Planning Fuel-Conservative Descents With or Without Time Constraints Using a Small Programmable Calculator

Algorithm Development and Flight Test Results

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SUMMARY

The Federal Aviation Administration (FAA) has implemented an automated, time-based metering form of air traffic control with profile descent procedures for arrivals into the terminal area. These concepts provide fuel savings by matching the airplane-arrival flow to the airport acceptance rate through time-control computations and by allowing the pilot to descend at his discretion from cruise altitude to a designated metering fix in an idle-thrust, clean (landing gear up, flaps zero, and speed brakes retracted) configuration. Substantial fuel savings have resulted from these procedures, but air traffic control (ATC) workload is high since the radar controller maintains time management for each airplane through either speed control or path stretching with radar vectors. Pilot workload is also high since the pilot must plan for an idle-thrust descent to the metering fix using various rules of thumb.

The National Aeronautics and Space Administration (NASA) has developed an airborne descent algorithm compatible with time-based metering procedures and profile descent procedures designed to improve the accuracy of delivering an airplane to a metering fix at a time designated by the ATC system. This algorithm provides open-loop guidance for an airplane to make an idle-thrust, clean-configured descent to arrive at the metering fix at a predetermined time, altitude, and airspeed. The algorithm may also be used for planning fuel-conservative descents when time is not a consideration.

The algorithm was programmed on a small, hand-held, programmable calculator. Flight tests were conducted using the calculator on a T-39A (Sabreliner) airplane. The results of these tests indicated that the open-loop guidance, provided to the pilot in the form of an indication on the distance-measuring equipment (DME) of the point to start the idle-thrust descent and Mach and airspeed indications during the descent, could be used to execute the descent as predicted by the algorithm. The resulting mean distance and time errors to actually achieve the predicted speed and altitude conditions at the end of the descent profile were 1.2 n.m. long and 1.4 sec early.

INTRODUCTION

Rising fuel costs, combined with other economic pressures, have resulted in industry requirements for more efficient air traffic control and aircraft operations. The Federal Aviation Administration (FAA) has implemented an automated form of air traffic control (ATC) for arrivals into the airport terminal area. The concept provides for increased airport capacity and fuel savings by combining time-based metering with profile descent procedures. Time-based metering procedures provide for sequencing arrivals to the airport through time control of airplanes to metering fixes located 30 to 40 n.m. from the airport. A time is computed for each incoming airplane to cross one of the metering fixes, based on an estimate of its arrival to the runway, with adjustments made to resolve conflicts where more than one airplane would be on the runway simultaneously. By time metering the airplanes to these fixes, the low-altitude vectoring (and associated fuel consumption) required to sequence the airplanes into a final queue for landing is reduced. In addition,
delays due to terminal area sequencing may be absorbed at higher altitudes, further minimizing fuel usage (refs. 1 and 2).

Profile descent procedures permit the initiation of the descent at the pilot's discretion. This procedure allows the pilot to plan a fuel-conservative descent, that takes into account the performance characteristics of his particular airplane, so that the airplane will pass the metering fix at a specified altitude and airspeed.

In the present operational concept of the time-based metering operations, the ATC controller changes the flight-path length by issuing radar vectors and/or commands speed changes, as required, so that each airplane crosses the metering fix at its assigned time. These operations have resulted in errors of between 1 and 2 min in airplane arrival time at the metering fix (ref. 3). The pilot is responsible for planning a fuel-conservative descent from cruise altitude to arrive at the metering fix at a designated altitude and airspeed. Since no guidance is computed, the pilots are forced to rely on experience and various rules of thumb to plan the descent. This results in high cockpit workloads and in the full potential of fuel savings from a planned descent not being obtained (ref. 4).

The National Aeronautics and Space Administration (NASA) has developed and flight tested, in its Advanced Transport Operating Systems (ATOPS, previously designated the Terminal Configured Vehicle) Boeing 737 research airplane, a flight-management descent algorithm (ref. 5) which provides closed-loop guidance to the pilot for a time-constrained, fuel-conservative descent to the metering fix. These flight tests demonstrated that the arrival time dispersion at the metering fix could be reduced to less than 10 sec and pilot workload could be reduced with the use of a flight-management system integrated with an advanced guidance and display system.

The favorable results of these tests encouraged additional research to examine the feasibility of providing open-loop guidance for airplanes with conventional cockpit instrumentation. This research was accomplished by programming a simplified version of the flight-management descent algorithm on a small programmable calculator. The Mach and airspeed indicators were used to provide open-loop guidance during the descent. Flight tests were conducted with a T-39A (Sabreliner) airplane to study the effects of algorithm simplification and open-loop guidance upon the accuracy of arriving at a metering fix at a predetermined time, speed, and altitude. This report describes the simplified descent-planning algorithm, documents the sensitivity of the descent profile to variations of pilot inputs used in the computations, and summarizes the T-39A flight test results.

SYMBOLS AND ABBREVIATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>ATC</td>
<td>air traffic control</td>
</tr>
<tr>
<td>ATOPS</td>
<td>Advanced Transport Operating Systems</td>
</tr>
<tr>
<td>CAS</td>
<td>calibrated airspeed, knots</td>
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<tr>
<td>CAS&lt;sub&gt;d&lt;/sub&gt;</td>
<td>calibrated airspeed used during descent, knots</td>
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<tr>
<td>CAS&lt;sub&gt;d,fixed&lt;/sub&gt;</td>
<td>descent calibrated airspeed fixed for speed iteration computations, knots</td>
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<tr>
<td>CAS&lt;sub&gt;d,i&lt;/sub&gt;</td>
<td>descent calibrated airspeed computed on ith iteration, knots</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
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<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\text{CAS}_{\text{d,initial}}$</td>
<td>computed initial descent calibrated airspeed, knots</td>
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<tr>
<td>$\text{CAS}_{\text{d,max}}$</td>
<td>maximum operational descent calibrated airspeed, knots</td>
</tr>
<tr>
<td>$\text{CAS}_{\text{d,min}}$</td>
<td>minimum operational descent calibrated airspeed, knots</td>
</tr>
<tr>
<td>$\text{CAS}_{\text{MF}}$</td>
<td>calibrated airspeed to cross metering fix, knots</td>
</tr>
<tr>
<td>DME</td>
<td>distance-measuring equipment</td>
</tr>
<tr>
<td>$D_{w,h}$</td>
<td>magnetic wind direction evaluated at altitude $h$, deg</td>
</tr>
<tr>
<td>$D_{w,s}$</td>
<td>magnetic wind direction computed for sea-level altitude, deg</td>
</tr>
<tr>
<td>$\frac{dD_{w}}{dh}$</td>
<td>wind direction gradient with respect to altitude, deg/ft</td>
</tr>
<tr>
<td>$\frac{dS_{w}}{dh}$</td>
<td>wind speed gradient with respect to altitude, knots/ft</td>
</tr>
<tr>
<td>GW</td>
<td>gross weight, lb</td>
</tr>
<tr>
<td>HD</td>
<td>airplane magnetic track angle along ground, deg</td>
</tr>
<tr>
<td>$h$</td>
<td>altitude, ft</td>
</tr>
<tr>
<td>$h_c$</td>
<td>cruise altitude, ft</td>
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<tr>
<td>$h_{GP}$</td>
<td>geopotential altitude, ft</td>
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<tr>
<td>$h_{MF}$</td>
<td>metering-fix altitude, ft</td>
</tr>
<tr>
<td>$h_p$</td>
<td>pressure altitude, ft</td>
</tr>
<tr>
<td>$h_{xo}$</td>
<td>altitude at transition from constant Mach descent to constant airspeed descent, ft</td>
</tr>
<tr>
<td>$\dot{h}$</td>
<td>rate of change of altitude, ft/sec</td>
</tr>
<tr>
<td>$\dot{h}<em>{\text{CAS}</em>{d}}$</td>
<td>rate of change of altitude evaluated at calibrated airspeed $\text{CAS}_{d}$, ft/sec</td>
</tr>
<tr>
<td>$\dot{h}_{\text{Md}}$</td>
<td>rate of change of altitude evaluated at Mach number $\text{Md}$, ft/sec</td>
</tr>
<tr>
<td>IDL</td>
<td>calculator display showing point where thrust should be reduced to flight idle, n.mi.</td>
</tr>
<tr>
<td>ISA</td>
<td>ICAO Standard Atmosphere</td>
</tr>
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</table>
\[ K \]
interpolation factor computed for speed iteration purposes

