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Final Report

by

Walter D. Pilkey  
and  
J. Kenneth Haviland

NASA Grant NAG-1-137-1  
1/16/81 - 1/15/83

Department of Mechanical & Aerospace Engineering  
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Several FORTRAN subroutines and programs were developed which compute complex eigenvalues of a damped system using different approaches, and which rescale mode shapes to unit generalized mass and make rigid bodies orthogonal to each other. An analytical proof of a Minimum Constrained Frequency Criterion (MCFC) for a single damper is presented. A method to minimize the effect of control spill-over for large space structures is proposed. The characteristic equation of an undamped system with a |                       |  |

generalized control law is derived using reanalysis theory. This equation can be implemented in computer programs for efficient eigenvalue analysis or control gain synthesis. Methods to control vibrations in large space structures are reviewed and analyzed. The resulting prototype, using electromagnetic actuator, is described.

**Table of Contents**

|      |   |     |
|------|---|-----|
| I.   | Introduction . . . . .  | 1   |
| II.  | Technical Developments . . . . .  | 2   |
|      | 1. Computer Programs . . . . .  | 2   |
|      | 2. Proof of Optimal Damper Location . . . . .   | 2   |
|      | 3. Locating Actuators for Minimum Spill-Over . . . . .                                    | 8   |
|      | 4. Characteristic Equations for Undamped System with<br>Generalized Control Law . . . . . | 11  |
|      | 5. Damper Design . . . . .  | 16  |
| III. | Conclusion . . . . .  | 38  |
| IV.  | Bibliography . . . . .  | 39  |
|      | Appendices . . . . .  | 41  |
|      | A-1 Subroutine ASMD1 . . . . .  | 43  |
|      | A-2 Subroutine ASMD2 . . . . .  | 53  |
|      | A-3 Subroutine SPSTGN Documentation . . . . .   | 64  |
|      | A-4 Program COPZ Documentation . . . . .  | 84  |
|      | A-5 Program NORMAL Documentation . . . . .  | 103 |
|      | A-5.1 Gram-Schmidt Orthogonalization Procedure . . . . .                                  | 106 |
|      | B Active Damper Analysis-Linear . . . . .   | 107 |
|      | C Use of Piezoelectric Materials . . . . .  | 113 |
|      | D Simulation of NASA Tests . . . . .  | 116 |
|      | E Bounds on Transient Responses . . . . .<br>Time Domain Optimal Design                   | 118 |
|      | F Formulation of Dynamical Matrix for LSS with<br>General Control Law . . . . .           | 122 |
|      | G Proposed Approaches for Actuator Failure . . . . .                                      | 124 |

# I. INTRODUCTION

It is expected that large space structures will be placed into orbit in the not-too-distant future. Such structures will lack the damping forces due to ground reactions or to hydraulic or aerodynamic forces available to earth-bound structures, thus, if they are excited dynamically by docking maneuvers or by control reactions, they could be expected to continue vibrating for hours or even for days.

It is this problem which is addressed in this report. The work to be discussed divides roughly into two areas. In the first, the questions such as how much damping is required, how are the dampers to be located, and how their required performance can be specified are examined. In the second, the design of the dampers is investigated, and is supported by the demonstration of a prototype damper.

Work on damping requirements resulted in five computer programs and proofs of two criteria for the optimum location of dampers. Also, a method of minimizing "control spill over" by locating actuators was demonstrated, and the characteristic equation of an undamped system using reanalysis theory was derived.

Work on damper design included a review of possible systems. Since hydraulic and pneumatic systems were eliminated for poor reliability, only electromagnetic or piezo-electric systems were considered. Passive dampers were eliminated because they need very fine tuning. Thus, only linear or rotational dampers using active electromagnetic or piezoelectric drives were left. A prototype linear damper using a moving magnet and a fixed coil was built and demonstrated using a current feedback proportional to local structure velocity.

## II. TECHNICAL DEVELOPMENTS

### 1. Computer Programs

Several FORTRAN subroutines and programs were developed under this grant. These are documented in Appendix A. The capabilities of the programs are enumerated below.

Subroutines ASMD1 and ASMD2 compute the complex eigenvalues of a damped system using the undamped modes of the structure as the assumed modes (assumed mode method). In subroutine ASMD1, a diagonal damping matrix must be used, whereas in ASMD2, a general damping matrix may be used.

Subroutine SPSTGN solves the damped eigenvalue problem using a reanalysis approach. It assumes a diagonal damping matrix.

Program COPZ computes the optimal (i.e., minimum) gains of a diagonal damping matrix for specified damping ratios.

Program NORMAL is designed to rescale the mode shapes to unit generalized mass and to make the rigid bodies orthogonal to each other with respect to the mass matrix.

These programs are stored in the Langley computer.

### 2. Proof of Optimal Damper Location

In previous papers [Ref. 1 and 2] we have proposed two criteria for the optimal location of a damper. They are the Minimum Constrained Frequency Criterion (MCFC) for a single damper [Ref. 1] and the Maximum Frequency Separation Criterion (MFSC) for multiple dampers [Ref. 2]:

MCFC: The optimal damper location is where the constrained frequency  $\alpha_1$  is a minimum.

MFSC: The optimal locations of dampers are where the constrained frequency associated with the damper location has the largest separation from the corresponding undamped natural frequency of the system.

In Ref. 1 and 2, these criteria are demonstrated using the Langley beam and grillage models. These demonstrations show that in choosing between sets of positions of dampers, the one that gives the best results is the one that gives the greatest separation between the undamped natural frequency and the corresponding constrained frequency. In this section, an analytical proof of MCFC for a single damper will be presented. The proof of the more general MFSC is still under investigation.

### Proof of MCFC

The proof of MCFC is based on the existence of fixed points in the frequency response curve when a single damper is introduced into an n-dof undamped system. This is an extension of the classical damped vibration absorber theory [Ref. 3], in which Den Hartog showed that when a damped vibration absorber is attached to a undamped SDOF main system, there are points on the frequency response curve which are independent of damping. These points are the fixed points [Ref. 3]. In this section, we will first show the existence of fixed points when a single damper is introduced into an undamped system. The responses at the fixed points can then be evaluated from the undamped equation of motion. The MCFC can then be proved using the characteristics of the resonance curve of the undamped system near resonance.

Consider the case of an n-dof undamped system under sinusoidal excitation

$$[M] \{x\} + [K]\{x\} = \{F\}e^{j\omega t} \quad (1)$$

where [M], [K] are mass and stiffness matrices and {x} is the displacement vector and {F} is the magnitude vector of the forcing function. When a single damper at dof J is introduced into this system, the equation of motion becomes

$$[M]\{x\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}e^{j\omega t} \quad (2)$$

where

$$[C] = c\{e_J\}\{e_J\}^T \quad (3)$$

$$\{e_J\}^T = [0 \ \dots \ 0 \ \underset{\substack{\Delta \\ J\text{th component}}}{1} \ 0 \ \dots \ 0]$$

Let the steady state solution of (2) be

$$\{x\} = \{X\}e^{j\omega t} \quad (4)$$

Then, use (3) and (4) the steady state response  $\{X\}$  can be computed from

$$[Z(\omega)]\{X\} = \{F\} - c\omega j\{e_J\}\{e_J\}^T\{X\} \quad (5)$$

where

$$[Z(\omega)] = -\omega^2[M] + [K]$$

Define

$$[R(\omega)] = [Z(\omega)]^{-1} \quad (6)$$

and note that

$$\{e_J\}^T\{x\} = x_J \quad (7)$$

$$[R]\{e_J\} = \{R_J\}$$

where  $\{R_J\}$  is the Jth column of  $[R]$ . Premultiply (5) by  $[R]$  and use (7) to obtain

$$\{X\} = \{X_0\} - j\omega c R_J X_J \quad (8)$$

Note that

$$\begin{aligned} \{X_0\} &= [R(\omega)]\{F\} \\ &= \text{steady state solution of the undamped system.} \end{aligned}$$

The Jth equation of (8) is

$$X_J = X_{0J} - j\omega c R_{JJ} X_J$$

or

$$X_J = \frac{X_{0J}}{1 + j\omega c R_{JJ}} \quad (9)$$

Equation (8) together with (9) are the general frequency response reanalysis equations when a damper  $c$  is introduced at dof  $J$ . It is interesting to note that at the frequency  $a_f$  at which

$$R_{JJ}(a_f) = 0 \quad (9a)$$

the response  $X_J$  is independent of damper gain  $c$ . Thus, we have shown the existence of fixed points for the response curve  $X_J(\omega)$ . Furthermore, those frequencies are the antiresonant frequencies for dof  $J$ . A typical frequency response curve for  $X_J$  (or  $\omega^2 X_J$ , the acceleration response) is shown in Fig. 2.1. In Fig. 2.1, point A is the fixed point corresponding to the smallest root of Eq. (9a). By definition of fixed point, all response curves pass through A

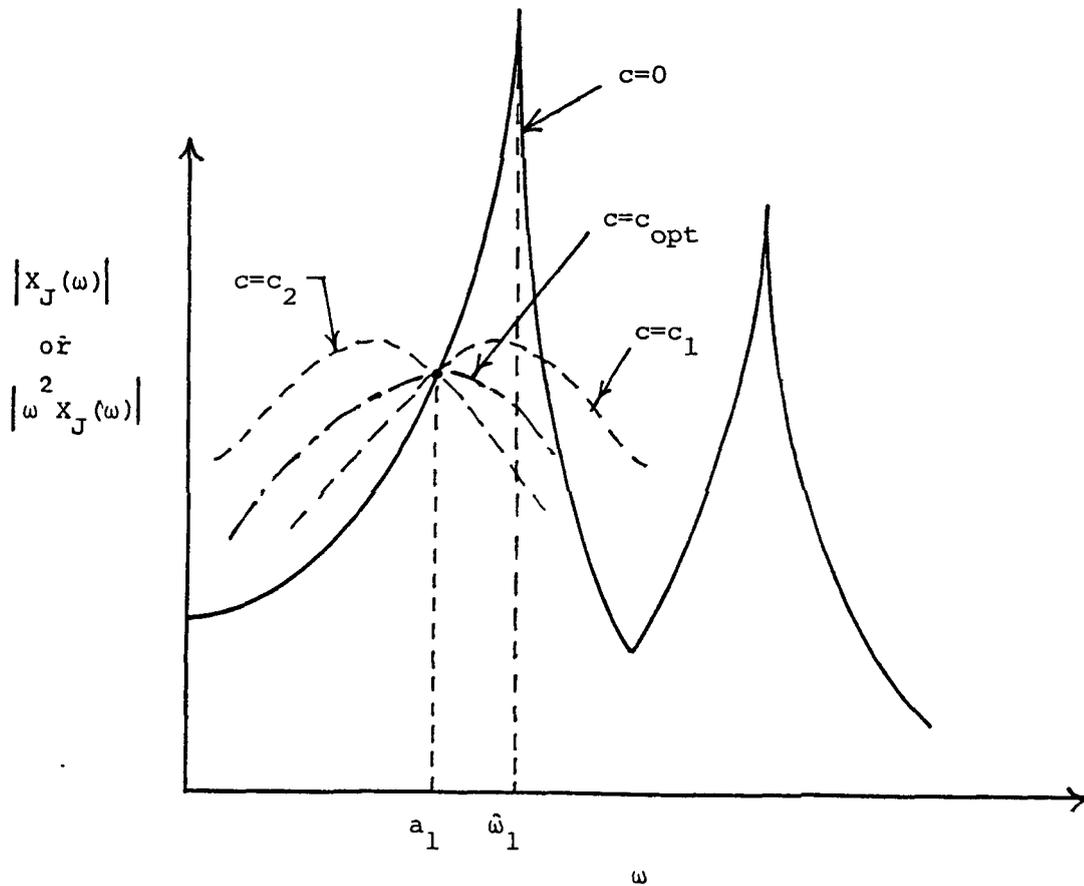


Fig. 2.1 Frequency Response Curve for  $|X_J|$  Showing fixed point A.  $a_1$  is the frequency corresponding to point A.

regardless of the damper gain  $c$ . Thus, it is obvious the optimal  $c$  (in terms of the smallest response) should produce a response curve that peaks at point A. In other words, the optimal response curve has a slope zero at the fixed point A.

From the above discussion, we know that once a damper location is selected, the minimum response  $X_J$  is determined by the undamped response at the fixed point frequency. Since near resonance, the undamped response is monotonically increasing as the frequency is approaching the undamped natural frequency from below, (see Fig. 2.2), we conclude that the optimal location for lowest amplification for a particular mode with undamped natural frequency  $\omega_n$  is where  $(\omega_n - a_n)$  is a maximum.

This proves MCFC for minimum frequency response. Our original MCFC pertains to the modal damping ratio. Since modal damping is closely related to the amplification factor we conclude that MCFC will produce a design with maximum damping ratio.

### Discussion

- (1) In general, there are  $(n-1)$  fixed points for response  $X_J$ , where  $n$  is the number of elastic modes of the system.
- (2) The responses  $X_J$  ( $I \neq J$ ) exhibits fixed points at frequencies where

$$|X_I|_{c=0} = |X_I|_{c=\infty} \quad (10)$$

or from (8) and (9), condition (10) becomes

$$|X_{OI}| = |X_{OI} - R_{IJ}(\omega)X_{OJ}| \quad (11)$$

which corresponds to the following two equations

$$X_{OI} = X_{OI} - R_{IJ}(\omega)X_{OJ} \quad (12a)$$

and

$$X_{OI} = -X_{OI} + R_{IJ}(\omega)X_{OJ} \quad (12b)$$

- (3) Our proof only pertains to  $X_J$ , however, experience showed that once  $X_J(\omega)$  is minimized, the other  $X_I(\omega)$ 's are almost optimal.
- (4) The proof cannot be extended to multiple dampers since no fixed points exist for the multiple damper case.

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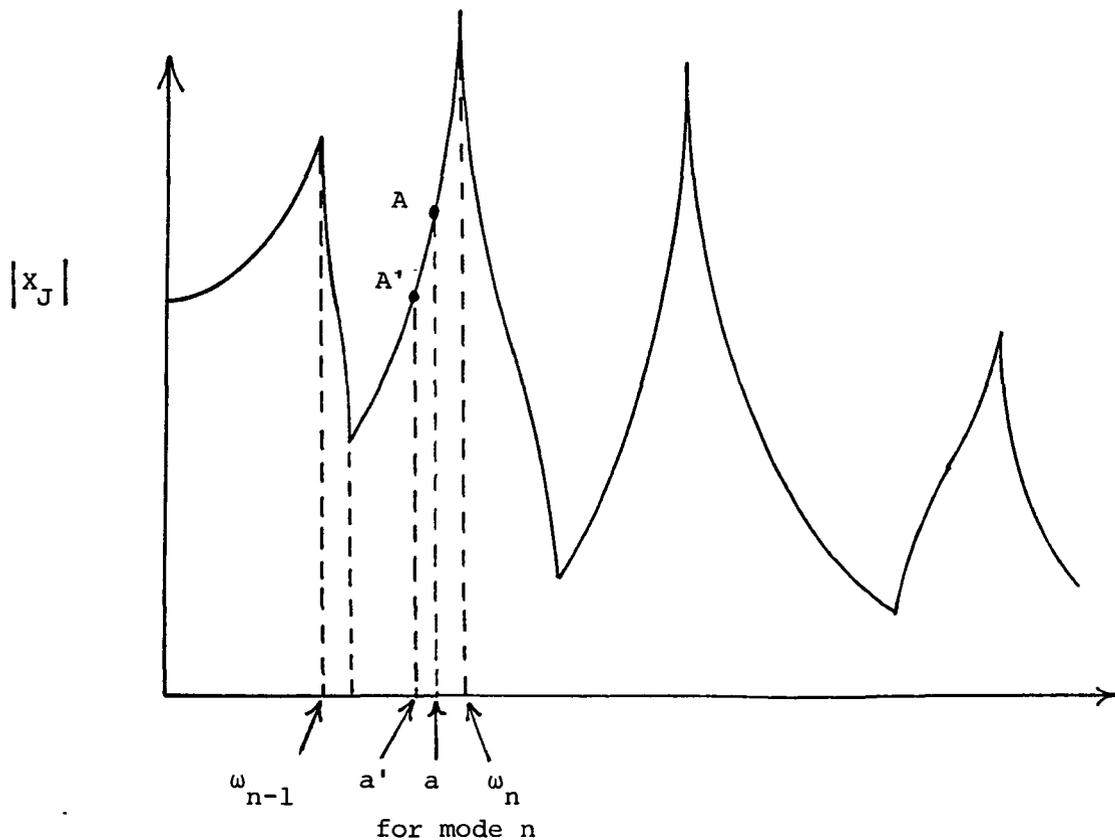


Fig. 2.2 Frequency response of undamped n-dof system.  
 $|x_J(\omega)|$  increases monotonically as  $\omega \rightarrow \omega_n$ .

### 3. Locating Actuators for Minimum Spill-Over

A frequently discussed problem in the control of LSS is that of "control spill-over". A method of minimizing the effect of control spill-over is proposed in this section. It is shown that to control  $n_c$  modes using  $n_a$  actuators, the spill over effect can be minimized by insuring that  $n_s$  secondary modes are not excited. This requires  $n_a > n_s$ . The method described allows only  $n' = n_a - n_s$  of the  $n_a$  actuators to be controlled independently. Conditions of actuator placement is also indicated.

Consider a proportionally damped system whose finite element model can be written as

$$\underline{M} \underline{\ddot{x}} + \underline{C} \underline{\dot{x}} + \underline{K} \underline{x} = \underline{B} \underline{u} \quad (13)$$

where  $\underline{M}$ ,  $\underline{C}$ , and  $\underline{K}$  are mass, damping, and stiffness matrices of the system respectively, and  $\underline{x}$  is the system displacement vector,  $\underline{u}$  is the control vector and  $\underline{B}$  is the actuator influence matrix.

In modal coordinates, (13) becomes

$$\underline{\ddot{q}} + 2\underline{E} \underline{\dot{q}} + \underline{\omega}^2 \underline{q} = \underline{\Phi}^T \underline{B} \underline{u} \quad (14)$$

where  $\underline{\Phi}$  is the modal matrix and is the solution of the undamped eigenvalue problem

$$\omega_i^2 \underline{M} \underline{\rho}_i = \underline{K} \underline{\rho}_i \quad (15)$$

$$\underline{E} = \underline{\Phi}^T \underline{C} \underline{\Phi} = \begin{bmatrix} \zeta_1 & & 0 \\ & \ddots & \\ 0 & & \zeta_n \end{bmatrix}$$

$$\underline{\omega}^2 = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^2 \end{bmatrix}$$

In general, only a few of the important modes are controlled. Designate these modes as  $\underline{q}_c$  and the remaining modes as  $\underline{q}_r$ , the residue modes. Then Eq. (14) can be written as

$$(16) \quad \ddot{q}_c + 2E \omega_c \dot{q}_c + \omega_c^2 q_c = \Phi_c^T B u$$

and

$$(17) \quad \ddot{q}_r + 2E \omega_r \dot{q}_r + \omega_r^2 q_r = \Phi_r^T B u$$

Now the physical response is given by

$$\underline{x} = \Phi_c q_c + \Phi_r q_r \quad (18)$$

Hence, if  $q_r \neq 0$ , the physical response is influenced by the uncontrolled mode. This effect is called control spill-over.

Examination of (17) shows that  $q_r \neq 0$  if  $\Phi_1^T B u \neq 0$ .

It would appear then if we choose B such that

$$\underline{\Phi}_r^T B = \underline{0} \quad (19)$$

the control spill over problem is solved. Unfortunately, condition (19) can not be met in general.

In the following, a simple technique of reducing the control spill-over effect is described. Partition the residue modes into a set of secondary modes  $\rho$  and higher modes, with the higher modes having negligible effects on the system. In this way, we have

$$\underline{x} = \underline{\Phi}_c q_c + \underline{\Phi}_s q_s \quad (20)$$

Now, to minimize control spill-over, minimize  $q_s$ , or ultimately make  $q_s = 0$ . The response  $q_s$  can be solved from

$$\ddot{q}_s + 2E_s \omega_s \dot{q}_s + \omega_s^2 q_s = \Phi_s^T B u \quad (21)$$

Now if

$$\Phi_s^T B u = 0 \quad (22)$$

then

$$q_s = 0$$

Rewrite (22) as

$$\underline{A} \underline{u} = 0 \quad \text{where} \quad A = \Phi_s^T B \quad (23)$$

Assume  $n_s < n_a$ , then partition  $\underline{A}$  and  $\underline{u}$  into

$$\underline{A} = [\underline{A}_i \quad \vdots \quad \underline{A}_d], \quad \text{with} \quad \underline{A}_d \quad \text{nonsingular}$$

$$\underline{u} = \begin{bmatrix} u_i \\ u_d \end{bmatrix} \quad (24)$$

Using (24), (23) becomes

$$\underline{A}_i u_i + \underline{A}_d u_d = 0 \quad (25)$$

From (25), solve for

$$u_d = -\underline{A}_d^{-1} \underline{A}_i u_i \quad (26)$$

Thus, if

$$\underline{u} = \begin{bmatrix} u_i \\ u_d \end{bmatrix} = \begin{bmatrix} I \\ -\underline{A}_d^{-1} \underline{A}_i \end{bmatrix} u_i \quad (27)$$

Then the condition (22) will be met. Check

$$\begin{aligned} \Phi_s^T B \underline{u} &= \underline{A} \underline{u} = [\underline{A}_i \quad \underline{A}_d] \begin{bmatrix} I \\ -\underline{A}_d^{-1} \underline{A}_i \end{bmatrix} u_i \\ &= (\underline{A}_i - \underline{A}_i) u_i = 0 \end{aligned}$$

Equation (25) implies that out of the  $n_a$  actuators, only  $n' = n_a - n_s$  of them can be controlled independently. Define

$$\underline{D} = \begin{bmatrix} I \\ -\underline{A}_d^{-1} \underline{A}_i \end{bmatrix} \quad (28)$$

then

$$\underline{u} = \underline{D} u_i \quad (29)$$

Substitute (29) into (16) to get

$$\ddot{\underline{q}}_c + 2E_c \omega_c \dot{\underline{q}}_c + \omega_c^2 \underline{q}_c = \begin{matrix} \underline{\Phi}_c^T & \underline{B} & \underline{D} & \underline{u}_i \\ n_c \times n & n \times n_a & n_a \times n' & n' \times 1 \end{matrix} \quad (30)$$

The design problem is then to choose  $\underline{u}_i$  to control  $\underline{q}_c$  in

$$\underline{q}_c + 2E_c \omega_c \dot{\underline{q}}_c + \omega_c^2 \underline{q}_c = \underline{\Phi}_c^T \underline{B}' \underline{u}' \quad (31)$$

where

$$\underline{B}' = \underline{B} \underline{D} = \underline{B} \begin{bmatrix} I \\ -A_d^{-1} A_i \end{bmatrix}$$

$$\underline{u} = \underline{u}_i$$

$$\underline{A} = [\underline{\Phi}_s^T \underline{B}]$$

Note that the placement on actuator should be such that one can find a non-singular  $n_s \times n_s$  matrix  $\underline{A}_d$  from the matrix  $\underline{A}$ .

#### 4. Characteristic Equations for Undamped System with Generalized Control Law

The characteristic equation of an undamped system with a generalized control law is derived using reanalysis theory in this section. This equation can be implemented in the computer programs for efficient eigenvalue analysis or control gain synthesis.

