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A NEW THEORY OF SOURCES OF BIRKELAND CURRENTS

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Abstract

A new approach to collisionless plasma (Cole 1983) shows the existence of current orthogonal to $\mathbf{B}$ along the low latitude boundary layer of the magnetosphere driven by electric field which is orthogonal to both $\mathbf{B}$ and the layer. In this case the relationship

$$p_1 + \frac{B^2}{8\pi} - \frac{\varepsilon E^2}{8\pi} = \text{constant},$$

holds on a line orthogonal to $\mathbf{B}$ and the layer, where $\varepsilon$ is the dielectric constant of the plasma for electric fields orthogonal to $\mathbf{B}$. Across the geomagnetic tail there flows a current in the direction of the dawn-dusk electric field, and in this case a relationship

$$p_1 + \frac{B^2}{8\pi} + \frac{\varepsilon E^2}{8\pi} = \text{constant},$$

holds along a line orthogonal to $\mathbf{E}$ and $\mathbf{B}$. Divergence of both these currents is shown to be a source of Birkeland currents. Also some of the boundary layer current is continuous with current across the tail. Electric currents of physically similar origin flow in interplanetary space, and when the magnetosphere interrupts them, additional Birkeland currents are driven.

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Introduction

In a recent paper a general theory of electromagnetic fields in plasmas in a quasi-steady state was proposed (Cole 1983). This theory is now applied to the problem of the sources of electric current responsible for geomagnetic disturbance. In the first part of the paper, some idealised configurations of electromagnetic fields in plasmas are considered. In the second part, these are applied to structures in the geomagnetic field and interplanetary space as part of a new theory of geomagnetic disturbance.

Though many components of earlier theories of geomagnetic disturbance (Axford and Mines, 1961; Dungey, 1961; Piddington, 1960, 1962; Cole, 1960, 1961) remain valid today, none of them is entirely satisfactory and all fail to be quantitative on the precise mechanism of transfer of energy from the solar wind into the magnetosphere and the atmosphere. Later, this author (Cole 1974) illustrated mechanisms whereby solar wind plasma could enter the geomagnetic field. These included "grad B capture" and plasma flow across the magnetopause caused by tangential electric fields and inertial drifts.

The tangential electric fields could be caused either as the result of grad B capture on the day side of the magnetopause (Cole 1974) or as the result of dynamo action in the high latitude ionosphere (Cole 1976).

In this paper an attempt is made to advance the theory of solar wind interaction with the magnetosphere invoking new understanding of currents generated by plasma flows (Cole, 1983) in interplanetary space, and in various regions of the magnetosphere and showing how the currents couple into the upper atmosphere. The units used are c.g.s., except where otherwise stated.
Idealized Field Configurations

In this paper the theory of the Maxwell stresses in a collisionless, 2-D plasma (Cole, 1983) is applied to discuss the sources of electric current responsible for geomagnetic disturbance. By treating a collisionless plasma as a medium with specifiable dielectric constant and magnetic permeability it has been shown that, in a steady state,

\[(\text{div} \mathbf{D}) \mathbf{E} + (\mathbf{V} \times \mathbf{H}) \times \mathbf{H} - \frac{1}{2} \mathbf{E} \mathbf{v} - \frac{1}{2} H^2 \nabla \Phi = 0,\]  \hspace{1cm} (1)

where \( \mathbf{E} \) = electric field,

\( \mathbf{B} \) = magnetic induction,

\( \mathbf{H} \) = magnetic field,

\( \mu = \frac{B}{H} \) = magnetic permeability,

\( \mathbf{D} = eE_\parallel + E_\perp \),

where \( E_\parallel, E_\perp \) denote components parallel and perpendicular respectively to \( \mathbf{B} \).

\[\varepsilon = 1 + \sum_{s} \frac{4 \pi n_s m_s c^2}{B^2},\]

where \( n_s \) = number of ions cm\(^{-3}\) of species \( s \),

and \( m_s \) = molecular mass of ions of species \( s \).

It has been shown elsewhere (Cole 1983) that in plasmas such as exist in the low latitude boundary layer and interplanetary space the force denoted by the term \((\text{div} \mathbf{D}) \mathbf{E} - \frac{1}{2} \mathbf{E} \mathbf{v} \) in equation 1 drives significant current. This current is referred to as dielectric current and arises principally because of the large value of \( \varepsilon \) in magnetised plasmas. In this paper divergence of currents of this kind in the geomagnetic environment are invoked as sources of Birkeland currents observed in the magnetosphere. First, some simple models of crossed electric and magnetic fields in plasmas are presented, which define the new currents invoked. Then these models are applied to the plasma environment of the earth in space to estimate the strength of Birkeland currents.
From equation (1)

\[
(d\text{iv}\ D) \quad E_x - \frac{1}{2} E_\perp \frac{\partial E}{\partial x} + \frac{1}{2} B \frac{\partial B}{\partial x} \left( \frac{1}{\mu} \right) - B_z \left( \frac{\partial (E/\mu)}{\partial x} - \frac{\partial (x/\mu)}{\partial z} \right) \\
+ B_y \left( \frac{\partial (B_y/\mu)}{\partial y} - \frac{\partial (B_y/\mu)}{\partial x} \right) = 0,
\]

(2)

\[
(d\text{iv}\ D) \quad E_y - \frac{1}{2} E_\perp \frac{\partial E}{\partial y} + \frac{1}{2} B \frac{\partial B}{\partial y} \left( \frac{1}{\mu} \right) - B_x \left( \frac{\partial (E/\mu)}{\partial y} - \frac{\partial (y/\mu)}{\partial x} \right) \\
+ B_z \left( \frac{\partial (B_z/\mu)}{\partial z} - \frac{\partial (B_z/\mu)}{\partial y} \right) = 0,
\]

(3)

and

\[
(d\text{iv}\ D) \quad E_z - \frac{1}{2} E_\perp \frac{\partial E}{\partial z} + \frac{1}{2} B \frac{\partial B}{\partial z} \left( \frac{1}{\mu} \right) - B_y \left( \frac{\partial (E/\mu)}{\partial z} - \frac{\partial (z/\mu)}{\partial y} \right) \\
+ B_x \left( \frac{\partial (B_x/\mu)}{\partial x} - \frac{\partial (x/\mu)}{\partial z} \right) = 0.
\]

(4)

when \( \varepsilon \gg 1 \), \( d\text{iv}\ D \approx \varepsilon E_\perp = \varepsilon(E_\perp - E_\parallel) \).

