ASPECTS OF COULOMB DAMPING IN ROTORS

SUPPORTED ON HYDRODYNAMIC BEARINGS

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ABSTRACT

The paper is concerned with the effect of friction in drive couplings on the non-sychronous whirling of a shaft. A simplified model is used to demonstrate the effect of large coupling misalignments on the stability of the system. It is concluded that provided these misalignments are large enough, the system becomes totally stable provided the shaft is supported on bearings exhibiting a viscous damping capacity.

SUMMARY

Rotating damping is well known to be a source of instability in shaft systems and many physical and mathematical explanations of the phenomenon exist. Published work seems mainly to be concentrated on a viscous or hysteretic damping model for the rotor, with simple dissipative bearings. The present paper considers non-linear damping of the Coulomb type in a simple rotor supported on 'realistic' linearized bearings, characterized by 4 stiffness and 4 damping coefficients.

The paper examines the nature of the sub-synchronous limit cycles resulting from interplay between the Coulomb damping within the rotor and viscous damping in the bearings. The rotating damping in the present model is assumed to arise from rubbing within gear type couplings at both ends of the rotor. The qualitative findings are however applicable to any mechanism producing friction forces depending on rotor flexure. It is also shown that as the couplings are misaligned in any plane, a level is reached above which the sub-synchronous oscillations collapse. A map of these limiting misalignments is compared with the shaft orbit under conditions of perfect alignment.

INTRODUCTION

Rotating damping has long been recognised as a source of instability in shaft systems. Both physical and mathematical explanations of the reasons for this instability have been advanced. A good summary has been given by Crandall (Ref. 1). Viscous and hysteretic damping is easily incorporated in a linear mathematical model. In many practical cases, however, the damping arises from relative motions within the rotating structure, the force-motion relationships obeying non-linear laws. A Coulomb damping model gives a reasonable representation of the energy exchanges resulting from slipping mechanisms, although to accurately incorporate such a model in a rotor/bearing system presents some problems in analysis.

Intuitively, one might suppose that instability due to any saturable type of rotating damping would be subject to limit cycles. This is only so if the system also contains positive damping elements with a more powerful energy-amplitude ratio. Such a system is a shaft with rotating Coulomb type damping supported on realistic, albeit linearized, hydrodynamic bearings. Very little published work exists on this very commonly occurring system and the present paper deals with a particular phenomenon of considerable practical significance.

The system dealt with is a rotor supported on two hydrodynamic bearings and driven by a gear type coupling. The rotating damping arises from movements between mating teeth of the coupling.

THE MODEL

The paper deals with an idealised model (Fig. 1). The degrees of freedom have been limited to 4, this being the minimum necessary to give a qualitatively adequate solution. The system comprises an isotropic non-damped shaft symmetric about its centre of span and characterised by a single flexural mode. The shaft is supported on two linearized hydrodynamic bearings, notionally represented as 8 stiffness and damping coefficients, each varying with running speed. Each end of the shaft moves within a coupling which has a fixed slope but is free to move in translation. The laws governing the force-velocity relationship within the couplings can be either of the viscous or Coulomb type. Provision is made for altering the coupling slope in both vertical and horizontal planes.

SYMBOLS

Axx_{11} etc.	kg	Elements of Inertia Matrix	Subscripts denote position in matrices.
Bxx_{11} etc.	Nsm^{-1}	Elements of Damping Matrix	
Exx_{11} etc.	Nm^{-1}	Elements of Stiffness Matrix	
B		Parameter concerned with viscous rotating damping (Eqn.7)	
Bxx etc.		Non dimensional oil film damping	
B^1xx etc.	Nsm^{-1}	Dimensional oil film damping	
b	Nsm^{-1}	Dimensional viscous damping of all coupling teeth moving axially	
C	m	Radial clearance of bearings	
Exx etc.		Non dimensional oil film stiffness	
E^1xx etc.		Dimensional oil film stiffness	
ex, ey	rad	Angular misalignment of couplings, Planes x, y	
F	N	Coulomb damping force generalised (Eqn.12)	
f	N	Coulomb damping force of all coupling teeth moving axially	
g	ms^{-2}	Acceleration due to gravity	
L	m	Half span of shaft	

M	kg	Half mass of shaft
m	kg/m^{-1}	Mass of increment of shaft
q_1x etc.		Generalised co-ordinate. Shaft centre x plane
r	m	Radius of coupling teeth
t	s	Time
V_x, V_y	ms^{-1}	Generalised velocity of rubbing of gear teeth
W_c	(Nm)	Work due to Coulomb damping/cycle
W_v	(Nm)	Work due to viscous damping/cycle
x, y		Horizontal and vertical planes
γ		Viscous rotating damping factor (Eqn.7)
Δ		Modal parameter (Eqn.4)
δx, δy		Misalignment parameters (Eqn.9)
ϕ		Mode shape of shaft
ϕ_0	m	Modal co-ordinate at shaft centre
ϕa^1	θ (rad)	Modal slope at shaft end
χ		Co-ordinate vector (Eqn.2)
ψ		Modal parameter (Eqn.6)
λ	$rad.s^{-1}$	Natural frequency of pinned-pinned shaft
ω	$rad.s^{-1}$	Lowest natural frequency
Ω	$rad.s^{-1}$	Running speed

