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Elastohydrodynamic Lubrication Theory

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ELASTOHYDRODYNAMIC LUBRICATION THEORY*

This chapter is devoted to the method used by the authors to solve the elastohydrodynamic lubrication problem for elliptical contacts (Hamrock and Dowson, 1976a). The problem has been discussed earlier in some detail and the relevant effects examined separately, so it remains only to seek a solution that reconciles the hydrodynamic and elastic equations. The problem is to calculate the pressure distribution in the contact and at the same time to allow for the effects that this pressure will have on the properties of the fluid and on the geometry of the elastic solids. The solution will also provide the shape of the lubricant film, particularly the minimum clearance between the solids. Thermal effects are, however, neglected. The variation of lubricant viscosity with temperature and the thermal expansion of the lubricant and the solids are not considered.

7.1 Reynolds Equation

The coordinate system to be used in defining the elastohydrodynamic problem is shown in Figure 7.1. The appropriate

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Reynolds equation for the coordinate system (\tilde{x}, \tilde{y}) can be written from equation (5.43) as

$$\frac{\partial}{\partial \tilde{x}} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial \tilde{x}} \right) + \frac{\partial}{\partial \tilde{y}} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial \tilde{y}} \right) = 12u \frac{\partial(\rho h)}{\partial \tilde{x}} \quad (5.43)$$

where $u = (u_a + u_b)/2$ is the mean surface velocity or the entraining velocity in the \tilde{x} direction. Letting

$$\left. \begin{aligned} X &= \frac{\tilde{x}}{b}, & Y &= \frac{\tilde{y}}{a}, & \bar{\rho} &= \frac{\rho}{\rho_0} \\ \bar{\eta} &= \frac{\eta}{\eta_0}, & H &= \frac{h}{R_x}, & P &= \frac{p}{E'} \end{aligned} \right\} \quad (7.1)$$

we can write equation (5.43) as

$$\frac{\partial}{\partial X} \left(\frac{\bar{\rho} H^3}{\bar{\eta}} \frac{\partial P}{\partial X} \right) + \frac{1}{k^2} \frac{\partial}{\partial Y} \left(\frac{\bar{\rho} H^3}{\bar{\eta}} \frac{\partial P}{\partial Y} \right) = 12U \left(\frac{b}{R_x} \right) \frac{\partial(\bar{\rho} H)}{\partial X} \quad (7.2)$$

where $U = u \eta_0 / E' R_x$ is a dimensionless speed parameter and $k = 1.03 (R_y / R_x)^{0.64}$ is an ellipticity parameter.

Equation (7.2) is a nonhomogeneous partial differential equation in two variables. This is generally a difficult equation to solve, and the degree of complexity depends on the form of the parametric functions $\bar{\rho}$, $\bar{\eta}$, and H and on the boundary conditions. In elastohydrodynamic lubrication the parameters are of the most difficult form. Therefore equation (7.2) is solved by approximate numerical methods. Before proceeding, however, it is necessary to write expressions for the dimensionless film

thickness H , the dimensionless density $\bar{\rho}$, and the dimensionless viscosity $\bar{\eta}$.

The influence of pressure on the density of fluids has been discussed in Section 5.5. From this section the dimensionless density can be written as

$$\bar{\rho} = 1 + \frac{A_2 PE'}{1 + B_2 PE'} \quad (5.57)$$

where A_2 and B_2 are constants that depend only on the fluid considered.

The viscosity of fluids has been discussed in Section 5.4, where attention was drawn to the work of Roelands (1966). For isothermal conditions the dimensionless viscosity of the fluid can be written from Section 5.4 as

$$\bar{\eta} = \left(\frac{\eta_\infty}{\eta_0} \right) \left[1 - (1+p/c)^{Z_1} \right] \quad (5.52)$$

where

$$\eta_\infty = 6.31 \times 10^{-5} \text{ N s/m}^2 \quad (9.15 \times 10^{-9} \text{ lbf s/in}^2)$$

$$c = 1.96 \times 10^8 \text{ N/m}^2 \quad (28\,440 \text{ lbf/in}^2)$$

η_0 = viscosity of lubricant at atmospheric pressure

Z_1 = viscosity-pressure index, a dimensionless constant

These dimensions should also be used to define the constants in equation (5.52).

7.2 Film Shape

The film thickness can be written simply as

$$h(\tilde{x}, \tilde{y}) = h_0 + s(\tilde{x}, \tilde{y}) + \tilde{\delta}(\tilde{x}, \tilde{y}) \quad (7.3)$$

where

h_0 = constant

$S(\tilde{x}, \tilde{y})$ = separation due to geometry of undeformed ellipsoidal-solids

$\tilde{\delta}(\tilde{x}, \tilde{y})$ = elastic deformation

The separation due to the geometry of the two undeformed ellipsoidal solids shown in Figure 2.18 can be described by an equivalent ellipsoidal solid near a plane. The geometrical requirement is that the separation of the two ellipsoidal solids in the initial and equivalent situations should be the same at equal values of \tilde{x} and \tilde{y} . Therefore from Figure 7.1 the separation due to the undeformed geometry of the two ellipsoids can be written as

$$s(\tilde{x}, \tilde{y}) = \frac{(\tilde{x} - \tilde{m}b)^2}{2R_x} + \frac{(\tilde{y} - \bar{l}_a)^2}{2R_y} \quad (7.4)$$

where

\tilde{m} = constant used to determine length of inlet region

\bar{l} = constant used to determine width of side-leakage region

For illustration the mesh described in Figure 7.1 is used, where $\bar{m} = 4$ and $\bar{\ell} = 2$. However, the equations developed are written in general form.

