) PRINT 9740, X(NF), AFY, AFZ, AFP 00012340
9740 FORMAT(//20X,19H10 = 13, STA, FY, Fz, FP = 4F10.3) 00012350
IF (INPUT(13) .NE. 0.AND. PER .NE. 0) PRINT 9743, PER 00012360
IF (INPUT(13) .NE. 0.AND. PER .NE. 0) NBF = 0 00012370
IF (INPUT(13) .NE. 0.AND. HB .GT. 1.AND. NBF .EQ. 0) PRINT 9741 00012380
IF (INPUT(13) .NE. 0.AND. HB .GT. 1.AND. NBF .NE. 0) PRINT 9742, NBF 00012390
9741 FORMAT(//3OX,7HALL BLADES ) 00012400
9742 FORMAT(//3OX,9HBLADE NO. 13) 00012410
9743 FORMAT(//20X,16H-COS FORCE FCR, F5.3, 24H OF ROTOR CYCLE (FROM 0)) 00012420
IF (INPUT(17) .NE. 2.AND. IC2 .EQ. 0) GO TO 4000 00012430
PRINT 9750, NLIH 00012440
9750 FORMAT(//20X,16H10 = 17, NLIH = , I3) 00012450
IF (NLIN .EQ. 0) PRINT 9760 00012460
IF (NLIN .EQ. 2) PRINT 9770 00012470
IF (NLIN .EQ. 1) PRINT 9755 00012480
9755 FORMAT(20X,27H*** 1-P MCA-LINEARITIES *** ) 00012490
9760 FORMAT(20X,27H*** ALL MCA-LINEARITIES *** ) 00012500
9770 FORMAT(20X,25H*** NO CCIRILIS TERMS *** ) 00012510
IF (NFLOQ .NE. 0) PRINT 9780 00012520
9780 FORMAT(20X,43H*** AUTOMATIC FLOQUET TRANSITION MATRIX *** ) 00012530
IF (NFLOQ .EQ. 2) PRINT 9785 00012540
9785 FORMAT(20X,55H*** STEACY FORCES DUE TO STRUCTURAL EFFECTS IGNORED *** ) 00012550
Normalization
C
4000 IF (INPUT(18) .NE. 2.AND. IC2 .EQ. 0) GO TO 5000 00012560
PRINT 9800, CYCLES, HINIT, ERROR, IYE, IYI, IYG, BERR 00012570
9800 FORMAT(//3X,20HIC = 18 CYCLES = F5.1, 4X, THHINIT = F5.1, 00012580
1 4X, TERROR = F6.3, 4X, 5HITYE = I4, 4X, 5HCIC = F5.2, 4X, 6HYIC = I4, 00012590
2 4X, 6HBBER = F6.2) 00012600
5000 RETURN
END 00012630
00012640
SUBROUTINE INT(A,B,A0,X,NX,ICNT)

C A(X) = INTEGRAL OF B(X) WITH BC = A0 AT X(1)
C X IS INDEPENDANT VARIABLE
C NX IS NUMBER OF STATIONS
C ICNT = 1 INTEGRAL FROM 0 TO X
C 2 INTEGRAL FROM X TO R (LAST X)
C
C TRAPEZOIDAL INTEGRATION

REAL A(1),B(1),X(1)
A(1)=AO
DO 10 I=2,NX
10 A(I)=A(I-1)+(B(I-1)+B(I))*(X(I)-X(I-1))/2
IF (ICNT.EQ.1) RETURN
C=A(NX)
DO 20 I=1,NX
20 A(I)=C-A(I)
RETURN
END
SUBROUTINE INVPS (B,N,A,D, IROW, ICOL, NRW, NCL)
C
A = INVERSE OF B UNDISTURBED
C
VARIABLE DIMENSIONS NCL MUST BE AT LEAST ONE GREATER THAN N
C
NRW MUST BE AT LEAST EQUAL TO N
C

REAL A(NRW,NCL),E(NRW,NCL),D(NRW,NCL)
INTEGER IROW(NCL),ICOL(NCL)

