Stirling Engine Design Manual

Second Edition

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1. **Summary**

The DOE Office of Conservation, Division of Transportation Energy Conservation, has established a number of broad programs aimed at reducing highway vehicle fuel consumption. The DOE Stirling Engine Highway Vehicle Systems Program is one such program. This program is directed at the development of the Stirling engine as a possible alternative to the spark-ignition engine.

Project Management responsibility for this project has been delegated by DOE to the NASA-Lewis Research Center. Support for the generation of this report was provided by a grant from the Lewis Research Center Stirling Engine Project Office.

For Stirling engines to enjoy widespread application and acceptance, not only must the fundamental operation of such engines be widely understood, but the requisite analytic tools for the simulation, design, evaluation and optimization of Stirling engine hardware must be readily available.

The purpose of this design manual is to provide an introduction to Stirling cycle heat engines, to organize and identify the available Stirling engine literature, and to identify, organize, evaluate and, in so far as possible, compare non-proprietary Stirling engine design methodologies. As such, the manual then represents another step in the long process of making available comprehensive, well verified, economic-to-use, Stirling engine analytic programs.

Two different fully described Stirling engines are presented. These not only have full engine dimensions and operating conditions but also have power outputs and efficiencies for a range of operating conditions. The results of these two engine tests can be used for evaluation of non-proprietary computation procedures.

Evaluation of partially described Stirling engines begins to reveal that some of the early but modern air engines have an interesting combination of simplicity and efficiency. These show more attractive possibilities in today's world of uncertain fuel oil supply than they did 20 years ago when they were developed.

The theory of Stirling engine is presented starting from simple cycle analysis. Important conclusions from cycle analysis are: 1) compared to an engine with zero unswept gas volume (dead volume), the power available from an engine with dead volume is reduced proportional to the ratio of the dead volume to the maximum gas volume, and 2) the more realistic adiabatic spaces can result in as much as a 40% reduction in power over the idealized isothermal spaces.

Engine design methods are organized as first order, second order and third order with increased order number indicating increased complexity.

First order design methods are principally useful in preliminary systems studies to evaluate how well-optimized engines may perform in a given heat engine application.

Second order design methods start with a cycle analysis and incorporate engine loss relationships that apply generally for the full engine cycle. This method assumes that the different processes going on in the engine interact very little.
A FORTRAN program is presented for both an isothermal second-order design program and an adiabatic second-order design program. Both of these are adapted to a modern four-piston Siemens type of heat engine.

Third-order methods are explained and enumerated. This method solves the equations expressing the conservation of energy, mass and momentum using numerical methods. The engine is divided into many nodes and short time steps are required for a stable solution. Both second- and third-order methods must be validated by agreement with measurement of the performance of an actual engine.

In this second edition of the Stirling Engine Design Manual the references have been brought up-to-date. There is a continual rapid acceleration of interest in Stirling engines as evidenced by the number of papers on the subject. A revised personal and corporate author index is also presented to aid in locating a particular reference. An expanded directory lists over 80 individuals and companies active in Stirling engines and details what each company does within the limits of the contributed information. About 800 people are active in Stirling engine development worldwide.
2. INTRODUCTION

2.1 Why Stirling?

Development of Stirling engines is proceeding world-wide in spite of their admittedly higher cost because of their high efficiency, particularly at part load, their ability to use any source of heat, their quiet operation, their long life and their non-polluting character.

For many years during the last century, Stirling engines occupied a relatively unimportant role among the kinds of engines used during that period. They were generally called air engines and were characterized by high reliability and safety, but low specific power. They lost out in the dollars-per-horsepower race with other competing machines. In the 1930's some researchers employed by the Philips Company, in Holland, recognized some possibilities in this old engine, provided modern engineering techniques could be applied. Since then, this company has invested millions of dollars and has created a very commanding position in Stirling engine technology. Their developments have led to smooth and quiet-running demonstration engines which have very high efficiency and can use any source of heat. They may be used for vehicle propulsion to produce a zero or low level of pollution. A great variety of experimental Stirling engines have been built from the same general principles to directly pump blood, generate electricity, or directly generate hydraulic power. Many are used as heat pumps and some can be used as both heat pumps and heat engines depending upon the adjustment. With a few notable exceptions of independent individuals who have done very good work, most of the work on Stirling engines has been done by teams of engineers funded by the giant companies of the world. The vital details of this work are generally not available. The United States government is beginning to sponsor the development of an open technology on Stirling engines and is beginning to spend large sums of money in this area. As part of this open technology, this design manual is offered to review all the design methods available in the open literature.

Consider the following developments which show that interest in Stirling engines is growing not just as a popular subject for research, but as a product that can be sold at a profit.

United Stirling of Sweden is committed to quantity production of their P-75, 75 kw truck engine.

Mechanical Technology, Inc., United Stirling and American Motors have teamed up to develop and evaluate Stirling engines for automobiles. The sponsor is the U.S. Department of Energy, via NASA-Lewis, at 4 million dollars per year.

- The Harwell thermo-mechanical generator, a type of super-reliable Stirling with three times the efficiency of thermo-electric generators has now operated continuously for four years.

- A Japanese government-industry team is designing and building a 800 hp marine engine. Funding is 5 million dollars for 5 years. A 10 kw and a 50 kw engine of reasonable performance have been built independently by Japanese firms.
Work has started by three organizations using the talents of long time Dutch, Swedish and German Stirling engine developers to design and eventually build a 500 to 2000 horsepower coal-fired Stirling engine for neighborhood heat and power generation.

Stirling Power Systems has equipped eight Winnebago motor homes with an almost silent and very reliable total energy system based upon a 6.5 kw Stirling engine generator. These systems are now ready for manufacture and sale.

Solar Engines of Phoenix, Arizona, have sold 20,000 model Stirling engines.

Sunpower of Athens, Ohio, has demonstrated an atmospheric air engine that produces 850 watts instead of 50 watts for an antique machine.

2.2 What Is A Stirling Engine?

Like any heat engine, the Stirling engine goes through the four basic processes of compression, heating, expansion, and cooling (See Figure 2-1). A couple of examples from everyday life may make this clearer. For instance, Figure 2-2 shows how an automobile internal combustion engine works. In this engine a gas-air mixture is compressed using work stored in the mechanical flywheel from a previous cycle. Then the gas mixture is heated by igniting it and allowing it to burn. The higher pressure gas mixture now is expanded which does more work than was required for the compression and results in net work output. In this particular engine, the gas mixture is cooled very little. Nevertheless, the exhaust is discarded and a cool gas mixture is brought in through the carburetor.

![Figure 2-1. Common Process for all Heat Engines.](image)
Figure 2-2. Example of Internal Combustion Engine. Figure 2-3. Example of Closed Cycle Gas Turbine Engine.
Another example of the general process shown in Figure 2-1 is the closed cycle gas turbine engine (See Figure 2-3). The working gas is compressed, then it passes through a steady-flow regenerative heat exchanger to exchange heat with the hot expanded gases. More heat is added in the gas heater. The hot compressed gas is expanded which generates more energy than is required by the compressor and creates net work. To complete the cycle, the expanded gas is cooled first by the steady flow regenerative heat exchanger and then the additional cooling to the heat sink.

In the first example (Figure 2-2), the processes occur essentially in one place, one after the other in time. In the second example (Figure 2-3), these four processes all occur simultaneously in different parts of the machine. In the Stirling machine, the processes occur sequentially but partially overlapping in time. Also the processes occur in different parts of the machine but the boundaries are blurred. One of the problems which has delayed the realization of the potential of this kind of thermal machine is the difficulty in calculating with any real degree of confidence the complex processes which go on inside of a practical Stirling engine. The author has the assignment to present as much help on this subject as is presently freely available.

A heat engine is a Stirling engine for the purpose of this book when:

1. The working fluid is contained in one body at nearly a common pressure at each instant during the cycle.
2. The working fluid is manipulated so that it is generally compressed in the colder portion of the engine and expanded generally in the hot portion of the engine.
3. Transfer of the compressed fluid from the cold to the hot portion of the engine is done by manipulating the fluid boundaries without valves or real pumps. Transfer of the expanded hot fluid back to the cold portion of the engine is done the same way.
4. A reversing flow regenerator (regenerative heat exchanger) may be used to increase efficiency.

