The Deep Space Network — Noise Temperature Concepts, Measurements, and Performance

C.T. Stelzried

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California Institute of Technology
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Extremely sensitive receiving systems are required for deep space communications. The NASA Deep Space Network is investigating the use of higher operational frequencies for improved performance. Noise temperature and noise figure concepts are used to describe the noise performance of these receiving systems. The ultimate sensitivity of a linear receiving system is limited by the thermal noise of the source and the quantum noise of the receiver amplifier. The sensitivity of a receiving system consisting of an "ideal" linear receiver with a 2.7 K source temperature (-194.3 dBm/Hz assuming hf<kT) degrades significantly at frequencies greater than about 50 GHz.

The atmosphere, antenna and receiver amplifier of an earth station receiving system are analyzed separately and as a system. Performance evaluation and error analysis techniques are investigated. System noise temperature and antenna gain parameters are combined to give an overall system figure of merit G/T. This parameter is useful for system sensitivity specifications and is conveniently evaluated using radio "stars".

Radiometers are used to perform radio "star" antenna and system sensitivity calibrations. These are analyzed and the performance of several types compared to an idealized total power radiometer. Thermal noise standards are useful for laboratory and field noise performance evaluations for all aspects of the receiving system. Proper account is taken of the transmission line degradation to realize the full potential of these devices.

The theory of radiative transfer is applicable to the analysis of transmission medium loss, which is useful for the evaluation of both thermal noise standards and the atmosphere. A power series solution in terms of the transmission medium loss is given for the solution of the noise temperature contribution.
1. Introduction

Extremely sensitive receiving systems are required in deep space communications. The NASA Deep Space Network is investigating the use of higher operational frequencies for improved performance. Noise temperature and noise figure concepts are used to describe the noise performance of these receiving systems.

Noise in a receiving system is defined as an undesirable disturbance corrupting the information content. Noise in the context of this report is assumed random with continuous spectral power density. The sources of noise can be separated into external noise and internal noise.

Sources of external noise (Mumford, 68, 3-6; CCIR, 78, 422-427) include: lightning, cosmic, solar, planetary, galactic, radio stars, emission from atmospheric constituents and man-made noise. Cosmic noise is considered to be the residual radiation (≈2.7 K) due to events occurring during the origin of the universe. Solar, planetary and radio "star" noise occur due to radio emission from these sources intercepted by the antenna. Man-made noise includes ignition systems, spark discharges, and transmission of noise and noiselike signals along power lines. Sources of internal noise include thermal, shot, current, Barkhausen, and quantum noise. In this report, thermal noise is treated as the limiting noise source for microwave receiving systems.

Thermal noise is caused by random motion of free electrons in a conductor excited by thermal agitation. The signal-to-noise performance of a receiving system composed of a source and an amplifier is expressed quantitatively most conveniently in terms of noise temperature concepts.

Noise temperature and antenna gain definitions, performance and measurement techniques are presented and analyzed for communications receiving systems. The atmosphere, antenna and receiver are treated individually and as a system.
System noise temperature and antenna gain parameters are combined to give an overall system figure of merit \( G/T \). This parameter is useful for system sensitivity specifications and is conveniently evaluated using radio "stars".

Radio "stars" are used to evaluate both antenna and total receiving system sensitivity performance by measurement of the increased output noise when the antenna is directed to these sources. Measurement error analysis techniques are necessary for proper interpretation of the results. Radiometers are used to perform radio "star" calibrations. These are analyzed and the performance of several types compared to an idealized total power radiometer. Thermal noise standards are useful for laboratory and field noise performance evaluations for all aspects of the receiving system. Proper account is taken of the transmission line degradation to realize the full potential of these devices.

The atmosphere is extremely important to the performance of receiving systems. Not only is there a direct loss due to rain, clouds and other constituents of the atmosphere, but in low noise systems the thermal emission can cause more signal-to-noise degradation than the direct loss. These effects are evaluated.

The theory of radiative transfer is applicable to the analysis of transmission medium loss, which is useful for the evaluation of both thermal noise standards and the atmosphere. A power series solution in terms of the transmission medium loss is given for the solution of the noise temperature contribution. The inverted solution for loss in terms of the measured thermal noise is useful for the determination of atmospheric loss from radiometric noise temperature measurements. These techniques can be used for calibrations of the atmospheric liquid water and water vapor content.
1. References


2. **Noise Temperature**

Thermal noise (Johnson, 28, 97; Nyquist, 28, 110) is caused by random motion of free electrons in a conductor excited by thermal agitation. The available thermal noise power $P_n$ (Mumford, 68, 4; Oliver, 65, 441) from a source at the amplifier output is given by ($G >> 1$)

$$P_n = kTBG \left( \frac{hf/kT}{e^{hf/kT} - 1} \right), \text{ W}$$  \hspace{1cm} (2-1)

where

- $T = \text{source temperature, K}$
- $h = \text{Planck constant} = 6.6262 \times 10^{-34} \text{ J-s}$
- $k = \text{Boltzmann constant} = 1.3806 \times 10^{-23} \text{ J/K}$
- $f = \text{operating frequency, Hz}$

$$S = \frac{1}{G} \int_0^\infty G(f) \, df = \text{noise bandwidth, Hz}$$

$G(f) = \text{available power gain, ratio}$

$G = \text{maximum available power gain, ratio}$

The amplifier output is approximately (disregarding the contribution of the amplifier; $hf << kT$)

$$P_n = kTBG$$  \hspace{1cm} (2-2)

Eqs. 2-1 and 2-2 are shown plotted in Fig. 2-1 for a large range of $hf/kT$ values. Most microwave applications are restricted to the region near the origin ($hf << kT$).

---

1Note that $(hf/kT) = 0.048 \, f(\text{GHz})/T(\text{K}) = 0.00048$ at 1 GHz and 100 K and $= 1$ at 208 GHz and 10 K indicates that Eq. (2-2) is an extremely good approximation for most microwave applications.
It is computationally convenient to define a temperature $T'$ such that

$$P_n = kT'BG = kTBG \left( \frac{h/\nu T}{e^{h/\nu T} - 1} \right)$$  \hspace{1cm} (2-3)

$T'$ can be conveniently found by subtracting a correction $T_c$ (Viggh, 76, 54) from $T$, so that

$$(P_n/kBG) = T' = T - T_c$$  \hspace{1cm} (2-4)

where

$$T_c = T \left( 1 - \frac{h/\nu T}{e^{h/\nu T} - 1} \right)$$

or conveniently

$$T_c = 0.024f(\text{GHz}) - 0.000192 \left[ \frac{f(\text{GHz})}{T+\ldots} \right]^2$$  \hspace{1cm} (2-5)

These correction terms are shown plotted in Fig. (2-2). At 32 GHz, the cosmic background temperature (Penzias, 65, 419; Otoshi, 75, 174) of $\approx 2.7 ^\circ K$, correctly defined for use with Eq. (2-1), is "corrected" to $\approx 2.0 ^\circ K$ for use with Eq. (2-2).

The quantum\footnote{Eq. (2-6) is appropriate only for linear amplification. At optical frequencies, using discrete photons, techniques may exist (Pierce, 60, 320) to circumvent this limitation. Equating $kTB$ to quantum noise $h/\nu B$ results in an equivalent quantum noise temperature, $T_q = (h/\nu B/k)$. Although $T_q$ is a fictitious temperature, it is useful for computational analysis and can be used with Eq. (2-2) to compute $P_n$ accounting for quantum noise (Stelzried, 82). This is discussed further by others (Siegman, 64, 412; Weber, 57, 540). Gulkis, 62, discusses the intermediate frequency ranges accounting for quantum and thermal noise for both linear and incoherent amplifiers.} noise limit (Oliver, 65, 450) of a linear amplifier (where both phase and amplitude information is retained), a manifestation of the quantum mechanics uncertainty principle, is given by
Fundamental limits of an ideal receiving system sensitivity are determined by the sum of the source thermal noise and the quantum noise limit of an ideal amplifier (since these noise sources are uncorrelated: Pierce, 80, 320).

\[ P_n = \frac{h \nu B G}{k T} \left( \frac{h \nu / k T}{e^{h \nu / k T} - 1} \right) + h \nu B G \]

or

\[ \left( \frac{P_n}{k B G} \right) = T' + T_q \]

This is plotted in Figs. (2-1, 3 and 4) as functions of \((h \nu / k T)\), frequency and temperature. The quantum noise limit and thermal noise are equal when \((h \nu / k T) = \ln 2 = 0.69\) as shown in Fig. 2-1. In Fig. 2-4 the value of \(P_n\) for \(h \nu >> k T\) is given by \(h \nu / k\) (or \(T_q\)) and for \(h \nu << k T\) by \(T + h \nu / 2k\) (or \(T + T_q / 2\)).
2. References


Fig. 2-1. Thermal and quantum noise vs (hf/kT)
Fig. 2-2. Thermal noise temperature correction $T_c$ vs frequency as a function of temperature.
Fig. 2-3. Thermal and quantum noise vs frequency as a function of temperature
Fig. 2-4. Thermal and quantum noise vs temperature as a function of frequency
3. **Antenna Noise Temperature**

The noise temperature of an antenna, from Eq. 2-2 (also Rusch, 70, 71), is defined, \( h \beta < k T \),

\[
T_a = \frac{i_A}{k B}, \text{K}
\]

(3-1)

where

\[
P_A = \text{noise power delivered by the antenna, over a bandwidth } B, \text{ into a matched termination, } W
\]

Also

\[
T_a = \frac{1}{4\pi} \int T(\Omega) G(\Omega) \, d\Omega
\]

(3-2a)

\[
= \int_4 \pi^2 T(\Omega) P(\Omega) \, d\Omega / \int_4 \pi^2 P(\Omega) \, d\Omega
\]

(3-2b)

where

\[
T(\Omega) = \text{equivalent blackbody temperature of area } d\Omega, \text{K}
\]

\[
P(\Omega) = \text{normalized antenna pattern gain in direction } \Omega, \text{ ratio}
\]

\[
G(\Omega) = \text{antenna gain in direction } \Omega, \text{ ratio}
\]

\[
G_m = \text{antenna maximum gain, ratio}
\]

\[
G_m = \frac{4\pi A_e}{\lambda^2}
\]

\[
A_e = \text{antenna effective area, } m^2
\]

\[
D_e = \text{antenna effective diameter, } m
\]

Measurement requirements dictate the appropriate method for evaluation of \( T_a \). Absolute antenna calibrations (Sec. 10) require thermal noise standards. The individual contributions to \( T_a \) include cosmic noise (Penzias, 65, 419), atmospheric noise (Sec. 4), spillover (Schuster, 62, 286), and ground contributions (Otoshi, 65, 262), all evaluated separately.
Antenna noise temperature calibrations involve the measurement of the increase (Sec. 14) in antenna temperature (antenna gain known) to evaluate an external noise source (such as a planet, radio star, the moon or the sun). Alternately, a known external noise source can be used to evaluate the antenna performance (radio stars: Freiley, 80; 16P; Baars, 73, 401; Baars, 77, 99; Sun: Kusseki, 76; Linsky, 73).

The increase in antenna temperature due to a "small" ($\Omega_s \ll \Omega_A$) source in the antenna beam far field ($R > 2D^2/\lambda$; Silver, 49, 199) is

$$\Delta T_a = T_s \Omega_s / \Omega_A$$

$$= T_s A_s / (R \lambda)^2$$

where

- $T_s$ = average temperature of source, K
- $\Omega_s$ = source solid angle, rad$^2$
- $A_s = \int d\Omega$ on-source
- $A_s / R^2$
- $R$ = range from antenna to source, m
- $A$ = antenna physical area, m$^2$
- $A_s$ = projected surface area of source, m$^2$
- $\Omega_A$ = antenna beam solid angle, rad$^2$
- $= \int P(\Omega) d\Omega$
- $= \lambda^2 / A_c$

3-2
Eq. 3-3 provides a convenient form for estimating $\Delta T_a$ for a distant radio source such as a planet (Fig. 3-1). For the JPL 64-m antenna at 8.5 GHz (Freiley, 80, 168 14, Fig. 3-2, $D_e = 45.6$ m), $\Delta T_a = 55$ K for Venus at inferior conjunction. It is frequently convenient to use (assuming an antenna which absorbs half of the incident energy of a randomly polarized source),

$$\Delta T_a = S A_e / 2k \quad \text{(3-4)}$$

where

$$S = \text{source flux density, J-m}^2$$

(1 Jansky ($J_y$) = $10^{-26}$ J-m$^2$)

$$= 2k T_S A_s / (R \lambda)^2, (\Omega \ll \Omega_A)$$

External contributions to antenna noise temperature as a function of frequency can be estimated from Fig. 3-3 (Smith, 82). At zenith, galactic noise (diurnal variations to $= 2900$ K at 0.1 GHz) dominates at the lower frequencies and atmospheric noise (discussed in Sec. 4) dominates at the higher frequencies. The lowest antenna noise temperatures (for ground-based antennas) occur between these regions from about 1.5 to 15 GHz.