DO 1 I=1,N
DO 1 J=1,N
1 A(I,J)=B(I,J)
M=N+1
DO 7 I=1,N
IROW(I)=I
7 ICOL(I)=I
DO 20 K=1,N
AMAX= A(K,K)
DO 10 I=K,N
DO 10 J=K,N
IF (ABS(A(I,J)) GREATER THAN AMAX)
CONTINUE
KI=ICOL(K)
ICOL(K)=ICOL(K)(I)
ICOL(K)=ICOL(I)(K)
IROW(K)=IROW(K)(J)
IROW(J)=IROW(J)(I)
IF (AMAX) 11,12,11
11 PRINT 13
12 FORMAT(* SOLUTION OF MATRIX NOT POSSIBLE*)
13 PRINT 13
GOTO 100
14 DO 14 J=1,N
E=A(K,J)
A(K,J)=A(K,J)(K)
14 A(I,J)=E
DO 15 I=1,N
E=A(I,K)
A(I,K)=A(I,J)(K)
15 A(I,J)=E
DO 16 I=1,N
IF(I-K) 18,17,18
17 A(I,M)=1.
GO TO 16
18 A(I,M)=0.
16 CONTINUE
PVT=A(K,K)
DO 8 J=1,M
8 A(K,J)=A(K,J)/PVT
DO 19 I=1,N
IF(I-K) 21,19,21
19 M=M+1
20 CONTINUE
21 STOP
END
21 AMULT=A(I,K)
DO 22 J=1,N
22 A(I,J)=A(I,J)-AMULT*A(K,J)
19 CONTINUE
DO 20 I=1,N
20 A(I,K)=A(I,K)
DO 25 I=1,N
DO 24 L=1,N
IF(IRCW(I-J-L)<=24,23,24
24 CONTINUE
0000530
23 DO 25 J=1,N
25 D(L,J)=A(I,J)
DO 26 J=1,N
DO 28 L=1,N
IF(ICOL(J)-L<=28,29,28
28 CONTINUE
0000620
29 DO 26 I=1,N
26 A(I,L)=D(I,J)
100 RETURN
END
0000720
SUBROUTINE MXV(A,B,C,M,N,NDIM,ICONT)
C MATRIX TIMES VECTOR  A(M) = B(M,N) * C(N) + A(M)  FOR ICONT = 0
C DIMENSION A(I),B(NDIM,J),C(I)
10 IF (ICONT.EQ.0) A(I)=0
DO 10 J=1,N
10 A(I)=A(I)+B(I,J)*C(J)
RETURN
END
SUBROUTINE OUTP(T, Y, DRY, IHLF, WDIM, PRMT, LY)
REAL M, KM, KP2, KA
LOGICAL LY(1)
REAL DATA (61, DATAT(3))
DIMENSION YV(1), DATAT(3)
LOGICAL JY(1, 1)
COMMON/INDAT/X(20), M(20), E(20), SEA(20), KM1(20), KM2(20), KA(20),
THP(20), YEP(20), GJ(20), EA(20), EB1(20), EB2(20), ECS(20), EIP(20),
NP, BP, YPP(20, 3), ZPP(20, 5), ZP(20, 5), PPP(20, 3), PPP(20, 3),
2 VV, Gw, GP, HE(3), PER
COMMON/HED/IC1, IC2, IC3, IC4, HEAC(19), IPAGE, INPUT(20), IEND, LINE, IC5
IF (NF LOQ. NE. 0) RETURN
CYCF = T*OMF / 628319
CYCR = T*OMF / 628319
NCYCR = CYCR
NCYCR = CYCR
CYCR = CYCR
NCYCR = CYCR
NCYCR = CYCR
DEGF = (CYCF - FLOAT(NCYCR)) * 360.
DEGR = (CYCR - FLOAT(NCYCR)) * 360.
LINE = LINE + NMAX * NE + 1
IF (NF4 GT. 1) LINE = LINE + NB
IF (NF4 GT. 1) LINE = LINE + NB
IF (NF4 GT. 1) LINE = LINE + NB
IF (NF4 GT. 1) LINE = LINE + NB
IF (NF4 GT. 1) LINE = LINE + NB
IF (NF4 GT. 1) LINE = LINE + NB
CALL HEADIN
PRINT 1000
1000 FORMAT ( /119 TIME, OMF, OMEGA, I, Y(I) DOT, YI)
DATA (5) = YV(II)
DATA (6) = YV(II+1)
DATA(3)=DATA(3)+DATA(6)
93 IF(I1.GT.1) GC TO 95
  IF(NB.EQ.1) GO TO 94
  IF(IB.EQ.1) PRINT 1004, T,NCYCF,DEGF,NCYCR,DEGR
  IF(IB.GT.1) PRINT 1005, IB
GO TO 95
1004 FORMAT (/1X,F6.3,2(I4,F6.1),10H #ELADE 1*)
1005 FORMAT (27X,7H #ELADE,I2,IH*)
94 PRINT 1010, T,NCYCF,DEGF,NCYCR,DEGR, I, DATA
1010 FORMAT (/1X,F6.3,2(I4,F6.1),13,3(1PE12.3,E13.3,8X))
GO TO 100
95 PRINT 1020, I, DATA
1020 FORMAT (20X,I10,3(1PE12.3,E13.3,8X))
100 CONTINUE
  IF(NMAX.GT.1) PRINT 1021, DATA
  IF(IC5.NE.0 .AND. IB.EQ.1) WRITE(9) CYCRP, DATA, YV(2), YV(4), YV(6)
110 CONTINUE
  IF (LY(1),CR.LY(3),CR.LY(5)) PRINT 1025, (YV(L),L=1,6)
1025 FORMAT (/4X,26XHUB XDOT,X, YDOT,Y,ZDOT,Z,3(1PE12.3,E13.3,8X))
  IF (PRMT(6).EQ.0) GO TO 200
  IF (ABS(YV(1Y)) .LT. PRMT(6)) GO TO 200
PRINT 1030
1030 FORMAT (/24H *** LIMIT EXCEEDED *** //)
PRMT(5)=1
200 RETURN
END
SUBROUTINE SCL(IPRT, YVAR, DERY, ILHF, LY)
INTEGER IRON(31), ICOL(31)
LOGICAL LY(1)
REAL PRMT(1), YVAR(1), DERY(1)
REAL AUX(8, 98), BFTEM(11), ERW(36), FLTM(30, 31), FLTMI(30, 31),
 WORK(30, 31)
REAL HFTEMP(3), FRTEMP(11)
COMMON/INDAT/X(201), H(201), E(201), EA(201), KM(201), KA(201),
 THP(BP), YPP(20, 3), ZPP(20, 5), Z(PE20, 3), PP(20, 3), PP(20, 3),
 OMEG, OM, E(20), NY, NZ, NP, NPY, IC4EMGS, OMFS, IDIM, NMAX, NLIN
COMMON/HED/ICL, IC2, IC3, IC4, HE(A19), IPAGE, INPUT(20), IFEND, LINE, IC5,
 COMMON/DIM/NINPUT, NSTAT, NMODE, NMODE, NMODE, NM1, NDIM, NBLADE
 COMMON/DER/TH(20), EV(20), EW(20), EP(20), Y(20, 3), Z(20, 5), P(20, 3)
EQUIVALENCE (AUX(1), WORK(11))
IF(NFOEQ.EQ.0) CALL RKGVS(IPRT, YVAR, DERY, IDIM, ILHF, AUX, LY)
IF(NFLOG.EQ.0) RETURN
NVAR = 0
DO 1 I = 1, IDIM
  IF(LY(I)) NVAR = NVAR + 1
10 ERY(I) = DERY(I)
  IF(NVAR.GT.30) CALL ERR (5010, 0)
  DO 20 I = 1, 11
    BFTEM(I) = BF(I)
    FRTEMP(I) = FR(I)
  FR(I) = 0.
20 BF(I) = 0.
  DO 25 I = 1, 3
    BFTEM(I) = BF(I)
25 BF(I) = 0.
  PRMT2 = PRMT(2)
  PRMT(2) = PRMT2/CYCLES
  CALL HEADIN
  PRINT 1000, PRMT(2)
1000 FORMAT (/30X, 3H FLOQUET TRANSITION MATRIX PERIOD(SEC) =)
I = 0
II = 0
  NC REPIETION OF SCLUTIONS FCR MULTIPLE BLADES
1000 DO 100 I = 1, NC
    IF(.NOT. ILY(I)) GO TO 100
    II = II + 1
100 CONTINUE
IF(I.GT.IE8) GO TO 101
DO 30 J=1,IDIM
DERY(J)=ERY(J)
30 YVAR(J)=0
YVAR(I)=1
CALL RKGSV(PRMT,YVAR,DERY,IDIM,IHLF,AUX,LY)
IF(IHLF.EQ.11) CALL ERR (5030,0)
IF(IHLF.EQ.12) CALL ERR (5031,0)
J=0
DO 30 J=1,IDIM
IF(.NOT.YVAR(J)) GC TO 50
JJ=J+1
YVAR(JJ)=0
DO 50 I=1,1,IHLF
PRINT 1010,YVAR(I)
50 CONTINUE
PRINT 1011,YVAR(1),YVAR(I)
1010 FORMAT(1X,13,1P10E12.3/(14X,10E12.3))
100 CONTINUE
GO TO 109
101 ID1 = II-NM2
ID11 = ID1-1
ICD1 = II
DO 108 J=2,NB
DO 108 J=2,NB
JJ = ID11+(J-1)*NM2+J
JREF = ID11+J
IF(ID11.EQ.0) GO TO 103
DO 102 I=1,ID11
PRINT 1010,YVAR(I)
102 FLTM(I,J) = FLTM(I,J-NM2)
103 DO 107 IB = 1,NB
IF(IB.EQ.JB) IREF = ID11
DO 107 I=1,NM2
II = ID11+(IB-1)*NM2+I
107 FLTM(I,J) = FLTM(IREF+I,JREF)
PRINT 1010,YVAR(I)
CONTINUE
108 CONTINUE
109 CONTINUE
DO 110 J=1,IDIM
DERY(J)=ERY(J)
110 YVAR(J)=0
DO 120 J=1,II
IF(NFLDG.EQ.2) GC TO 120
FR(J)=FRTEMP(J)
120 BF(J)=BFTEMP(J)
DO 125 I=1,3
125 BF(I)=BFTEMP(I)
IF(INPUT(13).NE.0) GO TO 115
IF(LY(1)) ANCY=ANCTEMP(1,AE,0) GO TO 115
IF(LY(3)) ANCY=ANCTEMP(2,AE,0) GO TO 115
IF(LY(5)) ANCY=ANCTEMP(3,AE,0) GO TO 115
RETURN
115 CALL RKGSV(PRMT,YVAR,DERY,IDIM,IHLF,AUX,LY)
130 FLTM(I,I)=FLTM(I,I)-1.
CALL INVRS(FLTM,ANVAR,FLTML,WRK,IRN,ICOL,30,31)
II=0
DO 140 I=1,IDIM
IF(.NOT.LY(I)) GC TO 140
II=II+1
YVAR(I)=YVAR(I)
140 CONTINUE
PRINT 1020,YVAR(I),I=1,NVAR
1020 FORMAT (/'30X,19HPARTICULAR SOLUTION /(4X,1P10E12.3))
CALL MXV(DERF,FLTM,YVAR,ANVAR,NVAR,30,0)
II=0
DO 150 I=1,IDIM
YVAR(I)=0.
IF(.NOT.LY(I)) GC TO 150
II=II+1
YVAR(I)=DERY(I)
150 CONTINUE
DO 160 I=1,IDIM
160 DERY(I)=ERW(I)
PRMT(2)=PRMT2
NFLT=NFLQ=0
CALL RKGVS(PRMT,YVAR,DERF,IDIM,IHLF,AUX,LY)
NFOQ=NFLT
IF(NFLOQ,NE,2) RETURN
DO 170 I=1,II
170 BF(I)=BFTEMP(I)
RETURN
END
SUBROUTINE RKGSV(PRMT,Y,DERY,ACIM, IHLF,AUX,LY) 00000010
C
SUBROUTINE RKGSV
C MODIFIED TO INCLUDE OPTIONAL COMPUTATION OF EACH Y(I)
C FCT, OUTP REMOVED FROM ARG LIST, THUS NO EXTERNAL SMT REQ
C PURPOSE
C TO SOLVE A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL
C EQUATIONS WITH GIVEN INITIAL VALUES.
C
USAGE
C CALL RKGSV (PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX,LY) 000000110
C PARAMETERS FCT and OUTP REQUIRE AN EXTERNAL STATEMENT.
C
DESCRIPTION OF PARAMETERS
PRMT - AN INPUT AND OUTPUT VECTOR WITH DIMENSION GREATER
C OR EQUAL TO 5, WHICH SPECIFIES THE PARAMETERS OF
C THE INTERVAL ARC OF ACCURACY AND WHICH SERVES FOR
C COMMUNICATION BETWEEN OUTPUT SUBROUTINE (FUNISHED)
C BY THE USER) AND SUBROUTINE RKGS, EXCEPT PRMT(5)
C THE COMPONENTS ARE NOT DESTROYED BY SUBROUTINE
C RKGS AND THEY ARE
C PRMT(1) - LOWER BOUND OF THE INTERVAL (INPUT),
C PRMT(2) - UPPER BOUND OF THE INTERVAL (INPUT),
C PRMT(3) - INITIAL INCREMENT OF THE INDEPENDENT VARIABLE
C (INPUT),
C PRMT(4) - UPPER ERROR BOUND (INPUT). IF ABSOLUTE ERROR IS
C GREATER THAN PRMT(4), INCREMENT GETS HALVED.
C IF INCREMENT IS LESS THAN PRMT(3) AND ABSOLUTE
C ERROR LESS THAN PRMT(4)/50, INCREMENT GETS DOUBLED.
C THE USER MAY CHANGE PRMT(4) BY MEANS OF HIS
C OUTPUT SUBROUTINE.
C PRMT(5) - ANY INPUT PARAMETER. SUBROUTINE RKGS INITIALIZES
C PRMT(5)=0. IF THE USER WANTS TO TERMINATE
C SUBROUTINE RKGS AT ANY OUTPUT POINT, HE HAS TO
C CHANGE PRMT(5) TO NON-ZERO BY MEANS OF SUBROUTINE
C OUTP. FURTHER COMPONENTS OF VECTOR PRMT ARE
C FEASIBLE IF ITS DIMENSION IS DEFINED GREATER
C THAN 5. HOWEVER SUBROUTINE RKGS DOES NOT REQUIRE
C AND CHANGE THEM, NEVERTHELESS THEY MAY BE USEFUL
C FOR HANDLING RESULT VALUES TO THE MAIN PROGRAM
C (CALLING RKGS) WHICH ARE OBTAINED BY SPECIAL
C MANIPULATIONS WITH OUTPUT DATA IN SUBROUTINE OUTP.
C
Y - INPUT VECTOR OF INITIAL VALUES, (DESTROYED)
C LATERON Y IS THE RESULTING VECTOR OF DEPENDENT
C VARIABLES COMPUTED AT INTERMEDIATE POINTS X.
C
DERY - INPUT VECTOR OF ERROR WEIGHTS, (DESTROYED)
C THE SUM OF ITS COMPONENTS MUST BE EQUAL TO 1.
C LATERON CERY IS THE VECTOR OF DERIVATIVES, WHICH
C BELONG TO FUNCTION VALUES Y AT A POINT X.
C
NDIM - AN INPUT VALUE, WHICH SPECIFIES THE NUMBER OF
C EQUATIONS IN THE SYSTEM.
C
IHLF - AN OUTPUT VALUE, WHICH SPECIFIES THE NUMBER OF
C
BISECTIONS OF THE INITIAL INCREMENT. IF IHLF GETS
GREATER THAN 10, SUBROUTINE RKGS RETURNS WITH
ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM. ERROR
MESSAGE IHLF=12 OR IHLF=13 APPEARS IN CASE
PRMT(3)=0 OR IN CASE SIGN(PRMT(3))#SIGN(PRMT(2)-0.0000570
PRMT(1)) RESPECTIVELY.

FCT - THE NAME OF AN EXTERNAL SUBROUTINE USED. THIS
SUBROUTINE COMPUTES THE RIGHT HAND SIDES DER(Y)
THE SYSTEM TO GIVEN VALUES X AND Y. ITS PARAMETER
LIST MUST BE X,Y,DER(Y),LY SUBROUTINE FCT SHOULD
NOT DESTROY X AND Y.

OUTP - THE NAME OF AN EXTERNAL OUTPUT SUBROUTINE USED.
ITS PARAMETER LIST MUST BE X,Y,DER(Y),IHLF,NDIM,PRMT
LY NONE OF THESE PARAMETERS (EXCEPT, IF NECESSARY,
PRMT(4),PRMT(5),...,SHOULD BE CHANGED BY
SUBROUTINE OUTP. IF PRMT(5) IS CHANGED TO NON-ZERO,
SUBROUTINE RKGS IS TERMINATED.

AUX - AN AUXILIARY STORAGE ARRAY WITH 8 ROWS AND NDIM
COLUMNS.