EQUATIONS OF MOTION

The system co-ordinates (Fig. 2) relate to movements in the horizontal and vertical planes. The coefficients of the equations of motion are expressed as elements of square matrices using a subscript code where x, y determine the plane and $_1$, $_2$ relate to the shaft and bearing respectively. Thus Axy_{12} would appear in a matrix as follows:-

$$\left[\begin{array}{cc|cc} - & - & - & A_{xy_{12}} \\ \hline - & - & - & - \end{array}\right] \left[\begin{array}{c} \ddot{X} \\ \hline \ddot{Y} \end{array}\right] \qquad (1)$$

In the systems with viscous rotating damping the equations of motion are:-

$$A\ddot{\chi} + B\dot{\chi} + E\chi = 0 \qquad (2)$$

Dimensional	Non Dimensional Inertia	
$Axx_{11} = Ayy_{11} = \Sigma m(\frac{\phi}{\phi_0})^2$	$\frac{\Sigma m\phi^2}{M\phi_0{}^2} = \Delta$	
$Axx_{22} = Ayy_{22} = M$	1	
$Axx_{12} = Ayy_{21} = Axx_{21} = Ayy_{21} = \Sigma m(\frac{\phi}{\phi_0})$	$\frac{\Sigma m\phi}{M\phi_0} = \psi$	
$Bxx_{22} = B^1xx$	$g\ Bxx/\Omega$	
$Bxy_{22} = B^1xy$	$g\ Bxy/\Omega C$	
$Byx_{22} = B^1yx$	$g\ Byx/\Omega C$	
$Byy_{22} = B^1yy$	$g\ Byy/\Omega C$	
$Bxx_{11} = Byy_{11} = \frac{br^2}{2}(\frac{\phi a^1}{\phi_0})^2$	$B = \frac{br^2}{2M}(\frac{\phi a^1}{\phi_0})^2$	(3)
$Exx_{11} = Axx_{11}\lambda^2$	$\lambda^2\ \Delta$	
$Eyy_{11} = Ayy_{11}\lambda^2$	$\lambda^2\ \Delta$	
$Exx_{22} = E^1xx$	$g\ Exx/C$	
$Exy_{22} = E^1xy$	$g\ Exy/C$	
$Eyx_{22} = E^1yx$	$g\ Eyx/C$	
$Eyy_{22} = E^1yy$	$g\ Eyy/C$	
$Exy_{11} = -Eyx_{11}{}^2 = \frac{br^2}{2}(\frac{\phi a^1}{\phi_0})^2\Omega$	$\frac{br^2}{2M}(\frac{\phi a^1}{\phi_0})^2\Omega$	

Regarding the shaft geometry, since the equations of motion are set up in terms of the pinned-pinned shaft modes ϕ, then it is convenient to assume a simple shaft geometry. The choices are a disc/weightless shaft combination or a uniform shaft. The latter model has been chosen and this results in a sinusoidal modal shape for which:-

$$\Delta = \Sigma m\phi^2/M\phi_0{}^2 = 0.5 \tag{4}$$

$$\psi = \Sigma m\phi/M\phi_0 = 2/\pi \tag{5}$$

$$\frac{\phi a^1}{\phi_0} = \pi/2L \tag{6}$$

Roots of equation 2 give both system frequencies and dampings. If the system damping factor is plotted against the rotating damping (Fig. 3) it is possible to find the reserve of viscous damping possessed by the system. The viscous rotating damping factor is conveniently expressed as:-

$$\gamma = \frac{r^2}{4\Delta}\left(\frac{\phi a^1}{\phi_0}\right)^2 \bullet \frac{b}{M\lambda} = \frac{B}{2\lambda\Delta} \tag{7}$$

γ being given as a proportion of critical damping of the shaft vibrating in flexure.

The damping B arises from the axial rubbing of the gear teeth in the coupling. These teeth are assumed to be continuously distributed on a pitch circle of radius r. The value b relates to the total viscous force/unit velocity obtained from sliding the coupling axially. The values of all parameters used in plotting the curve of Figure 3 are shown on that sheet and will be used for all further work in the paper. The bearing coefficients relate to a specific non-circular bearing profile of the fixed arc type.

Solutions of the linear equation of motion yield not only the system frequencies and dampings but the orbit shape, the size being, of course, arbitrary.

