LARGE SPACE STRUCTURES

CONTROL ALGORITHM CHARACTERIZATION

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Computation Consideration in LSS Control/Identification

- Algorithms
- Structures
- Computation considerations
MODEL

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

\[
A = \text{diag } H_j \quad H_j = \begin{bmatrix} u & 1 \\ \omega^2 & 0 \end{bmatrix}
\]

Separation to: controlled modes \( x_C \)

: residual modes \( x_R \)

\[
x = \begin{bmatrix} x_C \\ x_C \end{bmatrix}
\begin{bmatrix}
\dot{x} &= \begin{bmatrix} A_C & 0 \\ 0 & A_R \end{bmatrix} x + \begin{bmatrix} B_C \\ B_R \end{bmatrix} u \\
y &= C_C x_C + C_R x_R
\end{bmatrix}
\]

LAC/HAC

LAC: local feedback colocated sensor/actuator pairs

→ Augment damping

HAC: dynamic feedback to control a reduced order model

* frequency shaped K.F.
HAC/LAC Control Algorithm

LAC: \[ U_L = \bar{G} y \quad \bar{G} = \bar{G}^T > 0 \]

HAC: \[ \dot{x}_c = \Omega x_c + M x_c \]
\[ \dot{x}_c = A_c \hat{x}_c + BU + K(y - C_c \hat{x}_c) \]
\[ u_H = G_1 \hat{x}_c + G_2 \phi \]

\[ U = U_L + U_H \]

Rate: HAC rate = 1/2 LAC rate

LAC/HAC Based Computation Requirements

[Diagram of control system]
LQG APPROACH

Solution

\[
J = \int \left[ ||x_C||_Q^2 + ||u||_R^2 \right] dt
\]

Implementation

\[
U = T K \hat{x}_C
\]

\[
\hat{x}_C = A_C \hat{x}_C + B_C u + K \left( y - C_C \hat{x}_C \right)
\]

\[
K = -R^{-1} B_C^T P
\]

\[
P A_C + A_C^T P + Q - P T B_C R^{-1} T B_C^T P = 0
\]

\[
T: \quad B_R T = 0 \quad B_C T \neq 0 \quad \text{orthogonality}
\]

\[
\bar{T}: \quad \bar{T} C_R = 0 \quad \bar{T} C_C \neq 0 \quad \text{conditions}
\]

Closed loop:

\[
\frac{d}{dt} \begin{bmatrix} x_C \\ x_R \\ e \end{bmatrix} = \begin{bmatrix} A_C - B_C T K & 0 & B_C T K \\ 0 & A_R & 0 \\ 0 & 0 & A_C - K T C_C \end{bmatrix} \begin{bmatrix} x_C \\ x_R \\ e \end{bmatrix}
\]

\[
e = \Delta \hat{x}_C - \hat{x}_C
\]
STRUCTURES USED AS EXAMPLES

- 100-METER BEAM
- 50-METER REFLECTOR ANTENNA
Beam Instrumentation

3 clusters of x y accelerometer (sensor)
proof mass (actuator)
at: Top, Middle, Bottom

DIMENSION (M) OF SENSOR VECTOR AND ACTUATOR VECTOR = 3 x 2 = 6
13 CLUSTERS OF COLOCATED SENSORS/ACTUATORS AS FOLLOWS:

1. MAST/ORBITER ATTACHMENT:
   - SENSORS: 2 DOF ACCELEROMETER PKG (x, y)
   - ACTUATOR: 2 DOF PROOF MASS

2. REFLECTOR HUB (WHERE FEED SUPPORT MAST IS ATTACHED TO ANTENNA SUPPORT MAST)
   - SENSORS: 2 DOF ACCELEROMETER PKG (x, y)
   - 1 DOF RATE GYRO (TORSION AXIS)
   - ACTUATOR: 2 DOF PROOF MASS PKG
   - 1 DOF TORQUE WHEEL

3. 8 CLUSTERS OF INSTRUMENTS AROUND RIM OF REFLECTOR:
   - SENSORS: 2 DOF ACCELEROMETER (TANGENTIAL, + z) TENSIO METERS ON GUY WIRES
   - ACTUATORS: 2 DOF PROOF MASS (tangential + Z), Guy Tensioner

4. MIDDLE OF FEED SUPPORT:
   - SENSORS: 2 DOF ACCELEROMETER PKG (x, y)
   - ACTUATORS: 2 DOF PROOF MASS (x, y)
Antenna Instrumentation (cont.)