LY LOGICAL ARRAY, IF TRUE, CORRESPONDING Y(I)
IS CALCULATED

REMARKS
THE PROCEDURE TERMINATES AND RETURNS TO CALLING PROGRAM, IF
(1) MORE THAN 10 BISECTIONS OF THE INITIAL INCREMENT ARE
NECESSARY TO GET SATISFACTORY ACCURACY (ERROR MESSAGE
IHLF=11),
(2) INITIAL INCREMENT IS EQUAL TO 0 OR HAS WRONG SIGN
(ERROR MESSAGES IHLF=12 OR IHLF=13),
(3) THE WHOLE INTEGRATION INTERVAL IS WORKED THROUGH,
(4) SUBROUTINE OUTP HAS CHANGED PRMT(5) TO NON-ZERO.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
THE EXTERNAL SUBROUTINES FCT(X,Y,DER(Y)) AND
OUTP(X,Y,DER(Y),IHLF,NDIM,PRMT) MUST BE FURNISHED BY THE USER.

METHOD
EVALUATION IS DONE BY MEANS OF FOURTH ORDER RUNGE-KUTTA
FORMULAE IN THE MODIFICATION DUE TO GILL. ACCURACY IS
TESTED COMPARING THE RESULTS OF THE PROCEDURE WITH SINGLE
AND DOUBLE INCREMENT.
SUBROUTINE RKGS AUTOMATICALLY ADJUSTS THE INCREMENT DURING
THE WHOLE COMPUTATION BY HALVING OR DOUBLING. IF MORE THAN
10 BISECTIONS OF THE INCREMENT ARE NECESSARY TO GET
SATISFACTORY ACCURACY, THE SUBROUTINE RETURNS WITH
ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM.
TO GET FULL FLEXIBILITY IN OUTPUT, AN OUTPUT SUBROUTINE
MUST BE FURNISHED BY THE USER.
FOR REFERENCE, SEE
RALSTON/WILF, MATHEMATICAL METHODS FOR DIGITAL COMPUTERS,
WILEY, NEW YORK/LONDON, 1960, PP 110-120.
SUBROUTINE RKGSV(PRMT, Y, DERY, ACIP, IHLF, FCT, OUTP, AUX, LY)

DIMENSION Y(1), DERY(1), AUX(8, 1), A(4), B(4), C(4), PRMT(1)
LOGICAL LY(1)
DO 100 I = 1, NCIM
100 AUX(8, I) = 0.6666667*DERY(1)

X = PRMT(1)
XEND = PRMT(2)
H = PRMT(3)
PRMT(5) = 0.
CALL FCT(X, Y, DERY, LY, ACIM)

ERROR TEST
IF (H*(XEND-X) > 470, 460, 110)

PREPARATIONS FOR RUNGE-KUTTA METHOD

A(1) = 5
A(2) = 2.928932
A(3) = 1.707107
A(4) = 1.666667
B(1) = 2.
B(2) = 1.
B(3) = 1.
B(4) = 2.
C(1) = 5
C(2) = 2.928932
C(3) = 1.707107
C(4) = 5

PREPARATIONS OF FIRST RUNGE-KUTTA STEP
DO 120 I = 1, NCIM
IF (.NOT. LY(I)) GO TO 120
AUX(I, 1) = Y(I)
AUX(2, I) = DERY(I)
AUX(3, I) = 0.
AUX(6, I) = 0.
120 CONTINUE
IREC = 0
H = H + H
IHLF = -1
ISTEP = 0
IEND = 0

START OF A RUNGE-KUTTA STEP
130 IF ((XH - XEND) * H > 160, 150, 140
140 H = XEND - X
150 IEND = 1

RECORDING OF INITIAL VALUES CF THIS STEP
160 CALL CUTF(X, Y, DERY, IREC, NDIM, PRMT, LY)
IF (PRMT(5) > 490, 480, 470)
170 ITEST = 0
180 ISTEP = ISTEP + 1

126
C START OF INNERMOST RUNGE-KUTTA LOOP
J=1

190 AJ=A(J)
BJ=B(J)
CJ=C(J)
DO 200 I=1,NDIM
IF(,NOT._LY(I)) GC TO 200
RI=H*DERY(I)
R2=AJ*(1-BJ*AUX(6,I))
Y(I)=Y(I)+R2
R2=R2*R2+R2
AUX(6,I)=AUX(6,I)+R2-CJ*R1
200 CONTINUE
210 J=J+1
220 X=X+H
230 CALL FCT(X,Y,DERY,LY,NCIM)
240 CONTINUE
GO TO 190
C END OF INNERMOST RUNGE-KUTTA LOOP

C TEST OF ACCURACY
IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
250 DO 260 I=1,NDIM
IF(LY(I))AUX(4,I) = Y(I)
260 CONTINUE
ISTEP=ISTEP+ISTEP-2
270 IHLF=IHLF+1
X=X-H
H=H/2
DO 280 I=1,NDIM
IF(,NOT._LY(I)) GC TO 280
Y(I)=AUX(I,I)
DERY(I)=AUX(2,I)
AUX(6,I)=AUX(3,I)
280 CONTINUE
GO TO 180
C IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
290 IMOD=ISTEP/2
300 CALL FCT(X,Y,DERY,LY,ADIP)
310 CONTINUE
GO TO 180
C COMPUTATION OF TEST VALUE DELT
320 DELT=0
DO 330 I=1,NCIM
IF(,NOT._LY(I)) GC TO 330
330 CONTINUE
DELT = DELT + AUX(I, J) * ABS(AUX(I, J) - Y(I))

330 CONTINUE
   IF (DELT > PRMT(4)) 370, 370, 340
C ERROR IS TOO GREAT
C
340 IF (IHLF > L0) 350, 450, 450
350 DO 360 I = 1, NDI
   IF (LY(I)) AUX(I, J) = AUX(5, I)
360 CONTINUE
   ISTEP = ISTEP + ISTEP * 0
X = X - H
   IF (IHLP = 1) 370, 370, 370
   CONTINUE
STEP = STEP * 2
X = X - H
GO TO 270
C
RESULT VALUES ARE GOOD
370 CALL FC(5, X, Y, DERY, LY, NDI)
   DO 380 I = 1, NDI
   IF (NOT LY(I)) GC TO 380
   AUX(I, J) = Y(I)
   AUX(I, J-1) = DERY(I)
   AUX(I, J+1) = AUX(I, J+1)
   Y(I) = AUX(I, J)
   DERY(I) = AUX(I, J)
380 CONTINUE
   CALL OUTP(X, Y, DERY, IHLP, NDI, PRMT, LY)
   IF (PRMT(5)) 490, 390, 490
390 DO 400 I = 1, NDI
   IF (LY(I)) GC TO 400
   Y(I) = AUX(I, J)
   DERY(I) = AUX(I, J+1)
400 CONTINUE
   IREC = IHLP
   IF (IREC = 4, 10, 410, 480)
C INCREMENT GETS DOUBLED
410 IHLP = IHLP - 1
   ISTEP = ISTEP / 2
   H = H / 2
   IF (IHLP = 130) 420, 420, 420
420 I MOD = ISTEP / 2
   IF (STEP = I MOD - IMOD) 130, 430, 130
430 IF (DELT > 0.02 * PRMT(4)) 440, 440, 440
440 IHLP = IHLP - 1
   ISTEP = ISTEP / 2
   H = H / 2
   GO TO 130
C
RETURNS TO CALLING PROGRAM
450 IHLP = 11
   CALL FC(5, X, Y, DERY, LY, NDI)
   GO TO 480
460 IHLP = 12
   GO TO 480
470 IHLP = 13
   CALL OUTP(X, Y, DERY, IHLP, NDI, PRMT, LY)
480 RETURN

END

128
SUBROUTINE SLMODE(A,Q,PHI,STA,AX,N)
C MODE SUMMATION NSTA=DXESIS NX=NO OF STATIONS N=NO OF MJD
C
C A(I) = SUM(Q(J)*PHI(I,J))
C
REAL A(1),Q(1),PHI(NSTA,1)
DO 10 I=1,NX
A(I)=0.
DO 10 J=1,N
10 A(I)=A(I)+Q(J)*PHI(I,J)
RETURN
END
(1) HEADING

1 IGL = EQ 0 FIRST OR NORMAL RUN ALL INPUT
2 REPLACE MODES INPUT 3, 4, 5
2 ADD MODES INPUT 4, 5
8 NEW DP CODE ONLY INPUT 5
9 END OF RUN LAST CARD OF RUN

2 IC2 = EQ 1 PRINTS ORTHO CHECKS
2 AND NORMALIZES MODES
2 NOTE MODES ARE REPLACED
2 AFTER INPUT AND AFTER
2 RANDOM ERRORS.

3 IC3 = NE 0 PRINTS EOS FOR MASS IDENT
4 IC4 = NE 0 RESTORES INPUT MODES IF IGL = EQ 8

5-80 ARBITRARY HEADING HEAD (19)

(2) MASS DATA ONE CARD PER BLADE STATION 20 MAX

1-10 \{I\} STATION
11 \{SEE NOTE\} WM
12-20 \{SEE NOTE\} WM
21 \{SEE NOTE\} WE
22-30 E - CG OFFSET FROM EA + WHEN CG FORWARD
31 \{SEE NOTE\} WT
32-40 TH - PITCH ANGLE RAD
41 \{SEE NOTE\} WK
42-50 KM RADIUS OF GYRATION IN TORISON

* 1ST COL OF EACH WORD CONTAINS WEIGHTING FACTOR
* FROM 1-9 (0 = 1) HIGHER VALUE INDICATES GREATER CONFIDENCE

SEE 101 = 3 WDL

END WITH BLANK CARD

(3) CONTROL CARD MODES

1-10 CALV MULTIPLIES I-P MODE DEFL (0 = 1)
11-20 CALW MULTIPLIES O-P MODE DEFL (0 = 1)
21-30 CALP MULTIPLIES TOR MODE DEFL (0 = 1)
31-40 THO ROOT PITCH ANGLE RAD
41 ADDS TO TH (TH NOT CHANGED)
(4) MODES - STATIONS CORRESPOND TO MASS DATA

EACH MODE 1-10 FREQ NATURAL, RAD/SEC
11-20 OMEG ROTATIONAL, RAD/SEC
21-30 IF NE O TEMPORARILY REPLACES CALV
31-40 IF NE 0 TEMPORARILY REPLACES CALW
41-50 IF NE 0 TEMPORARILY REPLACES CALP