The coupling damping operates on the relative velocities of teeth arising from small vibrations in the system. It is possible to generalise these velocities in terms of the shaft co-ordinates as follows:-

$$\begin{aligned} Vx &= \dot{q}_1 x + \Omega\,(q_1 y + \delta y) \\ Vy &= \dot{q}_1 y - \Omega\,(q_1 x + \delta x) \end{aligned} \qquad (8)$$

where δx, δy relate to the angular misalignments of the coupling ex, ey thus:-

$$\delta x = ex\,\left(\frac{\phi_0}{\phi a^1}\right) \qquad \delta y = ey\,\left(\frac{\phi_0}{\phi a^1}\right) \qquad (9)$$

The energy reserve/cycle in viscous damping at the coupling is given by:-

$$Wv = B\int_0^{2\pi/\omega}(Vx\,\dot{q}_1 x \;+\; Vy\,\dot{q}_1 y)dt \qquad (10)$$

Coulomb Damping

Suppose now that the damping mechanism in the coupling is of the Coulomb type and that f is the force resisting axial movement of the teeth. The Coulomb energy/cycle is:-

$$Wc = F\int_0^{2\pi/\omega}\frac{(Vx\,\dot{q}_1 x + Vy\,\dot{q}_1 y)dt}{(Vx^2 + Vy^2)^{\frac{1}{2}}} \qquad (11)$$

where $$F = \frac{2r}{\pi}\;\left(\frac{\phi a^1}{\phi_0}\right)\,\frac{f}{M} = \frac{r}{L}\cdot\frac{f}{M} \qquad (12)$$

Note that strictly speaking Vx, Vy are not the same in the Coulomb case as in the viscous case, but for reasons which will be discussed later, the error involved in assuming both orbits to be identical ellipses is very small.

Plotting the work function (Wc - Wv) against orbit size for zero misalignment (Fig. 4) yields the magnitude of a limit cycle at the cross-over point. The fact that the cross-over slope is negative indicates that this is a stable cycle. The orbit size which refers to the length of the major axis of the ellipse at the shaft centre is proportional to f/M.

Misalignments

Having fixed the orbit size for the system under consideration, the values ex, ey are now systematically varied. Figure 5 shows that depending on the values of these parameters the character of the work function changes. Under some circumstances the function shows 2 cross-over points. The smaller orbit is unstable and the larger one is stable. Certain combinations of the misalignments give a totally stable system, there being a threshold at which a collapse from a finite orbit to zero takes place.

It is possible to plot a map of constant orbit combinations as in Figure 6. Here it will be seen that there exists a boundary in the ex/ey plane beyond which, for the model assumed, instability cannot exist. Within the boundary, limit cycles exist which are smaller than the limit cycle of the zero misaligned case.

DISCUSSION

It is first necessary to examine the validity of the model with regard to its general applicability. The semi-rigid modelling of the shaft implies certain constraints. This does not, however, affect the qualitative behaviour of the system, particularly that relating to the stabilizing effects of large misalignments. It does, however, mean that the system frequency and the orbit size will be incorrect, but to obtain a more accurate model it is only necessary to incorporate more shaft freedoms. Either a finite element or a modal representation could be used.

The present mathematical treatment of Coulomb damping also requires justification. It is reasonably easy, if tedious, to arrive at limit cycles by time marching methods or other slightly more elegant algorithms. Alternatively, a carefully constructed analogue computer circuit can solve the problem. All of these techniques have been used by the author on the particular problem presented in the present paper and for this case it can be shown that the orbits are very nearly elliptical with little harmonic content. Thus the assumptions used in deriving the energy expression (Wc - Wv) are valid. The reasons for this are fairly obvious. The energy supplied by the Coulomb damping is a small proportion of the inertial and strain energy in the system so that the orbits are largely controlled by these latter energy exchanges. This circumstance results from the fact that the shaft is very flexible, as evidenced by the small ratio of frequency: running speed (ω/Ω Figure 3).

The question arises as to what happens if the shaft is stiffer. As shaft stiffness increases, so does the stability margin. This margin is expressed in the present context as the viscous rotating damping necessary to reduce the system damping to zero. The author has found that reasonably accurate results can occur when this viscous damping margin is well over 15%.

Any rotating damping, whether viscous, hysteretic or Coulomb, reduces the stability margin of a rotating system. This means that non-sychronous whirling will start at a lower speed than that dictated solely by consideration of the oil film. Since most rotors contain some rotating damping mechanism, then extractions of the oil film coefficients from rotor/bearing systems may well be subject to inaccuracies.

More important, however, is what, in the authors' view, is the erroneous deduction from many tests and case histories, that oil film non-linearities are responsible for the small limit cycles often experienced prior to large non-sychronous orbits. The probability is that these limit cycles arise from the existance of friction in the drive mechanism. To be sure oil film non-linearities operate to limit journal orbits which are large compared with the bearing clearance, but this is a gross condition in which the oil film becomes effective around the complete bearing circumference.

