General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.
"DEVELOPMENT OF A CLEAN OPTICAL TELESCOPE"

NASA Grant No. NAGW-262

Final Report

Period: 1981 November 15 - 1982 November 14

April 15, 1983

Submitted to: Dr. William Brunk
Discipline Scientist
Code EL-4
NASA Headquarters
400 Maryland Avenue SW
Washington, DC 20546

Principal Investigator: Dr. Robert A. Brown*
Lunar & Planetary Laboratory
University of Arizona
Tucson, AZ 85721

* Current mailing address: Space Telescope Science Institute
Homewood Campus
Baltimore, Maryland 21218
Introduction

Particulate contamination on astronomical mirrors degrades performance in two ways: (1) by information loss by extinction of light; (2) background and noise from scattering, especially "forward" or Fraunhofer scattering. The proposal for this grant pointed out that these effects were not generally understood, and it outlined an ambitious pilot program to (1) measure particulate effects on telescope optical performance; (2) develop prophylactic and cleaning procedures suitable for groundbased observatories; (3) investigate by computational modelling the effects on telescopes in space; (4) communicate our results and concerns within the astronomical community.

In 1982 July the Principal Investigator joined the Space Telescope Science Institute, changing the profile of activity on this grant, but with important effect for Space Telescope. A timely assessment of the contamination problem for ST exposed a major concern that is now grouped with a handful of technical problems receiving serious attention for the first time. The ST mirror is now visibly dirty, and the problems of cleaning it and maintaining it free of dust are major challenges to the principal contractors. The work supported by this grant was instrumental in identifying and publicizing this problem.

Our program had three main thrusts: observational, engineering and theoretical. During the one year grant period we have (1) developed CCD procedures to document the optical efficiency and the magnitude and distribution of scattered light in the Whipple Observatory 24" telescope (2) contracted, installed and operated an electrostatic dust precipitator on that telescope; (3) computed numerically the optical effects of certain particulate populations on telescope performance with applications to our Whipple Observatory measurements and to the Space Telescope; (4) reported to the ST Contamination Control Committee aned ST Science Working Group on particulate effects on ST.

Observations

We used the SAO CCD and photometer to obtain star images to document the surface brightness profile in the range 3"-300" off axis. The 10:1 required dynamic range is achieved with several exposures of different durations. A Couder mask at the front of the telescope removes the secondary support diffraction spikes, so the images have cylindrical symmetry and the signal at an off-axis angular distance can be integrated in an angular strip. Figure A is such a result. The bump at 25" is due to an internal reflection: CCD up to window down to CCD again.

While we obtained believable scattered light profiles by combining sequential exposures, we did not achieve complete confidence in our absolute normalization procedures, so
comparisons between the light scattering on different days, before and after mirror washing, are not in hand.

Dust precipitator

The grant purchased the services of Dr. Stuart Hoenig's shop to construct an electrostatic precipitator unit for the closed-tube 24" telescope on Mt. Hopkins. This effort is described in Hoenig's article "Electrostatic dust protection for optical elements" in Applied Optics 21, pp565-569, 1982.

Theory

The appended report on ST particulate contamination is an exegesis of our current modelling efforts. It concluded that particulate contamination posed a threat to certain ST science programs and that the situation was not currently understood or under control. This problem is now receiving considerable attention in the ST Project.
Fig. A  A stellar scattered light profile obtained from five CCD images (A-E). \( \phi \) is the observed surface brightness normalized by the total stellar flux. Dust on the mirror surfaces causes the stellar profile to stand \( \sim2 \) orders of magnitude above the asymptotic diffraction pattern of the telescope aperture. When the telescope primary was cleaned, the measured profile indicated a factor 2 decrease in the scattered light, as indicated.
APPENDIX

REPORT ON ST PARTICULATE CONTAMINATION
7 September 1982
ROBERT A. BROWN, Instrument Scientist, ST ScI

TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 Introduction</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>2.0 Particulate Scattering</td>
<td></td>
<td>A2</td>
</tr>
<tr>
<td>2.1 ST Aperture Diffraction</td>
<td></td>
<td>A3</td>
</tr>
<tr>
<td>2.2 Diffuse Scattering</td>
<td></td>
<td>A4</td>
</tr>
<tr>
<td>2.3 Fraunhofer Diffraction</td>
<td></td>
<td>A4</td>
</tr>
<tr>
<td>2.3.1 Power Law Distribution</td>
<td></td>
<td>A5</td>
</tr>
<tr>
<td>2.3.2 LMSC Trial Distribution</td>
<td></td>
<td>A6</td>
</tr>
<tr>
<td>3.0 Impact of Particles on ST Science Investigations</td>
<td></td>
<td>A7</td>
</tr>
<tr>
<td>3.1 Near-In Science</td>
<td></td>
<td>A7</td>
</tr>
<tr>
<td>3.2 Far-Out Science</td>
<td></td>
<td>A8</td>
</tr>
<tr>
<td>4.0 Current Status and Planned Tests</td>
<td></td>
<td>A9</td>
</tr>
</tbody>
</table>
1.0 INTRODUCTION

The ST SWG has set a requirement that ST perform on orbit with <1% light loss on each of the primary and secondary mirrors due to particulate contamination. No more than half the total allowable degradation is allowed before launch. This optical efficiency requirement has been interpreted by the ST Contamination Control Committee as corresponding to <1% surface area coverage.

I find no evidence that Fraunhofer diffraction has been considered in developing ST particulate contamination requirements. That mechanism is shown below to (1) double the particle optical extinction (so the correct contamination requirements are 0.25%/0.5% surface area coverage before/after launch), and (2) produce "scattered" light that will impede certain observing programs.

It was reported at the 30 August 1982 meeting of the Contamination Control Committee that no procedures are currently in place at Perkin-Elmer to measure surface particulates and certify a cleanliness level. P-E reports that the ST primary mirror is currently not visibly clean.

I have taken the view that the Institute may have an (unstaffed) investigatory/advisory role with respect to the OTA similar to that described for the SIs in the ISB Management Plan. The following sections lay out (1) the physical principles behind particulate degradation of the ST optical performance including the Fraunhofer diffraction mechanism, and (2) my current understanding of prelaunch particulate contamination monitoring and control. I recommend that the Institute promote a fresh look
at this problem, possibly resulting in corrective procedures to be instituted at P-E, Lockheed, then KSC to block particulate accumulation in the prelaunch phase and to plan realistically for the optical impact of particulates on science programs.

2.0 PARTICULATE SCATTERING

This treatment of particulate scattering uses a circular disk (radius=a) as the fundamental element. The effects of more complex particles or of populations may be found by linear superposition. The disk lies flat on the primary mirror, and starlight is incident normal to the circular face, with flux= \( \tau F \). Kirchoff diffraction theory is valid for

\[
\frac{\text{disk circumference}}{\text{wavelength}} = \frac{2\pi a}{\lambda} = \frac{ka}{\lambda} \gg 1,
\]

and in that regime the particle extinets light from the incident parallel beam at rate:

\[
\text{extinction rate} = \pi F \cdot 2 \cdot \pi a^2 \text{ ph s}^{-1}.
\]

The factor two is composed of two equal parts as follows. One unit is intercepted on the disk surface then absorbed or back-scattered (diffusely or specularly). Attention to the second unit of extinction is the main contribution of the current work: this is forward scattering by Fraunhofer diffraction. Since the telescope mirror folds the "forward" onto the "backward" direction, both mechanisms direct scattered light toward the ST focal plane. However, the diffuse scattering will be distributed over a hemisphere whereas Fraunhofer diffraction is approximately collimated in a cone of half-angle 0.6 \( \lambda/a \). Since diffracted light is less baffled and dilute than light scattered by particle upper surfaces, it has greater potential for general fogging of the ST field.
Larger particles (a>100 μ) can threaten observations in high contrast regions, especially programs relying on the f/288 FOC apodizing pupil to reduce bright object aperture diffraction (calculated in the next section).