5. FEED MAST/SUPPORT MAST ATTACHMENT:

SENSORS: 2 DOF ACCELEROMETER (x,y)
1 DOF RATE GYRO (TORSION)

ACTUATORS 2 DOF PROOF MASS (x,y)
1 DOF TORQUE WHEEL (TORSION)

6. AT FEED:

SENSORS 2 DOF ACCELEROMETER (y,z)

ACTUATORS 2 DOF PROOF MASS (y,z)

- DIMENSION OF SENSOR/ACTUATOR VECTORS (M) = 2+3+2+3+2+8x3 = 36

LGG AND HAC/LAC COMPUTATIONAL SIZING

- THESE ALGORITHMS HAVE BEEN SIZED IN TERMS OF
  - FLOATING POINT OPERATION (FLOP) DEMANDS
  - STORAGE FOR VARIABLES
  - INPUT/OUTPUT DATA FLOW
- FLOP SIZING (PER CONTROL CYCLE) DONE AS A FUNCTION
  OF THE NUMBER OF CONTROL STATES AND THE NUMBER OF
  SENSOR/ACTUATOR PAIRS
- STORAGE FOR VARIABLES AND I/O SIZING DONE FOR
  SPECIFIC STRUCTURE EXAMPLES

Input/Output Data Flow Rules

**Assumption**
- Control bandwidth 50 Hz
  - Accuracy - 2 byte/word

\[
\begin{align*}
\text{Sampling frequency} & \quad 250 \text{ Hz} \\
\text{Command frequency} & \\
\end{align*}
\]

**Data Flow rate**

\[
\begin{align*}
\text{sensor} & \quad 500 \text{ [Bytes/sec]} \\
\text{actuator} & \\
\end{align*}
\]

**Total:**
- Beam: 3,000 [Bytes/sec]
- Antenna: 18,000 [Bytes/sec]
- 1553B bus capacity 48,000 [Bytes/sec]

189
### LOG SIZING

<table>
<thead>
<tr>
<th></th>
<th>BEAM</th>
<th>ANTENNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SENSOR/ACTUATOR PAIRS (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>CONTROL STATES (n_c)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>FLOP PER CYCLE*</td>
<td>1420</td>
<td>4420</td>
</tr>
<tr>
<td>VARIABLES**</td>
<td>752</td>
<td>2312</td>
</tr>
<tr>
<td>I/O PER CYCLE</td>
<td>12</td>
<td>72</td>
</tr>
</tbody>
</table>

*INCLUDES SENSOR COMPENSATION FLOP (120 FOR BEAM, 720 FOR ANTENNA)

**INCLUDES SENSOR COMPENSATION VARIABLES (60 FOR BEAM, 360 FOR ANTENNA)

![Diagram showing FLOP/CYCLE](image)

LOG ALGORITHM: \(2^n_2 + n_c + 4n_m\)

TYPICAL AP CAPACITY @ 250 cycles/s

NUMBER OF CONTROLLED STATES (n_c)
### HAC/LAC Sizing

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<tr>
<td>Control States (n_c)</td>
<td>12</td>
<td>12</td>
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<tr>
<td>Flop Per Cycle*</td>
<td>633</td>
<td>4608</td>
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<tr>
<td>Variables**</td>
<td>570</td>
<td>3060</td>
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<tr>
<td>I/O</td>
<td>12</td>
<td>72</td>
</tr>
</tbody>
</table>

*Includes sensor compensation flop (120 for beam, 720 for antenna)

**Includes sensor compensation variables (60 for beam, 360 for antenna)

### Flop/Lac Cycle

\[
\text{Flop/Lac Cycle} = \left( \text{LAC Flop} + \frac{1}{2} \text{HAC Flop} \right): \frac{16}{9} n_H^2 + \frac{7}{6} n_H + \frac{7}{3} n_H m + \frac{m}{2} + 2m^2
\]

