Optimum Parallel Step-Sector Bearing
Lubricated With an Incompressible Fluid

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The dimensionless parameters normally associated with a step sector thrust bearing are the film thickness ratio, the dimensionless step location, the number of sectors, the radius ratio, and the angular extent of the lubrication feed groove. The optimum number of sectors and the parallel step configuration for a step sector thrust bearing while considering load capacity or stiffness and assuming an incompressible fluid are presented.
lord Rayleigh (1918) determined the slider bearing profile that yielded a maximum load-carrying capacity per unit width for a given minimum film thickness and bearing length. The answer, revealed by an elegant application of calculus of variations, turned out to be the stepped parallel-surface bearing. Lord Rayleigh's analysis neglected side leakage and assumed an incompressible fluid. Archibald (1950) included side leakage effects while considering a rectangular and sector slider bearings lubricated with an incompressible fluid. His solution made use of Fourier series in providing one of the rare analytical solutions to lubrication problems when side leakage is considered. Hamrock (1972) used a linearized analysis in considering a finite-width rectangular step thrust bearing lubricated with a compressible fluid. The compressibility effects introduced an additional parameter, namely the compressibility number $\Lambda$. The optimum rectangular step bearing configuration for maximum load and stiffness was presented.

The present technical note will determine the optimum number of sectors and the parallel step configuration for a step-sector thrust bearing while considering load capacity or stiffness and assuming an incompressible fluid.

Figure 1 gives the configuration of the step-sector thrust bearing. The parameters to be used to define the dimensionless load and stiffness are:

1. $\bar{h}_i = h_i/h_0$, film thickness ratio
2. $n = e_i/(e_i + e_o)$, dimensionless step location
3. $N$, number of sectors
4. $\alpha = R_2/R_1$, radius ratio
5. $\theta_g$, angular extent of lubrication feed groove, rad

Note that the first four parameters are dimensionless and the fifth is above and dimensional and is expressed in radians.

The dimensionless load capacity in terms of the parameters listed according to the technique developed by Archibald (1950) is

$$p_z = \frac{p_z h_0^2}{n \omega R_1^2 R_2^2} = \sum_{m=1}^{\infty} \frac{F_m (1n \alpha)^2 [1 + e^{-2(-1)^m+1}]}{(4 + m^2 \pi^2)} \times \left\{ \frac{1 - \cosh(n \beta)}{\sinh(n \beta)} + \frac{1 - \cosh[(1 - n) \beta]}{\sinh[(1 - n) \beta]} \right\}$$

(1)

where

$$\beta = \frac{m \pi}{1n \alpha} \left( \frac{2n \pi - \theta_g}{N} \right)$$

(2)
The dimensionless load expressed in equation (1) can be written in dimensional terms as

\[ P_z = n \omega (R_1 R_2)^2 \sum_{m=1}^{\infty} \frac{\sum_{m=1}^{\infty} \left\{ h_0^3 \coth[(1 - n)\beta] + (h_o + h_s)^3 \coth(n\beta) \right\}}{m^\pi \left\{ \frac{1 - \cosh(n\beta)}{\sinh(n\beta)} + \frac{1 - \cosh[(1 - n)\beta]}{\sinh[(1 - n)\beta]} \right\}} \] (5)

where

\[ A = \frac{6a^2 K_m (\ln \alpha)^3[1 + e^{-2(-1)^{m+1}}]}{m^\pi(4 + m^2 \pi^2)} \left\{ \frac{1 - \cosh(n\beta)}{\sinh(n\beta)} + \frac{1 - \cosh[(1 - n)\beta]}{\sinh[(1 - n)\beta]} \right\} \] (6)

The stiffness can be expressed as \[ K = \frac{a P_z}{a h_0}. \] Making use of equation (5), we can write the dimensionless stiffness as

\[ K_h = \frac{K h_0^3}{n \omega (R_1 R_2)^2} = \sum_{m=1}^{\infty} \frac{-3A(h_i - 1) \left\{ \coth[(1 - n)\beta] + (h_i)^2 \coth(n\beta) \right\}}{\coth[(1 - n)\beta] + (h_i)^3 \coth(n\beta)}^2 \] (7)

Maximum Load Capacity

For radius ratios in the range 0.05 < \alpha < 0.95 and for angular extents of the lubrication feed groove equal to \pi/90, \pi/45, \pi/30, and 2\pi/45 the optimum values of the dimensionless step location parameter \( n \) and film thickness ratio \( h_i \) for maximum load are

\( (n)_{\text{opt}} = 0.558 \) and \( (h_i)_{\text{opt}} = 1.668 \) (8)

The optimum number of sectors to be placed into the thrust bearing for the range of \alpha and \theta_g given earlier can be expressed as

\[ (N)_{\text{opt}} = \frac{2\pi}{\theta_g + 2.24(1 - \alpha)} \] (9)
where \((N)_{opt}\) is rounded off to the nearest integer.

Figure 2 was obtained by using equation (9) to show the effect of radius ratio \(\alpha\) on the optimum number of sectors for angular lubrication feed groove extents of \(\pi/90\) and \(2\pi/45\). It is observed that, as the radius ratio increases, the difference in the optimum number of sectors increases for the two angular feed groove extents shown.

**Maximum Stiffness**

When maximum stiffness is considered, the only parameter that changes from the maximum-load results is the film thickness ratio. For maximum stiffness the optimum film thickness is

\[
(h_i)_{opt} = 1.467
\]  

(10)

This implies that, when stiffness is to maximized, the step height \(h_s\) is reduced by 12 percent from the optimum value obtained from the maximum-load results. The optimum step location \((n)_{opt}\) remains the same at 0.558, and the optimum number of sectors is obtained from equation (9).

**References**


Figure 1. - Configuration of step-sector thrust bearing.
Figure 2. - Effect of radius ratio $\alpha$ on optimum numbers of sectors for angular feed grooves of $\pi/90$ and $2\pi/45$. 

The graph shows the relationship between the radius ratio $\alpha = R_2/R_1$ and the number of sectors $N$, with lines representing different values of $\theta_g = \pi/90$ and $\theta_g = 2\pi/45$. 

- $\theta_g = \pi/90$: Dot-dashed line
- $\theta_g = 2\pi/45$: Solid line
**Abstract**

The dimensionless parameters normally associated with a step-sector thrust bearing are the film thickness ratio, the dimensionless step location, the number of sectors, the radius ratio, and the angular extent of the lubrication feed groove. The present paper will determine the optimum number of sectors and the parallel step configuration for a step-sector thrust bearing while considering load capacity or stiffness and assuming an incompressible fluid.

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