The general process shown in Figure 2-1 converts heat into mechanical energy. The reverse of this process can take place in which mechanical energy is converted into heat pumping. The Stirling engine is potentially a better cycle than other cycles because it has the potential for higher efficiency, low noise and no pollution.

Figure 2-4 shows a generalized Stirling engine machine as described above. That is, a hot and a cold gas space is connected by a gas heater and cooler and regenerator. As the process proceeds to produce power, the working fluid is compressed in the cold space, transferred as a compressed fluid into the hot space where it is expanded again, and then transferred back again to the cold space. Net work is generated during each cycle equal to the area of the enclosed curve.
In this publication the author would like to consider the classification of Stirling engines from a more basic standpoint. Figure 2-5 shows the various design areas that must be addressed before a particular kind of Stirling engine emerges. First some type of external heat source must be determined. Heat must then be transferred through a solid into a working fluid. There must be a means of cycling this fluid between the hot and cold portion of the engine and of compressing and expanding it. A regenerator is needed to improve efficiency. Power control is obviously needed as are seals to separate the working gas from the environment. Expansion and compression of the gas creates net indicated power which must be transformed by some type of linkage to create useful power. Also the waste heat from the engine must be rejected to a suitable sink.
A wide variety of Stirling engines have been manufactured. These old engines are described very well by Finkelstein (59 c) and Walker (73 j, 78 dc). Usually these involve three basic types of Stirling engines. One, the alpha type, uses two pistons (See Figure 2-4 and 2-6). These pistons mutually compress the working gas in the cold space, move it to the hot space where it is expanded and then move it back. There is a regenerator and a heater and cooler in series with the hot and cold gas spaces. The other two arrangements use a piston and displacer. The piston does the compressing and expanding, and the displacer does the gas transfer from hot to cold space. The displacer arrangement with the displacer and the power piston in line is called the beta-arrangement, and the piston offset from the displacer, to allow a simpler mechanical arrangement, is called the gamma-arrangement. However, all large size Stirling engines being considered for automotive applications employ what is variously called the Siemens, Rinia or double-acting arrangement. (See Figure 2-7.) As explained by Professor Walker (90 d, p. 109), Sir William Siemens is credited with the invention by Babcock (1885 a). (See Figure 2-8.) However, Sir William's engine concept was never reduced to practice. About 80 years later in 1949, van Weenan of the Philips company re-invented the arrangement complete with wobble plate drive. Because of the way the invention was reported in the literature, H. Rinia's name was attached to it by Walker (78 j).

Note in Figure 2-8 there are 4 pistons attached to a wobble plate which pivots at the center and is made to undergo a nutating motion by a lever attached to a crank and flywheel. This is only one way of getting these 4 pistons to undergo simple harmonic motion. Figure 2-7 shows these same 4 cylinders laid out. Note that the top of one cylinder is connected to the bottom of the next
by a heater, regenerator and cooler, as in the alpha-type of Figure 2-6. In the Siemens arrangement there are 4 alpha-arrangement working spaces with each piston double-acting, thus the name. This arrangement has fewer parts than any of the others and is, therefore, favored for larger automotive scale machines. Figure 2-9 shows an implementation of the Siemens arrangement used by United Stirling. United Stirling places 4 cylinders parallel to each other in a square. The heater tubes are in a ring fired by one burner. The regenerators and coolers are in between but outside the cylinders. Two pistons are driven by one crank shaft and two pistons are given by the other. These two crank shafts are geared to a single drive shaft. One end of the drive shaft is used for auxiliaries and one for the main output power.

**Figure 2-6. Main Types of Stirling Engine Arrangements.**

- **ALPHA-TYPE**
  - H = HEATER
  - R = REGENERATOR
  - C = COOLER
  - 1 = EXPANSION SPACE
  - 2 = COMPRESSION SPACE

- **BETA-TYPE**
  - H = HEATER
  - R = REGENERATOR
  - C = COOLER

- **GAMMA-TYPE**

**Figure 2-7. A Rinia, Siemens or Double-Acting Arrangement.**
2.4 Overview of Report

The chief aim of this design manual is to teach people how to design Stirling engines, particularly those aspects that are unique to Stirling engines. To this end in Section 3, two engines have performance data and all pertinent dimensions given (fully described). In Section 4 automotive scale engines, for which only some information is available, are presented. Section 5 is the heart of the report. All design methods are reviewed. A full list of references on Stirling engines to April 1980 is given in Section 7. Sections 8 and 9 are personal and corporate author indices to the references which are arranged according to year of publication. Section 10 is a directory of people and companies active in Stirling engines.

Appendix A gives all the property values for the materials most commonly used in Stirling engine design. The units employed are international units because of the worldwide character of Stirling engine development. Appendix B gives the nomenclature for the body of the report. The nomenclature was changed from the first edition to fit almost all computers. Appendices C, D and E contain three original computer programs. Appendix F presents a discussion of non-automotive present and future applications of Stirling engines.
Figure 2-9. Concept for United Stirling Production Engines.
3. FULLY DESCRIBED STIRLING ENGINES

Definition of Term "Fully Described"

Fully described does not mean that there is a complete set of prints and assembly instruction in hand so that an engine can be built just from this information. However, it is a lot more than is usually available which is power output and efficiency at a particular speed. Sometimes the displacement of the power piston and the operating pressure and the gas used in the engine are also given. What is meant by "Fully Described" is that enough is revealed so that the dimensions and operating conditions that the calculation procedure needs for input can be supplied. Also required is at least the reliably measured power output and efficiency for a number of points. If experimental measurements are not available, then calculated power output and efficiency are acceptable if they are done by an experimentally validated method. It is not necessary that this method be available for examination.

Two engines are presently well enough known in the open literature and of general interest to be "fully described." These are:

1) The General Motors GPU-3
2) The General Motors 4L23

All the necessary information for each engine will now be given.

3.1 The GPU-3 Engine

General Motors Research Corporation built the Ground Power Unit #3 (GPU-3) as a culmination of a program lasting from 1960 to 1966 with the U.S. Army. Although the program met its goals, quantity production was not authorized. Two of the last model GPU-3's were preserved and have now been tested by NASA-Lewis. One of the GPU-3's as delivered to the Army is shown in Figure 3-1.

3.1.1 Engine Dimensions

Figure 3-2 shows a cross section of the entire engine showing how the parts all fit together. The measurements for this engine (78 ad, pages 45-51; 78 o) have been superceded by later information (79 a). The following tables and figures are from this latter source. Table 3-1 gives the GPU-3 engine dimensions that are needed to input the computer program. Since dead volume is not only in the heater and cooler tubes and in the regenerator matrix, but is also in many odd places throughout the engine, the engine was very carefully measured and the dead volumes added up (see Table 3-2.) The total volume inside the engine was also measured accurately by the volume displacement method. By this method Table 3-2 shows an internal volume of 236 cc. Measurements accounted for 232.3 cc. In addition to the information given in Table 3-1 and 3-2, more information is needed to calculate heat conduction. This is given in Figure 3-3.
Figure 3-1. The General Motors GPU-3-2 Stirling Electric Ground Power Unit for Near Silent Operation (ref. 68 p.) Picture courtesy General Motors research.

Figure 3-4 defines the geometric relationship between piston position and crankshaft angle, which occurs in a rhombic drive machine.