Consider now different simple cases of equation 1 which can be integrated and applied to interplanetary space or the geomagnetic field.

**Current of Type I:**

Suppose \( B = (0, 0, B_z) \)

(5)

and \( E = (0, E_y(y), E_z) \)

(6)

It is shown in Cole (1983) that

\[
\mu = \frac{1}{\frac{\varepsilon E_\perp}{\mu B_\perp} + \frac{1}{B^2}}
\]

This case (see Fig. 1), discussed earlier (Cole, 1983) shows that a current
flows in the x direction, of magnitude given by

\[ j_x = \frac{1}{8\pi} \frac{\partial}{\partial y} [\varepsilon E_y^2 - B_z^2 - \bar{p}_l]. \]  

(7)

Also if \( E_z \ll E_y \) equation 3 integrates, when \( \frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0 \), to

\[ \frac{\partial}{\partial z} (B_z^2) + \varepsilon E_y^2 = \text{constant}. \]  

(8)

This corresponds to the circumstances of plasma flows and currents in the x direction. We investigate later the possibility that the flow in the low latitude boundary layer of the earth (Eastman, 1979; Eastman and Hones, 1979) is like this. Also we expect that flows describable by this model exist in interplanetary space (see applications, later in the paper).

**Current of Type II:**

In this case (see Fig. 2),

\[ \text{suppose } B = (0, 0, B_z), \]  

(9)

and

\[ E = (0, E_y(x), E_z). \]  

(10)

with \( \frac{\partial}{\partial y} = \frac{\partial}{\partial x} = 0 \), and \( E_z \ll E_y \) equation 2 yields

\[ \frac{\partial}{\partial x} \left( \frac{B_z^2}{\mu} \right) + E_y \frac{\partial E_y}{\partial x} = 0. \]  

(11)

Integrating equation 11, yields

\[ \frac{\partial}{\partial x} \left( \frac{B_z^2}{\mu} + \varepsilon E_y^2 \right) - \int \frac{\partial}{\partial x} (E_y^2) \, dx = \text{constant}. \]  

(12)
This case is appropriate to discussions of flow of current across the "neutral" sheet in the geomagnetic tail under the influence of a "cross-tail" electric field.

In this case the current in the $y$-direction driven by electric field is given by

$$\mathbf{j}_y = \frac{1}{8\pi} \frac{c}{B_y} E_y \frac{\partial \mathbf{E}}{\partial y}$$

(13)

In the special case, when $\nabla \times \mathbf{E} = 0$, equation 12 becomes

$$8\pi p_\perp + B_z^2 + cE_y^2 = \text{constant}. \quad (14)$$

In the case of cold plasma $p_\perp = 0$, and,

$$B_z^2 + \frac{c^2}{v_A^2} = \text{constant}, \quad (15)$$

where $v_A^2 = c^2/\varepsilon$ = square of Alfvén velocity

Equation (15) resembles the results obtained by Alfvén (1968) and Cowley (1973) in discussing properties of neutral surfaces. While the general solution of the problem of an infinite neutral sheet is contained in equation (14), the Alfvén (1968) type solution is contained as a special case (corresponding to $p_\perp = 0$), while the Harris (1961) type solution is another case, corresponding to $E_y = 0$. In the former case, a balance of forces is maintained by $-\nabla (cE_y^2)/8\pi$ and $\mathbf{j} \times \mathbf{B}$. In the case $E_y = 0$, a balance of $\mathbf{j} \times \mathbf{B}$ and $-\nabla p_\perp$ could be maintained.

Case II is, of course, relevant to discussion of magnetic "merging." Equation 14 can be interpreted as implying the existence of static structures of opposed magnetic fields in which plasma is driven towards the merging region by an applied electric field $E_y$. The plasma is turned away from the
neutral line before reaching it generating current in the direction of $E_y$.

The partitioning of energy between thermal energy of the plasma, as manifested by $\rho$, magnetic energy and electric field energy density, is dependent on the plasma density, or equivalently, the Alfvén speed (see equation 14). Clearly there is a limitless variety of possible partitions. Experiments looking for merging regions in the geomagnetic tail or at the magnetopause should take into account all three terms of equation 14. It appears that a steady state neutral sheet can exist in which no merging takes place. Additional factors to those considered here would appear to be necessary in order to generate merging.

A Variant of Case 1:

Assume $\mathbf{B} = (B_x, 0, B_z)$

$$E = (0, E_y(y), 0)$$

Equation 3 then yields, assuming $\frac{\partial}{\partial x} = 0 = \frac{\partial}{\partial z}$

$$\frac{3}{2Y} (\varepsilon E_y) E_y - \frac{1}{2} E_y^2 \frac{3\varepsilon}{3y} + \frac{1}{2} B_x^2 \frac{3B_x}{3y} B_x - B_x \frac{3(B_x/\mu)}{3y}$$

$$-B_z \frac{3(B_z/\mu)}{3y} = 0,$$

Therefore,

$$\frac{3}{3y} (\varepsilon E_y^2) - \frac{3}{3y} (\frac{\mu}{\mu}) = 0,$$

and it follows by integration along the $y$-direction, that,

$$\varepsilon E_y^2 - B_z^2 - 8\pi p_\perp = \text{constant.}$$

7
This is essentially the same result as case I which shows that the components of \( B \) perpendicular to \( E_y \) act independently but similarly in relation to \( E_y \).