\[ K_h \]
gross-weight multiplication factor for altitude rate for constant calibrated-airspeed descent

\[ K_{dh/dh} \]
gross-weight multiplication factor for change of altitude rate with respect to altitude for constant Mach descent

\[ K_1 = \left( \frac{dh}{dt} \right) M_d^{*} \text{ that has been corrected for gross-weight effects,} (\text{ft/sec})/\text{ft} \]

\[ K_2 \]
equation constant, \text{ft/sec}

\[ l_e \]
distance between entry fix and metering fix, n.mi.

\[ M \]
Mach number

\[ M/CAS \]
Mach number and calibrated airspeed

\[ MSL \]
mean sea level

\[ M_C \]
cruise Mach number

\[ M_d \]
descent Mach number

\[ M_d,\text{fixed} \]
descent Mach number fixed for speed iteration purposes

\[ M_d,\text{initial} \]
computed initial descent Mach number

\[ M_d,\text{max} \]
maximum operational descent Mach number

\[ M_d,\text{min} \]
minimum operational descent Mach number

\[ OAT \]
outside air temperature, °F

\[ S_{w,h} \]
wind speed evaluated at altitude \( h \), knots

\[ S_{w,s} \]
wind speed computed for sea-level altitude, knots

\[ TAS \]
true airspeed, knots

\[ TRK \]
airplane magnetic track angle along ground, deg

\[ T_0 \]
standard sea-level air temperature, °R

\[ T' \]
nonstandard sea-level air temperature, °R

\[ T_{st} \]
static air temperature, °R

\[ T_{st,c} \]
static air temperature measured at cruise altitude, °R

\[ t_E \]
time error for descent speed convergence criteria, sec

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DESCRIPTION OF FLIGHT-MANAGEMENT DESCENT ALGORITHM

Description of General Profile

The flight-management descent algorithm computes the parameters required to describe a seven-segment cruise and descent profile (fig. 1) between an arbitrarily located entry fix and an ATC-defined metering fix. The descent profile is computed based on linear approximations of airplane performance for an idle-thrust, clean-configured descent. Airplane gross weight, wind, and nonstandard-temperature effects are also considered in these calculations.

Figure 1 shows the vertical-plane geometry of the path between the entry fix and the metering fix. Each path segment, starting at the metering fix, is numbered according to the order in which it is calculated by the algorithm. To be compatible with standard airline operating practices, the path is calculated based upon the descent being flown at a constant Mach number with transition to a constant calibrated airspeed and all speed reductions made in level flight.

The first segment traversed on the profile is segment 7 which begins at the entry fix and is flown at constant cruise altitude and Mach number. Segment 6 is a relatively short, level flight path segment in which the pilot reduces thrust to flight idle so the airplane will slow from the cruise Mach number to the descent Mach number. Segment 6 is eliminated if the descent and cruise Mach numbers are the same. Once the descent Mach number is attained, the descent is started at the beginning of segment 5. Segment 5 is flown at a constant Mach number. As altitude is decreased along this path segment, the calibrated airspeed will increase because of increasing air pressure. Segment 4 begins when the desired calibrated airspeed is attained for descent. The descent is continued along this segment at the desired, constant calibrated airspeed. If the metering-fix altitude is below 10 000 ft MSL and the descent airspeed is greater than 250 knots, segments 2 and 3 are computed for the pilot to comply with the ATC-imposed airspeed limit of 250 knots, or less, below 10 000 ft MSL. Segment 3 is a level-flight segment at 10 000 ft MSL where the airspeed is reduced to 250 knots. The descent is then continued at 250 knots along segment 2. When the metering-fix altitude has been reached, the airplane is flown at a...
constant altitude along segment 1 and slowed from the descent airspeed to the designated calibrated airspeed over the metering fix. This path segment is eliminated if the descent and metering-fix airspeeds are the same.

The flight-management descent algorithm can be used in either of two modes. In the first mode the pilot can enter the desired M/CAS to be flown during descent. The descent profile is then computed based on this descent speed schedule without consideration of a constraint on metering-fix arrival time. This mode would be used when time-based metering is not being used.

The second mode was designed for time-metered operations. In this mode, instead of specifying the M/CAS descent schedule, the pilot enters the time that the entry fix was crossed and the metering-fix arrival time assigned by ATC. The descent profile is then calculated based on an M/CAS descent schedule, computed through an iterative process, that will closely satisfy the crossing times at both of these way points.

During the profile descent computations in the time-metered mode, checks are made to ensure that the descent Mach number (M_d) and the descent calibrated airspeed (CAS_d) are within the minimum and maximum speed limits for the particular airplane modeled. For the T-39A airplane, these limits were

\[
0.55 \leq M_d \leq 0.75 \\
180 \leq CAS_d \leq 350 \quad \text{(knots)}
\]

There was an additional constraint that CAS_d would not be less than the airspeed at which the airplane was to cross the metering fix, so that extra fuel would not be required to subsequently increase airspeed. If the ATC-assigned metering-fix crossing time requires a descent speed less than the airplane's minimum descent speed limit, the profile is computed based on the minimum allowable descent speeds and a message is displayed to the pilot to "hold" (delay) for an appropriate amount of time. A similar "late" message is displayed with the time error if a descent speed schedule greater than the maximum allowed is required.

Logic Flow of Profile Descent Algorithm

Figure 2 shows the logic flow of the profile descent computations. Pilot inputs used to compute the profile may be entered prior to flight and modified, as required, prior to the descent. These parameters include cruise altitude and Mach number, airplane gross weight, outside air temperature, entry-fix and metering-fix descriptions, and the course direction to the metering fix. In addition to these parameters, the pilot may enter either a particular Mach number and calibrated airspeed to be used during the descent or the entry-fix crossing time and the ATC-assigned metering-fix crossing time.

If the M/CAS descent speed schedule has been entered in the calculator, the computations will be based on a nonmetered traffic environment. The pilot initiates the computations by pushing the "compute" key. The descent profile is then computed in a single iteration, and the display gives the point where thrust should be reduced to flight idle to start the descent.
If the entry-fix crossing time and the ATC-assigned metering-fix crossing time have been entered in the calculator, the time required to fly between the fixes $\Delta t_{req}$ will be computed and subsequent calculations will be based on a time-metered traffic environment. Once the pilot has initiated the computations by pushing the "compute" key, an iterative process is started to determine an appropriate M/CAS descent speed schedule that will satisfy the time constraints.

The iterative process starts with the computation of an initial M/CAS descent speed schedule and the associated time $(\sum_{j=1}^{7} \Delta t_j)_{initial}$ required to fly between the entry and metering fixes at those speeds. To reduce the number of iterations required to satisfy the time convergence criteria, the initial descent speed schedule chosen is slightly less than the midpoint between the maximum and minimum speeds allowed for the descent, as shown in the following equations:

$$M_{d, initial} = M_{d, min} + 0.45(M_{d, max} - M_{d, min})$$

$$CAS_{d, initial} = CAS_{d, min} + 0.45(CAS_{d, max} - CAS_{d, min})$$

where

$$CAS_{d, min} = \begin{cases} 
CAS_{MF} \\
\text{or} \\
180 \text{ knots} 
\end{cases}$$

whichever is greatest.

