LARGE SPACE STRUCTURES

CONTROL ALGORITHM CHARACTERIZATION

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Computation Consideration in LSS Control/Identification

- Algorithms
- Structures
- Computation considerations

Diagram:

Spillover Effect

Diagram showing controlled modes and residual modes with input $u$, output $y$, and computer.
MODEL

\dot{x} = Ax + Bu \\
y =Cx

\text{A} = \text{diag } H_j

H_j = \begin{bmatrix} u & 1 \\ \omega^2 & 0 \end{bmatrix}

\text{Separation to: } \text{controlled modes } x_c

\text{residual modes } x_R

x = \begin{bmatrix} x_c \\ x_c \end{bmatrix}

\begin{cases}
\dot{x} = \begin{bmatrix} A_c & A_R \\ & \end{bmatrix} x + \begin{bmatrix} B_c \\ B_R \end{bmatrix} u \\
y = c_c x_c + c_R x_R
\end{cases}

LAC/HAC

LAC: local feedback colocated sensor/actuator pairs

→ Augment damping

HAC: dynamic feedback to control a reduced order model

* frequency shaped K,F.
HAC/LAC Control Algorithm

**LAC:** \[ U_L = \bar{G} y \quad \bar{G} = \bar{G}^T > 0 \]

**HAC:**
\[
\begin{align*}
\dot{\hat{X}}_C &= \hat{A}_C \hat{X}_C + B \bar{U} + K(y - C \hat{X}_C) \\
u_H &= G_1 \hat{X}_C + G_2 \varphi
\end{align*}
\]

\[ u = u_L + u_H \]

Rate: \( \text{HAC rate} = 1/2 \text{ LAC rate} \)

LAC/HAC Based Computation Requirements

[Diagram of the control system with annotations for D/A, A/D, high rate, low rate, and computer connections.]
LQG APPROACH

Solution

\[ J = \int \left[ \| x_c \|^2 + \| u \|^2 \right] dt \]

Implementation

\[ u = T K \hat{x}_c \]

\[ \dot{\hat{x}}_c = A_c \hat{x}_c + B_c u + \bar{K} T (y - C_c \hat{x}_c) \]

\[ K = -R^{-1} B_c T P \]

\[ P A_c + A_c^T P + Q - P T B_c R^{-1} T B_c^T P = 0 \]

\[ \begin{align*}
T &: B_r T = 0 \quad B_c T \neq 0 \\
\bar{T} &: T C_r = 0 \quad \bar{T} C_c \neq 0
\end{align*} \]

Closed loop:

\[ \frac{d}{dt} \begin{bmatrix} x_c \\ x_r \\ e \end{bmatrix} = \begin{bmatrix} A_c - B_c T K & 0 & B_c T K \\ 0 & A_r & 0 \\ 0 & 0 & A_c - K T C_c \end{bmatrix} \begin{bmatrix} x_c \\ x_r \\ e \end{bmatrix} \]

\[ e = \Delta \]

\[ e = x_c - \hat{x}_c \]
STRUCTURES USED AS EXAMPLES

- 100-METER BEAM
- 50-METER REFLECTOR ANTENNA
Beam Instrumentation

3 clusters of x y \{ \text{accelerometer (sensor)} \}
\{ \text{proof mass (actuator)} \}

at: Top, Middle, Bottom

DIMENSION (M) OF SENSOR VECTOR AND ACTUATOR VECTOR = 3 \times 2 = 6
13 CLUSTERS OF COLLOCATED SENSORS/ACTUATORS AS FOLLOWS:

1. MAST/ORBITER ATTACHMENT:
   - SENSORS 2 DOF ACCELEROMETER PKG (x, y)
   - ACTUATOR 2 DOF PROOF MASS

2. REFLECTOR HUB (WHERE FEED SUPPORT MAST IS ATTACHED TO ANTENNA SUPPORT MAST)
   - SENSORS 2 DOF ACCELEROMETER PKG (x, y)
   - 1 DOF RATE GYRO (TORSION AXIS)
   - ACTUATOR 2 DOF PROOF MASS PKG
   - 1 DOF TORQUE WHEEL

3. 8 CLUSTERS OF INSTRUMENTS AROUND RIM OF REFLECTOR:
   - SENSORS 2 DOF ACCELEROMETER (TANGENTIAL + Z) TENSIOMETERS ON GUY WIRES
   - ACTUATORS 2 DOF PROOF MASS (tangential +Z), Guy Tensioner