In the present mathematical example, a single running speed has been considered so far. Obviously as this running speed rises and its associated oil film co-efficients change, the stability margin reduces. According to our simple model the orbits should increase with running speed and at the 'resonant oil whirl' threshold, should grow without limit. Under these circumstances, however, it is obvious that constraints associated with bearing clearances etc. will take over so that the increase in orbit size may not be as dramatic as Figure 7 predicts. In any case the present theory deals with small vibrations and above a certain level the force/spatial relationships within the coupling may become much more complex than those assumed here.

The nearer the running speed approaches the 'true' oil film instability threshold, the easier it is to supply the small amount of friction damping necessary to promote limit cycles. Figure 8 shows a shaft weighing 700 kgs supported on 2, 100 mm dia hydrodynamic bearings. Figure 9 shows friction being introduced by 3 small steel 'fingers' supported on a freely rotating layshaft. The pressure between the fingers and the main shaft was introduced by elastic bands. This very small amount of damping produced from the dynamic slope of the shaft was quite sufficient to lower the instability threshold and produce stable limit cycles. The stabilizing effect of large misalignments was also demonstrated by varying the angle of the layshaft.

CONCLUDING REMARKS

The author has dealt with the subject of limit cycles in a shaft/hydrodynamic bearing system with rotating damping present in the drive coupling. The same conclusions, i.e. that limit cycles exist when saturable damping mechanisms are present, would apply whenever friction was caused by shaft flexure. The point brought out in the present paper is that the destabilizing effect of Coulomb damping can be 'biased out' by friction arising in a fixed plane and resulting from the rotation. In many practical situations such a biasing would be impossible in that very heavy fretting wear would be brought about by the gross misalignments necessary to achieve stability. Nevertheless, the mechanism is of some interest and may not only apply to couplings, but to other sources of rubbing in rotating elements.

REFERENCES

1. Crandall, S.H.: Physical Explanations of the Destabilizing Effect of Damping in Rotating Parts. NASA Conference Publication 2133, May 12-14, 1980, Pages 369-382.

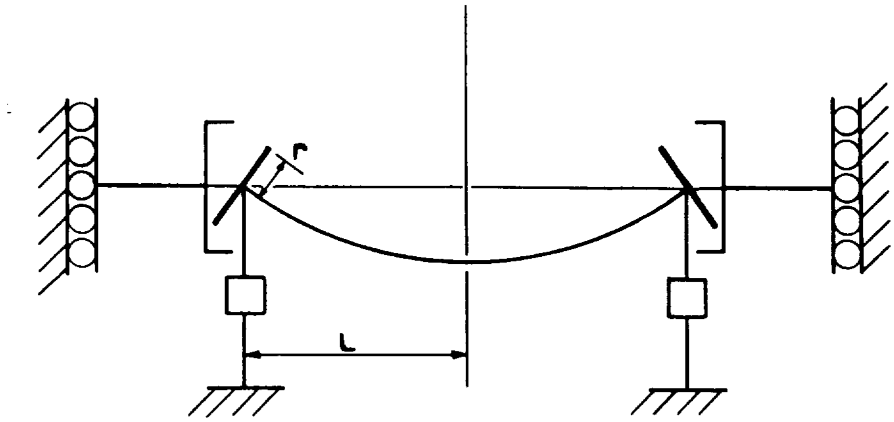

Figure 1. - Diagrammatic arrangement of shaft and couplings.

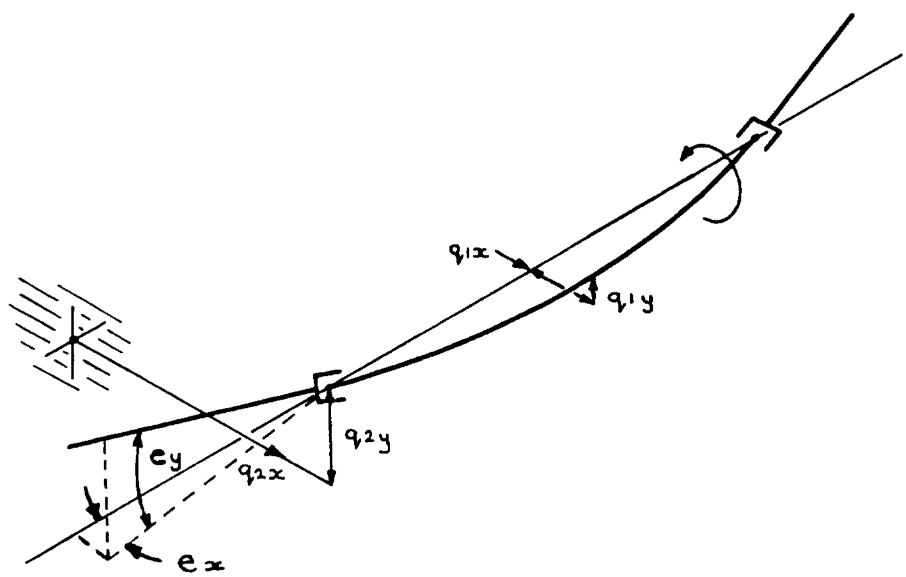

Figure 2. - Coordinate system.