2.1 ST APERTURE DIFFRACTION

It will be useful in the following to compare particulate scattering to the Airy pattern of the primary (including the central obscuration but ignoring the spider):

\[ I(\theta; \lambda) = \frac{\pi a_0^2 (1-\epsilon^2)}{\lambda^2} 4 \left( \frac{J_1(x)}{x} - \epsilon^2 \frac{J_1(\epsilon x)}{\epsilon x} \right)^2 \]  (1)

(Young, Appl. Opt. 2, pp. 1874-1878, 1970), where

\[ a_0 = 1.2 \times 10^6 \mu \text{ (outer radius of primary)}, \]

\[ \epsilon = 0.37 \text{ (radius obscuration ratio)}, \]

\[ x = \frac{2\pi a_0}{\lambda} \theta (= k_0 \theta). \]

Asymptotically,

\[ \frac{J_1(x)}{x} \xrightarrow{x>>1} \frac{0.8}{x^{3/2}} \cos(x-2.36) \approx \frac{0.4}{x^{3/2}} \text{ (average)}; \]

and

\[ I_A(\theta; \lambda) \xrightarrow{\theta>>1} 0.64 \pi F \frac{\pi a_0^2}{\lambda^2} \frac{(1-\epsilon^2)(1-\epsilon^2/2)}{x^3} \]

\[ = \left( \frac{0.64}{8\pi^2} \right) \frac{\pi F}{a_0} \frac{\lambda}{\epsilon^3} = 1.1 \times 10^{-3} \pi F \frac{\lambda}{a_0} \frac{1}{\epsilon^3} \]

for

\[ \theta >> \frac{\lambda}{2\pi a_0}. \]
or

\[ -2.5 \log \frac{I_A(0) \Omega_1}{\tau F} = 10.1 + 7.5 \log \theta \quad \text{mag arcsec}^{-2} \]

for 5000 A and \( \theta \) in arcsec, and the surface brightness is lower by 1 magnitude at 2000 A. \( \Omega_1 \) is the solid angle of one square arcsec.

2.2 DIFFUSE SCATTERING

Assume the particle upper surface is a Lambertian scatterer, albedo = 1 (worst case). The equivalent astronomical intensity due to one particle is

\[ I_L = \tau F \frac{r a^2}{A} (\cos \theta - 1) = \tau F \frac{f}{\tau} \quad , \quad (2) \]

where \( A \) is area of primary and \( f \) is fractional surface area covered by the particle. Since particle size is immaterial, the final expression works if \( f \) is the total surface fraction covered by dust. The brightness may be expressed

\[ -2.5 \log \frac{I_L \Omega_1}{\tau F} = \frac{f}{\tau} (4.848 \times 10^{-6})^2 = 27.8 - 2.5 \log f \quad \text{mag arcsec}^{-2}, \]

which is the surface brightness for zeroth magnitude flux onto the primary mirror. For \( f=0.01 \) the value is 32.8 mag arcsec\(^{-2}\), which is plotted with the Fraunhofer diffraction results developed below.

2.3 FRAUNHOFER DIFFRACTION

Figure 1 illustrates Babinet's principle: an obstruction has the same diffraction pattern as the complementary screen. This means that the particulate disk scatters light in the same pattern and total amount as if it were a same-sized hole in an otherwise totally black primary mirror coating.
Flammer (J.App. Phys. 24, pp. 1224-1231, 1953) shows the natural extension for $ka \to 0$ through the Mie and Rayleigh scattering regimes, but we treat here only the Fraunhofer case for which $ka \gg 1$, resulting in the Airy diffraction pattern of a telescope of radius $a$. The equivalent astronomical intensity is

$$I_F(\theta; \lambda, a) = \pi F \frac{\pi a^2}{\lambda^2} f \left[ \frac{2 J_1(x)}{x} \right]^2,$$

where $J_1$ = first order Bessel function and

$$x = \frac{2\pi a}{\lambda}\theta.$$

Figures 2 and 3 visualize the angular dependence of $I_F$, an enmeshed function of particle size and wavelength.