3.1.2 Engine Performance

Besides engine dimensions, a fully described engine has information available on engine performance. The original performance data was obtained from NASA-Lewis by private communication (78 q) to meet the operating point published in the first edition (78 ad, page 47.) Table 3-3 shows the measured performance for these eight points. In addition, NASA-Lewis did some additional tests which were compared with the NASA-Lewis computation method. Tabular
Table 3-1 GPU-3-2 Engine Dimensions and Parameters (79a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder bore at liner, cm (in.)</td>
<td>6.99 (2.751)</td>
</tr>
<tr>
<td>Cylinder bore above liner,* cm (in.)</td>
<td>7.01 (2.76)</td>
</tr>
<tr>
<td>Cooler</td>
<td></td>
</tr>
<tr>
<td>Tube length, cm (in.)</td>
<td>4.61 (1.813)</td>
</tr>
<tr>
<td>Heat transfer length, cm (in.)</td>
<td>3.55 (1.399)</td>
</tr>
<tr>
<td>Tube inside diameter, cm (in.)</td>
<td>0.108 (0.0425)</td>
</tr>
<tr>
<td>Tube outside diameter, cm (in.)</td>
<td>0.159 (0.0625)</td>
</tr>
<tr>
<td>Number of tubes per cylinder (or number of tubes per regenerator)</td>
<td>312 (39)</td>
</tr>
<tr>
<td>Heater</td>
<td></td>
</tr>
<tr>
<td>Mean tube length, cm (in.)</td>
<td>24.53 (9.655)</td>
</tr>
<tr>
<td>Heat transfer length, cm (in.)</td>
<td>15.56 (6.13)</td>
</tr>
<tr>
<td>Cylinder tube, cm (in.)</td>
<td>11.44 (4.503)</td>
</tr>
<tr>
<td>Regenerator tube, cm (in.)</td>
<td>12.89 (5.075)</td>
</tr>
<tr>
<td>Tube inside diameter, cm (in.)</td>
<td>0.302 (0.119)</td>
</tr>
<tr>
<td>Tube outside diameter, cm (in.)</td>
<td>0.483 (0.19)</td>
</tr>
<tr>
<td>Number of tubes per cylinder (or number of tubes per regenerator)</td>
<td>40 (5)</td>
</tr>
<tr>
<td>Cold end connecting ducts</td>
<td></td>
</tr>
<tr>
<td>Length, cm (in.)</td>
<td>1.59 (0.625)</td>
</tr>
<tr>
<td>Duct inside diameter, cm (in.)</td>
<td>0.397 (0.153)</td>
</tr>
<tr>
<td>Number of ducts per cylinder</td>
<td>8</td>
</tr>
<tr>
<td>Cooler end cap, cm (in.)</td>
<td>0.279 (0.0107)</td>
</tr>
<tr>
<td>Regenerators</td>
<td></td>
</tr>
<tr>
<td>Length (inside), cm (in.)</td>
<td>2.26 (0.89)</td>
</tr>
<tr>
<td>Diameter (inside), cm (in.)</td>
<td>2.26 (0.89)</td>
</tr>
<tr>
<td>Number per cylinder</td>
<td>8</td>
</tr>
<tr>
<td>Material</td>
<td>Stainless steel wire cloth</td>
</tr>
<tr>
<td>Number of wires, per cm (per in.)</td>
<td>79 x 79 (200 x 200)</td>
</tr>
<tr>
<td>Wire diameter, cm (in.)</td>
<td>0.004 (0.0016)</td>
</tr>
<tr>
<td>Number of layers</td>
<td>308</td>
</tr>
<tr>
<td>Fillet factor, percent</td>
<td>0.303</td>
</tr>
<tr>
<td>Angle of rotation between adjacent screens, deg</td>
<td>5</td>
</tr>
<tr>
<td>Drive</td>
<td></td>
</tr>
<tr>
<td>Connecting rod length, cm (in.)</td>
<td>4.60 (1.810)</td>
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<tr>
<td>Crank - radius, cm (in.)</td>
<td>1.38 (0.543)</td>
</tr>
<tr>
<td>Eccentricity, cm (in.)</td>
<td>2.08 (0.820)</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
</tr>
<tr>
<td>Displacer rod diameter, cm (in.)</td>
<td>0.952 (0.375)</td>
</tr>
<tr>
<td>Piston rod diameter, cm (in.)</td>
<td>3.22 (1.27)</td>
</tr>
<tr>
<td>Displacer wall thickness, cm (in.)</td>
<td>6.96 (2.740)</td>
</tr>
<tr>
<td>Displacer wall thickness, cm (in.)</td>
<td>0.159 (0.0625)</td>
</tr>
<tr>
<td>Displacer stroke, cm (in.)</td>
<td>3.12 (1.27)</td>
</tr>
<tr>
<td>Expansion space clearance, cm (in.)</td>
<td>0.163 (0.064)</td>
</tr>
<tr>
<td>Compression space clearance, cm (in.)</td>
<td>0.030 (0.012)</td>
</tr>
<tr>
<td>Buffer space maximum volume, cm³ (in³)</td>
<td>521 (21.78)</td>
</tr>
<tr>
<td>Total working space minimum volume, cm³ (in³)</td>
<td>233.5 (14.25)</td>
</tr>
</tbody>
</table>

*Top of displacer seal is at top of liner at displacer TDC.

Table 3-2 GPU-3 Stirling Engine Dead Volumes (79a)

Volumes are given in cu cm (cu in.)

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
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<tbody>
<tr>
<td>I. Expansion space clearance volume</td>
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</tr>
<tr>
<td>Displacer clearance (around displacer)</td>
<td>3.34 (0.206)</td>
</tr>
<tr>
<td>Clearance volume above displacer</td>
<td>7.41 (0.452)</td>
</tr>
<tr>
<td>Volume from end of heater tubes into cylinder</td>
<td>1.76 (0.106)</td>
</tr>
<tr>
<td>Total</td>
<td>12.5 (0.762)</td>
</tr>
<tr>
<td>II. Heater dead volume</td>
<td></td>
</tr>
<tr>
<td>Insulated portion of heater tubes next to expansion</td>
<td></td>
</tr>
<tr>
<td>space</td>
<td></td>
</tr>
<tr>
<td>Heated portion of heater tubes</td>
<td>47.46 (2.896)</td>
</tr>
<tr>
<td>Insulated portion of heater tubes next to regenerator</td>
<td>13.29 (0.813)</td>
</tr>
<tr>
<td>Additional volume in four heater tubes used for</td>
<td></td>
</tr>
<tr>
<td>instrumentation</td>
<td></td>
</tr>
<tr>
<td>Volume in header</td>
<td>2.74 (0.167)</td>
</tr>
<tr>
<td>Total</td>
<td>80.8 (4.933)</td>
</tr>
<tr>
<td>III. Regenerator dead volume</td>
<td></td>
</tr>
<tr>
<td>Entrance volume into regenerators</td>
<td>7.36 (0.449)</td>
</tr>
<tr>
<td>Volume within matrix and retaining disks</td>
<td>53.4 (3.258)</td>
</tr>
<tr>
<td>Volume between regenerators and coolers</td>
<td>2.59 (0.158)</td>
</tr>
<tr>
<td>Volume in snap ring grooves at end of coolers</td>
<td>2.18 (0.133)</td>
</tr>
<tr>
<td>Total</td>
<td>85.5 (5.398)</td>
</tr>
<tr>
<td>IV. Cooler dead volume</td>
<td></td>
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<tr>
<td>Volume in cooler tubes</td>
<td>13.13 (0.801)</td>
</tr>
<tr>
<td>V. Compression in space clearance volume</td>
<td></td>
</tr>
<tr>
<td>Exit volume from cooler</td>
<td>3.92 (0.239)</td>
</tr>
<tr>
<td>Volume in cooler end caps</td>
<td>2.77 (0.165)</td>
</tr>
<tr>
<td>Volume in cold end connecting ducts</td>
<td>3.56 (0.217)</td>
</tr>
<tr>
<td>Power piston clearance (around power piston)</td>
<td>7.39 (0.465)</td>
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<tr>
<td>Clearance volume between displacer and power piston</td>
<td>1.14 (0.070)</td>
</tr>
<tr>
<td>Volume at connections to cooler end caps</td>
<td>2.33 (0.142)</td>
</tr>
<tr>
<td>Volume in piston &quot;notches&quot;</td>
<td>0.06 (0.006)</td>
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<td>Volume around rod in bottom of displacer</td>
<td>0.11 (0.007)</td>
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<tr>
<td>Total</td>
<td>21.18 (1.293)</td>
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<tr>
<td>Total dead volume</td>
<td>193.15 (11.787)</td>
</tr>
<tr>
<td>Minimum live volume</td>
<td>39.18 (2.739)</td>
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<td>Calculated minimum total working space volume</td>
<td>232.3 (14.178)</td>
</tr>
<tr>
<td>Measured value of minimum total working space volume</td>
<td>232.5 (14.25)</td>
</tr>
<tr>
<td>(by volume displacement)</td>
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<tr>
<td>Change in working space volume due to minor engine</td>
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</tr>
<tr>
<td>modification</td>
<td>2.5 (0.15)</td>
</tr>
<tr>
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<td>-36.0 (1.640)</td>
</tr>
</tbody>
</table>
Figure 3-3. Schematic Showing Dimensions of GPU-3 Needed for Calculating Heat Conduction. (Regenerator, housing, cylinder, and displacer are 310 stainless steel. Dimensions are in cm (in.).)