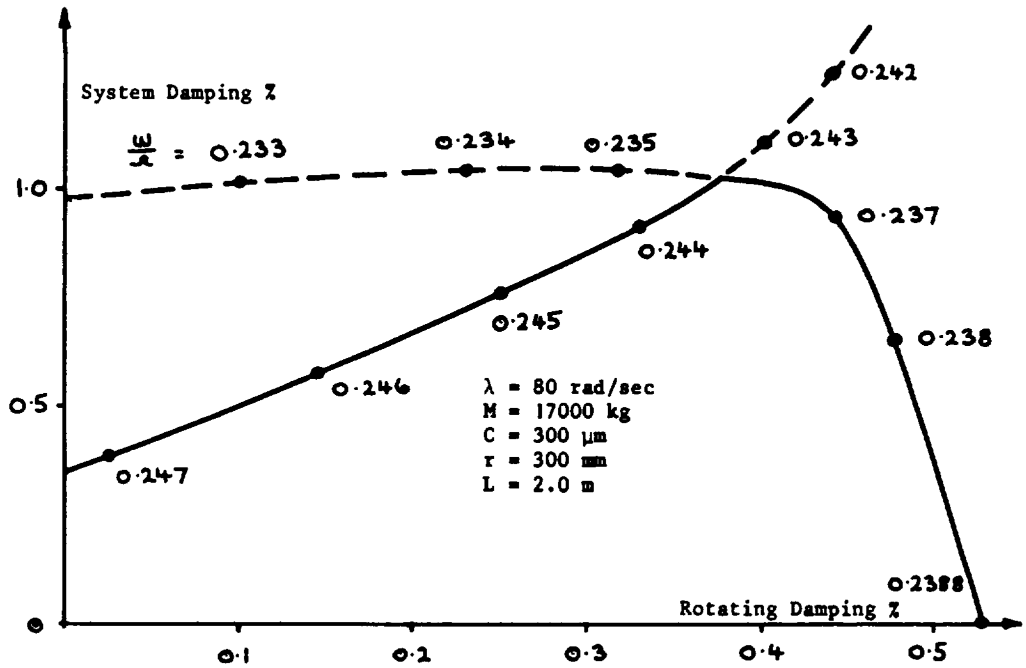

Figure 3. - Relationship between system damping and rotating viscous damping.

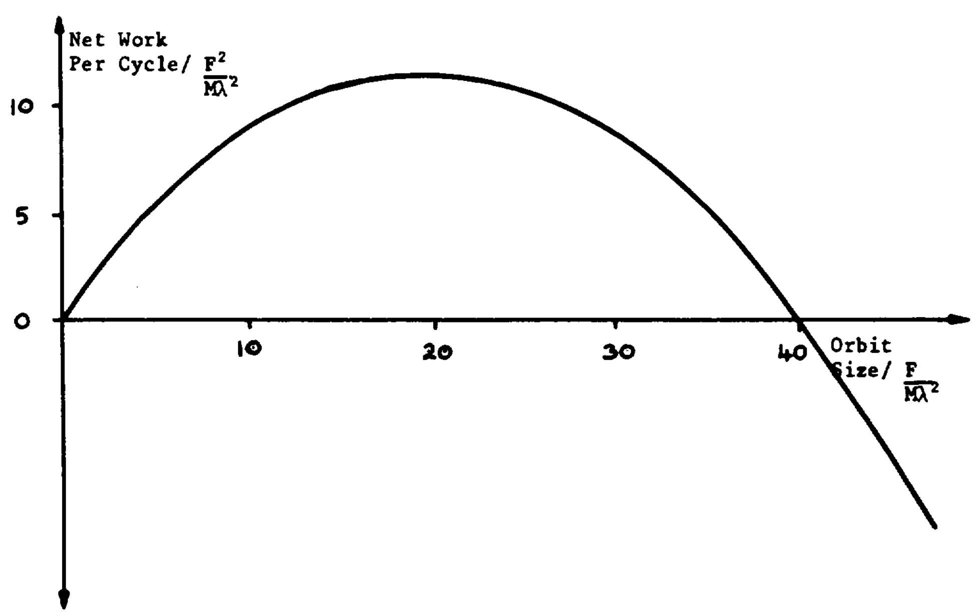

Figure 4. - Net work ~ orbit size.