Randomly placed particles of the same size additively combine intensity. Let $f(a)$ be the continuous density function giving the fractional area covered per unit $a$, then the total intensity is

$$I_F(\theta; \lambda) = \pi F \frac{\pi}{\lambda^2} \int_{a_1}^{a_2} f(a) a^2 \left[ \frac{2 J_1(x)}{x} \right]^2 da \quad (3)$$

where the integral extends from $a_1$ = smallest size for which $ka_1 \gg 1$ to $a_2 = 10^3 \mu$, the largest reasonable dust particle size. The Fraunhofer scattered intensity due to a variety of dust distributions (Figure 4) is presented in the following sections.

2.3.1 Power Law Distribution $f(a) = ka^\alpha$

The case $\alpha = -1$ is analytic and has special interest because (1) it has a possible connection to reality through the MIL-STD-1246A specification system (illustrated in Figure 4) and (2) the associated $\theta^{-2}$ scattering angular dependence is observed in ground-based telescopes (e.g. de Vaucouleurs, Ap. J. 128, pp. 486-488, 1958). Eq. 3 becomes:
\[ I_F(\theta; \lambda) = \pi F \frac{k}{(2\pi)^2} e^{-2} \left( \int_{x_1}^{x_2} \frac{4J_1^2(x)}{x} \, dx \right) \]

where \( k = f/\ln \left( \frac{a_2}{a_1} \right) \) and \( f = \) fractional obscuration due to all particles. Figure 5 shows the result for \( a_1 = 1 \mu, a_2 = 10^3 \mu, \) and \( \lambda = 2000 \) A and 5000 A. Also shown are the (1) mean asymptotic ST aperture diffraction pattern (Section 2.1) and (2) the diffuse scattering for \( f = 0.01 \) (Section 2.2). Note the wavelength independent section varying inversely as angle squared.

Figure 6 shows numerical integrations for the power laws \( \alpha = 0, -0.5, -1.0 \) for \( f = 0.01 \) and \( \lambda = 5000 \) A. The flatter distributions have relatively more large particles, producing greater central brightness.

2.3.2 LMSC Trial Distribution

The discrete size distribution in Table 1 was received 8/20/82 from D. Tenerelli of LMSC (through W. Fastie). Figure 7 shows the associated summation of Eq. (3) (labeled "original"), plus the result of an invented but reasonable extrapolation for \( \alpha = 100-500\mu \) (labeled "extended.") Adding the larger particles dramatically increases the central surface brightness. The "updated" LMSC trial data in Figure 4 is radically lower in surface coverage (\( fC \leq 0.1\% \)), but favors the larger particles. The scattered light from this distribution would resemble Figure 7 but at a lower level.
3.0 IMPACT OF PARTICLES ON ST SCIENCE INVESTIGATIONS

The foregoing work for a nominal 1% particulate contamination by area implies:

1. Particulates extinct $2 \times 1\% = 2\%$ of the incident light.

2. Fraunhofer diffraction generally dominates diffuse backscatter.

3. The dust-scattered light equals the ST primary Airy pattern at a point inside about 10" and dominates outside (sensitive to larger particles).

4. Wide-angle Fraunhofer diffraction (sensitive to smaller particles) is lower for shorter wavelengths.

5. The central brightness of the dust scattering pattern is higher for shorter wavelengths.

The following sections relate these conclusions to ST optical performance.