A check is made with the following transition-time inequality to determine if the time-convergence criteria $t_E$ has been satisfied. (For the purposes of these tests, $t_E = 5$ sec.)

$$\left| \Delta t_{req} - \sum_{j=1}^{7} \Delta t_j \right| < t_E$$

where

$$\Delta t_{req} = t_{MF} - t_{EF}$$

If this inequality is satisfied, the computations are complete and the idle-thrust descent point is displayed to the pilot. If the inequality is not satisfied, a revised M/CAS descent speed schedule and the associated descent time will be computed. This process will be repeated until the time inequality is satisfied.
The computation of the revised M/CAS schedule, graphically depicted in figure 3, shows a plot of the time required to fly between a specified entry fix and the metering fix for the complete speed range of the airplane. The M/CAS descent speed schedule is revised through a linear interpolation of the desired $\Delta t_{\text{req}}$ within a range of time bounded by a computed upper and lower value. The lower value of the time range $(\sum_{j=1}^{7} \Delta t_j)_{i-1}$ is variable and equal to the time computed for the descent on the last iteration $(i-1)$. The upper value of the time range $(\sum_{j=1}^{7} \Delta t_j)_{\text{fixed}}$ is fixed for the iteration process and equal to the time computed for the descent based on one of the following descent speed schedules:

$$M_{d,\text{fixed}} = M_{d,\text{initial}} \quad \text{if} \quad \Delta t_{\text{req}} < \left( \sum_{j=1}^{7} \Delta t_j \right)_{\text{initial}}$$

or

$$M_{d,\text{fixed}} = M_{d,\text{min}} \quad \text{if} \quad \Delta t_{\text{req}} > \left( \sum_{j=1}^{7} \Delta t_j \right)_{\text{initial}}$$

An interpolation factor $K$ is then computed for use in revising the M/CAS descent speed schedule

$$K = \frac{\Delta t_{\text{req}} - \left( \sum_{j=1}^{7} \Delta t_j \right)_{i-1}}{\left( \sum_{j=1}^{7} \Delta t_j \right)_{\text{fixed}} - \left( \sum_{j=1}^{7} \Delta t_j \right)_{i-1}}$$

The M/CAS descent schedule is then revised as follows:

$$M_{d,i} = M_{d,i-1} - K(M_{d,i-1} - M_{d,\text{fixed}})$$

$$\text{CAS}_{d,i} = \text{CAS}_{d,i-1} - K(\text{CAS}_{d,i-1} - \text{CAS}_{d,\text{fixed}})$$

where $i$ is the $i$th iteration.

Empirical Representation of Airplane Performance Characteristics

Computer memory limitations within the programmable calculator preclude the use of detailed aerodynamic and performance tables to represent the airplane for profile descent calculations. Instead, an empirical model of the performance of the T-39A test airplane was developed. The generic form of the equations that were needed to...
develop the empirical model was first determined from simulator and flight test data of a B-737 airplane (ref. 5). Actual flight test data generated with the T-39A test airplane executing idle-thrust clean-configured constant M/CAS descents and level-flight speed reductions were then used to determine the specific numerical constants and parameters used for the performance models.

The mathematical model representing the vertical speed of the airplane during the constant Mach portion of the descent was developed based on the descent rate changing linearly with decreasing altitude. The slope \( \frac{dh}{dh} \) of this linear representation depends upon the magnitude of the descent Mach number. The form of this model is

\[
\frac{h_d}{d_d} = h \left( \frac{dh}{dh} \right) + b
\]

in which \( b \) is the magnitude of \( h \) at sea level. The coefficients of the model were derived from altitude and time data recorded for constant Mach descents ranging from 0.55 to 0.75 in 0.05 increments. The altitude data for each descent were smoothed by computing a quadratic equation of altitude as a function of time. Figure 4 shows a typical set of data and the resulting curve fit for a constant Mach descent of 0.55. The quadratic equation was differentiated to obtain \( h \) so that vertical speed could be modeled as a function of altitude. Figure 5 shows a family of curves of vertical speed performance derived from the data recorded on the T-39A airplane. It was observed that each of the linearized constant Mach lines passed through a common point at an altitude of 45 000 ft and at \( h = 7 \) ft/sec. This relationship allowed \( b \) to be derived and resulted in the following equation for \( h \):

\[
\frac{h_d}{d_d} = h \left( \frac{dh}{dh} \right) + 7 - \left( 45 \ 000 \right) \left( \frac{dh}{dh} \right)
\]

[ft/sec]

Figure 6 shows the variation of the slopes \( \frac{dh}{dh} \) of the linearized vertical-speed performance of the T-39A airplane (Gross weight = 15 500 lb) as a function of Mach number. A quadratic-regression analysis was used to obtain the coefficients for the following equation for the variation of \( \frac{dh}{dh} \) as a function of the descent Mach number:

\[
\left( \frac{dh}{dh} \right) = A_0 + \left( A_1 \cdot d_d \right) + \left( A_2 \cdot d_d \right)^2
\]

[(ft/sec)/ft]

\[
A_0 = 0.019371
\]

\[
A_1 = -0.063457
\]

\[
A_2 = 0.062420
\]

A similar derivation was used to obtain a function of altitude rate for the constant calibrated-airspeed descents. It was observed that \( h \) was approximately constant at all altitudes for a given calibrated airspeed, but decreased as a
function of airspeed. The magnitudes of the vertical speeds obtained from the flight test data and a plot of the linearized model representing the data are shown in figure 7. The equation of this model is

\[ \dot{h}_{\text{CAS}} = -0.4624 \text{ CAS}_d + 66.68 \text{ [ft/sec]} \]

The effects of gross-weight variations on the airplane descent performance were accounted for with one multiplication factor to be applied to altitude rate \( \dot{h}_{\text{CAS}} \) for constant CAS descents and another to be applied to the altitude-rate variation with altitude \( \left( \frac{dh}{dh} \right)_{\text{M}_d} \) for constant Mach descents. A linear variation of descent performance as a function of gross weight was used for the T-39A airplane based on a nondimensionalized-linear-variation model for the B-737 airplane. The gross-weight-variation model for the T-39A airplane was normalized about a gross weight of 15 500 lb. The following equations define the multiplication factors for the T-39A airplane:

\[
K_h = -0.318697 \frac{GW}{15 500} + 1.318697 \quad \text{(Constant CAS)}
\]

\[
K_{dh/dh} = -0.9207 \frac{GW}{15 500} + 1.9207 \quad \text{(Constant Mach)}
\]

Deceleration performance data were obtained for idle-thrust clean-configured speed reductions on level flight paths for typical cruise and metering-fix altitudes. Calibrated airspeed and time data were recorded when the T-39A airplane was slowed from its maximum operating speed (\( M = 0.75 \) at \( h = 30 000 \text{ ft} \) and \( \text{CAS} = 350 \text{ knots} \) at 10 000 ft) to its minimum operating speed (\( M = 0.55 \) at \( h = 30 000 \text{ ft} \) and \( \text{CAS} = 180 \text{ knots} \) at 10 000 ft). The Mach numbers and the calibrated airspeeds were converted to true airspeeds to reduce computational requirements within the descent algorithm. Figures 8 and 9 show linear representations of these data for cruise altitude and metering-fix altitude, respectively. The magnitudes of the accelerations are slopes of these linear representations and are equal to

\[
x_c = -1.12 \text{ knots/sec} \quad (h = 30 000 \text{ ft})
\]

\[
x = -1.92 \text{ knots/sec} \quad (h = 10 000 \text{ ft})
\]