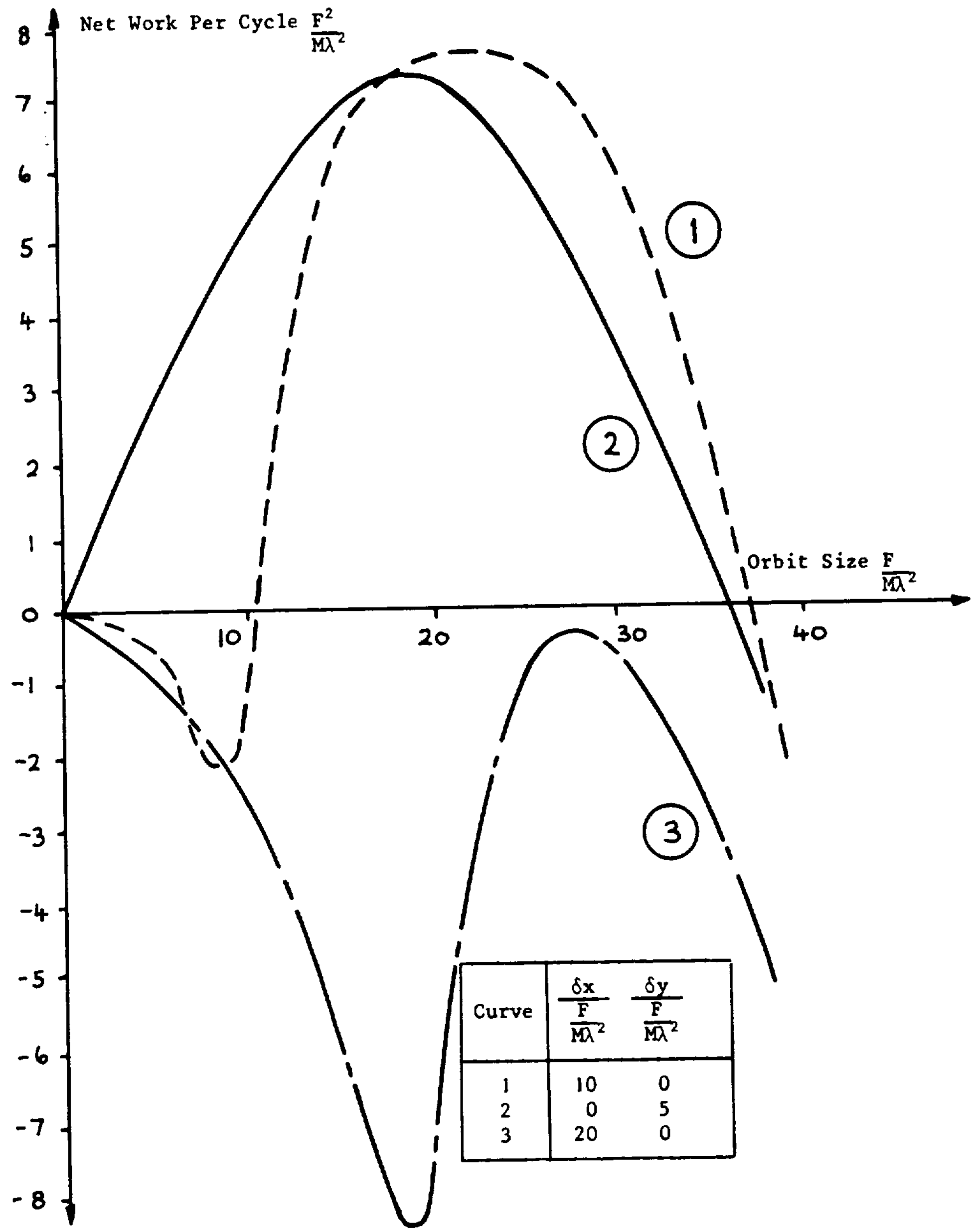

Figure 5. - Effect of misalignment on work function.

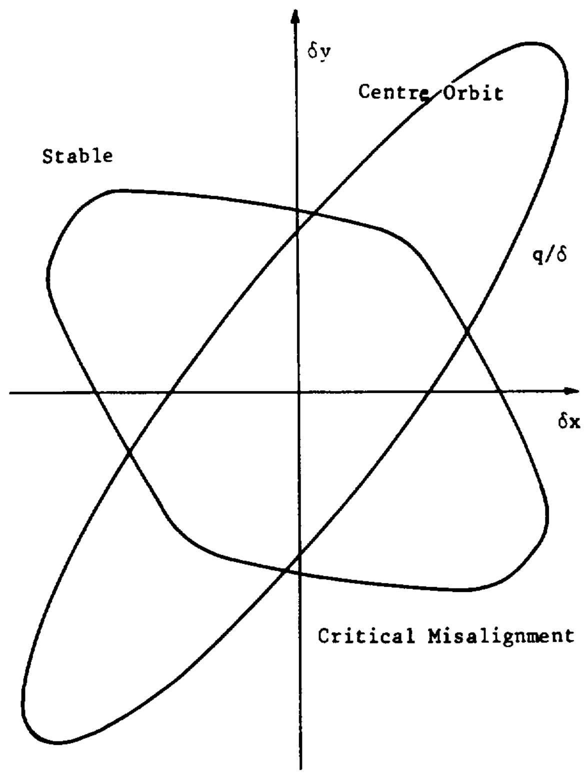

Figure 6. - Critical misalignment boundary compared with orbit of fully aligned system.