3.1 NEAR-IN SCIENCE

Here is meant observations in regions of high contrast, such as searches for stellar companions or studies of faint, extensive atmospheres of stars or planets. With 1% particulate contamination, the primary Airy pattern would supply the dominant background brightness inside about 5-10" for most SI's. However the f/288 FOC camera incorporates an apodizing mask on a re-imaged pupil plane specifically to reduce the Airy pattern. For imaging at $\theta > 1"$. The f/288 camera gives highest optical performance at $\lambda 1600\AA$. At that wavelength, the current scattered light...
calculations for 1% contamination indicate there will be no appreciable benefit for $\theta \lesssim 3''$, and it will be only a couple of magnitudes arcsec$^{-2}$ at $\theta=1''$.

This result is sensitive to particles of radius $a \sim 100\mu$, and Figure 8 dramatizes this point: it shows the number density and associated fractional contamination required to match the Airy diffraction pattern at $\theta=5''$ with diffraction from single-sized particles of radius $25\mu < a < 500\mu$.

### 3.2 FAR-OUT SCIENCE

Here is meant fogging of faint object observations due to wider-angle scattering due to both the Fraunhofer and the diffuse, upper-surface mechanisms. By the former mechanism, each stellar image has a halo characterized by Figs. 6-8 but with ordinate values $m + \Delta m (\theta)$, $m$=magnitude of the star. The typical net effect can be computed from a star count ($n_m$=# stars deg$^{-2}$ of magnitude $m$) and Table 2 shows the result using Allen’s mean values ($\S117$) with an average Figure 7 curve. $\theta_m$ is the radius brighter than 23 mag arcsec$^{-2}$ for a star of magnitude $m$. The sky fraction brightened above that level is

$$F = \sum_{m=0}^{\infty} F_m = n_m \pi \theta_m^2 (\text{deg}) = 1.0\%$$

This value varies approximately as fractional coverage to the $2/3$ power.

To find the field brightening due to diffuse scattering with 1% coverage, we use the coincidence that the average starlight falling on the ST mirror is equivalent to ~two $m=0$ stars in the visible:
Solid angle of tube opening = \(4.8 \times 10^{-2} \text{ sr} = 160 \text{ deg}^2\),
Mean starlight = \(119 \text{ m}=10 \text{ stars deg}^{-2}\),
Flux on ST primary = \(1.9 \text{ m}=0 \text{ stars}\),

so the "diffuse" line on Figs. 5-7 reads almost directly in typical surface brightness:

\[ I_L \approx 32.9 \text{ mag arsec}^{-2}, \] which is utterly negligible.

4.0 CURRENT STATUS AND PLANNED TESTS

I attended a meeting of the ST Contamination Control Committee at P-E in Danbury, CT on 8/31/82. I had been invited by J. Olivier, ST Chief Engineer, to present my concerns on particulate contamination, though that committee is not able to respond directly to out-of-channels concerns on ST optical performance.

P-E reviewed its understanding of particulate requirements: (1) that it is not contractually required to meet any hard specification (e.g. MIL-STD-1246A); (2) that it has no procedures for particulate measurements; (3) that it was working toward a "visibly clean" criterion but conceded that meeting it was unlikely and in any case difficult to verify.

P-E reports that after a cumulative exposure of \(-50\) hours to a class \(10^5\) clean room environment, the ST primary is not now visibly clean. This implies greater than \(0.03-0.10\%\) obscuration or \(0.06-0.2\%\) optical extinction. P-E plans to clean-off particulates at the last moment before baffling, and after that there are no planned cleanings or inspections. Baffling occurs about a year before shipment to Lockheed.