Figure 7.2 illustrates the film thickness and its components for an ellipsoidal solid near a plane. Substituting equation (7.4) into equation (7.3) while using equation (7.1) to make this equation dimensionless gives

$$H = H_0 + \frac{b^2(X - \bar{m})^2}{2R_x^2} + \frac{a^2(Y - \bar{\ell})^2}{2R_x R_y} + \frac{\tilde{\delta}(X, Y)}{R_x} \quad (7.5)$$

where H_0 is a constant that is initially estimated.

The way in which the elastic deformation of an equivalent ellipsoidal solid near a plane can be calculated has been outlined in Section 5.7. Therefore from Figure 7.1 and the results of Section 5.7 the elastic deformation can be written as

$$\tilde{\delta}_{\alpha, \beta}(X, Y) = \frac{2}{\pi} \sum_{j=1, 2, \dots}^{\frac{2\bar{\ell}X\bar{c}}{(\bar{m}+\bar{n})X\bar{d}}} \sum_{i=1, 2, \dots}^{(\bar{m}+\bar{n})X\bar{d}} P_{i, j} \bar{D}_{i^*, j^*} \quad (7.6)$$

where

\bar{c} = number of divisions in semimajor axis

\bar{n} = constant used to determine length of outlet region

\bar{d} = number of divisions in semiminor axis

$$i^* = |\alpha - i| + 1$$

$$j^* = |\beta - j| + 1$$

$$\begin{aligned} \bar{D} = & b \left(x + \frac{1}{2d} \right) \ln \left\{ \frac{k \left(y + \frac{1}{2c} \right) + \left[k^2 \left(y + \frac{1}{2c} \right)^2 + \left(x + \frac{1}{2d} \right)^2 \right]^{1/2}}{k \left(y - \frac{1}{2c} \right) + \left[k^2 \left(y - \frac{1}{2c} \right)^2 + \left(x + \frac{1}{2d} \right)^2 \right]^{1/2}} \right\} \\ & + a \left(y + \frac{1}{2c} \right) \ln \left\{ \frac{\left(x + \frac{1}{2d} \right) + \left[k^2 \left(y + \frac{1}{2c} \right)^2 + \left(x + \frac{1}{2d} \right)^2 \right]^{1/2}}{\left(x - \frac{1}{2d} \right) + \left[k^2 \left(y + \frac{1}{2c} \right)^2 + \left(x - \frac{1}{2d} \right)^2 \right]^{1/2}} \right\} \\ & + b \left(x - \frac{1}{2d} \right) \ln \left\{ \frac{k \left(y - \frac{1}{2c} \right) + \left[k^2 \left(y - \frac{1}{2c} \right)^2 + \left(x - \frac{1}{2d} \right)^2 \right]^{1/2}}{k \left(y + \frac{1}{2c} \right) + \left[k^2 \left(y + \frac{1}{2c} \right)^2 + \left(x - \frac{1}{2d} \right)^2 \right]^{1/2}} \right\} \\ & + a \left(y - \frac{1}{2c} \right) \ln \left\{ \frac{\left(x - \frac{1}{2d} \right) + \left[k^2 \left(y - \frac{1}{2c} \right)^2 + \left(x - \frac{1}{2d} \right)^2 \right]^{1/2}}{\left(x + \frac{1}{2d} \right) + \left[k^2 \left(y - \frac{1}{2c} \right)^2 + \left(x + \frac{1}{2d} \right)^2 \right]^{1/2}} \right\} \end{aligned} \quad (7.7)$$

Equation (7.8) and Figure 7.1 clarify the meaning of equation (7.6). The elastic deformation at the center of the rectangular area $\delta_{9,5}$ (Figure 7.1) caused by the pressure on the

various rectangular areas in and around the contact ellipse can be written as

$$\tilde{\delta}_{9,5} = \frac{2}{\pi} \begin{pmatrix} P_{1,1}\bar{D}_{9,5} + P_{2,1}\bar{D}_{8,5} + \cdots + P_{35,1}\bar{D}_{27,5} \\ P_{1,2}\bar{D}_{9,4} + P_{2,2}\bar{D}_{8,4} + \cdots + P_{35,2}\bar{D}_{27,4} \\ \vdots \\ P_{1,20}\bar{D}_{9,16} + P_{2,20}\bar{D}_{8,16} + \cdots + P_{35,20}\bar{D}_{27,16} \end{pmatrix} \quad (7.8)$$