---

3For completeness (Kraus, 66, 86), Planck's law:

$$S = 2 \, hf^3 \, \Omega_s / c^2 \, (ehf/kT_S - 1)$$

Rayleigh-Jean's law ($hf \ll kT$):

$$S = 2 \, kT_S \, \Omega_s / \lambda^2$$

Wein's law ($hf \gg kT$):

$$S = 2 \, hf^3 \, \Omega_s / c^2 (ehf/kT_S)$$
References


Fig. 3-1. $\Delta T_a$ vs $D_e$ (or $A_e$) for various radio source parameter values
Fig. 3-2. Photograph of JPL 64-m antenna, DSS 14, Goldstone, California
Fig. 3-3. Worldwide (0.1-100 GHz) minimum external noise levels (solid curves); other noise sources of interest are given by dashed curves (CCIR, 78)
4. Effect of the Atmosphere on Antenna Temperature

The earth's atmosphere (Smith and Waters, 81, 1-45) consists mostly of the dry components oxygen (~21% by volume), nitrogen (~78% by volume) and argon (~1% by volume), and wet components (water vapor, clouds and rain). Water vapor at 100% relative humidity is approximately 1.7% by volume assuming the U.S. Standard Atmosphere, 15°C, at sea level.

A communications link through the atmosphere suffers several forms of degradation (Crane 76, 177-200; Straiton, 75, 595; Damosso, 76, 98; Fogarty, 75, 441; Daywitt, 78, 1-39; Hogg, 59, 1417; Crane, 81, 196-209). The decreased signal-to-noise (SNR) ratio system performance due to atmospheric attenuation and increased noise temperature is discussed further in Sec. 7.

The total loss through the atmosphere (Fig. 4-1) is given by (neglecting scattering)

\[ L = e^\tau, \text{ ratio } (\geq 1) \]  \hspace{1cm} (4-1)

where

\[ \tau = \text{total atmospheric absorption (optical depth), nepers} \]

\[ L(\text{dB}) = (10 \log e) \tau = 4.343\tau \]

\[ = \int_0^L \alpha(x) \, dx \]

\[ \alpha(x) = \text{absorption coefficient of the atmosphere at } x, \text{ nepers/m} \]

\[ x = \text{slant distance along propagation path, m} \]

\[ (x = 0 \text{ at surface}, \tau = L \text{ at "top" of atmosphere}) \]

\[ L = \text{total atmospheric slant path length, m} \]

\[ ^4 \text{Scattering should be considered for rain at frequencies above } \approx 10 \text{ GHz, and for clouds at frequencies above } \approx 100 \text{ GHz (Tsang, 77, 650, Flock, 79, 176).} \]

\[ ^5 \text{It is sometimes convenient to integrate from the "top" of the atmosphere to the surface (Waters 76, 142).} \]
From the theory of radiative transfer (neglecting scattering, assuming \( hf < kT \), see Sec. 20, Stelzried, 81a, 73, Slobin 81a, Chandrasekhar, 60), the thermal noise temperature defined at the receiving system input is given by

\[
T'_s = (T_s/L) + \int_0^L T(x) \alpha(x)e^{-\tau(x)} dx, \quad K
\]  

(4-2)

where

- \( T_s \) = source temperature (usually \( \approx 2.7 \) K, due to the cosmic background noise temperature), K
- \( T(x) \) = physical temperature of medium at \( x \), K
- \( \tau(x) \) = attenuation between 0 and \( x \), nepers

\[
= \int_0^x \alpha(x') \, dx'
\]

Computed clear sky\(^6\) zenith atmospheric attenuation and noise temperature contributions are shown in Figs. 4-4, 5 and Table 4-1 as functions of frequency, surface water vapor density (i.e. absolute humidity), and elevation angle. Microwave absorption is directly proportional to the water vapor density (actually, the total integrated content along the line of sight). For temperate latitudes in summer (20°C), the average surface water vapor density is about 7.5 g/m\(^3\) (Van Vleck, 51, 647; Bean and Dutton, 68, 269).

At saturation (sea level, 20°C), the density is \( \approx 17 \) g/m\(^3\); 3-5 g/m\(^3\) may be more appropriate for arid regions such as Goldstone, CA. An approximate measured value of 9 K at 31.4 GHz is obtained from the tipping curve calibrations at Goldstone as shown in Fig. 4-6. The actual value for a particular day depends on the surface water vapor density; this results in the measurement spread shown. Operating frequencies are usually chosen to be well away from the water vapor line (\( \approx 22 \) GHz) and the oxygen line (\( \approx 60 \) and \( \approx 118 \) GHz).

Minimum attenuation occurs with low humidity, low frequency and high elevation angle (Fig. 4-1, \( Z = 0^\circ \)).

---

\(^6\)"Clear sky" indicates an atmosphere containing only gaseous constituents (oxygen, nitrogen and water vapor) with no liquid water (rain or clouds).
For a homogeneous, isothermal atmosphere, \( a(x) = a_0 \), and temperature 
\( T(x) = T_p \). From Eq. 4-2,

\[
T_s' = T_s + (1 - 1/L)(T_p - T_s)
\]

(4-3)

where

\[
L = e^{-\alpha T} = e^{-a}
\]

\( T_p = \) equivalent atmospheric physical temperature, K 
\((\approx 260-280 \text{ K}; \text{Stelzried}, 81b, 87)\)

Atmospheric loss is frequently determined using Eq. 4-3 and measuring \( T_s' \) radiometrically,

\[
i = 1 + (T_s' - T_s)/(T_p - T_s')
\]

(4-4)

Atmospheric loss measurements are investigated further in Section 15.

The effect (Ippolito, 81b) of clouds and rain is to both further attenuate the signal and increase the atmospheric noise temperature. The calculated effect of a one- or two-cloud model (Slobin, 81a, 25) is tabulated in Table 4-2 at S, X and K\(_a\) bands. This is shown as a function of frequency in Fig. 4-7 for cloud water particle density of 0.5 gm/m\(^3\). Figs. 4-8 to 4-11 show photographs of clouds taken from the area below the JPL Mesa antenna range (R. Clauss, 82). A two-frequency (20.7 and 32.4 GHz) water vapor radiometer observed the approximate increase in noise temperature due to the clouds shown in the circled region of the photographs (Clauss, 82, Slobin, 81c). The range of measured values is indicated in Fig. 4-7. Statistics of cloud noise temperature and attenuation at various sites in the United States, Alaska, and Hawaii are available in Slobin, 82.

Comprehensive treatments of the effects of rain in satellite communications have been published by Hogg (75, 1308-1331), Ippolito (81a, 697-727), Lin (80, 183-228) and Crane, (80, 1717-1733). The attenuation of a microwave signal propagating through rain is given by (Olsen, 76, 318).
\[
A = \int_0^L \alpha \, dx, \text{ dB} \quad (4-5)
\]

where

\[\alpha = \text{absorption coefficient, dB/km}\]
\[L = \text{length of rain cell, km}\]
\[= \text{specific attenuation of rain cell, dB/km}\]
\[a, b = \text{empirical coefficients}\]
\[R = \text{rain rate, mm/hour}\]

Table 4-3 tabulates \(a\) and \(b\) as a function of frequency and rain rate.

The combined effect (Ippolito, 81b) of variations in clouds, rain and clear sky surface water vapor density on the communication link performance is determined empirically. These effects are measured as a function of frequency, geographical location and time of year. Typical statistical multiyear 11-GHz attenuation data is shown in Table 4-4 for various geographical locations.

Fig. 4-12 shows the percent of time the zenith X-band atmospheric noise temperature is above the clear sky baseline for Goldstone, CA (data obtained with noise adding radiometers, Sec. 19). Fig. 4-13 shows similar data using a 30° elevation angle: curve 1, using CCIR data for an arid region, rain only; curve 2, DSS 13, measured X-band data; curves 3, 4 and 5, 810-5 Rev. D data (probably too pessimistic); curve 6, DSS 13 31.4-GHz data. Curve 6, the result of 1500 hours of measured data, taken at Goldstone, DSS 13, during typical winter weather indicates that noise temperature increases due to rain and clouds of more than 30 K occur less than 2% of the time. A comparison of predicted 8.5- and 31.4-GHz confidence levels (for atmospheric loss and noise temperature of the Goldstone region) is shown in Table 4-5 (see Sec. 7 for 8.5- and 31.4-GHz link performance comparison). For this comparison, the 8.5-GHz values are based on the CCIR model (Fig. 4-13) for arid regions; the 31.4-GHz values are based on results of the 1500 hours of measured data together with 396 radiosonde measurements at Edwards Air Force base (Clauss, 82). This comparison is believed to best characterize the effects of the weather upon the 8.5- and 31.4-GHz link performance without the degradation of non-weather effects.
The effects of water collecting on the antenna feed horn cover have been evaluated at X-band (Slobin, 82). Special techniques are required to minimize the resulting increased noise temperature, especially with "weathered" plaster horn cover material (Hoffman, 79).
4. References


4. References (cont.)

1979

1980

1980

1981

1981

1981

1981a

1981b

1981a

1981b

1981c

1981a

4-7


Table 4-1. Clear sky zenith atmosphere noise temperature as a function of frequency and surface water vapor density (W) for sea level and DSS 14 Goldstone, CA (Slobin, 81b, 161)

<table>
<thead>
<tr>
<th>Location and altitude</th>
<th>Atmosphere components</th>
<th>2.3 GHz</th>
<th>8.5 GHz</th>
<th>21.0 GHz</th>
<th>32.0 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$O_2 + W(g/m^3)$</td>
<td>T,K, A, dB</td>
<td>T,K, A, dB</td>
<td>T,K, A, dB</td>
<td>T,K, A, dB</td>
</tr>
<tr>
<td>Sea level</td>
<td>$O_2 + 0.0$</td>
<td>2.12</td>
<td>0.035</td>
<td>2.29</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 3.0$</td>
<td>2.13</td>
<td>0.035</td>
<td>2.48</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 7.5$</td>
<td>2.15</td>
<td>0.035</td>
<td>2.78</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 10.0$</td>
<td>2.16</td>
<td>0.036</td>
<td>2.94</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 15.0$</td>
<td>2.18</td>
<td>0.036</td>
<td>3.28</td>
<td>0.053</td>
</tr>
<tr>
<td>H = 0.000 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSS 14</td>
<td>$O_2 + 0.0$</td>
<td>1.68</td>
<td>0.028</td>
<td>1.80</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 3.0$</td>
<td>1.69</td>
<td>0.028</td>
<td>1.98</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 7.5$</td>
<td>1.70</td>
<td>0.028</td>
<td>2.24</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 10.0$</td>
<td>1.71</td>
<td>0.028</td>
<td>2.39</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>$O_2 + 15.0$</td>
<td>1.73</td>
<td>0.029</td>
<td>2.69</td>
<td>0.043</td>
</tr>
</tbody>
</table>
Table 4-2. Sample cloud models and S-, X-, and K-band zenith atmospheric noise temperature contributions
(Slobin, 8la, 23)

<table>
<thead>
<tr>
<th>Case</th>
<th>Lower Cloud</th>
<th>Upper Cloud</th>
<th>Remarks</th>
<th>S-band (2.3 GHz)</th>
<th>X-band (8.5 GHz)</th>
<th>Kα-band (32 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density g/m</td>
<td>Base km</td>
<td>Top km</td>
<td>Density g/m</td>
<td>Base km</td>
<td>Top km</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>1.0</td>
<td>1.2</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td>3.0</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0.5</td>
<td>3.0</td>
</tr>
<tr>
<td>9</td>
<td>0.7</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0.7</td>
<td>3.0</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>11</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.5</td>
<td>1.0</td>
<td>3.5</td>
</tr>
<tr>
<td>12</td>
<td>1.0</td>
<td>1.0</td>
<td>3.0</td>
<td>2.0</td>
<td>1.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Notes:
1. Cases 2-12 are clear air and clouds combined.
2. Antenna located at sea level.
3. Heights are above ground.
4. No cosmic background or ground contribution considered.
5. T(K) is atmospheric noise temperature at zenith.
6. A(dB) is atmospheric attenuation along vertical path from ground to 30 km above ground.
Table 4-3. Coefficients $a$ and $b$ as a function of frequency and rain rate for calculation of atmospheric rain attenuation (Ippolito, 81a, 701; Olsen, 76, 318)

Marshall-Palmer drop distribution rain temperature 0°C

<table>
<thead>
<tr>
<th>Frequency, GHz</th>
<th>Coefficient</th>
<th>$a$, dB/km, for $R$ specified in mm/h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td>2</td>
<td>$3.45 \times 10^{-4}$</td>
<td>0.891</td>
</tr>
<tr>
<td>4</td>
<td>0.00147</td>
<td>1.016</td>
</tr>
<tr>
<td>6</td>
<td>0.00371</td>
<td>1.124</td>
</tr>
<tr>
<td>12</td>
<td>0.0215</td>
<td>1.136</td>
</tr>
<tr>
<td>15</td>
<td>0.0368</td>
<td>1.118</td>
</tr>
<tr>
<td>20</td>
<td>0.0719</td>
<td>1.097</td>
</tr>
<tr>
<td>30</td>
<td>0.186</td>
<td>1.043</td>
</tr>
<tr>
<td>40</td>
<td>0.362</td>
<td>0.972</td>
</tr>
<tr>
<td>94</td>
<td>1.402</td>
<td>0.744</td>
</tr>
</tbody>
</table>
Table 4-4. Summary of 11 GHz annual attenuation measurements for various geographical locations (Ippolito, 81a, 707)