**General two dimensional plasma**

In this case,

\[
\text{assume } B = (0,0, B_z)
\]

and \( \frac{\partial B}{\partial z} = 0 \)

Equation 2 now yields

\[
\left[ \frac{\partial}{\partial x}(eE_x) + \frac{\partial}{\partial y}(eE_y) \right] E_x = \frac{1}{2} \mu \epsilon_0 \frac{\partial B_z}{\partial x} + \frac{1}{2} B^2 \frac{\partial B_z}{\partial y} = 0, \tag{26}
\]

and \( \frac{\partial}{\partial x}(eE_x) + \frac{\partial}{\partial y}(eE_y) \right] E_y = \frac{1}{2} \mu \epsilon_0 \frac{\partial B_z}{\partial y} = 0. \tag{27}

From (27)

\[
\left[ \frac{\partial}{\partial y}(eE_y) \right] E_x + \frac{\partial}{\partial x}(eE_x) - \frac{\partial}{\partial x} \left( \frac{E^2}{\mu} \right) - \frac{1}{2} \mu \epsilon_0 \frac{\partial B_z}{\partial y} = 0. \tag{28}
\]

From (28)

\[
\left[ \frac{\partial}{\partial x}(eE_x) \right] E_y + \frac{\partial}{\partial y}(eE_y) - \frac{\partial}{\partial y} \left( \frac{E^2}{\mu} \right) - \frac{1}{2} \mu \epsilon_0 \frac{\partial B_z}{\partial x} = 0. \tag{29}
\]
Since \( \vec{V} \times \vec{E} = 0 \), equation 29 yields
\[
\frac{\partial}{\partial x} (\varepsilon E_x^2) - \frac{1}{2} \frac{\partial}{\partial x} (\varepsilon E_y^2) - \frac{3}{2} \frac{\partial}{\partial x} (B_z^2) + \frac{3}{2} \varepsilon (\sigma E_x E_x) = 0, \tag{30}
\]
and equation 30 yields
\[
\frac{\partial}{\partial y} (\varepsilon E_y^2) - \frac{1}{2} \frac{\partial}{\partial y} (\varepsilon E_x^2) - \frac{3}{2} \frac{\partial}{\partial y} (B_z^2) + \frac{3}{2} \varepsilon (\sigma E_y E_y) = 0. \tag{31}
\]

It follows that
\[
\frac{\partial^2}{\partial x^2} \left[ \varepsilon E_x^2 - \frac{1}{2} \varepsilon E_y^2 - \frac{B_z^2}{\mu} \right] = \frac{\partial^2}{\partial y^2} \left[ \varepsilon E_y^2 - \frac{1}{2} \varepsilon E_x^2 - \frac{B_z^2}{\mu} \right] \tag{32}
\]

No simple interpretation can be given at this time in this case of a general two-dimensional plasma and it would appear to need computer solutions to produce further erudition. The symmetry of equation 32 is, however, noted.

**Intermediate \( V_x \) and \( V(1/\mu) \)**

Suppose a unidirectional electric field exists orthogonal to \( \vec{B} \) and let there be gradients of \( \varepsilon \) and \( \mu \) orthogonal to \( \vec{B} \), so that,
\[
\vec{B} = (0, 0, B_z), \tag{33}
\]
\[
\vec{E} = (0, E_y, 0), \tag{34}
\]
\[
\frac{\partial \varepsilon}{\partial z} = 0, \frac{\partial \varepsilon}{\partial y} \neq 0, \frac{\partial \varepsilon}{\partial x} \neq 0, \tag{35}
\]
\[
\frac{\partial \mu}{\partial x} \neq 0, \frac{\partial \mu}{\partial y} \neq 0. \tag{36}
\]

Equation (2) then yields, for the \( x \)-component,
\[
- \frac{1}{2} E_y^2 \frac{\partial \varepsilon}{\partial x} + \frac{1}{2} B^2 \frac{3}{2} \frac{\partial}{\partial x} (\frac{1}{\mu}) - B_z \left( \frac{3}{2} \frac{\partial (z/\mu)}{\partial x} \right) = 0, \tag{37}
\]
and for the $y$-component,

$$
\left[ \frac{\partial}{\partial y}(\varepsilon E_y) \right] y - \frac{1}{2} \varepsilon_y E_{2y} + \frac{1}{2} E_{2y} \frac{1}{y} - E \frac{3}{y^2} \frac{B_z}{\mu} = 0. 
$$

(38)

Equation 38 integrates to

$$
\varepsilon E_y^2 - n^2 - 8 \pi p_{\perp} = f(x),
$$

(39)

while equation 37 yields

$$
\varepsilon E_y^2 + B^2 + 8 \pi p_{\perp} = \phi(y). 
$$

(40)

Once again it appears that computer calculations could be required to make physical interpretation more accessible in this case.

**APPLICATIONS**

We are now in a position to discuss the various current regions of the magnetosphere. First we consider the regions near the magnetopause. Traditionally, currents at the magnetopause have been considered to be produced solely by a gradient of pressure pointing outwards from the magnetosphere. A Lorentz force $\mathbf{j} \times \mathbf{B}$ was considered to oppose the force due to this gradient of pressure (see Willis 1971 for a review). These currents are the Chapman-Ferraro currents. It is considered here that the traditional approach must now be modified. It will be seen to be of limited application in portions of the magnetosphere only, viz., over a small range of longitudes either side of the "nose" of the magnetosphere. In this region a negative pressure gradient acts from the solar wind into the magnetosphere. In vast regions of the magnetosphere, and the low-latitude boundary layer associated with it, there is a flow of solar wind plasma inside and roughly parallel to the magnetopause in which the speed decreases inwards (see e.g., Fig. 3 taken from Eastman (1979)), and in which the term $\varepsilon E_{\perp}^2$ is a considerable
fraction of $p$. In making this claim, we assume that $eE_\perp^2$ is estimated by $\frac{1}{2} n_i m_i V_{sw}^2$ (c.f., Cole, 1983).

**Boundary Layer**

It is claimed by the present author that in the boundary layer the current system is different from what might be expected on Chapman-Ferraro theory and the consequences are significantly different. One long recognized difficulty of the Chapman-Ferraro approach is that it predicts no energy or plasma flow into the magnetosphere at all. Nor does it account for the neutral sheet in the geomagnetic tail. The Chapman-Ferraro theory however appears applicable in a limited manner to some parts of the magnetopause as will be demonstrated. Let us now apply equations 7 and 8 of Case I to the low latitude boundary layer.

**The Boundary Layer Current**

Consider an idealized low latitude boundary layer in which the flow of solar wind plasma is laminar and parallel to the boundary. Let the velocity, pressure and density be functions of distance from the magnetosheath. The magnetopause is considered to be a tangential discontinuity of the magnetic field.