7.3 Phi (ϕ^*) Solution

Having defined the density, viscosity, and film thickness, we can return to the problem of solving for the pressure in the Reynolds equation. It is well known (e.g., Whomes, 1966) that the dimensionless pressure P of the Reynolds equation plotted as a function of x exhibits a very localized pressure field with high values of $\partial P / \partial X$ and $\partial^2 P / \partial X^2$. Such a condition with high gradients is not welcomed when performing numerical analysis by relaxation methods. Therefore, to produce a much gentler curve, a parameter ϕ^* is introduced where

$$\phi^* = PH^{3/2} \quad (7.9)$$

The pressure P is small at large values of film thickness H and large when the film thickness is small. This substitution also has the advantage that it eliminates all terms containing derivatives of the products of H and P or H and ϕ^* . Therefore from equation (7.9) and expansion of terms within equation (7.2) the Reynolds equation becomes

$$H^{3/2} \left[\frac{\partial}{\partial X} \left(\frac{\bar{\rho}}{\bar{\eta}} \frac{\partial \phi^*}{\partial X} \right) + \frac{1}{k^2} \frac{\partial}{\partial Y} \left(\frac{\bar{\rho}}{\bar{\eta}} \frac{\partial \phi^*}{\partial Y} \right) \right] - \frac{3\phi^*}{2} \left\{ \frac{\partial}{\partial X} \left(\frac{\bar{\rho}}{\bar{\eta}} H^{1/2} \frac{\partial H}{\partial X} \right) + \frac{1}{k^2} \frac{\partial}{\partial Y} \left(\frac{\bar{\rho}}{\bar{\eta}} H^{1/2} \frac{\partial H}{\partial Y} \right) \right\} = \frac{12U_b}{R_x} \frac{\partial(\rho H)}{\partial X} \quad (7.10)$$

Relaxation methods are usually used to solve equation (7.10) numerically. In the relaxation process the first step is to replace the differentials in equation (7.10) by finite difference approximations. The area to be considered is shown in Figure 7.1. The relaxation method relies on the fact that a function can be represented with sufficient accuracy over a small range by a quadratic expression. With this standard finite central-difference representation equation (7.10) can be rewritten as

$$A_{i,j}^* \phi_{i+1,j}^* + B_{i,j}^* \phi_{i,j-1}^* + C_{i,j}^* \phi_{i-1,j}^* + D_{i,j}^* \phi_{i,j+1}^* - L_{i,j}^* \phi_{i,j}^* - M_{i,j}^* = 0 \quad (7.11)$$

where

$$\mu^* = \frac{\bar{\rho}}{\eta}$$

$$A_{i,j}^* = 3\mu_{i+1,j}^* + \mu_{i-1,j}^*$$

$$B_{i,j}^* = \left(\frac{\bar{c}}{dk}\right)^2 (\mu_{i,j+1}^* + 3\mu_{i,j-1}^*)$$

$$C_{i,j}^* = \mu_{i+1,j}^* + 3\mu_{i-1,j}^*$$

$$D_{i,j}^* = \left(\frac{\bar{c}}{dk}\right)^2 (3\mu_{i,j+1}^* + \mu_{i,j-1}^*)$$

$$L_{i,j}^* = 4 (\mu_{i+1,j}^* + \mu_{i-1,j}^*) + 4 \left(\frac{\bar{c}}{dk}\right)^2 (\mu_{i,j+1}^* + \mu_{i,j-1}^*)$$

$$+ \frac{3}{2H_{i,j}^{3/2}} \left\{ \mu_{i+1,j}^* (H_{i+1,j})^{1/2} (3H_{i+1,j} - 4H_{i,j} + H_{i-1,j}) \right.$$

$$+ \mu_{i-1,j}^* (H_{i-1,j})^{1/2} (H_{i+1,j} - 4H_{i,j} + 3H_{i-1,j})$$

$$+ \left(\frac{\bar{c}}{dk}\right)^2 \left[\mu_{i,j+1}^* (H_{i,j+1})^{1/2} (3H_{i,j+1} - 4H_{i,j} + H_{i,j-1}) \right.$$

$$\left. + \mu_{i,j-1}^* (H_{i,j-1})^{1/2} (H_{i,j+1} - 4H_{i,j} + 3H_{i,j-1}) \right] \left. \right\}$$

$$M_{i,j}^* = \frac{24Ub}{\bar{d}R_x H_{i,j}^{3/2}} (\bar{\rho}_{i+1,j} H_{i+1,j} - \bar{\rho}_{i-1,j} H_{i-1,j})$$

Central differences are used in obtaining equation (7.11) because they produce better estimates than forward or backward differences. Central differences tend to average; forward and backward differences tend to "overshoot" and "undershoot" the value of the function. More details of the derivation of equation (7.11) are given in Hamrock (1976).

An example of a nodal structure that can be used in evaluating the pressure distribution in an elastohydrodynamic elliptical contact is shown in Figure 7.3. A nodal structure is considered to be suitable when the minimum film thickness within the conjunction does not change materially either when additional nodes are placed in the semimajor and semiminor axes or when the distances from the center of the contact to the edges of the computing zone are extended. Note that because of the dimensionalization of \tilde{x} and \tilde{y} the Hertzian contact area is represented by a circle of radius unity regardless of the ellipticity parameter.