<table>
<thead>
<tr>
<th>Location</th>
<th>Elevation angle</th>
<th>Time period</th>
<th>Attenuation (dB) for given percent outage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Waltham, MA</td>
<td>26°</td>
<td>Feb '77 - Jan '78</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Feb '78 - Jan '79</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Holmdel, N.J</td>
<td>27°</td>
<td>Jun '76 - Jun '77</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jun '77 - Jun '78</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jun '78 - Jun '79</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Groenbelt, MD</td>
<td>28°</td>
<td>Jul '76 - Jun '77</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jul '77 - Jun '78</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jul '78 - Jun '79</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Blacksburg, VA</td>
<td>33°</td>
<td>Jan '78 - Dec '78</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jan '77 - Dec '77</td>
<td>2</td>
</tr>
<tr>
<td>Austin, TX</td>
<td>49°</td>
<td>Feb '78 - Jan '79</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Munich, Germany</td>
<td>29°</td>
<td>Jan '78 - Dec '78 (91.2% coverage)</td>
<td>3</td>
</tr>
<tr>
<td>Fucino, Italy</td>
<td>30°</td>
<td>Jan '78 - Dec '78</td>
<td>1</td>
</tr>
<tr>
<td>Lahio, Italy</td>
<td>30°</td>
<td>Jan '78 - Dec '78</td>
<td>1</td>
</tr>
<tr>
<td>Nederhorst den Berg, Netherlands</td>
<td>27.5°</td>
<td>Aug '75 - Oct '75 (8100 hrs.)</td>
<td>.6</td>
</tr>
<tr>
<td>Kashima,</td>
<td>38°</td>
<td>Aug '78 - Jun '79</td>
<td>-</td>
</tr>
<tr>
<td>Japan</td>
<td>47°</td>
<td>May '77 - Apr '78</td>
<td>1</td>
</tr>
</tbody>
</table>
### Table 4-5.
Comparison of Goldstone 8.5 GHz (X-band) and 31.4 GHz (K_a band) predicted confidence level of troposphere loss and noise temperature at 30° elevation angle (R. Clauss, 82)

<table>
<thead>
<tr>
<th>Confidence level, %</th>
<th>0</th>
<th>50</th>
<th>80</th>
<th>90</th>
<th>95</th>
<th>98</th>
<th>99</th>
<th>99.5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tropospheric loss, dB</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-band</td>
<td>0.06</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
<td>0.11</td>
<td>0.13</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>K_a-band</td>
<td>0.16</td>
<td>0.31</td>
<td>0.41</td>
<td>0.47</td>
<td>0.58</td>
<td>0.72</td>
<td>0.91</td>
<td>1.23</td>
</tr>
<tr>
<td><strong>Tropospheric noise Temperature, K</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X-band</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>K_a-band</td>
<td>10</td>
<td>19</td>
<td>25</td>
<td>29</td>
<td>35</td>
<td>43</td>
<td>53</td>
<td>69</td>
</tr>
</tbody>
</table>
Fig. 4-1. Representation of a receiving system with signal propagating through lowary medium
Fig. 4-2. Clear sky zenith atmospheric attenuation as a function of frequency and surface water vapor density (assuming exponential decrease with 2-km scale height; Smith and Waters, 81, 39)
Fig. 4-3. Clear sky zenith atmospheric noise temperature as a function of frequency and surface water vapor density (assuming exponential decay with 2-km scale height; Smith and Waters, 81, 41)
Fig. 4-4. Clear sky atmospheric noise temperature as a function of frequency and elevation angle (7.5 gm/m$^3$ surface water vapor density, assuming exponential decrease with 2-km scale height; Smith and Waters, 81, 42)
Fig. 4-5. Zenith clear sky atmospheric noise temperature as a function of frequency (Jet Propulsion Laboratory, Goldstone, CA)
Fig. 4-6. Zenith clear sky atmospheric 31.5 GHz noise temperature measurements (obtained from tipping curve calibrations, Goldstone, CA from March 27–April 29, 1981)
Fig. 4-7. Graph of atmospheric noise temperature vs frequency (assuming 30° elevation angle, clear sky or clear sky and one or two cloud models with 0.5 gm/m³ cloud water particle density; after Slobin, 81a, 84); the vertical bars at 20.7 and 31.4 GHz represent the ranges of measured noise temperature increase indicated in Figs. 4-8 to 4-11.
Fig. 4-8. Photograph of clouds observed from below the JPL mesa area. The increase in noise temperature due to the clouds was approximately 0 and 1 K at 20.7 and 31.4 GHz respectively.
Fig. 4-9. Photograph of clouds observed from below the JPL mesa area. The increase in noise temperature due to the clouds was approximately 1 and 3 K at 20.7 and 31.4 GHz respectively.
Fig. 4-10. Photograph of clouds observed from below the JPL mesa area. The increase in noise temperature due to the clouds was approximately 4 and 9 K at 26.7 and 31.4 GHz respectively.
Fig. 4-11. Photograph of clouds observed from below the JPL mesa area. The increase in noise temperature due to the clouds was approximately 19 and 38 K at 20.7 and 31.4 GHz respectively.
Fig. 4-12. Cumulative distribution of X-band (8.5 GHz), all weather, zenith atmospheric noise temperature increase above quiescent clear sky baseline at DSS 13, Goldstone, CA (Slobin, 81b, 161)
Fig. 4-13. Cumulative distributions of 8.4 and 31.4 GHz, 30° elevation atmospheric noise temperature increase above baseline (Clauss, 82)
5. **Amplifier Input Noise Temperature**

The noise performance of an amplifier (Haus, 63, 436) can be characterized by its effective input noise temperature $T_e$. For a single response amplifier ($G \gg 1$),

$$T_e = \frac{N_e}{T_o} \frac{(T_i = 0 \text{ K})}{kBG} \quad \text{(5-1)}$$

where

$$N_e (T_i = 0 \text{ K}) = \text{total output noise power of receiver with input termination temperature at 0 K, W}$$

Alternately, $T_e$ can be defined as the input termination temperature of an "ideal" noiseless amplifier with gain $G$, and bandwidth $B$ which results in the same output noise power as the actual amplifier with an input termination temperature of 0 K.

For a multiple response receiver (Mumford, 68, 45)

$$T_e = \frac{N_e}{T_o} \frac{(T_i = 0 \text{ K})}{k(B_1G_1 + B_2G_2 + \ldots + B_nG_n)} \quad \text{(5-2)}$$

where

- $B_n = \text{bandwidth of } n\text{th response, Hz}$
- $G_n = \text{receiver gain to } n\text{th response ratio}$

Recent low noise performance parameters of microwave low noise amplifiers are shown in Fig. 5-1 and Table 5-1.

---

7. $I_v = 0$ for an "ideal" linear amplifier when $hf \ll kT$ and $hf/k$ otherwise (Stelzried, 82, 100).
5. References

1963

1968

1980

1980

1981
Lum, W., private communications.

1982
Clauss, R., private communications.

1982
Table 5-1. Recent (1981) S- and X-band microwave low noise amplifier noise temperature performance

<table>
<thead>
<tr>
<th>Frequency, GHz</th>
<th>Physical temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3 (S-band)</td>
<td>1.9 K, 12 K, 60 K</td>
</tr>
<tr>
<td>8.5 (X-band)</td>
<td>3.2 K, 50 K, 220 K</td>
</tr>
</tbody>
</table>

Maser\(^a\)  | 4, 77, 300 |
FET\(^b\)     |            |

\(^a\) In field use: R. Clauss, 82 Pasadena, CA.
\(^b\) Laboratory: D. Williams, 80 Berkeley, CA.
Fig. 5-1. Recent (1980) noise performance for low noise amplifiers as a function of frequency (Weinreb, 80)
6. **Input Noise Temperature of Cascaded Amplifiers**

Consider an amplifier consisting of two internal amplifiers in cascade (Humford, 68, 22; Fig. 6-1). Assume $G_1 \gg 1$, $G_2 \gg 1$, $B_2 \leq B_1$, $G = G_1 G_2$.

We have

$$N_{T_0} = k(T_1 + T_e)BG$$  \hspace{1cm} (6-1)

where

- $N_{T_0}$ = total output noise power of combined amplifier, W
- $T_e$ = effective input noise temperature of combined amplifier, K
- $T_1$ = noise temperature of input termination, K

Also

$$N_{T_0} = kT_eBG_2 + k(T_1 + T_e)BG_1G_2$$  \hspace{1cm} (6-2)

Equating Eqs. 6-1 and 6-2 and setting $T_1 = 0$,

$$T_e = T_{e1} + (T_{e2}/G_1)$$  \hspace{1cm} (6-3)

\footnote{Gain response of the second amplifier is centered within the gain response of the first amplifier.}

6-1
For multiple amplifiers,

\[
T_e = T_{e1} + \left( \frac{T_{e2}}{G_1} \right) + \left( \frac{T_{e3}}{G_1G_2} \right) + \ldots + \left( \frac{T_{en}}{G_1G_2\ldots G_{n-1}} \right)
\]  

(6-4)

If \( B = B_1 < B_2 \),

\[
T_e = T_{e1} + \left( \frac{T_{e2}B_2}{G_1B_1} \right) + \ldots
\]  

(6-5)
6. References

Fig. 6-1. Combined amplifier consisting of two amplifiers in cascade
7. **System Operating Noise Temperature**

The noise performance of a receiving system is determined by the sum of the input termination temperature and the receiver effective noise temperature. For a single $^9$ response receiver ($G > 1$, $hf << kT$),

\[
T_{op} = \frac{N_{io}}{kBG} \quad (7-1)
\]

where

\[
T_{op} = \text{system operating noise temperature, } K
\]

\[
N_{io} = \text{total output noise power of the receiving system, } W
\]

Also $^10$ (Haus, 63, 434)

\[
T_{op} = T_i + T_e \quad (7-2)
\]

For an "ideal" receiver $^10$, $T_e = 0$, and $T_{op} = T_i$.

For a communication receiving system, $T_i$ represents the antenna temperature, $T_a$. In general, $T_i$ could be a thermal noise standard (Sec. 20) or the output of the source termination at the defined reference plane (Sec. 8).

The signal to noise performance of a linear receiving system is given by, ($hf << kT$)

---

$^9$ The following assumes receivers with a single response; multiple responses require (Mumford 68, 33) modification of the equations.

$^10$ When $hf = << kT$, $T_{op} = T_i + T_e$, where $T_i = T_i \left(\frac{hf/kT_i}{e^{hf/kT_i} - 1}\right)$ and $T_{op} \text{ (ideal)} = T_i + (hf/k)$, (Sec. 2; Stelzried, 82, 100).
\[
S/N = \frac{S_i}{G/N_0} = \frac{S_i}{kT_{op}B} = \frac{S_i}{k(T_i + T_e)B} = \frac{S_i}{k(T_i + (F-1)T_0)B}
\]

where

\( S_i \) = input signal power, W

For \((0 < hf/kT) \leq 1\)

\[
(S/N) = \frac{S_i}{kT_{op}B} = \frac{S_i}{k(T_i' + T_e')B} = \frac{S_i}{k(T_i' + (F-1)T_0' + \nu T_q)B}
\]

and for \(hf > kT\)

\[
(S/N) = \frac{S_i}{(kT_eB)} = \frac{S_i}{kT_eB} = S_i/FkT_q = S_i/FhfB
\]

The performance of a receiving system composed of an "ideal" amplifier \(T_e = T_q\) and a source at the cosmic background temperature \((T_i = 2.7 K)\) is shown in Fig. 7-1 and Table 7-1 (from Stelzried, 82, 100). This demonstrates the loss in sensitivity at very high frequencies relative to low frequencies for a conventional receiving system with an "ideal" linear receiver.

\(^{11}\)A typical infrared detector produces an output voltage proportional to input power. The sensitivity of these detectors is frequently described by their NEP (Noise Equivalent Power, W-Hz\(^{1/2}\); Kruse, 62, 268, Gagliardi, 82). For these detectors \((hf > kT)\), \((S/N) = S_i^2/(\text{NEP})^2B\) referred to the input or voltage signal to noise ratio = VSNR = \(S_i/(\text{NEP})\sqrt{B}\) referred to the output. For comparison with Table 7-1, an ideal detector at 1 mm has an NEP = \(1.9 \times 10^{-19} W\sqrt{Hz}\) so that VSNR = \(S_i/(1.9 \times 10^{-19})\sqrt{B}\).
It is instructive to consider this relationship for a receiving system degraded by an additional 0.1 dB of input absorption loss (possibly due to atmospheric changes). With high system noise temperatures ($T_{op} >> 290 K$) this degrades $S/N$ by $\approx 0.1$ dB due to the additional direct signal attenuation. With low system noise temperatures ($T_{op} << 290 K$) this has a very much larger effect due to the increase in $T_{op}$ caused by the added thermal noise contribution. Assume

$$T_{op} = (T_{op})_0 + \Delta T_{op},$$

(7-6)

where

$T_{op} =$ system noise temperature assuming atmospheric loss $L, K$
$(T_{op})_0 =$ system noise temperature assuming a baseline atmospheric loss $L_0, K$
$\Delta T_{op} =$ increase in system noise temperature due to loss increase from $L_0$ to $L, K$

$$= (L_0^{-1} - L^{-1})(T_p - T_s)$$

$T_p =$ atmosphere equivalent physical temperature for loss $L, K$
$L =$ atmospheric loss, ratio
$L_0 =$ baseline atmospheric loss, ratio
$T_s =$ cosmic background noise temperature, $K$

The degradation in signal to noise ratio is given by

$$\Delta(S/N) = \Delta A + 10 \log \left[ \frac{T_{op}}{(T_{op})_0} \right], \text{ dB}$$

(7-7)

or

$$\Delta(S/N) = \Delta A + 10 \log \left[ 1 + (T_p - T_s)(1 - 10^{-\Delta A/10})/L_0(T_{op})_0 \right], \text{ dB}$$

(7-8)
where

\[ \Delta A = \text{increased atmospheric absorption, dB} \]
\[ = (A - A_0) \]
\[ = 10 \log \left( \frac{L}{L_0} \right) \]
\[ A = 10 \log L \]
\[ A_0 = 10 \log L_0 \]

