LARGE SPACE STRUCTURES

CONTROL ALGORITHM CHARACTERIZATION

E. Fogel
Charles Stark Draper Laboratory, Inc.
Cambridge, Massachusetts
Computation Consideration in LSS Control/Identification

- Algorithms
- Structures
- Computation considerations

Spillover Effect
MODEL

\[ \dot{x} = AX + BU \quad y = CX \]

\[ A = \text{diag} \quad H_J \]

\[ H_J = \begin{bmatrix} u & 1 \\ \omega^2 & 0 \end{bmatrix} \]

Separation to: controlled modes

\[ x_C \]

: residual modes

\[ x_R \]

\[ x = \begin{bmatrix} x_C \\ x_R \end{bmatrix} \]

\[ \begin{aligned} \dot{x} &= \begin{bmatrix} A_C & 0 \\ 0 & A_R \end{bmatrix} x + \begin{bmatrix} B_C \\ B_R \end{bmatrix} U \\ y &= c_C x_C + c_R x_R \end{aligned} \]

LAC/HAC

**LAC:** local feedback colocated sensor/actuator pairs

\[ \rightarrow \text{Augment damping} \]

**HAC:** dynamic feedback to control a reduced order model

* frequency shaped K.F.
HAC/LAC Control Algorithm

**LAC:** \[ U_L = \dot{y} \quad \ddot{G} = \dot{G}^T \geq 0 \]

**HAC:**

\[ \dot{\phi} = \Omega \phi + M \hat{x}_c \]

\[ \dot{\hat{x}}_c = A_c \hat{x}_c + BU + K(y - C_c \hat{x}_c) \]

\[ U_H = G_1 \hat{x}_c + G_2 \phi \]

\[ U = U_L + U_H \]

**Rate**

HAC rate = 1/2 LAC rate

LAC/HAC based computation requirements

![Diagram of LAC/HAC control system]
LQG APPROACH

Solution

\[ J = \int \left[ \| X_c \|^2 + \| U \|^2 \right] dt \]

Implementation

\[ U = T K \hat{X}_c \]

\[ \hat{X}_c = A_c X_c + B_c U + K^T (Y - C_c \hat{X}_c) \]

\[ K = -R^{-1} B_c^T P \]

\[ P A_c + A_c^T P + Q - P T B_c R^{-1} T B_c^T P = 0 \]

\[ T : \quad B_R T = 0 \quad B_c T \neq 0 \quad \text{orthogonality} \]

\[ \bar{T} : \quad \bar{T} C_R = 0 \quad \bar{T} C_c \neq 0 \quad \text{conditions} \]

Closed loop:

\[
\frac{d}{dt} \begin{bmatrix} X_c \\ X_R \\ e \end{bmatrix} = \begin{bmatrix} A_c - B_c T K & 0 & B_c T K \\ 0 & A_R & 0 \\ 0 & 0 & A_c - K T C_c \end{bmatrix} \begin{bmatrix} X_c \\ X_R \\ e \end{bmatrix}
\]

\[ e = X_c - \hat{X}_c \]
STRUCTURES USED AS EXAMPLES

- 100-METER BEAM
- 50-METER REFLECTOR ANTENNA
Beam Instrumentation

3 clusters of $x \ y$ \{ accelerometer (sensor) \\
proof mass (actuator) \}

at: Top, Middle, Bottom

DIMENSION (M) OF SENSOR VECTOR AND ACTUATOR VECTOR = $3 \times 2 = 6$
13 CLUSTERS OF COLOCATED SENSORS/ACTUATORS AS FOLLOWS:

1. MAST/ORBITER ATTACHMENT:
   SENSORS  2 DOF  ACCELEROMETER PKG (x, y)
   ACTUATOR 2 DOF  PROOF MASS

2. REFLECTOR HUB (WHERE FEED SUPPORT MAST IS ATTACHED TO ANTENNA SUPPORT MAST)
   SENSORS  2 DOF  ACCELEROMETER PKG (x, y)
   1 DOF  RATE GYRO (TORSION AXIS)
   ACTUATOR 2 DOF  PROOF MASS PKG
   1 DOF  TORQUE WHEEL

3. 8 CLUSTERS OF INSTRUMENTS AROUND RIM OF REFLECTOR:
   SENSORS  2 DOF  ACCELEROMETER (TANGENTIAL, + z) TENSIO METERs ON GUY WIRES
   ACTUATORS 2 DOF  PROOF MASS (tangential +Z), Guy Tensioner