Consider an undamped  $n$ -dof system with  $n_a$ -actuators, the equation of motion can be written as

$$\underline{M} \underline{x} + \underline{K} \underline{x} = \underline{B} \underline{u} \quad (32)$$

where  $\underline{M}$ ,  $\underline{K}$  are the  $n \times n$  mass and stiffness matrices respectively,  $\underline{u}$  is a  $n_a \times 1$  vector, i.e.,

$$\underline{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_{n_a} \end{bmatrix}$$

$\underline{B}$  is the  $n \times n$  Boolean distribution matrix with  $n_a$  nonzero rows. We will designate  $\underline{x}_a$  and  $\underline{x}_s$  as the dof locations for actuator and sensors respectively. Now define

$$\underline{x}_a = \underline{B}_a^T \underline{x} = n_a \times 1 \text{ vector} \quad (33)$$

$$\underline{x}_s = \underline{B}_s^T \underline{x} = n_s \times 1 \text{ vector}$$

then we conclude that

$$\underline{B} = \underline{B}_a \quad (34)$$

Furthermore, assume  $\underline{x}_a$  is contained in  $\underline{x}_s$ , and partition  $\underline{x}_s$  as

$$\underline{x}_s = \begin{bmatrix} \underline{x}_a \\ \underline{x}_r \end{bmatrix} \quad (35)$$

Now, the system dof can be partitioned either as

$$\underline{x} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_r \end{bmatrix} \quad \text{or} \quad \underline{x} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \\ \underline{x}_c \end{bmatrix}$$

Note that

$$\underline{x}_r = \begin{bmatrix} \underline{x}_b \\ \underline{x}_c \end{bmatrix}$$

Also note that

$$n = n_a + n_r = n_a + n_b + n_c$$

Now, express

$$\underline{x}_s = \underline{B}_a \underline{x} + \underline{B}_b \underline{x} \quad (36)$$

Assume the control law is

$$\underline{u} = -(\underline{c}_a \dot{\underline{x}}_a + \underline{c}_b \dot{\underline{x}}_b + \underline{k}_a \underline{x}_a + \underline{k}_b \underline{x}_b) \quad (37)$$

Substitute (6) into (1), so that for the closed loop system

$$\underline{M} \underline{\ddot{x}} + \underline{K} \underline{\dot{x}} = -\underline{B}_a \left( \underline{c}_a \dot{\underline{x}}_a + \underline{c}_b \dot{\underline{x}}_b + \underline{k}_a \underline{x}_a + \underline{k}_b \underline{x}_b \right) \quad (38)$$

$\begin{matrix} n \times n & n \times n \\ \underline{a} & \underline{a} & \underline{b} & \underline{a} & \underline{b} & \underline{a} & \underline{b} \end{matrix}$

or, assuming

$$\underline{x} = \underline{X} e^{st} \quad (39)$$

and letting  $\underline{R} = (s^2 \underline{M} + \underline{K})^{-1}$  equation (38) becomes:

$$\underline{x} = -\underline{R} \underline{B}_a (\underline{s} \underline{c}_a \underline{x}_a + \underline{s} \underline{c}_b \underline{x}_b + \underline{k}_a \underline{x}_a + \underline{k}_b \underline{x}_b)$$

or

$$\underline{x} = -\underline{R} \underline{B}_a [(\underline{s} \underline{c}_a + \underline{k}_a) \underline{x}_a + (\underline{s} \underline{c}_b + \underline{k}_b) \underline{x}_b] \quad (40)$$

and finally

$$\underline{x} = -\underline{R} \underline{B}_a [(\underline{s} \underline{c}_a + \underline{k}_a) (\underline{s} \underline{c}_b + \underline{k}_b)] \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix}$$

Now, premultiply (40) by  $\underline{B}_a^T$  to produce (41) and premultiply (40) by  $\underline{B}_b^T$  to produce (42).

$$\underline{B}_a^T \underline{x} = -\underline{B}_a^T \underline{R} \underline{B}_a \begin{bmatrix} \underline{H}_a & \underline{H}_b \end{bmatrix} \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix} \quad (41)$$

$$\underline{B}_b^T \underline{x} = -\underline{B}_b^T \underline{R} \underline{B}_a \begin{bmatrix} \underline{H}_a & \underline{H}_b \end{bmatrix} \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix} \quad (42)$$

Use the definition

$$\underline{x}_a = \underline{B}_a^T \underline{x}$$

$$\underline{x}_b = \underline{B}_b^T \underline{x}$$

$$\underline{H} = [\underline{H}_a \quad \underline{H}_c] = [s\underline{c}_a + \underline{k}_a \quad s\underline{c}_B + \underline{k}_b]$$

$$\hat{\underline{x}} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix}$$

Equations (41) and (42) become

$$\begin{matrix} \underline{x}_a \\ n_a \times 1 \end{matrix} = \begin{matrix} -\underline{R}_{ac} & \underline{H} \\ n_a \times n_a & n_a \times n' \end{matrix} \begin{matrix} \underline{x} \\ n' \times 1 \end{matrix} \quad n' = n_a + n_b \quad (43)$$

$$\begin{matrix} \underline{x}_b \\ n_b \times 1 \end{matrix} = \begin{matrix} -\underline{R}_{bc} & \underline{H} \\ n_b \times n_b & n_b \times n' \end{matrix} \begin{matrix} \underline{x} \\ n' \times 1 \end{matrix}$$

where

$\underline{R}_{aa}$  = portions of  $\underline{R}$  associated with  $\underline{x}_a$  only

$\underline{R}_{ba}$  = portion of  $\underline{R}$  associated with the coupling between  $\underline{x}_a$  and  $\underline{x}_b$

Rewrite (43) as

or

$$\begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix} = \begin{bmatrix} -\underline{R}_{aa} & \underline{H} \\ -\underline{R}_{ba} & \underline{H} \end{bmatrix} \hat{\underline{x}}$$

$$\hat{\underline{x}} = \begin{bmatrix} -\underline{R}_{aa} & \underline{H} \\ -\underline{R}_{ba} & \underline{H} \end{bmatrix} \hat{\underline{x}}$$

$$\det \left[ I + \begin{bmatrix} \underline{R}_{aa} & \underline{H} \\ \underline{R}_{ba} & \underline{H} \end{bmatrix} \right] = 0$$

thus

$$\det \begin{bmatrix} I + \underline{R}_{aa} \underline{H}_a & \underline{R}_{aa} \underline{H}_b \\ \underline{R}_{ba} \underline{H}_a & [I + \underline{R}_{ba} \underline{H}_b] \end{bmatrix} = 0 \quad (44)$$

is the characteristic equation of the closed loop system where

$$\underline{H}_a = s\underline{c}_a + \underline{k}_a \quad \underline{H}_b = s\underline{c}_b + \underline{k}_b$$

Example

For the 2-dof system shown in Figure (4.1)

$$u_1 = -(c_a \dot{x}_1 + c_b \dot{x}_2)$$

$$H_a = s c_a \quad H_b = s c_b \quad \underline{x}_a = x_1 \quad \underline{x}_b = x_2$$

$$R_{aa} = R_{11} \quad R_{bb} = R_{22} \quad R_{ba} = R_{21}$$

Thus, Eq. (14) becomes

$$\det \begin{bmatrix} 1 + s c_a R_{11} & s c_b R_{11} \\ s c_a R_{21} & s c_b R_{21} \end{bmatrix} = 0$$

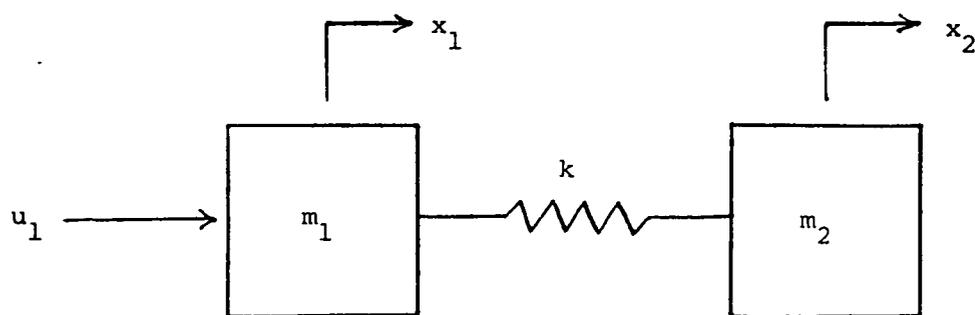


Fig. 4.1

## 5. Damper Design

Possible methods of controlling vibrations on large space structures have been reviewed. Two basic approaches are identified:

1. Modification of the control system so that the poles are well damped. This approach is based on the assumption that the major, if not only, cause of vibrations is the control system itself.
2. Addition of a separate damping system, which might be:
  - 2.1 Passive damping
  - 2.2 Active damping

The working medium of a ground based damper of either kind might be electromagnetic, but it would more probably be hydraulic or pneumatic, or, in the case of passive systems, magnetic or frictional. For space applications, hydraulic, pneumatic, and frictional systems must be ruled out as too unreliable in the space environment when little or no maintenance is possible. This leaves electromagnetic, magnetic, and possibly new systems based on the piezoelectric or other effects.

A list of possible systems is given in Table 5.1. The active damping systems using electromagnetics are assumed to incorporate coils moving in the fields of permanent magnets. The current in a coil would be provided by an electrical circuit in response to the output from one or more sensors. For the purpose of this study, such a circuit is assumed to introduce a negligible weight penalty, whereas the permanent magnet is considered to be a significant weight item. Similarly, the permanent magnet in the eddy current damper of a passive system is also considered to be a significant weight item. For systems which require a moving mass, such as the seismic mass of a passive system, or the mass in an active inertial system, it would therefore seem to be advantageous to combine the roles of moving mass and magnet in order to save weight. On this basis, the gyro concept might have a considerable weight disadvantage, because it would be impossible to combine the magnet and the flywheel.

Table 5.1. Possible Damping Systems

---

|         |  |
|---------|--|
| 1.0     | Modification of Control System               |
| 2.0     | Damping System                               |
| 2.1     | Passive Damping: Magnet and conductive strip |
| 2.1.1   | Seismic mass with stationary magnet          |
| 2.1.2   | Magnet as seismic mass                       |
| 2.1.3   | Gyro   |
| 2.1.4   | Two-force member                             |
| 2.2     | Active Damping: Electromagnetic              |
| 2.2.1   | Separate mass and magnet                     |
| 2.2.1.1 | Linear                                       |
| 2.2.1.2 | Rotating (inertia wheel)                     |
| 2.2.2   | Moving magnet                                |
| 2.2.2.1 | Linkage system                               |
| 2.2.2.2 | Rack and pinion                              |
| 2.2.3   | Gyro   |
| 2.2.4   | Two-force member                             |
| 2.3     | Active Damping: Piezoelectric                |

---

In comparing active against passive damping, the relative disadvantage of requiring an electrical system for the former must be weighed against the need to tune the seismic mass to a given frequency, so that the latter is essentially a narrow band device, whereas the active damper is relatively broadband.

The systems mentioned in Table 5.1 are discussed in more detail below.

#### **Modification of Control System (1.0)**

Modern control theory prescribes methods for designing control circuits having poles in any desired location, thus active damping can be achieved directly at the major source of disturbances. A very simple example is illustrated in Figure (5.1) which shows a one-dimensional system consisting of two masses connected by a spring, having one rigid-body translational mode, and one vibrational mode. Suppose that the impulse from a thruster is applied at one of the two masses. The vibrational mode will be excited and the system will move off with one mass coming to rest every half cycle. Now suppose that there are two thruster impulses applied one half-period apart, then the system will move without any internal vibration after the first half-period. Applied on a very much larger and more complex scale, such an approach could be used to maneuver a large space structure without any residual vibrations of serious amplitude.

#### **Passive Damping (2.1)**

The only reliable method of passive damping appears to be through the use of a magnet and a conductive strip. When the strip is passed between the poles of a magnet, eddy currents are induced in a direction normal to the velocity and to the magnetic field, and the effectiveness of such a damper is directly proportional to the product of electrical conductivity, magnetic field strength, and pole area. Unfortunately, the damping forces so produced are relatively small at frequencies of interest.

Four possible systems are shown in Figures (5.2) to (5.5). The first, system 2.1.1 in Figure (5.2), is a seismic damper, while for system 2.1.2 in Figure (5.3) the magnet doubles as the seismic mass. For system 2.1.3 in Figure (5.4), the magnetic damper is used in conjunction with a gyro to provide damping against angular motion of the structure. Finally, for system 2.1.4 in Figure (5.5), the magnetic damper is used in a two-force member spanning part of the large structure.

#### **Active Damping: Electromagnetic (2.2)**

In the active damping system, energy is removed from the structure by the action of an electromagnetic actuator or an electric motor. For the purpose of illustration, a voice-coil shaker of the type used for vibration testing has been assumed for the analyses of the active systems in the appendices. Such shakers are relatively heavy, because they contain powerful permanent magnets, although careful redesign for space applications might result in considerable

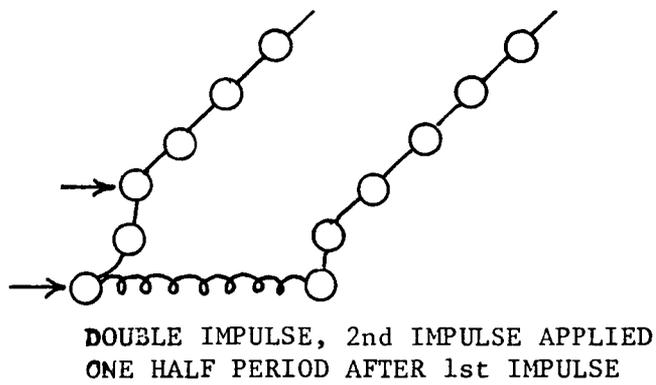
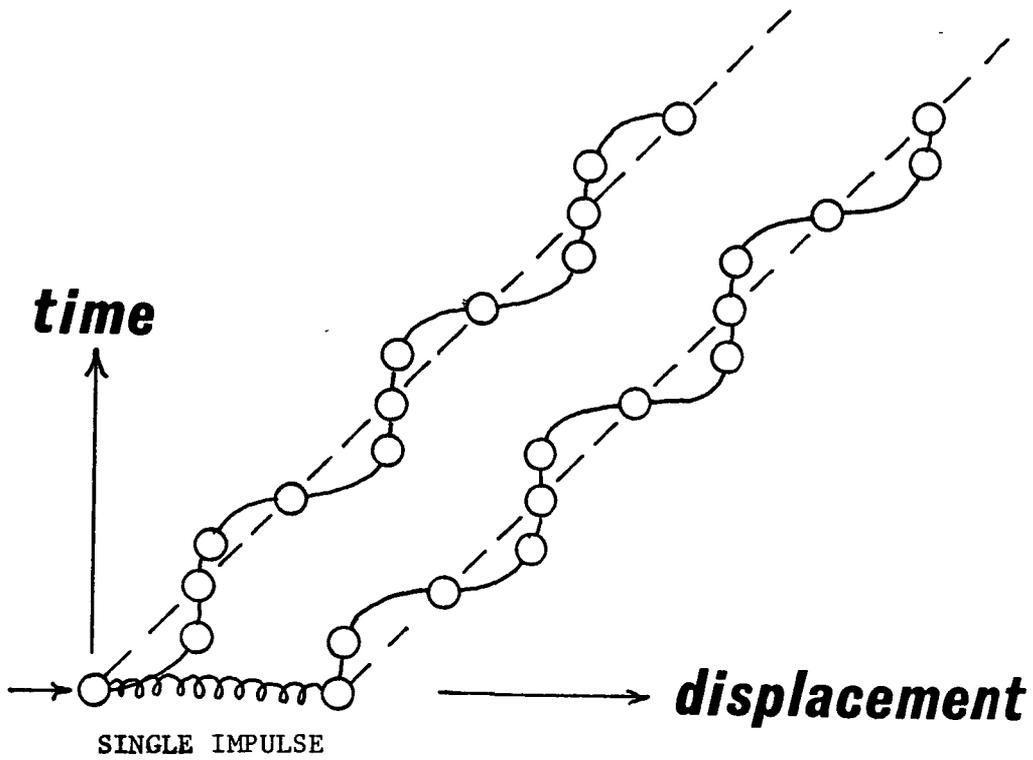


Fig. (5.1) 1.0 Modification of Control System

Figs. (5.2)-(5.5) Passive Damping Systems

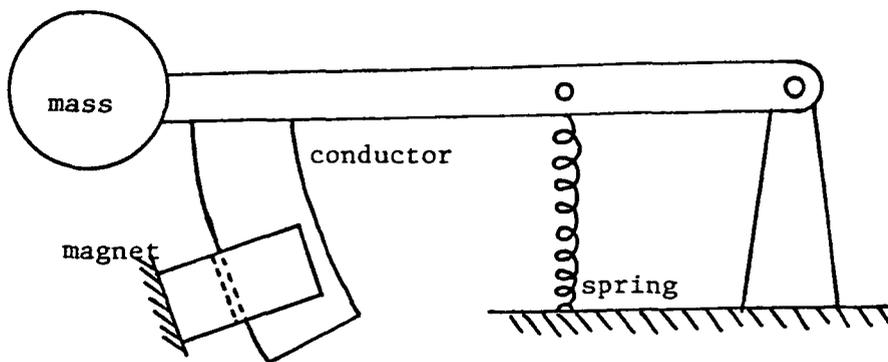


Fig. (5.2) Seismic Mass, Stationary Mass (2.1.1)

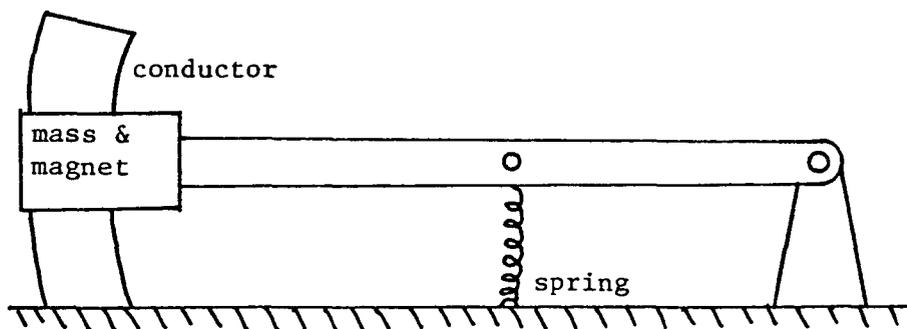


Fig. (5.3) (2.1.2) Magnet as Seismic Mass

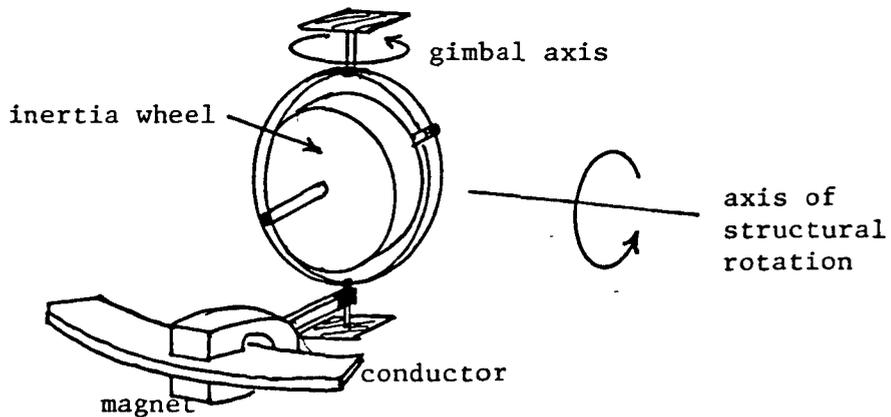


Fig. (5.4) Gyro System

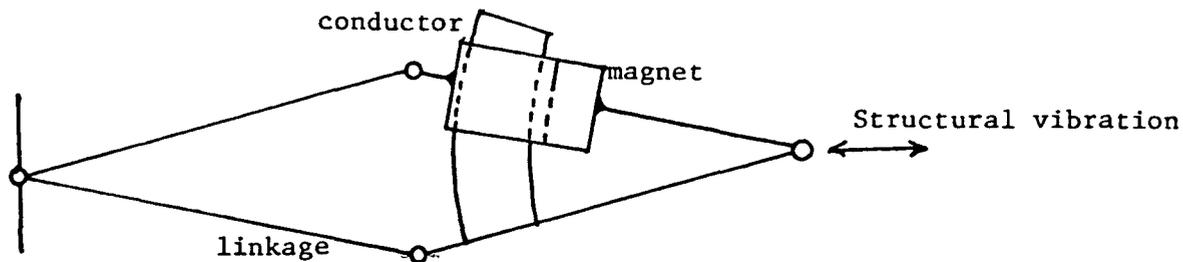


Fig. (5.5) (2.1.4) Two-Force Member

weight reductions. However, the weight penalty is directly proportional to the maximum energy which can be removed per cycle, as is shown in Appendix B. The design problem with a damper of this kind is that of ensuring that this maximum energy reduction per cycle is in fact available.

The system would be driven through a current amplifier from the difference between two integrated accelerometer signals, one from the structure, and one from the moving mass. Thus a force would be applied to the structure which would be directly proportional to the local velocity of the structure.

The major problem to be faced is how to provide an appropriate reaction on the structure so that as much energy is removed per cycle as is possible. One approach, shown as system 2.2.1.1 in Figure (5.6), employs a moving mass. As is shown in Appendix B, this system can be designed for peak performance at a given frequency. Above this frequency, performance is limited by the maximum available shaker force, while below this frequency, performance is limited by the maximum displacement available. An analogous rotary system, 2.2.1.2, shown in Figure (5.7) uses an electric motor driving an inertia wheel.

System 2.2.2.1 in Figure (5.8) is an improvement on the previous system, in which the shaker magnet doubles as the moving mass. A second version is shown in Figure (5.9). In Figure (5.8), the actuator motion is amplified by a "lazy tong" linkage. Such a linkage would not be practical, but could be used in a laboratory demonstration. In the second system, 2.2.2.2., Figure (5.9), the shaker is replaced by an electric motor which drives itself up and down a shaft with a rack-and-pinion gear.

System 2.2.3 shown in Figure (5.10) is similar to system 2.1.3 shown in Figure (5.4) except that the passive damper has been replaced by an active electromagnetic damper, while system 2.2.4 shown in Figure (5.11) is a two force member corresponding in a similar way with system 2.1.4 in Figure (5.5).

#### Active Damping: Piezoelectric (2.3)

Acoustical transducers have been made from polyvinylidene fluoride ( $PVF_2$ ) sheet, aluminized on both sides. The sheet is polarized, so that when a voltage differential is applied between the aluminized coatings, the material strains in one direction. Conversely, when it is strained, a voltage differential is induced. If the material is bonded to the surface of a structural element, a surface shearing force can be induced which will load the structure, and, if properly controlled, is capable of introducing damping. Proper use of such a material evidently depends on new approaches to structural dynamic analysis, but is certainly worthy of consideration.