Fig. 7-2 indicates the degradation of S/N as functions of the baseline system temperature, baseline atmospheric absorption and increase in atmospheric absorption. This analysis technique has been used to compare the confidence performance of 8.5- and 31.4-GHz receiving systems. Table 7-2 shows the link SNR degradation computed using the data from Table 4-5; the resultant performance improvement at 31.4 GHz is due to the higher antenna gain relative to 8.5 GHz. This analysis accounts only for weather effects—all other possible degradations are neglected.
7. References


1982 Gagliardi, R. M., Private communication

Table 7-1. Tabulation of the sensitivity of an "ideal" receiver \((T_e = T_q)\) with an input source temperature of 2.7 K as a function of frequency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.0</th>
<th>8.5 (3.5 cm)</th>
<th>32 (3.4 mm)</th>
<th>300 (1 mm)</th>
<th>3000 (0.1 mm)</th>
<th>300,000 (1 μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_e'), K</td>
<td>2.7</td>
<td>2.5</td>
<td>2.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(T_e - T_q), K</td>
<td>0.0</td>
<td>0.4</td>
<td>1.5</td>
<td>14.4</td>
<td>144.0</td>
<td>14400</td>
</tr>
<tr>
<td>(T_{op}), K</td>
<td>2.7</td>
<td>2.9</td>
<td>3.5</td>
<td>14.5</td>
<td>144.0</td>
<td>14400</td>
</tr>
<tr>
<td>(S_{in} \text{ dbm/Hz}^*)</td>
<td>-194.3 (3.73 x 10^{-23})</td>
<td>-194.0 (4.00 x 10^{-23})</td>
<td>-193.1 (4.83 x 10^{-23})</td>
<td>-187.0 (2.00 x 10^{-22})</td>
<td>-177.0 (1.99 x 10^{-21})</td>
<td>-157.0 (1.99 x 10^{-19})</td>
</tr>
</tbody>
</table>

\(*S_{in}\) is the input signal power required for \((S/N) = 1\). Since the energy of a photon is \(hf\), \(S_{in} = \frac{T_{op}}{T_q}\), photons/sec-Hz. Therefore \(S_{in} \approx 7, 2, 1\) photons/sec-Hz for this system at 8.5, 32 and >300 GHz respectively.
<table>
<thead>
<tr>
<th>Confidence level, %</th>
<th>0</th>
<th>50</th>
<th>80</th>
<th>90</th>
<th>95</th>
<th>98</th>
<th>99</th>
<th>99.5</th>
</tr>
</thead>
</table>

- Link SNR loss caused by the atmosphere, $K_a$-band relative to $X$-band, $\text{dB}^a$
  - 0.89
  - 2.21
  - 2.90
  - 3.31
  - 3.86
  - 4.51
  - 5.20
  - 6.14

- Link SNR improvement, $K_a$-band relative to $X$-band, $\text{dB}^b$
  - 10.62
  - 9.30
  - 8.61
  - 8.20
  - 7.65
  - 7.00
  - 6.31
  - 5.37

---

$a$ R. Claus, 82, assuming baseline system noise temperatures of 72 K with 4 K atmospheric contribution at 30° elevation angle for $X$-band and 26 K with 10 K atmospheric contribution at 30° angle for $K_a$-band.

$b$ This improvement accounts for atmospheric loss and noise temperature; antenna sizes and efficiencies and transmitted powers are assumed equal for both $X$- and $K_a$-bands.
Figure 7-1. $T_i'$, $T_q$, and $T_{op}$ vs frequency using an "ideal" receiver ($T_e = T_q$) with an input source temperature of 2.7 K.
Fig. 7-2. (S/N) degradation vs atmospheric absorption increase as a function of the baseline system noise temperature \((T_p - T_s) = 280\, \text{K}\).
8. **Shifting the Reference Plane of Cascaded Amplifiers**

The previous concepts of noise temperature \((T_i, T_e \text{ and } T_{op})\) are applicable to a defined reference plane in a typical receiving system. Assuming a linear receiving system and \(hf << kT\), the signal to noise (Eq. 7-3) ratio is invariant regardless of the chosen reference plane. Then (Fig. 8-1)

\[
(T_{op})_2 = (T_{op})_1/L \tag{8-1}
\]

and

\[
(T_{op})_3 = G(T_{op})_2 \tag{8-2}
\]

where

\[
(T_{op})_n = T_{op} \text{ defined at reference plane n, K}
\]

\[
= (T_i)_n + (T_e)_n
\]

\[
L = \text{attenuator loss, ratio (L≥1; assumed "matched")}
\]

\((T_i)_n, (T_e)_n = T_i \text{ and } T_e \text{ defined at reference plane n, K}
\]

Also (Humford, 68, 19)

\[
(T_i)_2 = \left[\frac{(T_i)_1}{L}\right] + (1 - 1/L) T_p \tag{8-3}
\]

where

\[
T_p = \text{physical temperature of attenuator, K}
\]

With Eqs. 5-1 and 7-2,

\[
(T_e)_2 = \left[\frac{(T_e)_1}{L}\right] - (1 - 1/L) T_p \tag{8-4}
\]
Also (Eq. 8-3),

\[(T_i)_1 = L(T_i)_2 + (L - 1) T_p\]  \hspace{1cm} (8-5)

and

\[(T_e)_1 = L(T_e)_2 - (L - 1) T_p\]  \hspace{1cm} (8-6)

The output signal to noise ratio is \((hf<><kT)\)

\[(S/N)_o = (S_i)_n / k(T_{op})_n\]  \hspace{1cm} (8-7)

where

\[(S_i)_n = \text{signal power at the receiving system reference plane } n, W\]

The reference plane chosen in a particular receiving system is usually dictated by physical constraints; the JPL Deep Space Network has historically chosen the reference plane at the receiver input.

The above techniques can be used to transfer \(T_i\), \(T_e\), or \(T_{op}\) defined at any given reference plane to any other chosen reference plane. However, \(T_i\), \(T_e\) and \(T_{op}\) must all be defined at the same reference plane.
8. References

Fig. 8-1. Receiving system representation
9. Relationship of Noise Figure to Noise Temperature

The definition (Haus, 63, 434) of an amplifier noise figure is (neglecting quantum effects; \( hf \ll kT \)),

\[
F = N_{TO} (T_i = T_o) / k T_o \tag{9-1}
\]

where

\( N_{TO} (T_i = T_o) \) = total output noise power of the receiver with the input termination at \( T_o \), W

\( T_o = 290 \text{ K} \)

From Eqs. 7-1, 7-2 and 9-1\(^{12} \),

\[
F = 1 + T_e / T_o \tag{9-2}
\]

or

\[
T_e = (F - 1) T_o \tag{9-3}
\]

In terms of \( F \),

\[
T_{op} = T_i + (F - 1) T_o \tag{9-4}
\]

For an "ideal" amplifier (\( hf \ll kT \)), \( T_e = 0 \) and \( F = 1 \) (see Sec. 2).

\(^{12}\)When \( hf \ll kT \), quantum effects cannot be ignored. Then, it is proposed (Stelzried, 82, 100) that \( F = (1 + T_e / T_o^0) / (1 + T_q / T_o) \) where \( T_o^0 = T_o (hf/kT_o) / (ehf/kT_o - 1) \) and \( T_q = (hf/k) \). Then, for an "ideal" receiver, \( T_e = (hf/k) \) and \( F = 1 \).
9. Reference


10. **Measurements of Antenna Temperature**

The evaluation of antenna noise temperature $T_a$ is usually an intermediate measurement. For example, we have

\[ T_{op} = T_a + T_e \]  \hspace{1cm} (10-1)

where

- $T_a = \text{antenna temperature, K}$
- $T_e = \text{background temperature, K}$
- $T_{op} = \text{operational temperature, K}$

Separate measurements of $T_a$ and $T_e$ can be used to obtain $T_{op}$. $T_a$ has multiple contributors,

\[ T_a = T_s + T_{atm} + T_g + ... \]  \hspace{1cm} (10-2)

where

- $T_s = \text{cosmic noise background temperature, K}$
- $T_{atm} = \text{atmospheric contribution to antenna noise temperature, K}$
- $T_g = \text{ground contribution (due to antenna spillover, etc.); Otoshi, 67, 143 to antenna noise temperature, K}$

$T_s$ is determined (Penzias, 65, 419-421; Otoshi, 75, 174) from

\[ T_s = T_a - T_{atm} - T_g - ... \]  \hspace{1cm} (10-3)

Antenna noise temperature $T_a$ can be measured using matched thermal noise standards. Switching sequentially between the thermal noise standards and the antenna (Fig. 10-1, $hf < kT$, matched antenna and thermal noise standards, linear receiving system),
\[ T_a = T_C + (T_H - T_C)(Y_2 - 1)/(Y_1 - 1) \] (10-4)

where

\[ Y_1 = \frac{P_H}{P_C}, \text{ ratio} \]
\[ Y_2 = \frac{P_A}{P_C}, \text{ ratio} \]
\[ P_A = \text{receiver output power when switched to antenna, W} \]
\[ P_H = \text{receiver output power when switched to } T_H, \text{ W} \]
\[ P_C = \text{receiver output power when switched to } T_C, \text{ W} \]
\[ T_H = \text{thermal noise temperature of calibration source } T_H, \text{ K} \]
\[ T_C = \text{thermal noise temperature of calibration source } T_C, \text{ K} \]

\( Y_1 \) and \( Y_2 \) can be determined directly from power meter measurements or with a precision attenuator by adjusting for equal output levels when the receiver input is switched between the appropriate noise sources. The thermal noise standards (see Sec. 20) \( T_H \) and \( T_C \) are usually calibrated by measuring the transmission line insertion loss. The physical temperatures of the terminations are monitored with thermometers or maintained at a constant temperature by submersion in a boiling liquid, such as liquid nitrogen (boiling temperature 77.36 K at 760 mm Hg). It is usually convenient to select one of the terminations to be at ambient temperature due to the ease of construction and low sensitivity to errors in waveguide loss calibrations.

It is sometimes convenient (Franco, 81, 80), to use a square law detector \( V_{output} \propto T_{op} \) and voltmeter to interface with an automated computer system. For this configuration,

\[ T_a = T_C + \frac{T_H - T_C}{V_H - V_C} (V_A - V_C) \] (10-5)

where

\[ V_A = \text{receiver output voltage when switched to antenna, V} \]
\[ V_H = \text{receiver output voltage when switched to } T_H, \text{ V} \]
\[ V_C = \text{receiver output voltage when switched to } T_C, \text{ V} \]
Antenna temperature can also be found using

\[ T_a = T_{op} - T_e \]  \hspace{1cm} (10-6)  

where \( T_{op} \) (Sec. 12) and \( T_e \) (Sec. 11) are measured separately. Assuming the errors in \( T_{op} \) and \( T_e \) are independent, the error in \( T_a \) is

\[ \delta T_a = \sqrt{(\delta T_{op})^2 + (\delta T_e)^2} \]  \hspace{1cm} (10-7)  

where

- \( \delta T_{op} \) = measurement error of \( T_{op} \), K  
- \( \delta T_e \) = measurement error of \( T_e \), K

A technique for measuring antenna temperature (or an unknown termination temperature) is illustrated in Fig. 10-2 which requires only one thermal noise standard \( T_C \) and a precision attenuator, frequently (Otoshi, 71, 843) a rotary vane attenuator. This technique has the advantage of eliminating receiver linearity errors (Sec. 24). Adjusting \( L \) for equal output noise levels (assuming \( T_a < T_C \))

\[ T_a = L T_C - (L - 1) T_P \]  \hspace{1cm} (10-8)  

where

- \( T_P \) = physical temperature of \( L \), K

Another technique (Schuster, 62, "86) for measurement of \( T_a \) which requires one thermal noise standard and a noise source is shown in Fig. 10-3. For this technique,

\[ T_a = T_c + T_N (Y_1 -1)/(Y_2 - 1) \]  \hspace{1cm} (10-9)  

10-3
where

\[ Y_1 = \frac{P_A}{P_C}, \text{ ratio} \]
\[ Y_2 = \frac{P_{CN}}{P_C}, \text{ ratio} \]
\[ P_A = \text{receiver output power, when switched to antenna,} \]
\[ \quad (\text{noise source turned off}), \text{ W} \]
\[ P_C = \text{receiver output power, when switched to } T_C, \]
\[ \quad (\text{noise source turned off}), \text{ W} \]
\[ P_{CN} = \text{receiver output power, when switched to } T_C \]
\[ \quad (\text{noise source turned on}), \text{ W} \]
\[ T_N = \text{increase in system noise temperature when noise source} \]
\[ \quad \text{turned on, K} \]

This technique requires calibration of \( T_N \), which can be accomplished by

use of another thermal noise standard or by the method discussed in Sec. 12

using a single ambient termination applicable if \( T_e \ll T_C \)

It is frequently desirable to measure the change in antenna temperature.

From Sec. 3,

\[
\Delta T_a = \frac{T_A A_T}{s e s} (R_\lambda)^2
\]

or

\[
\Delta T_a = \frac{S_A}{2k}
\]

Antenna efficiency is given by

\[
\eta = \frac{A_e}{A_p} = 2k \Delta T_a / S_{Ap}
\]

10-4
where

\[ A_p = \text{antenna physical area, m}^2 \]

Figure 10-4 shows antenna efficiency (1972) vs elevation angle and frequency for the DSS 14 (Goldstone) 64-m antenna. The 15 3-GHz (K\textsubscript{u}-band) measurements were performed using a noise adding radiometer (Se\textsuperscript{e}., 19) and the planets Jupiter and Saturn for calibration sources.
10. References


Fig. 10-1. Receiving system configuration for measuring antenna noise temperature using two thermal noise standards.