Figures 4a and 4b illustrate the situation in cross section in the dawn-side and dusk-side boundary layer. This configuration conforms to Case I. We realize from equation 7 that the dielectric current is sunward in the dusk side and anti-sunward in the dawn side. The presence of these currents has not hitherto been taken into account in the problem of solar wind interaction with the magnetosphere. Figure 5 illustrates the dielectric current in equatorial cross section. The current due to $-V_p$ is not illustrated.
The dielectric current flows in the opposite direction to the classical Chapman-Ferraro currents which are caused by the gradient of plasma pressure. The dielectric current is a dielectric current of type I. The complete current orthogonal to $\mathbf{B}$ in such flows leads to equation 8 and is specified by equation 7.

This theory calls into question the widespread view that the geomagnetic field is separated from the interplanetary medium solely by a current layer caused by the gradient of pressure of solar wind normal to the current layer. However the theory appears to be consistent with observations of the outer regions of the geomagnetic field where it interacts with the solar wind. Taking the ion plasma parameters for a typical dawn-dusk meridian in a sequence of steps through the plasma boundary layer (Eastman 1978) Table 1 is constructed.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Magnetosheath</th>
<th>PBL 1</th>
<th>PBL 2</th>
<th>PBL 3</th>
<th>PBL 4</th>
<th>Magnetosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ (cm$^{-3}$)</td>
<td>18</td>
<td>14</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>$V$ (km s$^{-1}$)</td>
<td>300</td>
<td>280</td>
<td>200</td>
<td>120</td>
<td>50</td>
<td>&lt;30</td>
</tr>
<tr>
<td>$\bar{E}$ (keV)</td>
<td>0.5</td>
<td>1</td>
<td>2.5</td>
<td>4</td>
<td>6.5</td>
<td>7</td>
</tr>
<tr>
<td>$\frac{1}{2} n V^2$ (keV cm$^{-3}$)</td>
<td>9.7</td>
<td>5.3</td>
<td>2.0</td>
<td>0.4</td>
<td>0.02</td>
<td>&lt;0.003</td>
</tr>
<tr>
<td>$\rho \bar{E}$ (keV cm$^{-3}$)</td>
<td>6</td>
<td>9.3</td>
<td>16</td>
<td>16</td>
<td>8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

In this table $\bar{E}$ is the mean energy per particle. Therefore there is a substantial dielectric current to deal with. There is a decrease of $\frac{1}{2} \rho V^2_{sw}$ (where $V_{sw}$ is the plasma velocity and $\rho$ the density) from the solar wind into the boundary layer.
It is evident from Table 1 that the component of current in the layer due to the gradient of \( E_1^2 \) is of comparable magnitude to those due to gradients of pressure (b) the component due to gradient pressure adds to that due to \( V(E_1^2) \) in the outer section of the plasma boundary layer (c) in the inner section of the layer the current due to gradient of pressure is in the direction of the classical Chapman-Ferraro current.

The strength of the dielectric current in the boundary layer can be estimated from equation 15 assuming \( E = c^{-1}V_{SW}B \).

Then, per cm along B, integrated through the layer it is estimated by

\[
J^E (\text{cm}^{-1}) = \frac{c}{2} \rho V_{SW}^2 B^{-1}. \tag{41}
\]

The total dielectric current \( J_T \) is given by integrating above and below the equatorial plane wherever the boundary layer exists. This is not known, but within a factor of 2 it is estimated to be \( 10 R_E \) above and below the equatorial plane.

So

\[
J_T^E = 10 R_E \rho V_{SW}^2 B^{-1} \quad \text{c.g.s.} \tag{42}
\]

Table II shows values of \( J_T^E \) for a variety of values of \( V_{SW}, B \) and \( n_s \)

where \( \rho = n_s m_s \) and \( m_s \) is assumed = mass of H atom.
TABLE II

<table>
<thead>
<tr>
<th>( n_B ) (cm(^{-3}))</th>
<th>( V_{SW} ) (km/s)</th>
<th>( B ) (G)</th>
<th>( J_T^E ) (amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>400</td>
<td>40</td>
<td>4.5 \times 10^6</td>
</tr>
<tr>
<td>10</td>
<td>400</td>
<td>10</td>
<td>1.8 \times 10^7</td>
</tr>
<tr>
<td>5</td>
<td>800</td>
<td>40</td>
<td>9 \times 10^6</td>
</tr>
</tbody>
</table>

These currents are geophysically significant. They exert an inwards force on the magnetosphere and would contribute to a decrease of the geomagnetic field earthward of them, and an increase of magnetic induction outside the layer in the solar wind. Together with the tail current (see later in the paper) these currents cause a perpetual "ring" current which should manifest itself at the earth's surface as a depression of the geomagnetic field. All currents in the earth environment need to be taken into account in the search for such an effect, including field-aligned currents, otherwise known as Birkeland currents. Equation 8 is derived on the assumption that the fields and medium parameters do not vary in the \( x \) direction. In the low latitude boundary layer we find that they do vary. This would mean that current generated locally according to equation 42 would have divergence. It is now suggested that the current in the low latitude boundary layer finds continuity partly with Birkeland currents into the polar ionosphere in the am sector and out in the pm sector. Later the continuity with electric current across the geomagnetic tail will be discussed. Consider the quantity

\[
\text{div} \ J_T^E = 10 R_E c V_{SW}^2 \text{div}(B),
\]  

(43)
we note that the boundary layer inside the geomagnetic field allows solar wind plasma to expand along the geomagnetic field towards the ionosphere. It follows that \((\rho B^{-1})\) decreases with distance from the "nose" of the magnetosphere. Approximately we can say that

\[
\frac{\rho}{B} = \frac{M_T}{V_T B} = \frac{M_T}{L_T A_T B'}
\]

where \(M_T, V_T, A_T, B\) = mass content, volume, length, cross sectional area and magnetic field (at equatorial plane) in a tube of magnetic field. Given \(M_T\) and \(A_T B\) (tube flux) constant during drift along the boundary layer, \(\nabla \cdot J_T\) is negative. It follows that Birkeland current must flow along the geomagnetic field to the pre-noon sector of the polar ionosphere to which field lines from the magnetopause are connected (see Fig. 6), there to be continued by ionospheric current. By symmetry Birkeland current would flow away from the polar ionosphere in the post-noon sector. It is suggested here that the part of this dielectric current which remains after flow along the flanks of the magnetosphere becomes continuous with current across the tail (see later).

