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LEHIGH UNIVERSITY

OFFICE OF RESEARCH

TECHNICAL REPORT

CRITICAL REVIEW OF THE TRAILING EDGE CONDITION IN STEADY AND UNSTEADY FLOW

NASA RESEARCH GRANT NSG3162

BLADE FLUTTER IN COMPRESSORS AND FANS: NUMERICAL SIMULATION OF THE AERODYNAMIC LOADING

BY

SAMIR F. RADWAN

DONALD O. ROCKWELL

STANLEY H. JOHNSON

DECEMBER 1982

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS

LEHIGH UNIVERSITY

BETHLEHEM, PENNSYLVANIA

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TABLE OF CONTENTS

		Page
	Abstract	1
I.	Introduction	2
II.	Kutta-Joukowski Condition	5
III.	Trailing-Edge Condition in Steady Flow	11
	A. Inviscid Analysis	11
	B. Viscous Analysis	16
	C. Multistructure Boundary-Layer Theory and Trailing-Edge Flow Structure	31
IV.	Trailing-Edge Condition in Unsteady Flow	42
	A. Unsteady Flow Generation	42
	B. Inviscid Analysis	47
	C. Viscous Analysis With Attached Flow	71
	D. Viscous Analysis With Boundary-Layer Separation	78
V.	Conclusions	84
	References	86
	List of Figures	95
	Figures	99
	Vita	145

ABSTRACT

This report is a critical review of the existing interpretations of the trailing-edge condition, addressing both theoretical and experimental works in steady, as well as unsteady flows. The work of Kutta and Joukowski on the trailing-edge condition in steady flow is reviewed. It is shown that for most practical airfoils and blades (as in the case of most turbo-machine blades), this condition is violated due to rounded trailing-edges and high frequency effects, the flow dynamics in the trailing-edge region being dominated by viscous forces; therefore, any meaningful modelling must include viscous effects. The question of to what extent the trailing-edge condition affects acoustic radiation from the edge is raised; it is found that violation of the trailing-edge condition leads to significant sound diffraction at the trailing-edge, which is related to the problem of noise generation. Finally, various trailing-edge conditions in unsteady flow are discussed, with emphasis on high reduced frequencies.

I. INTRODUCTION

Early progress in predicting aerodynamic forces on bodies in incompressible flow involved potential flow analysis. In such analyses, the Kutta-Joukowski condition [1902, 1906] is used to give the unique solution for both isolated airfoil and airfoils in cascade. This mathematical condition requires that the flow velocity at a sharp trailing-edge be finite. The resultant flow pattern and the predicted lift agree well with that observed at low angles of attack. However, many interpretations have been used instead of this condition, which can lead to widespread discrepancies in predicting the aerodynamics forces and moments. A critical review of these interpretations is presented.

However, this condition is violated when the trailing edge is not sharp, even though the flow is steady. In this case, the trailing-edge is dominated by viscous effects. For this class of trailing edges, the Taylor-Howarth criterion of "zero total flux of vorticity into the wake" is found to be the appropriate edge condition for steady flow, that establishes the circulation and the aerodynamic forces. Details are given on the nature of the trailing-edge flow structure, emphasizing the role of viscosity in smoothing the flow field in laminar non-separated flow, using multistructure boundary

layer theory of Stewarston.

The complexity of the problem increases when airfoils with rounded trailing-edges operate under unsteady conditions, as in turbomachinery applications. In such cases, there are all the previous theoretical difficulties encountered in steady flow and, in addition, the unsteady effects on the boundary layer and the vorticity eventually shed from the trailing-edge. These effects give rise to significant trailing-edge loading, as well as strong acoustic radiation from the trailing edge, especially when flow separation occurs, which is related to the problem of noise generation in turbomachines. In practice, the discrepancies in modelling this condition may lead to shortfalls of many tens of megawatts of generating capacity when they occur in the design of turbine nozzle blading for power plant stations. So, the correct theoretical modelling of the generalized trailing-edge condition is important in determining the acoustic radiation from trailing-edges of wings and blades in turbomachines, in understanding the mechanisms involved in certain classes of bird and insect flight, and as a prelude to analyzing trailing-edge stall on oscillating airfoils. A critical review and a clear picture about this complicated problem, especially unsteady aspects, seems essential; this

is the main purpose of this work. First, we start with various interpretations Kutta-Joukowski condition.

II. KUTTA-JOUKOWSKI CONDITION

The Kutta-Joukowski theorem [1902, 1906] does not explicitly provide a means of calculating the circulation (Γ) around the airfoil in two-dimensional steady incompressible potential flow, however, it provides the foundation for predicting the lift (L):

$$L = \rho U_\infty \Gamma \hat{j}$$

where

ρ = the density of the fluid

U_∞ = the undisturbed velocity at infinity

Γ = the value of circulation around the airfoil

$\hat{j}$ = unit vector perpendicular to the free stream direction

From early experimental work (Prandtl [1934]), it had been known that only airfoils with sharp trailing edges appear to have well-defined values of lift. The theoretical streamlines for flow without and with circulation past an airfoil are shown in Figure 1, and in general, for arbitrary values of the circulation (zero, or too large values) there will be flow around the trailing edge from one side to the other with an infinite velocity in the vicinity of the trailing edge, which appears as a sharp corner with angle greater than π as shown in Figure 2. But for a particular value of the circulation, given

the incidence α, the rear stagnation point is located at the trailing edge and the flow leaves the trailing edge smoothly, as in Figure 1(c), e.g., see Goldstein [1965].

So, for a given incidence α of the free stream past an airfoil with a sharp trailing edge, the actual flow has a defined circulation. That is, there is a definite relation between the condition at the trailing edge and the development of the circulation. In fact, the assumption of inviscid flow really represents the limiting case of a fluid whose viscosity is vanishing small at high Reynolds number. When the flow is attached, the effects of viscosity are confined to thin boundary layers on the surfaces of the airfoil and to the downstream wake formed by the merging of the upper and lower surface boundary layers at the airfoil trailing edge. As a first approximation, it is reasonable to assume that the flow can be regarded as inviscid as long as the flow in the region of the trailing edge remains attached. Given free stream velocity, the flow depends on a local Reynolds number, based on a length associated with the geometry of the trailing edge. Such a length is the radius of curvature of the trailing edge, r. Thus, an appropriate Reynolds number, Re is:

$$Re = \frac{r\,U_\infty}{\nu}$$

Now, assuming that ν can be varied at will, let it tend to zero.

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In order to maintain similarity, Re must be kept constant at the same time and this implies that the radius of curvature of the trailing edge also tends to zero. In other words, the trailing edge must be assumed sharp. At the same time, the velocity near the trailing edge must be bounded in the actual physical case.

Kelvin's theorem, dealing with the rate of change of the circulation about a closed path surrounding the same fluid elements (Lamb [1945]) states that:

$$\frac{D\Gamma}{Dt} = -\oint \frac{dP}{\rho}$$

where the circulation Γ is defined by:

$$\Gamma = \oint_C \underline{q} \cdot \underline{ds}$$

For an incromprssible or barotropic flow,

$$\frac{D\Gamma}{Dt} = 0$$

which is known as the law of conservation of circulation; in essence, this means that when vorticity is shed, the circulation of the vorticity round the airfoil is always equal and opposite to the shed vorticity at the trailing edge.

In summary, an airfoil with a sharp trailing edge (i.e. nonzero trailing edge angle), which is moving through a steady

inviscid incompressible fluid at small angle of incidence, will create about itself a circulation of strength just sufficient to hold the rear stagnation point at the trailing edge and the dividing streamline from the trailing edge bisects the tangents from the upper and lower surfaces at the trailing edge. For zero-trailing edge angle (cusped edge), the velocity remains at the cusp (Batchelor [1970]).

This condition was put forth by Kutta [1902] and independently by Joukowski [1906], and is known as the Kutta-Joukowski condition. This theoretical condition has been found to agree well with experimental predictions as long as the flow remains attached. If we examine the sequence of events observed experimentally when an airfoil with a rounded leading edge and a sharp trailing edge is set into uniform motion from rest through a real fluid such as air or water of low viscosity, we can see, immediately after the start of the motion, that the flow is irrotational everywhere, since the transport of vorticity away from the airfoil surface by viscous diffusion and convection takes place at a finite rate. For this initial irrotational flow, the circulation is zero and the rear stagnation point is on the upper surface of the airfoil. The fluid particles tend to flow around the trailing edge with very large velocity, and then rapidly decelerate to

to the stagnation point, leading to development of back flow in the boundary layer there and to separation of the boundary layer at the sharp trailing edge. Equivalently, no matter how small the viscosity, there will be a viscous force at the edge because of the large velocity gradient there. The effect of the vorticity generated at the trailing edge is to create a circulatory flow of fluid around the airfoil; this circulation continously modifies the flow pattern so that the velocity peak is reduced. This vortex is known as the starting vortex. As the airfoil proceeds, the strength of the starting vortex and that of the circulation around the airfoil grow simultaneously until the flow field around the airfoil is such that the fluid flows off smoothly from the trailing edge as shown in Figures 3,4,5. It has then practically no influence on the flow around the airfoil. Whenever the condition of the smooth flow at the trailing edge is disturbed, say by a change in the speed of the airfoil or in its angle of attack, a new starting vortex is formed, and a new value of the circulation is established such as to restore smooth flow at the trailing edge as shown in Figure 5. This explains the initial role of viscosity in the boundary layer in generating the well-defined circulation; once it is established, the effects of viscosity may be ignored in the subsequent steady motion since

no separation of the boundary layer occurs. From this, we can relate the hypothesis of Kutta-Joukowski: ..."The flow past an airfoil with a sharp trailing edge, in steady potential fluids, leaves the trailing edge smoothly with finite velocity..." to these experimental observations.

III. TRAILING-EDGE CONDITION IN STEADY FLOW

(A) Inviscid Analysis

The pioneering work of Kutta and Joukouski provided a mathematical constraint for the trailing edge region, involving a unique value for the lift on the airfoil using potential analysis. It is very important, at this point, to define exactly the limitations of the Kutta-Joukowski condition to preclude confusion. Both Kutta and Joukowski considered two-dimensional airfoils with a cusped trailing edge in steady, incompressible potential flow. So, the term "Kutta-Joukowski condition" should not be used indiscriminately to denote some kind of trailing edge condition, and any application calling for greater generality might refer to a "trailing edge condition". Gostelow [1975] pointed out that:

> The Kutta-Joukowski condition only pertains to the steady, incompressible potential flow around a two-dimensional airfoil having a cusped trailing edge. In such a case, the circulation is determined, for small angles of attack, by placing the rear stagnation point at the trailing edge, thus removing the singularity and a finite velocity is preserved at the cusp. The rear stagnation streamline, under these circumstances, will be tangential to the airfoil surface at the trailing edge, and the resulting flow predicted well the lift well and its chordwise pressure at low angles of attack.

In fact, we can generalize the Kutta-condition in steady incompressible potential flow around bodies with sharp trailing

edges as the flow velocity at the trailing edge must be finite for cusped trailing edges and be zero for trailing edges with finite angle; otherwise a surface of discontinuity (i.e. a vortex sheet) will emanate from the trailing edge, which cannot be permitted in steady flow whether the trailing edge is finite or zero. Consequently, the pressure difference between the top and bottom surface tends to zero at the trailing edge, see Robinson and Laurmann [1956], Tsien [1943] and Krishnamurtyk [1966].

Analytic solution of this class of problems, by conformal transformation techniques, has been carried out by a number of investigators, including Glauert [1947], using the Kutta-condition to obtain a unique solution. In this discussion, attention will be given to the validity and interpretation of the Kutta-condition, including its limitations.

Basu and Bancock [1978] point out that, although the velocities and pressures in their analytic solution remain finite at the trailing edge, the flow itself in this region is singular in the sense that the rates of change of the surface velocities are infinite in the vicinity of the trailing edge.

Considering another interpretation of the trailing edge condition in steady flow, Giesing [1969] puts forth a simple statement of the Kutta-condition applied to bodies with finite

trailing edge angles and to bodies with cusped trailing edges: ..."the velocities on the upper and lower surfaces at the trailing edge must be equal in magnitude, but opposite in tangential direction." In an equivalent interpretation, Whitehead [1973] states that one can choose between either the velocity difference or the pressure difference tending to zero.

For the inviscid flow past a steady airfoil where separation does not occur at the trailing edge, the analytic solutions give rise to singularities in the velocity and the pressure at the trailing edge, and the loading is also infinite at the trailing edge. Hess and Smith [1967] give a numerical procedure for this case. In their method, the profile is divided into several straight line elements. Sources of unknown strengths, each constant over a given element, are distributed arbitrarily. An unknown constant circulation is superimposed on the profile. The boundary condition is satisfied by equating the normal velocity component to zero, as shown in Figure 7. The Kutta-condition is interpreted by equating the tangential velocities on the two elements on either side of the trailing edge region, i.e.

$$(q_t)_1 = (q_t)_N$$

In fact, the flow characteristics are essentially averaged over the length of an element; thus the singular behavior in the neignborhood of the rear stagnation point at the trailing edge is averaged over the trailing-edge elements. This condition used in this procedure, is equivalent to stating that 'no vorticity' can be shed, i.e.

$$\gamma_{T.E.} = 0$$

and

$$\frac{d\Gamma}{dt} = 0$$

where $\gamma_{T.E.}$ is the instantaneous strength of the vortex shed from the trailing edge. This condition is consistent with the circulation around the airfoil remaining constant. It can also be interpreted as zero loading in the vicinity of the trailing edge, which is physically realistic. In fact, the actual trailing edge in their model is not a stagnation point; it is found that the velocities at the midpoints of the trailing-edge elements differ significantly from stagnation values. Thwaites [1960] also quotes zero loading as the trailing edge condition for steady flow. In addition, he has an original statement of the condition, "...the rear dividing streamline leaves the airfoil at the trailing edge"..., and another interpretation states as "...the tangent to the rear

dividing streamline passes through the interior of the airfoil, and the dividing streamline turns through an angle approximately equal to the incidence." Basu and Hancock [1978] tried to argue that there is no definite statement of the Kutta condition for a steady airfoil, they say:

> ...Analytical and numerical results for most aerofoils are virtually identical except in the region very close to the trailing edge, inspite of the alternative forms of the Kutta condition, ...each mathematical model requiring its own consistent 'Kutta' condition to ensure a unique solution, the relevant and appropriate Kutta condition needs to be formulated separately for each mathematical model.

In fact, this argument is similar to what Gostelow [1975] has mentioned about the difference between the original Kutta-condition and trailing edge condition. However, we can conclude that, for steady potential flow past an airfoil with sharp trailing edges of small incidences, there is really no contention. The statements of "zero loading" or "zero vorticity flux" shed from the trailing edge are equivalent, and they are consistent with the classical Kutta-Joukowski condition. We see later, through viscous analysis, how the steady trailing condition stated above is the correct viscous uniqueness criteria to apply to inviscid analysis in the limit that the Reynolds number tends to infinity. However, this condition is

violated when the extent of the region of separated flow is appreciable, as in the cases of high incidence or loading.

(B) Viscous Analysis and Real Airfoils

Most practical airfoils and blades have rounded trailing edges. Manufacturing considerations indicate that a true cusp cannot be produced, so, in practice, any airfoil will have finite curvature at the trailing-edge. Consequently, the Kutta-condition should, strictly speaking, not be applied to the manufactured profile of such an airfoil. Thwaites [1960] observes: "...if the rear of a body has no sharp trailing-edge, the Kutta-condition cannot be applied nor has any other criterion yet been generally accepted which renders unique the distribution of concentrated vorticity in the otherwise inviscid flow..." Gostelow [1976] shows that even an airfoil having a truly cusped trailing-edge could not operate in a purely potential flow and viscosity effects would be present. For example, the Kutta-Joukowski condition gives a finite nonzero velocity at the trailing-edge of a cusped airfoil, whereas any attempt to consider the effect of viscosity will give zero velocity on the airfoil surface. In the case where the airfoils have rounded trailing edges, the position of the rear stagnation point is indeterminate as there is no velocity singularity to be avoided; therefore the circulation must be

determined by accounting for the effects of viscosity in the region of the trailing edge. As a matter of fact, some investigators have attempted to solve this case by use of potential flow theory, but failed to obtain a unique or satisfactory solution. Schlichting [1955] replaces the actual trailing edge geometry by a substitute cusped edge, where the Kutta-Joukowsky condition may be applied at the expense of neglecting the original blade geometry. Methods such as of Martensen [1971] and Gostelow [1964a,b] treat the true trailing edge geometry. But, for a rounded trailing edge, their methods provide non-unique solutions.

In Gostelow's analysis [1964], he showed that for the case of two-dimensional potential flow through a cascade of blades having rounded trailing edges, the cascade outlet angle is extremely sensitive to small changes in rear stagnation point locations. Also, the pressure distribution was found to be sensitive to this change, especially as the trailing edge is approached, as shown in Figures 8,9. It was shown that a small movement of the rear stagnation point, over a distance of about 0.3% of the chord, resulted in a 10 degrees increase in the flow deflection imported by the cascade. Also, the surface velocity in the region of the trailing edge reaches high values when the rear stagnation point is moved to the upper or the

lower surface a small distance, as in cases a, c in Figures 8, 9 This emphasizes the fact that the potential flow around an airfoil with a blunt trailing edge in cascade is not completely determined by specification of the cascade configuration and the inlet angle. Indeed, the consensus of experimental evidence from low speed cascade testing is: for given inlet conditions, the downstream flow angle and a attendant blade pressure distributions are unique and repeatable Gostelow [1975]. Before making any conclusion here, it is appropriate to examine the work of Baskaran and Holla [1981] on the effect of rear stagnation point position and trailing edge bluntness on airfoil characteristics. They calculated the pressure distribution on the basic RAE 101 profile with a blunt trailing edge using the method of Hess and Smith, and the flow is steady, incompressible and two dimensional. The basic RAE 101 profile was divided into 104 straight line elements, and the rear stagnation point was moved on either side of the trailing edge up to 0.02 C in steps of 0.01 C. The blunt trailing edge is taken from the basic profile by flattening it at 0.94 C and rounding the trailing edge at 0.095 C. This blunt trailing edge is divided into 118 elements. The rear stagnation point is moved on either side of the trailing up to 0.003 C in steps of 0.0005 C, and the value of the angle of attack used was

4.09 degrees. Results are obtained for different stagnation point locations near the trailing edge satisfying the Kutta-Joukowsky condition. They found that the rear stagnation point position and bluntness of the separation edge have a strong effect on the pressure coefficient, lift coefficient, quarter chord moment coefficient, and the front stagnation point, as shown in Figure 10; a,b, and c. As shown in Figure 10b, the change in the value of lift coefficient for a given stagnation point location is relatively greater for the blunt trailing-edge profile than for the sharp trailing-edge profile. The same trend is observed with the moment coefficient behavior. They conclude that this behavior is attributed to the bluntness in comparison with the sharp trailing-edge profile due to a drastic change in local slope. From this analysis, we can conclude that it is the role of viscosity which exerts a dominant influence in determining the unique flow pattern in the case of cascade blades or airfoils with blunt trailing-edges since the potential analysis of both Gostelow and Baskaran are <u>inviscid</u> and the experimental observed flows are <u>viscid</u>. So, we can see that the processes of generation of circulation by viscosity are accounted for the trailing-edge condition. That is, we only know, up to now, the consequence of these viscous processes: removing any singularities in both velocity

and pressure at the trailing-edge. But, what exactly are these processes? Sears [1956] says:

> ...In fact, the recognition of this essentially viscous origin of circulation, lift, and induced drag might be said the emergence of aerodynamics as a science and to distinguish it from the purely mathematical fluid mechanics.

Howarth [1935] was the first who explained, by means of boundary layer concepts, the circulation and lift of an infinite cylinder of elliptic cross section for a range of incidence angles. His criterion for determining the circulation is that "...the total flux of vorticity into the wake must be zero for steady flow ..." in other words, "...equal amounts of vorticity, of opposite signs, must be shed into the wake from the upper and lower boundary layers at their separation points ... - " a theorem due to G.I. Taylor [1925]. The rate of vorticity shedding at a separation point where the free stream velocity is U_∞ is:

$$\int_0^\alpha u \frac{\partial u}{\partial y} dy = \frac{1}{2} U_\infty^2$$

He reduces this criterion, by means of boundary-layer approximations, to the condition:

$$q_{s_1}^2 = q_{s_2}^2$$

on the velocities at the separation points s_1, s_2. Figure 11 shows the points of separation on both the upper and lower surfaces. For the case of fully-laminar flow, it gives a rough idea of the size of the wake. His procedure is one of successive iteration. A value of the circulation is assumed and from the external potential flow analysis, the velocity at the surface of the cylinder is determined. Using this velocity as the mainstream velocity in the boundary layer calculation, the points of separation on the upper and lower surfaces can be determined, (The circulation has to be varied until the velocities at these points are the same.) Taylor first assumed laminar boundary layers and laminar separation, then made the analogous calculation with turbulent separation. In fact, the counterpart of the trailing edge in his analysis, for steady flow, is that "...the total flux of vorticity shed from both the upper and the lower surface into the wake must be zero ..."

Sears [1956] revised Howarth's criterion for the curved-surface boundary layer considered by Preston and Spence and

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showed that Howarth's criterion of vanishing total vorticity flux becomes

$$(\tfrac{1}{2})\, q_1^2 - (\tfrac{1}{\rho})\, \Delta p_1 = (\tfrac{1}{2})\, q_2^2 - (\tfrac{1}{\rho})\, \Delta p_2$$

where

$\Delta p_1 = p_1 - p_{TS}$

$\Delta p_2 = p_2 - p_{TP}$

p_1 and p_2 are static pressures at points 1 and 2

p_{TS} = pressure at T.E. from suction side

p_{TP} = pressure at T.E. from pressure side

q_1 and q_2 are the free stream velocities at both edges of the layer.

These parameters, and the boundary layer configuration at the trailing edge, are shown in Figure 12. Piercy, Preston, and Whitehead [1938] made an empirical allowance by emphasizing downstream sources for the wake region in Howarth's method. Hancock [1976], reviewed Basu's method [1973], where the boundary layer on the airfoil and wake displacement effects are considered. First, assuming inviscid flow, the pressure distribution is calculated using the Smith-Hess method, then the boundary layer calculation is performed along the upper and lower surface of the airfoil. Thwaites method [1949] is used for the initial laminar boundary layer and then either the

Horton [1969], or Green [1972] or the Bradshow [1966] method can be used for the turbulent boundary layers. Once the displacement thicknesses of the boundary layers over the airfoil are known, the displacement thickness of the downstream wake can either be simply assumed or calculated. Then, Smith's method is applied on the new profile plus the wake. The mathematical model used is shown in Figure 13. The predicted pressure distribution, compared with experiment on an RAE 101 airfoil, agrees well, as shown in Figure 14.

All the investigations mentioned above lie in the broad category of those stemming from the work of Howarth. These estimate the boundary-layer growth on airfoils and account for its effects on the circulation and pressure distribution.

Preston [1943, 1945, 1949] was the first to successfully employ detailed boundary layer and wake calculations to predict the circulation of airfoils. He modified the airfoil shape by the addition of the displacement thickness δ^* on the surface and along a line extending to infinity downstream, and attempted to calculate the potential flow about the new body by breaking the displacement thickness δ^* into symmetric and antisymmetric parts:

$$\delta^*_s = \frac{1}{2}(\delta^*_u + \delta^*_\ell)$$

$$\delta_c^* = \frac{1}{2}(\delta_u^* - \delta_\ell^*)$$

δ_c^* represents a cambered displacement of the airfoil centerline, equilvalent to a reduction of incidence, and δ_s can be represented by a symmetrical source distribution. This approach requires some empiricism in determining the final circulation so as to satisfy the vorticity condition. Another investigation along the same line was carried out by Spence [1954], who achieved some simplication of Preston's technique. His procedure, like Howarth's, is one of successive approximations. First of all, assuming that the boundary layer thickness is known, the potential flow outside the boundary layer is corrected for the displacement thickness of the boundary layer and the viscous wake by distribution of sources of proper strength along the airfoil contour and along the approximate position of the wake. Their criterion for circulation is: "...essentially, that the pressure at the trailing edge shall have the same value when determined from the potential flow values above and below the airfoil..." In fact, they pointed out that the pressure variation across the boundary layer is not negligible in such a singular region as that of the trailing edge. Thus, they estimated the pressure

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at the trailing edge shall have the same value when determined from the potential flow values above and below the airfoil. In fact, they pointed out that the pressure variation across the boundary layer is not negligible in such a singular region as that of the trailing edge. Thus, they estimated the pressure increments, say Δp_1 and Δp_2, such that:

$$p_1 - \Delta p_1 = p_2 - \Delta p_2$$

where p_1 and p_2 are static pressure at the edge of the upper and lower boundary layer on normals from the trailing edge. But outside the viscous layers, we have:

$$p_1 + (\tfrac{1}{2})\ \rho q_1^2 = p_2 + (\tfrac{1}{2})\ \rho q_2^2$$

Thus, their criterion becomes;

$$[\frac{q_1}{q_2}]^2 = \frac{1 - \bar{W}_2}{1 - \bar{W}_1} \doteq 1 + \Delta\bar{W}$$

where

$$\Delta\bar{W} = \bar{W}_1 - \bar{W}_2$$

$$\bar{W}_1 = \Delta p_1/(\tfrac{1}{2})\ \rho q_1^2$$

$$\bar{W}_2 = \Delta p_2/(\tfrac{1}{2})\ \rho q_2^2$$

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Spence finds that $\Delta\overline{w}$ is small of the order of 0.01. This theory gives the pressure increments, but states that they are of such order as to be neglected in the momentum equation for the velocity component along the surface. So, we get

$$q_1^2 = q_2^2$$

which provides an alternative interpretation of Howarth's vorticity condition.

In fact, Spence's criterion is exactly the same as Howarth's vorticity condition, "... zero total flux of vorticity must be shed into the wake..." . It can be demonstrated, from the linearized form of Kelvin's theorem, as given by Lamb [1932]:

$$\frac{d\Gamma}{dt} = -U_\infty \gamma_{T.E.}$$

Consequently, for the steady case, there is no net flux of vorticity out of a fixed closed circuit enclosing the airfoil and cutting the wake at a downstream location. Preston [1949] has pointed out that such a circuit must cut the wake streamline at right angles. Spence [1954], also represents pressure distribution on a Joukowsky airfoil at several Reynolds numbers

at an incidence angles of 6°. It is interesting that the distribution deviates from the inviscid fluid distribution for the same value of the circulation only near the trailing edge, as shown in Figure 15. Spence's conclusion, based on comparison with experiment, is that his method is accurate when the boundary-layer thickness can be accurately predicted, i.e. the limitations of the theory are the limitations of knowledge of the boundary-layer.

Their extension to subsonic compressible flows is straightforward. Spence [1970] has carried out additional modifications to his procedure, especially about the problem of the singularity in curvature of the streamline springing from the trailing edge in inviscid flow, which implies that the initial curvature of the wake in the real flow will be large enough to cause a modification to the potential flow. Using a viscous analysis, similar to the work of Brown and Stewartson [1970], he balanced the inner and outer flows so that the pressure rise across the wake is consistent with the streamline curvature which it induces. He found that the reduction in circulation below the Kutta-Joukowsky value is proportional to the curvature at the trailing edge; for laminar flow, this is of order $R^{-\frac{1}{2}}$ og(1/Re), and his solution contains an arbitrary constant which could be fitted only by examining the near wake.

v Extending Spence's method [1954] to separated flow is not accurate, especially when the region of separated flow becomes large enough to affect the potential flow field appreciably. In this case, the separation point will be unknown as the calculation begins, and will have to be determined by successive trials, as in Howarth's method.

Sears [1976] has considered all of these features in his prediction of unsteady motion of airfoils with boundary layer separation. He has shown that the condition that determines circulation about an airfoil with a boundary layer is identical with the usual inviscid flow condition based on the conservation of total circulation and the Kutta-Joukowski condition of zero static pressure difference in the region of the trailing edge, in both steady and unsteady flow.

Gostelow [1975] also showed that the condition which gives a unique flow solution is not only the condition of "zero static pressure difference as the trailing edge is approached from either side" but also the "first viscous approximation", which is simply fairing in the pressure distributions to avoid severe velocity peaks near the trailing edge, as shown in Figure 16. In fact, he arrived at this conclusion from a study of measured pressure distributions on compressor blades, where almost all pressure distributions indicated a linear change in

pressure over the last 15 percent of chord. As noted by Gostelow [1975], this conclusion agrees with the work of Spence and Beasley [1960].

This "fairing in" process is achieved in a real flow by means of the displacement effects of the boundary layer near the trailing edge; it results in modifications to the streamline curvature and less severe gradients in the measured pressure distribution. Miller [1973] has recommended Gostelow's method, in comparison with other methods, because it gives greater accuracy for most compressor blading or isolated airfoils, but not for turbine blading.