#### Electronics

The active systems would be driven by feedback circuits. Such a circuit is shown in Figure (5.12), which repeats system 2.2.1.2 of Figure (5.8) with the addition of a circuit which compares  $m\ddot{y}$  and  $\dot{y}$

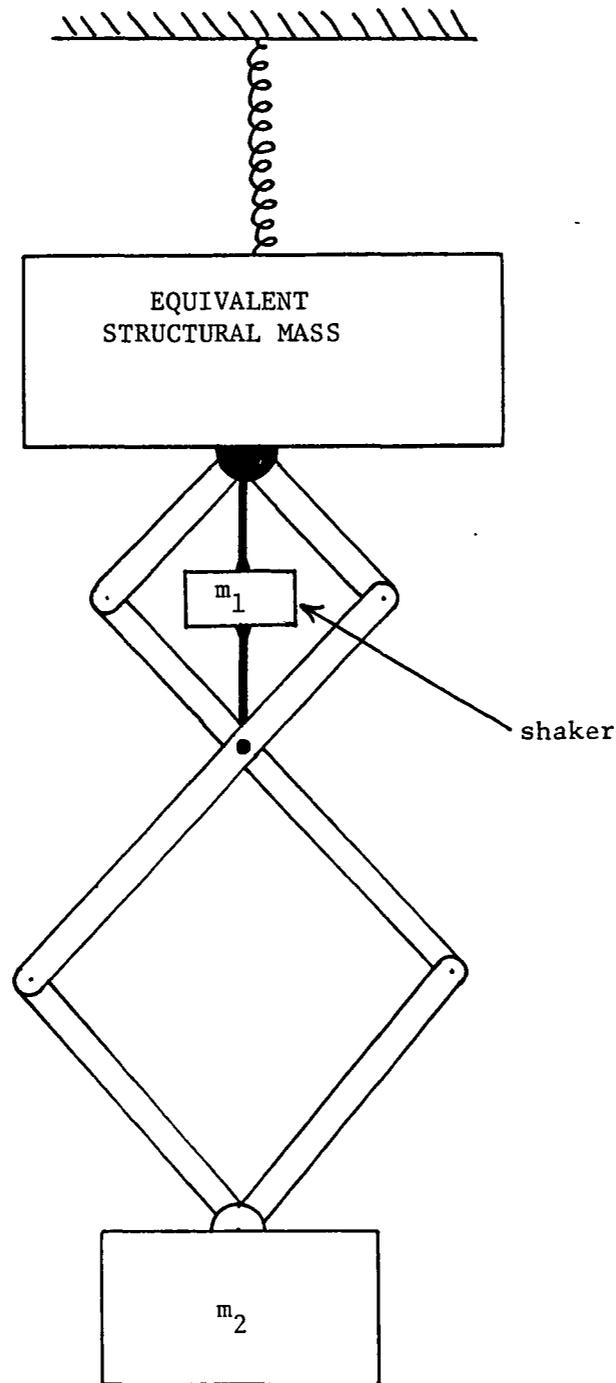


Fig. (5.6) System (2.2.1) Active Damper

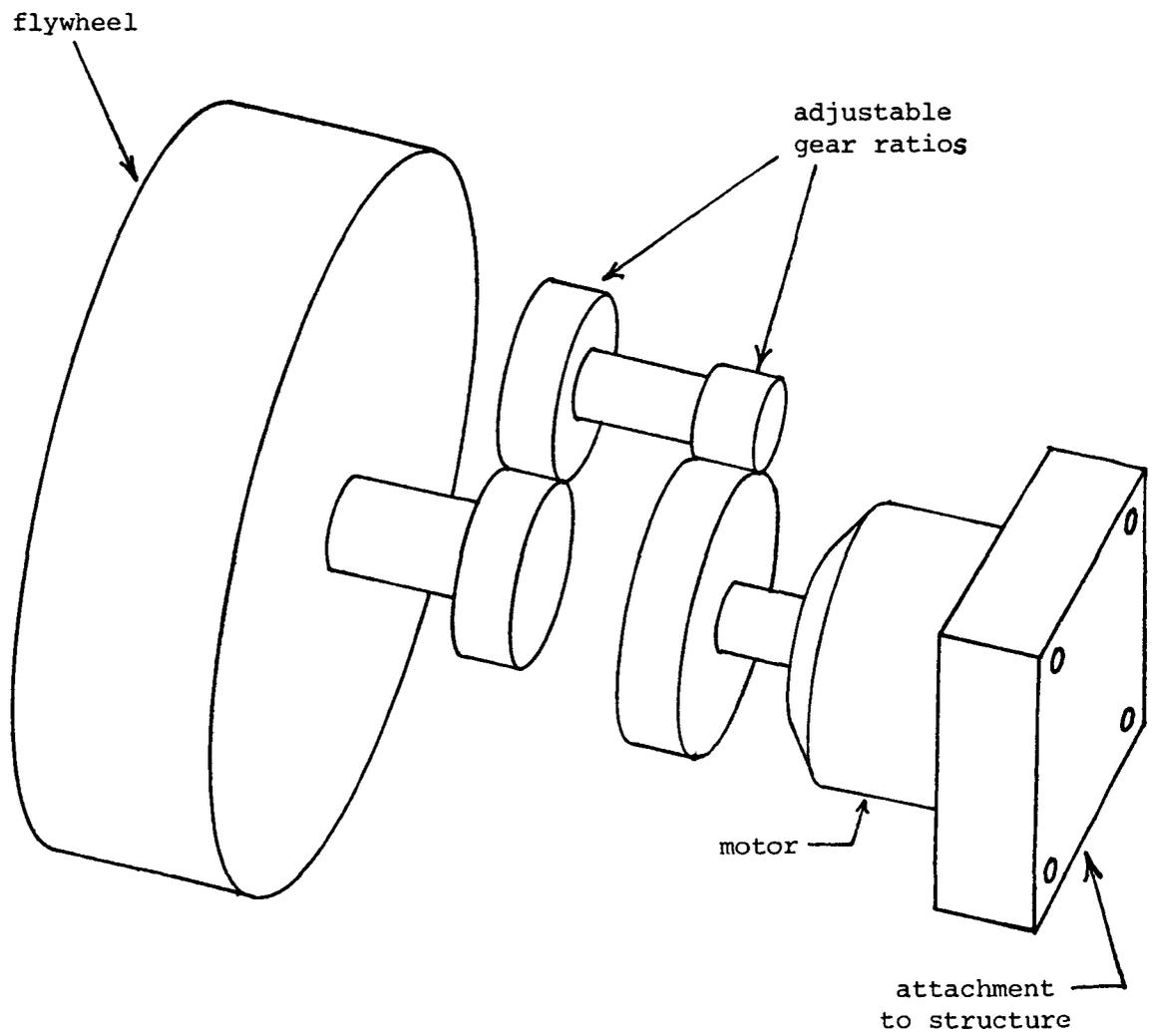


Fig. (5.7) Inertia Wheel System 2.2.1.2

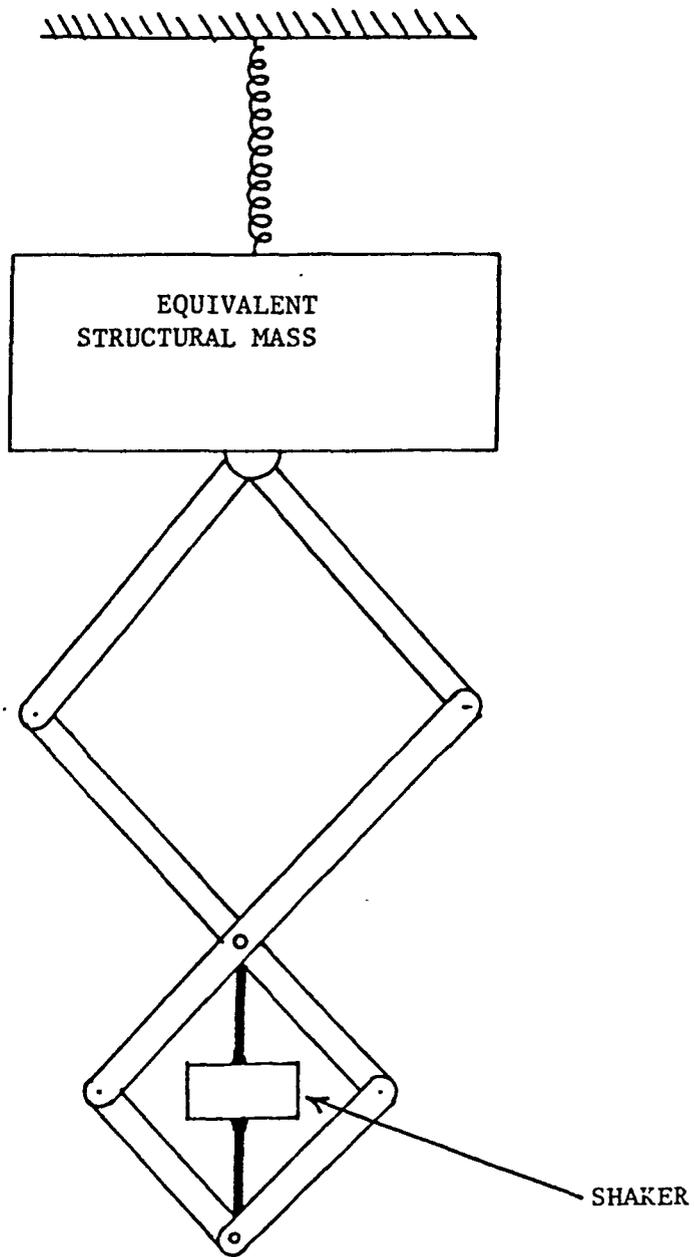


Fig. (5.8) System 2.2.2.1 Active Damper

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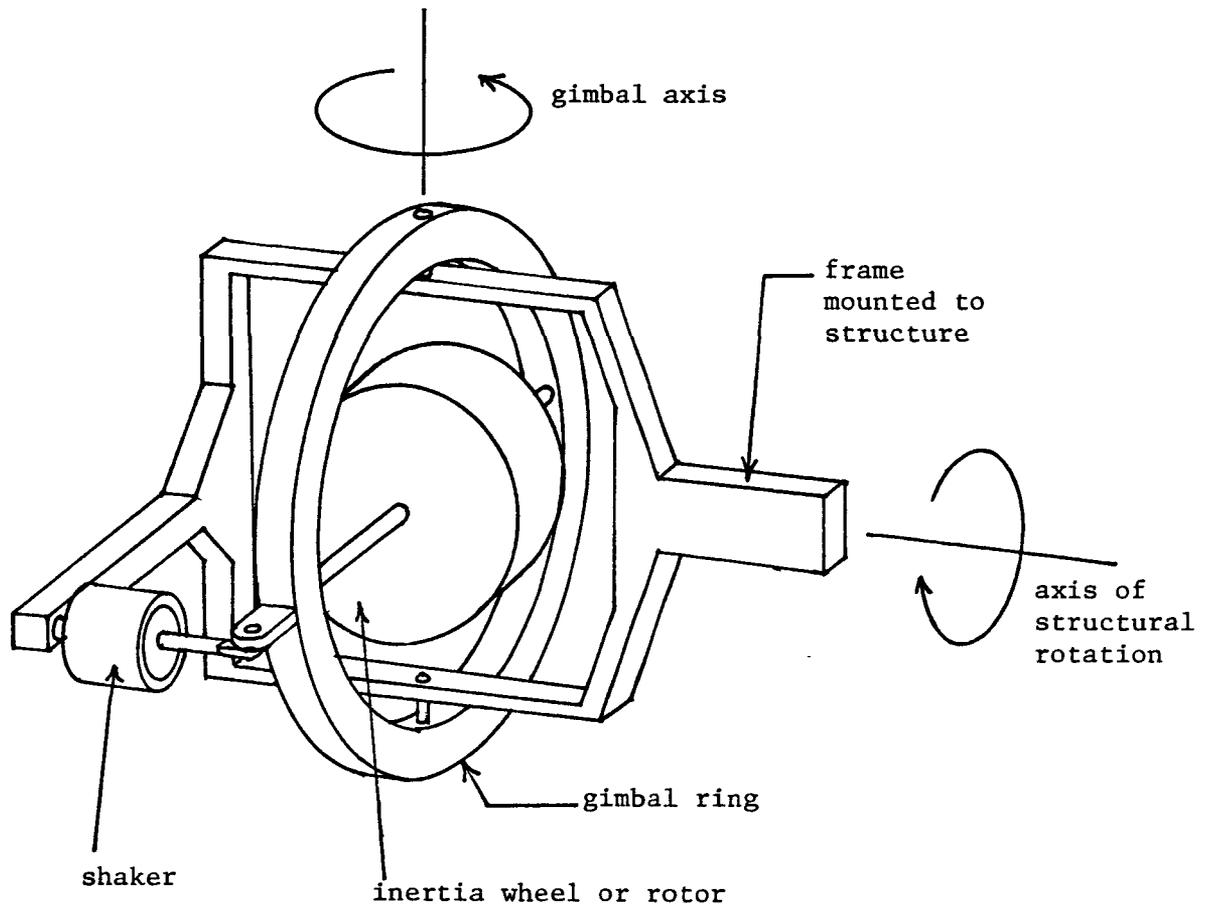


Fig. (5.10) System 2.2.3 Actively Damped Inertia Wheel

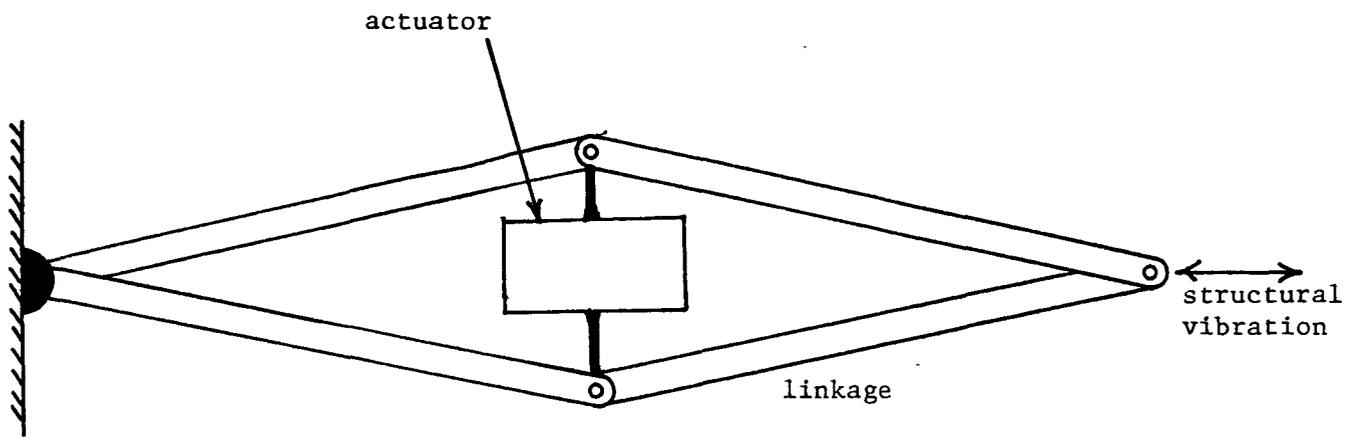


Fig. (5.11) System 2.2.4 Two Force Member Active Damper

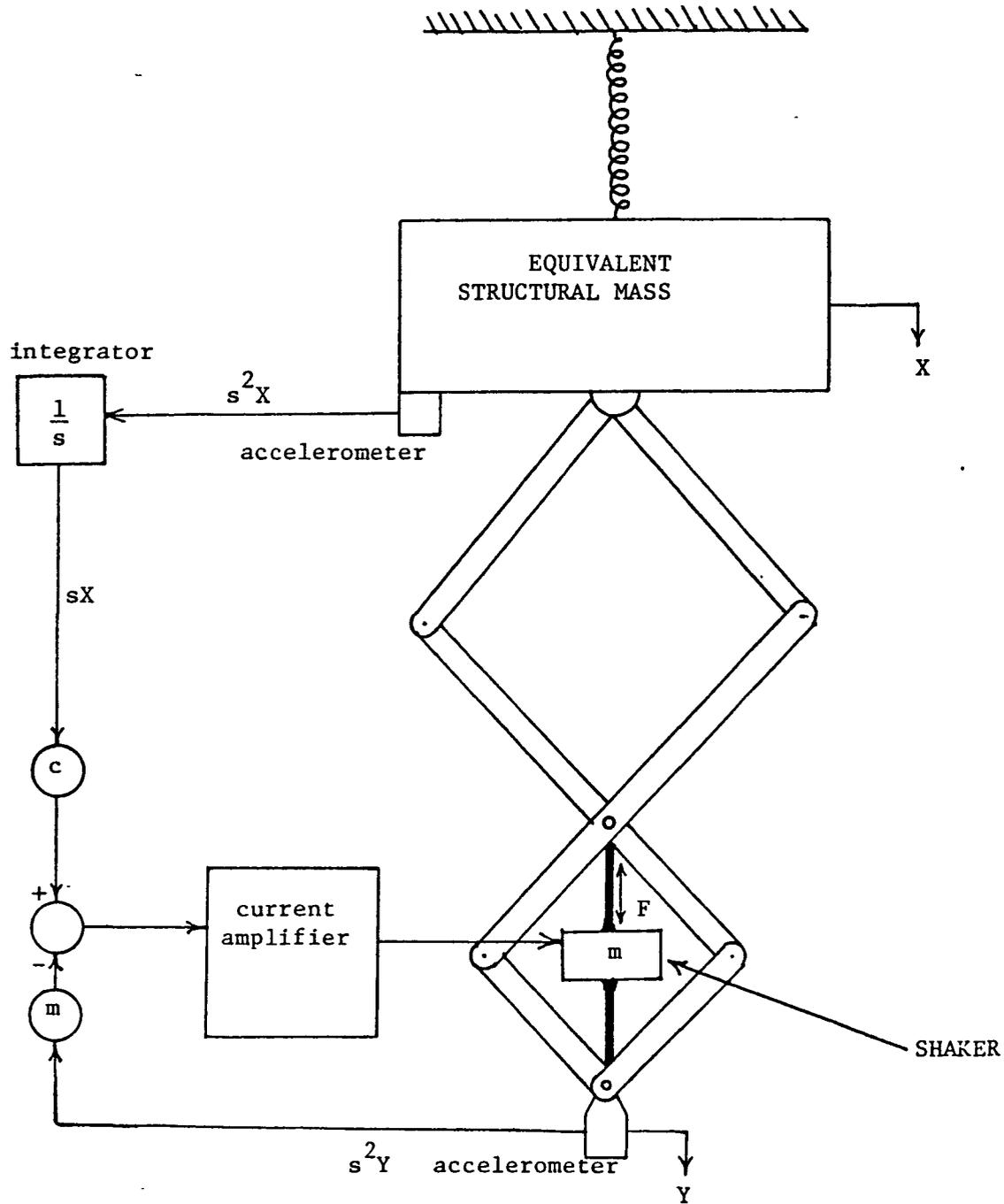


FIG. 12 SYSTEM 2.2.2.1 ACTIVE DAMPER Showing possible control loop added to system to yield desired characteristics

$cdx/dt$  as derived from two accelerometers, the difference being used to drive the voice-coil current through a amplifier.

### Comparative Performance

For a preliminary comparison of performance capability, two figures of merit can be considered.

1. Energy removed per cycle per unit mass at design frequency,  $W_D/m_T$
2. Bandwidth

In Appendix B, the value for  $W_D/m_T$  achieved with system 2.2.2.1 in Figure (5.8) under design conditions is shown to be .0619 J/kg, which is 79% of the maximum possible capability of a voice coil shaker. This was twice as good as the best system 2.2.1 arrangement of Figure (5.6). Therefore, it is tentatively concluded that other active damping systems, such as the inertia wheel, in Fig. (5.7), or the two force member, in Fig. (5.11), cannot be much better if they are based on electromagnetic action. Passive systems, being based on magnets, are probably no better, although this has not been investigated.

Performance of the active system, based on rate of energy removal, is constant above the design frequency, but rolls off below it. In contrast, the passive system is narrow-band and has to be tuned to a given frequency.

It is shown in Appendix C that the PVF<sub>2</sub> piezo-electric material has the potential for work per cycle of as much as 4 J/kg, which is more than 30 times that of the voice-coil shaker. On the other hand, using the minimum property values available, the work per cycle could be as low as .005 J/kg, which is less than one tenth as good as a voice-coil shaker. One of the most critical properties is the dielectric field strength,  $E_M$ . Since a dielectric breakdown causes an explosive failure, it might be necessary to allow quite a large safety margin, so that PVF<sub>2</sub> might prove no better than voice-coil shakers.

### Conclusion of Review

It is concluded from this series that an electromagnetic actuator, suspended in such a way that the magnet forms part of the moving mass, is the most promising approach for immediate application.

### Design of Experimental Damper

It was decided, as a result of the review, that an electromagnetic damper should be investigated experimentally, and that one should be developed for use on the scale model grillage at the NASA Langley Laboratory. The following design criteria were used initially

Maximum Force = 1 lbf = 4.45 N  
Design Amplitude = 0.5 in. = 0.0127 m.  
Design Frequency = 0.36 Hz

## Instrumentation

Accelerometer on moving mass - NASA supplied  
Accelerometer on grillage - NASA supplied  
Electronics - off grillage - NASA supplied

A design which meets these requirements is shown in Fig. (5.13).

### Description of Damper

The magnetic flux is supplied by the two toroidal samarium-cobalt magnets. The remainder of the magnetic circuit is temporarily completed with mild steel, which has a high saturation flux, and can therefore be designed for minimum weight. However, pure iron will be substituted when it becomes available. The annular gap is large to accommodate the windings of the fixed coil, this dictates a design which minimizes flux leakage at the gap. For example, two magnets are used where one would suffice if the gap were small, also, the internal diameter is larger than is required merely for clearance.

The entire magnet assembly moves along a hardened steel shaft on linear bearings, with one inch of useful travel. An accelerometer is attached to the moving magnet to feed back the acceleration of its mass, which is directly proportional to the damping force. The fixed coil has a Delrin core, and is designed to take ten layers of 26 gage magnet wire.

An outer case of polycarbonate tubing is used to keep the linear bearings clean. It also holds a track in which a small ball race moves to prevent the magnet from turning and thereby twisting the leads to the accelerometer. The end cap of the cover is removable so that the accelerometer leads can be connected to terminals.

### Magnet Analysis

The samarium cobalt magnets have the following specifications:

Outside Diameter = 0.75 in.  
Inside Diameter = 0.43 in.  
Magnetic Induction = 8000 Gauss  
= 0.8 Tesla

The following is calculated:

Magnetic Flux = 153 micromaxwells

The remaining circuit is designed so that the magnetic flux density in the mild steel does not exceed 66% of the 2.1 Tesla saturation flux density.

For the purpose of the subsequent analyses, the total 153  $\mu\text{M}$  flux is assumed to cut the coil windings.

- 7 Samarium Cobalt King Magnets
- 8 Linear Bearings
- 9 Stainless Steel Keeper
- 10 Accelerometer
- 11 Accelerometer Mount
- 12 3" External Casing and End Cap
- 13 Plastic Bumper
- 14 Linear Slide Bearings

N.A.S.A. Active Damper Actuator Prototype

- 1 Grillage Structure
- 2 Aluminum Base
- 3 1/8-inch Steel Shaft
- 4 Plastic Coil Frame and Coil Windings 4A
- 5 Iron Pole Piece
- 6 Iron Flux Loop

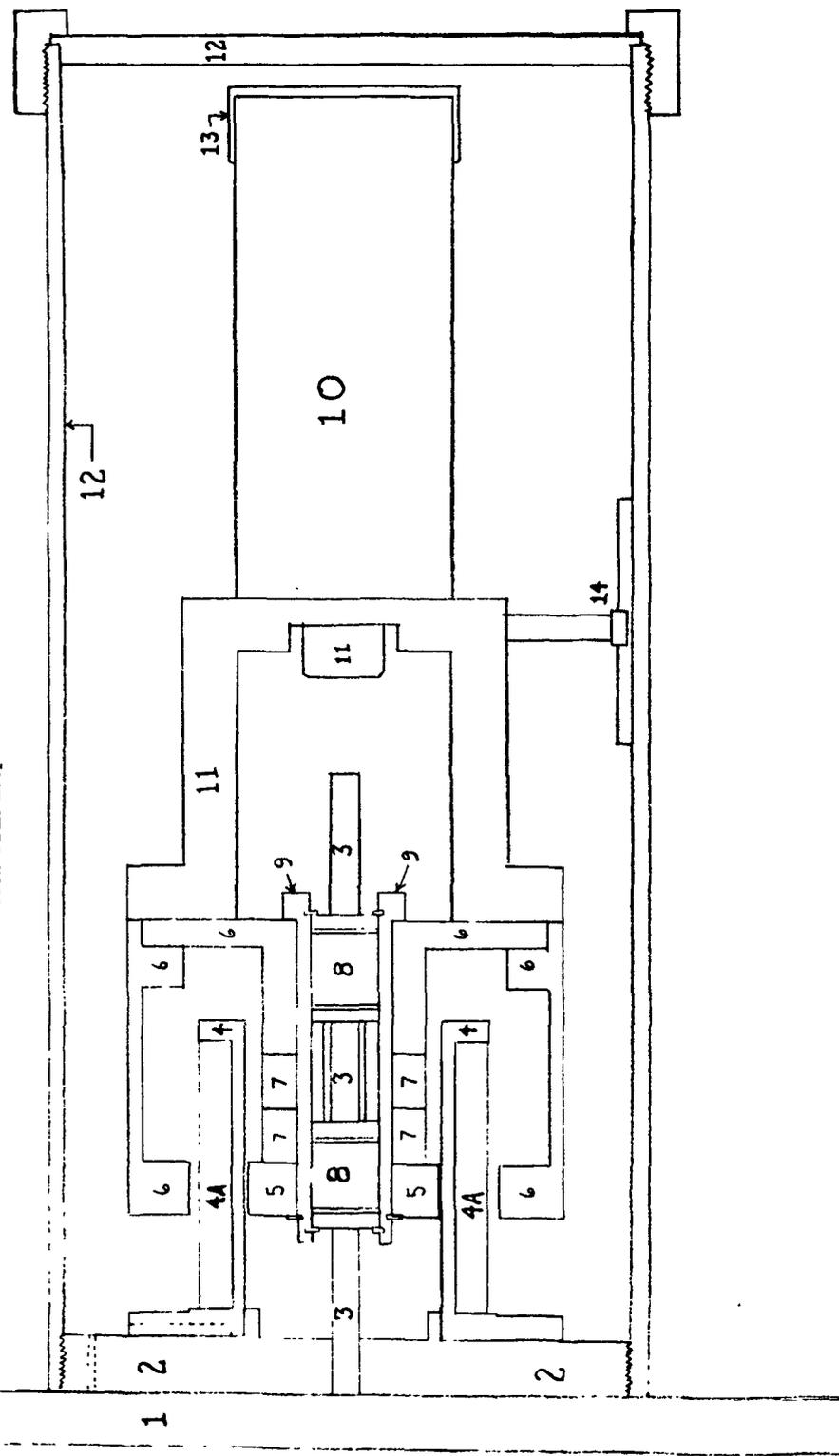


Fig. (5.13) N.A.S.A. Active Damper Actuator Prototype

## Analysis of Coil

It is assumed that each of the ten layers of 26 gage wire contains 67 turns, based on the following analysis

Spool length = 1.25 in.  
Diameter of bare wire = 0.015945 ins  
Diameter of enamelled wire (measured) = 0.01772 ins  
Winding efficiency =  $67 \times 0.01772 / 1.25 = 0.95 = 95\%$   
Turns per meter =  $10 \times 67 \times 39.37 / 1.25 = 21100 \text{ m}^{-1}$

The relationship between the force acting,  $F$ , the flux  $\Phi$ , the turns per meter,  $n$ , and the coil current  $A$  is developed as follows. Assume an effective coil diameter  $D$ , and a gap width  $w_g$ , then the flux in the gap is

$$(5.1) \quad \frac{B}{\text{gap}} = \frac{\Phi}{\pi D w_g}$$

The length of wire immersed in the gap is

$$(5.2) \quad \frac{L}{\text{gap}} = \pi D n w_g$$

and the force acting between the coil and the magnet is

$$(5.3) \quad F = I \frac{B}{\text{gap}} L = \Phi n I$$

For a force of one lbf, or 4.45 N

$$I = F / n \Phi = (4.45) / (21100)(153 \times 10^{-6}) = 1.38 \text{ A}$$

In operation, this would be a peak value, the actual DC current recommended for 26 gage wire is 0.51 A, based on 254 circular mils ( $\text{in}^2$ ) at 500 per A. Thus a damper operating continuously at maximum amplitude would be limited to a maximum force of

$$F_{\text{max}} = \sqrt{2} (0.51 / 1.38) = 0.52 \text{ lb}$$

However, during tests, peak amplitude will vary, so that the damping constant can be set to correspond to a much higher maximum force without causing excessive heating in the coil. The resistance of the coil, based on 0.0410 Ohms/ft., and a mean diameter of 1.2 ins, is

$$R = (0.0410)(10)(67)(1.2)\pi / (12) = 8.63 \text{ Ohms}$$

Other estimates, based on different values for resistivity, have been as high as 23 Ohms. Based on 8.63 Ohms, the peak voltage is

$$V_{max} = (1.38)(8.63) = 11.9 \text{ V}$$

$$V_{min} = -11.9 \text{ V}$$

### Analysis of Damping Constants

If we redefine K so that it is the ratio of structural amplitude  $|x_o|$  to damper amplitude,  $D_m$ , i.e.,

$$K = |X_o|/D_m \quad (5.4)$$

then, from Eq. (B.2) of Appendix B

$$1/K^2 = 1 + c^2/\omega^2 m_2^2 \quad (5.5)$$

also, with the linkage ratio R set to unity, Eq. (B.1) can be written as

$$F/F_m = \omega c D_m K/F_m \quad (5.6)$$

For the present design, the following values are anticipated

$$m_2 = 0.428 \text{ kg}$$

$$D_m = 1/2 \text{ inch} = 0.0127 \text{ m}$$

$$F_m = 1 \text{ lbf} = 4.45 \text{ N}$$

From Eq. (5.5), we can construct the following table for the damping constants, c.