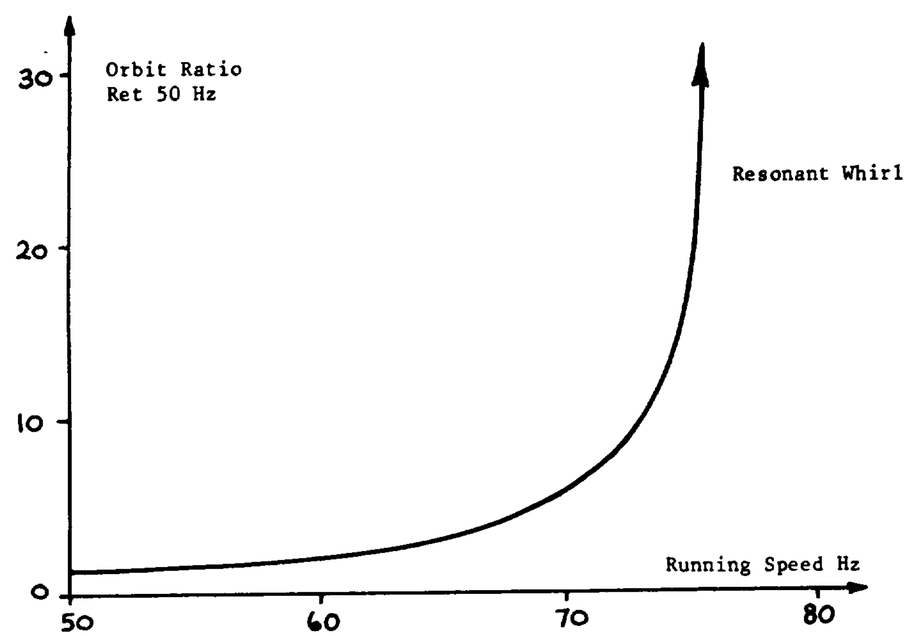

Figure 7. - Growth of orbit size with running speed.