Yates [1978] pointed out, through steady viscous analysis of thin airfoil theory, that the steady trailing edge condition of "zero loading at the trailing edge" is the correct viscous uniqueness criteria to apply to inviscid thin airfoil theory in the limiting case as the Reynolds number tends to infinity. So, the inviscid solution is obtained from the viscous solution by invoking the steady trailing edge condition as the Reynolds number tends to infinity. The most interesting point he mentioned, is that the Reynolds number correction to the inviscid lift curve slope is found to be of $O(1/\ell nRe)$; this correction is much greater than boundary layer thickness effects calculated with the inviscid parallel shear flow

boundary layer model, and is numerically of order 10 to 20 percent for Re between one and ten million. These results agree well with the experimental results for a variety of thin airfoils.

However, Wu [1981] represents a new and general theory for aerodynamics forces and moments, developed through a rigorous analysis of the viscous flow equations. He says:

> ...the circulation theory is known to predict the lift force accurately for certain types of solid; e.g., thin airfoil, under certain flow environment, e.g., small angle of attack. The scope of applicability of the circulation theory and its extensions has not been established precisely. Considerable uncertainties exist regarding the application of the theory in cases where the solid does not possess a sharp trailing edge, where the massive separation occurs, and where the solid is three dimensional and its motion is time dependent. These uncertainties arise mainly because of the perfect-fluid assumption used in the mathematical development of the theory. Nevertheless, it is often difficult to interpret the application of the circulation theory as an approximation of the viscous flow phenomena.

The distinguishing feature of this theory is that the concept of bound vortex, or that of singularity elements, is not embodied in the general formulating of the theory. Rather, the actual vorticity distribution of the flow region

enter these formulas. For example, it permits a precise definition of the circulation about a two-dimensional solid boundary not only for an unseparated flow, but also for a flow containing an appreciable region of separation. More details will be presented in the next sections.

(C) Multistructured Boundary Layer Theory and Trailing-Edge Flow Structure

Gostelow [1975], through a potential analysis, has shown that for a cusped trailing-edge the trailing-edge velocity is finite while the velocity gradient is infinite, and for the rounded trailing-edge, $\theta_T = \pi$, the velocity is zero and the velocity gradient is finite. Also, he pointed out the discontinuity in slope and curvature of the downstream stagnation streamline, associated with this singularity. But in real flows the effect of increasing displacement thickness over the trailing-edge region reduces the slope of the surface pressure distribution, reducing the degree of discontinuity. Gostelow reached the same result for the rounded trailing edge, i.e., the role of viscosity is to reduce the pressure gradient at the trailing-edge. He called this viscous process a "second viscous approximation", which can be accounted for by computation of a revised potential flow using the displacement surface as a boundary condition; this displacement surface

requires detailed descriptions of separation behavior and wake curvature, especially when the trailing-edge region is loaded.

Gostelow [1975] also pointed out the role of viscosity in making the downstream partition streamline more stable. He has shown by potential flow analyasis for a flat plate at zero incidence, that violation of the Kutta-Joukowski condition at the trailing-edge results ina partition streamline leaving the body orthogonally, its shape demonstrated to be hyperbolic. But the partition streamline will be a straight line when the Kutta-Joukowski condition is satisfied, and is unstable to small disturbances; a small change in incidence results in a change to a hyperbolic separation line, while the experiment of Fujita and Kovasnay [1971], however, shows that the location of the experimental partition streamline is more stable. So, in further analysis, we must look more closely at the role of viscosity as a stabilizing influence, for both steady and unsteady flows. In doing so, consideration should be given to recent advances, especially those of Stewartson [1969] and Messiter [1970].

In fact, the nature of the flow near the trailing-edge of an airfoil has been a subject of both theoretical and practical interest. The problem exhibits a singularity intriguing to the theoretician; the question of finite Reynolds

number effects on aerodynamics forces is of considerable importance.

Goldstein [1930] first treated this problem within the framework of laminar boundary layer. He showed that the continuation of the flat plate solution beyond the trailing-edge required introduction of a thin sublayer along the wake centerline with thickness of order $O(x^{1/3})$, where x is the streamwise distance from the trailing-edge. But, as Goldstein showed, the change of boundary conditions at the trailing-edge results in a singularity of the velocity component normal to the plate, being finite on the upstream side and infinite on the downstream side. As a consequence, the streamline, in the boundary layer experiences a sharp turn at the trailing-edge, which physically means a rapid acceleration of the fluid at the bottom of the boundary layer due to the termination of the plate; this effect abruptly draws fluid in towards the center from the edges of the boundary layer. Many authors have recognized that classical boundary layer theory fails near the trailing-edge, and that the flow field in that region cannot be constructed as single layer matched with both Blasius and Goldstein wake layers. However, an understanding of this problem was attained by the simultaneous revolutionary discoveries of Stewartson [1969] and Messiter [1970]: at high

Reynold numbers, the flow near the trailing-edge has a compound structure that Stewartson calls a "triple deck structure". It is of length $\varepsilon^3 L$ in x-direction, where L is the nondimensional length of the plate, and t is defined by;

$$Re = \frac{U_\infty L}{\nu} = \varepsilon^{-8} \quad , \quad \varepsilon << 1$$

In fact, Stewartson has shown that there exists a very small region enclosing the edge where the derivatives of the flow variables are of the same order in both directions, and the displacement effect of the boundary layer is not negligible, especially in a region like the trailing-edge, where the boundary conditions change from the condition of zero tangential velocity on the plate to zero stress on the center line of the wake. This region in the vicinity of the trailing-edge of order $O(\varepsilon^3)L$ intervenes between the region of validity of the Blasius solution [1908] and that of Goldstein's [1930] wake solution. Normal to the plate, this region has three layers; lower deck of thickness $O(\varepsilon^5)$, main deck of thickness $O(\varepsilon^4)$, and upper deck of thickness $O(\varepsilon^3)$, as shown in Figure 17. The main deck corresponds to Goldstein's outer wake, which to first order is the inviscid continuation of the Blasius boundary layer solution; the pressure variation across the deck is small, and it plays a relatively passive role in the

mechanism. The lower deck corresponds to Goldstein's inner viscous wake which is produced by the altered boundary condition at the trailing edge, and is controlled by the conventional boundary-layer equations. Broadly, the upper deck provides a pressure gradient which helps drive the lower deck. In turn, the lower deck produces changes in the displacement thickness of the boundary layer, and these generate the pressure gradient in the upper deck. The effect of this triple deck structure is to induce a favorable pressure gradient upstream of the trailing-edge, which tends to smooth out the discontinuity in transverse velocity, as well as displacement thickness, at the trailing edge.

Both Messiter and Stewartson presented uniformly valid asymptotic solutions, but the resulting pressure gradient has a discontinuity at the trailing edge region; this can be resolved by a finer substructure, ultimately of the $O(\varepsilon^6)$ scale of Hakkinen and O'Neil [1967]. Dennis and Chang [1969], and Dennis and Dunwoody [1966] find a trailing-edge region of influence that scales with (ε^3), in agreement with Stewartson [1969] and Messiter [1970]. As mentioned by Van Dyke [1975], this structure contributes a correction to Blasius drag which is of order $O(\varepsilon^7)$, and hence slightly more important than the displacement effects of order $O(\varepsilon^8)$ calculated by Kuo [1953] and Imai [1957].

The application of multi-structural boundary layers has been successful in providing insight into many problems. These include plates at incidence, bodies of nonzero thickness in steady and unsteady flows, noise generation and sound diffraction, separation, and others. As mentioned by Stewartson [1974], the essential requirement is that catastrophic separation does not occur, and in turn, this means that there must be a Reynolds-number dependent parameter defining the departure of the problem from that of the basic finite flat plate at zero incidence. Riley and Stewartson [1969] extend this theory to the flow in the trailing-edge region of airfoils with a finite-trailing edge angle at zero incidence, when there is pressure gradient imposed on the boundary layer. They establish a criterion for separation to occur, and make an estimate of the distance from the trailing-edge at which separation takes place; for the trailing-edge of the form of a wedge of small angle $2\pi\alpha$, flow separation occurs within a distance of order $O(\alpha^{3/2})$ if $\alpha\epsilon^{-2}<<1$, and the largest trailing-edge angle for which the flow will not separate is $O(\epsilon^2)$ i.e., $O(Re^{-1/4})$. It is believed this is the criterion for inhibition of separation. In general, separation can either be catastrophic or regular, but when the pressure gradient is prescribed externally it always appears to be catastrophic.

In addition, the theory of the triple-deck has been applied to study the viscous correction to the lifting forces on aerodynamics shapes at high Reynolds numbers, and to show that the trailing-edge condition, which determines the circulation and the lift for inviscid steady flow, can be embedded in a formal asymptotic expansion of the flow field in powers of ε. Brown and Stewartson [1970] extend the triple-deck structure to the case of a flat plate at incidence. Upstream of the trailing-edge, the boundary layer remains close to the Blasius profile over the majority of the plate, but then changes rapidly in the neighborhood of the trailing edge, in a similar way as before, then subboundary layers develop and are the geneses of the lower decks of the trailing-edge. The interaction between the adverse pressure gradient due to finite incidence angle, which threatens separation near the trailing-edge and induces a favorable pressure gradient on the lower side of the plate, is the main factor in flow separation on the upper side of the plate. If the angle of incidence (α^*) is large, i.e., $\alpha^* \gg O(\varepsilon^{1/2})$, the flow separates before it is influenced by the triple deck, then it provokes the phenomenon of trailing-edge stall. If the incidence angle is too small, i.e., $\alpha^* \ll O(\varepsilon^{1/2})$, the effect of the triple deck outweighs that of incidence and the boundary layer remains attached

until the trailing-edge. However, if the angle of incidence is of order $O(\varepsilon^{1/2})$ the two effects are comparable and trailing-edge stall is liable to occur. Brown and Stewartson estimated this critical angle of incidence $\alpha_s \approx 0.4$ which leads to rather low stalling angles $(\approx 2°)$ in realizable situations. These results emphasize the importance of viscous effects in this phenomenon. They also have shown that the critical angle of incidence for subsonic flow is $\alpha^* = O(\varepsilon^{1/2})$ and for supersonic $\alpha^* = O(\varepsilon^2)$. The asymptotic form of the pressure at the trailing-edge is

$$p_{\pm}(x) = \pm(-x)^{1/2} \pm \alpha b_1/(-x)^{1/2} + \ldots$$

$$\text{as } x \to -\infty$$

where, x is the streamwise coordinate inside the main deck of the triple deck structure at the trailing-edge, and is related to the outer flow coordinate x^* by:

$$x^* = \varepsilon^3 \lambda^{-5/6} L x \quad , \quad x = O(1)$$

$$\gamma \approx 0.3321$$

and b_1 is determined from over-all properties of the triple deck. For an approximate solution, $b_1 \approx 0.79$, the corrected viscous lift coefficient on the plate is given by

$$C_L = 2\pi\alpha^* (1 - 1.58\lambda^{-5/4} \varepsilon^3)$$

Stewartson [1974] has shown that the wake curvature just outside the triple deck is not important for the determination of the viscous correction to the lift, as formerly believed. The contribution of the wake curvature to C_L is of order $O(\varepsilon^4 \log \varepsilon)$, which is weaker than that of order $O(\varepsilon^3)$ due to the triple deck. It should be noted that Stewartson [1974] has reviewed this problem and others and extends his multistructured boundary layer theory to analysis of compressible situations, which are beyond the scope of this review.

It is worthwhile to present some results of the work of Daniels [1977] on the viscous mixing layers at the trailing-edge. He considered the problem of the laminar viscous mixing of two parallel streams of strength U_1 and U_2 $(U_1 \geq U_2)$ in the trailing-edge region of a flat plate at high Reynolds number, and found the structure of the trailing-edge is a generalization of the triple-deck theory of Stewartson [1969] and Messiter [1970], which is recovered in the limit as $U_2 \to U$. The major influence of the trailing-edge extends to a distance of order $O(\varepsilon^3)$ i.e., $O(Re^{-3/8})$ as $Re \to \infty$, but in the limit as $U_2 \to 0$ (which corresponds to stagnant fluid below

the plate), for an incompressible fluid, this influence is much weaker and the trailing-edge effect is confined to a small region of order $O(\varepsilon^6)$ in the vicinity of the trailing-edge. For this case $(U_2 = 0)$, he argued that the mainstream above the plate induces a velocity in the stagnant flow, the action of which is to draw fluid from below the plate, where a backward-facing boundary layer of thickness $O(\varepsilon^2)$ is set up along the plate in the stagnant fluid. This fluid is then drawn into the mixing layer at the trailing-edge, where it supports an upward curvature of the streamline from the trailing; that is, the streamline from the trailing-edge bends upwards and away from the stagnant fluid. This leads to the unique determination of the location of the dividing streamline in the wake. So the singularity in the displacement of the velocity profile is removed. This problem was originally investigated by Ting [1959] who showed that the solution obtained by requiring a continuity of the pressure across the mixing layer is not unique. But Daniels [1977] has shown that the nonuniqueness is removed when the outer inviscid and the boundary-layers flows are matched with a consistent solution in the vicinity of the trailing-edge; such a solution has no singularity in pressure at the trailing-edge and may be likened to the validity of the trailing-edge

condition for the inviscid solution.

It is also shown that similar theory is applicable to the flow of a uniform stream over a backward-facing step, and to the steady laminar flow at the nozzle of a jet. Figure 18 demonstrates this argument.

IV. TRAILING-EDGE CONDITION IN UNSTEADY FLOW

(A) Unsteady Flow Generation

In consideration of the unsteady trailing edge condition, attention must be given to all the previous theoretical difficulties encountered in steady flow, as well as those associated with the unsteady effect on the boundary layer and the eventual shedding of vorticity from the trailing edge.

In general, unsteadiness generated upstream of the leading edge of an airfoil in a turbomachine provides an additional dimension of complication. Such incident unsteadiness makes the airfoils response a function of the history of the vorticity field along the airfoil. As discussed by Horlock [1968], flow in axial turbomachines can give rise to upstream unsteadiness, i.e. disturbances, due to wakes shed by one stator impinging on a following stator, or to those entering the machine and impinging on the first row of rotating blades, and due to the relative movement of rotors and stators. Also, the blade flutter gives rise to unsteadiness to the flow in turbomachines as well as to the flow around wings and airfoils. Flutter, which is an aeroelastic instability, involves a transfer of excess energy from unsteady aerodynamics forces to a single or cascaded airfoil(s). The mechanism of flutter is

a net (kinetic) energy transfer, from the surrounding flow to the airfoil, that exceeds the amount of available mechanical damping generated either internally (material hysteresis) or by friction at the blade foundation. This mechanism may be understood by visualizing a spring-mass-dashpot system with excessive excitation energy. A survey article by Sabatiuk and Sisto [1956], defines two forms of flutter: self-excited and forced.

Self-excited flutter results when the unsteady forces acting on the blade are functions of the displacement, velocity or acceleration of the blade. From a small initial deflection or perturbation of the blade surface in a uniform incident flow, the unsteady forces feed energy into the system, yielding self-induced oscillations.

Forced flutter, on the other hand, is driven by a non-uniform incident flow. Therefore it is externally-excited. The nature of the forces acting on the airfoil are essentially independent of the blade displacement, velocity or acceleration. The operation schedule of an axial compressor is illustrated in Figure 19. Shown are four distinct flutter regions, determined from elastic rather than fluid dynamic measurements. More details on associated theoretical and experimental aspects can be founded in articles by Pratt and Whitney Aircraft

[1976], Jeffers and Meece [1979], Kerrebrock [1974], and Jones [1977]. In fact, the need to understand flutter has increased in the last twenty years. Active in this area is the NASA-Lewis Research Center. A program for full scale engine testing in June 1977 was carried out at NASA-Lewis (see Figure 20). Figure 21 shows visualization of blade displacement during flutter of the first fan stage of the F 100 using a fiber optic technique (PES System) at NASA-LERC (see Nieberding and Pollach [1977]).

Even for a stationary blade, the periodic vortex shedding and oscillating wake behind the trailing edge, which has been observed over a wide range of laminar and turbulent flows, give rise to unsteadiness in the flows around bodies especially when the trailing edge has a large thickness or wedge angle. These effects in the trailing edge region are very important in prediction of the turning angle and loss coefficients of turbine blades, and unsteady jet flow at the trailing-edge of a nozzle which is closely connected with the problem of noise generation.

An extensive review of many unsteady fluid dynamics problems is found in the review article of McCrosky [1977], and in the short review article of Hancock [1976].

To simplify matters as much as possible, the situation of

a disturbance-free incident free stream will be considered herein.

When an airfoil performs harmonic motion about its mean position at a relatively high frequency, the circulation, and hence the forces and moments acting on the airfoil, are also time-dependent. As a consequence of this motion, an unsteady wake is produced, which, in turn, influences the response of the airfoil. Since, according to Helmholtz's theorem, "... the total circulation round a closed contour enclosing the airfoil and the wake must be zero...", then each time-dependent change in circulation around the airfoil must be compensated by the shedding of vorticity from the trailing edge. This vorticity, which has the same strength as the change in the circulation but is of opposite sign, is convected downstream by the flow as shown in Figure 22. A measure of the flow unsteadiness is the reduced frequency k (or ν), defined as:

$$k = \frac{\omega c}{U_\infty}$$

where ω is the vibrational frequency of the blade, c is the blade semi-chord, U_∞ is the free stream velocity, and the wavelength λ;

$$\lambda = 2\pi \frac{U_\infty}{\omega}$$

Unsteady effects are important when some time scale of the

physical motion is comparable to the basic fluid-dynamic time scale, i.e. when $\omega L/U_\infty$ or L/Ut are of order 1 or greater. In general, the most important direct effects of unsteadiness are: 1) a phase difference between the aerodynamics forces and the motion producing them, and 2) an attenuation of the lift vector.

Figure 23 shows the time histories of the local pressures, as well as the resultant lift and moment on an airfoil performing oscillations in pitch in a subsonic flow. Both the pressures and the overall loads show sinusoidal variations about their mean values. For moderately subsonic and supersonic unsteady flows, a linear relationship exists between the displacement of the airfoil and the unsteady pressures at least as long as the flow remains attached. However, for transonic flow, particularly in the region of a shock wave, this is no longer true, the flow is nonlinear.

Generally speaking, unsteady flow problems can be linear or nonlinear in their behavior. In the former, the governing equations and the boundary conditions can usually be linearized, i.e. the fluid dynamic aspects can normally be approximated by small departures from steady behavior as in moderately subsonic and supersonic unsteady flows. In the latter case, either the equations of motion or the boundary

conditions, or both, contribute strong nonlinearities. This implies that the unsteady flow field can no longer be treated independently as a steady flow field, regardless of frequency. Also, the fluctuations are not always small in amplitude, and the unsteady airloads are no longer linear functions of the amplitude of motion. Most viscous flows as well as many inviscid transonic flows are nonlinear. Although the major share of problems that can be handled by linear theory are now fairly well understood, nonlinear aspects deserve further experimental and theoretical investigation.

(B) Inviscid Analysis

The first vivid demonstration of the importance of the trailing edge-and wake flow for unsteady aerodynamic theory associated with the flutter problem was carried out by Theodorsen [1935]. He separated the unsteady lift(L) and moment(M) of an oscillating airfoil in both pitching and plunging into: 1) noncirculatory components (L_{NC}, M_{NC}), where the influence of wake vortices on the flow is neglected and 2) circulatory components (L_c, M_c) accounting for the downstream wake. The importance of the circulatory components lies in restoration of a finite velocity at the trailing edge.

The circulatory components are generated by a continuous distribution of wake vortices, from the trailing edge to infinity, and are expressible as:

$$L_c \text{ or } M_c = \binom{\text{geometric and}}{\text{fluid properties}} C(k)$$

where C(k) is the so-called Theodorsen function. In Theodorsen's analysis, the wake is modelled as a continuous distribution of harmonically-oscillating free vortices shed downstream along the chord-line from the trailing edge to infinity. The model includes the assumptions that: a) the wake streamline coincides with the steady state streamline, i.e. the dividing streamline leaves the airfoil at the trailing edge and b) the pressure is continuous across the wake and at the trailing edge. Each element of the shed vortex in the wake may be traced back both spatially and temporally to its origin as a free vortex released from the trailing edge. The simple harmonic wake resulting from harmonic motion of the airfoil is expressed by:

$$\gamma_W(x^*,T) = \bar{\gamma}_W \; e^{i(\omega T - kx^*)}$$

where $\bar{\gamma}_W$ is the complex amplitude of the wake vorticity, ω is the vibration frequency, k the reduced frequency, and x^* is the

dimensionless distance downstream of the trailing edge, see Figure 24.

In fact, the trailing edge condition (zero pressure jump across the airfoil at the trailing edge), which had been used by Theodorsen, is typically embodied in unsteady potential flow predictions (e.g., Whitehead [1960]; Naumann and H. Yeh [1972]; Hancock [1972]; and Ni and Sisto [1976]. Although this view is satisfactory from a computational standpoint, it is an idealization of the real situation at large Reynolds, as shown in Figure 25.

In general, when the airfoil undergoes unsteady motion at moderate reduced frequency, and the flow near the trailing edge is assumed to be attached, then the viscous (actual) trailing edge flow can be effectively modelled as in Figure 25(a). In this interpretation, the effect of viscosity is taken to be confined to a thin boundary layer on the airfoil surface, and to the downstream wake formed by merging of the upper and lower surface boundary layers at the trailing edge (i.e. shed vorticity following Helmholtz's law); moreover the trailing edge is assumed to be sharp, as discussed by Prandtl [1961]. He suggests that the scale of the residual viscous effects, in the limit as the Reynolds number becomes large, would be of the order of the trailing edge radius, i.e. O(r),

so that it would vanish in the limit of a sharp trailing edge (i.e. $r \to 0$) and the flow at that point would tend to the ideal one of Figure 25(a). In the following, we shall discuss the trailing edge condition only in the limit of large Reynolds number, for completely attached flow approaching the trailing edge.

Several works have been somewhat successful in matching various geometric or dynamic conditions at the trailing edge with their mathematical formulation to obtain a compatible explanation of the flow dynamics in that region. Karman and Sears [1938] have shown, for a flat plate in unsteady motion, that "...the velocity difference across the trailing-edge is equal to the instantaneously shed vortex strength...", i.e.

$$\Delta V = \gamma_{T.E.}$$

In fact, this statement is equivalent to the statement that "...the bound vorticity around the airfoil must be equal to the shed vorticity at the trailing-edge for thin airfoil theory..." which is based on conservation of the total circulation.

Van der Vooren and Van der Vel [1964], in their elegant analytic solution for an oscillating airfoil using a conformal mapping technique, imposed a stagnation point at the trailing edge, (i.e. $V_{T.E.} = 0$ as trailing edge condition). In this case, the zero pressure difference across the trailing edge

is acceptable only if the trailing edge is cusped. However, for a non-zero trailing edge angle, a singularity in the form of an infinite velocity difference appears in their solution; the removal of this singularity results in a pressure discontinuity across the trailing edge region, as well as across the downstream wake. In essence, one must choose between zero loading across the trailing edge region or zero velocity difference at that point; both conditions cannot be satisfied simultaneously as noted by Whitehead [1973], in his discussion of the trailing edge condition in unsteady flow.

Giesing [1968] has proposed that the velocity difference at the trailing edge be zero, i.e. $\Delta V = 0$. In 1969, he showed that the velocity distributions with the condition $\Delta V = \gamma_{T.E.}$ and with $\Delta V = 0$ are almost the same except at the trailing edge. Also, he posed the dynamical conditions for vortex shedding in unsteady flows, involving shed vorticity composed of an unsteady part γ_i, which is proportional to the time rate of change of circulation, and a steady part γ_s which is proportional to the total head across the vortex sheet, i.e.:

$$\gamma = \gamma_i + \gamma_s$$

$$\bar{V}_{\gamma_s} = \Delta h$$

$$\bar{V}_{\gamma_i} = -\frac{d\Gamma}{dt}$$

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where $\bar{V}$ is the vortex-shedding velocity and Δh the total head across the vortex sheet. The vortex sheet is shed parallel to one side of the trailing edge or to the other, depending on the sense of the shed vorticity, and the shedding velocity $\bar{V}$ is equal to one half the strength of the vorticity at the trailing edge, except for zero trailing-edge angles. That is,

$$\bar{V} = \frac{1}{2}\gamma + \begin{cases} 0 & \delta > 0 \\ c & \delta = 0 \end{cases}$$

where δ = trailing edge angle

c = constant

It should be noted that this solution of Giesing gives a finite velocity at the trailing edge, with a finite pressure loading across the trailing edge and across the downstream wake as well, which is not acceptable on physical grounds, especially at low and moderate reduced frequencies, see Fleeter [1980]. More specifically, the rate of change of the circulation around airfoil is;

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\oint q\cdot dr$$

$$= \oint \frac{dq}{dt}\cdot dr + \oint \frac{dq}{2}$$

$$= -\oint \left(\frac{dP}{\rho} - \frac{dq}{2}\right)$$

So, for the flow leaving the trailing edge bisecting t the edge angle (stagnation trailing-edge), we have a pressure discontinuity associated with the rate of change of the circulation around the airfoil, i.e.

$$\frac{\Delta P}{\rho} = -\frac{d\Gamma}{dt}$$

which is unacceptable as mentioned before. Also, for the imposition of the trailing-edge condition in the form of zero pressure difference at the trailing edge, we have;

$$\frac{d\Gamma}{dt} = -\frac{(q_u + q_\ell)}{2}(q_u - q_\ell)$$

which requires that either the average velocity or the velocity difference at the trailing edge should not be zero.

For general unsteady flow, it is argued that the appropriate solution should satisfy both conditions of zero pressure loading across the trailing edge and finite velocities at the trailing edge. The condition of zero instantaneous trailing edge loading seems to be physically realistic and ensures consistency of the flow mechanics downstream of the trailing edge. However, some experimental measurements indicate deviations, especially for high frequency unsteadiness, as well as for blunt trailing edge airfoils. As previously mentioned, for the unsteady motion of an airfoil, there is a balance between the instantaneous rate of change of bound

vorticity about the airfoil and the rate of shedding of vorticity into the downstream wake, which implies zero pressure loading at the trailing edge. But when there is not a match between these rates, there may be shedding of vorticity arising from the low wavelength pressure fluctuations, and an instantaneous pressure loading appears across the trailing edge (Gostelow [1975]). This seems to be suggested by the results of Kadlec and Davis [1979]. As shown in Figure 40, for high reduced frequency, the wake is entirely distorted and vortex shedding results behind the trailing edge.

Maskell [1973] has argued, that in order to satisfy both the condition of zero pressure loading and finite velocity at the trailing edge, the flow must leave the trailing-edge parallel either to the upper surface or to the lower surface, depending on the sign of the instantaneously shed vorticity, provided that the pressure loading across the trailing edge and across the wake are zero and the velocities at the trailing edge remain finite; thus, separation occurs at the trailing edge. As shown in Figure 26d, when the shed vorticity is counter clockwise, the flow leaves the trailing edge parallel to the lower surface. In this case, the upper surface velocity tends to zero at the trailing edge ($q_u = 0$), while the lower velocity, q_ℓ, remains finite. So, from the previous

form of the rate of change of the circulation, we get:

$$\frac{d\Gamma}{dt} = + \frac{1}{2} q_\ell^2$$

here, $\frac{1}{2}q_\ell$ is the average velocity across the shed vortex sheet, which is the same result of Giesing [1969].

However, Gostelow [1975], in a discussion of the stability of vortex shedding from the trailing edge in steady flow and of the relation between the vortex street configuration and the trailing edge angle, has shown that the partition streamline in Maskell's model is unstable, and it is inappropriate in predicting the drag coefficient. Basu and Hancock [1978], have presented a numerical solution for an airfoil undergoing an arbitrary motion, using a procedure similar to A.M.O. Smith's [1967] method. They postulate that the flow separates at the trailing edge with zero loading across the shed vorticity just downstream of the trailing edge and zero load on the computational elements on the upper and the lower surfaces at the trailing edge. For an airfoil oscillating in pitch at high frequency (k=20), the resultant wake pattern involves vortices of opposite sign and it closely follows the one observed experimentally. Also, the trailing edge wake elements lie parallel to one surface or the other, depending on the direction of the shed vorticity. Moreover, it is

interesting to point out that the wake element follows Maskell's postulate as discussed above, as shown in Figures 27 and 28. Sears [1976], presented a generalized criterion for unsteady airfoils with boundary layer separation and has shown that, for airfoils with sharp trailing edge and which can be approximated by thin-airfoil (i.e. linear airfoil theory), that the flow is attached until the trailing edge and the condition there is:

$$\frac{d\Gamma}{dt} = -U_\infty \gamma_{T.E.} \text{ (to order } \varepsilon)$$

He also has shown that the condition that determines circulation about an airfoil with boundary layers is identical with the usual inviscid-flow condition based on conservation of total circulation and with the trailing edge condition in the form of zero pressure difference at the trailing edge. He says:

> All of this serves to remind us that the inviscid fluid model must represent the limiting case of vanishingly small viscosity and not the flow of a truly inviscid fluid. Thus, the viscous (boundary-layer) and inviscid models of an unsteady airfoil are identical; in both there is a continuous flux of vorticity from the trailing edge into the wake, and there is no discontinuity in vortex strength at the trailing edge. The trailing edge is just the chordwise station where the vortex distribution becomes "free" insteady of "bound", because there is no force on the wake.