Table 5.2 Values of Damping Constants

| K  | $ X_o $<br>(m) | c/ $\omega$<br>(kg) | f = .35Hz | c (Ns/m) |           |  |
|----|----------------|---------------------|-----------|----------|-----------|--|
|    |                |                     |           | f = 1 Hz | f = 3.5Hz |  |
| .8 | .0102          | .321                | .7059     | 2.017    | 7.059     |  |
| .5 | .0064          | .7413               | 1.630     | 4.658    | 16.3      |  |
| .3 | .0038          | 1.361               | 2.993     | 8.551    | 29.93     |  |
| .1 | .0013          | 4.259               | 9.37      | 26.76    | 93.7      |  |

and from Eq. (5.6) we can construct the following table for the corresponding force ratios,  $F/F_m$ .

Table 5.3 Force Ratios

| K  | $ X_o $<br>(m) | $F/F_M$            |                   |                    |
|----|----------------|--------------------|-------------------|--------------------|
|    |                | $f = .35\text{Hz}$ | $f = 1\text{ Hz}$ | $f = 3.5\text{Hz}$ |
| .8 | .0102          | .00354             | .0289             | .354               |
| .5 | .0064          | .00511             | .0418             | .511               |
| .3 | .0038          | .00563             | .046              | .563               |
| .1 | .0013          | .00588             | .0480             | .588               |

Note that the full force capability of the actuator is not required until the frequency exceeds 3.5 Hz.

In the proposed design, the damping constant  $c$  will be set by the electronic circuit. Suppose we pick a value of 2 Ns/m and examine the corresponding values for  $K$  and  $F/F_M$  as given by Eqs. (5.5) and (5.6). Note that a value for  $K$  of unity could correspond to a peak-to-peak structural amplitude of one inch while a value for  $F/F_M$  of unity would correspond to a maximum force of one lbf, the design value. The results are given in Table 5.4.

Table 5.4 Values for  $K$  and  $F/F_M$  when  $C = 2$  Ns/m, (Eqns. 2 and 3)

| $f(\text{Hz})$ | $K$  | $F/F_M$ |
|----------------|------|---------|
| .1             | .133 | .00048  |
| .2             | .261 | .00185  |
| .3             | .374 | .00403  |
| .35            | .426 | .00535  |
| .4             | .474 | .00678  |
| .5             | .558 | .01006  |
| .8             | .732 | .0210   |
| 1.0            | .802 | .0288   |
| 1.5            | .899 | 0.0484  |
| 2.0            | .937 | .0672   |
| 2.5            | .958 | .0859   |
| 3.0            | .971 | .1045   |
| 5.0            | .989 | .1773   |

Note that the maximum force is not approached even at 5 Hz. Had a larger damping constant been used, these forces would have been larger, but the value for  $K$  would have been smaller. For example, Table 5.5 shows how  $K$  varies with  $c$  at frequencies of 0.35, 1.0 and 3.5 Hz.

Table 5.5 K vs. c (Eqn. 2)

| c<br>Ns/m | K       |        |        |
|-----------|---------|--------|--------|
|           | 0.35 Hz | 1.0 Hz | 3.5 Hz |
| 0.1       | .994    | .999   | 1.000  |
| 0.2       | .978    | .997   | 1.000  |
| 0.3       | .953    | .994   | 1.000  |
| 0.4       | .920    | .989   | .999   |
| 0.6       | .843    | .976   | .998   |
| 0.8       | .762    | .958   | .996   |
| 1.0       | .685    | .937   | .994   |
| 1.5       | .532    | .873   | .988   |
| 2.0       | .426    | .802   | .978   |
| 2.5       | .352    | .732   | .966   |
| 3.0       | .299    | .607   | .953   |
| 5.0       | .185    | .474   | .883   |
| 10.0      | .094    | .260   | .685   |

None of the examples shown in Table 5.5 correspond to a value for  $F/F_m$  exceeding unity. Thus, the present design, if achieved, should be adequate for all of the cases shown, provided that c is set at 2 Ns/m or less.

#### Preliminary Tests

A prototype unit was built from the design shown in Figure 5.13. a photograph of this unit is shown in Figure 5.14, while the analog circuit used in an initial test of the unit attached to the NASA grillage is shown in Figure 5.15.

Because of the possibility of hitting the stops when the moving mass is not centralized, a spring was added to the system which gave it a natural frequency of about 2 Hz. Using switch no. 2, the system was excited at a natural frequency close to 4 Hz., then it was switched to feedback control, and the resulting damping was observed. Through switch no. 1, the inner feedback loop incorporating the accelerometer mounted on the moving mass could be included at will. When it was excluded, rapid damping of the 4 Hz. mode was observed, but when the inner loop was included, the system at first damped, and then went unstable at a frequency lower than 2 Hz.

It was concluded that the accelerometer on the inner loop could be discarded, but that an inner loop should be incorporated which included an LVDT to measure position. This way, a very low natural frequency could be simulated for the system, while a bias voltage could be applied to offset gravitational effects. At the same time, studies should continue on a system which incorporates an inner loop accelerometer.

#### Conclusions

An experimental damper has been designed in which a mass of .428 kg (.95 lbf) moves over a peak-to-peak amplitude of one inch under a programmed force which can be as much as one lbf. The design includes an accelerometer attached to the mass.

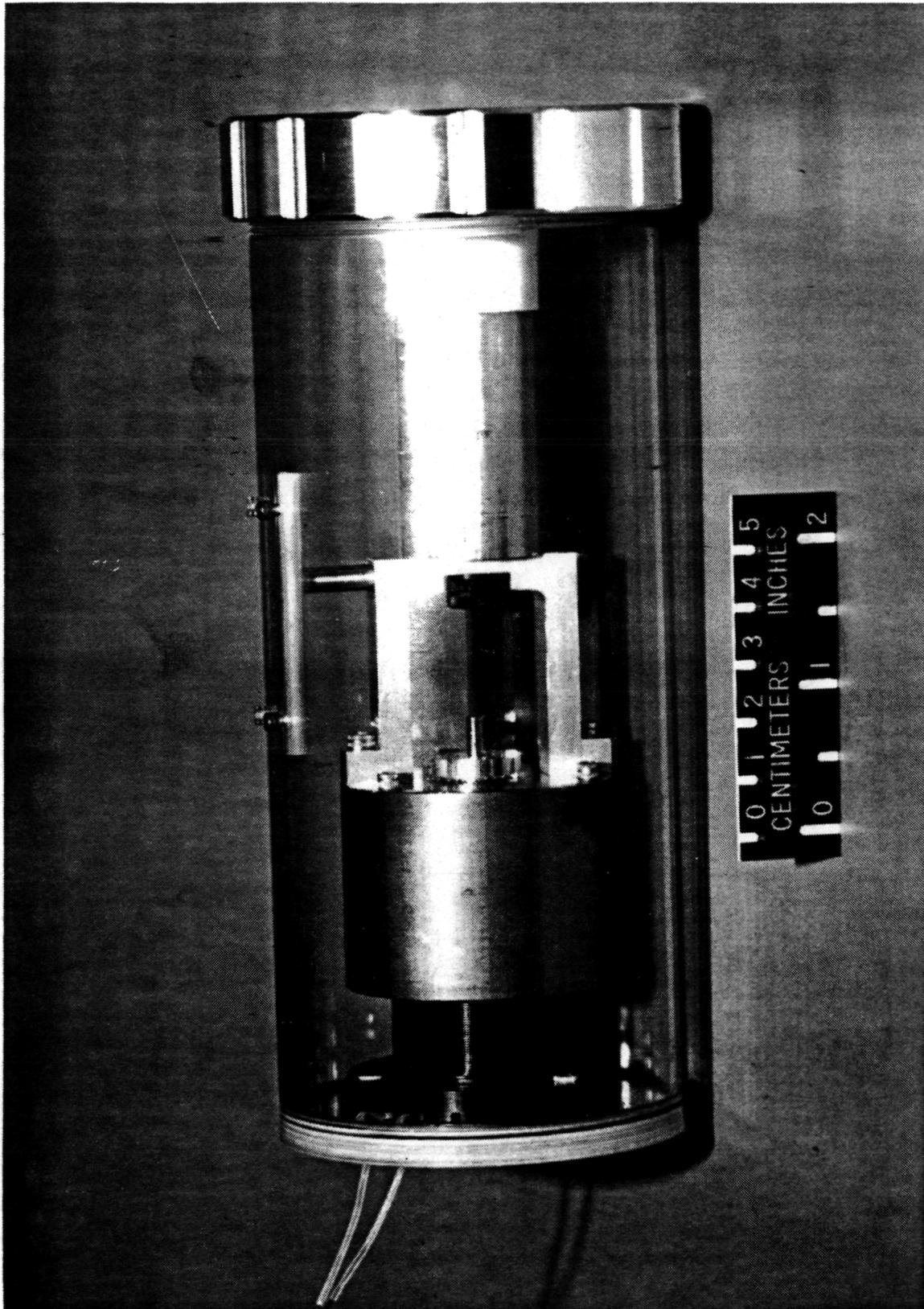


Fig. 5.14 Prototype Damper

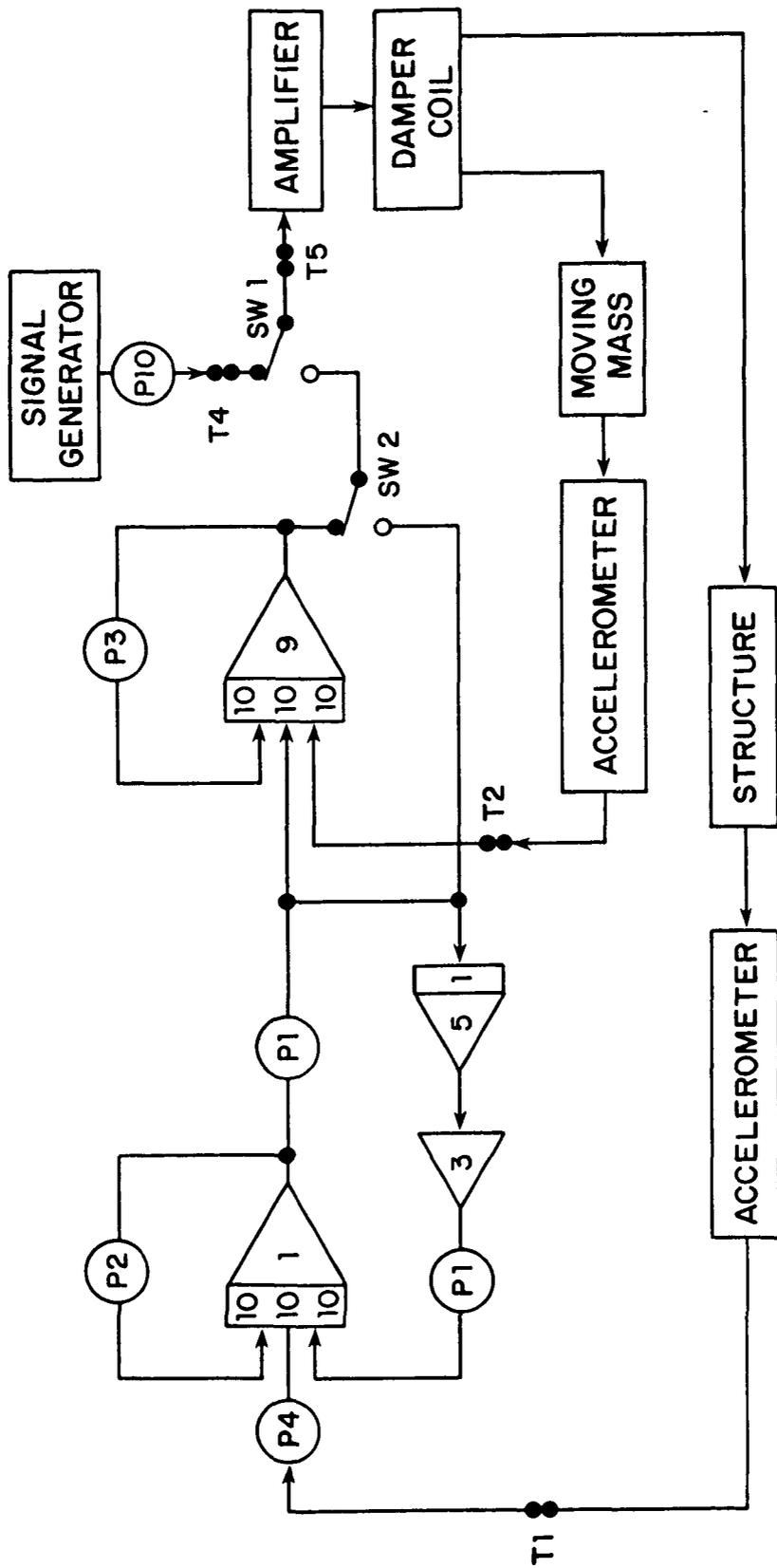


Fig. 5.15 Control Circuit for Prototype Damper

The programmed force is to be produced from the generation of a current in an amplifier driven by the difference between a signal proportional to the structural velocity and one proportional to the acceleration of the mass. Thus a damping force  $c$  will be generated on the structure.

Tentatively, the following design values are suggested

Damping constant = 2 Newton sec/meter

Design frequency = 0.35 Hz

Structural Amplitude = .43 inches peak-to-peak

From early experience with the prototype, it is concluded that the inner-loop accelerometer should be replaced by an LVDT if an early working system is required. However, research should continue into the use of the inner accelerometer, and an alternative to the LVDT should be found if dampers with longer strokes are found to be desirable.

### III. CONCLUSIONS

In conclusion, this work has covered two aspects of the problems involved in damping large space structures. On the one hand, the analytical problem of locating dampers has been investigated, while on the other, the problems of damper design have been reviewed. These considerations are summarized below

- (1) Five computer programs have been developed. They are
  - (i) ASMD1 - Assumed mode method with diagonal damping matrix
  - (ii) ASMD2 - Assumed mode method with full damping matrix
  - (iii) SPSTGN - Damped eigenvalues using reanalysis
  - (iv) COPZ - Optimization of damper gain
  - (v) NORMAL - Orthonormalization of mode shapes
- (2) Proof has been presented for two criteria for the optimal location of dampers. They are:
  - (i) MCFC - That the optimal single damper location is where the constrained frequency is a minimum
  - (ii) MFSC - That the optimal locations of several dampers are where the constrained frequency associated with the damper location has the largest separation from the corresponding undamped natural frequency of the system.
- (3) A method of minimizing the effect of control "spill-over" has been proposed and demonstrated.
- (4) The characteristic equation of an undamped system with a generalized control law has been derived using reanalysis theory.
- (5) A review of possible damper designs has been conducted. It was concluded that the most promising designs are active systems, using electromagnetic or piezoelectric actuators with linear or rotational motion.
- (6) A prototype linear electromagnetic damper was built and demonstrated using a moving permanent magnet. The damper is driven by feedback from an accelerometer, mounted in the structure, and integrated to provide a feedback force proportional to structural velocity. It was found necessary to incorporate a centering spring which will be replaced in future designs by a position feedback from an LVDT or other device.

#### IV. BIBLIOGRAPHY OF PAPERS PUBLISHED UNDER THIS CONTRACT

Optimal Damping for the Vibration Control of a Two-Dimensional Structure

B.P. Wang, W.D. Pilkey, G. Horner

AIAA Paper No. 81-1845

Optimal Damper Location in the Vibration Control of Large Space Structures

B.P. Wang, W.D. Pilkey, presented at the Third VPI/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft.



APPENDICES





$[ I ] = L \times L$  identity matrix  
 $[ \Lambda ] = [ \omega_l^2 ] = L \times L$  diagonal matrix  
 $[ c ] = [ \Phi ]^T [ c ] [ \Phi ] = L \times L$  full matrix

Set

$$\begin{aligned}
 \{ z \} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dot{q} \\ q \end{bmatrix} & \quad (A1.6) \\
 2L \times 1 &
 \end{aligned}$$

Then, (A1.5) becomes

$$\{ \dot{z}_1 \} = -[ \bar{c} ] \{ z_1 \} - [ \Lambda ] \{ z_2 \} \quad (A1.7a)$$

By definition (A1.6)

$$\{ \dot{z}_2 \} = \{ z_1 \} \quad (A1.7b)$$

Place (A1.7a), (A1.7b) together

$$\{ \dot{z} \} = [ A ] \{ z \} \quad (A1.8)$$

where

$$[ A ] = \begin{bmatrix} -[ c ] & -[ \Lambda ] \\ [ I ] & [ 0 ] \end{bmatrix} = \begin{bmatrix} -[ \Phi ]^T [ c ] [ \Phi ] & -[ \omega_l^2 ] \\ [ I ] & [ 0 ] \end{bmatrix}$$

A standard eigenvalue problem can be formulated from (8) and solved using the routine RESV in the NASA Langley library.

### III. INPUT PARAMETERS

#### III.1 Formal Parameters

Description of the 6 formal parameters

**N:** number of DOF's of the original system  
**NC:** number of dampers

IPRT: Dynamical matrix ([A] in (8)) printing flag,  
= 0 no printing  
= 1 print

### III.2 Common Blocks

In addition to the formal parameters, the following variables enter the subroutine ASMD through the common block set contained in file COMASMD. This file must be placed in the main program.

NTITLE(I): a holerith array of up to 80 columns which is used to define the title of the job

JC(I): damper location (DOF) of the Ith damper

C(I): initial damper values

WN(I): undamped natural frequency of the ith mode

PHI(J,I): mode shape coefficient at dof J of the Ith mode

NM: number of modes used in the assumed mode method

### IV. OTHER SUBROUTINES USED

ASMD calls NASA Library routine RESV to compute the eigenvalues of the dynamical matrix [A]. The computed complex eigenvalues are stored in array ER(I) and EI(I), which are the real and imaginary part of the Ith eigenvalue of matrix [A].

## V. USING SUBROUTINE ASMD

The following is a sample program that calls routine ASMD1.

```
PROGRAM TESTAS1(INPUT,OUTPUT,TAPE12)
C
C
      DIMENSION G(99)
COMMON / DATA1 / C(50)
COMMON / DATA2 / PHI(99,99),WN(99)
COMMON / DATA3 / EIGDATA(40,20,4),DAMP(40)
COMMON / DATA4 / NTITLE(80),JC(50)
COMMON / DATA5 / NM
COMMON / DATA6 / NDP
COMMON /DATA7/ER(99),EI(99)
C
C
      DO 1 I=1,50
1         C(I)=0.0
C
C      INPUT SECTION
C
      READ 10,(NTITLE(I),I=1,80)
10      PRINT 10,(NTITLE(I),I=1,80)
      FORMAT(80A1)
      READ *,N,NM,NC
      PRINT *,N,NM,NC
      DO 20 I=1,NC
      READ *,JC(I),C(I)
      PRINT *,JC(I),C(I)
20      CONTINUE
      READ *,IPRT
      PRINT *,IPRT
C
      DO 5010 I =1,NM
      READ*,   WN(I),G(I)
      PRINT*,WN(I),G(I)
      READ*, (PHI(J,I),J=1,N)
      PRINT*,(PHI(J,I),J=1,N)
5011     FORMAT(5E16.7)
5010     CONTINUE
      PRINT*, "N   WN   G"
      DO 99 I=1,N
99      PRINT*,I,WN(I),G(I)
      PRINT*, "PHI"
      DO 999 J=1,N
999     PRINT*,(PHI(J,I),I=1,N)
C
C
C
C
      CALL ASMD1(N,NC,IPRT)
C
C
      END
```



```

C          PRINT 1020,NM
C
          PRINT*, '***** DAMPERS *****'
          PRINT*, ' PLACEMENT(DOF)   VALUE'
          DO 666 I=1,NC
666        PRINT*, '                ', JC(I), '                ', C(I)
1020       FORMAT(/,10X,* ASSUMED MODE METHOD, USE *,IS,*,* MODES:/)
          PRINT 11
          PRINT*, 'THE DAMPED EIGENVALUES:'
C
          DO 333 II=1,NMD
333        PRINT*, II, ER(II), '+', EI(II), 'J'
          PRINT 11
          PRINT 11
9999      CONTINUE
END

```

## VII NUMERICAL EXAMPLE

To illustrate the use of ASMD1, the 2 dof system of Fig. (A1.1) is used. A damper of  $c = 2.3$  is attached to dof 2. The input data and control cards for this sample problem are shown in Table 1. The output is given in Table 2.

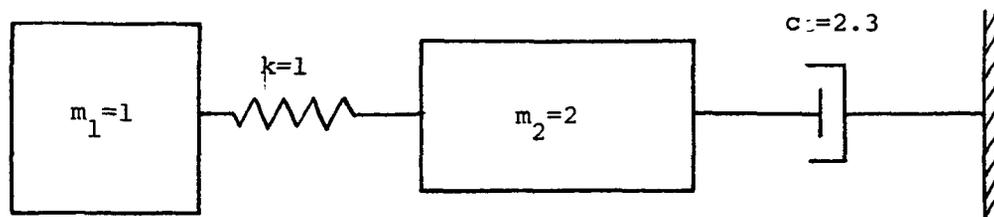


Fig. A1.1 A Damped 2-dof system

Table 1 Control Cards and Input Data for Sample Problem

The test program TESTAS1 can be run through the use of the following job file:

```

/JOB
BPW,CM300000,T1000.
USER,697121E.
CHARGE,102428,LRC.
DELIVER. 1293B D UVA
ATTACH,FTNMLIB/UN=LIBRARY.
GET,TESTAS1.
MAP(OFF).
FTN,I=TESTAS1,L=0,OPT=1,B=F.
LDSET,LIB=FTNMLIB.
F.
REWIND,OUTPUT.
COPYEI,OUTPUT,OTTAS1.
REPLACE,OTTAS1.
DAYFILE,L=DATAS1.
REPLACE,DATAS1.
EXIT.
REWIND,OUTPUT.
COPYEI,OUTPUT,XOTTAS1.
REPLACE,XOTTAS1.
DAYFILE,L=XDATAS1.
PACK,XDATAS1.
REPLACE,XDATAS1.
/EOR
TEST: 2-DOF FREE SYSTEM,TEST ASMD1 ← Title
2, 2, 1 ← 2 DOF, 2 MODES USED, 1 DAMPER
2, 2, 34 ← DAMPER: DOF 2, C=2.3
0 ← DYN. MAT. NOT PRINTED
0, 1 ←  $\omega_1=0, G_1=1$ 
0.577, 0.577 ←  $\phi_1$ 
1.2247, 1 ←  $\omega_2=1.2247, G_2=1$ 
0.316, -0.400 ←  $\phi_2$ 
EOI ENCOUNTERED.