Information as in Table 3-3 has not been released. Tables 3-4 to 3-8 give approximate and incomplete information by reading the graphs (79 a). If brake efficiency is given, it is not calculated by dividing the brake power by the heat input, but is determined by reading a separate graph. Since this work was done, a complete test report was published (79 b1) which includes 7 microfiche sheets of all the test data. The reader is referred to this report (79 b1) for more exact information.

NASA-Lewis also determined mechanical losses due to seal and bearing friction and similar effects. Figure 3-4 shows these losses for hydrogen working gas and Figure 3-6 shows the same losses for helium.

Percival (74 bc) gives two sets of curves for the power output and efficiency for the "best" GPU-3 engine tested in late 1969 (see Figures 3-7 and 3-8).
Figure 3-4 Schematic Showing Geometric Relations Between Piston Positions and Crankshaft Angle

Table 3-3 Measured Performance of the GPU-3 Engine Under Test at NASA-Lewis

<table>
<thead>
<tr>
<th>Measurements</th>
<th>H2</th>
<th>H2</th>
<th>H2</th>
<th>H2</th>
<th>He</th>
<th>He</th>
<th>He</th>
<th>He</th>
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<tr>
<td>Engine Speed, Hz</td>
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<td>Cooling Water flow, g/sec.</td>
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<td>134</td>
<td>141</td>
<td>143</td>
<td>124</td>
<td>122</td>
<td>126</td>
<td>124</td>
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<td>8.2</td>
<td>9.6</td>
<td>19.3</td>
<td>9.6</td>
<td>11.9</td>
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<td>281.1</td>
<td>281.1</td>
<td>281.1</td>
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<td>4.274</td>
<td>4.260</td>
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<td>1291</td>
<td>1560</td>
<td>1715</td>
<td>2314</td>
<td>1833</td>
<td>1408</td>
<td>1208</td>
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<tr>
<td>Average Temperatures, K</td>
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<td></td>
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<td></td>
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<tr>
<td>Heater tube*</td>
<td>990.7</td>
<td>990.8</td>
<td>1008.9</td>
<td>1020.0</td>
<td>1058.3</td>
<td>1023.9</td>
<td>1026.7</td>
<td>1007.8</td>
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<td>Expansion Space wall</td>
<td>876.1</td>
<td>883.9</td>
<td>905.6</td>
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<td>886.1</td>
<td>911.1</td>
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<td>Gas between heater and exp. space</td>
<td>891.7</td>
<td>897.8</td>
<td>917.8</td>
<td>931.6</td>
<td>950.6</td>
<td>912.8</td>
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<td>Gas midway thru heater</td>
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<td>Gas between cooler and compression space</td>
<td>320.6</td>
<td>315.6</td>
<td>311.3</td>
<td>316.7</td>
<td>378.3</td>
<td>348.9</td>
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<td>335.6</td>
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<td>Brake Efficiency T</td>
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<td>24.7</td>
<td>25.1</td>
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<td>18.8</td>
<td>15.9</td>
<td>18.3</td>
<td>20.7</td>
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*used in CALCULATIONS.
Table 3-4 Measurements of GPU-3 Engine Performance
by NASA-Lewis - Part I (79a)
Hydrogen Gas, 704C (1300F) Heater Gas Temperature,
15C (59F) Inlet Cooling Water Temperature

<table>
<thead>
<tr>
<th>Pt</th>
<th>Mean Press MPa</th>
<th>Engine SP HZ</th>
<th>Ind. Power KW</th>
<th>Brake Power KW</th>
<th>Heat Input* KW</th>
<th>Brake Eff.* %</th>
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</thead>
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<tr>
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<td>PSIa</td>
<td>RPM</td>
<td></td>
<td></td>
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<td></td>
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<td>0.39</td>
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<td>25</td>
<td>0.58</td>
<td>3.06</td>
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<td>5.64</td>
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<td>4.47</td>
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*Based upon energy balance at cold end.
Table 3-5  Measurements of GPU-3 Engine Performance  
by NASA-Lewis - Part II (79a)  
Hydrogen Gas, 65C (59F) Cooling Water Inlet Temperature,  
2.76 MPa (400 psia) Mean Pressure

<table>
<thead>
<tr>
<th>Pt</th>
<th>Heater Gas Temp. $\theta_C$</th>
<th>Engine Speed</th>
<th>Brake Power</th>
</tr>
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<td></td>
<td>$\theta_C$ $\theta_F$</td>
<td>HZ RPM</td>
<td>KW HP</td>
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<td>1</td>
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<td>16.67 1000</td>
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<td>2</td>
<td>704 1300</td>
<td>25 1500</td>
<td>1.49 2.00</td>
</tr>
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<td>704 1300</td>
<td>33.33 2000</td>
<td>1.95 2.62</td>
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<td>41.67 2500</td>
<td>2.35 3.15</td>
</tr>
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Table 3-6  Measurements of GPU-3 Engine Performance
by NASA-Lewis - Part III (79a)
Helium Gas, 704°C (1300°F) Nominal Heater Gas Temperature
13°C (56°F) Cooling Water Inlet Temperature

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<th>Pt</th>
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<th>Engine Speed</th>
<th>Ind. Power</th>
<th>Brake Power</th>
</tr>
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<td>HZ</td>
<td>RPM</td>
</tr>
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<td>400</td>
<td>16.67</td>
<td>1000</td>
</tr>
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<td>25</td>
<td>1500</td>
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<td>400</td>
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Table 3-7  Measurements of GPU-3 Engine Performance
by NASA-Lewis - Part IV (79a)
Helium Gas, 395°C (1100°F) Nominal Heater Gas Temperature
13°C (56°F) Cooling Water Inlet Temperature

<table>
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<th>Brake Power</th>
</tr>
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<td>KW/HP</td>
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<td>25/1500</td>
<td>0.93/1.25</td>
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<td>33.33/2000</td>
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<td>41.67/2500</td>
<td>2.96/3.97</td>
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<td>50/3000</td>
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<td>58.33/3500</td>
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Table 3-8 Measurements of GPU-3 Engine Performance by NASA-Lewis - Part V (79a)

Helium Gas, 649°C (1200°F) Nominal Heater Gas Temperature, 13°C (56°F) Cooling Water Inlet Temperature

<table>
<thead>
<tr>
<th>Pt</th>
<th>Mean Pressure</th>
<th>Engine Speed</th>
<th>Brake Power</th>
<th>Heat Input*</th>
<th>Brake Eff.* %</th>
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</thead>
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<td>PSIa</td>
<td>Hz</td>
<td>RPM</td>
<td>KW</td>
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<td>400</td>
<td>41.67</td>
<td>2500</td>
<td>1.21</td>
</tr>
<tr>
<td>5</td>
<td>2.76</td>
<td>400</td>
<td>50</td>
<td>3000</td>
<td>1.04</td>
</tr>
<tr>
<td>6</td>
<td>2.76</td>
<td>400</td>
<td>58.33</td>
<td>3500</td>
<td>0.56</td>
</tr>
<tr>
<td>7</td>
<td>4.14</td>
<td>600</td>
<td>25</td>
<td>1500</td>
<td>1.79</td>
</tr>
<tr>
<td>8</td>
<td>4.14</td>
<td>600</td>
<td>33.33</td>
<td>2000</td>
<td>2.20</td>
</tr>
<tr>
<td>9</td>
<td>4.14</td>
<td>600</td>
<td>41.67</td>
<td>2500</td>
<td>2.42</td>
</tr>
<tr>
<td>10</td>
<td>4.14</td>
<td>600</td>
<td>50</td>
<td>3000</td>
<td>2.35</td>
</tr>
<tr>
<td>11</td>
<td>4.14</td>
<td>600</td>
<td>58.33</td>
<td>3500</td>
<td>1.73</td>
</tr>
<tr>
<td>12</td>
<td>5.52</td>
<td>800</td>
<td>41.67</td>
<td>2500</td>
<td>3.28</td>
</tr>
<tr>
<td>13</td>
<td>5.52</td>
<td>800</td>
<td>50</td>
<td>3000</td>
<td>3.28</td>
</tr>
<tr>
<td>14</td>
<td>5.52</td>
<td>800</td>
<td>58.33</td>
<td>3500</td>
<td>2.76</td>
</tr>
<tr>
<td>15</td>
<td>6.9</td>
<td>1000</td>
<td>50</td>
<td>3000</td>
<td>3.93</td>
</tr>
<tr>
<td>16</td>
<td>6.9</td>
<td>1000</td>
<td>58.33</td>
<td>3500</td>
<td>3.37</td>
</tr>
</tbody>
</table>