With regard to experimental verification of the unsteady

trailing edge condition, most works have been directed towards corroboration of the assumption of zero unsteady pressure loading in the trailing edge region. In fact, proper theoretical modelling of the generalized trailing-edge condition is important in evaluating the unsteady lift and moment, especially for isolated airfoils and cascade blades. Although the unsteady loading variations in the trailing-edge region may not significantly affect the magnitude of the unsteady lift, it may affect the unsteady moment; also, aerodynamic phase lag variations in this region have an influence on noise generation. Some experimental studies with oscillating airfoils have revealed that as the reduced frequency increases, the validity of zero unsteady loading, in both magnitude and phase, breaks down. Greidanus, Van der Vouren, and Bergh [1952], working on an airfoil mechanically oscillated separately in heaving and pitching modes up to a value of reduced frequency parameter k=2.0, have reported on the non-validity of the kutta condition. The experimental disagreement with the Theodorsen potential theory [1975], was ascribed to the lack of validity of the trailing edge condition of zero pressure loading. Also, Satyanarayana [1977], through experimental investigation of an isolated airfoil, and airfoils in cascade subjected to a sinusoidally varying gust,

has concluded that the instantaneous pressure differential at the trailing edge region approaches zero at low reduced frequency, $k \leq 0.1$. But deviations from the linear potential theory (as mentioned above, the fluid dynamics aspects can be approximated by small departures from steady behavior) are reported in the phase angle, as shown in Figure 29. The basic airfoil employed in these experiments was an uncambered NGTE-10C4 section of 6 in. chord and 18 in. span, the maximum thickness-to-chord ratio was 10%, the Reynolds number based on the chord C was 160,000, and the mean incidence was $\alpha=0$. In a later work, involving an isolated cambered airfoil (NACA G4A010 airfoil with a 15-cm chord, an 25-cm span with sharp trailing edge) oscillating up to ±1-deg. incidence at a mean angle of attack of zero deg., over reduced frequencies ranging from 0.05 to 1.2, at Mach number M = 0.168 and a chord-Reynolds number of 560,000, Satyanarayana and David [1978], have reported that the zero pressure loading condition is valid with reduced frequency values less than 0.6; for reduced frequency values greater than 0.8, the measured loading in the trailing edge region deviates from that predicted by linear theory, and the pressure trace in the trailing edge region exhibits frequency doubling. There is a similar deviation for phase angle of the loading at the trailing edge,

as shown in Figure 30. The authors point out that this deviation is due to boundary layer displacement effects near the trailing edge. In fact, while the instantaneous pressure difference is large at the trailing edge, the amplitude of the time-averaged pressure fluctuation is quite small. It is felt that the discrepancies in phase lag of the absolute pressure are more significant than those of pressure difference near the trailing edge. This is due to periodic separation in this region, which would change the shape of the pressure distribution. Similar results for the case of an isolated airfoil subjected to a sinusoidal transverse gust at 9 deg. on incidence have been reported by Holmes [1972], as shown in Figure 31. In still another related investigation, Fujita and Kovasznay [1974] reported on the response of a stationary instrumented airfoil to the wake of an upstream rotating rod. The measured chordwise response was in good agreement with the linear theory over most of the chord except for the last 10%. In this region, theoretical agreement was poor, associated with finite loading at the trailing-edge. In fact, the trailing edge of the test airfoil in their experiment was clearly quite rounded; this results in significant viscous effects, and consequently loading at the trailing-edge. Ostiek [1975] also has reported that the measured pressure

distribution on airfoils in cascade at lower reduced frequencies up to 0.08 agrees well with the predicted one except in the trailing edge region. The unsteady flow field was created by oscillating the inlet section, the mean angle of attack being varied between 6-deg. to 12-deg. On the other hand, at high values of the reduced frequency parameter, $k>5$, Archibald [1975] measured the pressure differential near the trailing edge of a flat plate and an airfoil and concluded that the zero pressure loading at the trailing edge does not hold. In this case the unsteady flow was created by exciting two loudspeakers connected in antiphase. He pointed out the disagreement from the theoretically predicted zero trailing edge loading caused by viscous instabilities is found to be acoustically corrected vortex shedding, natural vortex shedding, Tollmien-Schlichting waves, and, by implication turbulent boundary-layer eddies. Also, another failure of the condition of zero trailing edge loading at high reduced frequencies ($k\approx5$), in connection with measurements of the noise generation, was reported by Davis [1976]. But Commerford and Carta in 1974, have reported, from experiments on a circular arc airfoil, where the periodic fluctuating flow field was produced by the natural shedding of vortices from a <u>transverse</u> cylinder to yield a reduced frequency parameter $k=3.9$, that

the individual pressure distributions at each angle of attack tended to zero at the trailing edge, indicating the validity of Kutta-condition of zero loading, even at this high reduced frequency.

The recent experimental investigation of Fleeter [1980], involves generation of an unsteady flow field by a rotor wake, characterized by a high reduced frequency, k = 8.0. The trailing-edge data, involving unsteady differential pressure, was correlated with predictions for a zero incidence flat plate cascade and an isolated flat plate airfoil; the theory employed was the compressible transverse gust analysis of Fleeter [1973] . For both experimental cases, the zero pressure difference at the trailing edge was found to be valid up to a reduced frequency of 8.0 for a wide range of incidence angles. However, for the case of a cambered airfoil cascade, it breaks down at higher reduced frequencies. His results show that the difference between the pressure-and the suction-side aerodynamic phase angle lag either remains constant or decreases as the trailing edge is approached for the isolated flat plate and the flat plate cascade, but increases or remains constant for the cambered airfoil cascade, as shown in Figure 32.

Obviously, from the above discussion, one cannot conclude

that the condition of zero loading holds for all values of the reduced frequency. However, the results of Fleeter [1980]. seem to indicate that the zero loading assumption is reasonably acceptable, especially for the flat plate and the flat plate cascade, at high values of reduced frequency up to 10. Further theoretical and experimental investigations should be carried out, especially for the cambered airfoil cascade, where the trailing edge is rounded rather than sharp.

The interaction between the instability wave in a free shear layer and the surface from which the shear layer is shed gives rise to the problem of noise generation; it has been found that the proper theoretical modelling of the edge condition plays a crucial role in understanding acoustic radiation from the trailing-edges of wings and rotating blades, and from flow nozzles. With regard to the stability of the vortex sheet emanating from a trailing-edge, the pioneering work of Helmholtz [1968] shows that the flow of two parallel uniform streams of different strengths is subjected to an instability in the form of a spatially growing time-harmonic oscillation of the vortex sheet which divides the streams. Orszag and Crow [1970] extended the work of Helmholtz by considering the effect of the semi-infinite plate which divides the flow upstream. They restricted their attention

to the case in which the fluid is at rest below the plate, and have introduced three alternative conditions at the trailing-edge to render a unique flow solution. The nature of their solution strongly depends on whether a Kutta condition is enforced at the trailing-edge. The first, or 'no' Kutta condition solution, predicts that the vortex sheet leaves the trailing-edge in the shape of a parabola which oscillates symmetrically above and below the line of the plate as time progresses; it involves a singularity in the pressure at the trailing-edge.

Secondly, based on physical arguments, they suggested that the vortex sheet should never leave the trailing-edge in such a way that an angle greater than π is turned. Movement of the vortex sheet between this limit and that of the "flapping parabola" constitutes what they term a "rectified Kutta condition". The pressure jump across the vortex sheet is zero and this solution was regarded physically as the most likely to occur, but again there is an inverse singularity in the pressure on the plate at the trailing-edge. In fact, viscous effects prevent such a turn in real fluid, vorticity being shed from the trailing-edge as shown in Figure 33(d), and the induced flow bending the vortex sheet up at the trailing-edge. More vorticity of the same sign will be shed, until

eventually a first-order mean circulation change will have been induced on the plate and a final state is asymptotically reached. The sheet never leaves the plate with a downward slope.

Finally, they showed that a 'full' Kutta condition is such that the vortex sheet leaves the trailing-edge smoothly. In this case, there is a singularity in the pressure at the trailing-edge and a reduction in the decay rate of the solution at infinity. Daniels [1978] supports this interpretation and was found that the pressure grows in the upstream direction at large distance from the trailing-edge.

When near the trailing-edge of an airfoil, in the presence of flow, sound may be generated either by turbulent eddies or by an external source. The sound may induce vorticity shedding which dominates, or at least provides a local ordering of the turbulent eddies. Ffowcs Williams and Hall [1970], and Crighton and Leppington [1970] have shown that the intensity of aerodynamic noise is greatly enhanced when the sound-producing turbulent quadrupoles are located near the sharp trailing-edge i.e., the edge acts as a scattering center in the absence of mean flow. They also argued that at sufficiently high frequencies, the edge flow will be dominated only by the diffracted field, while for low frequencies the

edge flow is determined by viscous effects. In the presence of mean flow, there is also the possibility that velocity fluctuations induced by convected turbulence will result in the generation of additional noise-producing vorticity shed from trailing-edge and subsequently swept downstream. Works have been carried out in a related area, noise generation by a jet due to turbulent eddies, by many investigators including Crighton [1972], Morgan [1975], Munt [1977], and Rienstra [1979]. Rienstra considered the problem of the interaction between subsonic jet flow issuing from a semi-infinite circular pipe, and a harmonic plane wave with small strouhal number. He showed that the Kutta condition plays a significant role; it appears to affect the magnitude of the reflection coefficient, but not the end correction. Besides diffraction and vortex shedding, reflection at the open pipe is present as well. Also, he concluded that, in the case of 'no Kutta condition', the induced energy is reflected at the pipe exit, while, in the case of a full Kutta condition, an amount of acoustic energy of order M (Mach number) is transmitted and is transformed into hydrodynamics energy (vortices), and therefore hardly felt in the far field. This conclusion has also been arrived at by Howe [1978], who studied the interaction of sound with different jet flows from various types of nozzles,

to explain the attenuation of the radiated sound power observed in practice for low strouhal number.

Crighton [1972, a-b] has extended the work of Orszag and Crow to compressible flow, and suggests that vortex sheet leaves the trailing-edge with zero gradient velocity at all times (i.e., the full Kutta condition is satisfied); he studied the edge diffraction radiation induced by the unstable oscillations of a vortex wake, and concluded that, at low mean-flow Mach number M, the application of a Kutta condition at the edge resulted in an increase in the acoustic intensity. A similar dependence on the mean-flow Mach number has been predicted by Davies [1975]. This may be contrasted with the conclusion of Jones [1972]; he has examined a model problem involving the generation of sound by a stationary line source located in the vicinity of the trailing-edge of a large airfoil, and reports no significant acoustic response arising from the imposition of the Kutta condition. Jones and Morgan [1974] presented a linear model of the interaction of sound with the vortex sheet, which physically limits its amplitude. In essence, the idea used is simply that once the instability waves on the vortex sheet are large enough for the nonlinear effects to be signigicant, they break, i.e., downstream of this point the two regions of

flow are separated by a region of turbulence which has a significant width compared with the wavelength of the triggering sound.

In fact, instabilities have particular theoretical significance for diffraction problems involving a surface edge shedding an unstable shear layer. This has been demonstrated by Crighton and Leppington [1974], and Morgan [1974]. They studied the problem of interaction of an acoustic source with a semi-infinite vortex sheet and reported no appreciable influence on the intensity of the radiated sound when the 'full Kutta condition' was imposed. Morgan found that there is no solution which satisfies the "full" Kutta condition for supersonic flows; an alternative "modified" Kutta condition is proposed to overcome this difficulty which is roughly that the solution must be as smooth as possible near the edge. The unique solution defined by this condition satisfies the full Kutta condition for subsonic flows, while for supersonic flows, the vortex sheet leaves the edge at an angle of less than 90°.

However, Howe [1976] has pointed out the reasons for these conflicting conclusions are the inadequacies in the mathematical modelling of the interaction of a real aerodynamic source with a trailing-edge. He examined a sequence of mathematical problems intended to model the mechanism by

which sound is generated as a turbulent eddy is convected in a mean flow past an airfoil, and has reported that the application of the 'full' Kutta condition leads to a complete cancellation of the sound generated when frozen turbulence convects past a semi-infinite plate, and to the cancellation of the diffraction field due to the trailing-edge in the case of an airfoil of compact chord. The cancellation is brought about by the shed vorticity, which smoothes out the flow in the vicinity of the trailing-edge. He claims that a 'full' Kutta condition of the type considered by Orszag and Crow is not relevant if there is no external flow.

Rienstra [1979] has studied three models concerning the interaction of a flow with diffracting sound waves at the trailing-edge. A uniform subsonic inviscid compressible flow on either, or only one, side of a semi-infinite thin plate flat plate; and in a semi-infinite thin-walled open tube. All cases are perturbed by a sound wave, as shown in Figure 34. He reports for the case of a flow on both sides of the plate that the application of a 'full' Kutta condition leads to an increase of the diffracted wave in a <u>downstream</u> arc and a decrease elsewhere; this effect is dependent upon the Mach number. Also he found that the effect of application of

the Kutta condition leads to a decrease of the diffracted outer field for the case of flow on one side of the plate. These agree well with the results of Howe [1976] and the experimental results of Heavens [1978]. The latter concluded that the diffracted wave is very weak when the flow at the trailing-edge is smooth i.e., where the Kutta condition is satisfied, as shown in Figure 35. On the other hand, the diffracted field is strongly visible when the Kutta condition is violated either by boundary layer separation or by unsteadiness in the flow; see for instance Figure 36 , and Heaven's Figures 3-5 and 7(a).

With regard to the experimental investigations of the trailing-edge condition of a nozzle in unsteady flow, Bechert and Pfizenmaier [1971] have pointed out that the 'full' Kutta condition is satisfied at low magnitudes of the fluctuating flow. Their experiments showed that no velocity singularity occurs at the trailing-edge of the nozzle at low reduced frequencies. In 1974, they pointed out the effect of the boundary-layer thickness at the nozzle discharge edge; at very low strouhal numbers S_θ (based on the momentum thickness of the boundary layer) the flow in the vicinity of the edge behaves like a steady flow, i.e., the 'full' Kutta condition is satisfied. While at high strouhal numbers S_θ 'no Kutta

condition' is to be expected. Moreover, they found, through experiments on the unsteady flow at a nozzle discharge edge, that the jet deflexion envelope has a nearly parabolic shape near the nozzle edge, i.e., the full Kutta condition is not satisfied, and the size of the 'parabolic' region decreased with decreasing strouhal number. But they mentioned that at low Strouhal number, the unsteady motion in the vicinity of the trailing-edge behaves linearly. Also they pointed out that a transition may occur from the 'full Kutta condition' to the 'no-Kutta condition' with increasing strouhal number. They proposed a 'mixed Kutta condition' obtained by a linear combination of both conditions for a sufficiently low strouhal number.

So, we can conclude that the trailing-edge condition of zero gradient or zero pressure difference at the trailing-edge is satisfied at low reduced frequency, where the acoustic field is very weak. As the reduced frequency increases, the smooth flow at the trailing-edge is disturbed, and the full Kutta condition is no longer satisfied. In this case, the trailing-edge vortex sheet has a parabolic shape, and the acoustic radiation field is strong.

C. Viscous Analysis With Attached Flow

All the previous investigations are an approximation to the real situation, especially when the flow remains attached until the trailing edge is reached. In this case, the viscous effects are confined to the laminar boundary layer on the airfoil surface and to the thin downstream wake. In fact, the inviscid problem is only the outer solution of a singular perturbation problem in which the Reynolds number tends to infinity. So, it is the consistency of the inner regions of the flow where viscosity has a significant effect, which may lead to unique determination of the inviscid solution. Gostelow [1975], in a discussion of the stability of the vortex shed from the trailing edge and the relation between the vortex street configuration and the trailing edge wedge angle, has shown that the partition streamline in Maskell's model is unstable if the Kutta condition (i.e. tangency of the streamline to the airfoil surface at the trailing edge) is violated. Consequently, the drag coefficient may be severely mispredicted, especially for blade cascades or airfoils with blunt trailing edges. He also proposed that the potential flow partition streamline should leave close to the trailing edge and normal to the orientation of the edge surface, and considered it as more meaningful than criterion of Maskell. The vortex street

drag in this case shows a strong dependence on the trailing edge, it tends to zero for a cusped trailing edge, as shown in Figure 37. In fact, it is the role of viscosity which makes Maskell's model inappropriate in predicting the drag coefficient. Also, it has been found that viscosity plays a central role in acoustic problems; it smooths the singularity in the flow field at the trailing edge by means of shedding of vorticity from the edge. These shed vortices change the total sound field because concentrations of vorticity moving near a solid edge generate sound (Crighton [1972-b]. This smoothing process is essentially a viscous effect, so we have to seek viscous models if we want a better understanding of this problem, which is related directly to noise generation problem. As mentioned before, in Chapter III, (Section C), for laminar flow at high Reynold number, the boundary layers are attached until the trailing edge is reached, but due to the change in boundary conditions there, the flow in the boundary layer accelerates when it passes the edge. This gives rise to a singularity in the flow field of the inviscid outer flow, which is smoothed out by the process in the inner viscous region. An understanding of the mechanisms of smoothing processes has been considerably deepened by the discovery of multi-boundary layer theory by Stewartson [1969] and Messiter [1970].

So, we review relevant works employing this procedure for unsteady flow around airfoils and aeroacoustics problems. In fact, this analysis gives some additional details about the condition at the trailing edge when a non-separating laminar boundary layer is considered in unsteady flows. Brown and Daniels [1975] extend the same theory discussed by Brown and Stewartson [1970] to the case of a flat plate oscillating in pitching or in plunging motion of small amplitude ($\alpha^*\ell$) or ($h^*\ell$) and of high frequency (ω^*) in a uniform incompressible flow, and in the limit as the Reynolds number tends to infinity. The same restrictions on the thickness of the airfoil imposed by Brown and Stewartson [1970], to ensure that the flow remains attached, were employed. Brown and Stewartson showed that, for oscillations of non-dimensional frequency $(S = \frac{\omega^* L}{U_a} = 0\ (\varepsilon^{-2}))$ and amplitude $(\alpha^* = 0(\varepsilon^{9/2}))$, the flow in the vicinity of the trailing edge on the upper side of the plate has a structure involving five distinct regions, as shown in Figure 38. Two additional layers called "the fore deck" of order $0\ (\varepsilon^2)$, which do not occur in steady flow, lie between the perturbed Blasius flow region and the triple deck. The full Kutta condition (i.e. zero pressure difference) leads to a consistent viscous flow field. In other words, it leads to an inner viscous flow region which matches uniformly to the

outer inviscid flow region without any singularities in the flow field. No complete solutions were obtained, but an estimate for the time-dependent viscosity correction to the circulation was made; the overall viscous effect correction to C_p was found to be of order $0\ (\varepsilon^4)$, and the contribution of the triple deck to the lift and pitching moment was of order $0\ (\varepsilon^5)$. Also, the viscous corrections to the lift and moment were found to lag the inviscid solutions (the leading-order terms) by an angle $\frac{\pi}{4}$. It emerges that there is a stagnation point of the outer flow at a distance of order $0\ (\varepsilon^7)$ from the trailing edge which moves from one side of the plate to the other with a phase lag of $\frac{\pi}{4}$ relative to the oscillation of the airfoil. In their analysis, in order to utilize the triple-deck structure for the trailing edges, they were obliged to scale the amplitude and reduced frequency to the order of magnitudes $\alpha^* = 0\ (\varepsilon^{9/2})$ and $S = 0\ (\varepsilon^{-2})$, in such way that the viscous effects due to the triple-deck at the trailing edge are balanced by the effects due to rapid oscillation of the airfoil. Shen and Crimi [1965] have pointed out the validity of the trailing edge condition of "zero pressure loading" at the trailing edge of an oscillating plate at high Reynold number. It holds as long as the flow is attached and flow separation is confined to the immediate vicinity of the trailing edge, i.e.,

the inviscid solution is really the limiting case as Reynold number tends to infinity (linear problem). This is still valid if the boundary layer flow is turbulent. However if extensive separation occurs, the unsteady trailing edge condition of zero pressure loading has no significance, because the boundary conditions change completely. That is, for the oscillating airfoils, involving problems of vortex shedding and acoustic radiation from the trailing edges, the failure of the inviscid analysis to give a unique solution and the singular behavior of the problem as Reynolds number tends to infinity suggests that the role of viscosity in the inner region renders the uniqueness. Daniels [1978] extended the works of Orszag and Crow [1970], taking viscous effects into account. A consistent viscous flow structure was established at the trailing edge, and the matching between the inviscid outer region of Orszag and Crow and the viscous inner region at the trailing edge led to uniqueness of the problem. For amplitudes of oscillation of order $O(\varepsilon^{7/2})$ and frequencies of order $O(\varepsilon^2)$ at large Reynolds number, it was found that application of the full Kutta condition of smooth flow at the trailing edge in the inviscid problem of Orszag and Crow leads to a consistent viscous flow field and predicts occurrence of flow separation for large amplitude oscillations. His results

also showed the inadequateness of the inviscid theory to determine the shape of the dividing streamline sufficiently close to the trailing edge. While the theory of Orszag and Crow suggests that, for the full Kutta condition, the dividing streamline leaves the trailing edge tangentially, the viscous flow structure, valid in the vicinity of the trailing edge, reveals oscillations having a parabolic amplitude envelope consistent with the experimental results of Bechert and Pfizenmaier [1975], who examined the exit condition of a weakly unsteady flow issuing from a circular nozzle. Also, the rectified or no Kutta conditions characterized by a parabolic oscillation of the vortex sheet at the trailing edge are found to be inconsistent for amplitudes of oscillation of the same order (i.e. $O(\varepsilon^{7/2})$); since they lead to solutions involving singularities of pressure within the triple-deck region. But for smaller amplitudes $O(\varepsilon^{13/2})$ their consistency appear to depend upon the existence of a solution of the full Navier-Stokes equations in a region of dimension $O(\varepsilon^{6})$ at the trailing edge (Daniels [1978]). The resulting viscous flow structure at the trailing edge is not similar to the structure on an oscillating plate determined by Brown and Daniels [1975], but more complicated, as shown in Figure 39. As previously shown in Section (B), the trailing edge condition has an

important effect on the noise generation. In these problems, the role of viscosity is accounted for in application of this condition. The effect of viscosity takes the form of vortex shedding from the trailing edge, which smooths the singularities in the acoustic flow field. It was shown before that the Kutta condition problem can be identified with the balance between that flow-induced pressure and the externally generated pressure perturbations (i.e. diffracting sound waves). When the pressure of the full Kutta condition solution is of the same order of magnitude near the edge as the flow-induced pressure, the viscous smoothing forces, prepared for the flow-induced pressure singularity, takes care of both and the Kutta condition is valid. Also, this is the case when the externally generated pressure is much lower (Daniels, [1978]). However, when the external pressure dominates the flow-induced pressure, separation is likely to occur and the Kutta condition is violated. The sound pressure singularity may be helped to overcome the smoothing forces (effect of viscosity) by another external effect (e.g. a plate at incidence, or a wedge-shaped edge) capable of generating a singularity at the edge. The two singularities cause violation of the Kutta condition earlier than if only one external singularity-inducing process were in action.

Rienstra [1979] has extended his analysis of the trailing edge influence on the interaction of a flow with diffracting sound waves to include viscous effects. Making use of the work of Brown [1975] and Daniels [1978], he derived an outer field correction, due to the viscous interaction at the trailing edge, for high Reynolds numbers; this viscous correction was small. This leads to the conclusion that the assumption of the Kutta condition is consistent with triple deck structure. It appears that an incident pressure wave with a dimensionless amplitude of order $O(\varepsilon^{\frac{1}{2}})$ and frequency of order $O(\varepsilon^{-2})$ almost satisfies the Kutta condition; a multiple of the singular eigensolution with an amplitude of order $O(\varepsilon^{3})$ is to be added. It is conjectured that when a pressure wave satisfies the Kutta condition, it behaves near the edge according to $P = \text{const.} + A(\omega,M)\sqrt{r}\,\exp(i\omega t)$, where $A = O(\varepsilon^{\frac{1}{2}})$.

(D) Viscous Analysis Associated With Boundary Layer Separation

All of the above analysis deal with flow that separates at the trailing-edge, which is an idealization of the actual flow dynamics, especially when the trailing-edge is not sharp and operating at high frequency. In this situation, the trailing-edge is, of course, buried in a turbulent flow which is often separated as

well, with a significant trailing edge loading and highly deformed wake. Figure 25 contrasts the actual flow dynamics with the idealized one. In fact, this situation is of special interest because it relates to the problem of noise generation from the edge; as mentioned before, the flow unsteadiness and boundary-layer separation at the trailing edge give rise to a strong diffracted wave e.g., figures (7a, 10b) of Heavens [1978]. This implies that the flow field in this situation is nonlinear; deviations from the linearized airfoil theory, especially at high reduced frequency, have been reported by many investigators as mentioned before, including Archibald [1975], Davis [1976], Satyanarayana [1977, 1978], Fleeter [1979], and Kadlec and Davis [1979]. The latter examined the structure of the near wake behind a pitching airfoil at amplitude ratios 0.02 and 0.4 of the airfoil chord, at a reduced frequency range of 1 to 10, and in the Reynolds number range 0.54×10^5 to 1.66×10^5. As shown in Figure 40, at a small value of reduced frequency (k=1) the wake distortion is small and the assumption of small disturbance theory (linear airfoil theory) that the wake elements coincide with airfoil chord line is valid, as in Figure 40a. But, as the reduced frequency increases, the wake distortion is greater (Figure 40b) and the larger trailing-edge velocity ratios indicate that the limits of the linear theory have already been exceeded. Figure 40c shows the case at very high reduced frequency; the wake becomes unstable

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and highly deformed into a vortex-like disturbance, linear theory failing to match it.

Sears [1976] has pointed out the condition which determines the circulation in the case of flow with boundary layer separation is such that "the net rate of vorticity transport at separation into the wake should be equal to the rate of change of the circulation around the airfoil", which is a generalization of the Howarth criterion, i.e.

$$\int_0^\delta \zeta\, U_{rel}\, dy = -\, d\Gamma/dt$$

which can be written in the form

$$\left[\tfrac{1}{2}U_1^2 - U_{sep}U_1\right]_B^A = -\, d\Gamma/dt$$

where U_{rel} denotes the difference $(U-U_{sep})$ and $[\ ]_B^A$ denotes the difference between values at points A and B of the expression in the brackets (see Figure 41), and denotes vorticity, positive clockwise. For the case of an airfoil with a rounded trailing edge, he proposed dual models for calculating the flow field, forces, and moments without developing actual solutions. A vortex-sheet model in the spirit of thin airfoil theory suffices for the calculation of pressure, lift, moment, etc. (Figure 42a). The circulation of this model has to be determined from a second

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model; in boundary-layer calculations, more details about the actual contour of the airfoil and its stagnation points are required (Figure 42b). In fact, as he mentioned, the flow field of each model depends upon the other; the flow field disturbances calculated in the first model are carried over into the second model and the circulation is determined by the generalized Howarth criterion, i.e., the two flows would have to be calculated iteratively. The lift is not equal to U, but is equal to

$$L(t) = \rho U \Gamma(t) - \rho \frac{d}{dt} \int_{L.E.}^{T.E.} \gamma\,(x,t)\; x dx$$

and the pitching moment about the trailing edge is

$$M_{T.E.} = \rho U \int_{L.E.}^{T.E.} \gamma(x,t)\; x dx - \frac{1}{2} \rho \frac{d}{dt} \int_{L.E.}^{T.E.} \gamma(x,t)\; x^2 dx$$

where x is measured (positive downstream) from the trailing edge. Figure 42b shows a comparison between linear airfoil theory and this dual model.

We cannot make any conclusion about the accuracy of this dual model since it has not been tested yet. Further works using this model are needed.

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Wu [1981] has developed a general theory for the aerodynamic force and moment through a rigorous analysis of the incompressible viscous equations. The main feature of this theory is the generalization of the formulas which relate aerodynamic force and moment acting on one or more solid bodies to rates of change of vorticity moments in the fluid and the solid regions; e.g., the aerodynamic force F, exerted by the fluid on N-solid bodies is:

$$F = F_L - \frac{d}{dt}\int_{RL} \rho \underline{v} dR + \sum_{j=1}^{N} \frac{d}{dt}\int_{R_j} \rho \underline{v} dR$$

where $\underline{v}$ is the velocity vector on the boundary B_L, F_L is the force acting on the boundary B_L, and R_L is the control volume bounded externally by B_L. For two-dimensional steady viscous flow about an airfoil, the lift is obtained from this general equation which can be simplified to:

$$L = \rho \frac{d}{dt}\int_{R_f}\int x\omega dxdy$$

where R_f is the fluid region in the control volume R_L, and is the vorticity vector, which is identical to the well-known Kutta-Joukowski theory. It seems that this new approach will give a new dimension in understanding and interpreting complex

aerodynamic phenomena and in computational fluid dynamics. With regard to this analysis of the trailing-edge flow separation, this author feels that this new procedure is fruitful, since it gives a precise definition of the circulation about two-dimensional solid boundaries for both unseparated flow and a flow containing an appreciable region of separation; also, there are not simplifying assumptions or approximations in deriving the general formulas of the theory. This would appear to make it more accuate than the Sears dual model.