P-E was given an action item to document the current particulate contamination of the ST primary mirror.
**Table 1: LMSC Suggested Particle Size Distribution (Tennelli 8/20/82)**

<table>
<thead>
<tr>
<th>a (μ)</th>
<th># ft⁻²</th>
<th>f₁</th>
<th>Δa (μ)</th>
<th>f(a) = f₁/Δa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>10⁴</td>
<td>8.4 x 10⁻⁸</td>
<td>0.5</td>
<td>1.7 x 10⁻⁷</td>
</tr>
<tr>
<td>1.0</td>
<td>10⁵</td>
<td>3.4 x 10⁻⁶</td>
<td>0.75</td>
<td>4.5 x 10⁻⁶</td>
</tr>
<tr>
<td>2.0</td>
<td>10⁵</td>
<td>1.4 x 10⁻⁵</td>
<td>2.0</td>
<td>7.0 x 10⁻⁶</td>
</tr>
<tr>
<td>5.0</td>
<td>10⁵</td>
<td>8.4 x 10⁻⁵</td>
<td>4.0</td>
<td>2.1 x 10⁻⁵</td>
</tr>
<tr>
<td>10.0</td>
<td>5 x 10⁴</td>
<td>1.7 x 10⁻⁴</td>
<td>5.0</td>
<td>3.5 x 10⁻⁵</td>
</tr>
<tr>
<td>15.0</td>
<td>10⁵</td>
<td>7.6 x 10⁻⁴</td>
<td>5.0</td>
<td>1.5 x 10⁻⁴</td>
</tr>
<tr>
<td>20.0</td>
<td>3 x 10⁵</td>
<td>4.1 x 10⁻³</td>
<td>7.5</td>
<td>5.5 x 10⁻⁴</td>
</tr>
<tr>
<td>30.0</td>
<td>10⁵</td>
<td>3.0 x 10⁻³</td>
<td>15.0</td>
<td>2.0 x 10⁻⁴</td>
</tr>
<tr>
<td>50.0</td>
<td>5 x 10⁴</td>
<td>4.2 x 10⁻³</td>
<td>35</td>
<td>1.2 x 10⁻⁵</td>
</tr>
</tbody>
</table>

| Extended |
|---------|--------|----|--------|-------------|
| 100.   | 9.5 x 10³| 3.2 x 10⁻³| 50    | 6.4 x 10⁻⁵  |
| 150.   | 4.2 x 10³| 3.2 x 10⁻³| 50    | 6.4 x 10⁻⁵  |
| 200.   | 2.4 x 10³| 3.2 x 10⁻³| 75    | 4.3 x 10⁻⁵  |
| 300.   | 1.1 x 10³| 3.2 x 10⁻³| 150   | 2.1 x 10⁻⁵  |
| 500.   | 3.8 x 10²| 3.2 x 10⁻³| --    | --           |

1. Integrates to 1.2% coverage, normalized to 1% in scattering calculations.

2. Integrates to 2.8% coverage, normalized to 1% in scattering calculations.

3. Plotted on Figure 4 normalized to 1% coverage.

**ORIGINAL FIGURES OF POOR QUALITY**

10 A
Table 2.

Computation of sky fraction brightened to < 23 mag arsec^{-2} by Fraunhofer diffraction (visible light).

<table>
<thead>
<tr>
<th>m</th>
<th>n_{m} (deg^{-2})</th>
<th>\theta_{m} (deg)</th>
<th>F_{m}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.9 x 10^{-5}</td>
<td>0.28</td>
<td>1.9 x 10^{-5}</td>
</tr>
<tr>
<td>1</td>
<td>3.2 x 10^{-4}</td>
<td>0.22</td>
<td>4.9 x 10^{-5}</td>
</tr>
<tr>
<td>2</td>
<td>1.4 x 10^{-3}</td>
<td>0.18</td>
<td>1.4 x 10^{-4}</td>
</tr>
<tr>
<td>3</td>
<td>4.9 x 10^{-3}</td>
<td>0.14</td>
<td>3.0 x 10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>1.8 x 10^{-2}</td>
<td>0.11</td>
<td>6.8 x 10^{-4}</td>
</tr>
<tr>
<td>5</td>
<td>5.0 x 10^{-2}</td>
<td>0.088</td>
<td>1.2 x 10^{-3}</td>
</tr>
<tr>
<td>6</td>
<td>1.4 x 10^{-1}</td>
<td>0.055</td>
<td>1.3 x 10^{-3}</td>
</tr>
<tr>
<td>7</td>
<td>4.0 x 10^{-1}</td>
<td>0.044</td>
<td>2.4 x 10^{-3}</td>
</tr>
<tr>
<td>8</td>
<td>1.1</td>
<td>0.035</td>
<td>4.2 x 10^{-3}</td>
</tr>
<tr>
<td>9</td>
<td>3.0</td>
<td>0.0</td>
<td>0.0010 = F</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total</td>
</tr>
</tbody>
</table>
Babinet's Principle