*Based upon energy balance at cold end.
Figure 3-5  Mechanical Loss As a Function of Engine Speed for Hydrogen Working Gas (Determined from Experimental Heat Balance)

Figure 3-6  Mechanical Loss As a Function of Engine Speed for Helium Working Gas (Determined from experimental heat balance.)
Figure 3-7. GPU-3 STIRLING THERMAL ENGINE PERFORMANCE - PRESSURE RUNS

- WORKING FLUID: HYDROGEN
- COMBUSTION FUEL: DF #1
- PISTON ROD SEALS: ROLLING POLYURETHANE
- HEATER GAS TEMP: 135°F
- COOLING WATER INLET TEMP: 100°F
- AIR FUEL RATIO: 25/1
- CRANKCASE OIL: LUBRITE 10
- ENGINE NO.: 122
- DISPL. CAP: 159
- GUNNER NO.: 137
- PRE-HEATER NO.: WC-510-517
- REGENERATORS: 186-193

Includes burner efficiency.
Figure 3-9.
Later in the General Motors papers on Stirling engines released in 1978, a graph giving the calculated performance for the GPU-3 engine was published (78 bh, section 2.116, page 6, March 1970). (See Figure 3-9.) Furnace and mechanical efficiency are stated so the indicated power and efficiency calculated by most design methods can be compared with the unpublished method used by General Motors. Examinations show that Figures 3-7 and 3-8 agree well and are probably different plots of the same experimental measurements. Figure 3-9 agrees fairly well with measurement near the design point of 3000 rpm 1000 psia.

<table>
<thead>
<tr>
<th>G.M. Calculation</th>
<th>G.M. Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3-9</td>
<td>Figure 3-8</td>
</tr>
<tr>
<td>Output BHP</td>
<td>11.6</td>
</tr>
<tr>
<td>Overall Efficiency</td>
<td>29.8</td>
</tr>
</tbody>
</table>

However, at 3000 rpm and 250 psi, the calculated power is 3.3 hp, but the measured is only 1.5 hp.

The GPU-3 engine now has considerable data on it. It is not completely understood but the engine has been thoroughly measured and carefully run. A full test report on this is available (79 bl).

3.2 The 4L23 Engine

According to Percival (74 bc), design for a four-cylinder double-acting engine was started in 1968. Eventually, the goal was to demonstrate an advanced Stirling engine of about 150 hp. The engine became known as the 4L23 because of the piston displacement of 23 cubic inches and having four cylinders in a line. A single crankshaft was used with cross heads and only one piston per cylinder was needed. Figure 3-10 shows a cross section through one of these cylinders. In this Rinia, or Siemens, arrangement, the gas leaves the hot space and goes through a series of tubes arranged in a circle similar to the way the GPU-3 engine is designed. The tubes go from the hot space up to a manifold at the top and then other tubes come down and enter one of six regenerator cans grouped around each engine cylinder. Figure 3-11 shows a top view of this engine showing the four cylinders and the 24 regenerator cans that were used. Below each porous regenerator is the tubular gas cooler. As in the GPU-3, the regenerator and gas cooler were made as a unit and slipped into place. From the bottom of the gas cooler the gas is not inducted into the same cylinder as in the GPU-3, but into another cylinder in the line. Figure 3-11 and 3-12 show the arrangement of these conducting ducts. Figure 3-11 shows how the cold space of cylinder 1 is connected to the gas coolers of cylinder 3. The cold space of cylinder 3 is connected to the gas coolers of cylinder 4. The cold space of cylinder 4 is connected to the gas coolers of cylinder 2; and finally, the cold space of cylinder 2 is connected to the gas coolers of cylinder 1 to complete the circuit. This particular arrangement is done for the purpose of balancing the engine. In addition to this "firing order" arrangement and the counter-weights shown in Figure 3-10, engine 4L23 had two balance shafts on either side of the main crankshaft which has weights on them that rotated in such a way as to attain essentially perfect balance. This made the crankcase wider at the bottom. Also from the drawings sent to NASA-Lewis from General Motors (1978 dk) the crankcase was much less compact than that shown in Figure 3-10. Also the corrugated metal air preheater sketched in Figure 3-10 turned
Figure 3-10. Cross Section of Single Crank In-Line Engine.
Figure 3-11. Arrangement of Regenerators and Hold Down Studs for In-Line Crankcase.
Figure 3-12. Diagram of Connecting Ducts for In-Line Engines.

"FIRING ORDER" 1-3-4-2-1
out to be a shell and tube heat exchanger about three times as large. No report quality cross sections or artists' renderings or pictures of hardware were ever released on this engine. Nevertheless this engine is important today because it is of a very modern design and has an adequate description as to dimensions and calculated performance. It is very similar to the P-40 or P-75 engine that United Stirling is now building and testing. In order to provide for future engine upgrading, the combustion system and crankcase, crankshaft and bearings were designed to accept 3000 psi mean pressure. The 4L23 was General Motors Research's first computer design (optimized engine.) The 4L23 was the first engine with the sealed piston. In other engines a small capillary tube allowed the inside of the piston to be pressurized at the mean pressure of the engine working gas. This was done in order to minimize the inventory of hydrogen gas and also to reduce heat leak by having air instead of hydrogen in the piston dome. The 4L23 was optimized for the use of Met Net regenerator material which was found by General Motors to be considerably less expensive to produce than the woven wire regenerator material which had been used up until that time.

Table 3-9 gives all the engine dimensions necessary to calculate the power output and efficiency of the 4L23. Most of these numbers come from GMR-2690 section 2.115 (78 bh) report dated 19 January 1970. Some come from additional drawings sent to NASA-Lewis from General Motors Research (78 dk). The list given by Martini (79 ad) has been revised somewhat. The final list is given in Table 3-9.

3.2.2 Engine Performance

Insufficient data is given in the General Motors reports to calculate static heat loss through the engine. Second order theory indicates that if the engine heat inputs are plotted against frequency the extrapolation to zero frequency should give the static heat loss. This process was done for the data given by Diepenhorst (see Figures 3-13 to 3-15.) It was found that the heat inputs were exactly proportional to frequency, but that the zero intercept was not consistent (see Figure 3-16.) Since the heat input was so perfectly proportional to frequency of operation, it was a shock that the zero intercepts did not follow any particular pattern. One would expect that the zero intercepts for hot tube temperature of 1400 F would be always higher than those for 1200 F, which would always be higher than those for 1000 F. There is also no reason for a dependence on average pressure because metal thermal conductivity is not affected by this, and gas thermal conductivity is almost not affected. This problem is only discussed in this section because there should be some information given from which the static thermal conductivity can be calculated. Table 3-10 gives the information needed to calculate static thermal conductivity. The engine cylinder and the regenerator cases are tapered to have a smaller wall thickness at the cold end. However, at this level of detail only an average wall thickness and an average thermal conductivity for the entire wall is desired.