NEXT CDS V I-P DISPLACEMENTS, BF10 UP TO 3 CARDS
NEXT CDS W 0-P START JV YEW CD
NEXT CDS P TOR

FOLLOW BY NEXT MODE - 8 MODES MAX AT ONE OMEG
-16 MODES MAX AT ALL OMEG
*** 30 Eqs MAX (NOT INCL INVARIANCES) ***

END WITH BLANK CARD

(5) OPERATION CODES COL 1-2 101,102

COL 1 101

1 MODIFY MODES WITH RANDOM ERRORS -- MODES REPLACED
WD1 PERCENT RANDOM + OR - RECTANGULAR DIST
WD2 PERCENT BIAS
WD3 INTEGER SEED TO START RANDOM SEQUENCE

*** FOLLOW BY NEXT OPERATION CARD (5) ***

2 SOLVE FOR MINIMUM MODAL CHANGES - MASS MATRIX UNCHANGE

ALL MODES MUST BE AT SAME OMEGA - 8 MAX
FIRST MODE UNCHANGED, LAST MODE WILL CHANGE MOST
MINIMUM SUM PERCENT CHANGES USED
WEIGHTING FACTORS NOT USED IN THIS OPTION

WD1.EQ.0 NO LIMIT ON CHANGES
WD1.EQ.1 LIMIT CHANGES - SCALE OPTION
WD2-8 MAX PCT CHANGE ALLOWED IN EACH MODE.
CHANGES ARE SCALING SO MAX CHANGE L.E. MAXIMUM
0 INDICATES NO LIMIT

WD1.EQ.2 LIMIT CHANGES - TRUNCATE OPTION
WD2-8 SAME AS FOR SCALE OPTION EXCEPT THAT ONLY
CHANGES WHICH EXCEED LIMITS ARE TRUNCATED.
OTHER CHANGES ARE NOT MODIFIED

131
3. INCOMP-MODEL-MASS CHANGES

WD1.EQ.1 WEIGHTING FACTORS ALL SET TO 1 (TEMP)
WD1.EQ.2-STATS WITH INVARIANT PARAM, READ 5(A)

THE FOLLOWING CONTROLS CAUSE THE CORRESPONDING PROPERTIES TO REMAIN INVARIANT IF .NE. 0

<table>
<thead>
<tr>
<th>COL</th>
<th>DESCRIPTION</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>TOTAL MASS M</td>
<td>M</td>
</tr>
<tr>
<td>30</td>
<td>RADIAL CG M*</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>CHORDWISE CG M*</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>FLAPPING MOM OF INERT M*K**2</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>FEATHERING MOM OF INERT M*K**2</td>
<td></td>
</tr>
</tbody>
</table>

COL 2 = I02

0 ABOVE OPERATIONS DO NOT DISTURB ORIGINAL DATA

1 ABOVE OPERATIONS REPLACE ORIGINAL DATA IN PREPARATION FOR SEQUENTIAL OPERATIONS

(5A) USED ONLY FOR INVAR STATS. SEE 3, ABOVE, WD1 = 2

COL1 = NO OF STATIONS (8 MAX)
WD1,WD2,... STATION NUMBERS, NO ZEROES

NEXT HEADING CARD

************************************************************************************************************

JJJ1  INTEGER HEAD(19),IROW(46),ICOL(46)
JJJ2  INTEGER IJEQ(40,2)
JJJ3  INTEGER NIN(0)
JJJ4  REAL X(21),WM(20),M(21),WE(20),E(21),X(20),THI(21),WK(20),XK(21),
1     OMEG(16),FREQ(16),V(16,20),W(16,20),P(16,20)
JJJ5  REAL DUM(8),V2(16,20),X2(16,20),P2(16,20),WE(20),NET(20),MK(20),
1     A(60,7),PI(60),B(60,7),C(7,8),D(7,8),WORK(7,8)
JJJ6  REAL WOR(60,7)
JJJ7  REAL GMASS(16),OCHECK(16,16)
JJJ8  REAL EQ(35,80),MA(80),WA(80)
JJJ9  REAL MK2(20),SM(5)
JJJ10 REAL VSAV(16,20),MSAV(16,20),PSAV(16,20)
JJJ11 REAL WM(80),OM(80),AH(35,36),AMH(35,36),DMH(35,36)
JJJ12 1 READ 1000,IC1,IC2,IC3,IC4,HEAD
JJJ13 1000 FORMAT(11,19A4)
JJJ14 IF (IC1.EQ.9) CALL EXIT
JJJ15 PRINT 1001,IC1,IC2,IC3,IC4,HEAD
JJJ16 1001 FORMAT(1H1,10X,100(1H#)/
1 /
2 1/19/77 10X,4I2,19A4//10X,100(1H#)/

132
133

J17 IF (C1.EQ.1) GO TO 29
J18 IF (C1.EQ.2) GO TO 29
J19 IF (C1.EQ.8.AND.C4.EQ.0) GO TO 100
J20 IF (C1.EQ.0) GO TO 9
J21 DO 5 I=1,NX
J22 DO 5 J=1,NM
J23 V(I,J)=VSAV(I,J)
J24 W(I,J)=WSAV(I,J)
J25 F(I,J)=PSAV(I,J)
J26 CONTINUE
J27 1006 FORMAT (/10X,31H*** ORIGINAL MODES RESTORED *** /)
J28 GO TO 100
J29
J30 9 NX=0
J31 DO 10 I=1,21
J32 READ 1005,X(I),IM(M(I),IE,E(I),IT,TH(I),IK,KM(I)
J33 1005 FORMAT (F10.0,4(I1,F9.0))
J34 IF (M(I).EQ.0) GO TO 20
J35 NX=NX+1
J36 WM(I)=AMAX0(I,1)
J37 WE(I)=AMAX0(I,1)
J38 WT(I)=AMAX0(I,1)
J39 -10 WK(I)=AMAX0(I,1)
J40 CALL ERR(10,0)
J41 20 PRINT 1010,(X(I),WM(I),M(I),WE(I),E(I),WT(I),TH(I),WK(I),KM(I)
J42 1 I=NX
J43 1010 FORMAT (10X,90HI STA W M W E
J44 2 4(OPF8.0,1PE12.3))
J45 25 READ 1015, CALV,CALW,CALP,THO
J46 1015 FORMAT (8F10.0)
J47 N2=2*NX
J48 N3=3*NX
J49 N4=4*NX
J50 IF (CALV.EQ.0) CALV=1
J51 IF (CALW.EQ.0) CALW=1
J52 IF (CALP.EQ.0) CALP=1
J53 PRINT 1016-THO
J54 1016 FORMAT (/10X,32HROOT PITCH ANGLE (ADDS TO TH) = 1PE12.3/ 1H1,
J55 1 10X,25HINPUT MODES (CARD IMAGES) /)
J56 NH=0
J57 PRINT 1017,CALV,CALW,CALP,THO
J58 1017 FORMAT (/10X,8F12.5)
J59 29 IF (C1.EQ.2) PRINT 1019
J60 1019 FORMAT (1H1,10X,2THADDED MODES CARD IMAGES //)
J61 30 READ 1015,F,0,CV,CW,CP
J62 PRINT 1017,F,0,CV,CW,CP
J63 IF (F.EQ.0.AND.OE.EQ.0) GO TO 70
J64 NH=NH+1
J65 FREQ(NH)=F
J66 QE(H,NH)=0
J67 READ 1015,(V(NH,I),I=1,NX)
J68 PRINT 1018,(V(NH,I),I=1,NX)
J69 1018 FORMAT (10X,8F12.5)
J70 70 READ 1015,(W(NH,I),I=1,NX)
C. APPLY CALIBRATION

0071 IF(CV.EQ.0) CV=CALV
0072 IF(CW.EQ.0) CW=CALW
0073 IF(CP.EQ.0) CP=CALP
0074 I=NM
0075 IF(CV .LE. EQ.1) GO TO 50
0076 DO 45 J=1,NX
0077 45 VI(I,J)=VI(I,J)*CV
0078 50 IF(CW .LE. EQ.1) GO TO 60
0079 DO 55 J=1,NX
0080 55 WI(I,J)=WI(I,J)*CW
0081 60 IF(CP .LE. EQ.1) GO TO 30
0082 DO 65 J=1,NX
0083 65 PI(I,J)=PI(I,J)*CP
0085 70 DO 41 I=1,NX
0086 41 VSAV(J,I)=V(J,I)
0087 DO 41 J=1,NM
0088 WSAV(J,I)=W(J,I)
0089 41 PSAV(J,I)=P(J,I)

C. PRINT-MODES

0090 PRINT 1020
0091 1020 FORMAT (1H1//50X,31HINPUT MODE SHAPES (CAL APPLIED)
0092 CALL MODES -(X V W P OMEG, FREQ, NM, NX, 16)
0093 90 AM =0
0094 AM =0
0095 AMET =0
0096 AMK =0
0097 SM(2)=0
0098 SM(3)=0
0099 SM(4)=0
0100 DO 95 I=1,NX
0101 ME(I) = M(I)*PE(I)
0102 ME(T) = ME(I)*(TH(I)+TH0)
0103 MK(I) = M(I)*KM(I)**2
0104 AM = AM+M(I)
0105 SM(2)=SM(2)+M(I)*X(I)
0106 SM(3)=SM(3)+ME(I)
0107 SM(4)=SM(4)+M(I)*X(I)**2
0108 AME = AME+ABS(ME(I))
0109 AMET = AMET+ABS(ME(T))
0110 AMK = AMK+MK(I)
0111 SM(1)=AM
0112 SM(5)=AMK
0113 AM = AM/NX
0114 AME = AME/NX
0115 AMET = AMET/NX
0116 AMK = AMK/NX
0117 IF(AM.EQ.0) CALL ERR(95,0)
0118 IF(AMK.EQ.0) CALL ERR(96,0)
119 IF (IC2.EQ.0) GO TO 1030
120 PRINT 1031
121 1031 FORMAT (1H1//30X,25HINPUT ORTHONORMALITY CHECK //
122 CALL ORTH(V,W,P,M,ME,M,FK,NM,NX,16,GMASS,OCHCK,16,IC2)
123 IF (IC2.EQ.2) PRINT 1032
124 1032 FORMAT (40X,42H*** MODES REPLACED BY NORMALIZED MODES *** //
125 IF (IC2.EQ.2) CALL PMODES (V,X,W,P,OMEG,FREQ,NM,NX,16)
126 IF (IC2.EQ.2.AND.101.NE.1) IC2=1
C READ PROGRAM OPTIONS
127 1035 FORMAT (2I1,F6.0,7F10.0)
128 GO TO (110,200,500,130),101
C FOR IOI=1
C WDI=UNIVERSALY-DISTRIBUTED-RANDOM-ERROR-HAVING-A
C +/- MAXIMUM OF PCT ON AMPLITUDE
C W02=BIAIS ERROR OF PCTB ON AMPLITUDE
C 1Z IS USED IN CALCULATING AN INTEGER RANDOM NUMBER
C USED IN SUBROUTINE RANDU
130 110 WDI=DUM(1)/100.
131 W02=DUM(2)/100.
132 IZ=DUM(3)
133 IX=IZ+21
134 CALL ERR1(V,W,DI,W02,NX,NM,IX,16)
135 CALL ERR1(W,W1,W02,NX,NM,IX,16)
136 CALL ERR1(P,V,W01,W02,NX,NM,IX,16)
137 PRINT 2050,DUM(1),DUM(2),IZ
138 2050 FORMAT (1H1//30X,27H**** RANDOM ERROR OPTION ****
139 13X,10HPCT ERROR=+F7.3+5X,11HBIAS-ERROR=+F7.3+5X,
140 21HRANDOM NO SEED=+110//)
141 IF (IC2.NE.0) CALL ORTH(V,W,P,M,ME,M,FK,NM,NX,16,GMASS,OCHCK,16,
142 1IC2)
143 PRINT 1036
144 1036 FORMAT (1H1//40X,43H**** MODES REPLACED BY MODES WITH ERRORS **** //
145 IF (IC2.EQ.2) PRINT 1032
146 IF (IC2.EQ.2) IC2=1
147 CALL PMODES (X,V,W,P,OMEG,FREQ,NM,NX,16)
148 GO TO 100
149 130 CALL ERR1(130,0)
C CORRECT MODES ONLY IOI = 2
C ORIGINAL MODES UNDISTURBED
C CORRECTED MODES IN .V2,.M2,P2
C CHECK FREQUENCIES, MODES
147 200 PRINT 1040
148 1040 FORMAT (1H1//30X,18HMODE CHANGE - OPTION //30X,26HPERCENTAGE CHANGES -
149 1V,W,P) //20X,16HMODE 1 UNCHANGED )
149 IF (DUM(1).EQ.1) PRINT 1043
150 IF (DUM(1).EQ.2) PRINT 1044
151 1043 FORMAT (20X,21HLIMIT OPTION - SCALED )
152 1044 FORMAT (20X,24HLIMIT OPTION - TRUNCATED )
IF (NM. GT. 8) CALL ERR(200,0)