1. $U_1(P), \quad I_1(P) = |U_1(P)|^2$

2. $U_2(P), \quad I_2(P) = |U_2(P)|^2$

3. $U_3(P) = 0 = U_1(P) + U_2(P)$
   $\therefore U_1(P) = -U_2(P)$
   $\therefore I_1(P) = I_2(P)$

Figure 1. Illustration of Babinet's Principle. The diffracted amplitudes in direction $P$ must sum to zero for screens 1 and 2, since the screen sum is opaque. This implies the intensities are equal. Thus the diffraction pattern for an obscuring particle is the same as that of a clear hole of the same size and shape.
FRAUNHOFER DIFFRACTION

$$\text{Intensity} = i(\theta, \lambda, a) = (\pi F \cdot \pi a^2) \cdot \frac{1}{\lambda^2} \left[ \frac{2 J_1(x)}{x} \right]^2$$

Intercepted power

$$x = \frac{2 \pi a}{\lambda} \theta$$

Examples

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\theta$ at $x = 3.83$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 \mu$</td>
<td>7500 $\mu$</td>
</tr>
<tr>
<td>$100 \mu$</td>
<td>750 $\mu$</td>
</tr>
<tr>
<td>$1 \text{ mm}$</td>
<td>75 $\mu$</td>
</tr>
<tr>
<td>$1 \text{ cm}$</td>
<td>7.5 $\mu$</td>
</tr>
<tr>
<td>$1 \text{ m}$</td>
<td>.075 $\mu$</td>
</tr>
</tbody>
</table>

Figure 2. Concerning the diffraction pattern of small circular apertures.
Figure 3. The base plane shows contours of the Airy function argument $x = \frac{2\pi a \theta}{\lambda}$ for $\lambda = 5000\text{A}$ and $a$ and $\theta$ spanning the range relevant for dust scattering on ST. The vertical dimension shows a slice of the Airy function for $\theta$ numerically equal to $a$. 
Figure 4. Various particle size distributions discussed in text.
Inverse first power law (analytic solution)

\[ f(a)da = k a^{-1}da \]
\[ a_{\text{min}} = 1 \mu \]
\[ a_{\text{max}} = 10^3 \mu \]

1% area covered

ST Primary Airy Profile

Wavelength independent

\[ -2.5 \log \left[ \frac{I_F \Omega_1''}{\pi F} \right] \]

Diffuse (1%)

\[ \sim e^{-3} \]

\[ \sim e^{-2} \]

Figure 5. Fraunhofer diffraction surface brightness vs. distance from a zeroth magnitude star for the ST primary 1% covered by dust with an inverse first power law size distribution \(1 \mu \leq \alpha \leq 10^3 \mu\). The solution is analytic. The average ST primary Airy profile and the brightness due to scattering from Lambertian upper particle surfaces are shown for comparison.
Figure 6. Fraunhofer diffraction surface brightness distributions due to 1% coverage by dust with a variety of power-law size distributions in the range 1u_\text{a} \leq 10u_\text{a}. These are numerical integrations of Eq. 5.
Figure 7. Fraunhofer diffraction surface brightness distributions due to 1% ST dust coverage with the discrete size distributions in Table 1 and graphed in the upper right corner. These are numerical summations equivalent to the integral in Eq. 5.
Figure 8. Single-sized particles required to scatter light equal to the Airy diffraction pattern of the ST primary mirror at $\delta=5''$. Lower curves reference the right-hand scale, and the upper read to the left.