From Figure 7.3 the features of this nodal structure are

$$\left. \begin{aligned} \tilde{m} = 4, \bar{n} = 1.15, \bar{\ell} = 1.6 \\ \bar{c} = 5, \bar{d} = 13 \end{aligned} \right\} \quad (7.12)$$

These values are used to define the nodal structure for most of the results presented in Chapters 8 and 9. The only exception is for a number of high-speed cases when the constant used to

determine the inlet distance \tilde{m} has to be extended from 4 to 6, with the other values being held constant.

7.4 Boundary and Initial Conditions

The following boundary conditions have been adopted:

(1) At the edges of the rectangular zone of computation (Figure 7.3) the pressure is zero. This implies that ϕ^* is also zero. Specifically this means that along the bottom of Figure 7.3 $\phi_{i,1}^* = 0$, along the left side $\phi_{1,j}^* = 0$, along the top $\phi_{i,16}^* = 0$, and along the right side $\phi_{67,j}^* = 0$.

(2) At the cavitation boundary

$$P = \frac{\partial P}{\partial X} = \frac{\partial P}{\partial Y} = 0$$

This condition, commonly known as the Reynolds condition, is satisfied by simply resetting $\phi_{i,j}^*$ to zero whenever it occurs as a negative value.

The pressure outside the Hertzian contact area is initially assumed to be zero, and therefore ϕ^* is also equal to zero.

That is, $\phi^* = 0$ when

$$(X - \tilde{m})^2 + (Y - \bar{l})^2 \geq 1$$

The pressure inside the Hertzian contact area is initially assumed to be Hertzian. Therefore from equation (3.5) the initial value of ϕ^* inside the contact area is

$$\phi^* = \frac{3FH^{3/2}}{2\pi abE'} \left[1 - (X - \tilde{m})^2 - (Y - \bar{\ell})^2 \right]^{1/2} \quad (7.13)$$

where

$$(X - \tilde{m})^2 + (Y - \bar{\ell})^2 < 1$$

7.5 Relaxation Method

Equation (7.11) represents a system of simultaneous equations. There will be as many unknown values of ϕ^* as there are nodal points and equations. This system of equations can thus be solved by the Gauss-Seidel iterative method. In calculating each nodal value of ϕ^* this method uses the most recently calculated values at the surrounding nodes involved in the application of the finite difference equation (7.11). That is, nodal values calculated on that cycle are used if available. A cycle is one complete set of calculations for all the nodes in the structure. The concept of the Gauss-Seidel approach is that the best information available should be used instead of waiting until the next cycle. If subscript n is the iterant and $\phi_{i,j}^*$ is the particular solution to be found, the relaxation formula can be expressed as

$$\phi_{i,j,n+1}^* = \phi_{i,j,n}^* - \lambda_c \left\{ M_{i,j}^* - A_{i,j}^* \phi_{i+1,j,n}^* - B_{i,j}^* \phi_{i,j-1,n+1}^* - C_{i,j}^* \phi_{i-1,j,n+1}^* - D_{i,j}^* \phi_{i,j+1,n}^* + L_{i,j}^* \phi_{i,j,n}^* \right\} / L_{i,j}^* \quad (7.14)$$

where λ_c is a relaxation factor.

Therefore equation (7.14) is used, starting with node (2,2) and then continuing with (3,2), . . ., (\bar{M} ,2) followed by (2,3), (3,3), . . ., (\bar{M} ,3) and ending with (2, \bar{N}), (3, \bar{N}), . . ., (\bar{M} , \bar{N}), where

$$\left. \begin{aligned} \bar{M} &= (\tilde{m} + \bar{n})\bar{d} - 1 \\ \bar{N} &= 2\bar{\ell}c - 1 \end{aligned} \right\} \quad (7.15)$$

The relaxation procedure described by equation (7.14) is continued until

$$\left. \begin{array}{l} \bar{N} \\ \text{Z} \\ j=2,3,\dots \end{array} \right\} \left. \begin{array}{l} \bar{M} \\ \text{Z} \\ i=2,3,\dots \end{array} \right\} \frac{|\phi_{i,j,n+1}^* - \phi_{i,j,n}^*|}{\phi_{i,j,n+1}^*} < 0.1$$

The relaxation method provides values of $\phi_{i,j}^*$ for every node within the nodal structure. Once a solution has been obtained, the dimensionless pressure can be computed from the relationship

$$P_{i,j} = \phi_{i,j}^* (H_{i,j})^{-3/2} \quad (7.16)$$

With these new values of the dimensionless pressure, revised values of the dimensionless viscosity, density, and film thickness can be evaluated. Thus the coefficients of equation (7.14) ($A_{i,j}^*$, $B_{i,j}^*$, $C_{i,j}^*$, $D_{i,j}^*$, $L_{i,j}^*$, and $M_{i,j}^*$) may also change. Accordingly it is necessary to reenter the relaxation loop. This process is continued until the following inequality is satisfied:

$$\sum_{j=2,3,\dots}^{\bar{N}} \sum_{i=2,3,\dots}^{\bar{M}} \frac{|P_{i,j,n+1} - P_{i,j,n}|}{P_{i,j,n+1}} < 0.1$$