V. CONCLUSION AND RECOMMENDATIONS

This review is a critical assessment of the existing works on the trailing-edge condition.

The works of Kutta and Joukowski and the other related interpretations have been reviewed. For cases where the airfoil has a sharp trailing-edge, there is no contention that the Kutta-Joukowski condition is satisfied. All interpretations discussed herein, in fact, are essentially identical and give good agreement with the experimental results.

It appears that for most blade cascades or isolated airfoils having a blunt trailing-edge, the Kutta-Joukowski condition has no relevance. For this class of trailing-edges, the Taylor-Howarth criterion of "zero total flux vorticity into the wake" is found to be the appropriate trailing-edge condition, and gives a unique flow solution. Also, details are presented on the role of viscosity in smoothing the flow field at trailing-edge via multistructure boundary layer theory.

For unsteady flow analysis, the situation is more complicated. However, it has been shown that the trailing-edge condition of "zero pressure loading at the trailing-edge" is the appropriate condition as long as the flow remains attached until the trailing-edge, provided the reduced frequency is low. In this case, the acoustic radiation field is very

weak. However, violation of this condition is pointed out by many investigators for cases where the flow separates and the unsteadiness is high, accompanied by strong acoustic radiation from the trailing-edge.

It seems that further efforts in this area are called for; the following possibilities are recommended:

(a) Further experimental studies covering a range of reduced frequencies and angles of attacks are needed to guide new theoretical analyses.

(b) The multistructure boundary layer theory should be extended as to gain an understanding of the trailing edge flow structure, especially when flow separates.

(c) Critical testing of these new approaches is needed, in order to define limits of their applicability.

REFERENCES

Archibald, F.S., 1975, "Unsteady Kutta Condition at High Values of the Reduced Frequency Parameters", J. Aircraft, Vol. 12, June.

Baskaran, V., and Holla, V.S., January 1981, "Effect of Rear Stagnation Point Position and Trailing Edge Bluntness on Air-Foil Characteristics", J. Aircraft, Vol. 18, No. 1, pp. 63-64.

Basu, B.C., 1973, "A Numerical Solution for a Steady Two-Dimensional Aerofoil in Incompressible Viscous Flow", QMC-EP 1008, August.

Basu, B.C., and Hancock, G.J., 1978, "The Unsteady Motion of a Two-Dimensional Aerofoil in Incompressible Inviscid Flow", J. Fluid Mech., Vol. 87, Part 1, pp. 159-178.

Batchelor, G.K., 1970, "An Introduction to Fluid Dynamics, Cambridge University Press, pp. 435.

Bechert, D. and Pfizenmaier, E., 1971, "On the Kutta Condition at the Nozzle Discharge Edge in a Weakly Unsteady Nozzle Flow [in German]. D.L.R. Rep. FB 71-89. (Trans. R.A.E. Lib. Trans. No. 1617).

Bechert, D. and Pfizenmaier, E., 1975, "Optical Compensation Measurements on the Unsteady Exit Condition at a Nozzle Discharge Edge", J. Fluid Mech., 71, 123-144.

Blasius, H., 1908, Z. Math. Phys. 56, 1.

Bradshow, P., Ferris, D.H. and Atwell, N.P., 1967, "Calculation of Boundary Layer Development Using the Turbulent Energy Equation", J. Fluid Mech., 28, pp. 593.

Brown, S.N. and Daniels, P.G., 1975, "On the Viscous Flow About the Trailing Edge of a Rapidly Oscillating Plate", J. Fluid Mech., 67, pp. 743-761.

Brown, S.N. and Stewartson, K., 1970, "Trailing-Edge Stall", J. Fluid Mech., 42, pp. 561-584.

Commerford, G.L. and Carta, F.O., 1974, "Unsteady Aerodynamics Response of a Two-Dimensional Airfoil at High Reduced Frequency", AIAA J., Vol. 12, Jan., pp. 43-48.

Crighton, D.G., 1972a, "Radiation Properties of the Semi-Infinite Vortex Sheet", Proc. R. Soc. Lond. A 330, pp. 185-198.

Crighton, D.G., 1972b, "The Excess Noise Field of Subsonic Jets", J. Fluid Mech., 56, pp. 683-694.

Crighton, D.G., and Leppington, F.G., 1970, J. Fluid Mech., 43, pp. 721.

Crighton, D.G., and Leppington, F.G., 1974, "Radiation Properties of the Semi-Infinite Vortex Sheet: The Initial Value Problem", J. Fluid Mech., 64, pp. 393-414.

Daniels, P.G., 1977, "Viscous Mixing at a Trailing Edge, Q. Jl. Mech. Appl. Math., 30, pp. 319-342.

Daniels, P.G., 1978, "On the Unsteady Kutta Condition", Q. Jl. Mech. Appl. Math., 31, pp. 49-75.

Davies, S.S., 1975, "Theory of Discrete Vortex Noise", AIAA J., pp. 375-380.

Davies, S.S., 1976, "Measurements of Discrete Vortex Noise in a Closed-Throat Wind Tunnel", AIAA Progress in Astronautics and Aeronautics-Aeroactoustics: STOL Noise; Airframe and Airfoil Noise, Vol. 45, edited by I.R. Schwartz, pp. 237-257.

Dennis, S.C.R. and Chang, G.Z., 1969, "Numerical Integration of the Navier-Stokes Equations for Steady Two-Dim. Flow", Phys. Fluids, Suppl. II, pp. 88.

Dennis, S.C.R. and Dunwoody, J., 1966, "The Steady Flow of a Viscous Fluid Past a Flat Plate", J. Fluid Mech., 24, pp. 577.

Ffowcs Williams, J.E. and Hall, L.H., 1970, "Aerodynamics Sound Generation by Turbulent Flow in the Vicinity of a Scattering Half Plane", J. Fluid Mech., 40, pp. 657-670.

Fleeter, S., 1973, "Fluctuating Lift and Moment Calculations for Cascaded Airfoils in a Nonuniform Compressible Flow", J. Aircraft, Vol. 10, Feb., pp. 93-98.

Fleeter, S., 1980, "Trailing Edge Conditions for Unsteady Flows at High Reduced Frequency", AIAA J., Vol. 18, No. 5, May.

Fujita, H. and Kovasznay, L.S.G., "Unsteady Response of an Airfoil to Wake Cutting", AIAA J., Vol. 12, Sept.

Giesing, J.P., 1968, "Non-Linear Two-Dimensional Unsteady Potential Flow With Lift", J. Aircraft, 5 (2), pp. 135.

Giesing, J.P., 1969, "Vorticity and Kutta Condition for Unsteady Multi-Energy Flows", Trans. A.S.M.E., J. Appl. Mech., 91, 608.

Glauert, H., 1947, "The Elements of Aerofoil and Aircrew Theory", 2nd Edn. Cambridge University Press.

Goldstein, S., 1930, "Concerning Some Solutions of the Boundary Layer Equations in Hydrodynamics", Proc. Camb. Soc., 26, pp. 1-30.

Goldstein, S., 1965, "Modern Developments in Fluid Dynamics", Dover Pub., Vol. I.

Gostelow, J.P., 1964-a, "Potential Flow Through Cascades: A Comparison Between Exact and Approximate Solutions", A.R.C., CP No., pp. 807.

Gostelow, J.P., 1964-b, "Potential Flow Through Cascades: Extensions to an Exact Theory", A.R.C. CP No., pp. 808.

Gostelow, J.P., 1975, "Trailing Edge Flow Over Turbomachine Blades and the Kutta-Joukowsky Condition", ASME Paper 75-GT-94, March.

Green, J.E., 1972, "Application of Head's Entrainment Method to the Prediction of Turbulent Boundary Layers and Wakes in Compressible Flow", RAE Technical Report 72079.

Gredianus, J.H., Van de Vooren, A.I., and Bergh, H., 1952, "Experimental Determination of the Aerodynamics Coefficients of an Oscillating Wing in Incompressible, Two-Dim. Flow", Parts I-IV, Depts. F. 101-104, National Luchvaart-Laboratorium, Amsterdam, The Netherlands.

Hakkinen, R.J. and O'Neil, E., 1967, Rep. DAC-60862, Los Angeles, Calif.: Douglas Aircraft Co.

Hancock, G.J. and Padfield, G., 1972, "Numerical Solution for Steady Two-Dimensional Aerofoils in Incompressible Flow", Queen Mary Coll. Rep. EP-1003.

Hancock, G.J., 1976, "Computational Methods and Problems in Aeronautical Fluid Dynamics", Academic Press, pp. 214.

Heavens, S.N., 1978, "An Experimental Study of Sound Diffraction at an Airfoil Trailing Edge", J. Fluid Mech., Vol. 84, Part 2, pp. 331.

Helmholtz, H. Von, 1968, Phil. Mag., Vol. 36, pp. 185.

Hess, J.L. and Smith, A.M.O., 1967, "Calculation of Potential Flow About Arbitrary Bodies", Prog. Aero. Sci., Vol. 8, pp. 1

Holmes, D.W., 1972, "Lift Fluctuations on Aerofoils in Transverse and Streamwise Gusts", Ph.D. Dissertation, Engineering Dept., Univ. of Cambridge.

Horlock, J.H., 1968, "Unsteady Flow in Turbomachines", Paper No. 2674, Third Australasian Conference on Hydraulics and Fluid Mech., Sydney, 25-29 Nov.

Horton, H.P., 1969, "Entrainment in Equilibrium and Non-Equilibrium Turbulent Boundary Layers", H.S.A. Hatfield/Research/1094/HPH (August 1969).

Howarth, L., 1935, "The Theoretical Determination of the Lift Coefficient for a Thin Elliptic Cylinder", Proc. R. Soc. Lond., Vol. 149A, pp. 558-586.

Howe, M.S., 1976, "The Influence of Vortex Shedding on the Generation of Sound by Convected Turbulence", J. Fluid Mech., 76, 711-740.

Howe, M.S., 1978, "Attenuation of Sound in a Low Mach Number Nozzle Flow", Bolt, Beranek and Newman Inc. Report No. 3762; also J. Fluid Mech., 91, 209-229, 1979.

Imai, I., 1957, "Second Approximation to the Laminar Boundary-Layer Flow Over a Flat Plate", J. Aero. Sci., Vol. 24, 155-156.

Jeffers, J.D. and Meece, C.E., 1975, "F100 Fan Stall Flutter Problem Review and Solution", J. Aircraft, Vol. 12, No. 4 April, pp. 350-357.

Jones, D.S., 1972, "Aerodynamic Sound Due to a Source Near a Half-Plane", J. Inst. Maths Applics, 9, pp. 114-122.

Jones, W.H., et. al., 1977, "Experimental Apparatus for Investigation of Fan Aeroelastic Instabilities in Turbomachinery", NASA TM X-3508, NASA-Lewis Research Center, Cleveland, June.

Jones, D.S. and Morgan, J.D., 1974. "A Linear Model of a Finite Amplitude Helmholtz Instability", Proc. R. Soc. Lond. A, 338, pp. 17-41.

Joukowsky, N.E., Collected Works, (Vol. 6) (in Russian), OGIZ, State Publ. House of Tech. Theo. Lit., Moscow, (1950).

Kadlec, R.A., and Davis, S., 1979, "Visualization of Quasi-periodic Flows", AIAA J.; Vol. 17, No. 11, November.

Karman, V. Th., and Sears, W.R., 1938, "Airfoil Theory for Non-Uniform Motion, J. Aero. Sci., Vol. 5, Aug., pp. 379-390.

Kerrebrock, J.L., "Assessment, Program Definition and Research in Turbomachine Aeroelasticity", Final Report (unpublished) NASA-NSG 3006, Massachusetts Institute of Technology, April-Dec. 1974.

Karamcheti, 1966, Principles of Ideal-Fluid Aerodynamics, John Wiley and Sons.

Kutta, W.M., 1902, "Lift Forces in Fluid Flow", Illustrierte Aeronautische Mitteilungen, p. 133.

Kuo, Y.H., 1953, "On the Flow of an Incompressible Viscous Past a Flat Plate at Moderate Reynolds Numbers", J. Math. and Phys., Vol. 32, pp. 83-101.

Lamb, H., 1945, Hydrodynamics, 6th ed., Dover Publications, New York.

Robinson, A., and Laurmann, J.A., 1956, Wing Theory, Cambridge Univ. Press.

Martensen, E., 1971, "The Calculation of the Pressure Distribution on a Cascade of Thick Airfoils by Means of Fredholm Integral Equations of the Second Kind", NASA TT F-702, July.

Maskell, E.C., 1973, "On the Kutta-Joukowski Condition in Two-Dimensional Unsteady Flow", ARC 33967, FM 4327; RAE Tech. Memo. 1451 (1973).

McCroskey, W.J., 1977, "Some Current Research in Unsteady Fluid Dynamics", Trans. ASME J. Fluids Engineering, Vol. 99, Series I, No. 1, March, pp. 8-39.

Messiter, A.F., 1970, "Boundary-Layer Flow Near the Trailing Edge of a Flat Plate", SIAM J. Appl. Math., Vol. 18, pp. 241-257.

Miller, M.J., 1973, "Some Aspects of Deviation Angle Estimation for Axial-Flow Compressors", Ph.D. Dissertation, Iowa State Univ.

Morgan, J.D., 1974, "The Interaction of Sound With a Semi-Infinite Vortex Sheet", Q. Jl. Mech. Appl. Math., 27, 465-487.

Morgan, J.D., 1975, "The Interaction of Sound With a Subsonic Cylindrical Vortex Layer", Proc. R. Soc. Lond. A, 344, pp. 341-362.

Munt, R.M., 1977, "The Interaction of Sound With a Subsonic Jet Issuing From a Semi-Infinite Cylindrical Pipe, J. Fluid Mech., Vol. 83, pp. 609-640.

Naumann, H., and Yeh, H., 1972, "Lift and Pressure Fluctuations of a Cambered Airfoil Under Periodic Gusts and Applications in Turbomachinery", Trans. ASME, J. Engr. for Power, Paper No. 72-GT.30.

Ni, R.H. and Sisto, F., 1976, "Numerical Computation of Non-stationary Aerodynamics of Flat Plate Cascades in Compressible Flow", Trans. ASME J. Engineering for Power, Series A, Vol. 98, No. 2, pp. 165-170.

Nieberding, W.C. and Pollack, J.L., 1977, "Optical Detection of Blade Flutter", ASME Paper No. 77-GT-66, Gas Turbine Conference and Products Show, 1977.

Orszag, S.A., and Crow, S.C., 1970, "Instability of a Vortex Sheet Leaving a Semi-Infinite Plate", Stud. Appl. Math., Vol. 49, No. 2, pp. 167.

Ostdiek, F.R., 1975, "A Cascade in Unsteady Flow", AGARD CP 177 Unsteady Phenomena in Turbomachinery.

Piercy, N.A.V., Preston, J.H., and Whitehead, L.G., 1938, "The Approximate Prediction of Skin Friction and Lift", Phil. Mag., XXVI, pp. 791.

Prandtl and Tietjens, 1934, Fundamental of Hydro and Aerodynamics, Dover Pub.

Riegels, F. (Editor), 1961, Ludwig Praneltl-Gesammelte Abhandlungen, Vol. II, Springer Verlog, pp. 752.

Preston, J.H., May, 1943, "The Approximate Calculation of the Lift of Symmetrical Aerofoils Taking Account of the Boundary Layer, With Application to Control Problems, R. and M. 1996.

Preston, J.H., 1945, "The Effect of the Boundary Layer and Wake in the Flow Past a Symmetrical Aerofoil at Zero Incidence", R and M 2107, July.

Preston, J.H., 1949, "The Calculation of Lift Taking Account of the Boundary Layer", R. and M. 2725, November.

Pratt and Whitney Aircraft, 1976, "Analytical and Experimental Study of Subsonic Stalled Flutter", AFSOR TR-76-0829 (PWA Internal Report No. PWA-5420), Washington: United States Air Force Office of Scientific Research, May.

Rienstra, S.W., 1979, "Edge Influence on the Response of Shear Layers to Acoustic Forcing", Ph.D. Thesis, University of Eindhoven.

Riley, N., and Stewartson, K., 1969, "Trailing Edge Flows", J. Fluid Mech., Vol. 39, Part 1, pp. 193-207.

Sabatiuk, A. and Sisto, F., 1956, "A Survey of Aerodynamic Excitation Problems in Turbomachines", Transactions of the ASME, Paper No. 55-SA-40, April, pp. 555-564.

Satyanarayana, B., 1977, "Unsteady Wake Measurements of Airfoils and Cascades", AIAA J., Vol. 15, No. 5, May, pp. 613-618.

Satyanarayana, B., and Davis, S., 1978, "Experimental Studies of Unsteady Trailing Edge Conditions", AIAA J., Vol. 16, No. 2, Feb.

Schlichting, H., 1955, "Berechnung det Reibungslosen Inkompressiblen Stromung fur ein Vorgege Benes Ebenes Schaufelgitter", V.D.I. Forschungsheft 447.

Sears, W.R., 1954, "Some Recent Developments in Airfoil Theory", J. Aero. Sci., Vol. 23, No. 5, pp. 490-498.

Sears, W.R., 1976, "Unsteady Motion of Airfoils With Boundary Layer Separation", AIAA J., Vol. 14, No. 2, pp. 613-618.

Shen, S.F., and Crimi, P., 1965, "The Theory for an Oscillating Thin Airfoil as Derived From the Oseen Equations", J. Fluid Mech., Vol. 23, Part 3, pp. 586-609.

Spence, D.A., 1954, "Prediction of the Characteristics of Two-Dimensional Airfoils", J. Aero Sci., Vol. 21, No. 9, Sep., pp. 577-587.

Spence, D.A., 1970, "Wake Curvature and the Kutta Condition", J. Fluid Mech., Vol. 44, Part 4, pp. 625-638.

Spence, D.A., and Beasley, J.A., 1960, Aero. Res. Counc. R. and M. 3137.

Stewartson, K., 1969, "On the Flow Near the Trailing of a Flat Plate II, Mathematika, Vol. 16, pp. 106-121.

Stewartson, K., 1974, "Multistructured Boundary Layers on Flat Plates and Related Bodies", Adv. Appl. Mechs., Vol. 14, pp. 145.

Taylor, G.T., 1925, Phil. Trans., A, Vol. 225, pp. 238.

Theodorsen, T., 1935, "General Theory of Aerodynamic Instability and the Mechanism of Flutter", NACA Report 496.

Thwaites, B., 1949, "Approximate Calculations of the Laminar Boundary Layer", Aero. Q., 1, pp. 245.

Thwaites, B., 1960, Incompressible Aerodynamics, Clarendon Press, Oxford.

Tijdeman, H., and Seebass, R., 1980, "Transonic Flow Past Oscillating Airfoils", Ann. Rev. Fluid Mech., Vol. 12, pp. 181-222.

Ting, L., 1959, J. Math. Phys., Vol. 38, pp. 153

Tsien, H.S., 1943, "Symmetrical Joukowsky Airfoils in Shear Flow", Q. Appl. Math., Vol. 1, No. 2.

Vooren, A.I. van der and Vel, H. van der, 1964, "Unsteady Profile Theory in Incompressible Flow", Arch. Mech. Stosowanej 3.

Van Dyke, M., 1975, Perturbation Methods in Fluid Mechanics, The Parabolic Press.

Whitehead, D.S., 1960, "Force and Moment Coefficients for Vibrating Aerofoils in Cascade", Aeronautical Research Council, R & M 3254.

Whitehead, D.S., 1973, "Note on the Trailing Edge Condition in Unsteady Flow", CUED/A-Turbo/TR44.

Wu, J.C., 1981, "Theory for Aerodynamics-Force and Moment in Viscous Flows", AIAA J., Vol. 19, No. 4, pp. 432-441.

Yates, J.E., 1978, "Viscous Thin Airfoil Theory and the Kutta Condition", AIAA 16th. Aerospace Sciences Meeting, January, Paper No. 78-162.

LIST OF FIGURES

Figure 1: Theoretical flow pattern of a flat plate in steady incompressible potential flow.

Figure 2: The flow around the trailing-edge which can be interpreted as a sharp corner with $\alpha > \pi$.

Figure 3: Sketch showing the development of the circulatory flow around an airfoil up to the Kutta-conditions flow.

Figure 4: Experimental interpretation of the development of the flow starting from rest up to smooth flow (Prandtl and Tietjens [1934]).

Figure 5: Flow past an airfoil as viewed from a space-fixed reference frame: (a) the starting vortex and the formation of the circulatory flow over the airfoil immediately after starting the airfoil; (b) the decay of the circulatory flow when the airfoil is stopped (Prandtl and Tietjens [1934]).

Figure 6: Trailing-edge condition for steady inviscid potential flow around airfoil with trailing-edge: (a) trailing-edge with a finite angle; (b) cusped trailing-edge.

Figure 7: Hess and Smith's interpretation of the trailing-edge condition (Hess and Smith [1967]).

Figure 8: Trailing-edge detail for blade having a rounded trailing-edge (Gostelow [1964]).

Figure 9: Effect of variation of rear stagnation point on pressure distribution calculated by exact potential theory (Gostelow [1964]).

Figure 10: Effect of the rear stagnation point position and trailing-edge bluntness on airfoil characteristics (Baskaran and Holla [1981]).

Figure 11: Separation points in fully laminar flow (Howarth [1935]).

C - 2

Figure 12: Boundary layer configuration at the trailing-edge in laminar flow.

Figure 13: Mathematical model allowing for the boundary layer and wake displacement effects (Basu [1973]).

Figure 14: Comparison between viscous flow solution of Basu and the experimental results.

Figure 15: Predicted pressure distribution by Spence's method (Spence [1954]).

Figure 16: First viscous approximation (Gostelow [1975]).

Figure 17: The triple-deck structure in the trailing-edge region (Stewartson [1969], and Messiter [1970]).

Figure 18: The trailing-edge flow structure for mixing shear layer for: (a) stagnant flow below the plate, i.e. $U_2 = 0$; (b) the case for the flow over a backward-facing step (Daniels [1977]).

Figure 19: Compressor map showing flutter boundaries of four types of flutter (Pratt and Whitney Aircraft [1976]).

Figure 20: Full scale engine research (NASA-LERC [1979]).

Figure 21: PES display during quiet running condition (left) and during flutter (right) (Nieberding and Pollack [1977]).

Figure 22: Flow around an oscillating airfoil with vorticity shed from the trailing-edge region.

Figure 23: Unsteady pressure signals and overall loads on an oscillating airfoil: (a) subsonic flow (linear-flow); (b) transonic flow (nonlinear flow) (Tijdeman [1980]).

Figure 24: Wake patterns for harmonically oscillating airfoil model (Theodorsen [1935]).

Figure 25: Trailing-edge condition for unsteady flow (McCroskey [1977]).

Figure 26: Various interpretations of the unsteady trailing-edge condition.

Figure 27: Comparison between the computed wake pattern for an oscillating airfoil at high frequency and the experimentally observed wake pattern.

Figure 28: The theoretical characteristics of the wake element from the trailing-edge for a 8.4% thick symmetrical Von Mises airfoil oscillating about the leading edge (Basu and Hancock [1978]).

Figure 29: Experiments and prediction for the pressure differential due to an incident gust (Satyanarayana [1978]).

Figure 30: Variation of unsteady pressure vs reduced frequency due to oscillating airfoil (Satyanarayana [1978]).

Figure 31: Instantaneous pressure traces from isolated airfoil exposed to a sinusoidal transverse gust due Holmes (Gostelow [1975]).

Figure 32: Experimental data for the unsteady surface pressure coefficient vs. reduced frequency, due to an incident gust (Fleeter [1979]).

Figure 33: Trailing-edge condition for a semi-infinite plate according to Orszag and Crow [1970]: (a) 'No' Kutta condition; (b) 'Rectified' Kutta condition; (c) 'Full' Kutta condition.

Figure 34: The interaction of sound with a trailing-edge flow in three different problems: (a) a semi-infinite flat plate in an inviscid compressible fluid flowing on both sides; (b) as in case (a), but, the fluid flows on one side; (c) compressible inviscid jet flow enamating from a semi-infinite pipe (Rienstra [1979]).

Figure 35: The "full Kutta condition" flow, where the interaction between the diffracted wave and flow at the trailing-edge is weak (Heavens [1978]).

Figure 36: The "no Kutta condition flow", where the interaction between the diffracted wave and trailing-edge flow is strong (Heavens [1978]).

Figure 37: Vortex shedding models and drag coefficient for wedge-shaped airfoils (Gostelow [1975]).

Figure 38: Trailing-edge flow structure on the upper side of a rapidly oscillating plate (Brown and Daniels [1975]).

Figure 39: Trailing-edge flow structure in unsteady flow past a plate (Daniels [1978]).

Figure 40: Near wake behind an oscillating airfoil at different Reynolds numbers and reduced frequencies (Kadlec and Davis [1979]).

Figure 41: Airfoil in unsteady motion with vortical wakes produced at boundary-layer separation points A and B (Sears [1976]).

Figure 42: Dual method for predicting the unsteady aerodynamic forces on airfoils with rounded trailing-edges (Sears [1976]): (a) bound and free vortex; (b) model for boundary-layer calculations; (c) velocity component u at upper and lower surface of vortex sheets, and corresponding vortex strength γ.