Percival gives a somewhat different calculated performance for the 4L23 engine (see Figure 3-17.) Figure 3-15 and Figure 3-17 have the same operating conditions and engine specifications, but the power output and efficiency are slightly different. Figure 3-17 quotes 25 GPM cooling water flow which is for
Table 3-9 - Specifications for the General Motors 4L23 Stirling Engine Type: 4 cylinder, single crank drive with double acting pistons

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Fluid</td>
<td>Hydrogen</td>
</tr>
<tr>
<td>Design Speed</td>
<td>2000 RPM</td>
</tr>
<tr>
<td>Design Pressure</td>
<td>1500 psia</td>
</tr>
<tr>
<td>Cylinders per engine</td>
<td>4</td>
</tr>
<tr>
<td>Bore</td>
<td>10.16 cm (4.0 in.)</td>
</tr>
<tr>
<td>Stroke</td>
<td>4.65 cm (1.83 in.)</td>
</tr>
<tr>
<td>Displacement (per cyl)</td>
<td>377 cu. cm (23 c. in.)</td>
</tr>
<tr>
<td>Diameter of roll sock seal</td>
<td>4.06 cm (1.6 in.)</td>
</tr>
<tr>
<td>Piston end clearance</td>
<td>0.0406 cm (0.016 in.)</td>
</tr>
<tr>
<td>Cooler (per cyl.)</td>
<td></td>
</tr>
<tr>
<td>Tube Length</td>
<td>12.9 cm (5.08 in.)</td>
</tr>
<tr>
<td>Heat Transfer Length</td>
<td>12.02 cm (4.73 in.)</td>
</tr>
<tr>
<td>Tube I. D.</td>
<td>.115 cm (0.045 in.)</td>
</tr>
<tr>
<td>Tube O. D.</td>
<td>.167 cm (0.065 in.)</td>
</tr>
<tr>
<td>Number of Tubes</td>
<td>312</td>
</tr>
<tr>
<td>Water Flow</td>
<td>25 GPM</td>
</tr>
<tr>
<td>Water Inlet Temp.</td>
<td>135°F</td>
</tr>
<tr>
<td>Heater (per cyl.)</td>
<td></td>
</tr>
<tr>
<td>Tube Length</td>
<td>41.8 cm (16.46 in.)</td>
</tr>
<tr>
<td>Heat Transfer Length</td>
<td>25.58 cm (10.18 in.)</td>
</tr>
<tr>
<td>Tube I.D.</td>
<td>.472 cm (0.18 in.)</td>
</tr>
<tr>
<td>Tube O.D.</td>
<td>.640 cm (0.25 in.)</td>
</tr>
<tr>
<td>Number of Tubes</td>
<td>36</td>
</tr>
<tr>
<td>Inside Wall Temp.</td>
<td>1400°F</td>
</tr>
<tr>
<td>Cold End Connecting Ducts (per cyl.)</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>71 cm (27.95 in.)</td>
</tr>
<tr>
<td>I.D.</td>
<td>.76 cm (0.30 in.)</td>
</tr>
<tr>
<td>Number</td>
<td>6</td>
</tr>
<tr>
<td>Isothermal Volume</td>
<td>5 percent</td>
</tr>
<tr>
<td>Adiabatic Volume</td>
<td>95 percent</td>
</tr>
<tr>
<td>Regenerators (per cyl.)</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>2.5 cm (0.98 in.)</td>
</tr>
<tr>
<td>Diameter</td>
<td>3.5 cm (1.38 in.)</td>
</tr>
<tr>
<td>Number</td>
<td>6</td>
</tr>
<tr>
<td>Material</td>
<td>Met Net .05-.20</td>
</tr>
<tr>
<td>Filler Factor</td>
<td>20 percent</td>
</tr>
<tr>
<td>Wire Diameter</td>
<td>.00432 cm (.0017 in.)</td>
</tr>
<tr>
<td>Drive</td>
<td></td>
</tr>
<tr>
<td>Connecting Rod Length</td>
<td>13.65 cm (5.375 in.)</td>
</tr>
<tr>
<td>Crank Radius</td>
<td>2.325 cm (0.915 in.)</td>
</tr>
<tr>
<td>Cooling Water</td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>25 GPM/cyl. @2000 RPM</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>135°F</td>
</tr>
<tr>
<td>Mechanical Efficiency</td>
<td>90 percent</td>
</tr>
<tr>
<td>For Bare Engine</td>
<td></td>
</tr>
<tr>
<td>Furnace Efficiency</td>
<td></td>
</tr>
<tr>
<td>Burner + air preheater</td>
<td>80 percent</td>
</tr>
<tr>
<td>Hot Cap</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>6.40 cm (2.52 in.)</td>
</tr>
<tr>
<td>Gap</td>
<td>0.0406 cm (0.016 in.)</td>
</tr>
<tr>
<td>Phase Angle</td>
<td>90°</td>
</tr>
<tr>
<td>Velocity Heads due to</td>
<td></td>
</tr>
<tr>
<td>Entrance and Exit and Bends</td>
<td></td>
</tr>
<tr>
<td>Heater</td>
<td>4.4</td>
</tr>
<tr>
<td>Cooler</td>
<td>1.5</td>
</tr>
<tr>
<td>Connecting T.</td>
<td>3.0</td>
</tr>
</tbody>
</table>
FIGURE 3-13.

4L23 CALCULATED PERFORMANCE
BHP, TORQUE AND EFFICIENCY VS. ENGINE SPEED
AT VARIOUS MEAN WORKING PressURES

100 GPM COOLING WATER FLOW (AT 2000 RPM)
135°F COOLING WATER INLET TEMPERATURE
80% FURNACE EFFICIENCY
90% MECHANICAL EFFICIENCY

1000°F INSIDE HEATER TUBE WALL TEMPERATURE

ENGINE SPEED - RPM

ENGINE OUTPUT - BHP

ENGINE TORQUE - FT. LBF.

FIGURE 3-13.
4L23 CALCULATED PERFORMANCE
BHP, TORQUE AND EFFICIENCY VS. ENGINE SPEED
AT VARIOUS MEAN WORKING PRESSURES

100 GPM
135°F
1000°F COOLING WATER FLOW (AT 2000 RPM)
COOLING WATER INLET TEMPERATURE
80% FURNACE EFFICIENCY
90% MECHANICAL EFFICIENCY

1200°F INSIDE HEATER TUBE WALL TEMPERATURE

ENGINE SPEED - RPM
ENGINE OUTPUT - BHP
ENGINE TORQUE - FT. LBF.

FIGURE 3-14.
4L23 CALCULATED PERFORMANCE
BHP, TORQUE AND EFFICIENCY VS. ENGINE SPEED
AT VARIOUS MEAN WORKING PRESSURES

ENGINE SPEED - RPM

ENGINE OUTPUT - BHP

ENGINE TORQUE - FT. LBF.

FIGURE 3-15.
Figure 3-16. Calculated Zero Intercepts of Heat Input Vs. Frequency.
CALCULATED PERFORMANCE
COMPACT STIRLING RESEARCH ENGINE
MODEL 4L23

Hydrogen Working Fluid at Various Mean Working Pressures

Figure 3-17
Table 3-10. 4L23 Engine Dimension for the Purpose of Calculating Static Heat Conduction

<table>
<thead>
<tr>
<th>Engine Cylinder</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>OD</td>
<td>~12.7 cm (5 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID</td>
<td>~10.2 cm (4 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>22.6 cm (8.9 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number per engine</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hot Cap.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>OD</td>
<td>10.211 cm (4.020 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID</td>
<td>9.45 cm (3.72 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔT Length</td>
<td>10.03 cm (3.95 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Radiation Shields</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regenerator</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number per cylinder</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case Length (ΔT)</td>
<td>2.79 cm (1.1 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case ID</td>
<td>3.5 cm (1.38 in.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case OD (avg.)</td>
<td>4.32 cm (1.7 in.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Matrix = Met Net | .05 - .20 |
| Thermal Conductivity of Matrix | 0.017 w/cmC* |

*78 bm, Section 6.006, page 7.

Each cylinder. Figure 3-16 quotes 100 GPM cooling water flow which is for all 4 cylinders and is proportional to speed.

The same data given in Figures 3-13 to 3-15 are replotted in the form of "muschel" diagrams in Figures 3-18 to 3-20. These are included because this is the common way engines are described today.
4L23 CALCULATED PERFORMANCE

LINES OF CONSTANT OUTPUT
LINES OF CONSTANT EFFICIENCY
LINES OF CONSTANT PRESSURE

100 GPM COOLING WATER FLOW (AT 2000 RPM)
125°F COOLING WATER INLET TEMPERATURE
80% FURNACE EFFICIENCY
90% MECHANICAL EFFICIENCY

1000°F INSIDE HEATER TUBE WALL TEMPERATURE

ENGINE TORQUE - FT. LBF.

ENGINE SPEED - RPM

FIGURE 3-18
4L23 CALCULATED PERFORMANCE

- Lines of constant output
- Lines of constant efficiency
- Lines of constant pressure

100 GPM cooling water flow (at 2000 RPM)
135°F cooling water inlet temperature
80% furnace efficiency
90% mechanical efficiency

1200°F inside heater tube wall temperature

0 500 1000 1500 2000 2500 3000 3500
ENGINE SPEED - RPM

0 100 200 300 400 500 600 700 800
ENGINE TORQUE - FT. LBF.