7.6 Normal Applied Load and Flow Rate

The initial value of the constant H_0 in equation (7.5) has to be estimated, but thereafter the task is to find the correct value. To do this, the integrated normal load must be evaluated, where

$$\bar{F} = E'_{ab} \int_0^{\bar{m}+\bar{n}} \int_0^{2\bar{\ell}} P \, dY \, dX \quad (7.17)$$

According to Simpson's rule this double integral can be written as

$$\bar{F} = \frac{4E'ab}{9cd} \sum_{i=2,3,\dots}^{\bar{m}+\bar{n}-1} 2^{q_1} \left(\sum_{j=2,3,\dots}^{2\bar{l}-1} 2^{q_2} P_{i,j} \right) \quad (7.18)$$

where

$$\begin{aligned} q_1 &= 0 && \text{if } i \text{ is odd} \\ q_1 &= 1 && \text{if } i \text{ is even} \\ q_2 &= 0 && \text{if } j \text{ is odd} \\ q_2 &= 1 && \text{if } j \text{ is even} \end{aligned}$$

A new value of H_0 can be estimated from the following expression and the current values of H_0 and \bar{F} and the applied load:

$$\bar{H}_0 = H_0 \left(\frac{F}{\bar{F}} \right) \quad (7.19)$$

The film thickness can now be recalculated at each node for the new value of \bar{H}_0 from equation (7.5), and reentry into the relaxation process is required. This process is continued until the following convergence is satisfied:

$$\frac{|\bar{F} - F|}{F} < 0.0005$$

Once this convergence criterion has been satisfied, the values of pressure and film thickness in and around the contact are established.

The mass flow rate per unit width in the \tilde{x} and \tilde{y} directions shown in Figure 7.1 can be written as

$$q_{\tilde{x}} = u\bar{p}h - \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial \tilde{x}}$$

$$q_{\tilde{y}} = - \frac{\rho h^3}{12\eta} \frac{\partial p}{\partial \tilde{y}}$$

These equations can be written in dimensionless form as

$$\bar{Q}_x = U\bar{\rho}H - \frac{\bar{\rho}H^3}{12\bar{\eta}} \left(\frac{R}{b}\right) \frac{\partial P}{\partial X} \quad (7.20)$$

$$\bar{Q}_y = - \frac{\bar{\rho}H^3}{12\bar{\eta}} \left(\frac{R}{a}\right) \frac{\partial P}{\partial Y} \quad (7.21)$$

Furthermore the total flow rates and flow angles at any node can be written as

$$\bar{Q} = \left(\bar{Q}_x^2 + \bar{Q}_y^2\right)^{1/2} \quad (7.22)$$

$$\gamma_a = \tan^{-1} \left(\frac{\bar{Q}_y}{\bar{Q}_x}\right) \quad (7.23)$$

7.7 Flow Charts

Figures 7.4 and 7.5 are flow charts for the numerical solution on the digital computer of the equations developed in the analysis. Figure 7.4 is the flow chart for the main program. There are essentially three loops within the main program: the relaxation loop, the pressure loop, and the normal-load loop. In the relaxation loop $\Phi_{i,j,n+1}^*$ is generated. In the pressure loop the new values of $\Phi_{i,j,n+1}^*$ of the relaxation loop result in new values of pressure $P_{i,j,n+1}$, which in turn result in new values of film thickness $H_{i,j}$, viscosity $\bar{\eta}_{i,j}$, and density $\bar{\rho}_{i,j}$. The final loop, the normal-load loop, ensures that the integrated normal applied load agrees with the initially specified value.

Figure 7.5 is the flow chart for the film thickness subroutine SUB6. A number of calculations occur only once and need not be repeated on reentering this subroutine. With each new pressure distribution the elastic deformation is recalculated, and this subroutine is left with a new film thickness and therefore a new value of $\Phi_{i,j,n+1}^*$.

7.8 Closure

In this chapter a procedure for the numerical solution of the complete, isothermal elastohydrodynamic lubrication problem

for elliptical contacts has been outlined. This procedure calls for the simultaneous solution of the elasticity and Reynolds equations. In the elasticity analysis the contact zone was divided into equal rectangular areas, and it was assumed that a uniform pressure was applied over each area. In the numerical solution of the Reynolds equation the parameter Φ^* equals $PH^{3/2}$, where P is the dimensionless pressure and H the dimensionless film thickness, was introduced in order to ease the relaxation process. The nodal structure and the boundary and initial conditions were given and a Gauss-Seidel relaxation method was used. The computer program has three major loops: the relaxation loop, the pressure loop, and the normal-load loop. The last loop requires the integrated hydrodynamic load-carrying capacity to be in agreement with the applied load within some specified tolerance. When all three loops converge, the pressure and film thickness in and around the elliptical contact area are established.