DO = OMEG(1)

DO 210 I = 2,NM

IF (OMEI(1), NE. OM) CALL ERR(21), J

210 CONTINUE

C CHANGED MODE IN V2. Y, 2, P2

DO 220 I = 1,NX

V(2) = V(1) + I

P(2) = P(1) + I

N = 1

C FORM A WITH COLUMN 1 IS COMPRESSED

250 M1 = N

N = N + 1

DO 260 I = 1, NX

A(I, M1) = M1 + V2(M1, I) + MET(I) + P2(M1, I)

A(NX + I, M1) = M1 + V2(M1, I) + ME(I) + P2(M1 + I)

260 A(N2 + I, M1) = MET(I) + V2(M1, I) + ME(I) + W2(M1, I) + NK(I) + P2(M1 + I)

C FORM COMPRESSED M TH MODE

DO 270 I = 1, NX

PHI(I) = V(N, I)

PHI(NX + I) = W(N, I)

270 PHI(N2 + I) = P(N, I)

DO 280 I = 1, N3

DO 280 J = 1, M1

280 Bi(I, J) = PHI(I) + A(I, J)

C C = BTRAN * B (M*M)

DO 290 I = 1, M1

DO 290 J = 1, M1

290 C(I, J) = C(I, J) + B(I, I) * B(I, J)

C INVERT C INTO D

IF (M1, NE, 1) GO TO 300

D(I, I) = 1.0/C(I, I)

GO TO 310

300 CALL INVRs (C, M1, D, WORK, IROW, ICOL, 7, 8)

C ATRAN * PHI

310 DO 320 I = 1, M1

WOR(I) = 0

DO 320 J = 1, N3

320 WOR(I) = WOR(I) + A(J, I) * PHI(J)

CALL MXV (WOR, D, WOR,M1, M1, 7, 0)

CALL MXV (WOR, B, WOR, M1, 60, 0)

C WOR = ~ FRACTIONAL CHANGE IN EACH ELEMENT

PRINT PERCENT CHANGES

EMAX = 0

DO 330 I = 1, N3

330 EMAX = AMAX1 (EMAX, ABS(WOR(I)))

PRINT 1050, NEMAX

1050 FORMAT (/20X, 5H, Mode 12, 10HMAX CHANGE F8.1)

IF (DUM(I), EQ, 0) GO TO 331

136
118  IF(DUM(N) .NE. 0) PRINT 1051, DUM(N)
119  1051 FORMAT (45X,1AHMAX ALLOWED CHANGE F6.2)
120  PRINT 1055, (WOR(I), I=1,NX)
121  I1 = NX+1
122  I3 = N+1
123  PRINT 1055, (WOR(I), I=1,N)
124  1055 FORMAT (/120X,10F10.3)
125  PRINT 1055, (WOR(I), I=I3,N3)
126  331 TEMP = .01
127  IF(DUM(1) .EQ. 0) GO TO 335
128  IF (DUM(N) .EQ. 0 OR MAX .LE. DUM(N)) GO TO 335
129  IF(DUM(1) .EQ. 2) GO TO 342
130  TEMP = .01*DUM(N)/MAX
131  335 DO 340 I = 1,N3
132  340 PHI(I) = PHI(I)*(.1+TEMP*WOR(I))
133  GO TO 349
134  342 DO 345 I = 1,N3
135  IF(WOR(I) .GT. 0) WOR(I) = AMIN(WOR(I), DJM(N))
136  IF(WOR(I) .LT. 0) WOR(I) = AMAX(WOR(I), -DUM(N))
137  345 PHI(I) = PHI(I)*(.1+TEMP*WOR(I))
138  349 DO350 I = 1,NX
139  V2(N+1) = PHI(I)
140  W2(N+1) = PHI(NX+1)
141  350 P2(N+1) = PHI(N+1)
142  IF(N,N,T,NM) GO TO 250
143  355 PRINT 1060
144  1060 FORMAT (1H1 // 30X,15HCORRECTED MODES //)
145  CALL PMODES (X,2,2W,2P,2OMEG,FREQ,NM,NX,16)
146  370 IF (IC2.EQ.0) GO TO 1
147  PRINT 1061
148  1061 FORMAT (-- // 30X,30HCORRECTED-ORTHOGONALITY CHECK --)
149  CALL ORTH (V2,2W,2P,2M,2ME,2MET,2MK,2NM,2N,2X,16,GMASS,OCHECK,16,IC2)
150  IF (IC2.EQ.2) CALL PHODES (X,2V,2W,2P,2OMEG,FREQ,NM,NX,16)
151  IF (IC2.EQ.0) GO TO -1
152  DO 380 I = 1,NX
153  DO 380 J = 1,NM
154  380 VI(J;I) = V2(J,I)
155  WJ(J;I) = W2(J,I)
156  380 PJ(J;I) = P2(J,I)
157  PRINT 1065
158  1065 FORMAT (/10X,47H*** ORIGINAL DATA REPLACED BY MODIFIED DATA ***)
159  /1//
160  GO TO 1
161  C
162  ORIGINAL MAss PARAMETERS UNdisturbed
163  C
164  CORRECTED VALUES IN-M2, E2, TH2, KM2
165  C
166  SET UP EQUATION PAIRS
167      500 NEQ = 0
168      NSI = 0
169      NM1 = NM-1
170      DO 510 I = 1,NM1
171      IF(I .EQ. I+1)
172      DO 510 J = I,NM
173      IF (OMEG(I;NE,OMEG(J)) GO TO 510
NEQ = -NEQ + 1
IJEQ (NEQ, 1) = I
IJEQ (NEQ, 2) = J
-510 CONTINUE
IF (NEQ, GT, 30) CALL ERR (510, 0)
PRINT 2000, (IJEQ (I, 1), IJEQ (I, 2), I = 1, NEQ)
2000 FORMAT (4H1, 30X, 13H MASS-CHANGE OPTION--/30X, 25HEQUATIONPAIRS--(M)
ID ENVOS) // (10X, 10I7, 14I1))
IF (DUM (1), EQ, 1,.0) PRINT 1999
1999 FORMAT (330X, 37HWEIGHTING FACTORS SET TO 1 {TEMP})
IF (NEQ, GT, 34) CALL ERR (511, 0)
IF (DUM (1), NE, 2,.0) GO TO 520
READ 1997, NSI, (NIN (J), J = 1, NSI)
1997 FORMAT (10I9, 7I10)
PRINT 1998, (NIN (J), J = 1, NSI)
1998 FORMAT (30X, 28NO CHANGES AT FOLLOWING STS--/30X, 8{2X, 13})
C SET UP EQUATION COEFFICIENTS

520 DO 550 I = 1, NEQ
150 = 1, JEQ (I, 1)
J = IJEQ (I, 2)
DO 550 J = 1, NX
570 DO 551 I = 1, NEQ
551 W (I) = 0.
571 IF (DUM (2), EQ, 0,.1) GO TO 553
573 PRINT 2001, SM (1)
574 2001 FORMAT (33X, 36HTOTAL MASS INvariant AT F10, 3)
575 NEQ = NEQ + 1
576 W (NEQ) = SM (1)
577 DO 552 I = 1, NX
578 EQ (NEQ, I) = 1, 0
579 DO 552 I = 1, NX
580 EQ (NEQ, I) = 0, 0
581 EQ (NEQ, I) = 0, 0
582 EQ (NEQ, I) = 0, 0
583 IF (DUM (3), EQ, 0,.0) GO TO 555
585 TEMP = SM (2) / SM (1)
584 PRINT 2002, TEMP
585 2002 FORMAT (30X, 36Hradial CG invariant AT F10, 2)
586 NEQ = NEQ + 1
587 W (NEQ) = SM (2)
588 DO 554 I = 1, NX
589 EQ (NEQ, I) = X (I)
590 EQ (NEQ, I) = 0, 0
591 EQ (NEQ, I) = 0, 0
592 EQ (NEQ, I) = 0, 0
593 EQ (DUM (4), EQ, 0,.1) GO TO 557
594 TEMP = SM (3) / SM (1)
595 PRINT 2003, TEMP
596 2003 FORMAT (30X, 36HCG invariant AT F10, 4)
597 NEQ = NEQ + 1
598 W (NEQ) = SM (3)
DO 556 I=1,NX
500 EQ(NEQ,I) = 0.0
501 EQ(NEQ,NX+I) = 1.0
502 EQ(NEQ,N2+I) = 0.0
503 556 EQ(NEQ,N3+I) = 0.0
504 557 IF(DUM(5),EQ,0) GO TO 559
505 PRINT 2004,SM(4)
506 2004 FORMAT (30X,34HFLAPPING MOM OF INERT INVARIANT AT F12.2)
507 NEQ=NEQ+1
508 W(V,NEQ)=-SM(4)
509 DO 558 I=1,NX
510 EQ(NEQ,I) = X(I)*W(2)
511 EQ(NEQ,NX+I) = 0.0
512 EQ(NEQ,N2+I) = 0.0
513 558 EQ(NEQ,N3+I) = 0.0
514 559 IF(DUM(6),EQ,0) GO TO 565
515 PRINT 2005,SM(5)
516 2005 FORMAT (30X,36HFEATHERING MOM OF INERT INVARIANT AT F10.4)
517 NEQ=NEQ+1
518 W(V,NEQ)= -SM(5)
519 DO 560 I=1,NX
520 EQ(NEQ,I) = 0.0
521 EQ(NEQ,NX+I) = 0.0
522 EQ(NEQ,N2+I) = 0.0
523 560 EQ(NEQ,N3+I) = 1.0
524 565 N41=N4
525 IF(DUM(1),EQ,2.) N41=N4-NSI
526 PRINT 2006,NEQ,N41
527 2006 FORMAT(30X,1HTOTAL EQUATIONS = I5,18H, NO OF UNKNOWNS = I4/
528 IF(C3,EQ,0) GO TO 580
529 PRINT 2010
530 2010 FORMAT (30X,35HEQUATION COEFFICIENTS FOR MASS SI /
531 DO 570 I=1,NEQ
532 570 PRINT,2020,(EQ(I,J),J=1,N4)
533 2020 FORMAT (/10X,1P10E12.3)