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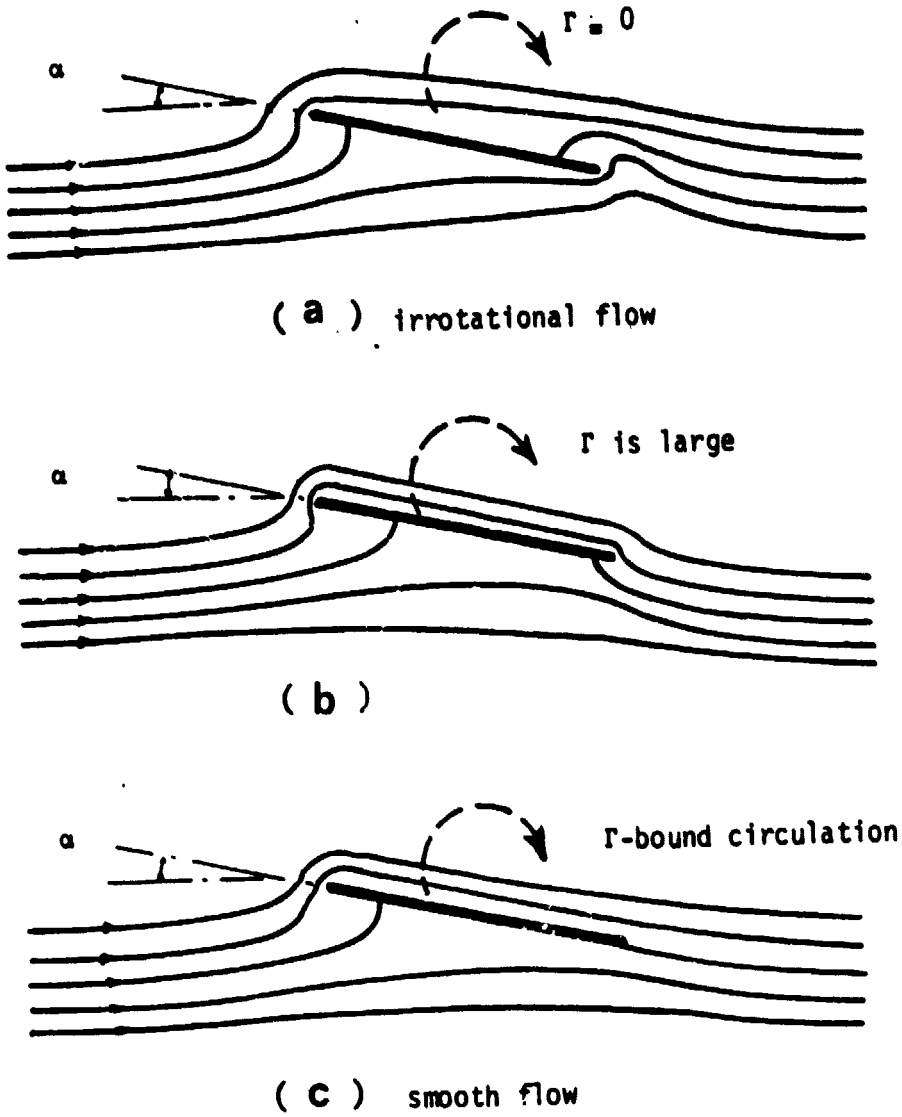

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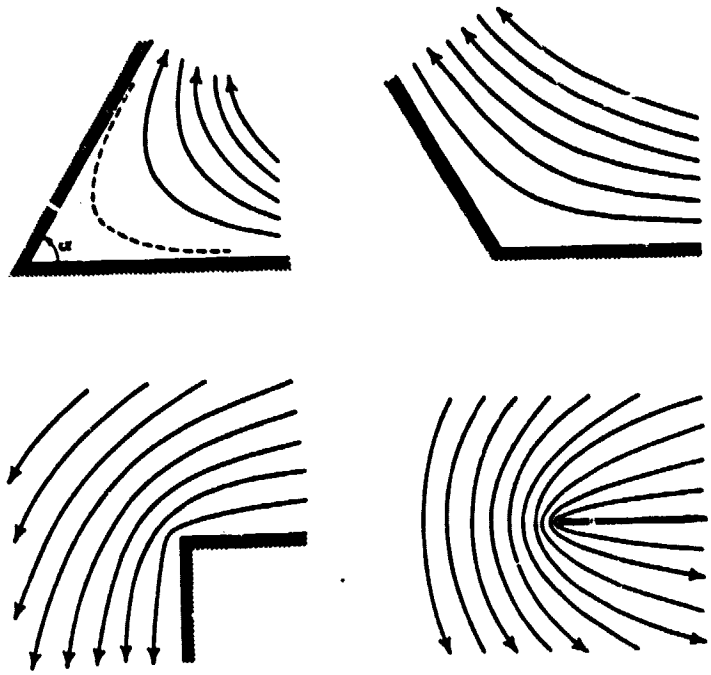

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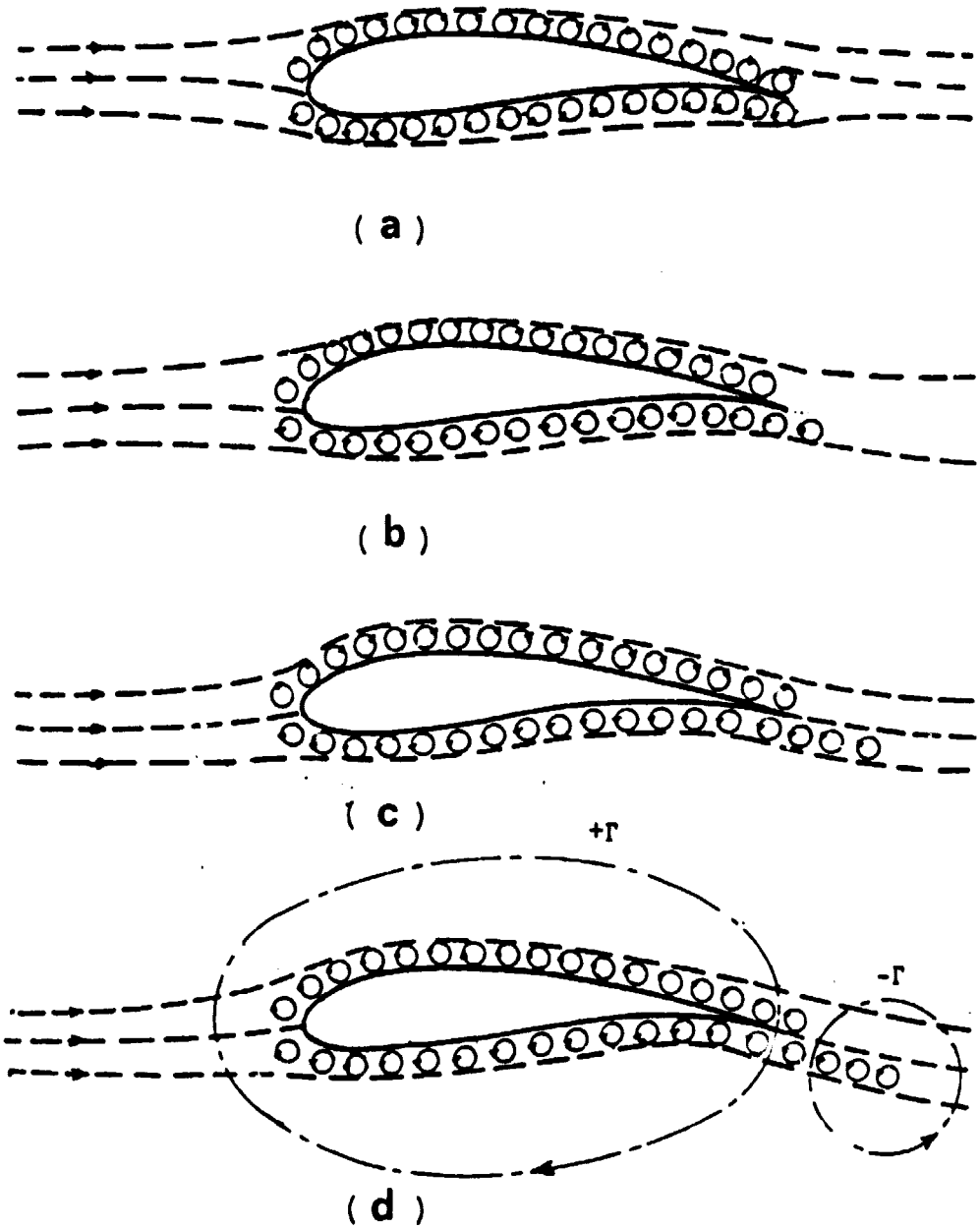

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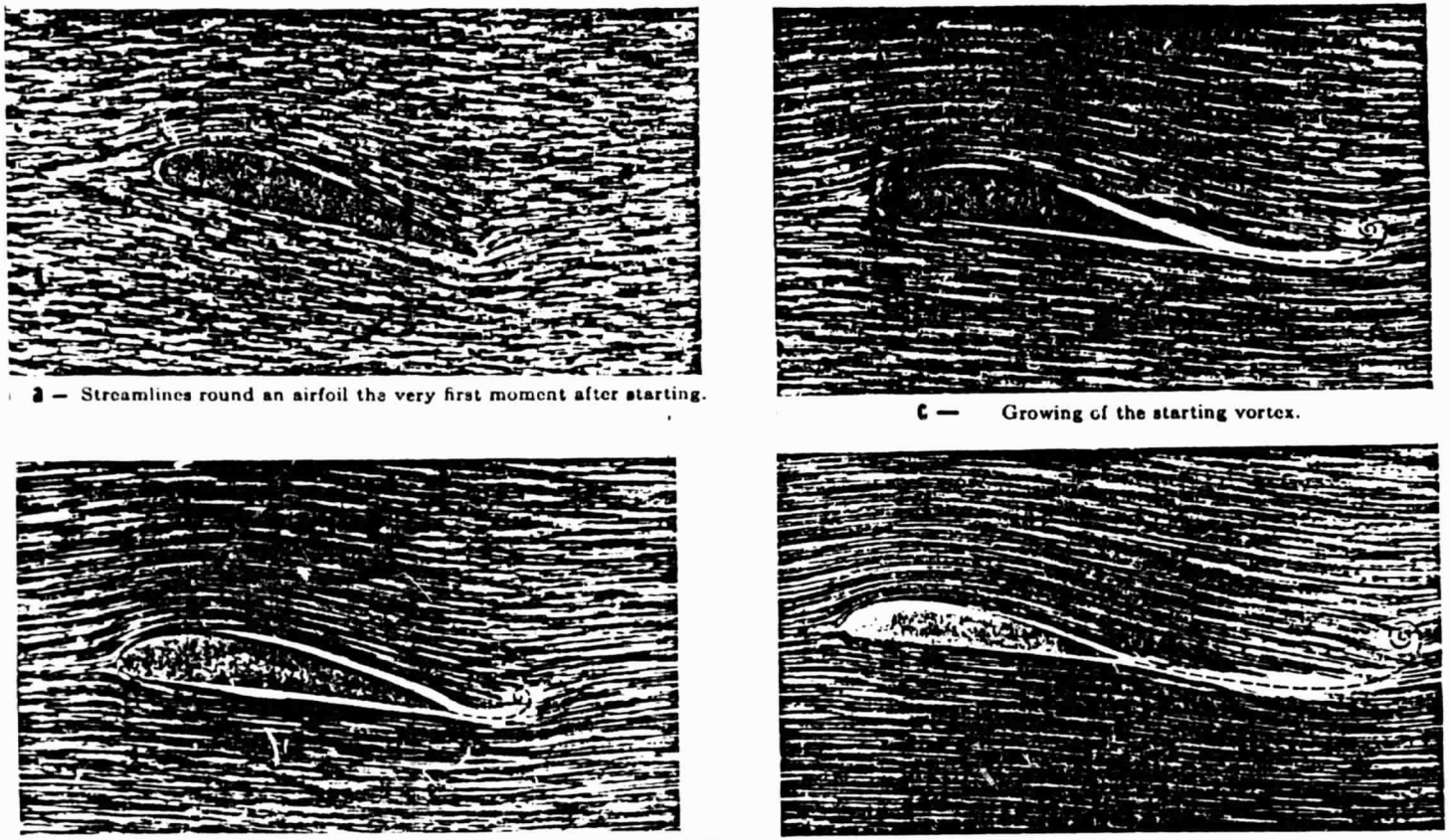

a — Streamlines round an airfoil the very first moment after starting.

b — Formation of the starting vortex which is washed away with the fluid.

c — Growing of the starting vortex.

d — Taken somewhat later than Fig. 50.

Figure 4: Experimental interpretation of the development of the flow starting from rest up to smooth flow (Prandtl and Tietjens [1934]).

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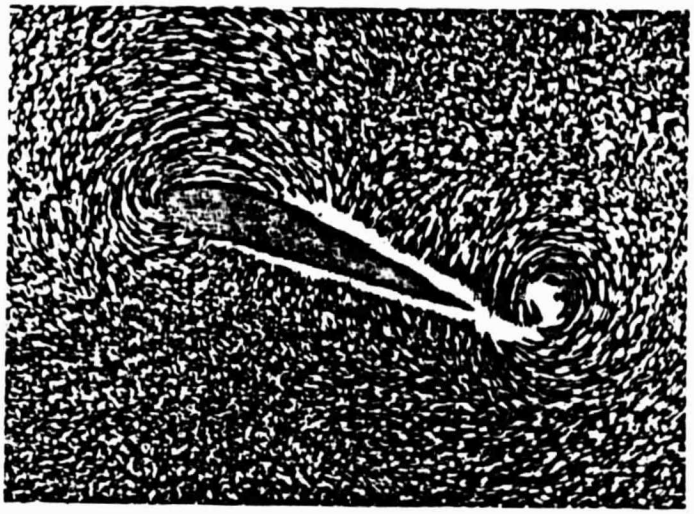
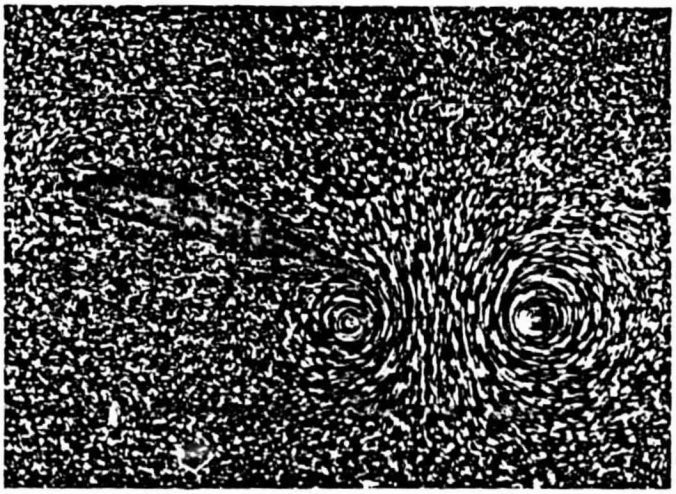


(a)



(b)

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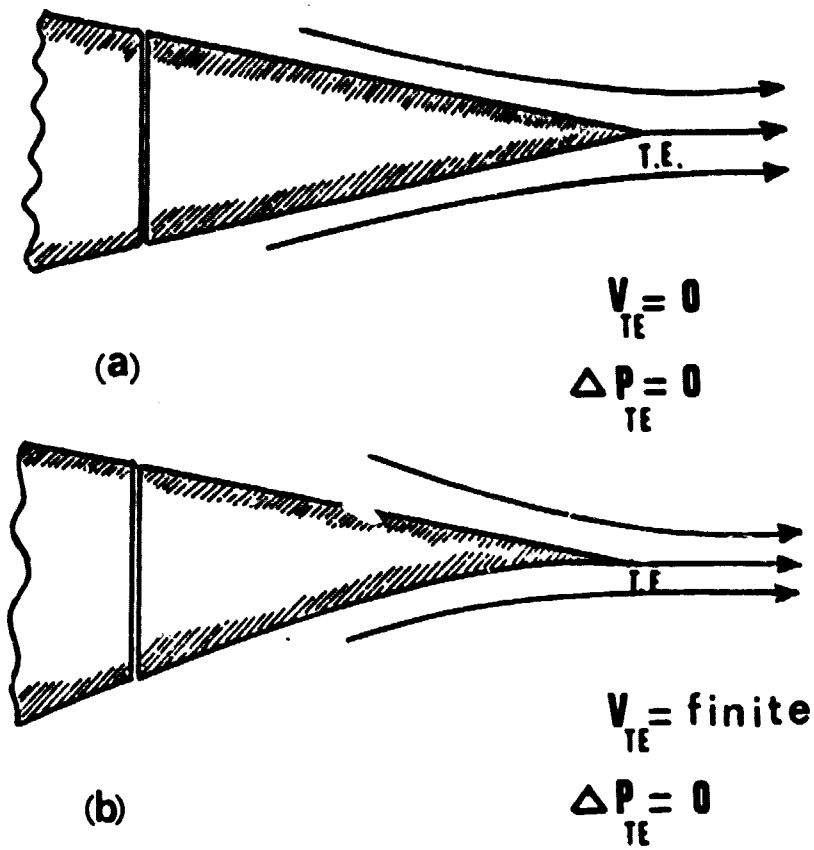

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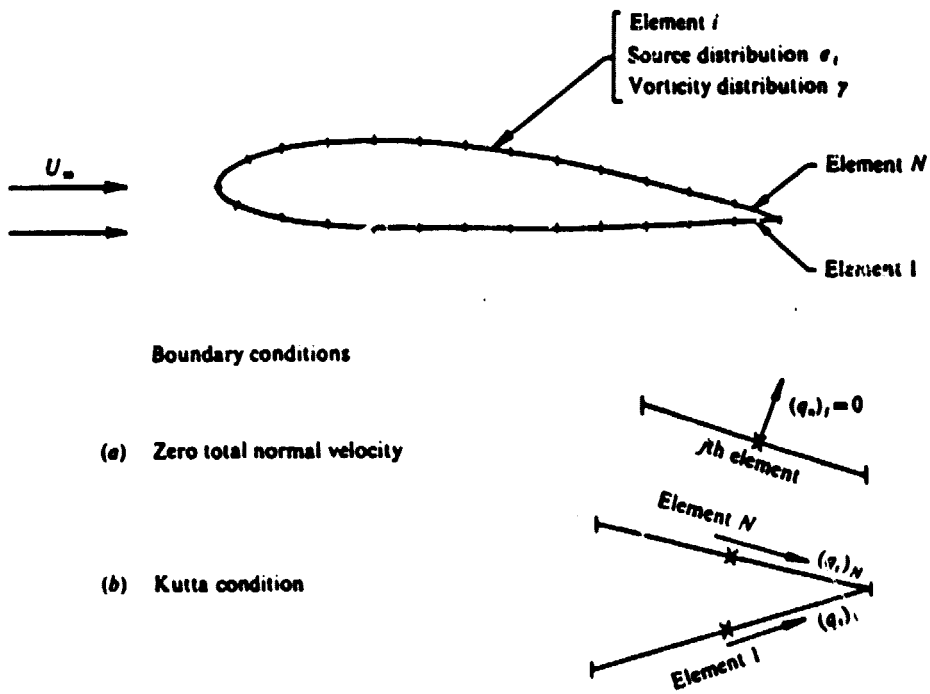

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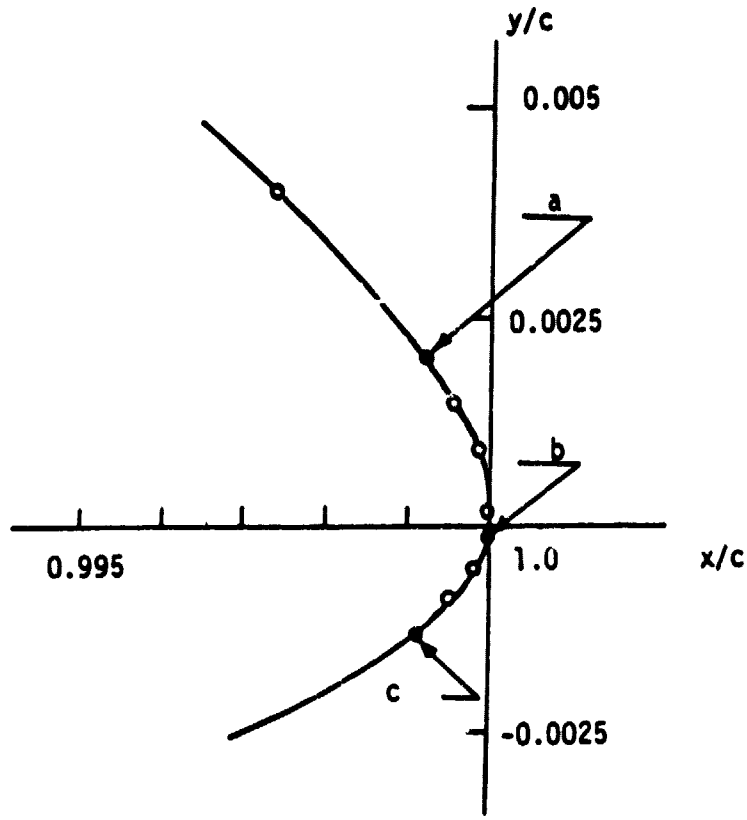

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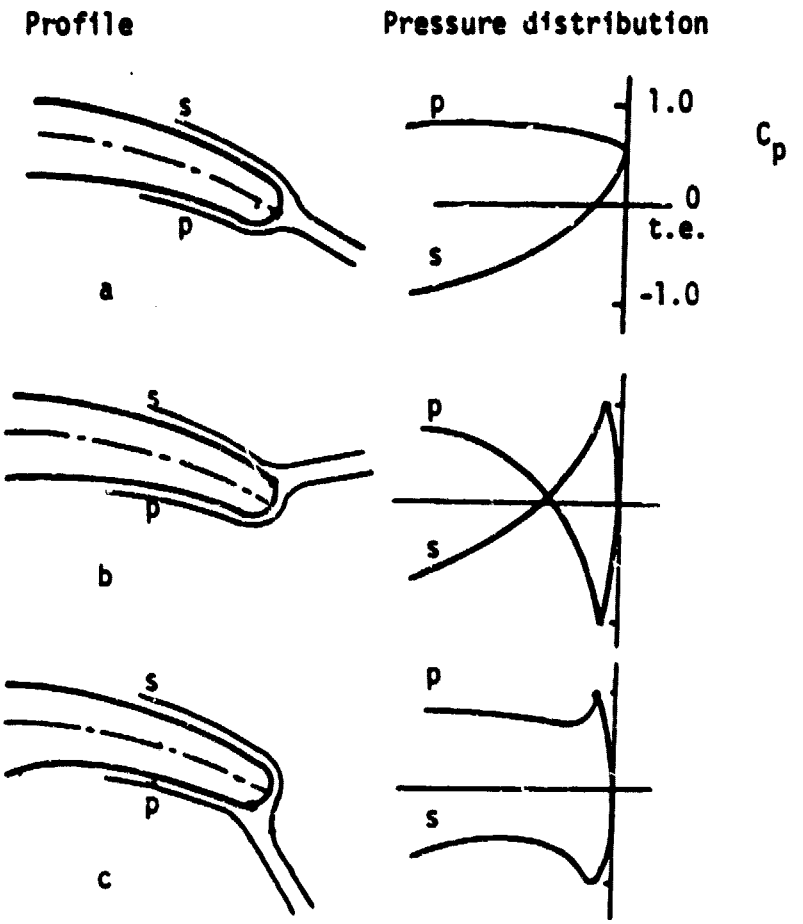

Figure 9: Effect of variation of rear stagnation point on pressure distribution calculated by exact potential theory (Gostelow [1964]).

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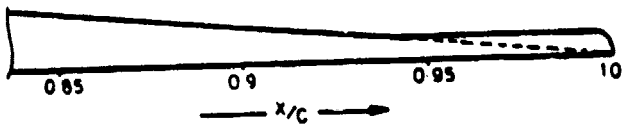
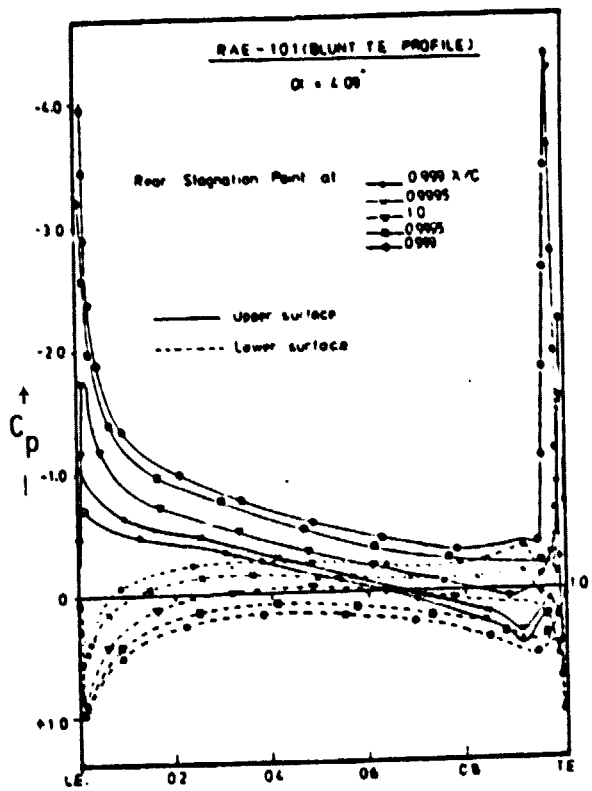
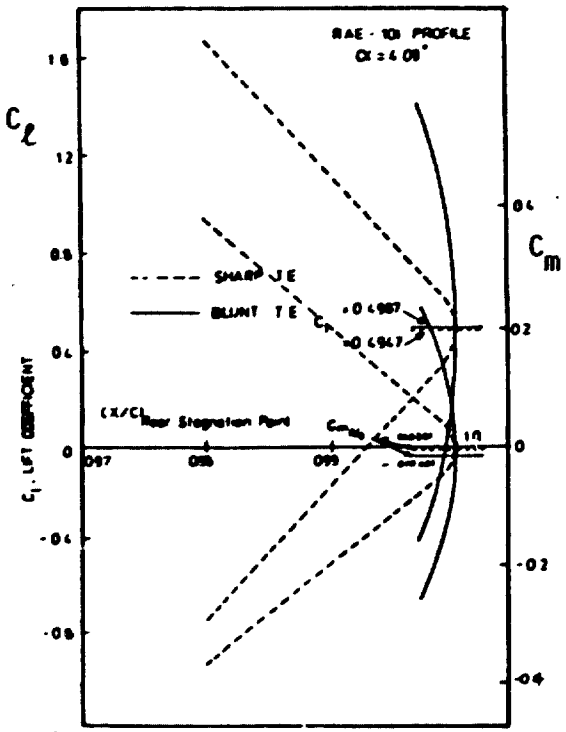


RAE 101 profile with sharp and blunt trailing edge.

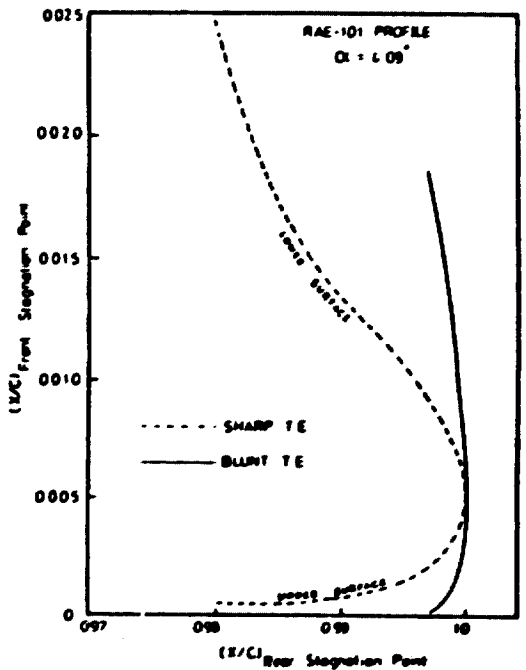


(a) Effect on pressure distribution.

Figure 10: Effect of the rear stagnation point position and trailing-edge bluntness on airfoil characteristics (Baskaran and Holla [1981])



(b) Effect on lift and moment coefficient.



(c) Effect on the location of front stagnation point.

Figure 10: Effect of the rear stagnation point position and trailing-edge bluntness on airfoil characteristics (Baskaran and Holla [1981])

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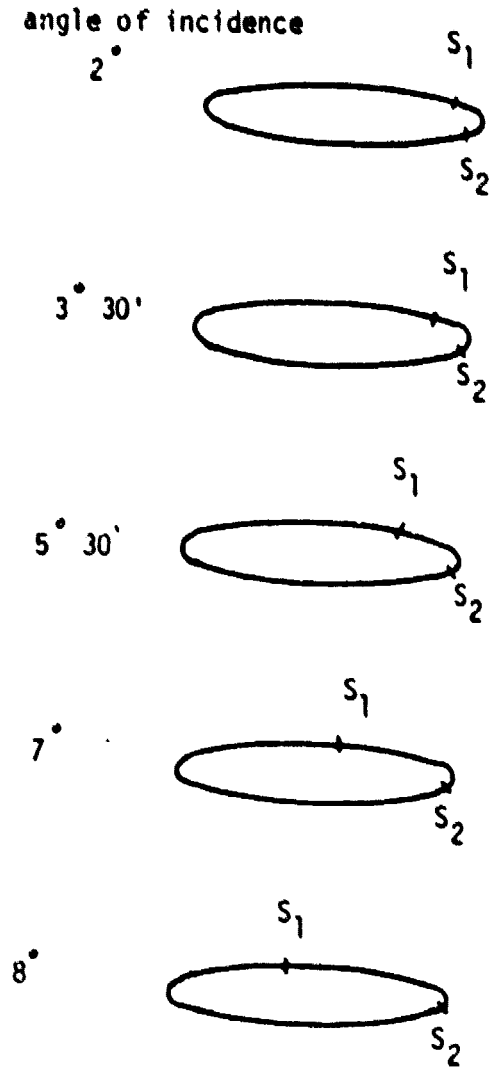

Figure 11: Separation points in fully laminar flow (Howarth [1935]).

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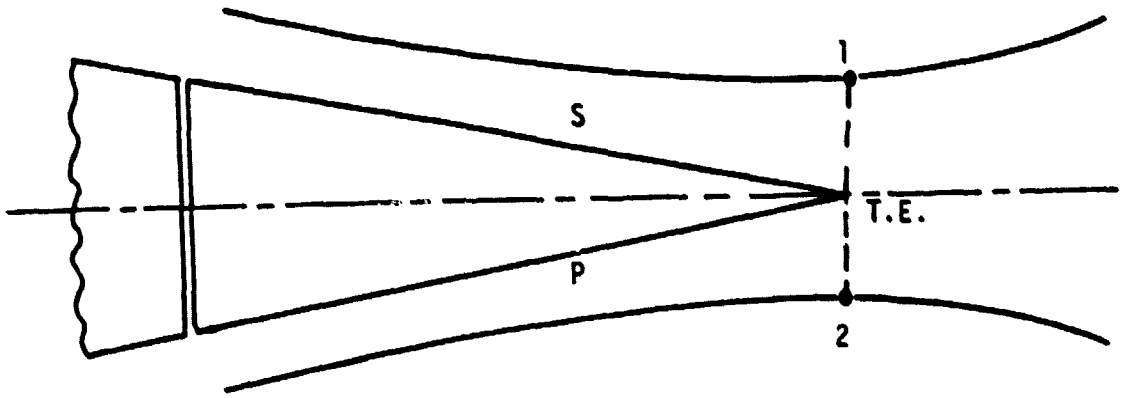

Figure 12: Boundary layer configuration at the trailing-edge in laminar flow.

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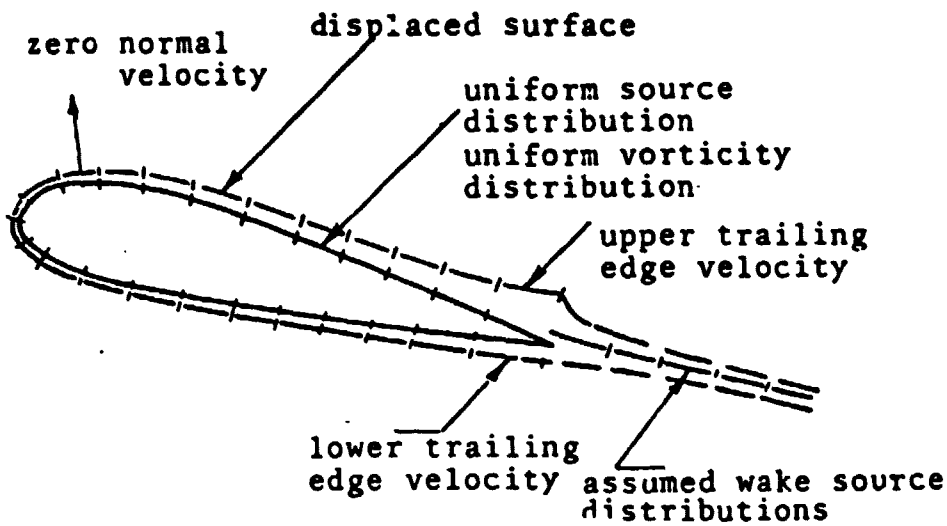

Figure 13: Mathematical model allowing for the boundary layer and wake displacement effects (Basu [1973]).