4. MIDDLE OF FEED SUPPORT:
   - SENSORS 2 DOF ACCELEROMETER PKG (x, y)
   - ACTUATORS 2 DOF PROOF MASS (x, y)
5. FEED MAST/SUPPORT MAST ATTACHMENT:

SENSORS:
- 2 DOF ACCELEROMETER (x,y)
- 1 DOF RATE GYRO (TORSION)

ACTUATORS:
- 2 DOF PROOF MASS (x,y)
- 1 DOF TORQUE WHEEL (TORSION)

6. AT FEED:

SENSORS
- 2 DOF ACCELEROMETER (y,z)

ACTUATORS
- 2 DOF PROOF MASS (y,z)

* DIMENSION OF SENSOR/ACTUATOR VECTORS (M) = 2+3+2+3+2+8x3 = 36

**LG AND HAC/LAC COMPUTATIONAL SIZING**

- THESE ALGORITHMS HAVE BEEN SIZED IN TERMS OF
  - FLOATING POINT OPERATION (FLOP) DEMANDS
  - STORAGE FOR VARIABLES
  - INPUT/OUTPUT DATA FLOW

- FLOP SIZING (PER CONTROL CYCLE) DONE AS A FUNCTION
  OF THE NUMBER OF CONTROL STATES AND THE NUMBER OF
  SENSOR/ACTUATOR PAIRS

- STORAGE FOR VARIABLES AND I/O SIZING DONE FOR
  SPECIFIC STRUCTURE EXAMPLES

**Input/Output Data Flow Rules**

**Assumption**
- Control bandwidth 50 Hz
- Accuracy - 2 byte/word

\[
\begin{align*}
\text{Sampling frequency} & \quad 250 \text{ Hz} \\
\text{Command frequency} & \quad \text{500 [Bytes/sec]} \\
\text{Data Flow rate} & \quad \frac{\text{sensor}}{\text{actuator}} \quad 500 \text{ [Bytes/sec]}
\end{align*}
\]

**Total:**
- Beam: 3,000 [Bytes/sec]
- Antenna: 18,000 [Bytes/sec]

**1553B bus capacity:** 48,000 [Bytes/sec]
LOG SIZING

<table>
<thead>
<tr>
<th></th>
<th>BEAM</th>
<th>ANTENNA</th>
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<tbody>
<tr>
<td>SENSOR/ACTUATOR PAIRS (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>CONTROL STATES (n_c)</td>
<td>20</td>
<td>20</td>
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<tr>
<td>FLOP PER CYCLE*</td>
<td>1420</td>
<td>4420</td>
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<tr>
<td>VARIABLES**</td>
<td>752</td>
<td>2312</td>
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<tr>
<td>I/O PER CYCLE</td>
<td>12</td>
<td>72</td>
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</tbody>
</table>

* INCLUDES SENSOR COMPENSATION FLOP (120 for beam, 720 for antenna)
** INCLUDES SENSOR COMPENSATION VARIABLES (60 for beam, 360 for antenna)

FLOP/CYCLE

LOG ALGORITHM: $2n_c^2 + n_c + 4n_c m$

TYPICAL AP CAPACITY @ 250 cycles/s

NUMBER OF CONTROLLED STATES (n_c)
HAC/LAC SIZING

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<tr>
<td>FLOP PER CYCLE</td>
<td>633</td>
<td>4608</td>
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<tr>
<td>VARIABLES**</td>
<td>570</td>
<td>3060</td>
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<td>I/O</td>
<td>12</td>
<td>72</td>
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</tbody>
</table>

*Includes sensor compensation flop (120 for beam, 720 for antenna)

**Includes sensor compensation variables (60 for beam, 360 for antenna)

FLOP/LAC CYCLE

\[
\text{(LAC FLOP + } \frac{1}{2} \text{ HAC FLOP): } \frac{16}{9} n_H^2 + \frac{7}{6} n_H + \frac{7}{3} n_H m + \frac{m}{2} + 2m^2
\]

STATES UNDER HIGH-AUTHORITY CONTROL (n_H)