![Graph showing states under high-authority control vs. flops](image)
<table>
<thead>
<tr>
<th>Structure</th>
<th>Algorithm</th>
<th>m</th>
<th>n</th>
<th>$n_c$</th>
<th>$n_H$</th>
<th>Rate (K Flops/sec)</th>
<th>% GPC capacity</th>
<th>% Typical AP capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>LQG</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td></td>
<td>55</td>
<td>67</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>16</td>
<td>10</td>
<td></td>
<td>112</td>
<td>135</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>HAC/LAC</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td></td>
<td>34.</td>
<td>41</td>
<td>4</td>
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<tr>
<td></td>
<td></td>
<td>6</td>
<td>16</td>
<td>10</td>
<td></td>
<td>57</td>
<td>69</td>
<td>7</td>
</tr>
<tr>
<td>Antenna</td>
<td>LQG</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td></td>
<td>260</td>
<td>300</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>HAC/LAC</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td></td>
<td>750</td>
<td>900</td>
<td>100</td>
</tr>
</tbody>
</table>

$m$ = # of sensors/actuators  
$n$ = # of modes in model  
$n_c = \{n_H \} = $ # of controlled modes

**SYSTEM IDENTIFICATION COMPUTATIONAL SIZING**

- ARMA-LEAST SQUARES ALGORITHM SIZED FOR FLOP AS FUNCTION OF MODEL ORDER ($n$) AND NUMBER OF SENSOR/ACTUATOR PAIRS ($m$)
- FLOP REQUIREMENTS FOR THIS ALGORITHM ARE SO LARGE THAT IMPLEMENTATION IN A FLIGHT SYSTEM OR ITS GTF ANALOG IS PRECLUDED
- EVEN IMPLEMENTATION IN GROUND-BASED COMPUTERS IS CONSIDERED QUESTIONABLE, BUT THIS STUDY ASSUMES A GROUND-BASED IMPLEMENTATION

- **NOTE:** SOME OTHER SYSTEM IDENTIFICATION ALGORITHM MAY BE IMPLEMENTABLE IN A FLIGHT SYSTEM
Algorithm Assessed - Least Squares

Motivation for choosing LS

- Relative high spectral resolution
- Comparable to other algorithms in computation complexity
  
  \[ \text{e.g.} : \text{Covariance algorithm} \]
  
  \[ : \text{Maximum Entropy} \]

- "Better" algorithms - considerably more complicated
- Less complex algorithms - considerable penalty in performance

- LS - robust to order reduction
- Useful for - control design
  - self tuning regulators

Identification Algorithm Sizing

Assume the ARMA model

\[ y_k = \sum_{i=1}^{N} A_i y_{k-i} + \sum_{i=0}^{N-1} B_i u_{k-i} \]

where \( y_k \) = vector of measurements (sensors) at cycle \( k \)

\( u_k \) = vector of control influence at cycle \( k \)

we can write

\[ y_k = \begin{bmatrix} A_1; \cdots; A_n; B_0; \cdots; B_{n-1} \end{bmatrix} \cdot \begin{bmatrix} y_{k-1} \\ \vdots \\ y_{k-n} \\ u_k \\ \vdots \\ u_{k-n+1} \end{bmatrix} = a \cdot z_k \]

Use least squares identification
### System Identification Algorithm Flop Requirements

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<tbody>
<tr>
<td>Sensor/Actuator Pairs (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>Modes Modeled (n)</td>
<td>12</td>
<td>42</td>
</tr>
<tr>
<td>Off-line Megaflop</td>
<td>354.2</td>
<td>297,779</td>
</tr>
<tr>
<td>For 4000 Cycles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Off-line Flop/Cycle</td>
<td>88,552</td>
<td>74,444,881</td>
</tr>
<tr>
<td>Off-line Megaflops (a 250 CPS)</td>
<td>22.1</td>
<td>18,611</td>
</tr>
<tr>
<td>On-line Flop/Cycle</td>
<td>169,784</td>
<td>73,601,174</td>
</tr>
<tr>
<td>On-line Megaflops (a 250 CPS)</td>
<td>42.5</td>
<td>18,400</td>
</tr>
</tbody>
</table>

### Avionics Data Processing

Throughput (MFLOPS)

- Existing Space Processor: SCP-24, CDC-66, IAP-11, L-4156
- Near-Term Processor: APF-224, ATAC
- μ-Processors: MECA-43, 9900
- Projected Trends (Avionics): 75, 80, 85, 90
- Existing Control
- LSS Control (ATB Models): [Model Dependent]
- LSS Identification
- Other Space RQTS: Housekeeping → Payload Processing → Signal Processing

194