C FORM COMPRESSED MA MATRIX

C
C 580 DO 590 I = 1,NX
581 MA(I)=M(I)
582 MA(NX+I)=ME(I)
583 MA(N2+I)=MET(I)
584 590 MA(N3+I)=MK(I)

C FORM INVERSE, COMPRESSED PERCENTAGE WEIGHTED WEIGHTING FUNCTION

C
C 0339 IF(DUM(1),EQ,1.) GO TO 602
0340 DO 600 I = 1,NX
0341 WA(I)=MK(I)/WK(I)
0342 WA(NX+I)=ME(I)/WE(I)
0343 IF(ME(I),EQ,0) WA(NX+I)=AME/WE(I)
0344 WA(N2+I)=MET(I)/WT(I)
0345 IF(MET(I),EQ,0) WA(N2+I)=AME/WT(I)
0346 WA(N3+I)=MK(I)/WK(I)

139
IF (MK(I) .EQ. 0) WA(N3+I) = AMK/WK(I)

CONTINUE

IF (NS1 .EQ. 0) GO TO 609

DO 601 I = 1, NS1

J = NIN(I)

IF (J .LE. 0 OR J .GT. NX) CALL ERR(601, 0)

WA(J) = 0

WA(NX + J) = 0

WA(N2 + J) = 0

601 CONTINUE

602 GO TO 609

603 DO 605 I = 1, NX

604 WA(I) = M(I)

605 WA(NX + I) = ME(I)

606 WA(N2 + I) = ET(I)

607 IF (ME(I) .EQ. 0) WA(NX + I) = AME

608 WA(N2 + I) = AMET

609 WA(N3 + I) = MK(I)

610 IF (MK(I) .EQ. 0) WA(N3 + I) = AMK

611 CONTINUE

C AWA = EQ*WA**(I-2)*EQ(I) (NEQ**NEQ)

609 DO 610 I = 1, NEQ

610 AWA(I, J) = AWA(I, J) + EQ(I, L)*EQ(J, L)*WA(L)*WA(J)

C NOTE DWA IS DUMMY ONLY, AWA IS FREE

C IF (IC3 .EQ. 0) GO TO 612

C PRINT 201

201 FORMAT (IH1 // 30X, 2HN MATRIX TO BE INVERTED //)

C DO 611 I = 1, NEQ

C PRINT 202

202 FORMAT (// 30X, 14H INVERSE //)

C DO 614 I = 1, NEQ

C PRINT 203

203 FORMAT (//30X, 14H (TRAN) //)

C DO 615 I = 1, NEQ

C PRINT 204

204 FORMAT (I5, 20I1)

C DO 618 I = 1, NEQ

C PRINT 205

205 FORMAT (I5, 20I1)

C DO 620 J = 1, NEQ

C FORM WV = EQ(I) * DM THEN DM = DELTA MASS

C DO 620 I = 1, NEQ

C DO 625 J = 1, NEQ

625 DM(I) = 0

626 DO 625 J = 1, NEQ

627 DM(I) = DM(I) + AWA(I, J) * WV(J)

628 DO 620 I = 1, NEQ

629 WV(I) = WV(I) + EQ(I, J) * DM(J)

C 140
C FORM CORRECTED CHARACTERISTICS

C DO 640 I = 1, NX
C M2(I) = M(I)*DM(I)
C ME2(I) = ME(I)+DM(NX+1)
C E2(I) = ME2(I)/M2(I)
C MET2(I) = MET(I)+DM(NX+1)
C IF(MET2(I), EQ, 0) GO TO 635
C TH2(I) = MET2(I)/ME2(I)-TH(I)
C GO TO 636
C 635 TH2(I) = TH(I)
C MK2(I) = MK(I)+DM(NX+1)
C TEMP = MK2(I)/M2(I)
C IF(TEMP EQ, 0) GO TO 639
C KM2(I) = SQR(T/TEMP)
C GO TO 640
C 639 KM2(I) = SQRT(TEMP)
C 640 CONTINUE
C COMPUTE PCT CHANGES IN AWA
C DO 650 I = 1, NX
C AWA(I, 1) = DM(I)/M(I)*100
C AWA(I, 4) = (KM2(I)-KM(I))/KM(I)*100
C IF(TH(I), EQ, 0) GO TO 667
C AWA(I, 3) = (TH2(I)-TH(I))/TH(I)*100
C GO TO 648
C 647 AWA(I, 3) = 100
C IF(TH2(I), EQ, 0) AWA(I, 3) = 0
C 648 IF(E2(I), EQ, 0) GO TO 649
C AWA(I, 2) = (E2(I)-E(I))/E(I)*100
C GO TO 650
C 649 AWA(I, 2) = 100
C IF(E2(I), EQ, 0) AWA(I, 2) = 0
C 650 CONTINUE
C PRINT CHANGED VALUES
C PRINT 2030
C 2030 FORMAT (14H1/130H-1, ORIG_M, NEW_M, PCT, ORIG_E
C 1 NEW_E PCT ORIG_TH NEW_TH PCT ORIG_KN
C 2NEW_KH_PCT /1
C 655 PRINT 2040, I, M(I), M2(I), AWA(I, 1), E(I), E2(I), AWA(I, 2),
C 1 TH(I), TH2(I), AWA(I, 3), KM(I), KM2(I), AWA(I, 4)
C 2040 FORMAT (13, 1PE13.3, 1E12.3, 0PE7.1)
C ORTH CHECK
C IF (IC2, EQ, 0) GO TO 1
C 36 PRINT 1061
C 1061 CALL ORTH (V, W, P, M2, ME2, MET2, MK2, NM, NX, 16, CM, OCH, 16, IC2)
C IF (IC2, EQ, 2) PRINT 1032
C 1032 CALL PHMOS (X, W, P, OM, FREQ, NM, NX, 16)
C IF (IC2, EQ, 2) GO TO 1
C 660 I = 1, NX
C 662 M(I) = M2(I)
C 664 E(I) = E(I)
C TH(I) = TH2(I)
KM(I) = KM2(I)
ME(I) = ME2(I)
MET(I) = MET2(I)
660  MK(I) = MK2(I)
PRINT 1065
GO TO 1
END
SUBROUTINE - RMODES (X, Y, Z, P, OMEG, FREQ, NH, NX, NDIM)

REAL X(1), Y(NDIM+1), Z(NDIM+1), P(NDIM+1), OMEG(1), FREQ(1)

IM0=1

IM1 = MINO(NM, 3)

75 PRINT 1025, (OMEG(I), I=IM0, IM1)

1025 FORMAT (/13X,"OMEGA =", F18.3, 2F39.3)

PRINT 1026, (FREQ(I), I=IM0, IM1)

1026 FORMAT (/13X,"FREQ =", F18.3, 2F39.3)

PRINT 1027

1027 FORMAT (/2X, "IH" STA, .3(39H "V" W IP "I")

DO 80 I = 1, NX

80 PRINT 1030, I, X(I), (Y(J), J=1, I), P(J), (Z(J), J=1, I), IM0, IM1

1030 FORMAT (1X, 12, 0P F10.3, 3(3X, 1P 3E12.3))

IF (IM0.GE.NM) GO TO 90

IM0=IM0+3

IM1 = MINO(NM, IM1+3)

IF (IM0.EQ.36 OR IM0.EQ.10 OR IM0.EQ.16) GO TO 75

PRINT 1020

1020 FORMAT (1H1, 50X, 11H METODE SHAPES //)

GO TO 75

90 RETURN

END

SUBROUTINE - ERR(N, I)

C I = 0, TERMINATES RUN I NE 0 WARNING ONLY, PRINTS I

10 PRINT 10, N

10 FORMAT (/10X,"ERRNO- NUMBER", 15, 5H ""

20 IF (I.NE.0) GO TO 20

CALL EXIT

30 FORMAT (20X, 20H "WARNING ONLY " IS//)

RETURN
SUBROUTINE CRTTH(V, W, P, M, ME, MET, MK, NM, NX, MDIM, GMASS, OCHECK, MCDIM, IP)

C PERFORMS ORTHOGONALITY CHECK
C GMASS ARE DIAGONAL ELEMENTS
C OCHECK IS NORMALIZED BY DIVIDING ROW, COL BY SQRT
C OF DIAGONAL
C
C IP.NE.0 GMASS, OCHECK ARE PRINTED
C IP.EQ.2 MODES ARE NORMALIZED (GEN MASS = 1.0)
C
REAL V(MDIM,1), W(MDIM,1), P(MDIM,1), ME(1), MET(1), MK(1), GMASS(1),
1 OCHECK(MCDIM,1), M(1)