FIGURE 3-19
4L23 CALCULATED PERFORMANCE

- LINES OF CONSTANT OUTPUT
- LINES OF CONSTANT EFFICIENCY
- LINES OF CONSTANT PRESSURE

1400°F INSIDE HEATER TUBE WALL TEMPERATURE

100 GPM COOLING WATER FLOW (AT 2000 RPM)
125°F COOLING WATER INLET TEMPERATURE
80% FURNACE EFFICIENCY
90% MECHANICAL EFFICIENCY

ENGINE SPEED - RPM
ENGINE TORQUE - FT-LBF.

FIGURE 3-20
4. PARTIALLY DESCRIBED STIRLING ENGINES

In this section will be given as much information as available on complete well-engineered engines which have some information on displacement, operating speed, operating temperatures, power and efficiency, but not enough data so that they can be classified as fully described engines. Information given elsewhere in the Design Manual will be referred to instead of being duplicated. This information will inform the readers what the state-of-the-art of Stirling engines is.

4.1 The Philips 1-98 Engine

About 30 Philips engines of this type have been built. They are the Rhombic drive type with a single power piston and displacer. The power piston displacement is 98 cm$^3$, and there is one power piston. Thus the name 1-98. The design of the heater, cooler and regenerator have not been disclosed. Probably there are many different kinds of 1-98 engines depending upon the intended use. Michels (76 e) has calculated the performance of the 1-98 engine for a variety of conditions. In each condition the heat exchangers of the engine are optimized for the best efficiency at each power point. Michels showed that for these optimized engines the indicated efficiency depends upon the heater temperature and cooler temperature and not upon the working gas used. Figure 4-1 shows this curve correctly labeled. Another way of describing the performance of the 1-98 engine is to relate the indicated efficiency to the Carnot efficiency for the particular heater and cooler temperature employed. Table 4-1 gives such information for the 1-98 engine. Table 4-2 gives similar computed information for the brake (shaft) efficiencies for the 1-98 Rhombic drive engine. These are correlated in Figure 4-2 in a way that might be applicable to other well-designed Stirling engines.

![Figure 4-1. Indicated Efficiencies for Philips 1-98 Engine Vs. Heater Temperature $T_H$ at Two Different Cooler Temperatures $T_C$. Engine Displacement 98 cm$^3$.](image-url)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>850</td>
<td>100</td>
<td>8</td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td>H₂</td>
<td>400</td>
<td>100</td>
<td>1</td>
<td>32</td>
<td>72</td>
</tr>
<tr>
<td>H₂</td>
<td>250</td>
<td>100</td>
<td>.35</td>
<td>18</td>
<td>63</td>
</tr>
<tr>
<td>He</td>
<td>850</td>
<td>100</td>
<td>6</td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td>He</td>
<td>400</td>
<td>100</td>
<td>1</td>
<td>30</td>
<td>67</td>
</tr>
<tr>
<td>He</td>
<td>250</td>
<td>100</td>
<td>.18</td>
<td>17</td>
<td>59</td>
</tr>
<tr>
<td>N₂</td>
<td>850</td>
<td>100</td>
<td>1.5</td>
<td>49</td>
<td>73</td>
</tr>
<tr>
<td>N₂</td>
<td>400</td>
<td>100</td>
<td>.35</td>
<td>31</td>
<td>70</td>
</tr>
<tr>
<td>N₂</td>
<td>250</td>
<td>100</td>
<td>Negative</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>H₂</td>
<td>850</td>
<td>0</td>
<td>10</td>
<td>57</td>
<td>75</td>
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<tr>
<td>H₂</td>
<td>400</td>
<td>0</td>
<td>2.8</td>
<td>45</td>
<td>76</td>
</tr>
<tr>
<td>H₂</td>
<td>250</td>
<td>0</td>
<td>1</td>
<td>34</td>
<td>71</td>
</tr>
<tr>
<td>He</td>
<td>850</td>
<td>0</td>
<td>8</td>
<td>58</td>
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<td>400</td>
<td>0</td>
<td>2</td>
<td>42</td>
<td>71</td>
</tr>
<tr>
<td>He</td>
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<td>60</td>
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</tbody>
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**Table 4-2**

(Reference 76(e))

Engine Optimized for Each Operating Point

4-98 Rismatic Drive Phillips

Computed Brake (shaft) Efficiencies for

Without auxiliaries

K, Watt, C, Temp, C, at Max, Eff, % Carnot Efficiency, Working, heater, Cool, % of Mechanical Power
Figure 4-2. Indicated and Brake Efficiency Factors for Optimized Philips 1-98 Engines (76 e).
engines. Note that when the efficiency is related to the Carnot efficiency for the temperatures over which the engine operates, this fraction of Carnot goes from 65 ± 6 percent at 250 C heater temperature to 75 ± 2 percent at 800 C heater temperature for the indicated efficiency. Lower numbers are shown for the brake efficiency which shows that the mechanical efficiency for this machine is generally about 80 percent (See Table 4-2).

4.2 Miscellaneous Engines

The size, weight, power and efficiency for a number of other engines mentioned in the literature are presented in Tables 4-3 and 4-4. It should be emphasized that the powers given are the maximum efficiency operating point, not the maximum power operating point. Note that the brake efficiencies range from 46 to 69 percent of Carnot.

Finegold and Vanderbrug (77 ae) used the data from the Philips 4-215 engine to conclude that the maximum brake efficiency is 52 percent of the Carnot efficiency. This factor is based upon 1975 data. Improvements have been made since then.

Net brake efficiency--the information presented in Tables 4-3 and 4-4 is for engines without auxiliaries. In Table 4-5 the performance and efficiencies are given for the engine powering all auxiliaries needed to have the engine stand alone. This includes cooling fan, the blower, the atomizer, the fuel burner and the water pump for the radiator. Table 4-5 shows that the maximum net brake efficiency is 38 to 65 percent of Carnot.

4.3 Early Philips Air Engines

The early antique Stirling engines, which were called air engines, were very ponderous, operated at a slow speed and were very heavy for the amount of power that they produced. They were operated at or near 1 atm pressure. In the late forties and early fifties, Philips developed a high speed air engine which was very much better than the old machines, but still was not competitive for the times. Philips never published any information on their early air engines. However, quite a number of these early machines were made and they were submitted for evaluation by at least one external laboratory. Even though they were not considered by Philips to be competitive, in today's world where the multi-fuel capability of the Stirling is much more keenly appreciated, the simplicity, the reasonable size for small scale stationary power using solid fuel and the reasonable efficiency of these early Philips air engines are attractive. The best documented account of one of these early air engines is given by Walker, Ward and Slowley (79 ao).

In the early Philips program, development of Stirling engines was concentrated on small engines of 1 KW or less. One machine was sufficiently developed to be made in quantities of several hundred. It was never put into regular production, however, and in the late 1950's, Philips disposed of the entire stock, largely to universities and technical institutes throughout Europe. A cross section of this engine is shown in Figure 4-3. Scaling of this drawing shows that the power piston has a diameter of about 4.8 cm and a stroke of about 3 cm, giving a displacement for the power piston of about 50 cm³. Twin connecting rods run
### Table 4-3

Maximum Brake Efficiencies for Various Stirling Engines

(Reference 1975 t)

<table>
<thead>
<tr>
<th>Engine Designation</th>
<th>Working Fluid</th>
<th>Mean Pressure</th>
<th>Cooler Temp</th>
<th>Maximum Efficiency Operating Point</th>
<th>Dimension cm</th>
<th>Engine Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturer</td>
<td>MPa</td>
<td>°F</td>
<td>KW</td>
<td>RPM</td>
<td>Brake* % of Carnot</td>
</tr>
<tr>
<td>Prototype</td>
<td>United Stirling</td>
<td>H2</td>
<td>14.5</td>
<td>691</td>
<td>71</td>
<td>2000 30 47</td>
</tr>
<tr>
<td>4-235</td>
<td>Prototype Philips</td>
<td>He</td>
<td>22.1</td>
<td>683</td>
<td>43</td>
<td>1800 31 46</td>
</tr>
<tr>
<td>40 HP</td>
<td>Prototype Philips</td>
<td>H2</td>
<td>14.2</td>
<td>649</td>
<td>16</td>
<td>725 38 55</td>
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<tr>
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<td>United Stirling</td>
<td>H2</td>
<td>14.5</td>
<td>719</td>
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<td>1200 35 54</td>
</tr>
<tr>
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<td>MAN-MWW</td>
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<td>10.8</td>
<td>633</td>
<td>41</td>
<td>1000 32 49</td>
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</table>