Approximation of True Airspeed

It is necessary to determine true airspeed from both Mach number and calibrated airspeed, as required by the path segment, so that a head-wind component can be added
to obtain ground speed for time calculations. True airspeed, as a function of Mach number and static air temperature $T_{st}$, was represented by the following equation:

$$TAS = 29.04(T_{st})^{1/2} M\text{[knots]}$$

True airspeed, as a function of calibrated airspeed and altitude $h$, was approximated as

$$TAS = \frac{CAS}{1 - (0.12 \times 10^{-4})h}\text{[knots]}$$

$$\left\{ \begin{array}{lcl} h < 35,000 \text{ ft} & \quad & 180 < \text{CAS} < 350 \text{ knots}\end{array}\right.$$ 

Wind Modeling Technique

A two-component linear wind model was used to represent the wind speed and the wind direction as functions of altitude. The coefficients of the wind model were determined by a linear regression analysis of the winds aloft forecast for the test area and entered into the calculator memory prior to each flight. The magnitude of the wind speed and the direction of the wind were computed with the following equations for each segment of the profile based on the middle altitude of each segment:

$$S_{w,h} = \left( \frac{dS_w}{dh} \right)h + S_{w,s}\text{[knots]}$$

$$D_{w,h} = \left( \frac{dD_w}{dh} \right)h + D_{w,s}\text{[deg]}$$

The headwind component for each segment was determined by combining the segment wind speed and direction with the ground track of the airplane as shown in the following equation:

$$W_{H,h} = S_{w,h} \cos(D_{w,h} - TRK)\text{[knots]}$$

Compensation for Effects of Nonstandard Atmospheric Temperature

Various flight instruments, including the Mach meter, the airspeed indicator, and the altimeter, are designed to display correct indications in a standard atmosphere. However, standard atmospheric conditions are rarely encountered. This results in slight errors in indicated altitude and speed. The profile descent algorithm compensates for nonstandard temperatures as they affect the Mach number calculations and altimeter indications.

A linear model was used to define atmospheric temperature as a function of altitude. The linear model uses a slope equal to a temperature-lapse rate of $-3.566 \times 10^{-3} \degree\text{R/ft}$. The model is completely defined with the static air temperature measured at cruise altitude and entered by the pilot. Static temperature required
for conversion of Mach number to true airspeed can then be determined for any altitude \( h \) by

\[
T_{st} = T_{st,c} - \left(3.566 \times 10^{-3}\right)(h - h_c) \quad [°R]
\]

Pressure altitudes \( h_p \) used to define the end points of each segment are corrected to geopotential altitudes by multiplying the pressure altitude by a temperature ratio of nonstandard and standard sea-level temperatures as follows:

\[
h_{GP} = h_p \left(\frac{T'}{T_o}\right) \quad [ft]
\]

The standard sea-level temperature \( T_o \) is 518.688°R; the nonstandard sea-level temperature \( T' \) is computed from the static-temperature model for \( h = 0 \).

Computations of Descent Path

The point where the pilot is to reduce power to idle thrust to start the descent was defined by summing the distances required to fly segments 1 to 6. Each segment length was determined by first computing the required time to traverse the segment and then multiplying by the average ground speed computed for the segment. Times for the level-flight segments requiring airspeed or Mach reductions were determined by dividing the required speed change by the deceleration capability of the airplane. Times for the path segments requiring descents were determined by dividing the required altitude change by the average rate of descent computed from the airplane performance model. The average ground speed at which the airplane was to fly each segment was determined by summing the computed true airspeed and the head-wind component evaluated for each segment.

The cruise segment (segment 7) at a level flight and a constant Mach number had no influence on the location of the point where idle thrust was to begin. This segment was significant only during the time-metered mode and was used for the calculations to satisfy the time constraints. Segments 2 and 3 were computed only if the ATC-imposed limit of 250 knots calibrated airspeed for flight below 10 000 ft MSL was applicable. The details of these calculations are presented in the following paragraphs.

M/CAS transition altitude.- As the airplane descends at a constant Mach number, the calibrated airspeed increases because of an increase in the air pressure. The altitude at which the desired descent calibrated airspeed is obtained is called the M/CAS transition altitude and defines the point at which the constant Mach segment ends and the constant CAS segment begins. The general equation for this transition altitude was determined by equating true airspeed as a function of calibrated airspeed and altitude with true airspeed as a function of Mach number and altitude. Solving for altitude results in the following equation to define the altitude for transition of Mach to calibrated airspeed

\[
h_{XO} = 1.77675 \times 10^5 - \left(\frac{8.90046 \times 10^9}{M_d} + 3.42936 \times 10^7\right)^{1/2} \quad [ft]
\]
Segment 1.- Segment 1 is a level-flight segment on which the airplane is slowed from the descent calibrated airspeed (or 250 knots if segments 2 and 3 are computed) to the metering-fix crossing speed. If the descent speed and the metering-fix crossing speeds are the same, this segment is not computed.

The equations for time and length in segment 1 are

\[ \Delta t_1 = \frac{(C_{AS_{MF}} - C_{AS_d})}{x[1 - (0.12 \times 10^{-4})h_{MF}]} \quad \text{[sec]} \]

where \( x = -1.92 \text{ knots/sec} \), and

\[ \Delta l_1 = \left[ \frac{(C_{AS_d} + C_{AS_{MF}})/2}{1 - (0.12 \times 10^{-4})h_{MF} - \frac{W_H}{h_{MF}}} \right] \frac{\Delta t_1}{3600} \quad \text{[n.mi.]} \]

Segment 2.- Segment 2 is an idle-thrust descent flown at a constant 250 knots from 10 000 ft MSL to the metering-fix altitude. Segment 2 is not computed if the metering-fix altitude is equal to or greater than 10 000 ft MSL or if the descent speed \( C_{AS_d} \) flown on segment 4 is 250 knots or less.

The equations for time and length in segment 2 are

\[ \Delta t_2 = \frac{(h_{MF} - 10 000)}{h \cdot (-48.915)} \quad \text{[sec]} \]

\( h = -48.915 \text{ ft/sec at } C_{AS_d} = 250 \text{ knots} \), and

\[ \Delta l_2 = \left[ \frac{250}{1 - (0.12 \times 10^{-4})(\frac{10 000 + h_{MF}}{2}) - \frac{W_H}{h_{MF}}(10 000 + h_{MF})/2} \right] \frac{\Delta t_2}{3600} \quad \text{[n.mi.]} \]

Segment 3.- Segment 3 is a level-flight segment on which the airplane is slowed from the descent speed \( C_{AS_d} \) to 250 knots. Segment 3 computations are made only if segment 2 is computed.

The equations for time and length in segment 3 are

\[ \Delta t_3 = \frac{(250 - C_{AS_d})}{x[1 - (0.12 \times 10^{-4})h]} \quad \text{[sec]} \]

where \( x = -1.92 \text{ knots/sec} \), and \( h = 10 000 \text{ ft} \)

\[ \Delta l_3 = \left[ \frac{(C_{AS_d} + 250)/2}{1 - (0.12 \times 10^{-4})h - \frac{W_H}{h}} \right] \frac{\Delta t_3}{3600} \quad \text{[n.mi.]} \]
Segment 4.— Segment 4 is an idle-thrust descent flown at a constant calibrated airspeed CAS\textsubscript{d}. The descent begins at the transition altitude \( h\textsubscript{XO} \) and ends at the metering-fix altitude (or at 10 000 ft MSL, if segments 2 and 3 are computed). 