```

Table 2 Output of Sample Problem

TEST: 2-DOF FREE SYSTEM, TEST ASMD1

ASSUMED MODE METHOD, USE 2 MODS

```
***** DAMPERS *****
  PLACEMENT(DOF)   VALUE
                2     2.3
```

THE DAMPED EIGENVALUES:

```
1 0.+0.J
2 -.8999940753023+0.J
3 -.1243049123488+1.122804973927.J
4 -.1243049123488+-1.122804973927.J
```

## Appendix A-2

### Subroutine ASMD2

#### Assumed Mode Method - Full Damping Matrix

##### I. INTRODUCTION

ASMD2 is a FORTRAN subroutine, based on the assumed mode method, that can be used to compute the eigenvalues of a damped system. The eigenvectors of the undamped system are used as the assumed modes. When all modes of the undamped system are used, the method is equivalent to a direct solution. Experience shows that retaining 1/3 to 1/2 of the modes of the original system in the assumed mode method usually leads to accurate eigenvalues while providing considerable savings in computer time. ASMD2 allows a full damping matrix to be used.

##### II. ASSUMED MODE FORMULATION

Let the free vibration of a damped system be described by

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\} \quad (\text{A2.1})$$

where  $[c]$  is a diagonal damping matrix. Let  $\{\rho_i\}$ ,  $\omega_i$  be the solution of the corresponding undamped problem

$$\omega_i^2 [m]\{\rho_i\} = [k]\{\rho_i\} \quad (\text{A2.2})$$

Furthermore, assume the mode shapes are normalized to unit generalized mass, i.e.,

$$\{\rho_i\}^T [m] \{\rho_i\} = 1.0 \quad (\text{A2.3})$$

Define

$$\{x\} = \sum_{l=1}^L \{\rho_l\} q_l = [\Phi] \{q\} \quad (\text{A2.4})$$

$$nx1 \qquad \qquad \qquad nxL \quad Lx1$$

where  $L$  is the number of modes used. Substitute (A2.4) into (A2.1) and premultiply by  $[\Phi]^T$  to get

$$[I]\{\ddot{q}\} + [\bar{c}]\{\dot{q}\} + [\Lambda]\{q\} = \{0\} \quad (\text{A2.5})$$

where

$[ I ] = L \times L$  identity matrix  
 $[ \Lambda ] = [ \omega_l^2 ] = L \times L$  diagonal matrix  
 $[ c ] = [ \Phi ]^T [ c ] [ \Phi ] = L \times L$  full matrix

Set

$$\{ z \} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dot{q} \\ q \end{bmatrix} \quad (A2.6)$$

2Lx1

Then, (A2.5) becomes

$$\{ \dot{z}_1 \} = -[ \bar{c} ] \{ z_1 \} - [ \Lambda ] \{ z_2 \} \quad (A2.7a)$$

By definition (A2.6)

$$\{ \dot{z}_2 \} = \{ z_1 \} \quad (A2.7b)$$

Place (7a), (7b) together

$$\{ \dot{z} \} = [ A ] \{ z \} \quad (A2.8)$$

where

$$[ A ] = \begin{bmatrix} -[ c ] & -[ \Lambda ] \\ [ I ] & [ 0 ] \end{bmatrix} = \begin{bmatrix} -[ \Phi ]^T [ c ] [ \Phi ] & -[ \omega_l^2 ] \\ [ I ] & [ 0 ] \end{bmatrix}$$

A standard eigenvalue problem can be formulated from (A2.8) and solved using the routine RESV in the NASA Langley library.

### III. FORMAL PARAMETERS AND CALLING ASMD2

The subroutine ASMD2 is called with the statement:

```
CALL ASMD2(N, NM, NROWC, C, PHI, WN, ER, EI, NTITLE, IPRT)
```

The formal parameters are defined as follows:

N= N.D.O.F. IN THE SYSTEM  
 NM= NO. MODES USED IN THE ASSUMED MODE CALC.  
 NROWC= NO. OF ROWS IN THE DIM. STATEMENT FOR C, PHI, WN, ER, EI  
 IN THE CALLING PROGRAM  
 C= THE DAMPING MATRIX  
 PHI= THE MODAL MATRIX-EIGENVECTORS NORMALIZED TO UNIT MASS.  
 WN= VECTOR OF UNDAMPED EIGENVALUES.  
 ER, EI= VECTORS CONTAINING THE REAL AND IMAG. PARTS OF THE EIGEN-  
 OF THE DAMPED SYSTEM. *values*  
 NTITLE= A VECTOR (DIM=80) CONTAINING THE TITLE OF THE RUN.  
 IPRT= A FLAG TO ALLOW PRINTING THE DYNAMICAL MAT. (1-PRINT, 0-NO

#### IV. OTHER SUBROUTINES USED

ASMD2 calls NASA Library routine RESV to compute the eigenvalues of the dynamical matrix [A]. The computed complex eigenvalues are stored in array ER(I) and EI(I), which are the real and imaginary part of the Ith eigenvalue of matrix [A].

## V. LISTING THE SUBROUTINE ASMD2

```

SUBROUTINE ASMD2(N,NM,NROWC,C,PHI,WN,ER,EI,NTITLE,IPRT)
C
C
C THIS PROGRAM USES THE ASSUMED MODE METHOD TO COMPUTE
C COMPLEX EIGENVALUES OF A DAMPED SYSTEM.
C A FULL DAMPING MATRIX MAY BE USED.
C
C THE PROGRAM IS A MODIFICATION OF PROGRAM CEIG 4/13/82.
C
C FORMAL PARAMETERS:
C
C N= N.D.O.F. IN THE SYSTEM
C NM= NO. MODES USED IN THE ASSUMED MODE CALC.
C NROWC= NO. OF ROWS IN THE DIM. STATEMENT FOR C,PHI,WN,ER,EI
C IN THE CALLING PROGRAM
C C= THE DAMPING MATRIX
C PHI= THE MODAL MATRIX-EIGENVECTORS NORMALIZED TO UNIT MASS.
C WN= VECTOR OF UNDAMPED EIGENVALUES.
C ER,LI= VECTORS CONTAINING THE REAL AND IMAG. PARTS OF THE EIGENVALUE
C OF THE DAMPED SYSTEM.
C NTITLE= A VECTOR(DIM=80) CONTAINING THE TITLE OF THE RUN.
C IPRT= A FLAG TO ALLOW PRINTING THE DYNAMICAL MAT.(1-PRINT, 0-NO PRINT
C
C DIMENSION C(NROWC,1),PHI(NROWC,1),WN(NROWC),ER(NROWC),
C E1(NROWC),NTITLE(80)
C
C DIMENSION DYEXAC(99,99),V(99,5),WK(270),A(99,99)
C
C
C MAX=99
C NDOF=N
C NMD=2*NM
C
C FORM DYNAMICAL MATRIX
C
C NN=2*NM
C DO 100 I=1,NN
C DO 100 J=1,NN
C A(I,J)=0.0
100 DYEXAC(I,J)=0.0
C DO 200 I=1,NM
C IN=I+NM
C DYEXAC(I,IN)=-WN(I)*WN(I)
C DYEXAC(IN,I)=1.0
200 CONTINUE
C
C DO 12 J=1,NDOF
C DO 12 L=1,NM
C DO 13 K=1,NDOF
13 A(J,L)=A(J,L)+C(J,K)*PHI(K,L)
12 CONTINUE

```

```

C
DO 14 K=1,NM
DO 14 L=1,NM
DO 15 J=1,NDOF
15 DYEXAC(K,L)=DYEXAC(K,L)-PHI(J,K)*A(J,L)
14 CONTINUE
C
PRINT*, "*****" OUTPUT "*****"
PRINT 11
PRINT 10, (NTITLE(I), I=1, 80)
10 FORMAT(80A1)
11 PRINT 11
11 FORMAT(//)
IF (IPRT.EQ.0) GO TO 210

PRINT 201
201 FORMAT(1H1, ///, * DYNAMICAL MATRIX IS *///)
DO 202 I=1, NM
202 PRINT 205, (DYEXAC(I, J), J=1, NND)
205 FORMAT(/, 5X, 5E15.6)
210 CALL RESV(MAX, NMD, DYEXAC, ER, EI, 0, 0, 0, WK, IERR)
C
PRINT 1020, NM
PRINT*, "THE DAMPING MATRIX IS:"
DO 666 I=1, NDOF
666 PRINT*, (C(I, J), J=1, NDOF)
C
1020 FORMAT(/, 10X, * ASSUMED MODE METHOD, USE *, IS, *, MODEL, *)
PRINT 11
PRINT*, "THE DAMPED EIGENVALUES:"
C
DO 333 II=1, NMD
333 PRINT*, II, ER(II), "+", EI(II), "J"
PRINT 11
PRINT 11
9999 CONTINUE

```

## VI. CONTROL CARDS AND TEST PROGRAM

The test program TESTAS4 can be run through the use of the job file of Table 1. Table 2 is a listing of the test program TESTAS4.

Table 1 Control Cards and Input Data for Sample Problem

```

/JOB                                1293B    UVA
BPW,CM300000,T1000.
USER,697121E.
CHARGE,102428,LRC.
DELIVER. 1293B D UVA
ATTACH,FTNMLIB/UN=LIBRARY.
GET,TESTAS4.
MAP(OFF).
FTN,I=TESTAS4,L=0,OPT=1,B=F.
LDSET,LIB=FTNMLIB.
F.
REWIND,OUTPUT.
COPYEI,OUTPUT,OTTAS4.
REPLACE,OTTAS4.
DAYFILE,L=DATAS4.
REPLACE,DATAS4.
EXIT.
REWIND,OUTPUT.
COPYEI,OUTPUT,XOTTAS4.
REPLACE,XOTTAS4.
DAYFILE,L=XDATAS4.
PACK,XDATAS4.
REPLACE,XDATAS4.
/EOR
TEST: 2-DOF FREE SYSTEM ← Title
2, 2 ← 2 DOF, 2 modes used in calc.
1.3, -.5, 0, 0 ← Damping matrix  $\begin{bmatrix} 1.3 & -0.5 \\ 0 & 0 \end{bmatrix}$ 
0 ← No printing of Dyn. mat.
0, 1 ←  $\omega_1 = 0, G_1 = 10$ 
0.577, -0.577 ←  $\phi_1$ 
1.2247, 1 ←  $\omega_2 = 1.2247, G_2 = 1$ 
0.816, -0.408 ←  $\phi_2$ 
0
EOI ENCOUNTERED.
/

```

Table 2 Listing of program TESTAS4

```

P*
PROGRAM TESTAS4(INPUT,OUTPUT)
C
C
      DIMENSION G(88),C(99,99),PHI(99,99),WN(99),ER(99),EI(99)
      DIMENSION NTITLE(80)
C
C
      DO 1 I=1,50
        DO 1 J=1,50
          C(I,J)=0.0
1
C
      INPUT SECTION
C
      READ 10,(NTITLE(I),I=1,80)
      PRINT 10,(NTITLE(I),I=1,80)
10
599  FORMAT(80A1)
      READ *,N,NM
      PRINT *,N,NM
C
      DO 20 I=1,N
        READ*,(C(I,J),J=1,N)
20
        PRINT*,(C(I,J),J=1,N)
      READ *,IPRT
      PRINT *,IPRT
C
C
      DO 5010 I =1,NM
      READ*, WN(I),G(I)
      PRINT*,WN(I),G(I)
      READ*, (PHI(J,I),J=1,N)
      PRINT*,(PHI(J,I),J=1,N)
5010  CONTINUE
      PRINT*, 'N   WN   G'
      DO 99 I=1,N
99    PRINT*,I,WN(I),G(I)
      PRINT*, 'PHI'
      DO 999 J=1,N
999   PRINT*,(PHI(J,I),I=1,N)
C
C
      CALL ASMD2(N,NM,99,C,PHI,WN,ER,EI,NTITLE,IPRT)
      PRINT*, '1=REPEAT,0=STOP'
      READ*,ITEST
      IF (ITEST.EQ.1) GO TO 599
C
C
      END
C

```

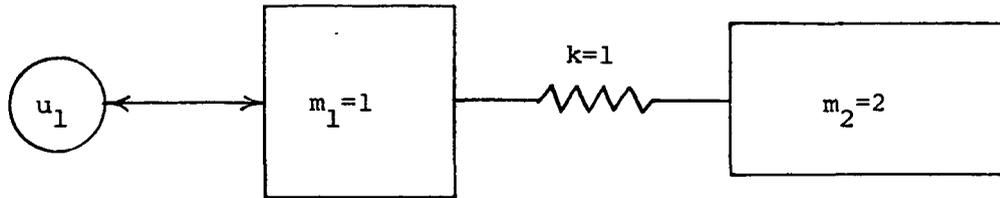


Fig. A2.1 Actively Controlled 2-dof system

## VII SAMPLE PROBLEM

Figure (A2.1) shows an actively controlled 2-dof free-free system. The control law is

$$u_1 = -1.3 \dot{x}_1 + 0.5 \dot{x}_2$$

The input data to test program TESTAS4 is shown in Table 1. Table 3 shows the output.

Table 3 Output of Sample Problem

\*\*\*\*\*  
OUTPUT \*\*\*\*\*

TEST: 2-DOF FREE SYSTEM

ASSUMED MODE METHOD, USE 2 MODES

THE DAMPING MATRIX IS:

1.3 -.5  
0. 0.

THE DAMPED EIGENVALUES:

1 0.+0.J  
2 -.3403281689367+0.J  
3 -.4790459155316+.971771301425J  
4 -.4790459155316+-.971771301425J

## Appendix A-3

### Subroutine SPSTGN Documentation

#### Damped Eigenvalues Using Reanalysis

##### I. Introduction

SPSTGN is a FORTRAN subroutine that can be used to compute the damped eigenvalues when damping is introduced into an originally undamped system through the introduction of a diagonal viscous damping matrix. Based on the modal data of the undamped structure, the damped eigenvalues are computed using an efficient reanalysis formulation. It is the purpose of this document to summarize the formulation and usage of the subroutine SPSTGN.

##### II. Summary of the Reanalysis Formulation of the Damped Eigenvalue Problem

Given an undamped system described by the following normalized modal data

$$\omega_i, \{\rho_i\}, \quad i = 1 \text{ to } L \leq \text{Number DOF} = \text{NELEM}$$

where

$$\begin{aligned} \omega_i &= \text{natural frequency of the } i\text{th mode, rad/sec} \\ &= \text{OMEGA}(I) \\ \{\rho_i\} &= \text{ith mode shape} \\ \rho_{j,i} &= \text{mode shape coefficient, } j\text{th component of the } i\text{th mode} \\ &= \text{PHI}(J,I) \end{aligned}$$

The normalization requirements are

$$\{\rho_i\}^T [M] \{\rho_j\} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

where  $[M]$  is the system mass matrix.

With the introduction of  $n_c$  damper at dof  $J_1, J_2, \dots, J_{n_c}$ , the damped eigenvalue problem can be formulated as the solution of the characteristic equation:

$$\det(I + \hat{R}(s)\hat{C}) = 0 \quad (\text{A3.1})$$

where

$$[\hat{R}] = n_c \times n_c \text{ condensed receptance matrix}$$



$n_d = \text{NODEPL} =$  the number of the particular damper in ISTATNO(I) being varied.

### III Input Parameters

#### III.1 Formal Parameters

Description of the 11 formal parameters:

NELEM: number of DOF's in the original system  
NMOD : the number of modes requested for the printout  $\leq L$   
RUNNO: your run number identifier  
IDOPT: 0 = all C values vary  
1 = only one value varies (see NODEPL)  
EPS : convergence parameter (used in the test for convergence in the Muller's Method root finding routine)  
NDAM : number of modified system modes desired ( $< L$ )  
NUMITER: maximum number of iterations per eigenvalue  
CDINIT: initial damper value (units should be consistent with mass and stiffness, e.g. lb sec/in)  
CDINC : increment in damper vlaue  
NCD : total number of damper increments  
NODEPL: the number of the varied damper in ISTATNO(I). NODEPL is an element of 1,2,...,NSTAT.

Note: if IDOPT = 0,

$$c_i = c_{i0} + \Delta c \text{ for } i = 1, 2, \dots, \text{NSTAT}$$

if IDOPT = 1

$$c_i = c_{i0} \quad i \neq \text{NODEPL}$$
$$c_i = c_{i0} + \Delta c, \quad i = \text{NODEPL}$$

#### III.2 Common Blocks

In addition to these variables, the following variables enter the subroutine through the common blocks and must be evaluated prior to calling SPSTGN. Also, the common block set contained in the file COMSPST must be placed in the main program.

NTITLE(I): a holerith array with a title of the job  
L: the number of modes used in the receptance calculation  
OMEGA(I): a vector containing the L values of the original  $\omega$   
PHI(J,I): a matrix whose L columns are the modeshapes of the original system (DIM: NELEM x L)  
Note: These modeshapes must be normalized to unit generalized mass.  
NSTAT: number of stations where dampers are placed  
ISTATNO(I): an array giving the location (DOF) of the attached dampers  $I = 1$  to NSTAT  
CARRAY(I): an array containing the damper value for position ISTATNO(I),  $I = 1$  to NSTAT

#### IV. Using Subroutine SPTGN

The data required are input through formal parameters (Section III.1) and through the common block set COMSPST, into SPSTGN (Section III.2). It should be noted that mode shapes have to be normalized to unit generalized mass. That is,

$$\{\rho_i\}^T [M] \{\rho_i\} = 1.0$$

#### V. SAMPLE PROBLEM

The Model is the NASA 88 D.C.F. grid.

The model data is read from TAPE 12 is NASSAGG2, which contains data for all 88 modes, including 3 rigid body modes.

Four dampers are placed at D.O.F. 1, 11, 77, 88.

The initial damper values are 0.2.

The approximate receptance elements are to be calculated on the basis of using  $L (= 30^{**})$  modes of the 88 modes.

The first 2 modes are to be printed out (NMOD = 2).

All the damper values are to vary (IDOPT = 0) with increments of CDINC = 0.1 for NCD 5 trials.

The convergence criterion for eigenvalues is  $|\det(A)| < \epsilon = 10^{-5}$ .

The test program TESTSP is designed to interactively accept the above data and then call subroutine SPSTGN to do the calculations and print out the results.

The following input values were given (the remark number corresponds to the number in the following TESTSP printout).

```
Remark 3  TEST SPST (title)
        2  88, 2, 30, 1 (88 DOF, 2 modes to be printed, use
           30 modes, Run No. = 1)
        3  0 (all damper values change)
        4  The modal data was read from TAPE 12  NASSAGG2
        5  4, 1.E-5, 2, 50 (4 dampers  $\epsilon$  (EPS) =  $10^{-5}$ ,
           computes 2 damped eigenvalues, max 50 iterations)
        6  1, 11, 77, 88 (damper locations)
        7  0.2, 0.2, 0.2, 0.2 (initial damper values)
        8  0.2, 0.1, 5, 1 (initial damper values, increment,
           5 increments, all dampers vary)
```

\*\*An arithmetic underflow condition will appear if a larger L is used. We have had such an experience by using  $L = 88$ . This is due to the formulation used in the receptance calculation. It can be improved if so desired. Our experience indicates that using  $L = 30$  provides very good results.

## VI. Program Listing and Sample Run

The subroutine SPSTGN is listed in Table 1. Table 2 lists a sample main program TESTSP which calls SPSTGN. The output of the sample problem is given in Table 3. The control cards necessary to run this sample problem is shown in Table 4.

**Table 1 Listing of Subroutine SPSTGN**

SUBROUTINE SPSTGN(NELEM,NMOD,RUNNO,IDOPT,EPS,NDAM,NUMITER,  
 CDINIT,CDINC,NCD,NODEPL)

SPSTG WRITTEN BY DR. A. PALAZZOLO AND DR. B. P. WANG MAY , 1981  
 COMPUTER CODE TO GENERATE THE COMPLEX EIGENVALUES USING  
 REANALYSIS FORMULATION.

INPUT CARD DESCRIPTION

NELEM: NO. OF ELEMENTS  
 NMOD: NO. OF MODES REQUESTED FOR PRINTOUT AND DAMPER SENSITIVITY  
 ANALYSIS AND PLOTTING @ 10 AND @ L  
 RUNNO: RUN NUMBER IDENTIFIER  
 IDOPT: 0=ALL C VALUES VARY  
 1= ONLY ONE VALUE VARIES  
 EPS: CONVERGENCE PARAMETER  
 NDAM: NUMBER OF MODIFIED SYSTEM MODES DESIRED @ L-2 AND D20  
 NUMITER: MAXIMUM NUMBER OF ITERATIONS PER EIGENVALUE  
 CDINIT: INITIAL DAMPER VALUE FOR ROOT LOCUS PLOT LP. SEC./IN  
 THIS VALUE WILL OVEPRIDE THE VALUE IN POSITION NODEPL , ENTERED INTO  
 CARRAY  
 CDINC: DAMPER INCREMENT FOR ROOT LOCUS PLOT LP.SEC./IN  
 NCD: TOTAL NUMBER OF DAMPER INCREMENTS FOR ROOT LOCUS PLOT  
 NODEPL: THE NUMBER OF THE VARIED DAMPER IN ISTATNO(I)  
 I.E. NODEPL IS AN ELEMENT OF 1,2,...,NSTAT.  
 THIS TELLS WHETHER NODEPL IS FIRST , SECOND ETC. IN ISTATNO

IN ADDITION THE FOLLOWING VARIABLES ENTER THE SUBROUTINE THROUGH THE  
 COMMON BLOCK:

L: NO. OF MODES FOR RECEPTANCE CALCULATION  
 ISTATNO(I): THE NSTAT DAMPER LOCATIONS (NODE NUMBERS)  
 CARRAY(3): THE DAMPER VALUES CORRESPONDING TO ISTATNO(I)  
 NSTAT: NUMBER OF STATIONS WHERE DAMPERS ARE ATTACHED  
 PHI(I,J): AN ARRAY CONTAINING THE UNDAMPED SYSTEMS EIGENVECTORS  
 NORMALIZED TO UNIT GENERALIZED MASS  
 OMEGA(I): A VECTOR CONTAINING THE FREQUENCIES OF THE UNDAMPED SYSTEM  
 NTITLE: AN ALPHA ARRAY CONTAINING THE TITLE OF THE PROJECT

INTEGER ISTATNO  
 DIMENSION IRDW(100),TEMP(100)  
 COMPLEX CIGEN,SLAMIN,VLAM,RLAM,COEIG,SLAMNXT,CMCOEIG,FII  
 COMPLEX FHAT  
 LOGICAL TEST

Table 1 Continued

```

C
C
COMMON/BLK1/AEJ(50),ARLJ(50),ATJ(50),ARHJ(50),ARD(50)
COMMON / BLK2 / SK(1,1)
COMMON/BLK3/SSTR(88),SSROT(88)
COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
6 SM(1,1)
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
COMMON/BLK8/L
COMMON/BLK10/SMINV(1,1)
COMMON/BLK12/WREX(105),WIEX(105)
COMMON/BLK13/CMCOEIG(105)
COMMON/BLK14/SENSD(1,1)
COMMON/BLK15/A(1,1)
COMMON/BLK16/PMR1
COMMON/BLK20/IAUTO,REMAX,AIMAX
COMMON/BLK21/STOM(88)
COMMON/BLK22/FACT
COMMON/BLK23/ISTATNU(16),NSTAT,CARRAY(16)
COMMON/BLK24/R1VEC(16)
COMMON/BLK25/IME
COMMON/BLK26/FHAT(10,10)
COMMON / BLK27 / NTITLE(80)
C
C *****
C
C *****
C
C DM=1.0
C BETA=1.0
C IDIAG=0
C
C READ *,NELEM,NMOD,NRIG,L,IFLAG,R,BETA,RUNNO
C PRINT 1,NELEM,NMOD,NRIG,L,IFLAG,R,BETA,RUNNO
C 1 FORMAT(5I10,3F10.3)
C N=NELEM
C NNM=NELEM+1
C
C READ *,JNOPT,IDOPT
C PRINT 20,INOPT,IDOPT
C IF(INOPT.EQ.0) GO TO 140
C GO TO 250
C 140 CONTINUE
C
C
C 20 2010 1 - 1 - L
C
C READ(12,2011) OMEGA(I),GM
C SFOM(I) OMEGA(I)
C
C READ(12,2011) (PHI(J,I),J=1,N)
2010 CONTINUE
2011 FORMAT(2E16.9)
2011 FORMAT(5Y,2E16.9)
C GO TO 240
C 240 CONTINUE
C
C DO I = 1,NNM
C READ *,SSTR(I),SSROT(I)
C PRINT 3,SSTR(I),SSROT(I)
C 6 FORMAT(2F10.3)
C 5 CONTINUE
C DO 10 I 1,NELEM