Fig. 10-2. Configuration for measuring antenna noise temperature using one thermal noise standard and a precision attenuator.

Fig. 10-3. Configuration for measuring antenna noise temperature using a thermal noise standard and a noise source.
Fig. 10-4. Antenna efficiency vs elevation angle and frequency for the Goldstone, CA, DSS 14 64-m antenna ($L_o$ is the zenith atmospheric loss)
11. **Measurement of Receiver Input Noise Temperature**

Receiver noise temperature (Humford, 68, 58; Arthur, 74, 1-188) can be evaluated with thermal noise standards or auxiliary noise sources (gas-discharge tubes, noise diode, etc.). These measurements require careful attention to instrumentation detail such as "matched" thermal noise standards and "linear" amplifiers.

Switching (Stelzried, 82, 100; Wait, 73b, 25; Wait, 73a, 1-25) between the thermal noise standards (Fig. 11-1; \(h\nu<<kT\), \(P_H\) and \(P_C\) defined in Sec. 10)\(^{13}\)

\[
T_e = \frac{(T_H - YT_C)}{(Y-1)}
\]

(11-1)

where

\[
Y = \left(\frac{P_H}{P_C}\right), \text{ ratio}
\]

It is sometimes convenient to use a calibrated noise source (Fig. 11-2) to calibrate the receiver noise temperature. Turning the noise source on and off,

\[
T_e = \left[\frac{T_N}{(Y-1)}\right] - T_i
\]

(11-2)

---

\(^{13}\) If \(h\nu \ll kT\), verify that \(T_H\) and \(T_C\) are properly calibrated (Stelzried, 82, 100); accounting for quantum noise does not impact the measurement technique or the equations; it simply sets a lower limit \(T_q = (h\nu/k)\) for a linear amplifier to the value obtainable for \(T_e\) using a linear amplifier.
11. References


Fig. 11-1. Configuration for measuring receiver noise temperature using two thermal noise standards

Fig. 11-2. Configuration for measuring receiver noise temperature using a noise source
12. **Measurement of System Temperature**

Using Eq. 7 \( \therefore \),

\[
T_{op} = T_a + T_e
\]  \hspace{1cm} (12-1)

System temperature can be obtained from the measurement results of Secs. 10 and 11 of antenna and receiver noise temperatures. More directly, using the concepts of Sec. 10 (Fig. 10-1; \( P_A \), \( P_C \) and \( P_H \) defined in Sec. 10),

\[
T_{op} = \frac{(T_H - T_C)}{(Y_2 - Y_1)}
\]  \hspace{1cm} (12-2)

where

\[
Y_1 = \frac{P_C}{P_A}, \text{ ratio}
\]

\[
Y_2 = \frac{P_H}{P_A}, \text{ ratio}
\]

Thermal terminations at liquid helium (Fig. 12-1) and liquid nitrogen temperatures were used to determine the noise temperature of a maser system terminated in a liquid helium cooled termination. This 2.3-GHz maser system has a noise temperature of 15 K (Clauss, 64, 619).

Although, in general, calibration terminations at two different temperatures are required to solve for \( T_{op}, T_a, \) or \( T_e \), it is sometimes convenient to use a single termination (Stelzried, 71, 41). This technique requires that \( T_e \ll T_H \) for reasonable accuracy.

Switching between \( T_H \) and the antenna (Fig. 12-2)

\[
T_{op} = \frac{(T_H + T_e)}{Y}
\]  \hspace{1cm} (12-3)

where

\[
Y = \frac{P_H}{P_A}
\]
For this calibration technique, the physical temperature of the standard termination \( T_H \) (usually an ambient termination) is monitored for \( T_H' \); \( T_e \) is considered known (from a previous laboratory measurement before installation in an operating receiving system). Assuming \( T_e \ll T_H \), it is seen that a fairly large error in \( T_e \) does not have much effect on the measurement error of \( T_{op} \). For example, if \( T_H = 295 \text{ K} \), \( T_e = 5 \text{ K} \), and \( Y = 10 \), then \( T_{op} = 30 \text{ K} \). A 100% error in \( T_e \) (i.e. \( T_e \) really 10 K) results in \( T_{op} = 31 \text{ K} \) (an error of only 3.3%). This technique of monitoring system noise temperature using a single ambient reference is used by the JPL Deep Space Network with good results at a tremendous cost savings. A simplified technique for evaluating \( T_{op} \) using an aperture ambient termination is illustrated in Figs. 12-3 and 12-4. The noise temperature contributions for this 8.5-GHz system is shown in Table 12-1.

A summary of the JPL Goldstone maser receiving system’s measured noise performance is given in Table 12-2.

\( T_{op} \) is commonly measured by providing a noise source \( T_N \) (Fig. 12-5). \( T_{op} \) is given by

\[
T_{op} = T_N (Y - 1)
\]  

(12-4)

where

\[
Y = \frac{P_{AN}}{P_A}, \text{ ratio}
\]

\[
P_A = \text{receiver output power, noise source off, W}
\]

\[
P_{AN} = \text{receiver output power, noise source on, W}
\]

The initial calibration of \( T_N \) can be performed by determination of \( T_{op} \) by some other means, such as switching between thermal noise standards; then (Stelzried, 80, 98)

\[
T_N = \frac{T_{op}}{(Y - 1)}
\]  

(12-5)

where

\[
(T_{op})_K = \text{known system operating noise temperature, K}
\]
12. References


1980 Clauss, R. C., private communication
Table 12-1. Noise temperature performance of 8.5-GHz JPL horn-maser receiving system located on the ground (Clauss, 80)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Noise temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{op}$</td>
<td>10.5 (measured)</td>
</tr>
<tr>
<td>$T_{M}$</td>
<td>3.5 (measured)</td>
</tr>
<tr>
<td>$T_{COSMIC}$</td>
<td>2.5 (measured)</td>
</tr>
<tr>
<td>$T_{HORN + ATMOSPHERE}$</td>
<td>4.5 (implied)</td>
</tr>
</tbody>
</table>

Table 12-2. Summary of JPL Goldstone, CA, maser receiving systems noise temperature performance (1972); $T_e$ evaluated in the laboratory and $T_{op}$ evaluated with the single ambient termination technique, Eq. 12-3

<table>
<thead>
<tr>
<th>System</th>
<th>Freq (GHz)</th>
<th>$T_{op}$ (K)</th>
<th>$T_e$ (K)</th>
<th>$T_M$ b (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldstone 64-m antenna</td>
<td>2.3</td>
<td>15.6</td>
<td>4.3</td>
<td>4.1</td>
</tr>
<tr>
<td>&quot;</td>
<td>8.5</td>
<td>20.0</td>
<td>6.1</td>
<td>6.0</td>
</tr>
<tr>
<td>&quot;</td>
<td>15.3</td>
<td>27.0</td>
<td>8.5</td>
<td>8.4</td>
</tr>
<tr>
<td>Minimum horn/maser a</td>
<td>2.3</td>
<td>10.7</td>
<td>4.3</td>
<td>4.2</td>
</tr>
<tr>
<td>&quot;</td>
<td>8.5</td>
<td>12.4</td>
<td>6.1</td>
<td>6.0</td>
</tr>
<tr>
<td>&quot;</td>
<td>15.3</td>
<td>18.5</td>
<td>8.7</td>
<td>8.4</td>
</tr>
</tbody>
</table>

aSystem located on ground.

b$T_M$ = noise temperature of maser amplifier only.
Fig. 12-1. Photograph of traveling wave maser system consisting of a liquid helium cooled termination and a 2.3-GHZ maser.

Fig. 12-2. Configuration for measuring system noise temperature using a single thermal noise standard.
Fig. 12-3. JPL 8.5-GHz horn-maser receiving system noise temperature measurement system; $T_{op}$ is evaluated by switching between the sky and an aperture ambient termination.
Fig. 12-4. JPL 8.5 GHz horn-ma-er receiving system showing noise measuring instrumentation
Fig. 12-5. Representation of receiving system with calibration noise source $T_N$.
13. **Measurement of G/T**

For typical communications links (Fig. 13-1) the receiving system output signal to noise ratio is

\[
(S/N)_0 = \frac{P_T G_T G_R G_A / L_S}{k T_0 B G_A}
\]

(13-1)

where

\begin{itemize}
  \item $P_T$ = transmitter power, W
  \item $G_T$ = transmitter antenna gain, ratio
  \item $G_R$ = receiver antenna gain, ratio
  \item $G_A$ = amplifier gain, ratio
  \item $L_S = \text{space loss, ratio}$
  \item $= (4\pi R/\lambda)^2$
  \item $R = \text{antenna separation, m}$
  \item $\lambda = \text{wavelength, m}$
\end{itemize}

Inspection of Eq. 13-1 indicates that a figure of merit (Wait, 77, 49, Lawton, 82) for the ground system is

\[
M = G/T
\]

(13-2)

where $G = G_R$ and $T = T_{op}$ is understood. $M$ can be determined indirectly by separate determinations of $G_R$ and $T_{op}$ (Sec. 7). However, there is usually considerable advantage to a direct measurement using radio star sources.

Consider the receiving system shown in Fig. 13-2. Moving the antenna beam on and off the source,

\[
(G/T) = 8wk (Y - 1)/k_1 k_2\lambda^2 S
\]

(13-3)

---

14. The JPL Deep Space Network does not use this technique due to the scheduling difficulty of routine performance monitoring.

13-1
where

\[ Y = \left( \frac{P_{ON}}{P_{OFF}} \right), \text{ ratio} \]

- \( P_{ON} \) = receiver output power when on source, W
- \( P_{OFF} \) = receiver output power when off source, W
- \( k_1 \) = atmospheric transmission factor, ratio (\( \approx 1; \) Daywitt, 78, 1-39; Sec. 4)
- \( k_2 \) = radio star shape factor, ratio (\( \approx 1; \) Kanda, 76, 173)

\[
\int T(\omega) P(\omega) d\omega / \int T(\omega) d\omega
\]

The error sources \( k_1, k_2 \) and others (such as antenna pointing, radiometer bandwidth and radio star polarization) have been analyzed (Daywitt, 76, 1-17; Daywitt, 77, 1-25; Kanda, 76, 173). As an example, a brightness temperature contour for Cassiopeia A is shown in Fig. 13-3. The correction \( k_2 \) for Cassiopeia A is obtained using this contour and the antenna pattern with the definition of \( k_2 \) above. It has been shown (Kanda, 76, 177) that Cassiopeia A can be treated as a uniform disk of about 4.6' diameter for antennas with a half-power beam width greater than about 4.6'.

Recent accurately calibrated radio star flux values for 2.3-GHz (S-band) and 8.4-GHz (X-band) are available (Klein, 76, 1078; Turegano, 80, 46). Radio star flux values are also available for a wider range of frequencies (Baars, 77, 99).

The communication link performance can be evaluated for a given ground system with known \( G/T \) using Eq. 13-1. \( G/T \) is a useful specification for a ground system contract (Wait, 77, 49). Tradeoffs can be made between receiver and antenna performance to meet an overall system specification. For very large antennas, \( T_{op} \) should be designed as low as possible due to the high cost of large antenna structures. Table 13-1 shows the figure of merit (expressed in dB).
\[ M(dB) = 10 \log (G/T) \]  

(13-4)

for the JPL Deep Space Network receiving systems from pre-1960 to post-1980. The 26-m antennas with mixer receivers were updated with maser amplifiers (servicing maser, Figs. 13-4 and 13-5) in 1960. The present (1982) 64-m antennas have very low noise masers with reliable cryogenic refrigerators. Further improvement in G/T will be obtained at higher frequencies and increased aperture using array techniques.
13. References


### Table 13-1. Summary of JPL DSN receiving system figure of merit (G/T) performance

<table>
<thead>
<tr>
<th>System</th>
<th>Receiving system figure of merit (dB)</th>
<th>Improvement over 1960 (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre 1960</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26-m antenna</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>960 MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{top}$ = 2000 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26-m antenna</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>960 MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{top}$ = 72 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64-m antenna</td>
<td>47</td>
<td>34</td>
</tr>
<tr>
<td>2300 MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{top}$ = 25 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post 1980</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64-m antenna</td>
<td>71 (optimistic)</td>
<td>58 (optimistic)</td>
</tr>
<tr>
<td>31 GHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{top}$ = 25 K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 13-1. Representation of typical deep space communications link
Fig. 13-2. Representation of receiving system configuration for G/T calibrations.
Fig. 13-3. Brightness temperature contour map of Cassiopeia A (coordinates for Epoch AD 1950.0, after Rosenberg, 70, 109-122)
Fig. 13-4. Photograph of 1960 960-MHz liquid-helium-cooled maser amplifier installed on 26-m antenna (JPL DSS 11 Goldstone, CA)
Fig. 13-5. Close-up photograph of 1960 960-MHz maser amplifier installation on 26-m antenna (JPL, DSS 11, Goldstone, CA)

Knowledge of the flux density (Baars, 77, 99; Klein 76, 1078; Juancey 82) of radio stars can be used to evaluate antennas or receiving system (previous Sec.) performance. Alternately, a receiving system with known antenna gain can be used for flux density calibration of radio stars (Baars 73,461; Freiley 77). Referring to Fig. 13-2, the increase in antenna temperature obtained when the radio source is in the beam vs not in the beam is given by (Y as defined in Sec. 13)

\[ \Delta T_a = T_{op} (Y - 1) \]  \hspace{1cm} (14-1)

where

\[ T_{op} = \text{system operating noise temperature (off source), K} \]

The source temperature \( T_s \) or flux density \( S \) is evaluated using Eqs. 3-3 or 3-4. \( T_{op} \) can be evaluated as before (Sec. 12). Then (using terms as defined in Sec. 3)

\[ S = 2k \frac{\Delta T_a}{A_e} \]  \hspace{1cm} (14-2)

One of the best methods (minimizing antenna gain and atmospheric loss calibrations; Klein, 76, 1078) is to use a previously calibrated reference radio source. Then, the flux density of the unknown source is

\[ S = S_R \frac{\Delta T_a}{(\Delta T_a)_R} \]  \hspace{1cm} (14-3)

where

\[ S_R = \text{flux density of reference radio source, J-m}^2 \]

\( (\Delta T_a)_R \) = increase in antenna temperature due to reference radio source, K
14. References


15. Measurement of Transmission Loss

It is shown in Sec. 7 that the receiving system sensitivity of a low noise system can be degraded severely, even with a relatively small amount of transmission line loss. Transmission line loss can be measured very precisely (Stelzried, 66, 98). These direct techniques require precision transmission lines that are well matched with precision connectors or waveguide flanges. Fig. 15-1 shows a photograph of typical instrumentation components used for these measurements. Fig. 15-2 shows a WR 430 waveguide flange being hand lapped prior to precision insertion loss calibrations. Verification by the National Bureau of Standards indicates that these calibration techniques are accurate to better than 0.001 dB (Otoshi, 70, 406).