The equations for the time and length of segment 4 are

\[ \Delta t_4 = \frac{(h - h\textsubscript{XO})}{K_h(-0.4624 \text{ CAS}_d + 66.68)} \quad \text{[sec]} \]

and

\[ \Delta l_4 = \left[ \frac{\text{CAS}_d}{1 - (0.12 \times 10^{-4})(h\textsubscript{XO} + h)^2} - \frac{W_e(h\textsubscript{XO} + h)/2}{\text{CAS}_d} \right] \frac{\Delta t_4}{3600} \quad \text{n.mi.} \]

where

\[ h = \begin{cases} h\textsubscript{MP}, & \text{if segments 2 and 3 are not computed} \\ 10 000 \text{ ft}, & \text{if segments 2 and 3 are computed} \end{cases} \]

Segment 5.— Segment 5 is a constant Mach descent flown at idle-thrust power settings. The constant Mach segment is described by a first-order differential equation of the form

\[ h\textsuperscript{\prime}_M + K_1 h + K_2 = 0 \]

This equation results directly from the altitude-rate relationship for constant Mach-number descents previously discussed. The resulting time and distance relations are

\[ \Delta t_5 = \frac{\ln \left[ \frac{(K_2 + K_1 h_c)/(K_2 + K_1 h\textsubscript{XO})}{K_1} \right]}{K_1} \quad \text{[sec]} \]

and

\[ \Delta l_5 = \left[ 29.04(T_{st,5})^{1/2} \text{ M}_d - \frac{W_e(h_c + h\textsubscript{XO})/2}{\text{CAS}_d} \right] \frac{\Delta t_5}{3600} \quad \text{n.mi.} \]

where \(-K_1\) is the change in altitude rate with altitude evaluated at the descent Mach number, corrected for gross-weight effects with the multiplication factor \( K\textsubscript{dh} \), as follows:

\[ K_1 = -\left( \frac{d h}{d h} \right) \frac{dh}{dh} M_d \quad \text{[(ft/sec)/ft]} \]

and \( K_2 \) is given by the following empirical relation:

\[ K_2 = 7 - 45 000 K_1 \quad \text{[ft/sec]} \]
The static temperature $T_{st,c}$ and the head-wind component are evaluated at the average altitude between the cruise and transition altitudes.

Segment 6.- Segment 6 is a level-flight speed change from the cruise Mach number to the descent Mach number. If the cruise and descent Mach numbers are the same, this segment is not computed. The equations for time and length of segment 6 are

$$\Delta t_6 = 29.04(T_{st,c})^{1/2} \left| \frac{M_d - M_c}{x_c} \right| \quad \text{[sec]}$$

where $x_c = -1.12$ knots/sec, and

$$\Delta l_6 = \left[ 29.04(T_{st,c})^{1/2} \frac{(M_c + M_d)}{2} - W_{H,h} \right] \frac{\Delta t_6}{3600} \quad \text{[n.mi.]}$$

Segment 7.- Segment 7 is the remaining path between the entry fix and the beginning of segment 6. The length of segment 7 is the difference between the total distance between the entry fix and metering fix $l_i$ and the sum of the distances of the remaining six segments. The length is given as follows:

$$\Delta l_7 = l_i - \sum_{j=1}^{6} \Delta l_j \quad \text{[n.mi.]}$$

Segment 7 time $\Delta t_7$ is found by dividing the distance to be flown by the ground speed, as follows:

$$\Delta t_7 = \frac{3600 \Delta l_7}{29.04(T_{st,c})^{1/2} M_c - W_{H,h}} \quad \text{[sec]}$$

Input/Output Requirements

Data required for the profile descent equations are obtained from the preprogrammed calculator memory and from pilot entries through the keyboard shown in figure 10. Though all data necessary to compute the descent may be entered prior to takeoff, some operational parameters such as cruise altitude, speed, gross weight, temperatures, and entry-fix and metering-fix arrival times may be required to be updated during the flight to obtain more accurate results. Entries normally made prior to takeoff include the performance constants for the airplane being flown and the wind model. Airplane performance data are normally entered automatically through a magnetic-card reader, but can be entered manually through the keyboard. The wind-model coefficients must be entered into the proper memory locations via the keyboard.

Operational parameters affected by ATC constraints or pilot desires, and not accurately known until prior to the start of descent, were designed to be single-key inputs. To enter those data, the pilot would press the particular key dedicated to the parameter to be changed. After the key has been pressed, the display will show the name of the parameter and its current value stored in the calculator. Another numerical value may be keyed on the display and then stored in the proper memory location by simply pressing the "New Entry" key. If the current value shown is satisfactory, no more keyboard actions would be required for that parameter.
The operational parameters may be inserted in any order, or may be changed at any time prior to initiating the descent calculations. When the magnitudes of the parameters are satisfactory to the pilot, he may start the profile descent computations by pressing the "compute" key. Computations typically require less than 2 min for completion in the time-metered mode of operation and approximately 25 sec in the nonmetered mode.

Operational parameters to be entered through single-key inputs and their symbology as presented on the keyboard and the display, are as follows:

<table>
<thead>
<tr>
<th>Keyboard symbology</th>
<th>Display symbology</th>
<th>Operational parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cruise M</td>
<td>Mc</td>
<td>Cruise Mach number</td>
</tr>
<tr>
<td>Cruise H</td>
<td>Hc</td>
<td>Cruise altitude, ft</td>
</tr>
<tr>
<td>Descent M</td>
<td>Md</td>
<td>Descent Mach number</td>
</tr>
<tr>
<td>Descent CAS</td>
<td>CASd</td>
<td>Descent calibrated airspeed, knots</td>
</tr>
<tr>
<td>Time MF</td>
<td>MF TM</td>
<td>Time assigned by ATC to cross metering fix, hr:min:sec</td>
</tr>
<tr>
<td>Time EF</td>
<td>EF TM</td>
<td>Time to cross entry fix, hr:min:sec</td>
</tr>
<tr>
<td>GW</td>
<td>GW</td>
<td>Airplane gross weight at top of descent, lb</td>
</tr>
<tr>
<td>OAT</td>
<td>OAT F</td>
<td>Static outside air temperature, °F</td>
</tr>
<tr>
<td>MF H</td>
<td>H MF</td>
<td>Crossing altitude at metering fix, ft</td>
</tr>
<tr>
<td>MF CAS</td>
<td>CAS MF</td>
<td>Calibrated airspeed to cross metering fix, knots</td>
</tr>
<tr>
<td>MF DME</td>
<td>MF DME</td>
<td>DME indication defining metering-fix location, n.mi.</td>
</tr>
<tr>
<td>EF DME</td>
<td>EF DME</td>
<td>DME indication defining entry-fix location — this mileage must be relative to same DME station used to define metering fix, n.mi.</td>
</tr>
<tr>
<td>HD</td>
<td>TRK</td>
<td>Magnetic ground track from entry fix to metering fix, deg</td>
</tr>
</tbody>
</table>

If the descent speed schedule has been specified, the entry-fix and metering-fix crossing times must remain unassigned. If these times are specified through the keyboard, the proper descent speed schedule will be computed and stored in the correct memory location for recall by the pilot.

When the computations are completed, the display will normally show the DME indication where thrust should be reduced to flight idle for the descent to the metering fix. If the assigned metering-fix crossing time cannot be attained in the
time-metered mode because of airplane operational speed limitations, a message will be displayed indicating the amount of time required to delay (hold) before starting the descent or the amount of time that the airplane will arrive late at the metering fix.