Birkeland current must flow along the geomagnetic field to the pre-noon sector of the polar ionosphere to which field lines from the magnetopause are connected (see Fig. 6), there to be continued by ionospheric current. By symmetry Birkeland current would flow away from the polar ionosphere in the post-noon sector. It is suggested here that the part of this dielectric current which remains after flow along the flanks of the magnetosphere becomes continuous with current across the tail (see later in the paper).

The strength of the current per cm of longitude into the auroral ionosphere can be estimated by

\[
j_{\parallel} \text{(cm}^{-1}) = \kappa \nabla \cdot J_T^{E}
\]
where \( f \) is the separation at the boundary layer of the lines of force which are 1 cm apart in longitude near the cusp at the ionosphere. Approximately \( f = 15 \). Estimating the length scale of variation of \( \rho/B \) around the boundary using equation 45, a value of 10 \( R_E \) is adopted. Then, at the ionosphere,

\[
\frac{j_{\parallel} (cm^{-1})}{15cV_{sw}^2 \rho} = \frac{B}{\rho},
\]

(46)

where the parameter \( \rho \) and \( B \) are of course boundary layer values.

With \( B = 40 \gamma \), \( V_{sw} = 4 \times 10^7 \text{ cm sec}^{-1} \), \( \rho = 10 \times 1.6 \times 10^{-24} \text{ gm cm}^{-3} \),

\( j_{\parallel} = 1.0 \times 10^{-2} \text{ amp cm}^{-1} \). Such currents are in the range actually observed over the polar ionosphere (Sugiura and Potemra, 1976).

Let us return for a moment to the Chapman-Ferrero currents. In the region around the noon-midnight plane of the magnetopause the gradient of pressure of the solar wind may be considered as the dominant non-magnetic force and reflection of solar wind particles would cause a surface current which is towards dusk, equatorwards of the geomagnetic cusps and towards dawn, polewards of the cusps (see Fig. 7). Outside of this region the term \( \varepsilon E_y^2/8\pi \) appears to play a progressively more significant role increasing up to a substantial fraction of \( \rho \). This is suggested by the work of Eastman (1979), who shows that in the numerous low latitude boundary layer crossings reported generally \( \frac{1}{2} \rho V_{sw}^2 \) is between one half and one third of \( \rho \). In the present context we estimate

\[
\frac{1}{2} \rho V_{sw}^2 = \varepsilon E_y^2/8\pi.
\]

(47)

It is clear that the magnitude of Chapman-Ferraro currents near the noon plane will dominate the dielectric current elsewhere. This is so because the
reflection of solar wind particles in that region is equivalent to a gradient of pressure which, integrated across the region gives an equivalent pressure \( p \) of approximately \( 2 \rho v_{sw}^2 \). The region in which reflection is the dominant interaction of the solar wind with the magnetosphere is here called the "median" strip (illustrated in Fig. 7).

Grad \( p \) current exists completely around the magnetosphere in the layer with density given by

\[
\mathbf{j}(\mathbf{V}) = \frac{\mathbf{B} \times \mathbf{V}}{B^2} \]

Integrated throughout the boundary layer it is estimated by

\[
\mathbf{j}_{T}(\mathbf{V}) = \frac{p_{sw}}{B_{sw}}
\]

where \( p_{sw} \) and \( B_{sw} \) are values in the magnetosheath rather than the boundary layer. In the magnetosheath, along the flanks of the magnetosphere, the flow is expected to be approximately laminar with no variation of \( p_{sw}/B_{sw} \) and therefore no divergence of \( \mathbf{j}_{T}(\mathbf{V}) \). It is suggested that near the "nose", spatial variation of \( p_{sw}/B_{sw} \) may be such that the ratio increases away from the nose. This would form field aligned (Birkeland) currents towards the ionosphere in the a.m. sector and opposite in the p.m. sector. Also the Birkeland currents would tend to cause a depression of the magnetopause surface in the "median" strip between the cusps (see Fig. 9). However, polewards of the cusps, the Birkeland current from the Chapman-Ferraro current would reinforce that from the dielectric current, producing a dawn-to-dusk electric field at ionospheric levels and at intermediate heights in the magnetosphere.
Historically, the increase of geomagnetic field at the earth's surface at the commencement of some magnetic storms has been interpreted in terms of an increase of the Chapman-Ferraro currents at the magnetopause particularly on the day side. This explanation would appear to be qualitatively valid in the present theory however the effect is complicated by the diminishing effect of dielectric currents and \( V_p \) currents in the opposite direction to the conventional \( V_p \) current of Chapman and Ferraro.

In the region of the cusp of the magnetosphere and the median strip polewards of the cusp it is conceivable that a mixture of Chapman-Ferraro current and dielectric current could flow if there is penetration of solar wind plasma there with a component of velocity orthogonal to \( B \). In this "zone of confusion" one could expect considerable variability in these currents and associated Birkeland currents. This is suggested as the origin of the irregular Birkeland currents observed at ionospheric altitudes at the site of the dayside cusp. (Ledley and Farthing, 1974).

The "Distant" Cross-Tail Current

Far downstream from the earth at distances greater than about 15 \( R_E \) the geotail is known to contain an almost neutral sheet and attempts to explain it have been in two classes. The first, exemplified by the work of Harris (1963) employs a set of "hot" particles only and essentially produces the result, on a line orthogonal to the sheet,

\[
8 \tau p + B_z^2 = \text{constant.} \tag{48}
\]

The second approach exemplified by Alfven (1968), works with cold plasma, and essentially produces the result
\[ B_z^2 + \varepsilon E_y^2 = \text{constant} \quad (49) \]

In a sense both approaches are valid but the most general approach combines the two, and is embodied in equation 14, viz.,

\[ 8\pi p + B_z^2 + \varepsilon E_y^2 = \text{constant}, \quad (50) \]

where \( E_y \) is now the cross-tail electric field, \( B_z \) the geotail magnetic field, and \( p \) the pressure of the entrapped plasma perpendicular to \( B_z \).

It is proposed here that the cross-tail electric field is created under "ordinary" conditions by the dielectric current in the flanks of the low latitude boundary layer of the magnetosphere. Any mismatch of the current down the flanks and current across the tail would be taken up by Birkeland current at their junctions discharging through the polar ionosphere.