```

Table 1 Continued

```

C      READ *,AEJ(I),ARLJ(I),ATJ(I),ARHJ(I),ARO(I)
C      PRINT 15,AEJ(I),ARLJ(I),ATJ(I),ARHJ(I),ARO(I)
C 15  FORMAT(5F10.3)
C 10  CONTINUE
240   CONTINUE
C      READ *,NSTAT,EPS,NDAM,NUMITER,IDIAG
C      PRINT 20,NSTAT,EPS,NDAM,NUMITER,IDIAG
C 20  FORMAT(I10,E10.3,3I10)
C      READ 115,IMDPLT
C 115 FORMAT(I10)
C      READ *,(ISTATNO(I),I=1,NSTAT)
C      PRINT 75,(ISTATNO(I),I=1,NSTAT)
C 75  FORMAT(16I5)
C      READ *,(CARRAY(I),I=1,NSTAT)
C      PRINT 77,(CARRAY(I),I=1,NSTAT)
C 77  FORMAT(3F10.3)
C      READ *,CDINIT,CDINC,NCD,NODEPL
C      PRINT 55,CDINIT,CDINC,NCD,NODEPL
C 55  FORMAT(2F10.3,2I10)
C 65  FORMAT(2I10,2F10.3)
C
C
      PRINT 101,T
101  FORMAT(1H-,10X,*TIME 1 = ,F10.3)
C      *****
C      *****
C      REANALYSIS SECTION
C      *****
C      CARRAY(NODEPL)=CDINIT-CDINC
C      CALL INPR2(L,EPS,NDAM)
C      DO 30 I=1,NCD
C      CARRAY(NODEPL)=CARRAY(NODEPL)+CDINC
C      CALL INPR2(L,EPS,NDAM)
C      DO 50 INDEIG 1,NDAM
C      FACT=1.0
C      CALL INITIAL(SLAMIN,INDEIG,BETA,T)
C      DO 40 ITER=1,NUMITER
C      CALL SULLER(SLAMNXT)
C      CALL UPDATE(SLAMNXT,INDEIG)
C      CALL CONVCHK(ITER,EPC,TEST)
C      TIME=ITER
C      IF(TEST) GO TO 70
60  CONTINUE
70  CALL EIGSTOR(INDEIG)
C      CALL HEXMODF(SLAMIN,INDEIG)
80  CONTINUE
C      CALL APEISOT(NDAM)
C      CALL APEIPRT(NDAM)

      PRINT 103,T
      IF(IDOPT.EQ.0) GO TO 1100
      CARRAY(NODEPL)=CARRAY(NODEPL)+CDINC
      GO TO 30

```

Table 1 Continued

```

1100      DO 1000 ID=1, NSTAT
1000          CARRAY(ID)=CARRAY(ID)+CDINC
103  FORMAT(1H ,2X,*TIME 3 = *,F10.3)
30  CONTINUE
          RETURN
      END
C
C
C
      SUBROUTINE INPR2(L, EPS, NDAM)
      INTEGER ISTATNO
      COMMON/BLK23/ISTATNO(15), NSTAT, CARRAY(16)
      PRINT 1
1  FORMAT(1H ,40X,*BEGIN REANALYSIS PORTION OF CODE*)
      PRINT 20
20  FORMAT(1H0,5X,*NODE*,10X,*DAMPER VALUE*)
      DO 30 I=1,NSTAT
      PRINT 35,ISTATNO(I),CARRAY(I)
35  FORMAT(1H0,6X,I2,15X,E10.3)
30  CONTINUE
      PRINT 3,L, EPS
3  FORMAT(1H ,10X,*NO. OF MODES FOR RECAP. CALC.,L = *,
4  15-5X,*EPS = *,E10.3)
      PRINT 10,NDAM
10  FORMAT(1H ,10X,*NUMBER OF MODIFIED SYSTEM MODES DESIRED = *,I2)
      RETURN
      END
C
      SUBROUTINE INITIAL(SLAMIN,INDEIG,BETA,I)
      CALCULATE THREE STARTING VALUES FOR MULLER ITERATION
      INTEGER ISTATNO
      COMPLEX CIGEN,SLAMIN,VLAM,PLAM,COEIG,SLAM,RL,CMPLX,EE
      COMMON/BL7/ULAM(3),RLAM(3),COEIG(20)
      COMMON/BL13/L
      COMMON/BL121/STON(99)
      COMMON/BL22/FACT
      COMMON/BLK23/ISTATNO(15),NSTAT,CARRAY(16)
      COMMON/BLK25/IME
          IME=0
          IDIAG=0
      ICE=INDEIG
      IF (ABS(.LOG(1-ICE/INDEIG+1)
      IF (IDIAG.EQ.0) GO TO 10
      PRINT 1,ICE
1  FORMAT(1H ,40X,*ITERATION FOR EIGENVALUE *,I3)
10  CONTINUE
      ZZ=STON(ICE)
      EE=CMPLX(-0.5,ZZ)
      IME=IME/BETA
      IF (IME.EQ.1) SLAMIN=EE
      IF (IME.EQ.1) SLAMIN=COEIG(INDEIG)/BETA
      VLAM(3)=SLAMIN*(1.0,0.001)
      VLAM(2)=-1.001*SLAMIN
      VLAM(1)=SLAMIN
      SLAM=VLAM(3)
      CALL FOFLA(SLAM,RL,INDEIG)

```

Table 1 Continued

```

RLAM(3)=RL
SLAM=VLAM(2)
CALL ROFLA(SLAM,RL,INDEIG)
RLAM(2)=RL
SLAM=VLAM(1)
CALL ROFLA(SLAM,RL,INDEIG)
RLAM(1)=RL
AB=CABS(RLAM(1))
TP=L
TP=1.0/TP
CB=AB**TP
FACT=1.0/CB
RLAM(3)=RLAM(3)/AB
RLAM(2)=RLAM(2)/AB
RLAM(1)=RLAM(1)/AB
RETURN
END

```

```

SUBROUTINE MULLER(SLAMNXT)
C   CALCULATE NEXT EIGENVALUE GUESS BY MULLER METHOD
COMPLEX SLAMNXT,VLAM,RLAM,COEIG,CSQRT,FZJ3J2,FZJ1J2,FZJ1J3,
& FJ1J2J3,W,SQ,AC1,AC2,DEN
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
FZJ3J2=(RLAM(2)-RLAM(3))/(VLAM(2)-VLAM(3))
FZJ1J2=(RLAM(2)-RLAM(1))/(VLAM(2)-VLAM(1))
FZJ1J3=(RLAM(3)-RLAM(1))/(VLAM(3)-VLAM(1))
FJ1J2J3=(FZJ3J2-FZJ1J2)/(VLAM(3)-VLAM(1))
W=FZJ1J2+FZJ1J3-FZJ3J2
SQ=W**2-4.0*RLAM(1)*FJ1J2J3
SQ=CSQRT(SQ)
AC1=W+SQ
AC2=W-SQ
T1=CABS(AC1)
T2=CABS(AC2)
IF(T1.GE.T2) DEN=AC1
IF(T2.GT.11) DEN=AC2
SLAMNXT=VLAM(1)-2.0*RLAM(1)/DEN
RETURN
END

```

```

SUBROUTINE UPDATE(SLAMNXT,INDEIG)
COMPLEX SLAMNXT,VLAM,RLAM,COEIG,RL
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
C   UPDATE THE LAMBDA (VLAM) AND P OF LAMBDA (RLAM) ARRAYS
VLAM(3)=VLAM(2)
VLAM(2)=VLAM(1)
VLAM(1)=SLAMNXT
CALL ROFLA(SLAMNXT,RL,INDEIG)
RLAM(3)=RLAM(2)
RLAM(2)=RLAM(1)
RLAM(1)=RL
RETURN
END

```

```

SUBROUTINE CONVCHK(ITER,EPS,TEST)
LOGICAL TEST
COMPLEX VLAM,RLAM,COEIG,SLAM,TEST1,TEST2,TEST3,FII

```

Table 1 Continued

```

COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
COMMON/BLK25/IME
C     TEST FOR CONVERGENCE IN MULLER METHOD
      IDIAG=0
      EPS2=0.01*EPS
      SLAM=VLAM(1)
      TEST1=VLAM(1)-VLAM(2)
      TEST2=RLAM(1)
      TEST3=VLAM(3)-VLAM(1)
      CHECK1=CABS(TEST1)
      CHECK2=CABS(TEST2)
      CHECK3=CABS(TEST3)
      TEST=.FALSE.
      IF(CHECK1.LT.EPS.AND.CHECK2.LT.EPS) TEST=.TRUE.
      IF(CHECK1.LT.EPS2) TEST=.TRUE.
      IF(CHECK3.EQ.0.0) TEST=.TRUE.
      IF(IDIAG.EQ.0) GO TO 5
      PRINT 1,ITER,VLAM(1),RLAM(1),CHECK1-CHECK2,CHECK3
1  FORMAT(1H ,1X,I2,1X,E12.5,1X,E12.5,3X,E12.5,1X,E12.5,3(5X,E12.5))
5  CONTINUE
      RETURN
      END
SUBROUTINE EIGSTOR(INDEIG)
COMPLEX VLAM,PLAM,COEIG
C     STORE MODIFIED SYSTEM EIGENVALUE
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
COEIG(INDEIG)=VLAM(1)
RETURN
END
SUBROUTINE NEXMODE(SLAMIN,INDEIG)
COMPLEX SLAMIN,VLAM,PLAM,COEIG
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
COMMON/BLK25/IME
C     THIS SUBROUTINE IS CURRENTLY NOT BEING USED.
C     SUBROUTINE INITIAL IS NOW MAKING THE INITIAL GUESS
ICE INDEIG

      IDIAG=0
      ITER=0
      IF(ITER.EQ.1) ICE=INDEIG+1
C     CALCULATE INITIAL CENTRAL EIGENVALUE GUESS FOR NX MODE
      SLAMIN=1.001*COEIG(INDEIG)
      PRINT 5,ICE,IME
C  FORMAT(1H ,50X,'EIGENVALUE ',I3,5X,'NO. OF ITERATIONS = ',I5)
      RETURN
      END
SUBROUTINE HOFILA(SLAM,FI,INDEIG)
      INTEGER ISTATNO
      COMPLEX SLAM=PL,VLAM=PLAM,COEIG=FI+1,CONJG,PHOT=ABC(10,10)
C     SUBROUTINE TO EVALUATE THE ITERATED FUNCTION
C  COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
6 SM(1,1)
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
COMMON/BLK8/L

```

Table 1 Continued

```

COMMON/BLK22/FACT
COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
COMMON/BLK26/FHAT(10,10)
C          PRINT 320,FACT
320       FORMAT(10X,*FACT=*,2E16.5)
C          PRINT 100,SLAM
100       FORMAT(10X,* SLAM + *,2E16.5/)
          NRIG=0
          BETA=1.0
          CALL RECFP(SLAM)
C          PRINT 100,SLAM
          DO 20 I=1,NSTAT
          DO 30 J=1,NSTAT
30       BDC(I,J)=FHAT(I,J)*CARRAY(I)*SLAM*BETA
          BDC(I,I)=BDC(I,I)+(1.0,0.0)
20       CONTINUE
          ISTAT=NSTAT
          IF(NSTAT.EQ.1) G=BDC(1,1)
          IF(NSTAT.EQ.1) GO TO 50
          CALL CDETER(ISTAT,BDC,G)
50       CONTINUE
C          PRINT 100,SLAM
C          PRINT 310,G
310       FORMAT(10X,* G      *,2E16.5/)
          RL G
          IF(NRIG.EQ.1) RL=G/SLAM
C          PRINT 310,G
C          PRINT 330,BETA
330       FORMAT(10X,*BETA=*,2E16.5)
          DO 1 I=1,L
C          PRINT 340,I
340       FORMAT(10X,* I=*,I5)
1         RL=PL/(SLAM**2*(BETA**2)+OMEGA(I)**2)+FACT
          IF(INDEIG.EQ.1) GO TO 10
          INT=INDEIG-1
          DO 2 I=1,INT
C          PRINT 200,I,COEIG(I)
200       FORMAT(10X,*I,COEIG(I)*,I5,2E16.5)
          RL=PL/(SLAM-CONJG(COEIG(I)))
          MA=ATNAG(COEIG(I))
          IF(MA.EQ.0.0) GO TO 2
C          PRINT 110,CONJG(COEIG(I))
110       FORMAT(10X,*CONJG **,2E16.5)
          RL=PL/(SLAM-CONJG(COEIG(I)))
2         CONTINUE
10        CONTINUE
          RETURN
END
SUBROUTINE RECFP(SLAM)
C          COMPUTE RECEPTANCES
C          MODES NORMALIZED TO UNIT MODAL MASS
          INTEGER ISTATNO
          COMPLEX SLAM,FII
          COMPLEX DEN,FHAT

```

Table 1 Continued

```

C COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
6 SM(1,1)
COMMON/BLK8/L
COMMON/BLK16/PMR1
COMMON/BLK23/1STATNO(16),NSTAT,CARRAY(16)
COMMON/BLK26/FHAT(10,10)
      BETA=1.0
      DO 5 I=1,NSTAT
      IP=1STATNO(I)
      DO 10 J=1,NSTAT
      FHAT(I,J)=(0,0,0,0)
      JP=1STATNO(J)
      DO 20 K=1,L
20 FHAT(I,J)=FHAT(I,J)+PHI(IP,K)*PHI(JP,K)
      4 (SLAM**2*(BETA**2)+OMEGA(K)**2)
10 CONTINUE
5 CONTINUE
RETURN
END
SUBROUTINE APEISOT(NDAM)
COMPLEX VLAM,PLAM,COEIG,DEI
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
ARRANGE MODIFIED SYSTEM EIGENVALUES (APPROX)
ACCORDING TO INCREASING IMAG. PARTS, DE- TIME SCALE EIGENVALUES
      BETA=1.0
      DO 10 I=1,NDAM
10 COEIG(I)=COEIG(I)*BETA
      NFC=NDAM-1
      DO 1 I=1,NFC
      JSTOR=0
      TMIN=1000000.0
      IOT=1
      DO 2 J=1,NDAM
      AT=AIMAG(COEIG(J))
      AT=ABS(AT)
      IF(AT-LE,TMIN) TMIN=AT
      IF(AT-GE,TMIN) JSTOR=J
2 CONTINUE
      DEI=COEIG(I)
      COEIG(I)=COEIG(JSTOR)
      COEIG(JSTOR)=DEI
1 CONTINUE
RETURN
END
SUBROUTINE APEIPRT(NDAM)
COMPLEX VLAM,RLAM,COEIG
COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
PRINT THE MODIFIED SYTEM APPROX. EIGENVALUES
PRINT 1
1 FORMAT(1H,12X,1MODIFIED SYSTEM APPROXIMATE EIGENVALUES?,12X,
6 *ZETA?,13X,*OMEGAN SEC-1?)
      NRIG=0
      DO 10 I=1,NDAM
      ICE=1

```

Table 1 Continued

```

      IF(NRIG.EQ.1) ICE=I+1
      OMEGAN=CABS(COEIG(I))
      ZETA=-REAL(COEIG(I))/OMEGAN
      PRINT 20,ICE,COEIG(J),ZETA,OMEGAN
20  FORMAT(1H ,10X,I5,10X,E12.5,3X,E12.5,13X,F13.4,11X,E12.5)
10  CONTINUE
      RETURN
      END
      SUBROUTINE CDETER(NST,A,D)

```

```

C
C      DETERMINANT OF AN NST BY NST COMPLEX MATRIX WHERE NST>1
C      COMPLEX A(10,10),D
C      COMPLEX PMULT,BIG,DET,BG,TEMP
C      NSIZ=NST
C      ICOUNT 0
C      NUMSYS=NSTZ-1
C      DO 14 I=1,NUMSYS

```

```

C
C      SCAN FIRST COLUMN OF CURRENT SYSTEM FOR LARGEST ELEMNT
C      CALL THE POW CONTAINING THIS ELEMENT, ROW NBGRW
C

```

```

      NN=I+1
      BIG=A(I,I)
      NBGRW I
      DO 5 J=NN,NSIZ
      IF(CABS(BIG)-CABS(A(J,I))) 6,5,5
6  BIG=A(J,I)
      NBGRW=J
5  CONTINUE
      BG=1.0/BIG

```

```

C
C      SWAP ROW I WITH ROW NBGRW UNLESS I=NBGRW
C

```

```

      IF(NBGRW.NE.I) ICOUNT=ICOUNT+1
      IF(NBGRW-I) 7,10,7

```

```

C
C      SWAP A MATRIX ROWS
C

```

```

7  DO 8 J=I,NBGRW
      TEMP=A(NBGRW,J)
      A(NBGRW,J)=A(I,J)
8  A(I,J)=TEMP

```

```

C
C      ELIMINATE UNKNOWNIS FROM FIRST COLUMN OF CURRENT SYSTEM
C

```

```

10 DO 13 K=NN,NSIZ

```

```

C
C      COMPUTE PIVOTAL MULTIPLIER
C

```

```

      PMULT=-A(K,1)*BG

```

```

C
C      APPLY PMULT TO ALL COLUMNS OF THE CURRENT A MARRIX ROW
C

```

```

      DO 11 J=NN,NSIZ
11  A(K,J)=PMULT*A(I,J)+A(K,J)
13 CONTINUE

```

Table 1 Continued

```

14 CONTINUE
   D=(1.0,0.0)
   DO 100 I=1,NSIZ
100  D=D*A(I,I)
      JCOUNT=(ICOUNT/2)*2
      IF(JCOUNT.NE.ICOUNT) D=-D
      RETURN
   END
   SUBROUTINE HEADNG
      INTEGER ISTATNO
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      PRINT 1
1   FORMAT(1H1,40X,*EISPACK(EXACT) SOLUTION OF THE MODIFIED SYSTEM*)
      PRINT 20
20  FORMAT(1H0,5X,*NODE*,10X,*DAMPER VALUE*)
      DO 30 I=1,NSTAT
      PRINT 35,ISTATNO(I),CARRAY(I)
35  FORMAT(1H0,6X,I2,16X,E10.3)
40  CONTINUE
      RETURN
   END
   SUBROUTINE LABRT(ISW,LHOL,INX)
      DIMENSION LHOL(5)
      LOGICAL PS,TS
      IF((ISW.EQ.0).OR.(ISW.GT.5))RETURN
      GO TO (1,2,3,4,5) ISW
      DATA NP/10/,PS/,TRUE./,TS/,FALSE./
7   IF(PS.AND.(NP.GT.0)) PRINT 27-LHOL,INX
27  FORMAT(1H0,9X,5A10,3X,06)
      NP=NP-1
      IF(PS) CALL EXI1
      RETURN
1   PS=.FALSE.
      RETURN
2   PS=.TRUE.
      NP=10
      RETURN
3   PS=.TRUE.
      RETURN
4   PS=.FALSE.
      RETURN
5   PS=.FALSE.
      RETURN
      END

```





Table 3 Output of Sample Problem

```

? TFST SPSI
TEST SPSI
? 002777.1
NLLM,NMOD,L,RUNNO :80 2 20 1
? 0
? 4.1E-5
? 2750
NSTAT,EFS,NEM,NUMITER :4 .00001 2 50
? 1.1179:88.
? 1 11 78 28
? 27.27(27.2
.200 .200 .200
? 27.195.1
.200 .100
TIME 1 : -1
? 0 NOISE DAMPER VALUE
? 0 1 .200E+00
? 0 11 .200E+00
? 0 78 .200E+00
? 0 88 .200E+00
NO. OF MODES FOR RECEP, CALC.,L = 30 EPS = .100E-04
NUMBER OF MODIFIED SYSTEM MODES DESIRED = 2
MODIFIED SYSTEM APPROXIMATE EIGENVALUES
1 -.10939E-04 -.61737E-09 ZETA 1.0000
2 .16191E-04 .81334E-05 ZETA -.8957
OMEGA SEC-1
.10939E-04
.15378E 01
? 0 NOISE DAMPER VALUE
? 0 1 .300E+00
? 0 11 .300E+00
? 0 78 .300E+00
? 0 88 .300E+00
NO. OF MODES FOR RECEP, CALC.,L = 30 EPS = .100E-04
NUMBER OF MODIFIED SYSTEM MODES DESIRED = 2
MODIFIED SYSTEM APPROXIMATE EIGENVALUES
1 -.99533E-07 -.18945E-08 ZETA .9998
2 .14859E-04 -.10327E-06 ZETA -.8213
OMEGA SEC-1
.99533E-07
.18104E-06
TIME 3 : -1

```

Table 3 Continued

BEGIN REANALYSIS PORTION OF CODE

```

0      0      DAMPER VALUE
0      1      .500E+00
0      11     .500E+00
0      78     .500E+00
0      88     .500E+00
      NO. OF MODES FOR RECEP. CALC., L = 30      EPS = .100E-04
      NUMBER OF MODIFIED SYSTEM MODES DESIRED = 2
      MODIFIED SYSTEM APPROXIMATE EIGENVALUES
      1      .27572E-07      ZETA      - .8773
      2      .19009E-07      .26869E-07      .8777
      TIME 3
  
```

OMEGAN SEC-1  
.315461E-07  
.56067E-07

BEGIN REANALYSIS PORTION OF CODE

```

0      0      DAMPER VALUE
0      1      .500E+00
0      11     .500E+00
0      78     .500E+00
0      88     .500E+00
      NO. OF MODES FOR RECEP. CALC., L = 30      EPS = .100E-04
      NUMBER OF MODIFIED SYSTEM MODES DESIRED = 2
      MODIFIED SYSTEM APPROXIMATE EIGENVALUES
      1      .21997E-07      ZETA      .8397
      2      .16118E-07      .16081E-07      -.7085
      TIME 3
  
```

OMEGAN SEC-1  
.25191E-07  
.22789E-07

BEGIN REANALYSIS PORTION OF CODE

```

0      0      DAMPER VALUE
0      1      .500E+00
0      11     .500E+00
0      78     .500E+00
0      88     .500E+00
      NO. OF MODES FOR RECEP. CALC., L = 30      EPS = .100E-04
      NUMBER OF MODIFIED SYSTEM MODES DESIRED = 2
      MODIFIED SYSTEM APPROXIMATE EIGENVALUES
      1      .14814E-07      ZETA      .9368
      2      .12677E-07      .79000E-09      -.2491
      TIME 3
  
```

OMEGAN SEC-1  
.15044E-07  
.14956E-07

3.821 OF SECONDS EXECUTION TIME.

Table 4 Control Cards for Using SPSTGN

```

COPY - COPY
/JOB
SPU - 04300000 - 11000
USER - 6P7121E
CHARGE - 102100 - 1 PC
GET - SPSTGN
GET - INP 12 - NAB0012
GET - DATA - SP001
SPSTGN (DATA)
PEL INP - OUTPUT
COPY 1 - OUTPUT - SP001
PAC 1 - SP001
REPLACE - SP001
PAYFILE - L - SP001
REF - 100 - SP001
EXIT
PEL INP - OUTPUT
GET (PEL - OUTPUT) - SP001
COPY - XCP001
REF - 100 - XCP001
PAYFILE - L - XCP001
REF - 100 - SP001
EXIT
EXIT - END OF UNIT (1)

```

## Appendix A-4

### Program COPZ Documentation

#### Damper Gain Optimization with Fixed Modal Damping Ratios

##### I. INTRODUCTION

COPZ is a damping optimization program to compute minimum damping gains for specified modal damping ratios using a diagonal damping matrix. Subroutine COMMIN is used in the optimization process.

##### II. PROBLEM FORMULATIONS

When a diagonal damping matrix [C] is introduced into an originally undamped system, the damped eigenvalue can be found by solving

$$f(s) = \det[I + [\hat{R}(s)][\hat{C}]] = 0 \quad (\text{A4.1})$$

where

$s$  = complex eigenvalue of the damped system

$[\hat{C}]$  = submatrix of  $[\Delta C]$  that contains only the non-zero terms of  $[\Delta C]$

$[\hat{R}]$  = corresponding sub-matrix of the receptance matrix R

$$\hat{R}_{ij} = R_{ij} = \sum_{\ell=1}^n \frac{\rho_{j\ell} \rho_{i\ell}}{s^2 + \omega_{\ell}^2}$$

Let

$$[\hat{C}] = \begin{bmatrix} c_1 & & 0 \\ & \cdot & \\ 0 & & c_{N_c} \end{bmatrix}$$

The optimization problem is:

$$\text{find } c_i, i = 1 \text{ to } N_c$$

such that  $J_1 = \sum_{i=1}^{N_C} c_i$  is minimized subject to the constraint

$$f(S_\ell) = 0, \quad \ell = 1 \text{ to } N_E$$

where

$$S_\ell = -\zeta_\ell \omega_{n_\ell} \pm j\sqrt{1 - \zeta_\ell^2} \omega_{n_\ell}$$

To convert the above constrained optimization problem into one without constraints, a penalty function approach is used. The problem solved by COPZ can then be rephrased as

$$\text{find } C_i, \quad i = 1 \text{ to } N_C$$

such that

$$J = \sum_{i=1}^{N_C} c_i + \sum_{\ell=1}^{N_E} W(f(S_\ell))$$

is minimized, where  $W$  is weighting function. Experience shows  $W = 1000$  yields good results.