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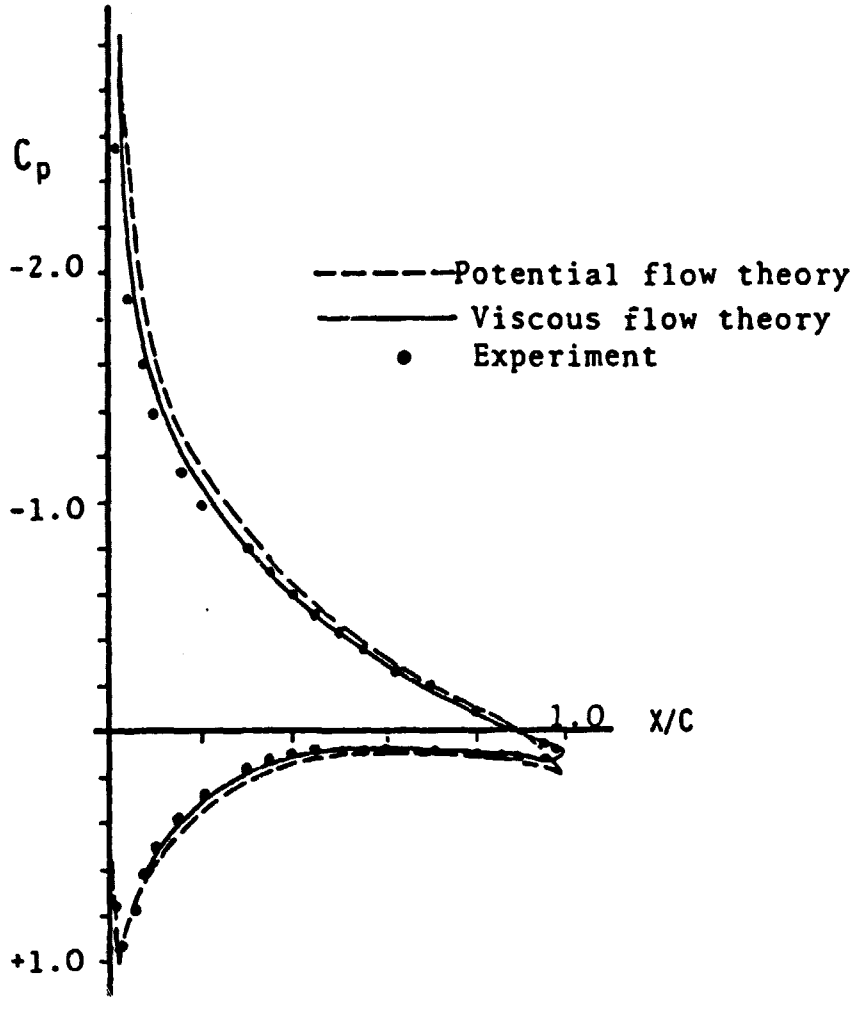

Figure 14: Comparison between viscous flow solution of Basu and the experimental results.

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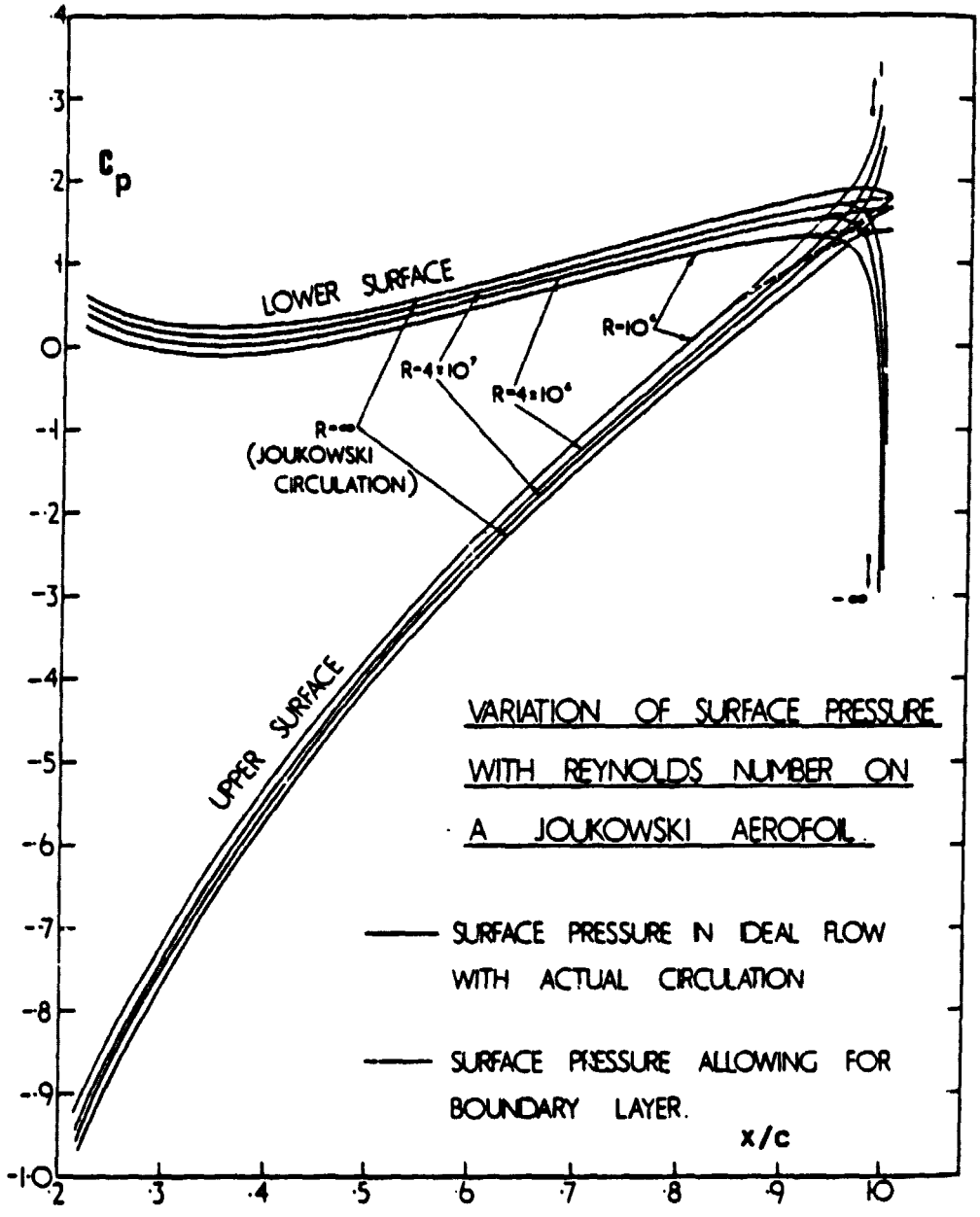

Figure 15: Predicted pressure distribution by Spence's method (Spence [1954]).

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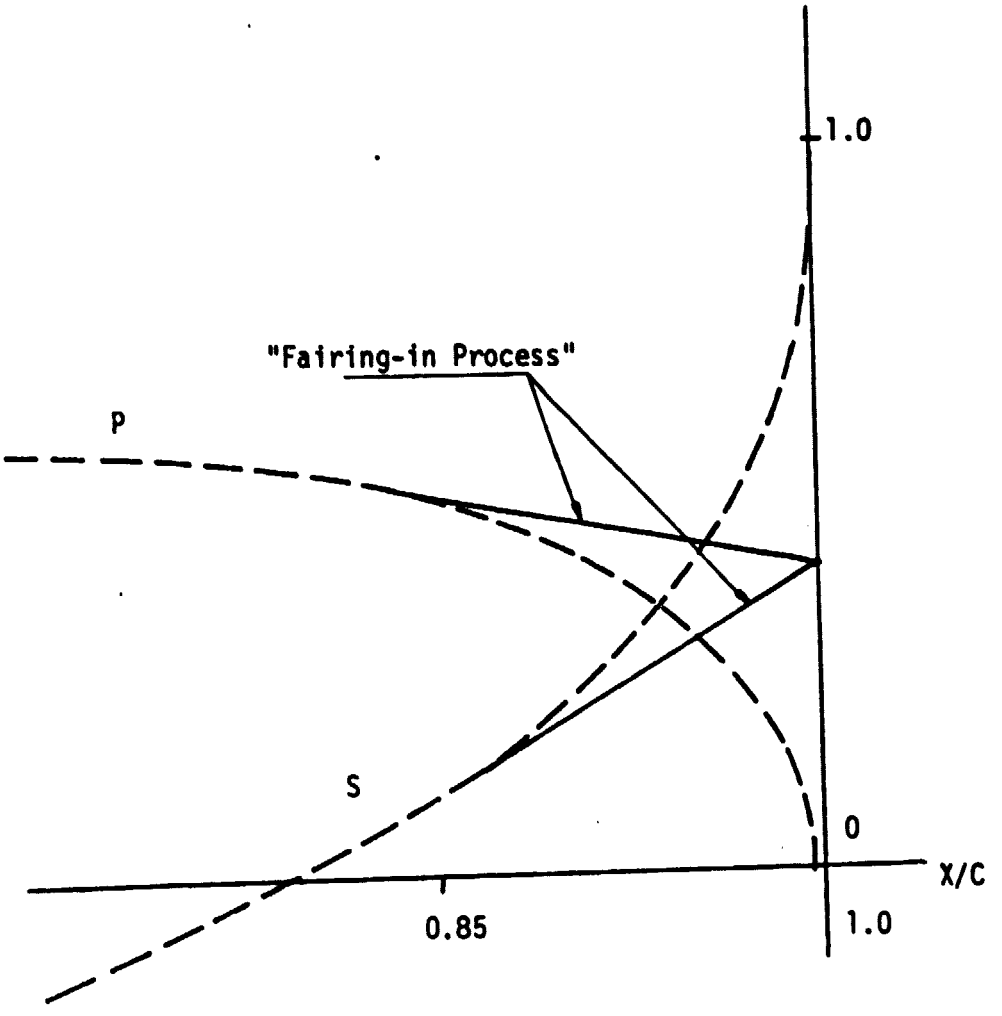

Figure 16: First viscous approxmation (Gostelow [1975]).

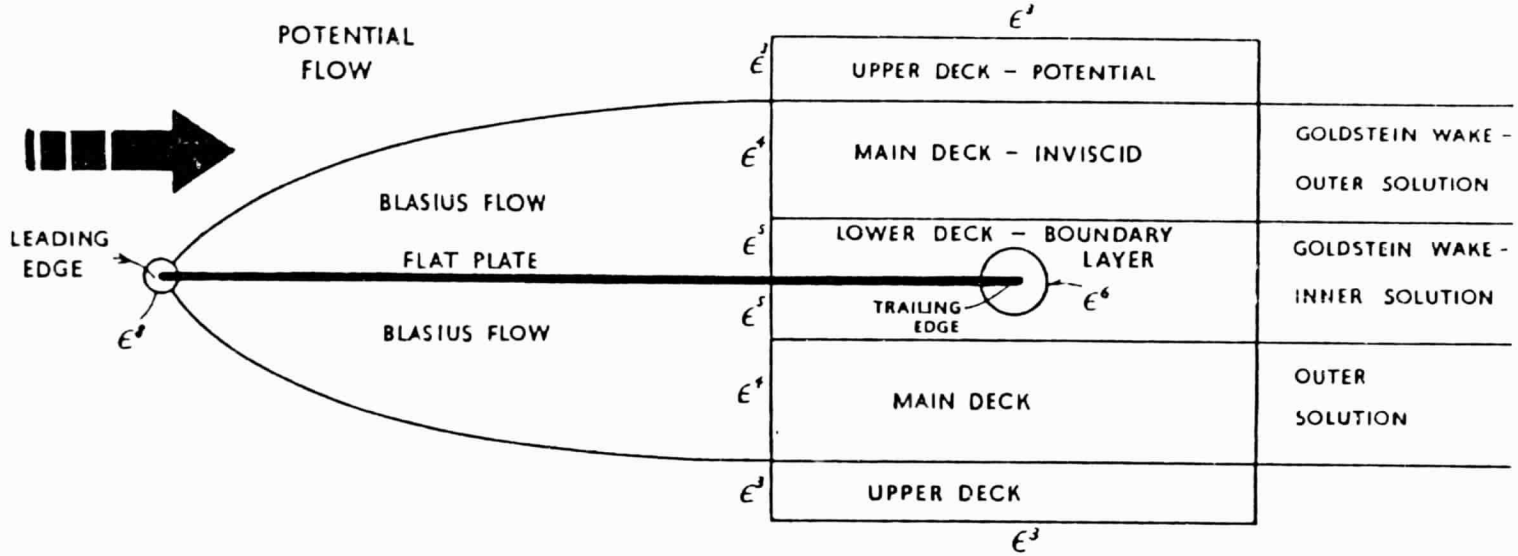

Figure 17: The triple-deck structure in the trailing-edge region (Stewartson; [1969] and Messiter [1970]).

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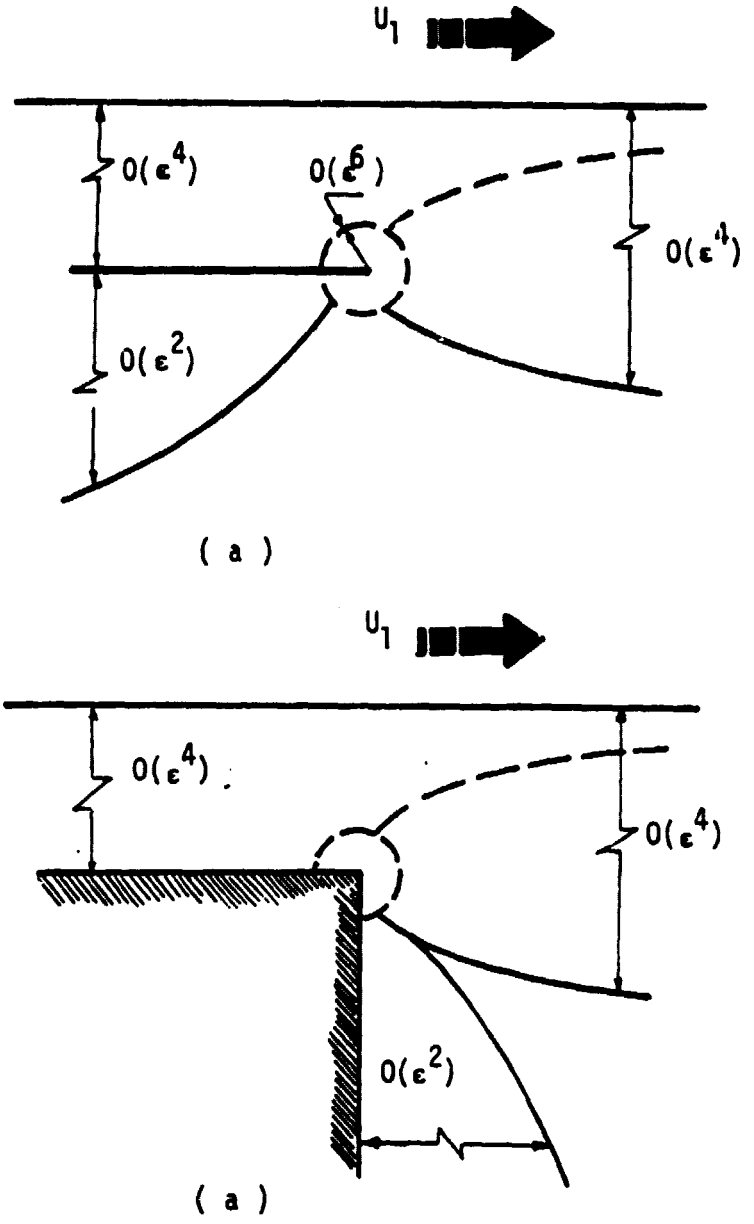

Figure 18: The trailing-edge flow structure for mixing shear layer for: (a) stagnant flow below the plate, i.e. $U_2 = 0$; (b) the case for the flow over a backward-facing step (Daniels, [1977]).

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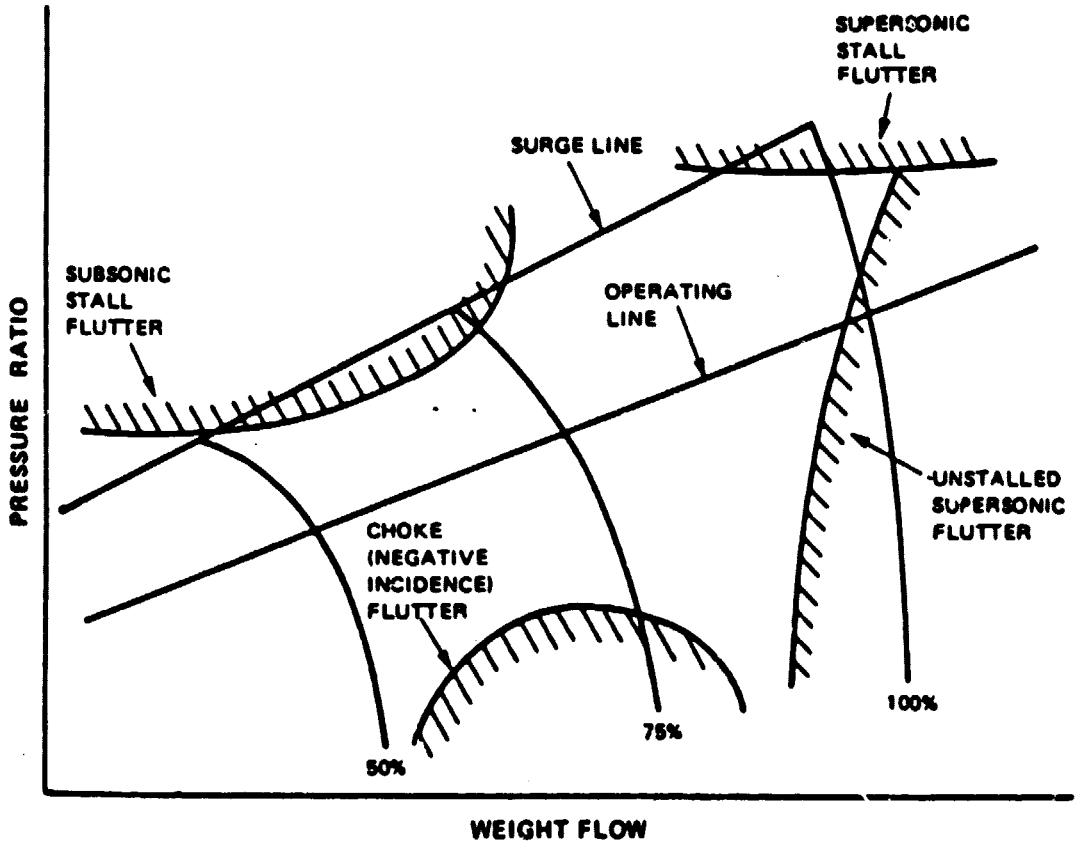

Figure 19: Compressor map showing flutter boundaries of four types of flutter (Pratt and Whitney Aircraft [1976]).

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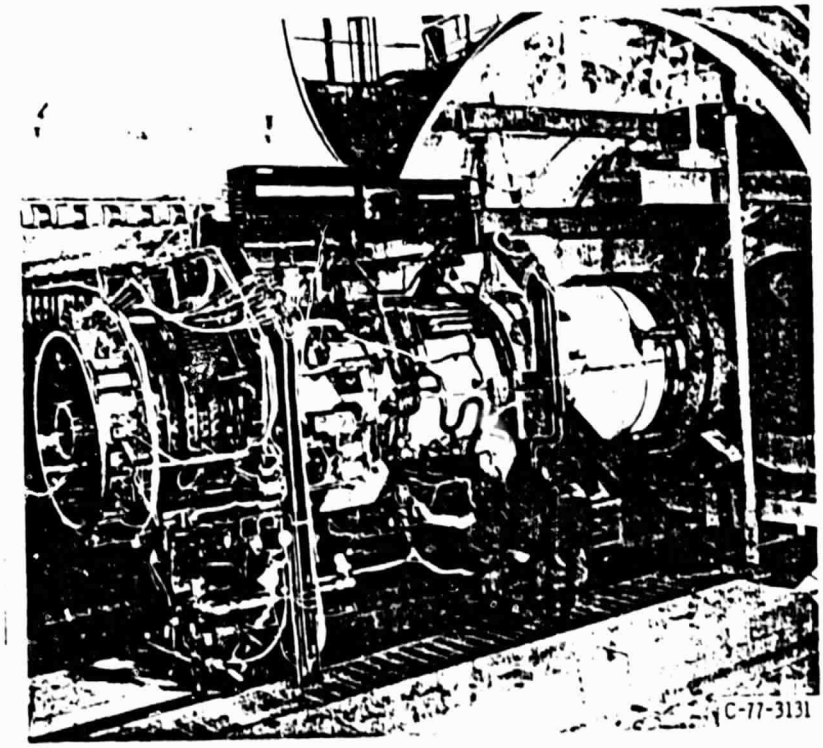

Figure 20: Full scale engine research (NASA-LERC [1979]).

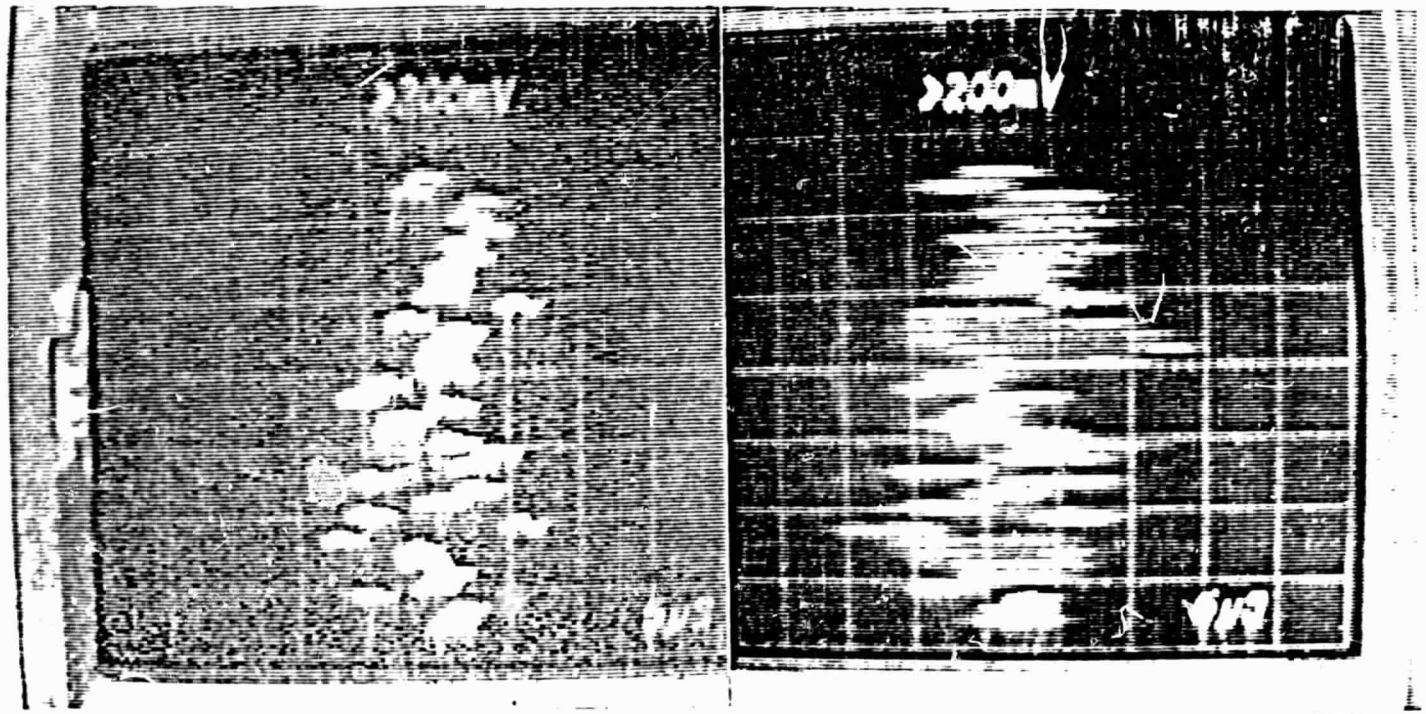

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Figure 21: PES display during quiet running condition (bottom) and during flutter (top) (Nieberding and Pollack [1977]).

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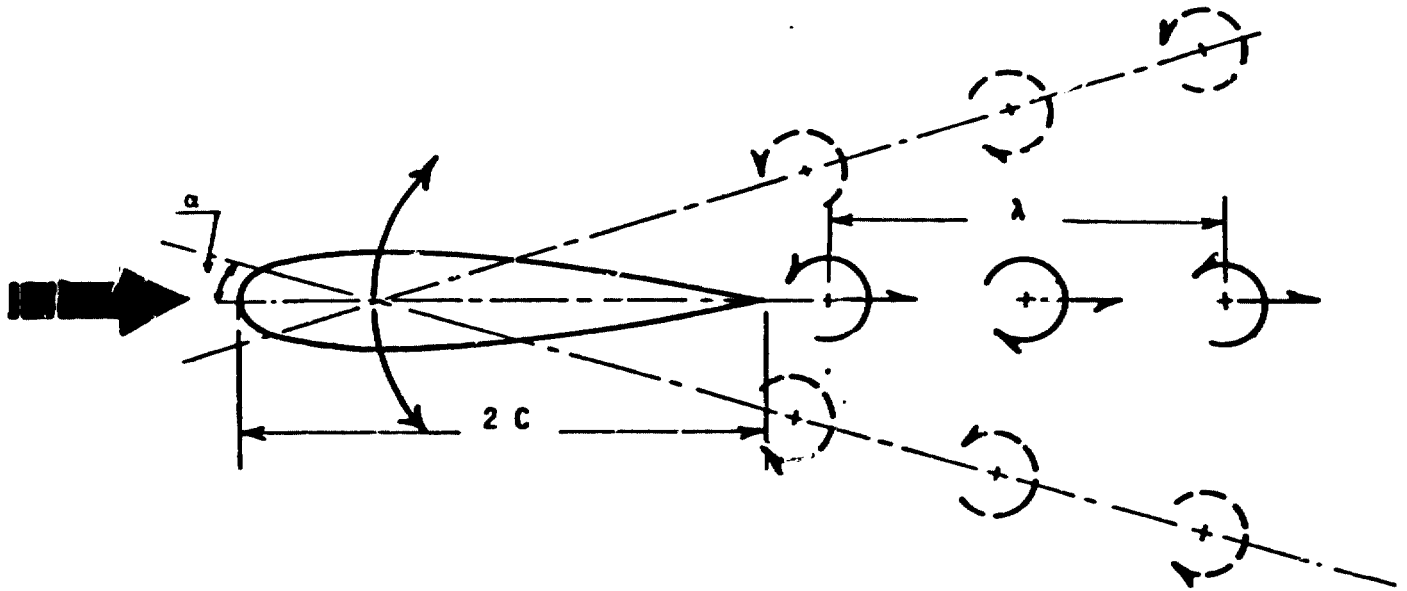

Figure 22: Flow around an oscallating airfoil with vorticity shed from the trailing-edge region.

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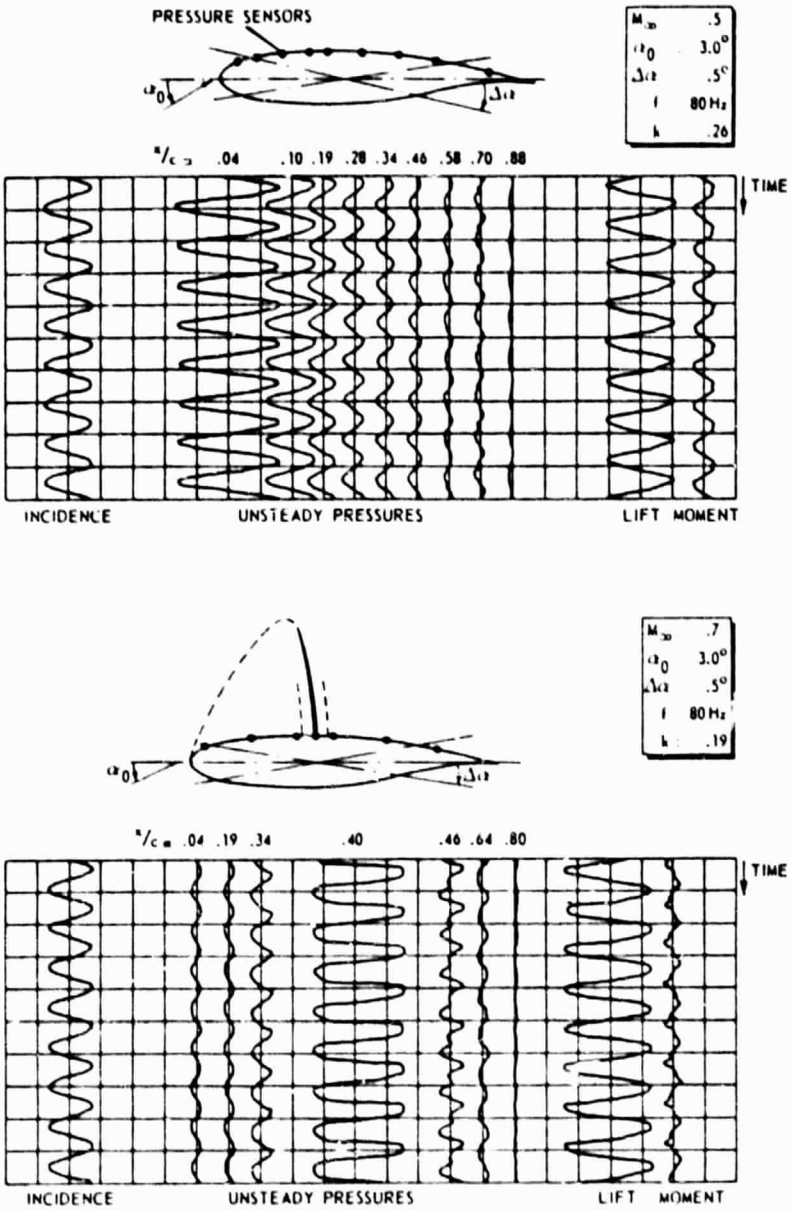

Figure 23: Unsteady pressure signals and overall loads on an oscillating airfoil: (a) subsonic flow(linear flow) ; (b) Transonic flow (nonlinear flow),(Tijdeman [1980]).