The current density across the tail from the dawn side to the dusk side is given by

\[ j_y = -\frac{c}{B} \frac{\partial p}{\partial x} - \frac{1}{8\pi B} \frac{\partial}{\partial x} \varepsilon E_y^2 \quad (51) \]

The component of \( j_y \), per unit of tail length, may be estimated as,

\[ j_y \, (\text{cm}^{-1}) = \frac{5B}{2\pi} \, \text{amps cm}^{-1}. \quad (52) \]

So that, with \( B = 10 \, \gamma \), \( j_y \, (\text{cm}^{-1}) = 8 \times 10^{-5} \, \text{amp cm}^{-1} \). For a tail of scale length 40 \( R_E \), the total cross tail current from this cause would be \( 2 \times 10^6 \) amps.
By a similar argument made in the case of the divergence of boundary layer currents to obtain Birkeland current, a case is now made for such currents derived from the divergence of cross-tail current using equation (52). This equation gives the locally produced current in the tail. It is assumed that changes of local parameters across the tail in the dawn-dusk (y) direction lead to divergence of $j_y$ which leads to Birkeland current. Firstly, let us note the symmetries of the terms in $j_y$. Both $\partial p/\partial x$ and $\partial \epsilon/\partial x$ are expected to point away from the neutral sheet, and since $B$ reverses sign there, divergence of $j_y$ will not change sign with change of position above or below the sheet. However the divergence of $j_y$ would be expected to change sign depending on whether the position is on the dawn side or the dusk side of the center line of the sheet, if there is symmetry of the local parameters about this line. It is expected on average that at any given distance $X$ from the neutral sheet, $\partial p/\partial x$, $\partial \epsilon/\partial x$ and $B$ would each increase in magnitude from the central meridian plane of the tail to the magnetopause. The divergence of $j_y$ would then be such as to produce Birkeland current towards the polar ionospheres from the "dawn" half of the tail, and from the ionospheres in the "dusk" half. However great departures from this simple pictures may be expected because of the dependence of $\text{div } j_y$ on $\partial p/\partial x$, $\partial \epsilon/\partial x$ and $B$. This perhaps accounts for the complexity of Birkeland currents near the "Harang" discontinuity.

The divergence of tail current may be estimated using equation 52, by

$$\text{div } j_y = f_T \frac{5BL}{2\pi W} = j_\parallel \ (\text{amps cm}^{-1}),$$  

where $W$, $L =$ half width, length of tail $\approx 15R_E$, and $f_T =$ separation in the tail of two lines of force 1 cm apart in longitude near the midnight auroral
Therefore, for $B = 10\gamma$ and $f_T = 20$, the Birkeland current per cm to each polar ionosphere would be $\approx 1.2 \times 10^{-3}$ amps cm$^{-1}$. Birkeland currents of this magnitude are observed in the night time auroral belt (Potemra et al. 1979).

This distribution of Birkeland electric current would also be compatible with Sugiura's (1975) inference from observations of magnetic fields regarding the connection of Birkeland currents near the polar ionosphere with those from the tail. These currents would cause the magnetic field lines to be splayed outwards away from the earth between them and the neutral sheet and to be oppositely splayed between the currents and the tail mantles of the magnetosphere (Fig. 8). There would be a tendency for the field aligned currents in the tail to produce a component of $B$, orthogonal to the neutral sheet, between the neutral sheet and the mantles, pushing the magnetic field out at the middle of the tail mantles. This should cause a "fin" to exist in the magnetic field down the middle meridian of the mantles. (see Fig. 9).

The electric field across the tail from dawn-side to dusk-side, will cause the drift of plasma towards the earth and the consequent energization of trapped plasma; production of partial ring current and Birkeland currents at places of divergence of the partial ring current (Harel et al. 1981). It is not the purpose of this paper to examine these internal processes of the magnetosphere but to discuss principally the external origin of the Birkeland currents, and the dawn-dusk electric field.

Additional Currents from the Interplanetary Medium

It is clear that equation 1 predicts significant electric currents in the interplanetary medium (Cole, 1983), and that such currents may find conduction paths through the geomagnetic plasmas if the magnetosphere encounters them.
The strength of these currents will now be estimated and their interaction with the geomagnetic field discussed. The currents of interest are those associated with shears in the flow of interplanetary plasma and in neutral sheets. These are examples of types I and II treated in Section 1. Both of these cases may apply to neutral sheets. In type I the electric field is perpendicular to both B and the neutral sheet. In type II the electric field is perpendicular to B but parallel to the neutral sheet.

In interplanetary space it is generally expected that \( p << \frac{eE_{y}^{2}}{y} \) so the current can be estimated in magnitude in both cases by

\[
    j = \frac{1}{8\pi} \frac{c}{B} \frac{eE_{y}^{2}}{L},
\]

where \( L \) is the scale of variation of a critical parameter \( \epsilon \) or \( E_{y}^{2} \) of the system. When \( L > c^{2} E_{m}/Be_{y} \), the plasma drift velocity is \( cE_{y}/B \) (Cole 1983), so we can say, in that case,

\[
    j = \frac{1/2n_m V_{sw}^{2}c}{B L},
\]

where \( V_{sw} \) is the component of the solar wind orthogonal to \( B \). Fig. 10 shows a range of values of \( j \) for a variety of values of \( V_{sw}, B \) and \( L \). Assuming \( n = 10 \text{ cm}^{-3} \). There may be occasions, too, when currents due to pressure are significant.

It is assumed that an interplanetary current, if intersected by the magnetosphere, will find continuity across the magnetopause with field aligned current to the polar ionosphere and out again, or may cross the tail of the earth via the neutral sheet. This would be done by a build up of space charge at the boundaries to adjust the electric fields to give current continuity (see Fig. 11). This seems a reasonable proposition.
Suppose the effective area of the magnetosphere accessible to the interplanetary current is \((15 \, R_E)^2\), if \(L > 15 \, R_E\) and \(15 \, L \, R_E\) if \(L < 15 \, R_E\).

Then we see that for the structures characterized in Fig. 10, may produce currents through the magnetosphere of order \(10^6 - 10^7\) amp. Now observations of magnetic bays (Cole 1962a) show that their durations range from about 20 mins. to about 200 mins., though some are outside these values. By assuming these bays are caused by changes to the magnetosphere/ionosphere system brought about by interplanetary currents flowing through the system, it is inferred that structures of scale length greater than about \(5 \times 10^{10}\) cm have sufficient current to account for geomagnetic bays.