4. MIDDLE OF FEED SUPPORT:
   SENSORS  2 DOF  ACCELEROMETER PKG (x, y)
   ACTUATORS 2 DOF  PROOF MASS (x, y)
5. FEED MAST/SUPPORT MAST ATTACHMENT:

SENSORS: 2 DOF ACCELEROMETER (x,y)
1 DOF RATE GYRO (TORSION)

ACTUATORS 2 DOF PROOF MASS (x,y)
1 DOF TORQUE WHEEL (TORSION)

6. AT FEED:

SENSORS 2 DOF ACCELEROMETER (y,z)
ACTUATORS 2 DOF PROOF MASS (y,z)

• DIMENSION OF SENSOR/ACTUATOR VECTORS (M) = 2+3+2+3+2+8x3 = 36

LOG AND HAC/LAC COMPUTATIONAL SIZING

• THESE ALGORITHMS HAVE BEEN SIZED IN TERMS OF
  • FLOATING POINT OPERATION (FLOP) DEMANDS
  • STORAGE FOR VARIABLES
  • INPUT/OUTPUT DATA FLOW

• FLOP SIZING (PER CONTROL CYCLE) DONE AS A FUNCTION
  OF THE NUMBER OF CONTROL STATES AND THE NUMBER OF
  SENSOR/ACTUATOR PAIRS

• STORAGE FOR VARIABLES AND I/O SIZING DONE FOR
  SPECIFIC STRUCTURE EXAMPLES

Input/Output Data Flow Rules

Assumption
• Control bandwidth 50 Hz
• Accuracy - 2 byte/word

→ \{ Sampling frequency 250 Hz
   Command frequency \}

Data Flow rate

<table>
<thead>
<tr>
<th></th>
<th>sensor</th>
<th>500 [Bytes/sec]</th>
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</thead>
<tbody>
<tr>
<td>Beam:</td>
<td>3,000</td>
<td>[Bytes/sec]</td>
</tr>
<tr>
<td>Antenna:</td>
<td>18,000</td>
<td>[Bytes/sec]</td>
</tr>
<tr>
<td>1553B bus capacity</td>
<td>48,000</td>
<td>[Bytes/sec]</td>
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</table>
**LOG SIZING**

<table>
<thead>
<tr>
<th></th>
<th>BEAM</th>
<th>ANTENNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>SENSOR/ACTUATOR PAIRS (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>CONTROL STATES (n_c)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>FLOP PER CYCLE*</td>
<td>1420</td>
<td>4420</td>
</tr>
<tr>
<td>VARIABLES**</td>
<td>752</td>
<td>2312</td>
</tr>
<tr>
<td>I/O PER CYCLE</td>
<td>12</td>
<td>72</td>
</tr>
</tbody>
</table>

*INCLUDES SENSOR COMPENSATION FLOP (120 FOR BEAM, 720 FOR ANTENNA)

**INCLUDES SENSOR COMPENSATION VARIABLES (60 FOR BEAM, 360 FOR ANTENNA)

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**FLOP/CYCLE**

LOG ALGORITHM: \(2n^2 + n_c + 4n_c m\)

![Graph showing the relationship between number of controlled states and flop/cycle](image)

_TYPICAL AP CAPACITY @ 250 cycles/s_
### HAC/LAC Sizing

<table>
<thead>
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<th>ANTENNA</th>
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<td>SENSOR/ACTUATOR PAIRS (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>CONTROL STATES (n_C)</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>FLOP PER CYCLE*</td>
<td>633</td>
<td>4608</td>
</tr>
<tr>
<td>VARIABLES**</td>
<td>570</td>
<td>3060</td>
</tr>
<tr>
<td>I/O</td>
<td>12</td>
<td>72</td>
</tr>
</tbody>
</table>

*Includes sensor compensation flop (120 for beam, 720 for antenna)

**Includes sensor compensation variables (60 for beam, 360 for antenna)

![FLOP/LAC CYCLE Graph](image)

\[
\text{(LAC FLOP + } \frac{1}{2} \text{ HAC FLOP): } \frac{16}{9} n_H^2 + \frac{7}{6} n_H + \frac{7}{3} n_H m + \frac{m^2}{2} + 2m^2
\]
### COMPUTATION LOAD