0033 DO 20 I = 1, NM
0034 DO 20 J = 1, NM
0035 OCHECK(I, J) = 0
0036 DO 20 20 = 1, NM
0037 20 OCHECK(I, J) = OCHECK(I, J) + V(I, L) * M(L) * V(J, L) - P(I, L) * MET(L) * V(J, L)
1 + W(I, L) * M(L) * W(J, L) + P(I, L) * ME(L) * W(J, L) - V(I, L) * MET(L) * P(J, L)
2 + W(I, L) * ME(L) * P(J, L) + P(I, L) * MK(L) * P(J, L)
0038 DO 30 I = 1, NM
0039 GMASS(I) = OCHECK(I, I)
0040 SQ = SQRT(GMASS(I))
0041 IF (IP.NE.2) GO TO 29
0042 DO 25 L = 1, NX
0043 V(I, L) = V(I, L) / SQ
0044 W(I, L) = W(I, L) / SQ
0045 P(I, L) = P(I, L) / SQ
0046 DO 30 J = 1, NM
0047 30 OCHECK(I, J) = OCHECK(I, J) / SQ
0048 IF (IP.EQ.0) RETURN
0049 PRINT 100, GMASS(I), I = 1, NM
0050 100 FORMAT (20X, 4HDIAGONAL ELEMENTS OF ORTHO CHECK MATRIX /)
0051 1 (10X, 1PE14.3))
0052 PRINT 200
0053 200 FORMAT (1//20X, 3HNORMALIZED ORTHO CHECK MATRIX /)
0054 DO 40 L = 1, NM
0055 40 PRINT 300, (OCHECK(I, J), J = 1, NM)
0056 300 FORMAT (/2X, 16F8.3)
0057 IF (IP.EQ.2) PRINT 350
0058 350 FORMAT (1H1, 30X, 16HNORMALIZED MODES //)
0059 RETURN
0060 END
SUBROUTINE INVRS(B,N,A,0,IROW,ICOL,NRW,NCL)
C A = INVERSE OF B   B UNDISTURBED
C VARIABLE DIMENSIONS   NCL MUST BE AT LEAST ONE GREATER THAN NRW
C NRW MUST BE AT LEAST EQUAL TO N
C IR0W, IC0L ARE VECTORS OF LENGTH NCL
REAL A(NRW,NCL),B(NRW,NCL),D(NRW,NCL)
INTEGER IR0W(NCL),ICOL(NCL)
DO 1 I=1,N
DO 1 J=1,N
1 A(I,J)=B(I,J)
M=N+1
DO 7 I=1,N
IF(IR0W(I))7,8,9
7 IC0L(I)=I
AMAX=A(I,K)
DO 10 I=1,N
10 AMAX=A(I,J)
M=MIN(M,AMAX)
IF(M.ALT.0.0)7,8,9
9 AMAX=A(I,J)
IC=I
IF(IR0W(IR0W(I)))10,11,12
10 CONTINUE
KI=IC0L(K)
IC0L(K)=IC0L(IC)
IF(M.GT.10)11,12,13
11 DO 14 J=1,N
E=A(K,J)
14 A(K,J)=-A(IC,J)
15 A(IC,J)=E
E=A(I,K)
16 A(I,K)=A(I,JC)
17 A(I,JC)=E
18 DO 19 I=1,N
19 IF(A(K,I))18,19,20
20 GO TO 16
12 PRINT 13
13 IF(M)1,2,3
1 IF(M.ALT.10)
1 CONTINUE
PVT=A(K,K)
DO 20 J=1,N
20 A(K,J)=A(K,J)/PVT
AMULT=A(I,K)
DO 21 J=1,N
21
22 A(I,J) = A(I,J) - AMULT*A(K,J)
19 CONTINUE
DO 20 I = 1, N
20 A(I,K) = A(I,N)
DO 25 I = 1, N
DO 24 L = 1, N
24 CONTINUE
DO 25 J = 1, N
25 D(L,J) = D(I,J)
DO 26 J = 1, N
DO 28 L = 1, N
28 CONTINUE
DO 26 I = 1, N
26 A(I,L) = D(I,J)
100 RETURN
END
SUBROUTINE MXV(A,B,C,M,N,NDIM,ICNT)

C MATRIX TIMES VECTOR A(M)*B(N)*C(J) FOR ICNT = 0
C +A(M) FOR ICNT = 1

DIMENSION A(1), B(NDIM,1), C(1)
DO 10 I=1,M
IF (ICNT.EQ.0) A(I)=0
DO 10 J=1,N
10 A(I)=A(I)+B(I,J)*C(J)
RETURN
END

SUBROUTINE ERRAL A,PCT,PCTB,NJ,NM,IX,NDIM

C A BIAS ERROR PCTB(RATIC) ON AMPLITUDE AND A UNIFORMLY DISTRIBUTED
C RANDOM ERROR HAVING A +/- MAXIMUM OF PCT(RATIO) ON AMPLITUDE

DIMENSION A(NDIM,1)
IF (PCT.NE.0) GO TO 110
100 IF (.NOT.PCTB.EQ.0) GO TO 130
110 DO 120 K=1,NM
120 DO 120 I=1,NJ
CALL RANDU(IX,IY,YFL)
IX=IY
E=1.*2.*PCT*1.5+PCTB
A(K,I)=A(K,I)*E
RETURN
END
SUBROUTINE RANDU (IX, IY, YFL)

C USAGE
C CALL RANDU ( IX, IY, YFL )

C COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN
C 0 AND 1.0 AND RANDOM REAL INTEGERS BETWEEN 0 AND 2**31.

C EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER AND
C PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.

C VARIABLES
C IX= FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER NUMBER
C WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY IX SHOULD BE
C THE PREVIOUS VALUE OF IX COMPUTED BY THIS SUBROUTINE.
C IY= A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT ENTRY
C TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS BETWEEN 0 AND 2**31
C YFL= THE RESULTANT UNIFORMLY DISTRIBUTED FLOATING POINT RANDOM
C NUMBER IN THE RANGE 0 TO 1.0

C
C 0002       IY=IX*65539
C 0003       IF(IY) 100,110,110
C 0004     100 IY=IY+2147483647+1
C 0005     110 YFL=IY
C 0006      YFL=YFL*.4656613E-9
C 0007     RETURN
C 0008     END
APPENDIX D
NORMAL MODES AND NATURAL FREQUENCIES OBTAINED FROM VACUUM WHIRL DATA*

INTRODUCTION

The rotor was forced vertically along the axis of rotation with no other external forces. The natural frequencies of the symmetric flapping modes with infinite hub impedance are the driving point antiresonant frequencies along the rotational axis. These frequencies were identified and a modal analysis done to determine the mode shapes using strain/hub acceleration transmissibility in the following manner.

Strain readings, calibrated in terms of bending moments, and hub vertical accelerations were recorded simultaneously on analog tape at the selected rotational speeds of 0, 5.24, 10.47 and 15.71 rod/sec. (0, 50, 100 and 150 RPM). The time domain hub acceleration signal was fed from the tape reader to the force input of a Fast Fourier Transform Digital Signal Analyzer, type Hewlett Packard 5420, while the time domain strain signal from the jth station along the blade was fed to the response input of the Digital Signal Analyzer for stations j = 1 to j = 12 at each of the rotor RPM settings. Over a narrow band of frequency covering each hub antiresonant frequency, determined approximately from broad band analysis in which the hub driving point antiresonant frequencies appear in the Fourier Transform in the form of natural frequencies, a Fourier Transform of 2^8 frequency line was obtained for each strain/hub acceleration transfer function. The narrow band data were then analyzed for global properties.

The transmissibility residues for the 12 blade stations in a given mode were found to be complex, due to the nature of the transfer function, but complex normalization showed the bending moment modes to be real (classical). The deflection modes were obtained from the bending moment modes by simple double trapezoidal integration of the curvature from the root to the tip.

*The tests from which this data were obtained are described in Ref. 9

149
The Antiresonant Method. - It is obviously impossible to achieve infinite terminating impedance in practice but the modal effects of infinite terminating impedance along a single motion coordinate can be obtained quite accurately through antiresonance theory even though the terminating coordinate never reaches absolutely zero motion. It never reaches absolute zero because, and only because, in this case, the rotor dissipates energy to a sink. The nature of this energy dissipation, called "damping", is not known. If the rotor were undamped the vertical motion along the axis of rotation, the coordinate of sole external excitation, would be absolutely zero at the natural frequencies of the symmetric flapping modes of infinite hub impedance regardless of the actual hub impedance. The sum of the inertial forces of the undamped rotor acting vertically on the hub would, at these frequencies, be exactly equal to the sole excitation force acting vertically at the hub, regardless of its magnitude (within the linear range) or phasing to any base, in the steady state. This is the principle of the undamped vibration absorber of 1909; its notable early 19th century predecessor, the una corda or "soft" pedal on aftersound of the concert grand piano; the Thearle invention of the 1930 on which shaft and turbinebalancing machines are based; the 1947 method of stabilization by Thor which made spin dry home washing machines practical and the many obvious helicopter applications along with the less obvious one recently in which a military helicopter initially had little pilot seat vibration at the expense of intolerable tail fatigue.

Mathematically, a damped antiresonance is merely a zero of zero magnitude. In the case at hand the single excitation along the axis of rotation is unknown (because the measured applied force in the rig is below the hub with an intervening unknown impedance) but as it is the same for hub vertical acceleration and blade bending moment the quotient of blade bending mobility and hub acceleration mobility involves cancellation of the pole roots leaving the denominator a polynomial whose roots are hub driving point zeros the undamped parts of which are the desired antiresonances. These can be determined from the Fourier Transform of the transfer function as will be shown below.
From elementary considerations of complex variable theory it is easily seen that the residues are without physical significance in themselves because the polynomial quotient has an arbitrary factor. For this reason one cannot use this procedure to obtain physically meaningful orthonormal modes. However, in normalizing on a station on the blade the arbitrary factor of the multiplying factor cancels, being the same for each station, and a valid bending-moment mode shape can be readily obtained. That is, the validity of the quotient of residues is maintained. This is precisely the same as ratioing the vectorial chords of the Nyquist plots of each blade station between given frequencies in the zero root range of the hub mobility to that of any given blade station.

Because the complex chordal vectors between given frequencies are parallel to the modal diameter of any transmissibility having the hub driving point product of roots of the zeros in the denominator and because the length of such chords are necessarily proportional to their associated diameters each it follows that the ratio of the complex chordal vectors is the same as that of the complex diametral vectors. In other words, if one were to transfer the Nyquist axes to an origin corresponding to the antiresonant frequency, do a bilinear transformation and ratio the distances of the resulting lines to the origin for any station to a given blade station one would find a canonical invariance of the polynomial in the poles and the frequency invariant factor for any given pole.
Finding the Natural Frequency. - Most often one will find three peaks in mobility associated with a mode, two in the real and one in the imaginary or vice versa. If the angle of a complex mode is near $45^\circ$, $135^\circ$, $225^\circ$ or $315^\circ$ there will be only two sharp peaks, one in the real and one in the imaginary.