Typical AP capacity @ 250 LAC/s

m = 36 (Antenna)
m = 30
m = 24
m = 18
m = 12
m = 6 (Beam)
COMPUTATION LOAD

<table>
<thead>
<tr>
<th>Structure</th>
<th>Algorithm</th>
<th>m</th>
<th>n</th>
<th>n_c ( (n_H) )</th>
<th>rate K Flops/sec</th>
<th>% GPC capacity</th>
<th>% typical AP capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>LQG</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td>55</td>
<td>67</td>
<td>7</td>
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<tr>
<td></td>
<td>LQG</td>
<td>6</td>
<td>16</td>
<td>10</td>
<td>112</td>
<td>135</td>
<td>14</td>
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<tr>
<td></td>
<td>HAC/LAC</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td>34.</td>
<td>41</td>
<td>4</td>
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<tr>
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<td>HAC/LAC</td>
<td>6</td>
<td>16</td>
<td>10</td>
<td>57</td>
<td>69</td>
<td>7</td>
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<tr>
<td>Antenna</td>
<td>LQG</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td>260</td>
<td>300</td>
<td>36</td>
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<tr>
<td></td>
<td>HAC/LAC</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td>750</td>
<td>900</td>
<td>100</td>
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</tbody>
</table>

\( m = \# \) of sensors/actuators  
\( n = \# \) of modes in model  
\( n_c \) \( n_H \) - \# of controlled modes

SYSTEM IDENTIFICATION COMPUTATIONAL SIZING

- ARMA-LEAST SQUARES ALGORITHM sized for FLOP AS FUNCTION OF MODEL ORDER (n) AND NUMBER OF SENSOR/ACTUATOR PAIRS (m)
- FLOP REQUIREMENTS FOR THIS ALGORITHM ARE SO LARGE THAT IMPLEMENTATION IN A FLIGHT SYSTEM OR ITS GTF ANALOG IS PRECLUDED
- EVEN IMPLEMENTATION IN GROUND-BASED COMPUTERS IS CONSIDERED QUESTIONABLE, BUT THIS STUDY ASSUMES A GROUND-BASED IMPLEMENTATION

- NOTE: SOME OTHER SYSTEM IDENTIFICATION ALGORITHM MAY BE IMPLEMENTABLE IN A FLIGHT SYSTEM
• Algorithm Assessed - Least Squares

Motivation for choosing LS

• Relative high spectral resolution
• Comparable to other algorithm in computation complexity
  e.g.: Covariance algorithm : Maximum Entropy
• "Better" algorithms - considerably more complicated
• Less complex algorithms - considerable penalty in performance
• LS - robust to order reduction
• Useful for - control design
  - self tuning regulators

Identification Algorithm Sizing

Assume the ARMA model

\[ y_k = \sum_{i=1}^{N} A_i y_{k-1} + \sum_{i=0}^{N-1} B_i u_{k-i} \]

where \( y_k \) = vector of measurements (sensors) at cycle \( k \)
\( u_k \) = vector of control influence at cycle \( k \)

we can write

\[ y_k = \begin{bmatrix} A_1 & \cdots & A_n & B_0 & \cdots & B_{n-1} \end{bmatrix} \begin{bmatrix} y_{k-1} \\ \vdots \\ y_{k-n} \\ u_k \\ \vdots \\ u_{k-n+1} \end{bmatrix} = a \cdot z_k \]

Use least squares identification
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<tr>
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<tr>
<td>SENSOR/ACTUATOR PAIRS</td>
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<td>MODES MODELED</td>
<td>12</td>
<td>42</td>
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<tr>
<td>OFF-LINE MEGAFLROP</td>
<td>354.2</td>
<td>297,779</td>
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<td>FOR 4000 CYCLES</td>
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<tr>
<td>OFF-LINE FLOP/CYCLE</td>
<td>88,552</td>
<td>74,444,881</td>
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<tr>
<td>OFF-LINE MEGAFLOPS</td>
<td>22.1</td>
<td>18,611</td>
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<td>(a 250 CPS)</td>
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<td>ON-LINE FLOP/CYCLE</td>
<td>169,784</td>
<td>73,601,174</td>
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<td>ON-LINE MEGAFLOPS</td>
<td>42.5</td>
<td>18,400</td>
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<td>(a 250 CPS)</td>
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**AVIONICS DATA PROCESSING**

throughput (MFLOPS)

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