After the profile descent computations have been completed, the value of any operational parameters, including those required for input, may be displayed by pressing the particular designated key on the keyboard. Parameters that may be displayed after the descent computations, and their designated names, are:

<table>
<thead>
<tr>
<th>Keyboard symbology</th>
<th>Display symbology</th>
<th>Operational parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descent M</td>
<td>Md</td>
<td>Descent Mach number required to satisfy entry-fix and metering-fix time constraints</td>
</tr>
<tr>
<td>Descent CAS</td>
<td>CASd</td>
<td>Descent calibrated airspeed required to satisfy entry-fix and metering-fix time constraints, knots</td>
</tr>
<tr>
<td>Idle DME</td>
<td>IDL</td>
<td>DME indication showing point where thrust should be reduced to flight idle, n.mi.</td>
</tr>
<tr>
<td>Late</td>
<td>LATE</td>
<td>Amount of time that airplane will be late crossing metering fix, min:sec</td>
</tr>
<tr>
<td>Early</td>
<td>HOLD</td>
<td>Amount of time that airplane must delay before starting descent if crossing-time constraint at metering fix is to be satisfied, min:sec</td>
</tr>
</tbody>
</table>

TEST OBJECTIVES

The objectives of the flight tests were (1) to document the difference between the predicted and actual time and distance required to obtain the airspeed and altitude for crossing the metering fix, (2) to determine the feasibility of using conventional flight instrumentation for open-loop guidance to fly a fuel-conservative descent, and (3) to obtain an understanding of the additional workload required to enter the necessary parameters into the calculator in a flight environment. These objectives were achieved by using subjective data in the form of pilot and copilot comments and quantitative data in the form of time, speed, and altitude data recorded with a voice recorder during the test flight.

DESCRIPTION OF AIRPLANE AND COCKPIT INSTRUMENTATION

The test vehicle, a T-39A airplane, is a twin-engine subsonic jet representative of a typical corporate-executive transport. The airplane is normally flown with a crew of two pilots, with up to seven additional passengers, and with a maximum gross weight of 17 760 lb.

Specific flight instrumentation used during the descents included the airspeed indicator, a Mach number indicator, and a pressure altimeter. All of these flight
instruments were driven directly from pressure sources in the airplane pitot-static system. No instrument corrections were applied through the use of an air-data computer.

The airspeed indicator, shown in figure 11, has a conventional pointer/dial format with the addition of a maximum-allowable-airspeed pointer which indicates the maximum speed at which the airplane can be flown at any particular altitude. In addition, a vernier scale, visible through a window at the top of the main dial, makes one revolution for each 100-knot change in airspeed. Graduations on this scale are provided for each 2 knots.

The Mach number indicator, shown in figure 12, is an independent instrument displaying only Mach number. A single pointer indicates the Mach number on a dial face graduated in hundredths.

DATA RECORDING

A portable voice recorder, a stop watch, and conventional flight instruments were used to collect data for descent-performance modeling. Altitude, speed, temperature, and time were recorded at altitude increments of approximately 500 ft during the descents. Speed, temperature, and time were recorded at 10-sec intervals during constant-altitude speed changes. Gross weights were recorded at the beginning and end of each test run.

Flight notes were recorded by the copilot during the flight tests when the calculator was used for descent planning. These data included the following:

(1) Time and DME indication when thrust was reduced to flight idle

(2) Time and DME indication when descent was started

(3) Time and DME indication when transition from constant Mach to constant airspeed occurred during descent

(4) Time and DME indication at beginning and ending points of level-flight deceleration path segments, including the segment at 10,000 ft, if required

(5) All parameters entered into calculator used to compute descent profile

TEST PROCEDURES

The experiment task required the flight crew to operate the airplane as a typical arrival to an airport terminal area. Each test run was started with the airplane established inbound on a specified VOR radial at a cruise Mach number of 0.72 and at an altitude of 35,000 ft. The pilot was expected to null the lateral tracking errors throughout each test run.

Prior to passing the entry fix established at 130 n.mi. DME from the VOR, the copilot would enter the appropriate parameters into the calculator and compute the descent profile. He would then prepare a descent data card with the following information for the pilot: (1) the DME indication to reduce thrust to flight idle for the descent, (2) the Mach number and airspeed to be used during the descent, and (3) the
airspeed, altitude, and DME indications at which the airplane should cross the metering fix.

The pilot's task was to fly the airplane along the idle-thrust descent at the M/CAS indications written on the descent data card. He would reduce the thrust at the DME indication specified on the data card and slow the airplane to the descent Mach number. He would then start the descent and maintain the constant descent Mach number by adjusting the airplane's pitch attitude. After the calibrated airspeed rose to the desired value, pitch attitude would be adjusted with reference to the airspeed indicator.

The pilot was instructed to conform to the ATC-imposed 250-knot airspeed limit, if required. On test runs where the metering-fix altitude was 5000 ft and the descent airspeed was greater than 250 knots, the pilot would level the airplane at 10000 ft and slow to 250 knots. Then he would continue the descent at 250 knots until leveling the airplane at the metering-fix altitude. The airplane would then be slowed to the metering-fix crossing speed. The test run would be completed at this point.

RESULTS AND DISCUSSION

Profile Descent Flight Performance

The prime indicator used to evaluate the performance that resulted with the open-loop guidance was the accuracy in terms of time and distance required to achieve the desired metering-fix conditions. This accuracy was quantified with the time and distance errors that resulted when the desired airspeed and altitude for the metering-fix crossing were attained and with the time error that resulted when the metering fix was crossed. These errors are shown for each of 12 test runs conducted on 4 different days (3 test runs per day) in table I. The mean and the standard

<table>
<thead>
<tr>
<th>Test run number</th>
<th>M/CAS, knots</th>
<th>CAS, ft</th>
<th>Time error to attain desired CAS and h, sec (a)</th>
<th>Distance error to attain desired CAS and h, n.m. (b)</th>
<th>Time error to cross metering fix, sec (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.70/300</td>
<td>250</td>
<td>1.2</td>
<td>-.2</td>
<td>16.9</td>
</tr>
<tr>
<td>2</td>
<td>.72/340</td>
<td>250</td>
<td>10.0</td>
<td>1.4</td>
<td>-8.2</td>
</tr>
<tr>
<td>3</td>
<td>.70/300</td>
<td>250</td>
<td>10.0</td>
<td>1.4</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>.70/300</td>
<td>210</td>
<td>10.0</td>
<td>1.4</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>.70/300</td>
<td>210</td>
<td>10.0</td>
<td>1.4</td>
<td>12.5</td>
</tr>
<tr>
<td>6</td>
<td>.70/300</td>
<td>210</td>
<td>5.0</td>
<td>-5.2</td>
<td>17.6</td>
</tr>
<tr>
<td>7</td>
<td>.65/300</td>
<td>5.0</td>
<td>13.0</td>
<td>-.2</td>
<td>-5.2</td>
</tr>
<tr>
<td>8</td>
<td>.60/300</td>
<td>210</td>
<td>10.0</td>
<td>-9.6</td>
<td>-5.2</td>
</tr>
<tr>
<td>9</td>
<td>.60/300</td>
<td>210</td>
<td>5.0</td>
<td>-5.2</td>
<td>17.6</td>
</tr>
<tr>
<td>10</td>
<td>.60/300</td>
<td>210</td>
<td>10.0</td>
<td>-9.6</td>
<td>-5.2</td>
</tr>
<tr>
<td>11</td>
<td>.60/300</td>
<td>210</td>
<td>5.0</td>
<td>-5.2</td>
<td>17.6</td>
</tr>
<tr>
<td>12</td>
<td>.60/300</td>
<td>210</td>
<td>10.0</td>
<td>-9.6</td>
<td>-5.2</td>
</tr>
</tbody>
</table>

Minus values indicate conditions attained later than predicted. 

Mean: 1.4

Standard deviation: 13.0

Minus values indicate conditions attained past metering fix.

Minus values indicate metering fix crossed later than predicted.
deviation for these errors are presented at the bottom of the table. The M/CAS
descent speed schedule and the desired crossing airspeed and altitude at the metering
fix are also shown in the table for each test run.

The time error that resulted while attaining the desired metering-fix crossing
speed $\text{CAS}_{MF}$ and altitude $h_{MF}$ was an indication of the accuracies with which the
airplane descent performance data had been modeled and how closely the airplane had
been flown on the predicted profile with the open-loop guidance. This particular
form of time error was not affected by winds since it was a time associated with
attaining airspeeds and altitudes (air-mass reference frame). The mean and the stan-
dard deviation of this time error for these tests were 1.4 and 13.0 sec, respec-
tively, for descents with time durations between 5 and 11 min. Based on these time
errors, it was concluded that the airplane descent performance data were adequately
modeled and that the time required to attain the desired airspeed and altitude could
be adequately predicted with the open-loop guidance provided by the Mach and airspeed
indicators.