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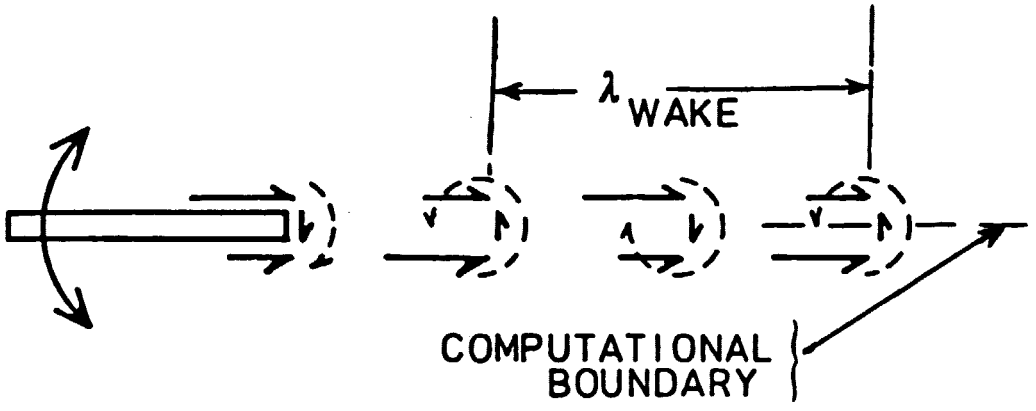

$$P_u = P_\ell$$

$$\gamma_W(X^*, T) = \bar{\gamma}_W e^{i(\omega T - KX^*)}$$

Figure 24: Wake patterns for harmonically oscillating airfoil model (Theodorsen [1935]).

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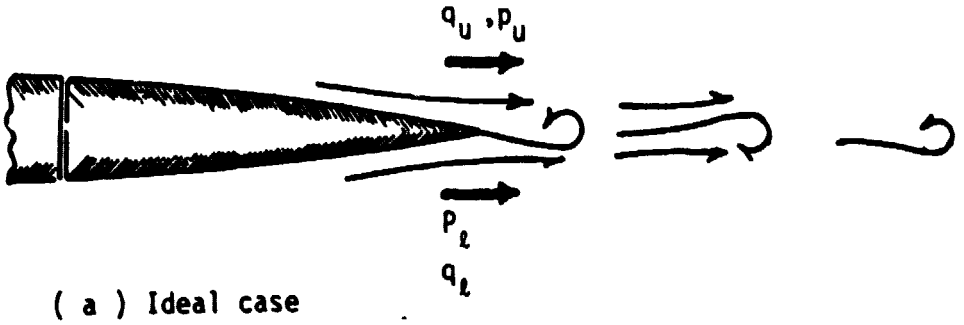
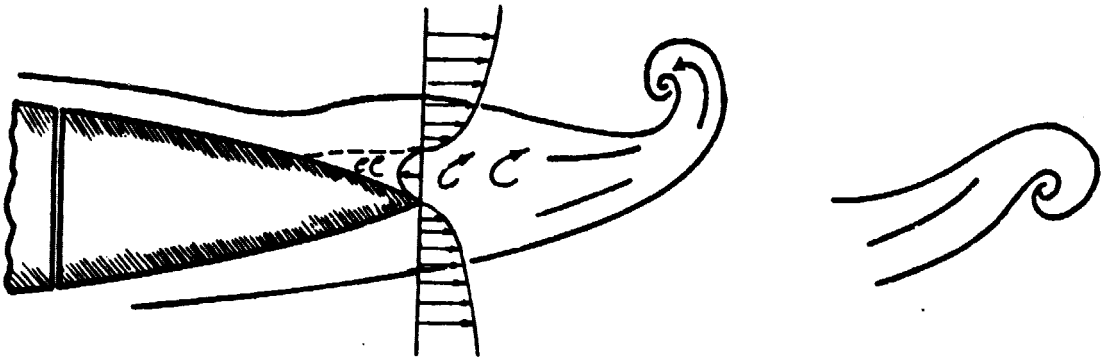


(a) Ideal case

$$\Delta P_{T.E.} = 0$$

$$\frac{d\Gamma}{dt} = +\frac{1}{2}\left(q_u^2 - q_\ell^2\right)$$



(b) Real case

$$P_{T.E.} \neq 0$$

$$\frac{d\Gamma}{dt} = \int_{\delta_\ell}^{\delta_u} q\,\zeta\,dy$$

Figure 25: Trailing-edge condition for unsteady flow (McCroskey [1977]).

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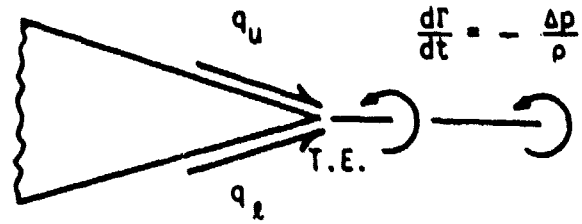
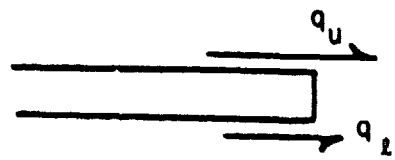
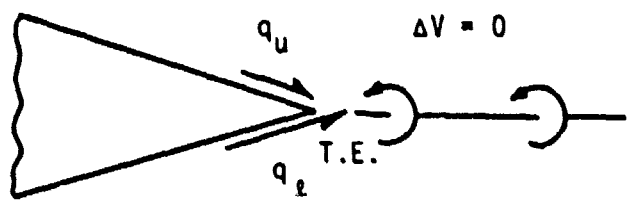


(a) Flow stagnates at trailing-edge , leads to pressure discontinuity (Van der Vooren and Vander Vel [1964]).



(b) Flat plate - velocity difference equal to the instantaneously shed vortex strength (Von Karman and Sears [1938]).



(c) Zero velocity difference at the trailing-edge (Giesing [1968]).

Figure 26: Various interpretations of the unsteady trailing-edge condition.

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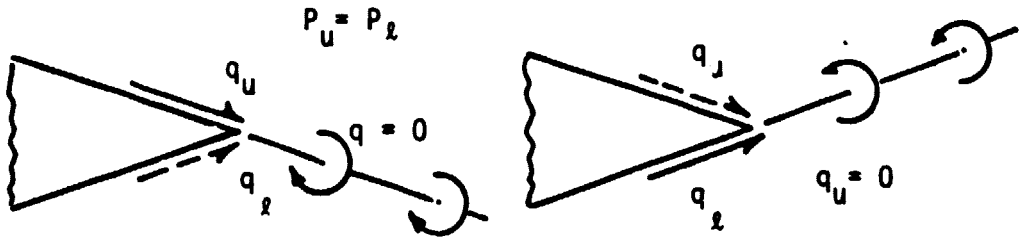
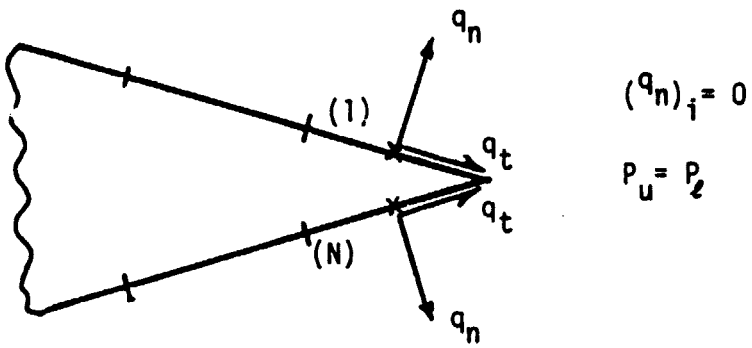


(d) Sense of shed vortex determines direction of instantaneous streamline - Zero pressure loading at trailing -edge (Maskell [1973], Giesing [1969]).



(e) Zero pressure loading at the trailing-edge elements- Zero normal velocity at each computational element (Basu and Hancock [1978]).

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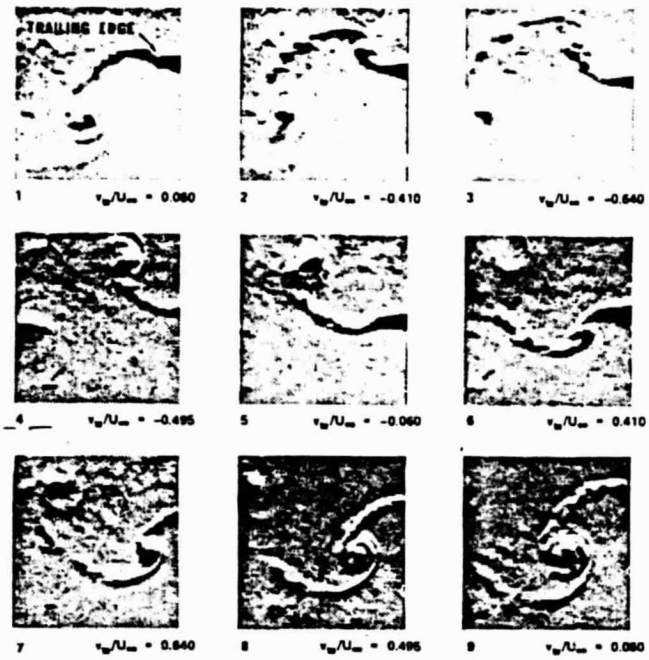
Vortex position at $tU_\infty = 1.85$ of 8.4 symmerical Von Miss aerofoil in pitching oscallation; $\nu=20$.(Basu and-Hancock [1978]).



Flow near the trailing-edge of an aerofoil oscillating at high frequency; pitch axis at $x/c = 0.30$ and $\nu=8.0$ (Kadlec and Davis [1979]).

Figure 27: Comparison between the computed wake pattern for an oscillating airfoil at high frequency and the experimentally observed wake pattern.

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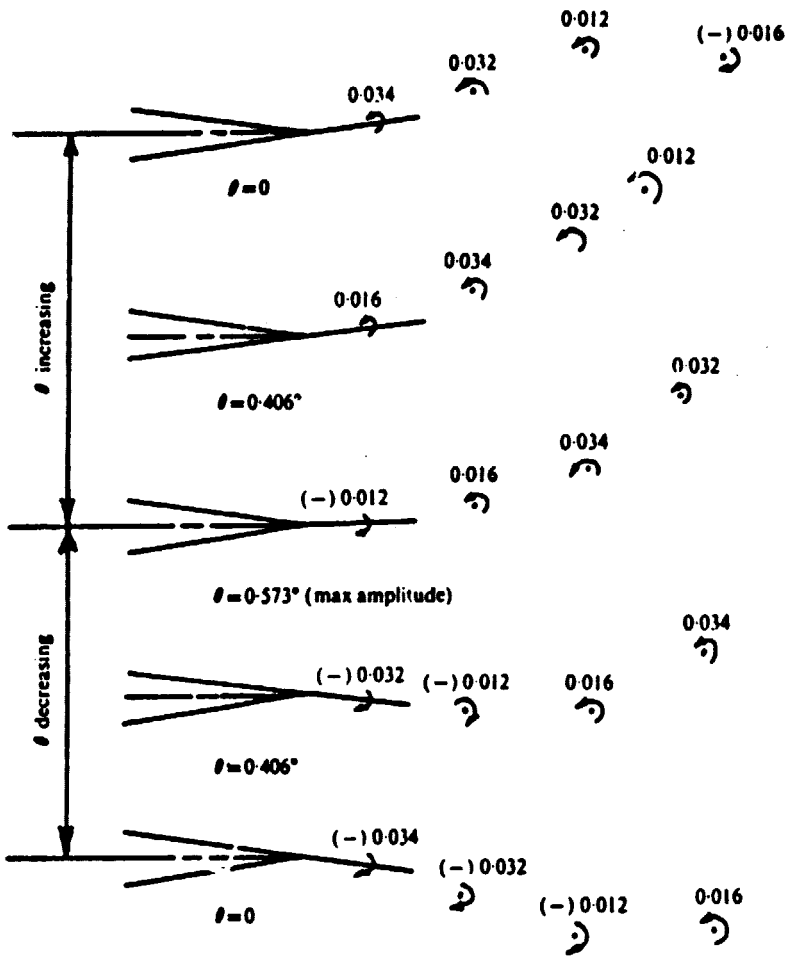
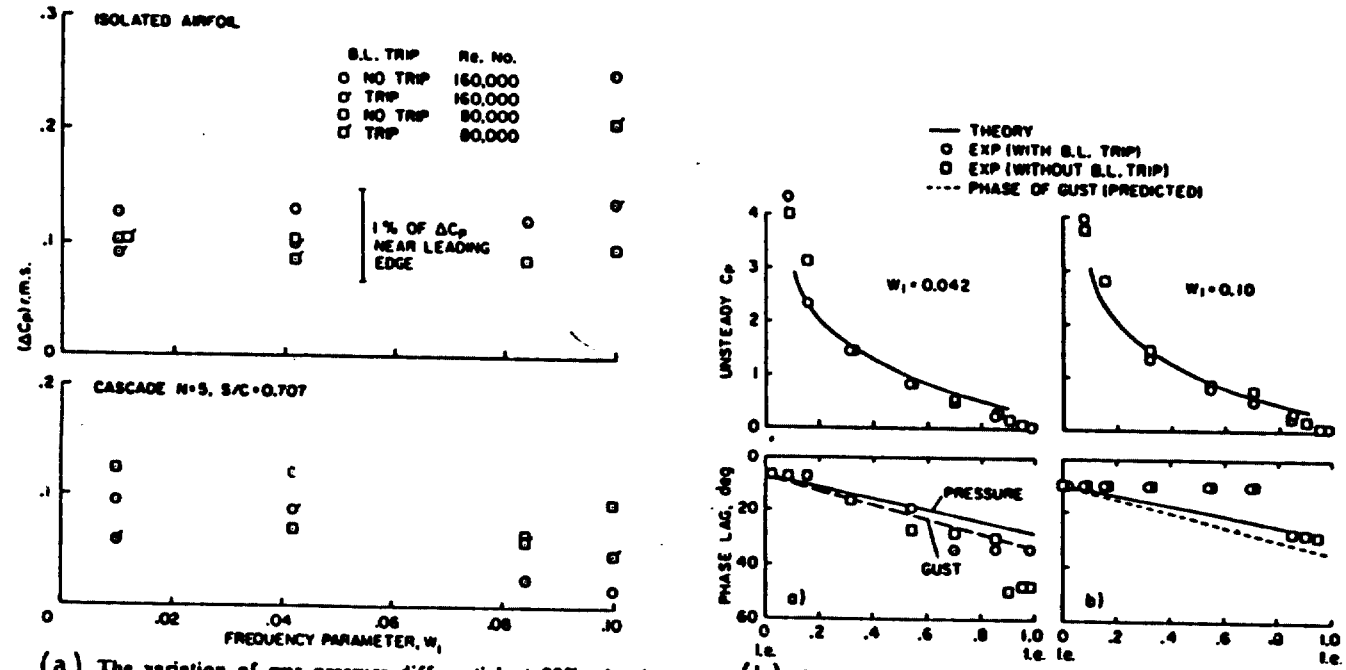


Figure 28: The theoretical characteristics of the wake element from the trailing-edge for a 8.4% thick symmetrical Von Mises airfoil oscillating about the leading-edge (Basu and Hancock [1978]).



(a) The variation of rms pressure differential at 98% chord pressure with frequency parameter.

(b) Comparison of experimental unsteady pressure and phase distribution with predicted results for an isolated airfoil.

Figure 29: Experiments and prediction for the pressure differential due an incident gust (Satyanarayana [1977]).

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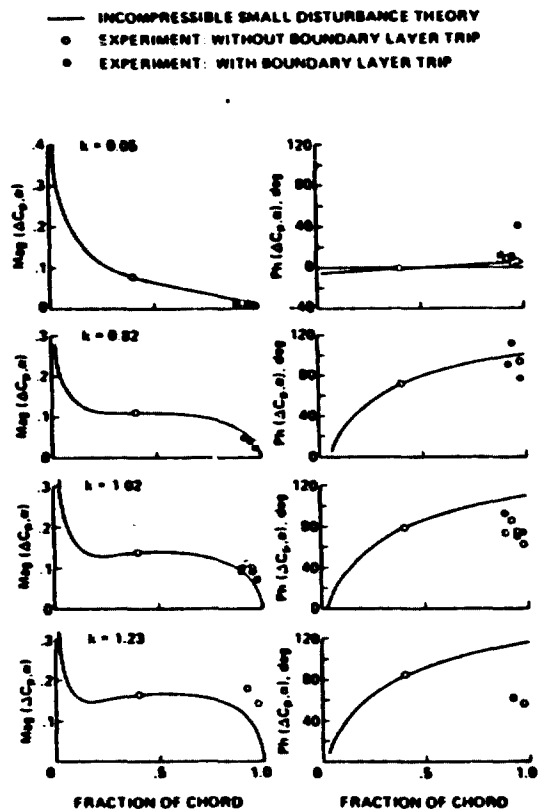
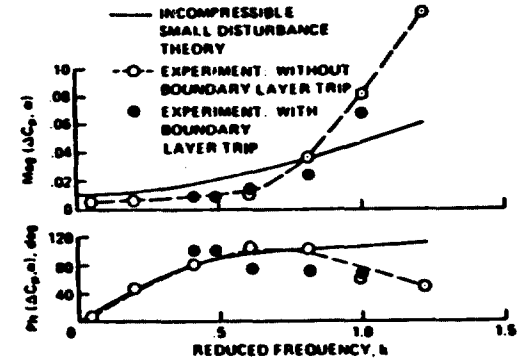
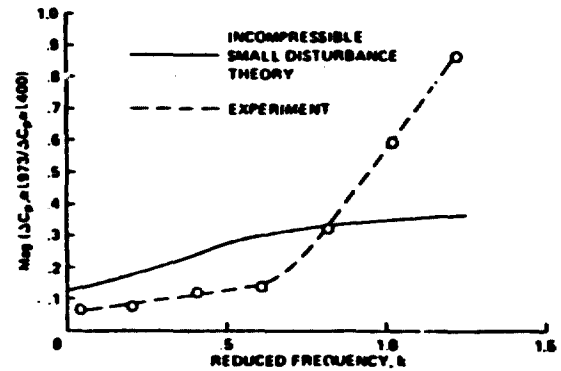


(a) Comparision of experimental chordwise unsteady pressure and phase distribution with predicted results at various reduced frequencies.



(b) Variation of unsteady loading with reduced frequency at 97.3% chord.



(c) Variation of the ratio of trailing-edge loading to 40% chord loading, with reduced frequency.

Figure 30: Variation of unsteady pressure vs reduced frequency due to oscillating airfoil (Satyanarayana [1978]).

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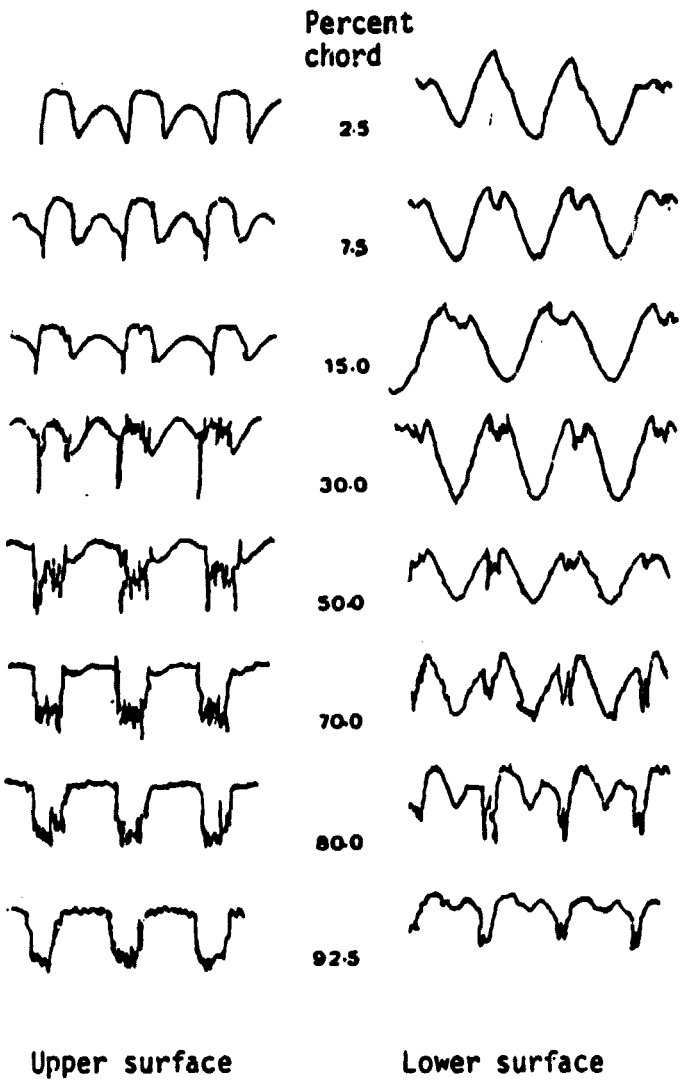
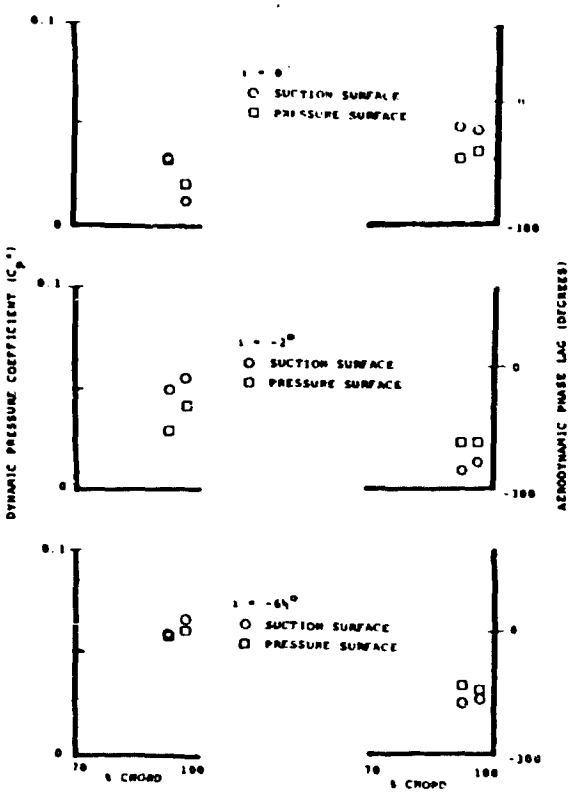
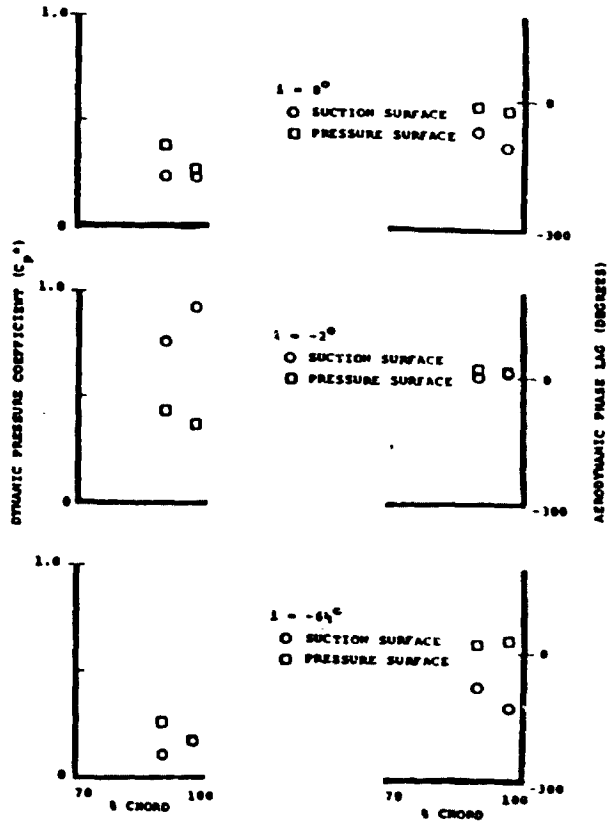


Figure 31: Instantaneous pressure traces from isolated airfoil exposed to a sinusoidal transverse gust due Holmes (Gostelow [1975]).



(a) Trailing edge region unsteady surface pressure variations for the flat plate cascade.



(b) Trailing edge region unsteady surface pressure variations for the cambered airfoil cascade.

Figure 32: Experimental data for the unsteady surface pressure coefficient vs reduced frequency, due to an incident gust (Fleeter [1980]).

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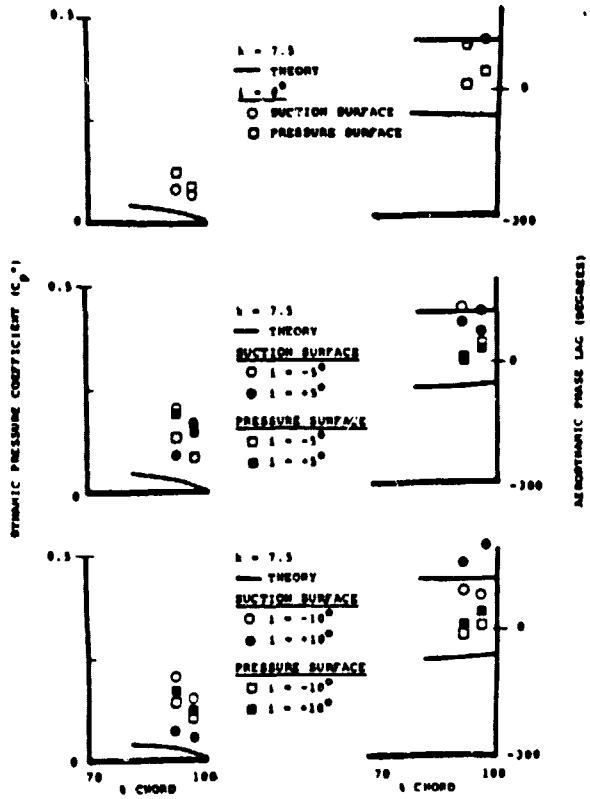

(c) Trailing edge region unsteady surface pressure variations for the isolated flat plate.

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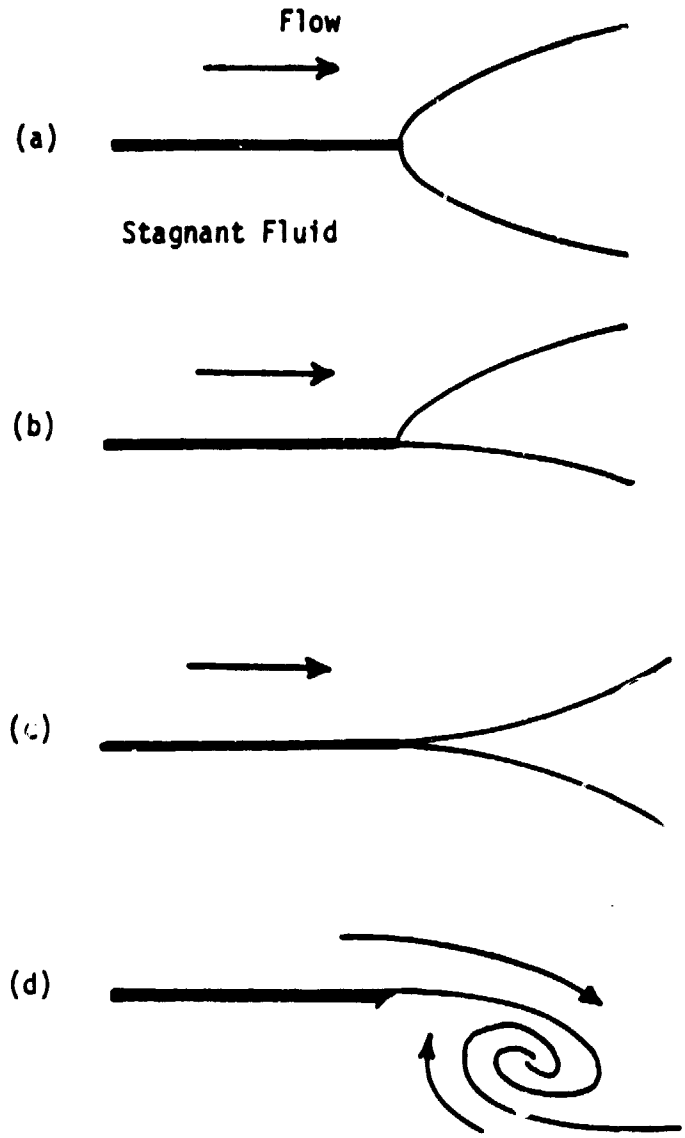

Figure 33: Trailing-edge condition for a semi-infinite plate according to Orszag and Crow [1970]: (a) "No Kutta condition"; (b) "Rectified Kutta condition"; (c) "Full Kutta condition".