Let us now consider the morphology of large scale interplanetary structures corresponding to types I and II. Take a system of cartesian coordinates in which \(x\) is radially outwards from the sun, \(y\) is azimuthal and \(z\) perpendicular to the plane of the ecliptic. Let the solar wind velocity be given by

\[
\mathbf{V}_{sw} = (V_x, V_y, V_z).
\]  

(56)

Let \(\mathbf{B} = (B_x, B_y, B_z)\).

(57)

For \(L \gg m \frac{E_c}{\mathcal{E}},\)

\[
\mathbf{E} = (V_y B_z - V_z B_y, V_z B_x - V_x B_z, V_x B_y - V_y B_x),
\]

(58)

\[
= \mathbf{0}, -V_y B_z', V_x B_y',
\]

and,

\[
\mathbf{E} \times \mathbf{B} = V_x (B_x^2 + B_y^2), V_x B_y x', V_z B_z x'.
\]

(59)
The magnitude of the current \( j \) is then approximated by

\[
\frac{e V_{sw}^2 (B_y^2 + B_z^2)}{8\pi L B_c} = \frac{c M^2 (B_y^2 + B_z^2)}{8\pi L B},
\]

where \( M \) is the ratio of the solar wind speed to the Alfvén speed.

In type I the direction of the current is given by \( \pm E \times B \) depending on whether \( \nabla(\epsilon E_\perp^2) \) is in the same or opposite direction to \( E_\perp \) (Cole 1976). In this case, \( E \) is in the \( +z \) direction if \( B \) is away from the sun and \( j \) will be in the plane of the ecliptic in the direction \( \pm E \times B \) if \( \nabla(\epsilon E_\perp^2) \) is in the \( \pm z \) direction (respectively).

In type II the direction of the current is in the direction of \( E \) for a gradient of \( \epsilon E_\perp^2 \) in the plane of the ecliptic and \( j \) will be in the \( \pm z \) direction depending on whether \( B \) is away from or towards the sun. In this case we should expect current to flow through the magnetosphere from N to S or vice versa depending on whether \( B \) is towards or away from the sun, producing significant hemispherical asymmetries in magnetic disturbances observed at the earth's surface.

Generally for type I currents (i.e., corresponding to Fig. I geometry, if \( \nabla(\epsilon E_\perp^2) \) is in the direction of \( \pm E \), \( j \) is in the direction of \( \pm E \times B \) respectively, and if \( L \gg mE c^2/Be \), we can say that the direction of \( j \) is \( \pm \) that given by equation 59. Whereas, for type II currents, if \( \nabla(\epsilon E_\perp^2) \) is in the direction of \( \pm E \times B \), \( j \) is in the direction of \( \pm E \) respectively, i.e., as specified by \( \pm \) that given by equation (58).

The amount of current converted to Birkeland current from the tail would depend upon the angle (\( \alpha \)) between the current direction in interplanetary
space and the direction defined by the vector cross product of the geomagnetic axis direction and the normal to the geomagnetic neutral sheet. However, Birkeland current from the day sectors of the magnetopause would be controlled more by the angle ($\delta$) between the current direction and the geomagnetic axis. The precise description of the amount of Birkeland current generated by divergence of interplanetary currents depends on the configuration of the draping of interplanetary magnetic induction lines over the magnetopause. The detailed configurations of the plasma in this coupling process will be the subject of future work.

Recently Iijima and Potemra (1982) correlated a variety (20 in number) of interplanetary quantities with morning and afternoon Birkeland current densities and found that some are better correlated than others. One of the best correlations is with the quantity $\left[n V_{sw} (B_y^2 + B_z^2) \sin \theta / 2 \right]^{1/2}$, where $\theta$ is the angle between the geomagnetic axis and the z-direction. This interplanetary quantity is clearly closely related "mathematically" to the current $j$ defined by equation 60, although its origin in physical discussion is different.

The theory of the present paper would explain a permanent Birkeland current system associated with divergence of dielectric current in the boundary layer and the geotail plus a variable component associated with interplanetary parameters via equation 60 and the angles $\alpha$ and $\delta$. To test this in the manner of Iijima and Potemra (1982) it would be preferable to use total Birkeland current per cm of auroral oval rather than current density.

On the Question of Magnetic Merging

The physics of case I and its variant (equation 22) are similar.
In case I there is no merging of magnetic field because the velocity of plasma is parallel to the discontinuity.

Type II could describe a "tangential" discontinuity but we see in this simple geometry that there is no merging. Additional factors, perhaps time variation or curvature of $B$ or dissipation may be needed to be incorporated in the model to manifest merging. It is clear that even without merging geophysically significant electric current flow in the boundary layer and geomagnetic tail.

The Bow Shock

That part of the bow shock which is called a quasi-perpendicular shock resembles the conditions for type II in the foregoing, while in the regions in which it is quasi-parallel, the conditions are those like type I. Since the thermal pressure of solar wind plasma is $\frac{1}{2} \rho V_{sw}^2$ ($= eB^2$) it is clear that the electric currents in the bow shock are dielectric currents of the kind discussed in this paper, viz., type II currents in quasi-perpendicular shocks and type I currents in quasi-parallel shocks. The size of the currents in the bow shock are estimated to be of the order of $10^6 - 10^7$ amps and represent another potential source of Birkeland currents from them to and through the magnetosphere to the ionosphere. This could happen if there were field lines connected to the shock, at places of divergence of the dielectric current draping over the magnetosphere.

Evaluation of Ring Current and Ionospheric Dynamo Currents

Traditionally the ring current has been ascribed mostly to trapped energized magnetospheric plasma on close geomagnetic field lines and manifested itself by a decrease of the geomagnetic field at the earth's
surface. The Chapman-Ferraro currents had the opposite effect. Given the validity of the new currents in space around the earth a re-evaluation of the component of depression of the geomagnetic field at the earth's surface due to conventional magnetospheric ring current will have to be done. Likewise a re-evaluation of the compression due to Chapman-Ferraro currents will need to be done. This would also affect the amount of geomagnetic variation attributable to dynamo action in the ionosphere.