The distance error that resulted while attaining the desired metering-fix cross-
ing speed and altitude was an indicator of the same error components associated with
the time error and, in addition, an indicator of the accuracy of wind modeling. The
resulting mean and standard deviation for the distance error for these test runs were
-1.2 n.mi. and 2.2 n.mi., respectively.

The distance error data for test runs 7 to 9 were considered high relative to
the data for the other test runs. For these test runs, actual ground speed, computed
from the flight data for the level-flight segment at cruise altitude, was compared
with the ground speed computed for that same segment with the profile descent algo-
rithm. In each case, the actual ground speed was approximately 28 knots greater than
predicted, thus indicating improperly modeled winds. The mean and the standard
deviation of the distance error, excluding runs 7 to 9, were -0.2 n.mi. and 1.6 n.mi.,
respectively.

Experience obtained during the profile descent tests with the ATOPS B-737 air-
plane (ref. 5) and the data obtained from these flight tests indicate that the two-
component, linear wind model was sufficient to model the wind during the descent,
provided that the model was based on reasonably accurate wind data. The data from
runs 7 to 9, however, have shown that it may be necessary for the pilot to apply
corrections to the wind model during flight to reduce the crossing errors at the
metering fix. An additional subroutine has been developed (but not flight tested)
that will allow the pilot to enter a head-wind-component correction factor based on
the actual ground speed obtained at cruise altitude. The profile descent algorithm
then adds the wind correction component (linearly reduced to zero from cruise alti-
tude to sea level) to the winds computed for each of the segments in the descent
profile.

The last column of table 1 shows the time error that resulted when the metering
fix was crossed. The mean and the standard deviation of this error were 20.2 and
25.5 sec, respectively. With the exception of test runs 7 to 9, all of the test runs
resulted in an improvement of the 1- to 2-min crossing error currently achieved
through control from the ground. The mean and the standard deviation of these data,
with runs 7 to 9 excluded, were 8.3 and 15.7 sec, respectively.

Oscillations about the desired descent speed schedule were observed during the
descents. They were typically less than 0.02 Mach during the constant-Mach descent,
and 5 knots calibrated airspeed during the constant-CAS descent. It was felt that
since the oscillations were low and about the desired speeds, their effects on achieving the predicted profile conditions were minimal. The pilot felt that he could have reduced the oscillations about the desired Mach and airspeed during the descents if a larger attitude indicator (only a 2 1/2" standby indicator was available during these tests) were used.

The additional workload associated with entering the required information into the calculator, computing the descent profile, and flying the descent with conventional cockpit guidance was reported by the flight crew to be acceptable. All calculator inputs and computations were made by the copilot during level cruise flight where normal workload is typically low. The pilot reported that the workload during the descents was normal.

**Parametric Sensitivity Analysis**

A parametric sensitivity analysis was conducted to determine the effects that uncertainties in the magnitudes of the operational parameters input for the descent computations would have upon the time and distance predictions for crossing the metering fix. This analysis was conducted by comparing the output (i.e., time and distance predictions) of the descent computations, using nominal inputs, with the output of a descent profile constructed to reflect the flight path resulting from an off-nominal flight condition. Each off-nominal descent was constructed using the predicted time and distance computed with the same inputs used for the nominal case, except for the specific input parameter to be examined. This descent information was combined with the idle-thrust descent point computed for the nominal case, so that the resulting time and airspeed errors for crossing the metering fix could be calculated. Each input parameter was varied individually by an amount judged to be typical of the accuracy that could result during routine flight operations.

**Nominal case**.- The magnitudes of the parameters used to compute the nominal case are:

1. Cruise altitude = 35 000 ft
2. Cruise Mach number = 0.72
3. Outside static air temperature = -65.8°F (ISA at 35 000 ft)
4. Winds = 0 knots at all altitudes
5. Descent Mach number = 0.70
6. Descent airspeed = 300 knots
7. Airplane gross weight = 15 500 lb
8. Metering-fix crossing altitude = 10 000 ft
9. Metering-fix crossing speed = 210 knots
10. Entry-fix location = 100 n.mi. from metering fix

The following results were obtained with the profile descent computations using the nominal inputs. The time required to fly between the entry fix and the metering
fix was 902.2 sec. Thrust was to be retarded to flight idle to start the descent 42.7 n.mi. from the metering fix. The time to fly from the point that the thrust was retarded to the metering fix was 405.5 sec.

The sensitivity of each parameter on the descent predictions was quantified (1) by the time error, (2) by the speed error when the metering fix was crossed, and (3) by the distance from the metering fix when the desired metering-fix airspeed and altitude were obtained. The variation of each parameter examined and the corresponding effects on the metering-fix-crossing accuracy is summarized in table II.

<table>
<thead>
<tr>
<th>Parameter changed</th>
<th>Parameter variation</th>
<th>Crossing time error, sec</th>
<th>Descent distance error, n.mi.</th>
<th>Crossing airspeed error, knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind modeling</td>
<td>20-knot head wind</td>
<td>62.1 late</td>
<td>2.3 prior</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20-knot tail wind</td>
<td>52.0 early</td>
<td>2.3 past</td>
<td>48 fast</td>
</tr>
<tr>
<td>Gross weight</td>
<td>500 lb heavier</td>
<td>4.6 early</td>
<td>.8 past</td>
<td>17 fast</td>
</tr>
<tr>
<td></td>
<td>500 lb lighter</td>
<td>4.0 late</td>
<td>.7 prior</td>
<td>0</td>
</tr>
<tr>
<td>Static air</td>
<td>ISA +5°C</td>
<td>10.3 early</td>
<td>1.5 past</td>
<td>31 fast</td>
</tr>
<tr>
<td>temperature</td>
<td>ISA -5°C</td>
<td>8.3 late</td>
<td>1.2 prior</td>
<td>0</td>
</tr>
<tr>
<td>Descent Mach</td>
<td>0.02 fast</td>
<td>6.0 late</td>
<td>1.5 prior</td>
<td>36 fast</td>
</tr>
<tr>
<td>number</td>
<td>0.02 slow</td>
<td>4.6 early</td>
<td>1.7 past</td>
<td>36 fast</td>
</tr>
<tr>
<td>Descent airspeed</td>
<td>10 knots fast</td>
<td>1.9 early</td>
<td>.7 prior</td>
<td>24 fast</td>
</tr>
<tr>
<td></td>
<td>10 knots slow</td>
<td>.1 late</td>
<td>1.1 past</td>
<td>24 fast</td>
</tr>
</tbody>
</table>

\(^a\)Thrust was required to maintain the desired speed when the crossing airspeed error was zero and the crossing airspeed and altitude were attained prior to the metering fix.

Wind-modeling sensitivity. The effects of wind-modeling errors were calculated first, assuming a constant 20-knot error in the head-wind component for the entire flight path between the entry fix and the metering fix. These calculations were then repeated assuming a 20-knot tail-wind error.

With an unknown 20-knot head wind, the resulting ground speed would be decreased proportionately and would result in a crossing-time error of 62.1 sec later than predicted. The desired airspeed and altitude for crossing the metering fix would be obtained 2.3 n.mi. prior to the fix. Thrust would be required to maintain airspeed and altitude to cross the metering fix. With the 20-knot tail wind, the metering fix would be crossed 52.0 sec earlier than predicted, but with an airspeed 48 knots faster than desired. The desired final airspeed and altitude would be achieved 2.3 n.mi. past the metering fix.