SYMBOLS

A	constant used in equation (3.113)
$\left. \begin{array}{l} A^*, B^*, C^* \\ D^*, L^*, M^* \end{array} \right\}$	relaxation coefficients
A_V	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
\bar{a}	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
\bar{b}	$b/2\bar{m}$
C	dynamic load capacity, N
C_V	drag coefficient
C_1, \dots, C_8	constants
c	$19,609 \text{ N/cm}^2$ ($28,440 \text{ lbf/in}^2$)
\bar{c}	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
\tilde{D}	material factor
\bar{D}	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
\bar{d}	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d'_e	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2 \left/ \left(\frac{1 - \nu_a^2}{E_a} + \frac{1 - \nu_b^2}{E_b} \right) \right.$, N/m^2
E _a	internal energy, m^2/s^2
\tilde{E}	processing factor
E ₁	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
\mathcal{E}	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F*	normal applied load per unit length, N/m
\tilde{F}	lubrication factor
\bar{F}	integrated normal applied load, N
F _c	centrifugal force, N
F _{max}	maximum normal applied load (at $\psi = 0$), N
F _r	applied radial load, N
F _t	applied thrust load, N
F _{ψ}	normal applied load at angle ψ , N
\mathcal{F}	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f _b	rms surface finish of ball, m
f _r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G*	fluid shear modulus, N/m^2
\tilde{G}	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
\hat{H}	dimensionless film thickness, $H(W/U)^2 = F^2 h/u^2 \eta_0^2 R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
\tilde{H}_c	dimensionless central film thickness obtained from least-squares fit of data
\tilde{H}_{min}	dimensionless minimum film thickness obtained from least-squares fit of data
\bar{H}_c	dimensionless central-film-thickness - speed parameter, $H_c U^{-0.5}$
\bar{H}_{min}	dimensionless minimum-film-thickness - speed parameter, $H_{min} U^{-0.5}$
\bar{H}_0	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, $m N s^2$
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
\bar{J}	polar moment of inertia, $m N s^2$
K	load-deflection constant
k	ellipticity parameter, a/b
\bar{k}	approximate ellipticity parameter
\tilde{k}	thermal conductivity, $N/s \text{ } ^\circ C$
k_f	lubricant thermal conductivity, $N/s \text{ } ^\circ C$
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
L_1, \dots, L_4	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, $N s^2/m$
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
\tilde{m}	dimensionless inlet distance (Figures 7.1 and 9.1)
\bar{m}	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
\bar{n}	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
P_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$P_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
\bar{Q}	dimensionless mass flow rate per unit width, $q_{n0}/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_C	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
$r_{ax}, r_{bx},$ r_{ay}, r_{by} }	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
\bar{r}	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
\tilde{T}	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u / E' R_x$
u	surface velocity in direction of motion, $(u_a + u_b) / 2$, m/s
\bar{u}	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F / E' R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x / R_x
Y	dimensionless coordinate, y / R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \bar{x}, \bar{x}_1$	coordinate system
$y, \tilde{y}, \bar{y}, \bar{y}_1$	
$z, \tilde{z}, \bar{z}, \bar{z}_1$	
α	pressure-viscosity coefficient of lubrication, m^2 / N
α_a	radius ratio, R_y / R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, $N / m^2 \cdot s$
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{\circ}\text{C}^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x , m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ϵ	coefficient of determination
ϵ_1	strain in axial direction
ϵ_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_{∞}	$6.31 \times 10^{-5} \text{ N s/m}^2$ (0.0631 cP)
θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3 \alpha_a)^{-1}$
λ_c	relaxation factor

μ	coefficient of sliding friction
μ^*	$\overline{\rho/\eta}$
ν	Poisson's ratio
∇	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, $N\ s^2/m^4$
$\overline{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, $N\ s^2/m^4$
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$\rho H^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_L	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_0	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c angular velocity of ball around shaft center, rad/s

ω_s ball spin rotational velocity, rad/s

Subscripts:

a solid a

b solid b

c central

bc ball center

IE isoviscous-elastic regime

IR isoviscous-rigid regime

i inner race

K Kapitza

min minimum

n iteration

o outer race

PVE piezoviscous-elastic regime

PVR piezoviscous-rigid regime

r for rectangular area

s for starved conditions

x,y,z coordinate system

Superscript:

($\bar{\quad}$) approximate

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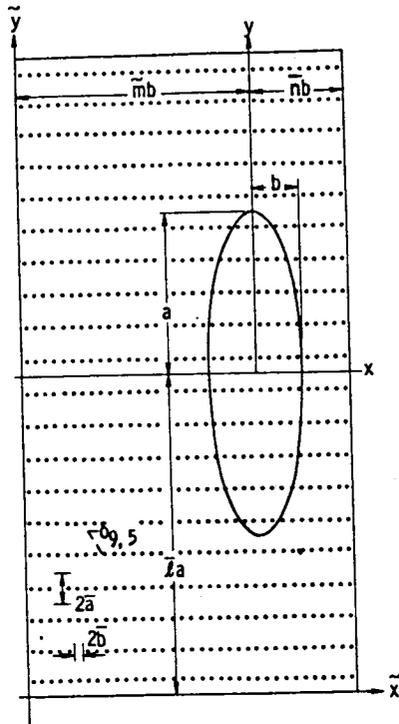


Figure 7.1. - Division of area in and around contact zone into equal rectangular areas.