**Ionospheric Dynamo Action**

The fact that neutral winds at ionospheric levels can produce significant electric fields and currents (Nagata et al. (1950); Fukushima, (1953); Cole, (1960, 1961) should be taken into account at all times.

These fields and currents are particularly significant (1) when currents and fields applied from the boundary layer and tail are weak i.e., under conditions of relative geomagnetic "quiet", (2) as a non-negligible component of disturbances, because externally applied electric fields drive ionospheric electric currents, which in turn accelerate the neutral gas (Cole, 1962b) by the Lorentz force (\( \mathbf{J} \times \mathbf{B} \)), otherwise known in the ionosphere as "ion drag". This, in turn, produces an ionospheric dynamo electric field. (3) When solar wind fields are "turned off" after a period of activity the newly established winds in the ionosphere drive currents orthogonal to \( \mathbf{B} \) in the ionosphere which are in the opposite direction to those driven by the solar wind fields (Cole, 1966). They will tend to set up electrostatic fields which cause convection of the magnetosphere also in the reverse sense to that caused by the solar wind fields. Moreover, the mismatch of electrostatic potentials, tended to be established this way, between the northern and southern hemisphere will cause field-aligned magnetospheric electric currents (cf. Cole, 1971).
Discussion

Having applied the unabridged Maxwell equations to the discussion of the electrodynamics of plasmas (Cole 1983) new "dielectric" currents have been predicted in the magnetosphere and interplanetary space to be of geophysical significance. The low latitude boundary layer is the site of one such current in the direction \( \mathbf{E} \times \mathbf{B} \) (type I) caused by an electric field \( \mathbf{E} \) orthogonal to \( \mathbf{B} \) accompanied by a gradient of \( \varepsilon \mathbf{E} \perp \mathbf{E} \). Divergence of this current produces Birkeland current to the polar ionosphere and the remainder is continuous with current across the geomagnetic tail. In the tail, the cross-tail electric field is accompanied by a gradient of \( \varepsilon \mathbf{E} \perp \mathbf{E} \) perpendicular to \( \mathbf{E} \) and \( \mathbf{B} \) producing a current in the direction of \( \mathbf{E} \). In addition there is cross tail current caused by \( \nabla \times \mathbf{p} \). Divergence of the cross-tail field appears to be responsible for the Birkeland currents to the auroral region of the ionosphere which were inferred to exist by Sugiura (1975).

In addition to these currents, dielectric currents of origin in interplanetary plasma structures may be conducted across the magnetopause through the magnetosphere by Birkeland currents, and also across the tail by alteration of the cross tail electric field. The paper does not address the problem of how interplanetary currents negotiate the bow shock to enter the magnetosphere.

This theory gives a new approach to the investigation of geomagnetic disturbance. It is of interest to note that the type II current was in fact invoked from a different standpoint by Alfvén (1968). It is now seen to be one of a wider class of electric current hitherto overlooked in plasma physics. The currents come about when a plasma is subject to electric fields (charge distributions) and currents of external origin. The bulk of plasma physics appears to be concerned with completely internal processes and it is
usual to assume electrical neutrality exists except in special circumstances, thus ignoring the kinds of new effects discussed here.

The theory so far developed using this new approach to collisionless plasmas (Cole, 1983) has been applied only to situations in which $B$ is unidirectional. To develop the theory further one needs to express the "$\mu"$ of the plasma in terms appropriate to general magnetic induction $B$, i.e. to allow for effects of "parallel" pressure of plasma in addition to "perpendicular" pressure to which this paper is presently restricted.
References


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Figure Captions

Fig. 1 Geometry for type I current.

Fig. 2 Geometry for type II current.

Fig. 3 Example of parameters measured in low latitude boundary layer, taken from Eastman (1979). The boundary layer is identified between the vertical full and dashed lines.

Fig. 4 Schematic representation of velocity, electric field energy density and dielectric current (type I) in low latitude boundary layer.
Fig. 5 Scheme of dielectric currents in low latitude boundary layer, and tail. Currents due to $V_p$ not illustrated.

Fig. 6 Schematic representation of Birkeland currents produced by divergence of currents in low latitude boundary layer and geotail.

Fig. 7 Illustration of the "median" strip enclosed by dashed curve on day side magnetopause.

Fig. 8 Illustration of "spalying" of geomagnetic field in tail due to Birkeland currents otherwise known as field-aligned currents (F.A.C.).

Fig. 9 Illustration of dayside "valley" and night side "fin" on magnetopause surface.

Fig. 10 Curves of measure of dielectric current in interplanetary structures assuming $n m_i = 1.67 \times 10^{-23} \text{ g cm}^{-3}$. $L =$ scale length across $B$ of structure.

Fig. 11 Illustration of interplanetary currents impacting on the magnetosphere. They are an additional source of Birkeland currents inside the magnetosphere.
FIG. 3A

11 APR 73
0400
0430
0500
0530

MAGNETIC FIELD
PLASMA

300 km ALTSIWARD
PROTONS

log ENERGY DENSITY (eV/cm³)

BULK FLOW VELOCITY

log DENSITY (cm⁻³)

ORIGINAL PAGE IS OF POOR QUALITY
Figure 3B

11 APR 73  \( \phi_{GSM} = 302^\circ, \lambda_{GSM} = 33^\circ, r = 8.8 R_E \)
(a) DAWN

(b) DUSK

FIG. 4
DIRECTION OF $B$

\[ \rightarrow \text{BETWEEN F.A.C. AND MAGNETOPAUSE} \]

\[ \rightarrow \text{BETWEEN F.A.C. AND NEUTRAL SHEET} \]

NORTHERN HALF OF TAIL

SOUTHERN HALF OF TAIL

**FIG. 8**
FULL LINE $B = 10 \gamma$
DOTTED LINE $B = 5 \gamma$

$$L = 10^9 \text{ cm}$$

$$L = 10^{10} \text{ cm}$$

$$L = 10^{11} \text{ cm}$$

$$L = 10^{12} \text{ cm}$$

$$j = \frac{n m_i V_{SW\perp}^2}{2 BL} \text{ emu}$$

$n m_i = 1.67 \times 10^{-23} \text{ g cm}^{-3}$

(⊥) SOLAR WIND SPEED km s$^{-1}$