Radiometric techniques are useful where these conditions are not realizable such as in bulk material (Seidel, 68, 625; Stelzried, 69, 172). The increase in system temperature (Fig. 15-3) is given by

\[
(\Delta T_{op}) = (1 - L_m^{-1}) (T_m - T_{atm} - T_s/L)
\]

(15-1)

where

\[
L = \text{atmospheric loss, ratio (} \geq 1\text{)}
\]
\[
L_m = \text{transmission loss of lossy material, ratio (} \geq 1\text{)}
\]
\[
T_m = \text{physical temperature of lossy material, K}
\]
\[
T_s = \text{cosmic background noise temperature, K (} \approx 2.7\text{ K)}
\]
\[
T_{atm} = \text{atmospheric noise temperature contribution, K}
\]

Then

\[
L = 1 / \left[1 - (\Delta T_{op})_m / (T_m - T_{atm} - T_s/L)\right]
\]

(15-2)

It is frequently convenient to perform a radiometer "tipping measurement" to determine \( T_{atm} \) and the atmospheric loss. Assuming a flat earth (Fig. 4-1) with a stratified atmosphere.
\[ L = L Z \sec Z \]  

where

\[ Z = \text{zenith angle} \]

\[ L, L Z = \text{atmospheric loss at zenith angle } Z \text{ and zenith respectively, ratio, } (L, L Z) \]

For the "tipping" measurement, the system temperature \( T_{\text{op}} \) is measured at zenith and \( Z \). Assuming an antenna with no sidelobes and an infinitely narrow beamwidth,

\[ \Delta T_{\text{op}} = \left( L Z - L Z \sec Z \right) \left( T_p - T_s \right) \]  

For many applications, it is convenient to measure \( \Delta T_{\text{op}} \) between \( Z = 0 \) and \( 60^\circ \). Then (see Stelzried, 8 , for additional terms in the expansion),

\[ T_{\text{atm}} = \Delta T_{\text{op}} \left[ L Z (\text{d}B)/4.343 \right] (T_p - T_s) + \ldots \]  

so that

\[ L Z (\text{d}B) = \left[ \frac{4.343 \Delta T_{\text{op}}}{(T_p - T_s)} \right] + \ldots \]  

This measurement, performed near the water vapor resonance (\( \approx 22 \text{ GHz} \)), is convenient for monitoring atmospheric water vapor content in the antenna beam line of sight with a "water vapor radiometer".

Two frequency water vapor radiometers are frequently used to monitor the water vapor and liquid in the troposphere. The increased loss through the troposphere due to the water content is given by (Stelzried, 82)
\[
\tau = \left[ \frac{L(\text{dB})}{4.343} \right] \left[ \frac{(T-T_s)/(T_n-T_s)}{(T-T_s)/(T_p-T_s)} \right]^{\text{nepers}} \\
-(B/A) \left[ \frac{(T-T_s)/(T_p-T_s)}{2 + \cdots} \right], \text{ nepers} \quad (15-7)
\]

where \((T_p\) and \((B/A)\) defined precisely in Stelzried, 821

\[T = \text{measured increased noise temperature, K} \]
\[T_p = \text{troposphere physical temperature, K} \]
\[(B/A) = (1/2)\]

This loss consists of a stable oxygen component \(t_0\) assumed known and water vapor and liquid components \(t_v\) and \(t_L\), assumed unknown,

\[\tau = t_v + t_L + t_0, \text{ nepers} \quad (15-8)\]

This can be evaluated by measurements of \(\tau_1\) and \(\tau_2\) at two frequencies \(f_1\) and \(f_2\).

At frequencies \(f_1\) and \(f_2\), from Eq. (15-3)

\[\tau_1 = \left[ \frac{(T_1 - T_s)/(T_p - T_s)}{2 + \cdots} \right] + (1/2) \left[ \frac{(T_1 - T_s)/(T_p - T_s)}{2 + \cdots} \right], \text{ nepers} \quad (15-9)\]

and

\[\tau_2 = \left[ \frac{(T_2 - T_s)/(T_1 - T_s)}{2 + \cdots} \right] + (1/2) \left[ \frac{(T_2 - T_s)/(T_1 - T_s)}{2 + \cdots} \right], \text{ nepers} \quad (15-10)\]

and from Eq. (15-8),

\[\tau_1 = t_v + t_L + t_0, \text{ nepers} \quad (15-11)\]

and

\[\tau_2 = t_v + t_L + t_0, \text{ nepers} \quad (15-12)\]
The total precipitable water vapor through the tropospheric line of sight is

\[ M_v = \frac{\tau_{V1}}{k_{V1}} \]
\[ = \frac{\tau_{V2}}{k_{V2}} \text{ cm} \] (15-13)

and the total precipitable water liquid is

\[ M_L = \frac{\tau_{L1}}{k_{L1}} \]
\[ = \frac{\tau_{L2}}{k_{L2}} \text{ cm} \] (15-14)

where

\[ k_v, k_L = \text{proportionality constants relating precipitable water to attenuation, nepers/cm} \]

Also

\[ \frac{\tau_{V2}}{\tau_{V1}} = \frac{k_{V2}}{k_{V1}} \]
\[ = k_v \] (15-15)

and

\[ \frac{\tau_{L2}}{\tau_{L1}} = \frac{k_{L2}}{k_{L1}} \]
\[ = k_L \] (15-16)

Combining and solving for \( M_v \) and \( M_L \), in terms of \( T_1 \) and \( T_2 \) assuming all constants are "known",

\[ M_v = a_1(T_1 - T_s) + a_2(T_1 - T_s)^2 \]
\[ + a_3(T_2 - T_s) + a_4(T_2 - T_s)^2 + a_5 \text{ cm} \] (15-17)
This allows monitoring of $M_V$ and $M_L$ from measurements of $T_1$ and $T_2$. S. C. Wu 78, 67, has investigated optimum frequency selection. The constants in Eqs. (15-13) and (15-14) can either be evaluated from the definitions above or from direct tropospheric calibrations (from radiosonde balloons, etc.). At frequencies $f_1 = 20.6 \text{ GHz}$ and $f_2 = 31.6 \text{ GHz}$ (Stelzried, 82)\textsuperscript{15},

\[
M_V = 0.11 (T_1 - T_s) + 0.0002G (T_1 - T_s)^2 \\
- 0.048 (T_2 - T_s) - 0.000086 (T_2 - T_s)^2 \\
- 0.064 \cos Z \\
\text{cm} \quad (15-19)
\]

\textsuperscript{15}Hogg, 80, 281, has $M_V = 0.11 T_1 - 0.053 T_2 - 0.18$ and $M_L = 0.00117T_1 + 0.0027T_2 - 0.17$ appropriate for the zenith climatology of Denver, CO. The biggest difference between these expressions is the constant term for $M_L$ (accounting for $T_s = 2.7 \text{ K}$).
The increase in propagation path length due to both the precipitable water vapor and liquid is (Flock, 81, 71)

\[
\Delta t = 6.48 \hat{M}_v + 1.45 \hat{M}_l, \text{ cm (15-21)}
\]

or, in terms of \( T_1 \) and \( T_2 \)

\[
\Delta t = 0.71 (T_1 - T_s) + 0.0013 (T_1 - T_s)^2 \\
- 0.31 (T_2 - T_s) - 0.00056(T_2 - T_s)^2 \text{ cm (15-22)}
\]

Inspection of the above equations indicates that most of the tropospheric delay is due to the water vapor and very little from the liquid water. The primary effect of the liquid water is to alter the noise temperature measurements.

If it is required that the constants of Eqs. (15-17) and (15-18) be determined from direct radiometer calibrations, the constants \( a_2 \), \( a_4 \), \( b_2 \) and \( b_4 \) might best be determined analytically.

Figure 15-4 shows a comparison of the Jet Propulsion Laboratory water vapor radiometer system with the Socorro, New Mexico, Very Large Array (7-km baseline) VLBI tropospheric delay measurements.

Single frequency water vapor radiometers can be used to monitor clear sky water vapor in the troposphere. Simple "tipping" measurements (Eq. 15-4) can be used for continuous calibrations. Although the single frequency water vapor radiometer will perform well in clear sky conditions, serious degradation occurs during cloudy weather. Some applications allow the selection of good data, while discarding poor data.
15. References


Fig. 15-1. Photograph of WR 430 waveguide insertion test set calibration components
Fig. 15-2. Photograph of WR 430 waveguide flanges being hand lapped prior to precision insertion loss calibrations.
Fig. 15-3. Representation of receiving system used to evaluate lossy material; (a) basic receiving system, (b) same as (a) except lossy material placed over antenna.
Fig. 15-4, Comparison of a JPL water vapor radiometer and the New Mexico Very Large Array (7-km baseline VLBI) tropospheric delay measurements (July 23, 1981; G. Resch, 82)
16. **Noise Temperature Measurements**

A wide variety of techniques and instrumentation is required and available for noise temperature measurements. Ideally, a measurement system is designed and assembled to perform the required measurements to the accuracy required with minimum expense and difficulty. Due to the inherent difficulty in most noise temperature measurements, obtaining more accuracy (see Secs. 22 through 25) than required, usually results in a waste of time and resources. Many times, the technique used is forced by the equipment available. Resourcefulness, experience and attention to detail are mandatory.
17. **Total Power Radiometer**

The total power radiometer is the simplest and potentially the most sensitive of all microwave radiometers. As shown in Fig. 17-1 it consists of the input source termination (or antenna) followed by amplification $G$ and a square law detector. The output noise voltage is given by

$$E_o = k T_{op} BG$$

(17-1)

where

$$T_{op} = \text{system operating noise temperature, K}$$

$$= T_I + T_e$$

Solving for $T_{op}$,

$$T_{op} = (E_o/kBG)$$

(17-2a)

$$= aE_o$$

(17-2b)

where

$$a = \text{scale factor, K/Volt}$$

The radiometer scale factor $a$ can be determined with the use of thermal noise standards (Section 20) or other calibration techniques (Baars, 73). $T_{op}$ is determined by measuring $E_o$. Assuming "perfect" amplification, the measurement resolution, or minimum detectable signal, of a total power radiometer is given by (Krauss, 66, 244)

$$\delta T_{op} \min = T_{op}/\sqrt{\tau B}$$

(17-3)

where

$$\tau = \text{integration time, seconds}$$

This is the standard by which all other radiometer types are compared (Colvin, 61). Gain instability is a serious problem with total power radiometers which other radiometers attempt to circumvent, usually with a resultant loss in
sensitivity. To account for gain and bandwidth instability Eq. 17-2a is differentiated and the instabilities treated as random variables (Price, 65, 210). Then, with Eq. 17-3

$$\delta T_{op} = T_{op} \sqrt{(1/\tau B) + (\delta G/G)^2 + (\delta B/B)^2}$$

(17-4)

where

- \(\delta G/G\) = radiometer amplification gain instability, ratio.
- \(\delta B/B\) = radiometer bandwidth instability, ratio.

The consequences of amplifier gain and bandwidth instabilities depend on the radiometer application. Slow drifts can usually be removed from the baseline of single radio source scan measurements. However, measurement of the galactic background temperature profile is difficult with such a system. For most total power radiometer applications, the gain instability of Eq. 17-4 dominate and

$$\delta T_{op} \approx T_{op} \left| \frac{\delta G}{G} \right|$$

(17-5)

This explains the requirement for various other radiometer types discussed in subsequent sections used to reduce or circumvent the effect of gain instability.
17. **Reference**

1961  
Colvin, R. S., "A Study of Radio-Astronomy Receivers."  
Publication No. 18A, Stanford Radio Astronomy Institute,  
Stanford, CA, (October 31, 1961).

1966  

1976  
Price, R. M., "Radiometers", *Methods of Experimental Physics*,  
Fig. 17-1. Representation of total power radiometer
18. Dicke Radiometer

The Dicke radiometer (Dicke, 46, 268) was possibly the earliest microwave radiometer scheme to successfully reduce the effects of radiometer gain instability. This radiometer is still in wide usage with minor variations (Walt, 67, 127) due primarily to its simplicity.