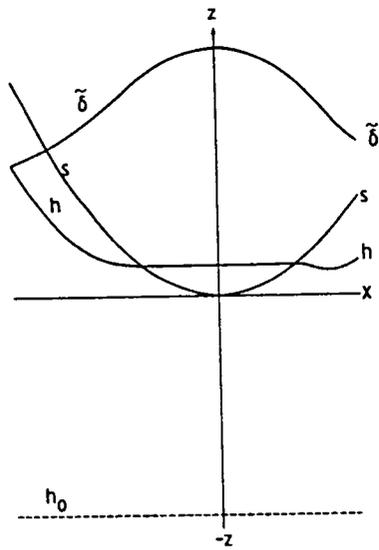


Figure 7.2. - Components of film thickness for ellipsoidal solid near plane.

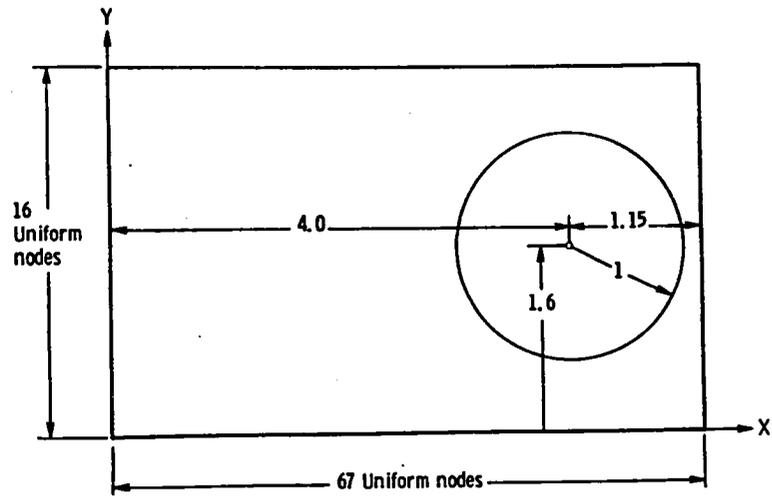


Figure 7.3. - Nodal structure used for numerical calculations.

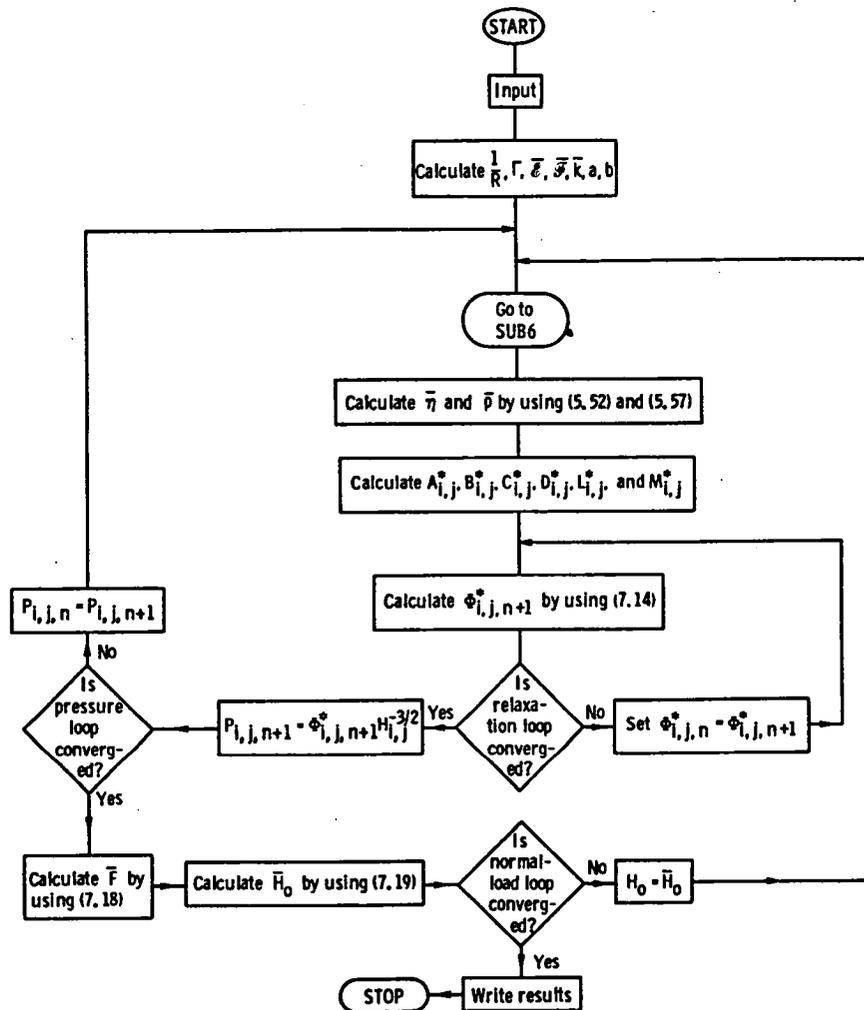


Figure 7.4. - Flow chart of main program.