<table>
<thead>
<tr>
<th>Structure</th>
<th>Algorithm</th>
<th>m</th>
<th>n</th>
<th>( n_c ) (( n_H ))</th>
<th>Rate K Flops/sec</th>
<th>% GPC capacity</th>
<th>% typical AP capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>LQG</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td>55</td>
<td>67</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>LQG</td>
<td>6</td>
<td>16</td>
<td>10</td>
<td>112</td>
<td>135</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>HAC/LAC</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td>34</td>
<td>41</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>HAC/LAC</td>
<td>6</td>
<td>16</td>
<td>10</td>
<td>57</td>
<td>69</td>
<td>7</td>
</tr>
<tr>
<td>Antenna</td>
<td>LQG</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td>260</td>
<td>300</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>HAC/LAC</td>
<td>36</td>
<td>42</td>
<td>6</td>
<td>750</td>
<td>900</td>
<td>100</td>
</tr>
</tbody>
</table>

\( m = \# \) of sensors/actuators  
\( n = \# \) of modes in model  
\( n_c, n_H \) = \# of controlled modes

### SYSTEM IDENTIFICATION COMPUTATIONAL SIZING

- ARMA-LEAST SQUARES ALGORITHM SIZED FOR FLOP AS FUNCTION OF MODEL ORDER \( (n) \) AND NUMBER OF SENSOR/ACTUATOR PAIRS \( (m) \)
- FLOP REQUIREMENTS FOR THIS ALGORITHM ARE SO LARGE THAT IMPLEMENTATION IN A FLIGHT SYSTEM OR ITS GTF ANALOG IS PRECLUDED
- EVEN IMPLEMENTATION IN GROUND-BASED COMPUTERS IS CONSIDERED QUESTIONABLE, BUT THIS STUDY ASSUMES A GROUND-BASED IMPLEMENTATION

- NOTE: SOME OTHER SYSTEM IDENTIFICATION ALGORITHM MAY BE IMPLEMENTABLE IN A FLIGHT SYSTEM
• Algorithm Assessed - Least Squares

Motivation for choosing LS

• Relative high spectral resolution
• Comparable to other algorithms in computation complexity
e.g.: Covariance algorithm
      : Maximum Entropy
• "Better" algorithms - considerably more complicated
• Less complex algorithms - considerable penalty in performance
• LS - robust to order reduction
• Useful for - control design
   - self tuning regulators

Identification Algorithm Sizing

Assume the ARMA model

\[ y_k = \sum_{i=1}^{N} A_i y_{k-1} + \sum_{j=0}^{N-1} B_j u_{k-1} \]

where \( y_k \) = vector of measurements (sensors) at cycle k
\( u_k \) = vector of control influence at cycle k

we can write

\[ y_k = \begin{bmatrix} A_1 & \cdots & A_n & B_0 & \cdots & B_{n-1} \end{bmatrix} \begin{bmatrix} y_{k-1} \\ \vdots \\ y_{k-n} \\ u_k \\ \vdots \\ u_{k-n+1} \end{bmatrix} \]

= \( a \cdot z_k \)

Use least squares identification
### System Identification Algorithm Flop Requirements

<table>
<thead>
<tr>
<th>Description</th>
<th>BEAM</th>
<th>ANTENNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor/Actuator Pairs (m)</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>Modes Modeled (n)</td>
<td>12</td>
<td>42</td>
</tr>
<tr>
<td>Off-Line Megaflop for 4000 Cycles</td>
<td>354.2</td>
<td>297,779</td>
</tr>
<tr>
<td>Off-Line Flop/Cycle</td>
<td>88,552</td>
<td>74,444,881</td>
</tr>
<tr>
<td>Off-Line Megaflops (a 250 CPS)</td>
<td>22.1</td>
<td>18,611</td>
</tr>
<tr>
<td>On-Line Flop/Cycle</td>
<td>169,784</td>
<td>73,601,174</td>
</tr>
<tr>
<td>On-Line Megaflops (a 250 CPS)</td>
<td>42.5</td>
<td>18,400</td>
</tr>
</tbody>
</table>

### Avionics Data Processing

Throughput (MFLOPS)

- Existing Space Processor
- Near-Term Processor
- μ-Processors
- Projected Trends (Avionics)
- Existing Control
- LSS Control (ATB Models)
- LSS Identification
- Other Space RQTS

- Housekeeping → Payload Processing → Signal Processing
- Model Dependent

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