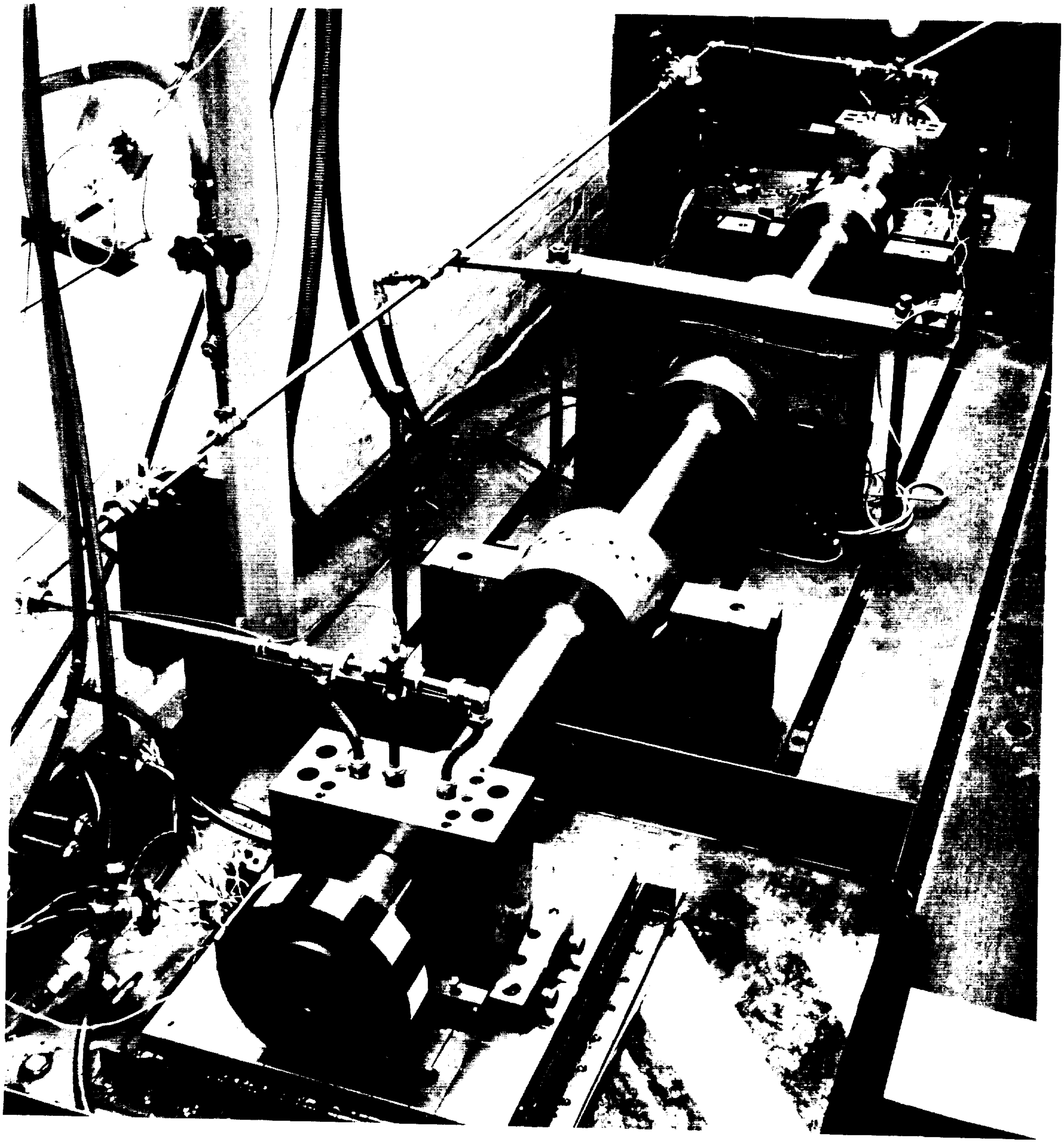

Figure 8. -Model rotor supported on 100mm dia. hydrodynamic bearings.

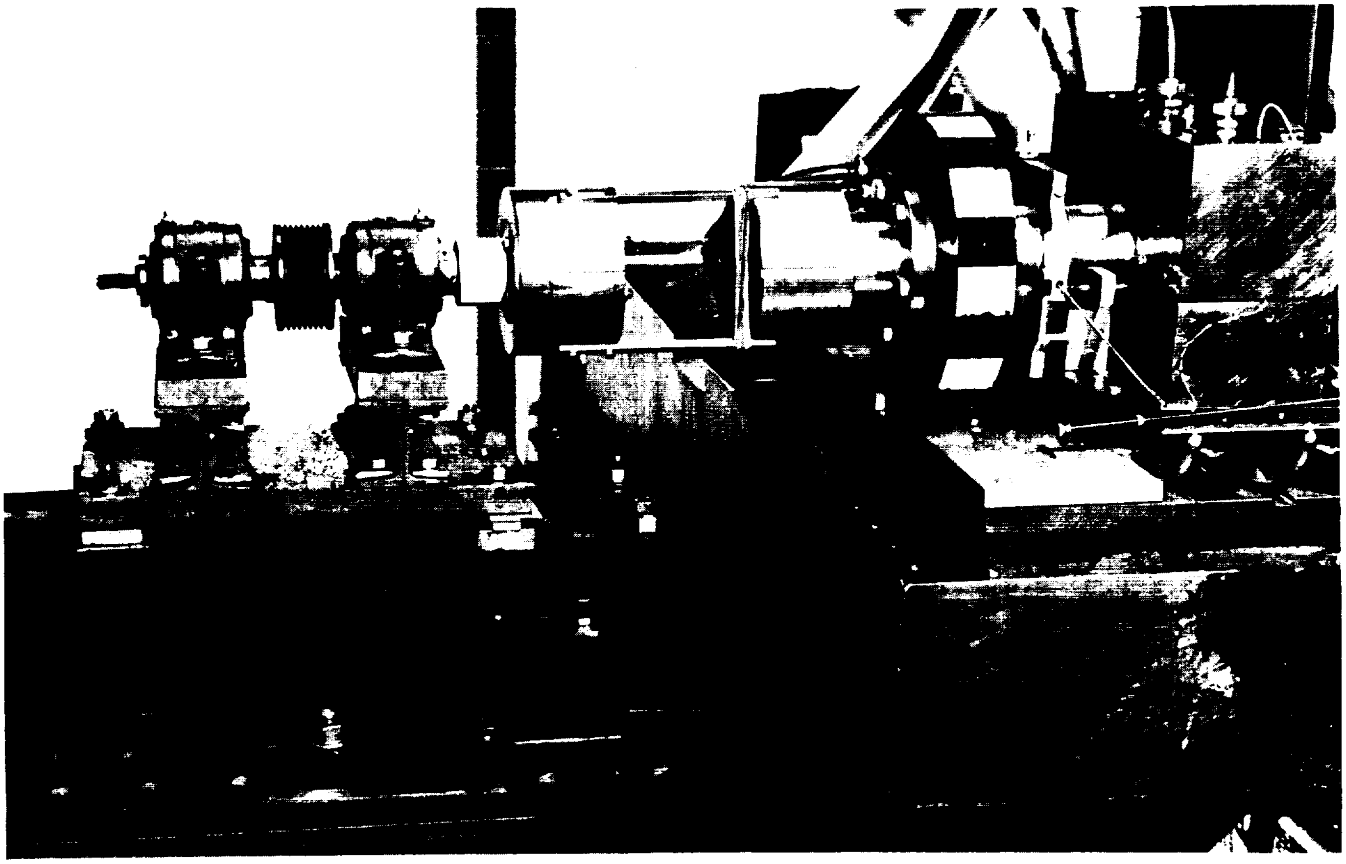

Figure 9. - Method of introducing friction damping into model rotor.