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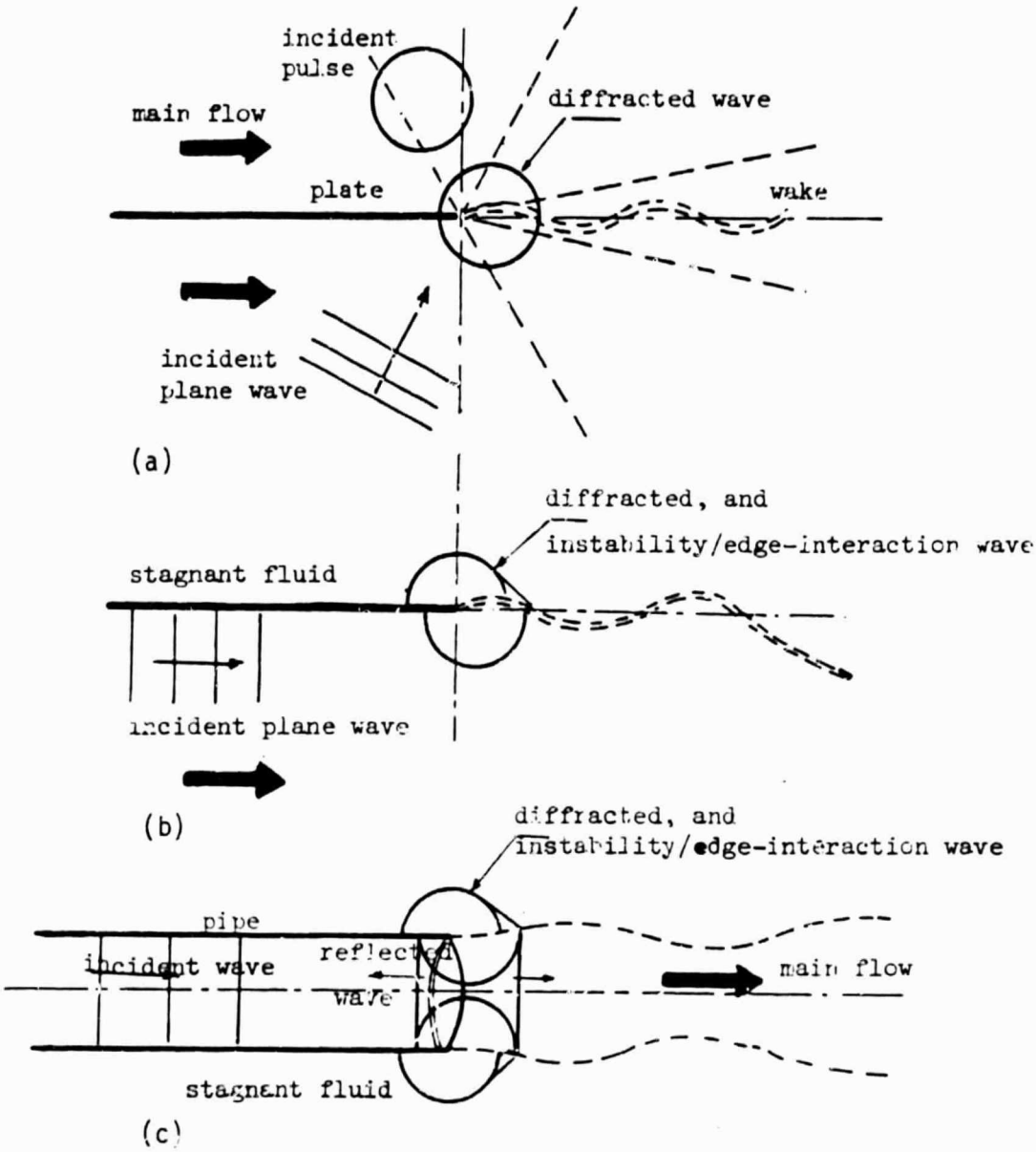
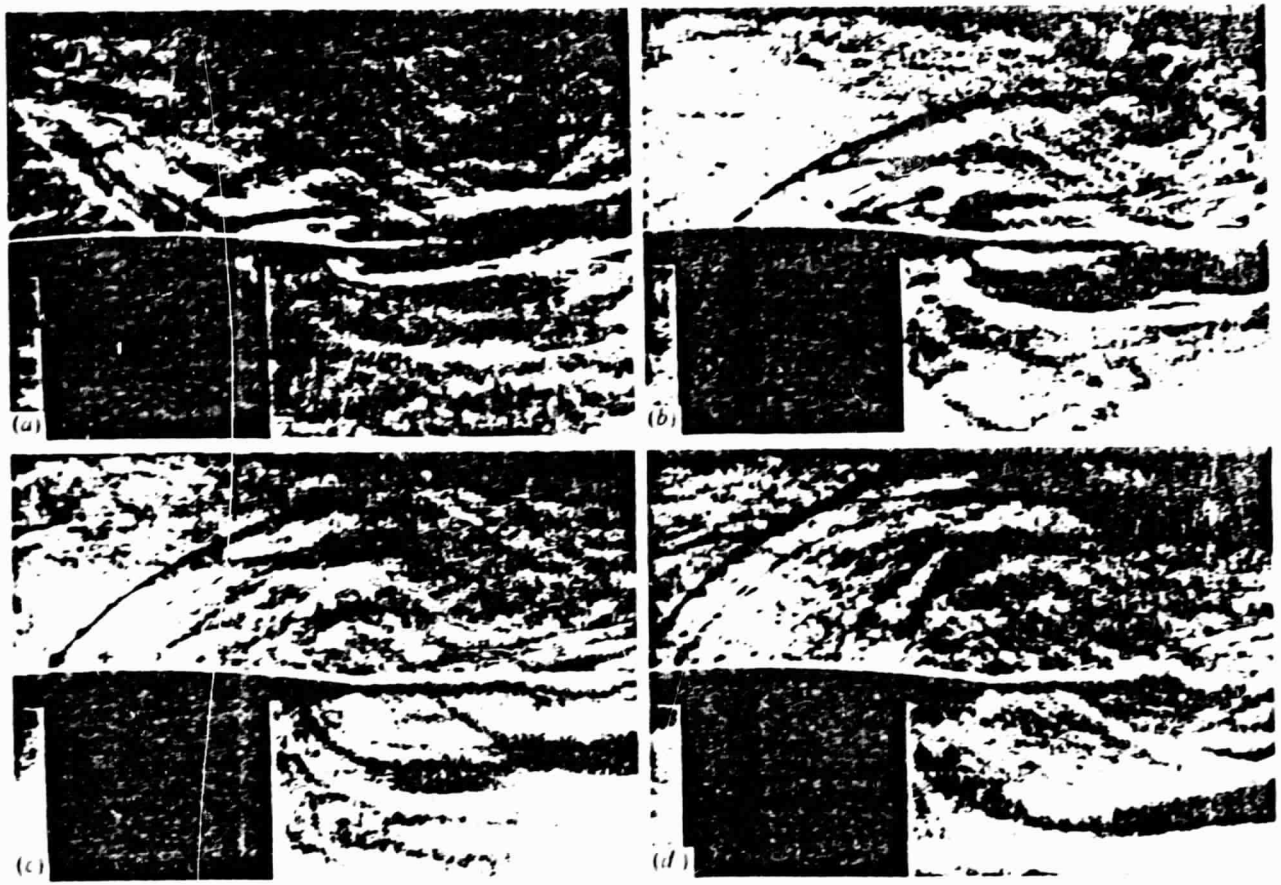


Figure 34: The interaction of sound with a trailing-edge flow in three different problems: (a) a semi-infinite flat plate in an inviscid compressible fluid flowing on both sides; (b) as in case (a), but , the fluid flows on one side ; (c) compressible inviscid jet flow emanating from a semi-infinite pipe (Rienstra [1979]).



Diffraction of the plane wave at zero incidence, Mach 0.34. In frame (a) the incident wave is passing through the wake. The diffracted wave in frames (b)–(d) is comparatively weak. Time between frames = 25 μs, knife-edge horizontal, transparent side upstream.

Figure 35: The "full Kutta condition" flow, where the interaction between the diffracted wave and flow at the trailing-edge is weak (Heavens [1978]).

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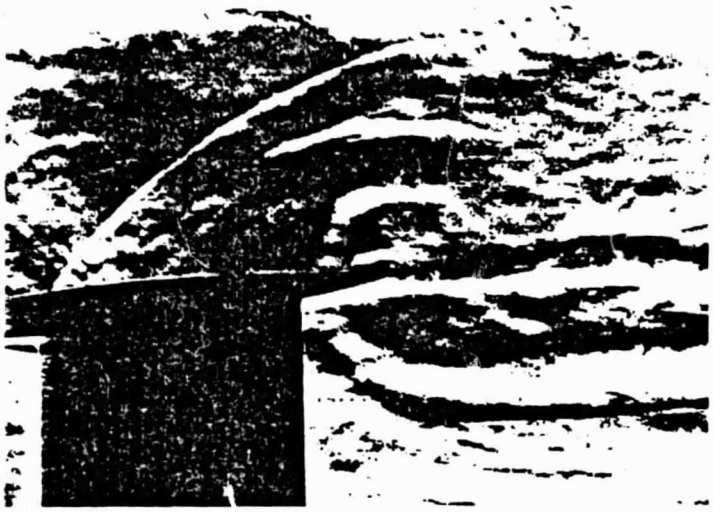
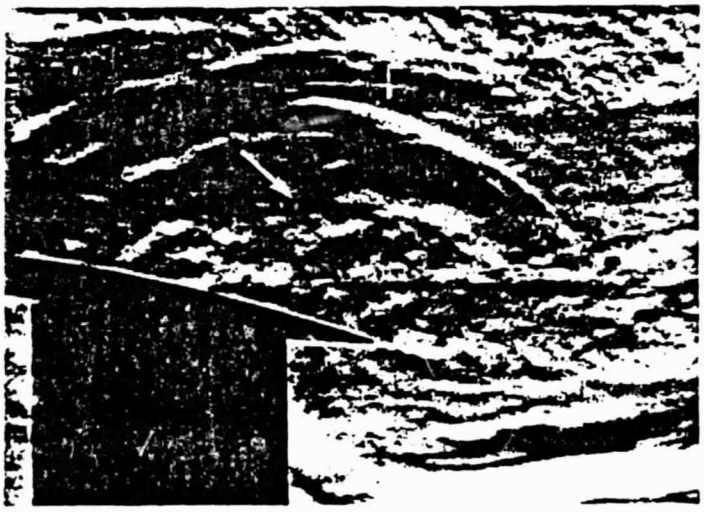


Further intensification of the diffracted wave at Mach 0·33. Angle of incidence = −5·7°; knife edge horizontal, opaque side uppermost.



Appearance of the strong diffracted wave (arrowed) as it emerges from the turbulent boundary layer at 11·3° incidence, Mach 0·22. Knife edge horizontal, transparent side uppermost.

Figure 36: The "no Kutta condition flow," where the interaction between the diffracted wave and trailing-edge flow is strong (Heavens [1978]).

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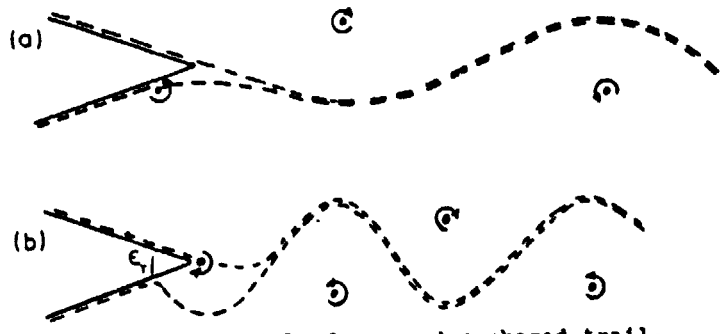
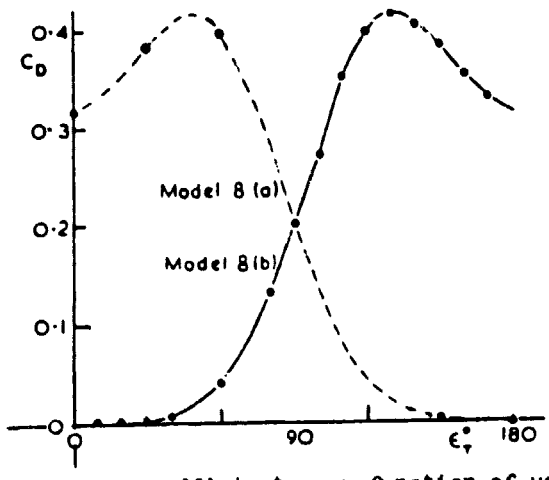


Shedding models for a wedge-shaped trailing edge



Drag coefficient as a function of wedge angle

Figure 37: Vortex shedding models and drag coefficient for wedge-shaped airfoils (Gostelow [1975]).

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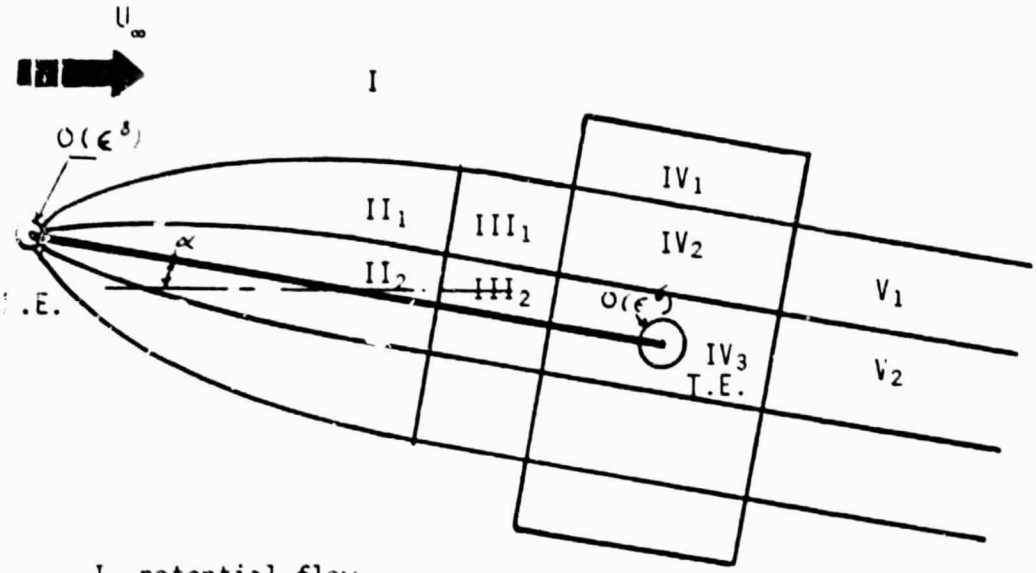

I- potential flow
II- perturbed Blasius flow and
inner Stokes layer
III- the fore deck
IV- the triple deck
V- modified Goldstein wake

Figure 38: Trailing-edge flow structure on the upper side of a rapidly oscillating plate (Brown and Daniels [1975]).

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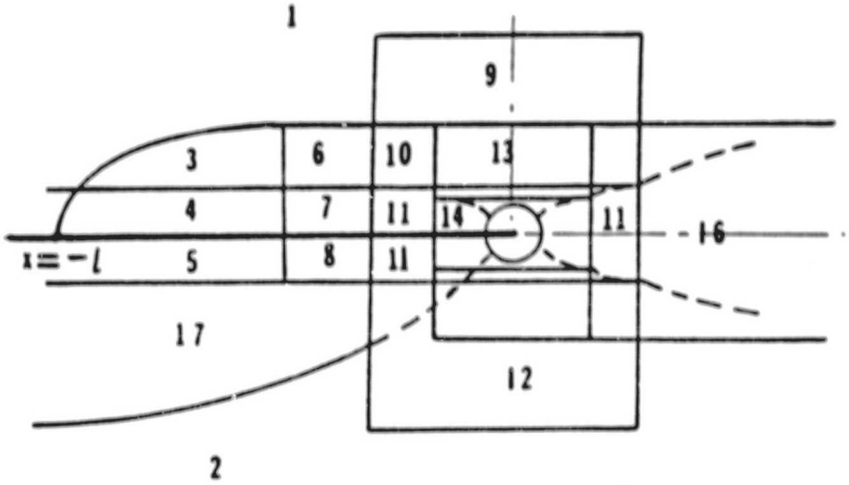
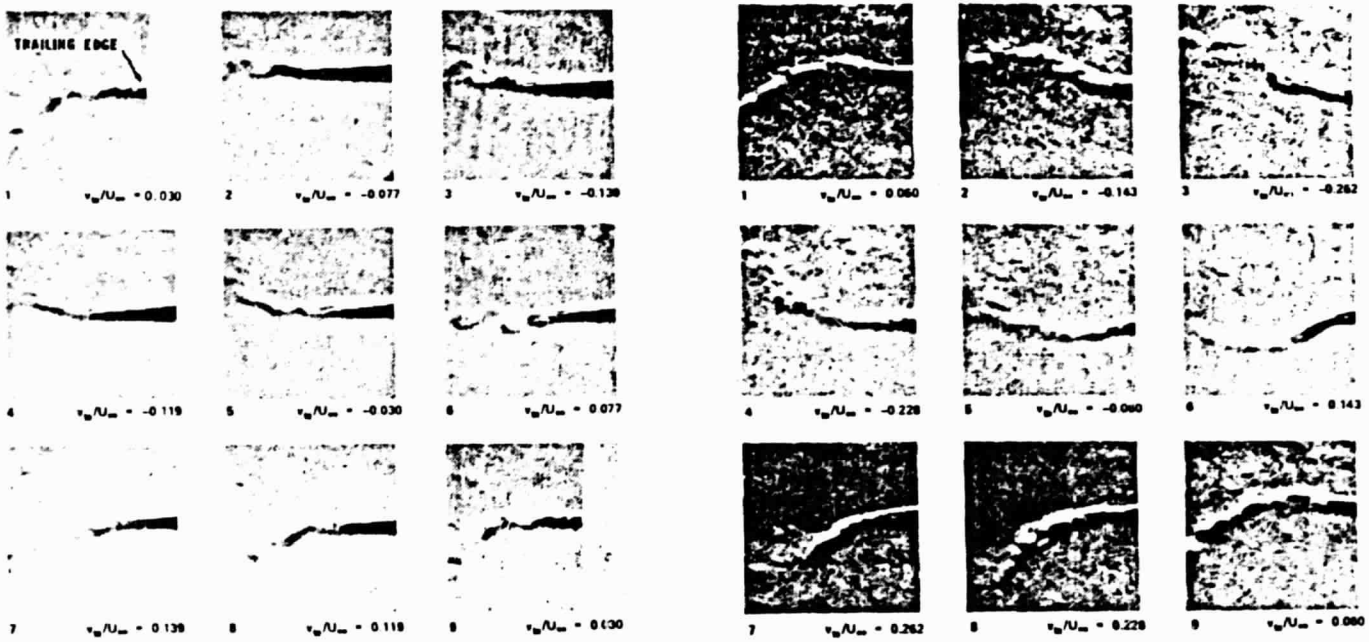


1,2 : Outer inviscid flow
3 : Blasius boundary layer
4,5 : Stokes layers
6 : Fore deck (main deck)
7,8 Fore deck (lower deck)
9,12 Triple deck (upper deck)
10 : Triple deck (main deck)
11 : Triple deck (lower deck)
13 : Inner region (main deck)
14 : Inner region (sublayer)
15 : Full Navier-Stokes region
16 : Mixing layer
17 : Displacement boundary layer

Figure 39: Trailing-edge flow structure in unsteady flow past a plate (Daniels [1978]).



Visualization of the near wake behind a pitching airfoil: Re = 166,000;ν=1.12

Visualization of the near wake behind a pitching airfoil : Re = 53,700; ν=3.47

Figure 40: Near wake behind an oscillating airfoil at different Reynolds numbers and reduced frequencies (Kadlec and Davis [1979]).

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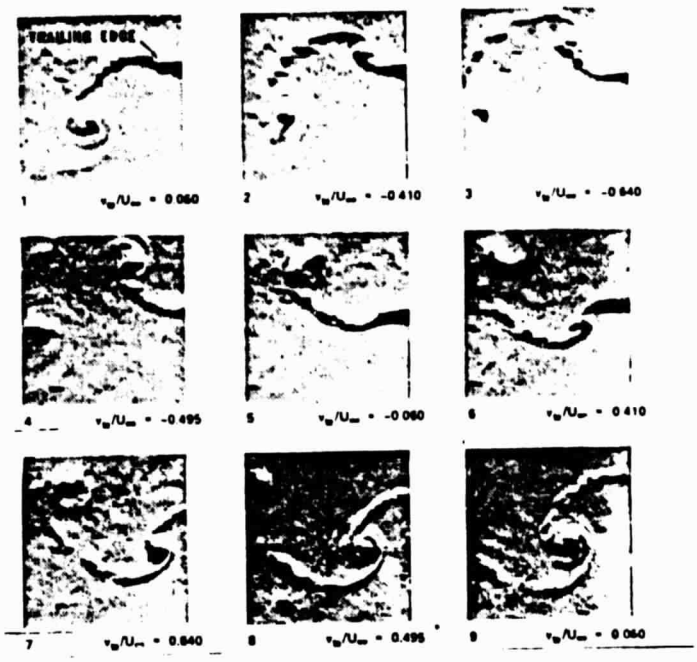
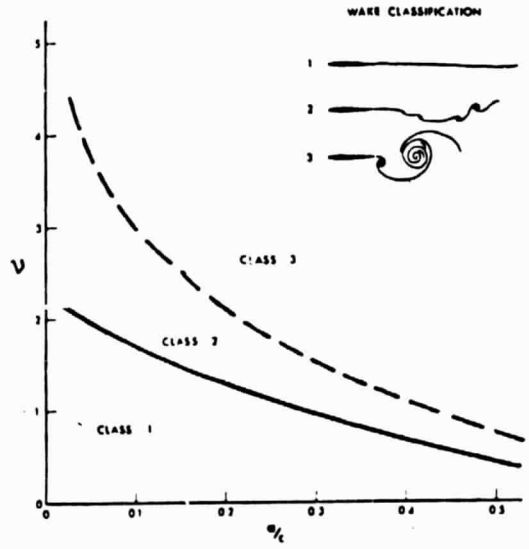


Visualization of the near wake behind a pitching airfoil: Re = 34,300; ν=8.0



Characterization of near-wake flow patterns

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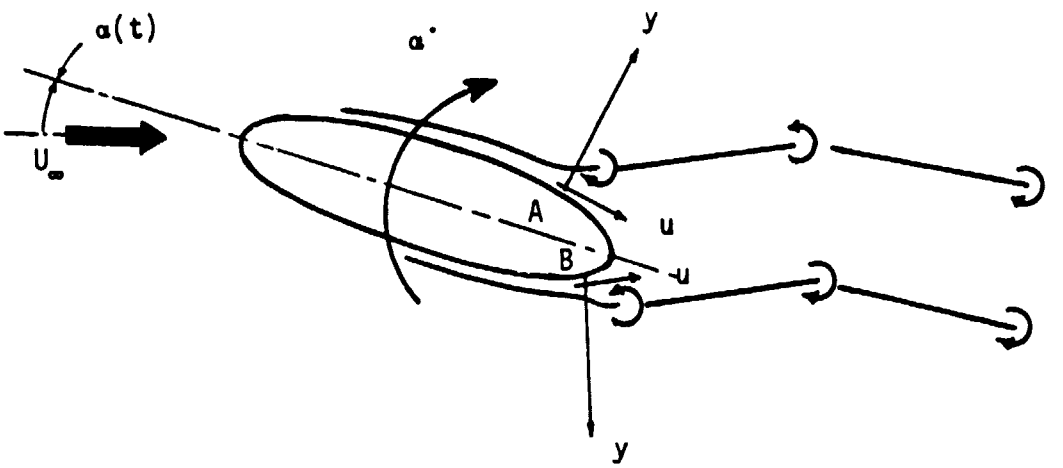

Figure 41: Airfoil in unsteady motion with vortical wake produced at boundary-layer separation points A and B (Sears [1976]).

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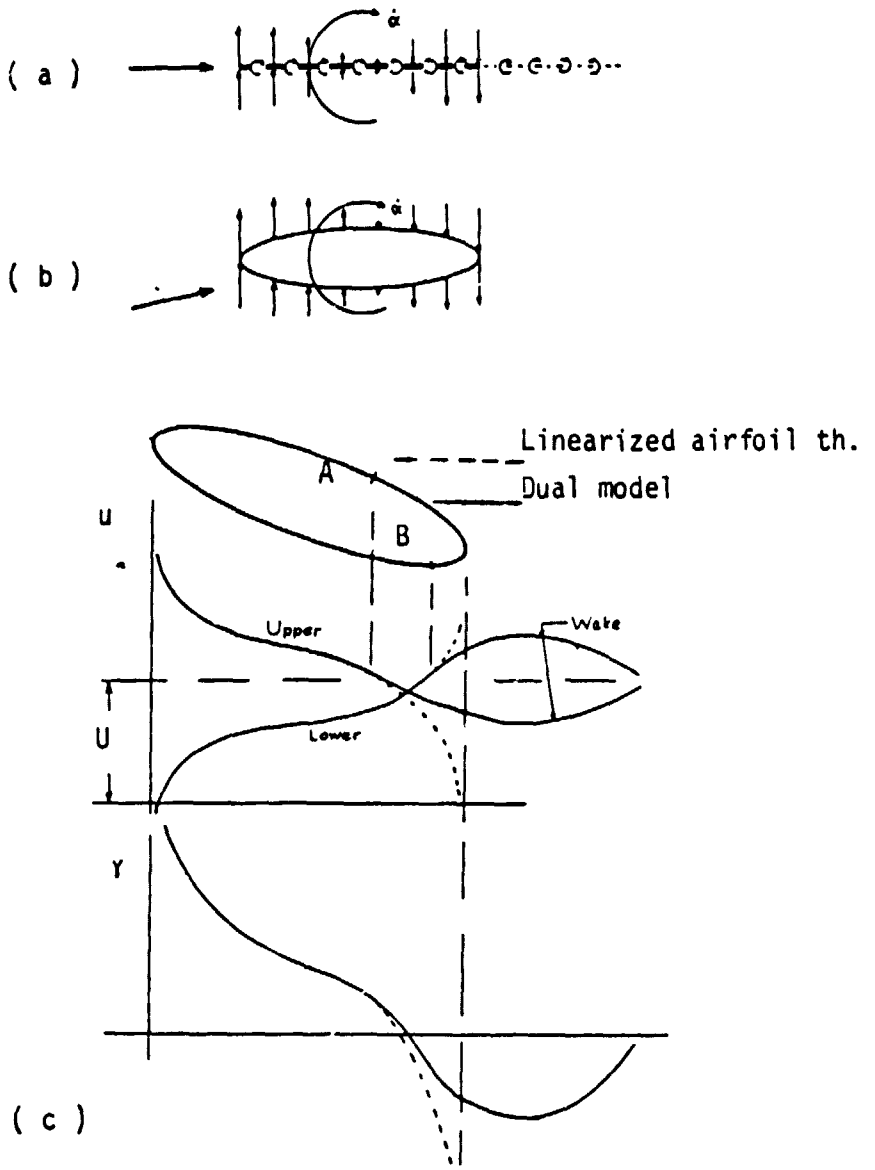

Figure 42: Dual method for predicting the unsteady aerodynamic forces on airfoils with rounded trailing-edges (Sears [1976]): (a) bound and free vortex; (b) model for boundary layer calculations; (c) velocity component u at upper and lower surface of vortex sheets, and corresponding vortex strength γ.

VITA

The author was [redacted]. He spent most of his life in Alexandria, where he was educated.

The author graduated with honors from Alexandria University in 1975, receiving the degree of Bachelor of Science in Mechanical Engineering. In 1979 he also received the Bachelor of Science in Mathematics from the same University. The author received a National Aeronautics and Space Administration assistantship for the period September 1979 through December 1980 under the auspices of the Mechanical Engineering and Mechanics Department of Lehigh University in Bethlehem, Pennsylvania. He is presently attending this institution, where he expects to receive the Master of Science in Mechanical Engineering degree in October 1981. The author will continue his graduate study at Georgia Institute of Technology in the summer of 1981.