The receiver input is rapidly switched between the source input termination (or antenna) and reference termination (Fig. 18-1). The receiver output is switched synchronously by the switch controller resulting in an output voltage

\[ E_o = k (T_1 - T_R) \text{BG} \]  

(18-1)

where

\[ T_1 = \text{antenna or unknown input termination temperature, } F \]

\[ T_R = \text{reference termination temperature, } K \]

Then

\[ T_1 = (E_o/k\text{BG}) + T_R \]

\[ = aE_o + b \]  

(18-2a)

(18-2b)

where

\[ a = \text{scale factor, } K/\text{volt} \]

\[ b = \text{bias constant, } K \]

The radiometer constants \( a \) and \( b \) are determined in the same manner as for the total power radiometer (Sec. 17).

The effect of amplification instability is proportional to \( (T_1 - T_R) \text{amp} \) instead of \( T_\text{amp} \) as in a total power radiometer. Differentiating Eq. 18-2a, using Eq. 17-1 with \( (t/2) \) for each switch position and assuming random instabilities.

18-1
If the radiometer is "balanced" \((T_1 = T_R)\),

\[
\delta T_1 = \sqrt{\left[ \frac{2(T_{op})^2}{T_B} \right] + \left[ \frac{2(T_{op})^2}{T_B} \right] + (T_1 - T_R)^2 \left[ \left( \frac{\delta G}{G} \right)^2 + \left( \frac{\delta n}{n} \right)^2 \right]}
\]  (18-3)

Eqs. 18-3 and 4 are applicable for square wave modulation. Other modulation schemes or signal filtering (Colvin, 31) result in a slightly increased instability.

The advantage of a Dicke radiometer relative to a total power radiometer is degraded unless \(T_1 = T_R\). Various modifications to the basic Dicke scheme such as the Kyle-Vonburg method (Ryle, 48) are used to further improve performance (Kraus, 66, 248-254). Another method is the beam "nodding" radiometer (Slobin, 70, 439) where the antenna pointing serves as the switch. The antenna beam is rapidly pointed on and off the source. Of course, balancing is only obtained for "weak" sources.

An actual radio source calibration measurement for most radiometers requires separate on and off source measurements. The radio source temperature is then given by

\[
T_s = (T_A)_{on} - (T_A)_{off}
\]  (18-5)

where

- \((T_A)_{on}\) = antenna temperature measured with antenna beam pointed on source, K
- \((T_A)_{off}\) = antenna temperature measured with antenna beam pointed off source, K
Assuming random measurement instabilities

\[
\delta T_s = \sqrt{(\delta T_A)^2_{\text{on}} + (\delta T_A)^2_{\text{off}}} \tag{18-6}
\]

Then, for the "ideal" total power radiometer, ignoring amplifier gain instabilities and assuming a "weak" source

\[
(\delta T_s)_{\text{min}} = T_{\text{op}} \sqrt{(1/\tau_1 B) + (1/\tau_2 B)} \tag{18-7}
\]

where

\[\tau_1, \tau_2 = \text{radiometer measurement times on and off source}\]

For \(\tau_1 = \tau_2 = \tau\)

\[
(\delta T_s)_{\text{min}} = T_{\text{op}} \sqrt{2/\tau B} \tag{18-8}
\]

Eqs. 18-6 and 18-7 are doubled for the balanced Dicke radiometer.

Potentially, the beam nodding radiometer can provide superior performance to the conventional Dicke radiometer since the radiometer is always "looking" at either the source or the sky. Bias errors can be removed by alternating the "on" and "off" source antenna beam positions. This observing strategy provides near ideal performance.
18. References


Fig. 18-1. Representation of Dicke radiometer
19. **Noise Adding Radiometer**

Very low noise receiving systems cannot tolerate the loss normally found in Dicke radiometer receiver input switches (= 0.1 dB loss increasing the system noise temperature ~6-7 K; see Secs. 7 and 20). The noise adding radiometer (Fig. 19-1) is particularly advantageous for this type of receiving system. The switch and reference termination of the Dicke radiometer is replaced in the noise adding radiometer by a transmission coupler and noise source. Square-wave-modulated noise at the receiver input is injected through the transmission line coupler. Then (Ohm, 63, 2047)

\[ T_{op} = T_N / (Y - 1) \]  

(19-1)

where

- \( Y = (P_{AN}/P_A) \), ratio
- \( T_N \) = excess noise at receiver input from noise source, K
- \( P_{AN} \) = receiver power output, noise source on, W
- \( P_A \) = receiver power output, noise source off, W

\( T_N \) can be calibrated with thermal noise standards or with a single ambient load termination (Stelzried, 71) applicable to systems with very low noise temperatures.

For this radiometer scheme (Batelaan, 70, 66-69)

\[ \delta T_{op} = 2T_{op} (1 + T_{op}/T_N) / \sqrt{T_B} \]  

(19-2)

This approaches the performance of a Dicke radiometer if \( T_{op}/T_N \). \( T_{op}/T_N \) is usually reduced to about 0.1, limited by the receiver dynamic range. If the modulation rate for \( T_N \) is high enough (10 Hz is satisfactory for many systems), this system is virtually immune to gain changes without a requirement for "balancing."

---

1. This is generally a commercial, solid-state, fast-switching noise source using a stabilized power supply installed in a temperature-stabilized oven (Kanda, 77, 676).
Eq. 19-2 assumes that the noise source $T_N$ is perfectly stable. Noise temperature bias errors are caused by incorrect calibration of $T_N$ and receiver nonlinearity (Stelzried, 80, 98).

Noise adding radiometer performance is illustrated in Fig. 19-2. In this example, two noise adding radiometers are operating simultaneously at 2.3 and 8.5 GHz. This figure shows standard "drift" curves obtained by "locking" the antenna and letting the radio source drift through the antenna beam with the earth's rotation. Noise temperature measurement resolution of $10^{-6}$ K during cosmic background measurements has been reported by Carpenter, 73, L61 using a noise adding radiometer on the Goldstone, CA 64-m antenna.
19. References

1963  

1970  

1970  

1973  

1973  

1973  

1976  

1977  

1980  
Fig. 19-1. Noise adding radiometer configuration
Fig. 19-2. Noise adding radiometer; simultaneous drift scan of 3C123 radio source through the 2.3-GHz and 8.5-GHz beams of the DSS 14 reflex feed (DSS 14 Goldstone, CA, 64-antenna)
20. Thermal Noise Standards

Thermal noise standards (Stelzried, 68, 646; Trembath, 68, 709; Daywitt, 72, 1-148; Yokoshima, 77, 1-140) provide the foundation for absolute radiometer rise temperature calibrations. A typical (Fig. 20-1) thermal noise standard consists of a matched source termination at temperature $T_s$ (as determined with a thermometer or by immersing the termination in a boiling liquid such as liquid helium (Stelzried, 61, 1224) or liquid nitrogen (Figs. 20-2 and 20-3). The transmission line is required to thermally isolate the output connector from the source termination.

The total loss through the transmission line (Fig. 20-1) is given by (assume "matched", $hf << kT$, Stelzried 82; the following is also applicable to the atmosphere, Sec. 4)

$$ L = e^{-\tau}, \text{ ratio (dB)} $$  \hspace{1cm} (20-1)

where

\[ \tau = \text{total transmission line absorption} \quad \text{[L (dB) = (10 log e) \tau]} \]

\[ = \int_0^L \alpha(x) \, dx \]

\[ \alpha(x) = \text{absorption coefficient of the transmission line at} \]

\[ x, \text{ nepers/m} \]

\[ x = \text{distance along the transmission line, m} \]

\[ (x = 0 \text{ at output} = L \text{ at source}^{17}) \]

\[ L = \text{total path length, m} \]

\[ ^{17}\text{It is sometimes convenient to integrate from the source to the output.} \]
From the theory of radiative transfer (neglecting reflections; Chandrasekhar, 60), the thermal noise temperature defined at the output is

\[ T_s' = \left( \frac{T_s}{L} \right) + \int_0^L T(x) \alpha(x) e^{-\tau(x)} \, dx, \text{ K} \]  

(20-2)

where

- \( T_s \) = source thermal noise temperature, K
- \( T(x) \) = physical temperature of medium at \( x \), K
- \( \tau(x) \) = absorption between 0 and \( x \), nepers

\[ \tau(x) = \int_0^x \alpha(x') \, dx' \]

This can be integrated directly or solved stepwise (Stelzried, 61, '224). For small transmission loss, Eq. (20-2) can be approximated by expanding \( e^{-\tau} \) and \( e^{\tau(x)} \) in power series.

\[ T_s' = T_s + A\tau + B\tau^2 + C\tau^3 + \ldots \]  

(20-3)

\( A, B \) and \( C \) can be solved and treated as "constants" assuming the physical temperature and loss distributions of the transmission line are known. For a transmission medium composed of discrete sections (\( l = n\Delta x \) and \( x = i\Delta x \)) and assuming uniform loss, \( \alpha(x) = \alpha_0 \) and \( \tau = \alpha_0 l \), usually applicable to thermal - case standards: 18

\[ A = T_p - T_s \]  

(20-4)

where

\[ T_p = \text{average physical temperature of the transmission medium, K} \]

\[ = \frac{1}{L} \int_0^L T(x) \, dx = \frac{1}{n} \sum_{i=1}^n T_i \]

18See Sec. 4 for nonuniform loss.
The last terms of Eq. (20-3) are small and provide an indication of the number of terms required and the accuracy of the power series expansion. Only 1 or 2 terms are required for most applications. In many applications, it is suitable to use two terms with \((B/A) = -(1/2)\).

Consider the solution of Eq. (20-3) for a linear physical temperature distribution, \(T(x) = T_I + (T_2 - T_I)x/a\) (applicable to a thermal noise standard consisting of a source at temperature \(T_s\) and a transmission line with a linear temperature distribution between \(T_I\) and \(T_2\)). We have

\[
A = (T_1 + T_2 - 2T_s)/2
\]

\[
B = (T_1 + 2T_2 - 3T_s)/6
\]

\[
C = (T_1 + 3T_2 - 4T_s)/24
\]

resulting in

\[
T'_s = T_s + L(dB)/4.343 \left(\frac{T_1 + T_2 - 2T_s}{2}\right)
- \frac{L(dB)}{4.343} \left(\frac{T_1 + 2T_2 - 3T_s}{6}\right)
+ \frac{L(dB)}{4.343} \left(\frac{T_1 + 3T_2 - 4T_s}{24}\right) + \ldots
\]

in agreement with Stelzried, 68, 648 (Case 4).

The error in \(T'_s\) due to an error in \(L(dB)\) is (using the first two terms of Eq. (20-3))

\[
\delta T'_s = 0.23 \left(\frac{T_p - T_s}{T_s}\right) \delta L(dB)
\]
This relationship is useful for estimating the required\(^\text{19}\) accuracy for loss calibrations; for \(\delta T_s' < 0.1 \text{ K}\), it is necessary that \(L(\text{dB}) < 0.0015 \text{ dB}\) (assumes \((T_p - T_s) = 290 \text{ K})\).

Similarly, the error in \(T_s'\) due to an error in \(T_p\) \((T_s' < T_p)\) is given by

\[
\delta T_s' = 0.23026 (\delta T_p) L(\text{dB})
\]

(20-9)

For \(\delta T_s' < 0.1 \text{ K}\) it is necessary to monitor \(T_p\) within \(\pm 4 \text{ K}\) (assumes \(L(\text{dB}) = 0.1 \text{ dB})\).

The model sophistication necessary for calibration depends on the accuracy requirement of the output noise temperature.

For some applications it may be necessary to measure the temperature distribution along the transmission line and compute the noise temperature at the output with an iterative technique. This technique can account for non-uniform transmission line loss with an overall accuracy of better than 0.2 K for a 78.1 K liquid-nitrogen-cooled termination calibrated at 2.3 GHz (Stelzried, 68, 650).

\(^{19}\)Precision transmission line loss measurement error has been reported at less than 0.001 dB (Stelzried, 70, 23). Precision calibrations require that special care be taken with matching (Sec. 23) and the use of precision connectors or waveguide flanges. Loss calibrations can also be performed with the "short circuit" method (Engen, 69, 1-23; Beatty, 65, 642; Otoshi, 70, 406; Yokoshima, 77, 1-140; Yokoshima, 76, 138).
20. References


Fig. 20-1. Representation of a thermal noise standard consisting of a source and a lossy transmission line.
Fig. 20-2. Photograph of liquid-nitrogen-cooled X-band microwave thermal noise standard
Fig. 20-3. Photograph of liquid-nitrogen-cooled S-band microwave thermal noise standard
21. **Noise Measuring Instrumentation**

Considerable time and effort can be invested in noise measurements. The required instrumentation can be fabricated as required by the experimenter. However, the wide availability of commercial noise instrumentation can reduce this time investment in many cases.