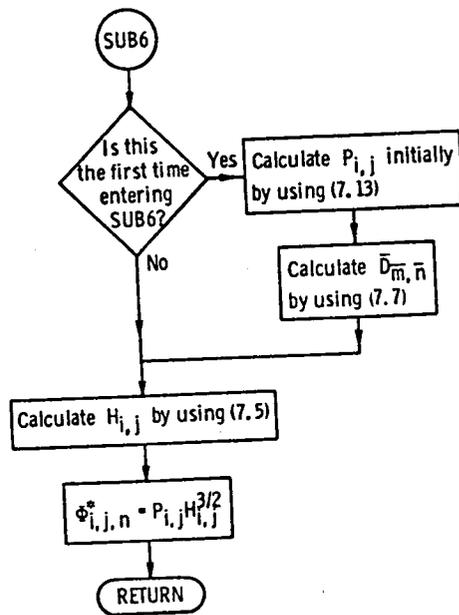
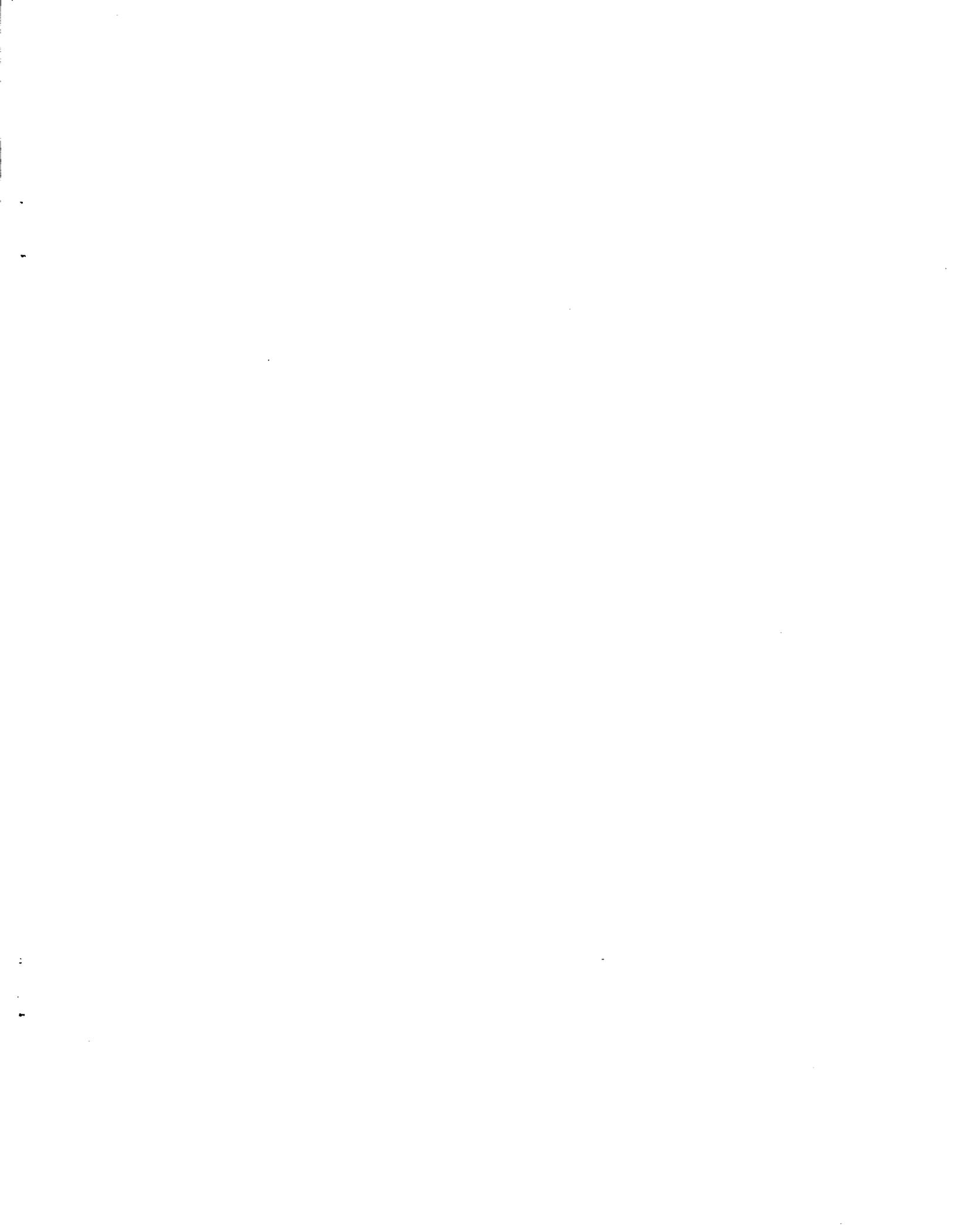


Figure 7.5. - Flow chart of subroutine SUB6.



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16. Abstract The isothermal elastohydrodynamic lubrication (EHL) of a point contact was analyzed numerically by simultaneously solving the elasticity and Reynolds equations. In the elasticity analysis the contact zone was divided into equal rectangular areas, and it was assumed that a uniform pressure was applied over each area. In the numerical analysis of the Reynolds equation, a phi analysis (where phi is equal to the pressure times the film thickness to the 3/2 power) was used to help the relaxation process. The EHL point contact analysis is applicable for the entire range of elliptical parameters and is valid for any combination of rolling and sliding within the contact.			
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