It should be noted that the time error is accumulated in the descent and cruise portions of flight between the entry and metering fixes. Hence, this time error will
also be a function of distance between the fixes. If the time error incurred during the cruise portion is not considered, the time error for the head-wind case would be 37.1 sec late and, for the tail-wind case, 29.1 sec early.

The potential for significant errors occurring in wind modeling is high because of a constantly changing atmosphere and the vagaries of forecasting winds aloft. However, the wind-modeling accuracy problem can be reduced if the pilot can modify the wind model based upon the winds actually encountered during the cruise portion of his flight.

Gross-weight sensitivity.—The effects of gross-weight variations were calculated for both 500 lb heavier and 500 lb lighter than nominal gross weight. This variation represented a 3.2-percent deviation from a nominal gross-weight condition of 15 500 lb.

The descent parameter most significantly affected by gross-weight variations was the point at which thrust should be retarded to flight idle for beginning the descent. The heavier-than-nominal airplane would require a path 0.8 n.mi. longer than predicted for the descent, since it must be flown at a slightly higher lift-to-drag ratio (same indicated airspeeds), which would result in a shallower descent angle. The lighter-than-nominal gross-weight condition would result in a path 0.7 n.mi. shorter than predicted.

The metering-fix crossing errors due to gross-weight variations occurred because the descent would be initiated at the wrong point. For the 500 lb heavier-than-nominal case, the metering fix would be crossed 4.6 sec early and 17 knots faster than desired. The desired airspeed would be achieved 0.8 n.mi. past the fix. For the 500 lb lighter-than-nominal case, the desired crossing speed and altitude would be achieved 0.7 n.mi. prior to crossing the metering fix. This would result in an arrival-time error of 4 sec late.

Static-air-temperature sensitivity.—Static air temperature is used in the descent computations to convert pressure altitude to geopotential altitude and to compute true airspeed (knots) from Mach number. The effects of static-air-temperature error on the descent profile were calculated by biasing the nominal temperature profile 5°C warmer and 5°C colder. Temperatures warmer than standard resulted in an increase in geopotential altitudes and an increase in true airspeed; colder than standard resulted in decreases.

With a temperature profile 5°C warmer than standard, the increased altitudes and airspeeds would cause the metering fix to be crossed 10.3 sec early, at an airspeed 31 knots faster than desired. The desired airspeed would be attained 1.5 n.mi. past the metering fix. For a temperature profile 5°C colder than standard, the desired crossing speed would be attained 1.2 n.mi. prior to the metering fix. The fix would be crossed 8.3 sec later than predicted.

Descent Mach number sensitivity.—The effects on the descent profile caused by variation of the Mach number during the descent segment were calculated for a 0.02-Mach increment faster and a 0.02-Mach increment slower than the nominal-descent Mach number of 0.70. The most significant effect upon the descent predictions was the change in distance required to descend from the cruise altitude to the metering-fix altitude. The faster Mach number (0.72) would result in the descent being completed 1.5 n.mi. earlier than for the nominal case. Although the time to descend would be shorter with the faster Mach number, the total time to fly to the metering
fix, with the additional 1.5 n.mi. to fly at the metering-fix altitude, would result in the metering fix being crossed 6.0 sec later than predicted.

For the slower Mach number (0.68), the metering fix would be crossed 4.6 sec early, at an airspeed 36 knots faster than desired. The desired crossing speed would be attained 1.7 n.mi. past the metering fix.

Descent-airspeed sensitivity.—The effects that variation of the descent airspeed had upon the descent predictions were calculated for 10-knot increments faster and slower than the nominal 300-knot descent speed. The faster descent speed (310 knots) would cause the desired crossing speed and altitude to be attained 0.7 n.mi. prior to the metering fix. The metering fix would be crossed 1.9 sec earlier than predicted. At the slower descent speed (290 knots), the fix would be crossed 0.1 sec later than predicted, but at an airspeed 24 knots faster than desired. The desired crossing conditions would be attained 1.1 n.mi. past the fix.

CONCLUDING REMARKS

A simple, airborne, flight-management descent algorithm, designed to aid the pilot in planning a fuel-efficient time-constrained descent, was programmed into a small programmable calculator. In a time-metered mode, the airborne algorithm computes the specific Mach number, airspeed, and point for the pilot to begin the descent to arrive at a metering fix at a predetermined airspeed, altitude, and time assigned by air traffic control (ATC). In the nonmetered mode, the algorithm computes the point to begin the descent based on the Mach and airspeed descent speed schedule input by the pilot.

Flight test data obtained with a T-39A airplane indicated that the time and distance to descend could be satisfactorily predicted with the use of relatively simple equations and airplane performance characteristics modeling. The flight data also have shown that the descent profile could be satisfactorily flown with open-loop guidance provided by conventional cockpit Mach and airspeed indicators. The crew workload required to compute and fly descents was reported to be acceptable.

A parametric sensitivity analysis has shown that improper wind modeling has the potential to produce significant prediction errors. These errors may be reduced by providing the flight crew with the capability to modify the wind model based on actual winds encountered at cruise altitudes.

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REFERENCES


Dashed line if the 250-knot speed limit below 10000 ft MSL is required

![Diagram of descent and speed changes](image-url)

- **Fig 1.** Vertical-plane geometry of computed descent path.

- **CAS** = 250 kt
- **CAS** descent at idle thrust
- **Top of descent**
- **Entry fix**
- **Constant-cruise Mach**

**Legend:**
- **Δt** = Time intervals
- **Δl** = Vertical intervals
- **h** = Height
- **EF** = Entry fix

- **T** = Time at various points
- **M** = Mach number
- **C** = Cruise
- **D** = Descent

**Equations:**

- **CASd = 250 kt**
- **Constant-Mach descent at idle thrust**
- **Speed change from cruise to descent Mach at idle thrust**

**Note:** The diagram illustrates the vertical plane geometry of the computed descent path with specific time and height intervals.
Figure 2.- Logic flow of flight-management descent algorithm.
Figure 3.- Mach number calibrated-airspeed descent schedule selection via interpolation.
Figure 4.— Quadratic curve fit for altitude of T-39A airplane executing constant 0.55-Mach descent.
Figure 5.- Vertical speed of T-39A airplane during idle-thrust, clean-configured, constant Mach descent. Gross weight = 15 500 lb.
\[
\frac{dh}{M_d} = A_0 + A_1 M_d + A_2 M_d^2
\]

\begin{align*}
A_0 &= 0.019371 \\
A_1 &= -0.063457 \\
A_2 &= 0.062420
\end{align*}

Figure 6.-- Variation of \( \frac{dh}{M_d} \) as function of descent Mach number for T-39A airplane. Gross weight = 15 500 lb; idle thrust.
Figure 7.- Vertical speed of T-39A airplane during idle-thrust, clean-configured, constant airspeed descent. Gross weight = 15,500 lb.
Figure 8.- Idle-thrust, clean-configured speed reduction from $M = 0.75$ to 0.55 at 30 000 ft.
Figure 9.- Idle-thrust, clean-configured speed reduction from 350 to 180 knots CAS at 10000 ft.

\[ \ddot{x} = -1.92 \text{ knots/sec} \]
Figure 10.- Calculator keyboard configured for profile descent computations.
Figure 11. - Airspeed indicator in T-39A test airplane.

Figure 12. - Mach number indicator in T-39A test airplane.
A simplified flight-management descent algorithm, programmed on a small programmable calculator, has been developed and flight tested. It was designed to aid the pilot in planning and executing a fuel-conservative descent to arrive at a metering fix at a time designated by the air traffic control system. The algorithm may also be used for planning fuel-conservative descents when time is not a consideration. The descent path was calculated for a constant Mach/airspeed schedule from linear approximations of airplane performance with considerations given for gross weight, wind, and nonstandard temperature effects. This report describes the flight-management descent algorithm, and presents the results of flight tests flown with a T-39A (Sabreliner) airplane.