This equipment includes microwave antennas, thermal noise standards (cryogenically cooled, ambient and hot), transmission line components (supplied with precision match and loss calibrations) and precision measurement instrumentation. Typical manufacturers include: Maury Microwave Corporation (Cucamonga, CA), Airborne Instrument Laboratories (Deer Park, Long Island, NY), Weinschel Engineering (Gaithersburg, MD), Scientific Atlanta Inc (Atlanta, GA), Merrimac Inds. Inc (West Caldwell, NJ), Microlab/FXR, (Livingston, NJ), Rhode and Schwarz (Fairfield, NJ), Telonic/Berkeley (Laguna Beach, CA), Hewlett-Packard Co. (Palo Alto, CA), Narda Microwave Corp. (Plainview, NJ), and Coax Devices (Chelsea, MA).
22. Measurement Resolution Error

The measurement resolution of noise temperature calibrations can be analyzed from the radiometer equation appropriate to the system analyzed. Consider the typical configuration for measuring receiver noise temperature (Fig. 11-1). From Sec. 11

\[ T_e = \frac{(T_H - Y T_C)}{(Y - 1)} \quad (22-1) \]

where

\[ Y = \frac{P_N}{P_C} \]

The measurement resolution of \( T_e \) is determined by the measurement resolution of \( Y \),

\[ \frac{\delta T_e}{R} = \left| \frac{\partial T_e}{\partial Y} \right| \frac{\delta Y}{R} \quad (22-2) \]

where

\[ \frac{\delta Y}{R} = \text{measurement resolution of } Y \]

\[ = \sqrt{(\frac{\partial Y}{\partial P_H})^2 (\delta P_H)^2 + (\frac{\partial Y}{\partial P_C})^2 (\delta P_C)^2} \]

\[ \left| \frac{\partial T_e}{\partial Y} \right| = (T_H - T_s)/(Y - 1)^2 \]

\[ = (T_{op})^2_c/(T_H - T_C) \]

\[ (\frac{\partial Y}{\partial P_H})^2 = 1/P_C^2 \]

\[ (\frac{\partial Y}{\partial P_C})^2 = P_H^2/P_C^4 \]
This results in

\[ \frac{\delta T_e}{K} = \frac{(T_{op})_H (T_{op})_C}{(T_H - T_C)^2} \sqrt{2/\tau_B} + 2(\delta G/G)^2 \]  

(22-3)

For \( \delta G=0 \)

\[ (\delta T_e/K)_{\text{min}} = \frac{(T_{op})_H (T_{op})_C}{(T_H - T_C)^2} \sqrt{2/\tau_B} \]  

(22-4)

This technique is applicable to other configurations. As examples, for antenna noise temperature calibrations (Fig. 10-1) (Franco, 81, 80), the measurement resolution of \( T_a \) is given by

\[ \frac{(\delta T_a)}{K} = \sqrt{(T_{op})_a^2 + \left( \frac{T_C - T_a}{T_H - T_C} \right)^2 (T_{op})_C^2 + \left( \frac{T_H - T_a}{T_H - T_C} \right) (T_{op})_C^2} \sqrt{2/\tau_B} \]  

(22-5)

and for system noise temperature calibration (Fig. 12-1) (Stelzried, 71, 41), the measurement resolution of \( T_{op} \) is given by

\[ \text{Obtained from the total power radiometer resolution equations (Sec. 17)} \]
\[
\frac{\delta T_{op}}{R} = \frac{(T_{op})_H}{(T_{op})_a} \sqrt{2 \left[ \frac{1}{\tau B} + (\delta G/G)^2 \right]}
\]

(22-6)

where

\( (T_{op})_H \) = system noise temperature when switched to \( T_H \), K
\[ = 300 \text{ K} \]

\( (T_{op})_a \) = system noise temperature when switched to \( T_a \), K
\[ = 30 \text{ K} \]

\( \tau \) = integration time, sec
\[ = 10 \text{ sec} \]

\( B \) = receiving system bandwidth, Hz
\[ = 10^7 \text{ Hz} \]

\( (\delta G/G) \) = gain instability, ratio
(calculation during measurement period)
\[ = 0.23 \delta G \text{ (dB)} \]
\[ = 0.23 (0.01) \]
\[ = 0.0023 \]
22. References


23. **Mismatch Error**

The previous sections assume that the source impedance, transmission line and receiver are "matched" with no signal reflections. Mismatched components cause multiple reflections with resultant noise calibration errors. Careful analysis of these reflections can be used to correct the calibrations (Otoshi, 68, 675, Wait, 68, 670, Nemoto, 68, 866). In many applications, it is easier (and customary) to reduce the reflections to a suitable level and then ignore the effect.

Consider a transmission line with characteristic impedance $Z_o$ and termination impedance $Z_L$ as shown in Fig. 23-1. From standard transmission line theory (Ramo, 53, 27; Stelzried, 61, 812), the reflection coefficient is given by

$$\rho = \frac{V'}{V}$$

$$= \frac{(Z_L - Z_o)}{(Z_L + Z_o)}$$

where

$V = \text{forward traveling voltage wave, } V$

$V' = \text{reflected traveling voltage wave, } V$

The voltage standing wave ratio (VSWR or $S$) is given by

$$S = \frac{V_{\text{max}}}{V_{\text{min}}}$$

$$S = \frac{1 + |\rho|}{1 - |\rho|}$$

where

$V_{\text{max}} = \text{maximum voltage amplitude, } V$

$$= \sqrt{V^2 + V'^2}$$

---

21 The following analysis is restricted to VSWR theory. Scattering parameters are also frequently used (Otoshi, 68, 675; Hecken, 81, 997).
so that

\[ V_{\text{min}} = \text{minimum voltage amplitude, } V \]

\[ = |V| - |V'| \]

\[ |\rho| = \frac{S-1}{S+1} \quad (23-3) \]

The error in noise temperature calibrations is proportional to power or \(|\rho|^2\). As shown in Fig. 23-2, a termination with a VSWR of 1.1 results in \(|\rho|^2 = 0.0023\). This \(\approx 0.23\%\) power reflection could result in \(\approx 0.7\) K error for an ambient (290 K) thermal standard termination. Precision calibrations usually require transmission components and terminations with VSWR's less than \(= 1.1\).

It is usually necessary to analyze each microwave noise calibration configuration to verify satisfactory performance. Multiple reflections without knowledge of the voltage phase relationships (i.e., using VSWR magnitude information only) require worst case analysis. Consider the configuration for measuring \(T_{\text{op}}\) using a single thermal termination standard (Fig. 12-3). For this configuration, the maximum error for a measurement of \(T_{\text{op}}\) due to mismatches \(S_a\) (antenna VSWR), \(S_p\) (source termination mismatch) and \(S_e\) (receiver mismatch) is (Stelzried, 71, 41; Otoshi, 68, 675),

\[
\Delta T_{\text{op/mm}} = \frac{T_{\text{op}}}{T_p + T_o} \left\{ \text{Max} \left[ 1 - \frac{S}{S_a} \left( \frac{S S_e + 1}{S_p S_c + 1} \right)^2 \right] T_p + \left[ 1 - \frac{S}{S_a} \left( \frac{S S_e + 1}{S_p S_c + 1} \right)^2 \right] T_e \right\}
\]

or:

\[
\left( 1 - \frac{S}{S_a} \left( \frac{S_a + S e}{S_p + S e} \right)^2 \right) T_p + \left[ 1 - \frac{S}{S_a} \left( \frac{S_a + S e}{S_p + S e} \right)^2 \right] T_e
\]

or:

\[
\left( 1 - \frac{S}{S_a} \left( \frac{S a + S e}{S_p + S e} \right)^2 \right) T_p + \left[ 1 - \frac{S}{S_a} \left( \frac{S a + S e}{S_p + S e} \right)^2 \right] T_e
\]
For example, if $T_p = 295$ K, $T_e = 5$ K, $T_{op} = 30$ K, $S_p = 1.02$, $S_e = 1.15$ and $S_a = 1.15$, the maximum error in $T_{op}$ due to mismatch is

$$\Delta T_{op}/mm = \frac{30}{295 + 5} \left[ 1 - \frac{1.02}{1.15} \left( \frac{1.15 \times 1.15 + 1}{1.02 \times 1.15 + 1} \right)^2 \right] 295$$

$$+ \left[ 1 - \frac{1}{1.15} \left( \frac{1.15 \times 1.15 + 1}{1.15 + 1} \right)^2 \right] 5$$

$$= 0.50 \text{ K peak}$$

The above analysis determines the peak or worst case error due to mismatch. In a statistical sense this is an $\sigma$ error. Assume

$$\delta T_{op}/mm = (\Delta T_{op}/mm)/3$$

$$= 0.17 \text{ K}$$

It is sometimes advisable to monitor the source termination VSWR in an operational system (Stelzried, 67).
23. Reference


Fig. 23-1. Representation of transmission line with termination
Fig. 23-2. Plot of $|\rho|^2$ vs VSWR from Eq. 23-3
24. Linearity Error

Previous sections assumed perfectly linear receiver amplifiers. It is important that noise calibration instrumentation be operated in the linear region of the receivers. If the receivers are non-linear, either the error contribution of the non-linearity must be known, or a calibration is necessary for a subsequent correction of the measurement results (Stelzried, 80, 98). Fig. 24-1 shows a typical test configuration for an amplifier linearity verification evaluation. The receiver output power \( P_{\text{OUT}} \) is plotted versus input power \( P_{\text{IN}} \) as shown in Fig. 24-2. Receiver saturation is the usual source of non-linearity. Another source of non-linearity error is the precision attenuator used for \( Y \) factor measurements (Sec. 10). Typical precision attenuator linearity specifications are 0.02 dB/dB (Stelzried, 71, 41).

As an example of the effect of a receiving system non-linearity consider the measurement of system noise temperature (Sec. 12, Fig. 12-1). For this configuration,

\[
T_{\text{op}} = \frac{(T_{\text{HL}} + T_{\text{L}})}{Y} \tag{24-1}
\]

Differentiating with respect to \( Y \),

\[
\left| \frac{dT_{\text{op}}}{\partial Y} \right| = \frac{T_{\text{op}}}{Y} \tag{24-2}
\]

The measurement error of \( T_{\text{op}} \) due to the error in \( Y \) is given by

\[
\frac{\sigma T_{\text{op}}}{Y} = (T_{\text{op}}) \frac{\delta Y}{Y} \tag{24-3}
\]

\[
= 0.23 (T_{\text{op}}) \delta Y \text{ (dB)}
\]

or

\[
\frac{\sigma T_{\text{op}}}{Y} = 0.23 (T_{\text{op}}) (\epsilon_L) Y \text{ (dB)} \tag{24-4}
\]

\[
= 0.23 (30) (0.0067) (10) \approx 0.46 \text{ K}
\]

\[22\] For this example, assume \( (T_{\text{op}})_H = 30 \text{ K} \), \( (T_{\text{op}})_L = 300 \text{ K} \), \( \epsilon_L = 0.0067 \text{ dB/dB} \).
where

\[ \delta Y(dB) = (c_L) Y(dB) \]

\[ c_L \text{ is linearity error, dB/dB} \]

\[ Y(dB) = 10 \log \left( \frac{T_{op}}{T_{top}} \right) \]

\[ T_{top} \text{ is system noise temperature when switched to the antenna, K} \]

\[ (T_{op})_H \text{ is system noise temperature when switched to } T_H, \text{ K} \]
24. References


Fig. 24-1. Test configuration for amplifier linearity test

Fig. 24-2. Typical result of amplifier linearity test
25. Cascading of Errors

The previous sections treated the individual error sources separately. In an actual receiving system, multiple errors are present. These separate error sources usually range somewhere between being completely correlated to completely uncorrelated with each other.

Usually, it is not possible to determine the degree of error correlation, nor to even know the type of statistics (Gaussian, Poisson, etc.) applicable to the individual error sources. It is usually satisfactory to indicate error bounds between treating the individual errors as if totally correlated

\[ \epsilon_c = |\epsilon_1| + |\epsilon_2| + |\epsilon_3| + \ldots + |\epsilon_n| \]  

(25-1)

or totally uncorrelated

\[ \epsilon_u = \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \ldots + \epsilon_n^2} \]  

(25-2)

As an example, continue with the analysis of the system operating noise temperature measurement scheme (Fig. 12-3) discussed in Secs. 12, 22, 23, and 24. We had

\[ T_{op} = (T_H + T_e)/Y \]  

(12-1)

where

\[ Y = \left( \frac{P_H}{P_A} \right) \]

\[ T_{op} = T_a + T_e \]

Then

\[ \delta T_{op}/K = 1 \sigma \text{ error in } T_{op} \text{ due to measurement resolution, } K \text{ (Sec. 22)} \]

\[ \approx 0.03 \text{ K} \]

---

\[ ^{23} \text{Assumes (Stelzried, 71, 41) 1} \sigma \text{ errors, } T_{op} = 30 \text{ K, } (T_{op})_H = 300 \text{ K, } \delta T_H = 0.33 \text{ K, } \delta T_e = 0.1 \text{ K, } \epsilon_L = 0.0067 \text{ dB/dB}. \]
\[ \frac{\delta T_{op}}{T_H} = \text{error in } T_{op} \text{ due to an error in } T_H, K \]
\[ = \frac{T_{op}}{(T_{op})_H} \delta T_H \]
\[ = 0.03 \text{ K} \]

\[ \frac{\delta T_{op}}{T_e} = \text{error in } T_{op} \text{ due to an error in } T_e, K \]
\[ = \frac{T_{op}}{(T_{op})_e} \delta T_e \]
\[ = 0.01 \text{ K} \]

\[ \frac{\delta T_{op}}{Y} = \text{error in } T_{op} \text{ due to an error in } Y, K \]
(Sec. 24)
\[ = 0.46 \text{ K} \]

\[ \frac{\delta T_{op}}{mm} = \text{error in } T_{op} \text{ due to mismatch errors, K} \]
(Sec. 23)
\[ = 0.17 \text{ K} \]

results in an estimated overall measurement error between \( \epsilon_u = 0.5 \text{ K} \)
and \( \epsilon_c = 0.7 \text{ K} \).
25. References