The following is done for acceleration mobility. $q$ refers to a frequency in the imaginary and $p$ to a frequency in the real. The subscript $x$ refers to an acceleration mobility maximum and $m$ to an acceleration mobility minimum.

If the modal angle is in the range from about $-40^\circ$ to about $+40^\circ$ or narrower there will be a maximum in the acceleration imaginary and a minimum and maximum in the real.

\[
2 q_x^2 - \frac{p_x^2 + p_m^2}{2} = \Omega^2 [1 + g(2 \tan \phi/2 - \tan \phi)]
\]  \hspace{1cm} (D-1)

Let the natural frequency be approximated by

\[
\Omega^2 \approx 2 q_x^2 - \frac{p_x^2 + p_m^2}{2}
\]  \hspace{1cm} (D-2)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$g = .02$</th>
<th>$g = .05$</th>
<th>$g = .10$</th>
<th>$g = .20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40^\circ$</td>
<td>0.22%</td>
<td>0.56%</td>
<td>1.11%</td>
<td>2.22%</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.08%</td>
<td>0.21%</td>
<td>0.41%</td>
<td>0.83%</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>0.02%</td>
<td>0.06%</td>
<td>0.11%</td>
<td>0.23%</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.003%</td>
<td>0.006%</td>
<td>0.01%</td>
<td>0.03%</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>$-10^\circ$</td>
<td>0.003%</td>
<td>0.006%</td>
<td>0.01%</td>
<td>0.03%</td>
</tr>
<tr>
<td>$-20^\circ$</td>
<td>0.02%</td>
<td>0.06%</td>
<td>0.11%</td>
<td>0.23%</td>
</tr>
<tr>
<td>$-30^\circ$</td>
<td>0.08%</td>
<td>0.21%</td>
<td>0.41%</td>
<td>0.83%</td>
</tr>
<tr>
<td>$-40^\circ$</td>
<td>0.22%</td>
<td>0.56%</td>
<td>1.11%</td>
<td>2.22%</td>
</tr>
</tbody>
</table>
If the modal angle is in the range from $50^\circ$ to $130^\circ$ one will observe a $p_m$, $q_x$ and $q_m$ with the identical errors over the range as given in Table D-I by adding $90^\circ$ to the angle. Similarly for the other cases.

$$\Omega^2 = 2 p_m^2 - \frac{q_x^2 + q_m^2}{2}$$  \hfill (D-3)

$$\Omega^2 = 2 q_m^2 - \frac{p_x^2 + p_m^2}{2} \quad 140^\circ \text{ to } 220^\circ$$  \hfill (D-4)

$$\Omega^2 = 2 p_x^2 - \frac{q_x^2 + q_m^2}{2} \quad 230^\circ \text{ to } 310^\circ$$  \hfill (D-5)

Equation D-2, D-3, D-4 and D-5 involve frequencies merely as twice the square of the single peak frequency less half the sum of the squares of the double peak frequencies.

Figure D-1. A diagram of acceleration mobility peak frequencies.
Two Peaks Only - Natural Frequency. - If there is only one real and one imaginary peak associated with a mode, the modal angle must be near $45^\circ + n*90^\circ$ for $n = 0, 1, 2, 3$ as seen in Figure D-1.

For $n = 0$

$$q_x^2 + p_m^2 = \Omega^2 \left[2 + g \left(\tan \frac{\phi}{2} - \cot \frac{\phi+\pi/2}{2}\right)\right]$$

(D-6)

Let the natural frequency be approximated by

$$\Omega^2 = \frac{q_x^2 + p_m^2}{2}$$

(D-7)

| TABLE D-II. INHERENT ERROR IN EQUATION 7. $\frac{\Omega^2}{2} - \Omega^2$ |
|-----------------|---------------|---------------|---------------|---------------|
| $\phi$          | $g = .02$     | $g = .05$     | $g = .10$     | $g = .20$     |
| nx90° + 35°     | 0.206%        | 0.516%        | 1.04%         | 2.096%        |
| nx90° + 40°     | 0.102%        | 0.257%        | 0.514%        | 1.034%        |
| nx90° + 45°     | 0%            | 0%            | 0%            | 0%            |
| nx90° + 50°     | 0.102%        | 0.257%        | 0.514%        | 1.034%        |
| nx90° + 55°     | 0.206%        | 0.516°        | 1.04%         | 20.096%       |

The actual inherent error in natural frequency is about half those in Table D-II.

Local Spectrum Analysis of a Complex Mode Given the Natural Frequency

This procedure may be used over any portion of the modal arc. In an acceleration mobility Kennedy-Pancu plot let $N$ be the natural frequency and $f_1$ be any frequency on the modal arc selected by the operator. The chord from frequency $f_1$ at $N\sqrt{1-b}$ to frequency $f_2 = N\sqrt{1+b}$ over an arc of $180^\circ$ or less is perpendicular to a diameter through the natural frequency, $b$ is an arbitrary number less than unity. See Figure D-2.
The modal angle is \( \phi \).

\[
\frac{c/2}{D-h} = \tan \frac{\alpha}{2}
\]  

(D-8)

For practical purposes (see the mensuration section of any standard engineering handbook)

\[
\frac{c/2}{D-h} = \frac{2h}{c} = \tan \frac{\alpha}{2}
\]  

(D-9)

and

\[
c = D \sin \alpha.
\]  

(D-10)

\[
c = \sqrt{\left(y_2^R - y_1^R\right)^2 + \left(y_2^I - y_1^I\right)^2}
\]  

(D-11)
\[ \gamma_A^R = \left( \frac{\gamma_2^R + \gamma_1^R}{2} \right) \]
\[ e^2 = \sqrt{\left( \gamma_N^R - \gamma_A^R \right)^2 + \left( \gamma_N^I - \gamma_A^I \right)^2} \]
\[ e_1 = \sqrt{\left( \gamma_N^R - \gamma_1^R \right)^2 + \left( \gamma_N^I - \gamma_1^I \right)^2} \]
\[ e_2 = \sqrt{\left( \gamma_N^R - \gamma_2^R \right)^2 + \left( \gamma_N^I - \gamma_2^I \right)^2} \]

If \( e_2/e_1 \geq 1.0 \) then \( N \) is not the natural frequency for points 1 and 2 on the modal arc. If \( e_2/e_1 < 1.0 \) then the natural frequency is less than \( N \), if \( e_2/e_1 > 1.0 \) then the natural frequency is greater than \( N \).

\[ \frac{f_2^2}{N^2} = 1 + g \tan \frac{\alpha}{2} \]  
\[ \frac{f_1^2}{N^2} = 1 - g \tan \frac{\alpha}{2} \]  
\[ \frac{f_2^2 - f_1^2}{N^2} = 2g \tan \frac{\alpha}{2} \]

\[ g = \frac{1}{2} \frac{f_2^2 - f_1^2}{N^2 \tan \alpha/2} \]
The natural frequencies determined from HP 5420 data using Equations D-2 through D-5 are shown in Table D-III in comparison to the natural frequencies found by NASA. The strain data for 100 RPM was quite noisy and was therefore not analyzed. Figures D-3 through D-11 show the bending moment normal modes and Figures D-12 through D-20 show the normalized deflection mode shapes.

<table>
<thead>
<tr>
<th>TABLE D-III. NATURAL FREQUENCIES Hz (cassette number, record number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 rad/s (0 RPM)</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td><strong>2nd Flapping</strong></td>
</tr>
<tr>
<td>8.2 NASA</td>
</tr>
<tr>
<td>8.18 (2,41)</td>
</tr>
<tr>
<td>8.18 (2,41)</td>
</tr>
<tr>
<td><strong>21.8 NASA</strong></td>
</tr>
<tr>
<td>21.71 (1,10)</td>
</tr>
<tr>
<td>21.82 (3,1)</td>
</tr>
<tr>
<td>21.81 (2,48)</td>
</tr>
<tr>
<td><strong>24.1 NASA</strong></td>
</tr>
<tr>
<td><strong>21.82 (3,1)</strong></td>
</tr>
<tr>
<td><strong>41.73 (3,5)</strong></td>
</tr>
<tr>
<td><strong>26.6 NASA</strong></td>
</tr>
<tr>
<td><strong>26.41 (1,28)</strong></td>
</tr>
<tr>
<td><strong>27.02 (3,40)</strong></td>
</tr>
</tbody>
</table>
RECOMMENDATIONS

If this test were to be repeated it would be useful to measure strain on the hub near the center of rotation to provide the initial condition for integration of strains and it would be practical to calibrate in terms of the differential strains of the bending bridges, instead of bending moment, to eliminate the need for theoretical EI values in the integration.

In the photographic method of obtaining mode shapes the assumption is that the modes are uncoupled, that is, that the shaking excites only one mode. With that assumption, a promising method of obtaining rotating mode shapes is that pioneered by Hassal of the Royal Aircraft Establishment:

\[ \{q(R)\} = \Phi \{\phi(\epsilon)\} + \{\epsilon(R)\} \]

where \( \epsilon^{(R)} \) is the vector of blade strains measured in rotation
\( \Phi \) is the matrix of nonrotating normalized normal translational modes
\( \phi(\epsilon) \) is the matrix of nonrotating normalized normal strain modes

Normalization of the Left Hand Side at a natural frequency, given very light damping and widely separated natural frequencies, would be the rotating normal mode. \( \Phi \) and \( \phi(\epsilon) \) are obtained in a nonrotating shake test after which the accelerometers are removed from the blade and have the same number of columns but not necessarily the same number of rows. The strains used need not be directly related to bending moments.

CONCLUSIONS

The rotating and nonrotating modes in flatwise bending for the cantilever condition were found to be real. The natural frequencies found in bending moment modal analysis agreed closely with those found by other methods.
2nd FLAPWISE BENDING MOMENT MODE

0 RPM

BLADE STATION
Figure D-3
4th FLAPWISE BENDING MOMENT MODE

50 RPM

BLADE STATION

Figure D-8
2nd FLAPWISE BENDING MOMENT MODE

BLADE STATION

Figure D-9
4th FLAPWISE BENDING MOMENT MODE

150 RPM

BLADE STATION

Figure D-11

X/R
4th FLATWISE MODE SHAPE

Figure D-17

X/R
The work presented in this report was performed in order to develop methods of using rotor vacuum whirl data to improve the ability to model helicopter rotors. The work consisted of the following: (1) formulation of the equations of motion of elastic blades on a hub using a Galerkin method; (2) development of a general computer program for simulation of these equations; (3) study and implementation of a procedure for determining physical parameters based on measured data; (4) application of a method for computing the normal modes and natural frequencies based on test data.