There are  $N_C + N_E$  unknowns in the optimized problems. The first  $N_E$  of the unknowns are the unknown  $\omega_{n_\ell}$  in  $S_\ell$  (with  $\delta_\ell$  specified). The last  $N_C$  unknowns are the damper gains.

## II. INPUT TO COPZ

A. Input from terminal (or input from TAPE 5). All are free format.

1. Title. One line description of the problem.
2. NDOF, NMODE, NE, NC  
 NDOF = no. of system dof  
 NMODE = no. of modes to be used  
 NE = no. of specified  $\zeta$   
 NC = no. of dampers.
3. JD(I), I = 1, NC)  
 JD(I) = Ith damper location
4. X(I), I = 1, NC  
 initial damper constant
5. NM(I), ETA(I), (I = 1 to NE)  
 NM(I) = mode no.  
 ETA(I) =  $\zeta_i$  = specified damping ratio
6. WEIGHT  
 WEIGHT =  $W$  is objective function  $J = 1000$  usually

B. Modal Data Needed: (same as routine SPSTGN)

Read from TAPE 12:

For I = 1 to NMODE

WN(I),GM (Format 5X,2E16.9)

(PHI(JJ,I),I=1,NDOF) (Format 5E16.9)

Note:  $WN(I) = \omega_i$ , GM(noticed),  $PHI(J,I) = \phi_{j,i}$

#### IV. SAMPLE RUN OF COPZ

Table 1 shows the job file to run COPZ, include data for a sample problem. For this sample problem, we place 6 dampers at dof 1, 11, 39, 50, 78, and 88 of the 88-dof grillage model. We specify the model dampings to be 0.7 and 0.6 respectively for mode 4 and 5 (the first vibration modes). The results are:

$$\begin{aligned} C_1 &= 0.214, & C_{11} &= 0.268, & C_{39} &= 0.146 \\ C_{50} &= 0.146, & C_{78} &= 0.207, & C_{88} &= 0.267 \end{aligned}$$

These results can be found in the end of Table 2.

Table 1 CONTROL Cards for use of Program COPZ

```

COPZ=10000
/00
RPM=16300000,1000,
USER=697121E,
NUMBER=102428 LRD,
BTLINER=1E9E30 T UUA
ATTACH=FTNALIB (UN=LIBRARY
LIB=APR10=NOFAPBL
LIB=PSLMCON/UN=862E10N,
DEF=ENHWC (UN=012486N,
DEF=LOPZ,
FTN, (=COPZ, PL=10000, E=F,
L=SET, LIP=FTNALIB,
LOAD(=,PSLMCON,ENHWC)
EXECUTE,
FORM=OUTPUT
COPZ=ET, OUTPUT=OTCOPZ,
HALT=OTCOPZ,
REPLACE,OTCOPZ
DAYFILE=L=DAACOPZ,
REPLACE,DAACOPZ,
EXIT,
FORM=OUTPUT
COPZ=ET, OUTPUT=OTCOPZ,
HALT=OTCOPZ,
REPLACE,OTCOPZ,
DAYFILE=L=XDACOPZ,
REPLACE,XDACOPZ,
/END

```

JASA BRILLIAGE MODEL PUMPING OPTIMIZATION-VARY NMODE

```

DE=0.01
/11=ET, NMODE
/12=1-0.01, N1=0.1
/13=
/14=
/15=
/16=
/17=
/18=
/19=
/20=
/21=
/22=
/23=
/24=
/25=
/26=
/27=
/28=
/29=
/30=
/31=
/32=
/33=
/34=
/35=
/36=
/37=
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/79=
/80=
/81=
/82=
/83=
/84=
/85=
/86=
/87=
/88=
/89=
/90=
/91=
/92=
/93=
/94=
/95=
/96=
/97=
/98=
/99=
/100=

```





Table 2 Continued

CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

79 P100

CONTROL PARAMETERS

IPRINT NDV ITHAX NCON NSIDE ICONDIP NSCAL NFDG  
 5 3 20 0 1 9 9 0

LINDEL ITRM N1 N2 N3 N4 N5  
 0 3 10 16 17 17 34

CT CTMIN DTL DTLMIN  
 -.10000E+00 .40000E-02 -.10000E-01 .10000E-02

THETA PHI DELFUN DAFUN  
 .10000E+01 .50000E+01 .10000E-03 .10000E-04

FICH FICHM ALPHAX ABORJI  
 .10000E-02 .10000E-03 .10000E+00 .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLS)  
 1) .10000E-02 .10000E-02 .10000E-02 .10000E-02 .10000E-02  
 7) .10000E-02 .10000E-02

UPPER BOUNDS ON DECISION VARIABLES (VUB)  
 1) .27100E+02 .32771E+02 .10000E+00 .10000E+05 .10000E+05  
 7) .10000E+05 .10000E+05

INITIAL FUNCTION INFORMATION

OBJ .32771E+02

DECISION VARIABLES (X-VECTOR)  
 1) .12710E-02 .12710E+00 .10000E+00 .10000E+00 .10000E+00  
 7) .10000E+00 .10000E+00

DESIGN ITERATION NUMBER 1

NEW POINTING VECTOR / SCAL  
 .12710E+02 .12710E+00 .10000E+00 .10000E+00 .10000E+00 .10000E+00  
 .10000E+00

Table 2 Continued

```

X(I)-
  .1375E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.627771052219,9.822291127974)
  FOR II=      1      ,DET= .1668E+00 -1.4041E-01
X(I)-
  .1375E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.8221503357471,13.09540516330)
  FOR II=      2      ,DET= .1524E+00 -1.3935E-01
X(I)-
  .1374E+02      .1639E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.61815289932,9.812478349325)
  FOR II=      1      ,DET= .1668E+00 -1.4044E-01
X(I)-
  .1374E+02      .1639E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.831375428615,13.10850057149)
  FOR II=      2      ,DET= .1524E+00 -1.3941E-01
X(I)-
  .1374E+02      .1637E+02      .1001E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.61815289932,9.812478349325)
  FOR II=      1      ,DET= .1667E+00 -1.4043E-01
X(I)-
  .1374E+02      .1637E+02      .1001E+00      .1000E+00      .1000E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.821553397474,13.09540516330)
  FOR II=      2      ,DET= .1523E+00 -1.3934E-01
X(I)-
  .1374E+02      .1637E+02      .1001E+00      .1001E+00      .1001E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.61815289932,9.812478349325)
  FOR II=      1      ,DET= .1667E+00 -1.4043E-01
X(I)-
  .1374E+02      .1637E+02      .1001E+00      .1001E+00      .1001E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.821553397474,13.09540516330)
  FOR II=      2      ,DET= .1523E+00 -1.3934E-01
X(I)-
  .1374E+02      .1637E+02      .1001E+00      .1001E+00      .1001E+00
  .1000E+00      .1000E+00      .1000E+00
SI-
(-9.61815289932,9.812478349325)
  FOR II=      1      ,DET= .1668E+00 -1.4044E-01

```

Table 2 Continued

(11) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1001E+00  
 .1000E+00 .1000E+00 .1000E+00  
 FOR II= 2 -DET= .1524E+00 -.3933E-01

(12) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 1 -DET= .1668E+00 -.4044E-01

(13) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 2 -DET= .1524E+00 -.3933E-01

(14) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 1 -DET= .1668E+00 -.4044E-01

(15) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 2 -DET= .1524E+00 -.3933E-01

(16) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 1 -DET= .1668E+00 -.4044E-01

(17) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 2 -DET= .1524E+00 -.3933E-01

(18) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 1 -DET= .1668E+00 -.4044E-01

(19) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 2 -DET= .1524E+00 -.3933E-01

(20) -  
 .1374E+02 .1637E+02 .1000E+00 .1000E+00 .1000E+00  
 .1001E+00 .1000E+00 .1000E+00  
 FOR II= 1 -DET= .1668E+00 -.4044E-01

Table 2 Continued

\* \* \* CONSTRAINED ONE-DIMENSIONAL SEARCH INFORMATION \* \* \*

PROPOSED DESIGN  
 ALPHA = .46436E-01  
 VECTOR  
 X(I) = .1370E+01 .1046E+00 .1046E+00 .1007E+00 .1007E+00  
 (1)  
 .1370E+02 .1479E+02 .1046E+00 .1046E+00 .1007E+00  
 .1007E+00 .1046E+00 .1046E+00  
 SI  
 (-9.784874592703, 9.982670818414)  
 FOR II = 1 -DET = .1506E+00 -.3630E-01  
 X(I) =  
 .1370E+02 .1479E+02 .1046E+00 .1046E+00 .1007E+00  
 .1007E+00 .1046E+00 .1046E+00  
 SI  
 (-9.81125470040, 10.10233384004)  
 FOR II = 2 -DET = .1349E+00 -.3746E-01  
 OBJ = .287811407  
 END--ONE-DIMENSIONAL SEARCH

PROPOSED DESIGN  
 ALPHA = .03212E+00  
 VECTOR  
 X(I) = .1197E+00 .1144E+00 .1032E+00 .1032E+00 .1032E+00 .1032E+00  
 (1)  
 .1197E+02 .1144E+02 .1032E+00 .1032E+00 .1032E+00 .1032E+00  
 .1032E+00 .1032E+00 .1032E+00  
 SI  
 (10.055001174, 10.1224357874)  
 FOR II = 1 -DET = .9458E-01 -.3021E-01  
 X(I) =  
 .1197E+02 .1144E+02 .1032E+00 .1032E+00 .1032E+00 .1032E+00  
 .1032E+00 .1032E+00 .1032E+00  
 SI  
 (10.055001174, 10.1224357874)  
 FOR II = 2 -DET = .3707E-01 .0009E-01  
 OBJ =  
 FOR II = 2 -DET = .2339E-01 .5109E-01  
 END--ONE-DIMENSIONAL SEARCH

Table 2 Continued

SI-  
 Y(I)-  
 .1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00  
 .1859E+00      .2671E+00      .2141E+00

SI-  
 (-11.20133457956, 12.20303273078)  
 FOR II= 1      DET= -.5981E-03      .4868E-03

Y(I)-  
 .1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00  
 .1859E+00      .2671E+00      .2141E+00

SI-  
 (-9.788474052277, 13.0512967373)  
 FOR II= 2      DET= -.2748E-05      -.9043E-06

DEI = .20290E+01

TWO-POINT INTERPOLATION

REPROSED DESIGN

ALPHA .17001E+05

Y(I)-

.1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00      .1457E+00  
 .1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00  
 .1859E+00      .2671E+00      .2141E+00

SI  
 (-11.20137297082, 12.2030412926)  
 FOR II= 1      DET= -.5981E-03      .4868E-03

Y(I)-  
 .1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00  
 .1859E+00      .2671E+00      .2141E+00

9.788474052277, 13.0512967373  
 FOR II= 2      DET= -.2748E-05      -.9043E-06

DEI = .20290E+01

TWO-POINT INTERPOLATION

REPROSED DESIGN

ALPHA .17001E+05

Y(I)-

.1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00      .1457E+00  
 .1709E+00      .1631E+02      .2141E+00      .2679E+00      .1457E+00  
 .1859E+00      .2671E+00      .2141E+00

9.788474052277, 13.0512967373  
 FOR II= 2      DET= -.2748E-05      -.9043E-06

Table 2 Continued

X(I)  
 .1709E+02      .1631E+02      .2145E+00 =  $C_1$       .2679E+00 =  $C_1$       .1457E+00 =  $C_{39}$   
 .1450E+00      .2671E+00      .2141E+00  
 GI =      =  $C_{50}$       =  $C_{78}$       =  $C_{88}$   
 (-.788816344956, 13.05135512561)  
 FOR I1 =      2      ,DET = -.2650E-05      -.9123E-06

OBJ = .00290E+01

\*\*\* END OF ONE-DIMENSIONAL SEARCH

CALCULATED ALPHA = .33881E-20

OBJ = .002897E+01      NO CHANGE ON OBJ

PRECISION VARIABLES (X-VECTOR)

|    |            |            |            |            |            |
|----|------------|------------|------------|------------|------------|
| 1) | .17098E+02 | .16314E+02 | .21446E+00 | .26794E+00 | .14565E+00 |
| 2) | .26712E+00 | .21408E+00 |            |            |            |

FINAL OPTIMIZATION INFORMATION

OBJ = .002897E+01

PRECISION VARIABLES (X-VECTOR)

|    |            |            |            |            |            |
|----|------------|------------|------------|------------|------------|
| 1) | .17098E+02 | .16314E+02 | .21446E+00 | .26794E+00 | .14565E+00 |
| 2) | .26712E+00 | .21408E+00 |            |            |            |

NUMBER OF      0 ACTIVE SIDE CONSTRAINTS







Program Listing Continued

```

SUBROUTINE FUN NEW-Y, NCON, DEJ-G)
    COMPLEX SI-DET,H
    COMMON /DATA2/F(30,88),WN(88)
    COMMON /DATA3/ETA(30),HM(30)
    COMMON /DATA4/NE-NC-WEIGHT
        DIMENSION N(30)
    DIMENSION G(1)
    COMMON /DATA5/EPF
    COMMON /DATA7/H(3,6)

    IS=0
    DEJ=0
    NIT=NDINE
    TO=1000 II=1,100
    A=ETA(II)
    TM=HM(II)
    SIR=100Y II
    OLI=(OBT(1,0-AIAC),LIVITY)
    II=OBT(1,0-AIAC)

    WRITE(4,*) II
    WRITE(4,*) SIR
    WRITE(4,*) OLI
    PRINT 4, 'I'
    WRITE(4,*) SI
    CALL HROP(4,IX)

    EVALUATE COMPLEX DETERMINANT

    TO=1000 II=1,100
    PRINT 4, 'I'
    PRINT 4, SIR
    PRINT 4, OLI
    CALL HROP(4,IX)
    WRITE(4,*) II
    WRITE(4,*) SIR
    WRITE(4,*) OLI

    NFI=NEAR
    TO=1000 II=1,100
    DEJ=DEJ+1

    RETURN
END

```

Program Listing Continued

```

SUBROUTINE DETMIN(NBT-3)
C
C     DETERMINANT OF AN NBT x NBT COMPLEX MATRIX (SHARE 1971)
C     COMMON DATA: (A(1-3)
C     COMPLEX, I=1)
C     COMPLEX PAULT, B(10-DET, 33-TEAF
C     *B(11-NBT)
C     DO 300 I=1-NBT
C300      PRINT 210, I, A(I, I), B(I, I), NBT)
C310      FORMAT(10, 'A', I, ' = ', A, 'B', I, '
C     ICOUNT =
C     IUNBYE = I(12-1)
C     DO 18 I=1, IUNBYE
C
C     SCAN FIRST COLUMN OF CURRENT MATRIX FOR LARGEST ELEMENT
C     CALL THE ROW CONTAINING THIS ELEMENT FOR PIVOT
C
C     N(1)=1
C     I(1)=1
C     I(2)=1
C     I(3)=1
C     I(4)=1
C     I(5)=1
C     I(6)=1
C     I(7)=1
C     I(8)=1
C     I(9)=1
C     I(10)=1
C
C     N(1)=1
C     I(1)=1
C     I(2)=1
C     I(3)=1
C     I(4)=1
C     I(5)=1
C     I(6)=1
C     I(7)=1
C     I(8)=1
C     I(9)=1
C     I(10)=1
C
C     IF (NEGPW.NE.I) ICOUNT = I(1)+1
C     IF (NEGPW-1) 7, 10, 7
C
C     SWAP A MATRIX ROWS
C
C     I(1)=I(1)+1
C     I(2)=I(2)+1
C     I(3)=I(3)+1
C     I(4)=I(4)+1
C     I(5)=I(5)+1
C     I(6)=I(6)+1
C     I(7)=I(7)+1
C     I(8)=I(8)+1
C     I(9)=I(9)+1
C     I(10)=I(10)+1
C
C     I(1)=I(1)-1
C     I(2)=I(2)-1
C     I(3)=I(3)-1
C     I(4)=I(4)-1
C     I(5)=I(5)-1
C     I(6)=I(6)-1
C     I(7)=I(7)-1
C     I(8)=I(8)-1
C     I(9)=I(9)-1
C     I(10)=I(10)-1
C
C     PRINT I(1), I(2), I(3), I(4), I(5), I(6), I(7), I(8), I(9), I(10)
C
C     I(1)=I(1)+1
C     I(2)=I(2)+1
C     I(3)=I(3)+1
C     I(4)=I(4)+1
C     I(5)=I(5)+1
C     I(6)=I(6)+1
C     I(7)=I(7)+1
C     I(8)=I(8)+1
C     I(9)=I(9)+1
C     I(10)=I(10)+1
C
C     PRINT 200, I, A(I, I), B(I, I)
C300      FORMAT(100, 'A', I, ' = ', A, 'B', I, ' = ', B, '

```

Program Listing Continued

```
100 PRINT(I-1)
    ICOUNT=(ICOUNT/2)*2
    IF (ICOUNT NE ICOUNT) P--P
    RETURN
END
EOI ENCOUNTERED.
```

## Appendix A-5

### Program NORMAL Documentation

#### Mode Shape Ortho Normalization

##### I. INTRODUCTION

NORMAL is a FORTRAN program that can be used to orthonormalize the mode shapes computed by EISPACK routines (such as RESV). For zero-frequency rigid body modes, Gram-Schmidt orthogonalization procedure (see the Appendix A5.1) is used to make the rigid body modes orthogonal to each other with respect to the mass matrix. All modes are then normalized to unit generalized mass. Currently up to 3 rigid body modes can be accepted.

##### II. INPUT TO NORMAL

The following data is needed to use NORMAL:

1. Input N, NR, L where N = no. of dof of the system

NR = no. of rigid body

( $\leq 3$ ) mode

L = number of elastic modes to be normalized

2. Read from TAPE 20

For I = 1 to L

WN(I), (V(J,I), J = 1, N)

where

WN(I) =  $\omega_i$  = natural frequency of the ith mode, rad/sec

V(J,I) =  $\phi_{ji}^i$  = mode shape coefficient at dof j of ith mode

3. Read from TAPE 10

NJ, (AM(I), I = 1, NJ)

NJ = no. of system dof

where

AM(I) =  $m_i$  = lumped mass of ith dof

Note: Both TAPE 10 and TAPE 20 are created by unformatted WRITE in a prior analysis

### III. PROGRAM LISTING

```

COPY 105M
PROGRAM NORM(INPUT,OUTPUT,TAPE10,TAPE12-TAPE20)
C
C
C      N=NO. OF THE SYSTEMS D.O.F.(UNNORMALIZED),
C      NR=NO. OF RIGID BODY MODES(=3)
C      L=NO. OF ELASTIC MODES TO BE NORMALIZED TO UNIT GEN. MASS
C      NATURAL FREQ.(RAD./SEC.) AND MODE SHAPES ARE STORED ON TAPE12
C      (USE FREE FORMAT)
C      DIAGONAL ELEMENTS OF THE MASS MATRIX ARE STORED ON TAPE10.
C      THE NORMALIZED MODAL MATRIX IS WRITTEN ON TAPE12.
C
C
C      DIMENSION WN(99),V(99,99),Z(99,3),AM(99),PHI(99,99),TT(99)
C
C      PEALY,N-NR,L
DO 300 I=1,L
500  PEAP(20) WN(I)-( V(J,I),J=1,N )
      READ(10) NJ, ( AM(I),I=1,NJ )
C
C      NORMALIZE MODAL MATRIX
C
DO 1000 I=1,N
      G2 = 0.0
DO 2000 J=1,N
2000  G2= G2 + V(J,I)*V(J,I)*AM(J)
      GM = SQRT(G2)
DO 3000 J=1,N
3000  V(J,I) = V(J,I)/GM
1000  CONTINUE
C
C      ORTHONORMALIZE RIGID BODY MODES
C
      IF (NR.LE.1) GO TO 9999
DO 4100 I=1,N
4100  Z(I,1) = V(I,1)
      A = 0
DO 4200 I=1,N
4200  A= A - V(I,1)*AM(I)*V(I,2)
      G2 = 0
DO 4300 I=1,N
4300  Z(I,2) = A*V(I,1) + V(I,2)
      G2 = G2 + Z(I,2)*Z(I,2)*AM(I)
      GM = SQRT(G2)
DO 4400 I=1,N

```

```

4400  V(I,2) = Z(I,2)/GM
      IF (NR.LT.3) GO TO 9999
      C2 = 0
      C1 = 0
      DO 5100 I=1,N
      C1 = C1 - Z(I,1)*AM(I)*V(I,3)
5100  C2 = C2 - Z(I,2)*AM(I)*V(I,3)
      G2 = 0
      DO 5200 I=1,N
      Z(I,3) = C1*Z(I,1) + C2*Z(I,2) + V(I,3)
5200  G2 = G2 + Z(I,3)*Z(I,3)*AM(I)
      GM = SQRT(G2)
      DO 5300 I=1,N
5300  V(I,3) = Z(I,3)/GM
      PRINT * , WN(1), (V(J,1),J=1,N)
      GM = 1.0
      DO 6000 I=1,N
      WRITE(12) WN(I), GM
6000  WRITE(12) ( V(J,I),J=1,N )
6011  FORMAT(5E16.9)
6011  FORMAT(5X,2E16.9)
9999  CONTINUE
      STOP
      END

```

## APPENDIX A5.1

### Gram-Schmidt Orthogonalization Procedure

$n$  independent vectors  $\{v_i\}$  can be transformed into vectors  $\{p_i\}$  orthogonal with respect to a matrix  $[m]$  using the following recursion equations:

$$\begin{aligned} \{p\}_1 &= \{r\}_1 \\ \{p\}_i &= \{r\}_i + \alpha_{i1}\{p\}_1 + \alpha_{i2}\{p\}_2 + \dots + \alpha_{i,i-1}\{p\}_{i-1} \end{aligned}$$

where

$$\alpha_{ij} = - \frac{\{p\}_j^T [m] \{r\}_i}{\{p\}_j^T [m] \{r\}_j}$$

## APPENDIX B

### Active Damper Analysis - Linear

Figure (B.1) shows system 2.2.2.1, it is essentially the same as Figure (5.6) except that definitions of displacement coordinates have been added. Note that the concept is essentially different to that used in the prototype design, which uses a long-stroke actuator without a linkage.

#### Definitions

- C - Damping constant
- D - Voice coil, i.e., shaker, displacement
- f - Frequency
- F - Shaker force
- K - Ratio of maximum structural amplitude to maximum voice-coil amplitude at design conditions
- $m_1$  - shaker mass
- $m_2$  - Moving mass
- $P^2$  - Rate of energy absorption
- R - Linkage ratio
- W - Energy absorbed from structure
- x - Displacement of structure
- y - Displacement of  $m_2$
- z - Relative displacement
- $\omega$  - Radial frequency

#### Subscripts

- AL - Amplitude limited
- CH - Characteristic
- C - Per cycle
- D - Design
- FL - Frequency limited
- M - Maximum
- O - Complex amplitude
- T - Total

Referring to Figure (B.1), the basic damping equation is

$$F/R = cdx/dt = m_2 d^2 y/dt^2 = m_2 (d^2 x/dt^2 + d^2 z/dt^2) \quad (B.1)$$

and, assuming SHM with complex amplitudes of  $x_0$ ,  $z_0$ , respectively

$$|z_0| = |x_0| \{1 + (C^2/\omega^2 m_2^2)\}^{1/2} \quad (B.2)$$

If motion is amplitude-limited, so that

$$|z_0|_{AL} = RD_M \quad (B.3)$$

then

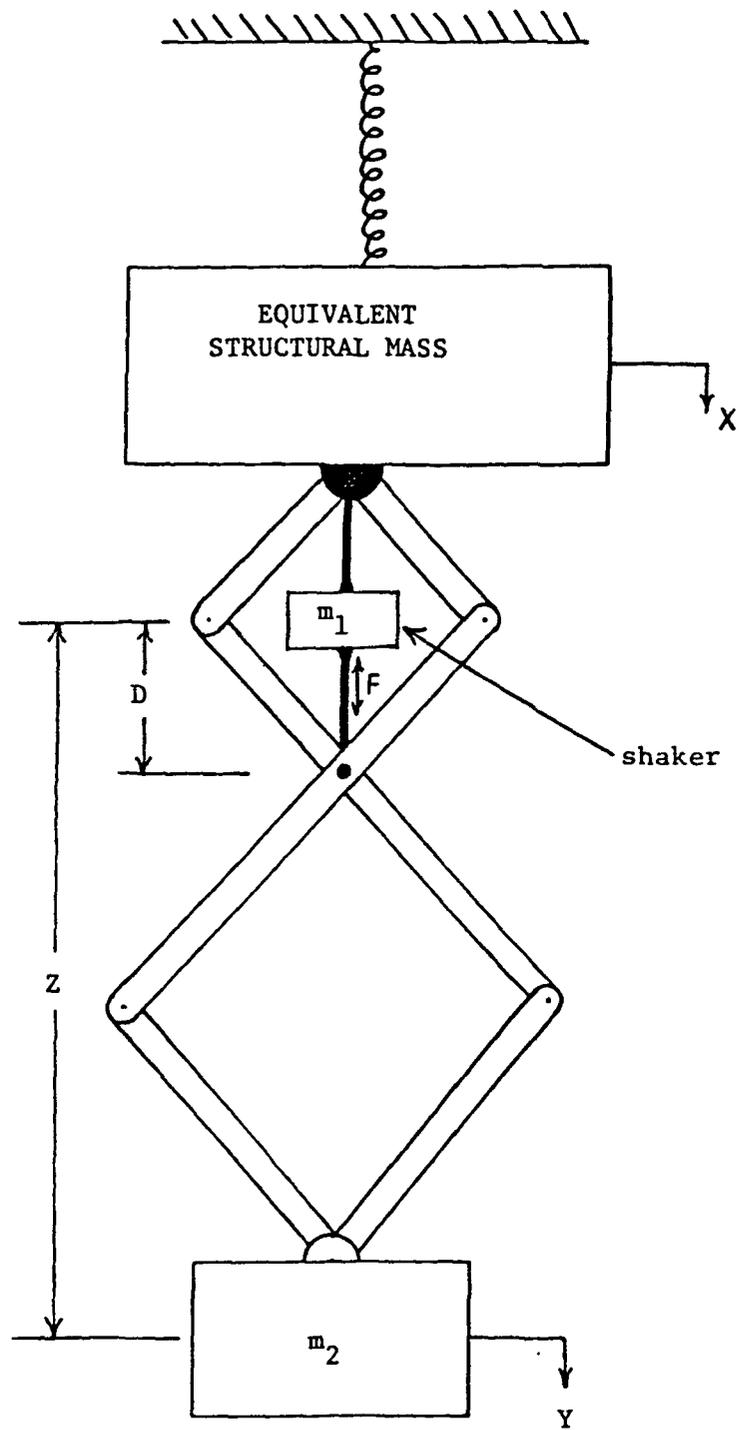


Fig. (B.1) System 2.2.1 Active Damper

$$|x_o|_{AL} = RD_M / \{1 + (C^2 / \omega^2 m_2^2)\}^{1/2} \quad (B.4)$$

whereas, if the motion is force-limited, we get, from B1

$$|x_o|_{FL} = F_M / \omega c R \quad (B.5)$$

Defining design conditions as those for which amplitude limits and force limits coincide, so that

$$|x_o|_{AL} = |x_o|_{FL} = |x_o|_D \quad (B.6)$$

and introducing the ratio

$$K = |x_o|_D / D_M \quad (B.7)$$

one can eliminate  $c$  from (B.4) and (B.5) to get the biquadratic equation

$$(R/K)^4 - (R/K)^2 - (f_{CH}/Kf_D)^4 = 0 \quad (B.8)$$

where  $f_{CH}$  is the characteristic frequency given by

$$f_{CH} = \frac{1}{2\pi} \{F_M / m_2 D_M\}^{1/2} \quad (B.9)$$

and  $f_d$  is the design frequency. Solving A8

$$(R/K)^2 = 1/2 + 1/2 \{1 + 4(f_{CH}/Kf_D)^4\}^{1/2} \quad (B.10)$$

thus, assuming that  $K$ ,  $f_d$ , and  $f_{CH}$  are given, the linkage ratio  $R$  can be found, and the linkage can be designed. The damping constant  $c$  can be found by substituting back into (B.4) and assuming design conditions

$$C = 2\pi f_D m_2 \{(R^2/K^2) - 1\}^{1/2} \quad (B.11)$$

Note that the electronic system is designed so that the damping constant  $c$  is a constant regardless of frequency. Thus, on rearranging (B.4), and substituting from (B.11)

$$\frac{|x_o|_{FL}}{|x_o|} = \frac{R/K}{\{(R/K^2 - 1)(f_D^2/f^2) + 1\}^{1/2}} ; f < f_D \quad (B.12)$$

Equation (B.12) defines the ratio of structural amplitude at frequency  $f$  to the design amplitude. It is less than unity. For frequencies above the design frequency, the structural amplitude is force limited, so that, according to (B.1)

$$\frac{|x_o|_{FL}}{|x_o|_D} = \frac{f_D}{\bar{f}} ; f > f_D \quad (B.13)$$

The above two equations define structural amplitudes for which the damping system would remain linear under conditions of SHM. If the structure were to vibrate at a single frequency, they would constitute an envelope of permissible amplitudes. The situation with complicated structural responses would be much more difficult to analyze.

The energy removed from the structure per cycle is

$$W_C = \oint c \dot{x} dx = 2\pi^2 f c |x_o|^2 \quad (B.14)$$

thus, at design conditions

$$W_D = 2\pi^2 f_D c |x_o|_D^2 \quad (B.15)$$

and, on substitution from (B.5) and (B.7)

$$W_D = W_m (R/K) \quad (B.16)$$

where

$$W_M = \pi F_M D_M \quad (B.17)$$

is the maximum energy which can be removed by the shaker. Since  $R/K$  is always greater than unity,  $W_D$  is always less than  $W_M$ , the discrepancy being due to phase differences between the structural and damper motions.

From (B.14) and (B.15)

$$\frac{W_C}{W_D} = \frac{f}{\bar{f}_D} \frac{|x_o|^2}{|x_o|_D^2}$$

thus, for the amplitude limited case of (B.12)

$$\frac{W_C}{W_D} = \frac{(f/f_D)(R^2/K^2)}{(R^2/K^2 - 1)(f_D^2/f^2) + 1} ; f < f_D \quad (B.19)$$

while for the force-limited case of (B.13)

$$\frac{W_C}{W_D} = \frac{f_D}{\bar{f}} ; f > f_D \quad (B.20)$$

The rate of removal of energy is given by

$$P = f W_C \quad (B.21)$$

thus, above the design frequency

$$P = P_D = f_D W_D; \quad f > f_D \quad (B.22)$$

while, below the design frequency, we have from (B.19)

$$P/P_D = \frac{(f^2/f_D^2)(R^2/K^2)}{(R^2/K^2 - 1)(f_D^2/f^2) + 1} \quad (B.23)$$

Taking the Goodman V102 shaker as representative of current practice, we use the following values as typical:

$$\text{mass, } m_1 = 2 \text{ lb}_m = 0.9072 \text{ kg}$$

$$\text{maximum force, } F_M = 2 \text{ lb}_f = 8.890 \text{ N}$$

$$\text{P-P deflection, } 2D_M = 0.2" = 0.00508 \text{ m}$$

$$\text{Max work per cycle, } W_M \text{ (from (B.17))} = 0.0709 \text{ Joules}$$

$$\text{Max work per cycle per unit mass, } W_M/m = 0.0782 \text{ J/kg}$$

As an example, consider a design condition of one inch structural amplitude,  $|x_0|_D$ , at a frequency  $f_D$  of one Hertz. Then from (B.7)

$$K = |x_0|_D / D_M = 10$$

The following three sets of calculations are for three values of the moving mass  $m_2$

| Case                 | 1       | 2      | 3      | Dimensions |
|----------------------|---------|--------|--------|------------|
| $m_2$                | 0.09072 | 0.9072 | 9.072  | kg         |
| $f_{CH}^2$ (from A9) | 31.26   | 9.886  | 3.126  | Hz         |
| $R/K$ (from A10)     | 3.207   | 1.264  | 1.005  |            |
| $c$ (from A11)       | 1.737   | 4.407  | 5.707  | Ns/m       |
| $W_D$ (from A16)     | 0.0221  | 0.0561 | 0.0705 | J          |
| $m_T = m_1 + m_2$    | 0.9979  | 1.8144 | 9.9792 | kg         |
| $W_D/m_T$            | 0.0221  | 0.0309 | 0.0071 | J/kg       |
| $P/m_T$ (from A22)   | 0.0221  | 0.0309 | 0.0071 | W/kg       |

It will be noted that case 2, for which the magnet and moving mass are equal, gives the best performance in terms of watts of energy removed per kg for frequencies above one Hz. The damping constant  $c$  achieved is also close to the maximum. If system 2.2.2.1 in Figure (5.7) were used instead, the total mass  $m_T$  would be 0.0972

kg, and the performance of the system would otherwise be identical to case 2 above. Thus, values for  $W_D/m_T$  and  $P/m_T$  would be 0.0619 J/kg and 0.619 w/kg respectively. This value for  $W_D/m_T$  is close to the maximum value of  $W_M/m$  obtainable with this voice-coil shaker.

## APPENDIX C

### Use of Piezoelectric Materials

Polyvinylidene flouride (PVP<sub>2</sub>) exhibits the following relationship between strain  $e_1$ , normal electric field,  $E_3$ , and stress  $\sigma_1$  (see Figure (C.1))

$$e_1 = -d_{31}E_3 + (1/C_{11}^E)\sigma_1 \quad (C.1)$$

where  $d_{31}$  = transverse piezoelectric charge coefficient

$C_{11}^E$  = modulus of elasticity

Assuming SHM with maximum values of  $e_M$ ,  $E_M$  and  $\sigma_M$ , such that  $e_1$  and  $\sigma_1$  are 90° out of phase with each other, and 45° out of phase with  $E_3$ .

$$\sigma_M = d_{31}C_{11}^E E_M/\sqrt{2} \quad (C.2)$$

$$e_M = d_{31} E_M/\sqrt{2} \quad (C.3)$$

and the maximum energy absorbed per unit volume is

$$w_M = \pi \sigma_M e_M = \pi d_{31}^2 C_{11}^E E_M^2/2 \quad (C.4)$$

while

$$W_M/m = w_M/\rho = \pi d_{31}^2 C_{11}^E E_M^2/2\rho \quad (C.5)$$

Typical values are

$$d_{31} = 5 \text{ to } 37 \times 10^{-12} \text{ m/V}$$

$$C_{11}^E = 1.6 \text{ to } 3.8 \times 10^9 \text{ N/m}^2$$

$$E_M = 12 \text{ to } 30 \times 10^6 \text{ V/m}$$

$$\rho = 1.8 \times 10^3 \text{ kg/m}^3$$

which gives the following range of values

$$W_M/m = .005 \text{ to } 4.1 \text{ J/Kg}$$

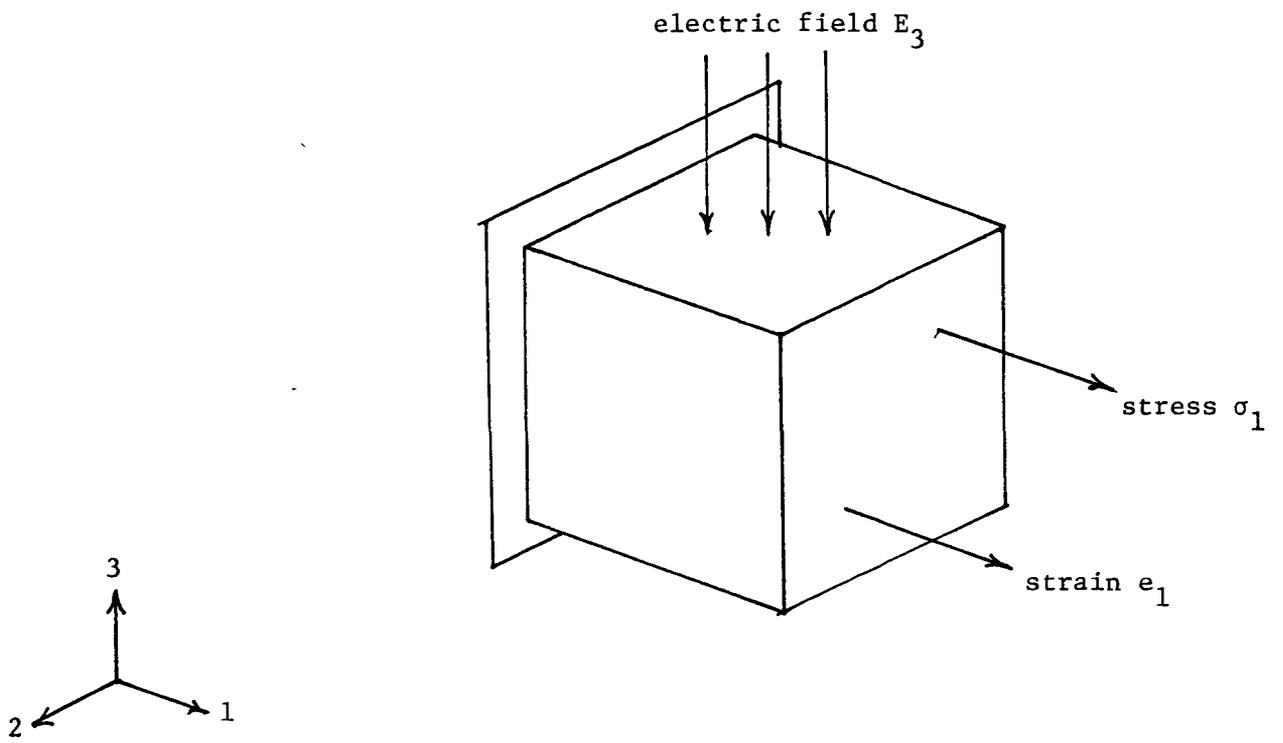


Fig. (C.1) Unit Cube of Polyvinylidene Fluoride (PVF2) Piezoelectric Material

The most critical property is that of  $E_M$ . Unless a considerable safety margin is allowed, there is a severe danger of dielectric breakdown in a space application, particular after a micrometeorite has damaged the material.

## APPENDIX D

### Simulation of NASA Tests

To help the actuator hardware design, a model of the prototype actuators will be put on the finite element model to simulate a vibration control test. Initially, a linear simulation will be performed. The purpose is to determine the magnitude of the displacement and the actuator force under designed conditions. The assumptions of this simulation are

- (1) the system is linear
- (2) the FEM of the NASA grillage (or beam) will be used as the undamped structure
- (3) the actuator dynamics will be ignored. That is, the actuator will be assumed to be a perfect device capable of producing force  $f = cx$  under all conditions.
- (4) the weight of each damper will be included as a discrete mass added to the system
- (5) the excitation will be an initial displacement and/or initial velocity of the FEM.

#### Formulations

Let  $[M]$ ,  $[K]$  be the mass and stiffness of the undamped structure. Introducing  $n_c$  dampers at dof  $J_1, \dots, J_{n_c}$ , the damping matrix is

$$[c] = [B][\hat{C}][B]^T \quad (D.1)$$

where

$$[B] = [ \{e_{J_1}\} \mid \{e_{J_2}\} \mid \dots \mid \{e_{J_{n_c}}\} ]_{n \times n_c}$$

$$[\hat{C}] = \begin{bmatrix} c_1 & & & & 0 \\ & c_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & c_{J_{n_c}} \\ 0 & & & & \end{bmatrix}$$

and

$\{e_j\}$  = null vector except the  $j$ th element has a value of 1

In expanded form,



## APPENDIX E

### 1. Bounds on Transient Responses

#### 2. Time Domain Optimal Design

##### 1. Bounds on Transient Responses

The disturbances acting on an Large Space Structure is mission dependent. However, one can say that these disturbances are usually of a transient type, e.g. firing of control thrusters for attitude control etc. The vibration control system must be designed to suppress these transient vibrations. One way of measuring the performance of a vibration control system is to evaluate how fast the control system can bring the response from some initial disturbed state to within an "acceptable" level. The response could be the displacement, velocity, or acceleration at certain "critical" locations of the LSS. For a large structure, it may not be easy to define which specific response is critical and the designer usually resorts to some aggregation of responses as the performance measure. One commonly used criterion is the minimization of

$$V(\{z\}) = \{z\}^T [P] \{z\}; \quad z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (\text{E.1})$$

where [P] is the weighting matrix. The estimation of transient behavior for the performance index can be derived according to Lyapunov's 2nd method, given a linear system governed by  $\dot{z} = [A]z$  with initial state  $\{z_0\}$  at  $t = t_0$  and where  $V(\{z\}, t_0)$

is given, the system attains a value of  $v$  given by

$V(\{z\}, t) = V(\{z_0\}, t_0) e^{-\eta(t-t_0)}$  in  $t-t_0$  seconds using Lyapunov's second method (Ref. 1,2). The following is a description of the use of Lyapunov's second method in estimating transient responses (Ref. 6) and then indicate how it is applied to the control of LSS. Finally, some research issues will be raised.

Given initial states  $\{z_0\}$  and the initial value  $V(\{z_0\}, t_0)$ , the system reaches the region

$$V(\{z\}, t) = V(\{z_0\}, t_0) e^{-\eta(t-t_0)} \quad (\text{E.2})$$

in  $t-t_0$  seconds.

For a linear time invariant system governed by the following state equations

$$\{\dot{z}\} = [A]\{z\} \quad (E.3)$$

the parameter  $\eta$  in Eq. (E.2) is the minimum eigenvalue of the matrix  $[Q][P]^{-1}$  where  $[P]$  and  $[Q]$  are related by

$$[A]^T[P] + [P][A] = -[Q] \quad (E.4)$$

and  $[P]$  is the weighting matrix in the performance index, Eq. (E.1). Note that if  $[Q]$  is symmetric, so will be  $[P]$ .

The above results can be applied to the control of LSS by letting

$$[A] = \left[ \begin{array}{c|c} 0 & I \\ \hline -M^{-1}k & -M^{-1}c \end{array} \right] \quad (E.5)$$

where  $[A]$  is the dynamical matrix of the closed loop system. That is, in Eq. (E.5) the mass matrix  $[M]$  include the mass of the control devices;  $[k]$  is the stiffness matrix of the system plus the control law that is proportion to the displacement ( $\{x\}$ ) and  $[c]$  is the damping matrix which corresponds to the portion of control law that is proportional to rate ( $\{\dot{x}\}$ ), see Appendix F for detailed formulation.

It should be noted that the use of Lyapunov's second method requires that all the eigenvalues of the matrix  $[A]$  have negative real parts. For a free-free structure with no position feedback, the system matrix  $[A]$  has some zero eigenvalues. Thus, to use the above results directly we need to include positive feedback terms in the control law.

The Lyapunov's second method provides a sound basis for transient performance estimates. To apply it to the design of vibration control of LSS, the following research needs to be carried out.

- (1) matrix  $[Q]$  must be positive definite, otherwise its eigenvalue could be zero or negative which renders the estimate (Eq. (E.2)) useless. This implies the performance index  $V(\{z\}, t)$  is restricted to quadratic form with weight matrix  $[P]$  that leads to a positive definite  $[Q]$ . Thus,  $[P]$  may not be specified arbitrarily. The question of how to pick matrix  $[P]$  needs further research.

- (2) Alternatively, we can specify a positive definite  $[Q]$  and solve for  $[P]$  by Eq. (E.4). This requires the solution of a set of  $n(n+1)/2$  equations, where  $n$  is the order of matrix  $[A]$ . An efficient solution scheme of Eq. (E.4) needs to be developed (or found in the literature).
- (3) modify the formulation to allow it to accept LSS whose control law consists of rate feedback only. This may be achieved by formulating equations of motion for elastic modes only by filtering out the rigid body motion.
- (4) application of the formulation in a truncated modal space. This will reduce the computational burden drastically.
- (5) Formulate a synthesis procedure using the basic results. This may be achieved by interactive methods or by a direct formulation. The results of the above researches can also be used in the next section.

## 2. Time Domain Optimal Design

Traditionally, time optimal design using optimal control theory with quadratic performance results in the solution of a higher order Riccati equation, (Ref. 3). This will result in a control law with full state feedback which may be difficult to implement. For a vibration control system, it may be advisable to take another approach. That is one could fix the form of the control law then find the pairs that optimize some performance index. In general, nonlinear programming approaches can be used to find the optimal gains. This will be extremely costly since many transient responses are to be evaluated. If we limit ourselves to a quadratic performance index, then the Lyapunov's second method can be used to evaluate the performance index when the system is driven by initial conditions. The general results (Ref. 6) will be stated first and then their application to LSS will be indicated.

Consider the system

$$\dot{\{z\}} = [A]\{z\} \quad (E.6)$$

where all eigenvalues of  $[A]$  have negative real parts, or the original  $\{0\}$  is asymptotically stable. The design problem is to adjust the elements in matrix  $[A]$  such that the performance index

$$J = \int_0^{\infty} \{z\}^T [Q] \{z\} dt$$

is minimized. It has been shown that

$$J = \int_0^{\infty} \{z\}^T [Q] \{z\} dt = \{z_0\}^T [P] \{z_0\} \quad (E.7)$$

where  $[P]$  is the solution of the equation

$$[A]^T[P] + [P][A] = -[Q] \quad (E.8)$$

Note that [P] is symmetrical. The significance of Eq. (E.7) is that to evaluate the performance index, no transient analysis has to be performed at all. Rather, we just solve a set of  $n(n+1)/2$  linear Eqs. (E.8) and then perform matrix multiplication. Since the elements of [P] is an implicit function of matrix [A] through Eq. (3), the following optimal procedure can be proposed:

- (1) from trial value of design parameters, form matrix [A]
- (2) solve [P] using Eq. (E.8)
- (3) evaluate J using Eq. (E.7)
- (4) repeat the above process until J is minimized.

When applied to LSS, we simply use the dynamic [A] as defined in Appendix F. The results of the research is used in the previous section is applicable here also.

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2. K. Ogata, Modern Control Engineering, Prentice-Hall, Inc. Englewood Cliffs, New Jersey.
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## APPENDIX F

### Formulation of Dynamical Matrix for LSS with General Control Law

Consider the actively controlled LSS

$$[M]\{\ddot{x}\} + [k_0]\{z\} = [B]\{u\} \quad (F.1)$$

where

$[k_0]$  is the stiffness matrix of the uncontrolled structure ( $n \times n$ )

$[M]$  is the mass matrix of the controlled system including the mass and inertia of the control devices (sensors and actuators, etc) ( $n \times n$ )

$[B]$  is the control distribution matrix ( $N \times N_u$ )

$\{u\}$  is the control input ( $N_u \times 1$ )

Let the control law be

$$\{u\} = -[\hat{\Delta}k]\{y\} - [\hat{\Delta}c]\{\dot{y}\} \quad (F.2)$$

where

$$\{y\} = [B_d]\{x\} = \text{displacement measurement } (N_d \times 1) \quad (F.3)$$

$$\{\dot{y}\} = [B_u]\{\dot{x}\} = \text{velocity measurement } (N_u \times 1)$$

$$[\hat{\Delta}k] = \text{displacement gain matrix } (N_u \times N_d)$$

$$[\hat{\Delta}c] = \text{velocity gain matrix } (N_u \times N_u)$$

$$[B_d] = \text{displacement measurement matrix } (N_d \times N)$$

$$[B_u] = \text{velocity measurement matrix } (N_u \times N)$$

Substitute (F.3) into (F.2)

$$\{u\} = -[\hat{\Delta}k][B_d]\{x\} - [\hat{\Delta}c][B_u]\{\dot{x}\} \quad (F.4)$$

Substitute (F.4) into (F.1)

$$[M]\{\ddot{x}\} + [K_0]\{x\} = -[B\hat{\Delta}k B_d]\{x\} - [B\hat{\Delta}c B_u]\{\dot{x}\} \quad (F.5)$$

Equation (F.5) can be simplified to

$$[M]\{x\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\} \quad (F.6)$$

where

$$\begin{aligned} [c] &= [B][\hat{\Delta}c][B_v] \\ [K] &= [K_0] + [\Delta K] \\ [\Delta K] &= [B][\hat{\Delta}k][B_d] \end{aligned} \quad (F.7)$$

Note that in general, matrix [K] and [C] are arbitrary matrices, they may not be symmetric.

Finally, we can put (F.6) in state space form as

$$\{\dot{z}\} = [A]\{z\} \quad (F.8)$$

where

$$[A] = \left[ \begin{array}{c|c} 0 & I \\ \hline -M^{-1}K & -M^{-1}c \end{array} \right] \quad (F.9)$$

$$\{z\} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

## Appendix G

### Proposed Approaches for Actuator Failure

#### Detection

The actuator failure modes can be classified as:

- (a) total failure: actuator does not produce a force in response to a command signal
- (b) random failure: actuator produces random force for a given input signal.

The total failure mode may be considered in the design of electronics for the actuator.

The random failure mode is more complex and may not be detected using hardware alone. In this case, a monitoring system to keep track of the performance of each actuator may be needed. Through the performance monitoring an adaptive control approach may be derived to detect and isolate failed actuators. Such an approach has been used in the detection of rate gyro failure in flight control systems [Ref. 1]. The feasibility of this approach to our problem is being investigated. Literature search will be made to look for other approaches.

#### References

1. Wagdi, M.N., "An Adaptive Control Approach to Sensor Failure Detection and Isolation," Journal of Guidance and Control, Vol. 5, No. 2, pp. 118-123, 1982.