*without auxiliaries
Table 4-4

Maximum Brake Efficiencies for Various Stirling Engines

<table>
<thead>
<tr>
<th>Engine Designation</th>
<th>Working Fluid</th>
<th>Mean Pressure</th>
<th>Heater Temp</th>
<th>Cooler Temp</th>
<th>Maximum Efficiency Operating Point</th>
<th>Dimension cm</th>
<th>Engine Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPU-3 H2</td>
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<td>816</td>
<td>10</td>
<td>8.1</td>
<td>2000 9.5 39 53</td>
<td>28 x 29 x 27</td>
<td>Piston-Displ. 1</td>
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<tr>
<td>General Motors H2</td>
<td>4.1</td>
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<td>10</td>
<td>6.0</td>
<td>2500 8 38.5 52</td>
<td>28 x 29 x 27</td>
<td>Piston-Displ. 1</td>
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<tr>
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<td>10</td>
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<td>28 x 29 x 27</td>
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<td>816</td>
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<td>19.4</td>
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<td>44 x 43 x 86</td>
<td>Rinia 4</td>
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<tr>
<td>Philips H2</td>
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<td>816</td>
<td>10</td>
<td>17.2</td>
<td>1200 23 50 68</td>
<td>44 x 43 x 86</td>
<td>Rinia 4</td>
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<td>(Ref. 69 f)</td>
<td>6.2</td>
<td>816</td>
<td>10</td>
<td>14.9</td>
<td>1400 20 49 67</td>
<td>44 x 43 x 86</td>
<td>Rinia 4</td>
</tr>
<tr>
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<td>10</td>
<td>11.2</td>
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<td>6.0</td>
<td>1800 8 45 61</td>
<td>44 x 43 x 86</td>
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* without auxiliaries
Table 4-4 (continued)

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<th>Engine Designation</th>
<th>Working Fluid</th>
<th>Mean Pressure</th>
<th>Heaters Temp</th>
<th>Coolers Temp</th>
<th>Maximum Efficiency Operating Point</th>
<th>Dimension cm</th>
<th>Engine Type</th>
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<tr>
<td>Maker</td>
<td>MPa</td>
<td>C</td>
<td>C</td>
<td>RPM</td>
<td>Brake %</td>
<td>wt, kg</td>
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<tr>
<td></td>
<td>psia</td>
<td>F</td>
<td>F</td>
<td>BHP</td>
<td>% of Carnot</td>
<td>No. of cylinders</td>
<td></td>
</tr>
<tr>
<td>Rinia 1/4</td>
<td>150 HP</td>
<td>H2</td>
<td>10.3 1500 1500</td>
<td>10 130 1400 44 60</td>
<td>94 x 50 x 84</td>
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<td>4</td>
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<tr>
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<td>General Motors</td>
<td>H2</td>
<td>8.3 1200 1500</td>
<td>10 78 1500 44 60</td>
<td>94 x 50 x 84</td>
<td>**</td>
<td>4</td>
</tr>
<tr>
<td>Rinia 1/4</td>
<td>Research (Ref. 69 f)</td>
<td>H2</td>
<td>6.2 900 1500</td>
<td>10 75 1800 44 60</td>
<td>94 x 50 x 84</td>
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<td>4</td>
</tr>
<tr>
<td>Rinia 1/4</td>
<td>H2</td>
<td>6.2 900 1500</td>
<td>10 75 1800 44 60</td>
<td>94 x 50 x 84</td>
<td>**</td>
<td>4</td>
<td></td>
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<tr>
<td>Rinia 1/4</td>
<td>H2</td>
<td>4.1 600 1500</td>
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<td></td>
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<td>H2</td>
<td>2.1 300 1500</td>
<td>10 30 2000 40 54</td>
<td>94 x 50 x 84</td>
<td>**</td>
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<tr>
<td>10-36</td>
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<td>H2</td>
<td>6.9 1000 1400</td>
<td>24 1800 26.3 28</td>
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<tr>
<td>4S1210</td>
<td>General Motors</td>
<td>H2</td>
<td>10.3 1500 1202</td>
<td>33 750 35 52</td>
<td>188 x 102 x 193</td>
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<tr>
<td>1-51050</td>
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<td>Electro Motive</td>
<td>H2</td>
<td>9.9 1436 1270</td>
<td>38 1200 28 30</td>
<td>91 x 70 x 165</td>
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<td>2W17A</td>
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<td>Electro Motive</td>
<td>H2</td>
<td>7.6 1100 1100</td>
<td>38 900 28.4 31</td>
<td>92 x 158 x 215</td>
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* Bare engine with preheater. ** Without flywheel.
### Table 4-5

Maximum Net Brake Efficiencies for Various Stirling Engines

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Designation</th>
<th>Working Fluid</th>
<th>Mean Pressure</th>
<th>Heater Temp</th>
<th>Cooler Temp</th>
<th>Maximum Efficiency Operating Point</th>
<th>Dimension cm</th>
<th>Engine Type</th>
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<tr>
<td>Philips (Ref. 75 t)</td>
<td>4-215</td>
<td>H_2</td>
<td>9.6</td>
<td>705</td>
<td>8</td>
<td>1100 56 32 50</td>
<td>340</td>
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<td>Philips (Ref. 75 T)</td>
<td>He</td>
<td>22.1</td>
<td>~760</td>
<td>71</td>
<td>500 43 65 149 x 131 x 67</td>
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<td>General Motors (Ref. 75 t)</td>
<td>GPU-3</td>
<td>H_2</td>
<td>6.89</td>
<td>700</td>
<td>83</td>
<td>1900 26.5 40 40 x 40 x 73</td>
<td>75</td>
<td>Piston-Displ. 1</td>
</tr>
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<td>United Stirling (Ref. 77 bj)</td>
<td>P-40</td>
<td>H_2</td>
<td>15.2</td>
<td>721</td>
<td>52</td>
<td>1250 35 52 ---</td>
<td>---</td>
<td>Double Acting Dual Crank 4</td>
</tr>
<tr>
<td>Model IV</td>
<td>MIT/Sunpower (Ref. 77 s)</td>
<td>He</td>
<td>5.0</td>
<td>594</td>
<td>23</td>
<td>960 25 38 ---</td>
<td>---</td>
<td>Free Piston Free Displ.</td>
</tr>
<tr>
<td>Farwell (Ref. 75 1)</td>
<td>TMG(03)</td>
<td>He</td>
<td>0.1</td>
<td>594</td>
<td>40</td>
<td>6000 0.0375 16.9 26.5</td>
<td>---</td>
<td>Oscillating diaphragm: sprung displacer 1</td>
</tr>
</tbody>
</table>

* with auxiliaries
from the power piston to the crank shaft. In between these rods a flexible connecting rod drives the displacer through a bell crank linkage to a connecting rod radiating from the crank at about 90° from the main power crank (See Figure 4-3). This bell crank also operates an air compressor needed to keep the engine pumped up. Figure 4-4 shows the same engine installed in an electric power generating package which was made in a self-contained unit designed for 200 W (e) output. This unit incorporated a gasoline or kerosene fuel tank, a cooling fan, and engine controls by mean pressure. In the tests done by Walker, Ward and Slowley at the University of Bath in Somerset, England, the engine was removed from the frame of the generator set and was mounted on a test rig. The engine was coupled to an electric swing-field dynomometer capable of acting as a generator or as a motor. The combustion equipment was modified to allow the use of liquified petroleum gas and air rather than the normal liquid kerosene or gasoline as fuels. Provision was made for accurate measurement of the gas-air consumption and engine shaft speed and brake power input or output of the engine.

The principle modification of the engine was to substitute water cooling for the original air cooling around the compression space of the cylinder. The
Figure 4-4. Stirling Cycle Air Engine/Generator Set.
temperature and flow rate of cooling water was measured. Chromel-alumel thermocouples were brazed to the engine cylinder head to measure the nominal cylinder heater head temperature. In normal practice the air acting as a working fluid is compressed by a small crank-driven air compressor before delivery to the working space. For the tests reported here provision was made for the air pressure to be supplied and controlled from laboratory air supplies.

In the motoring tests the working space was connected to a large tank thereby increasing the internal dead volume of the engine by a large factor. Therefore, during operation there was no substantial change in the pressure level of the working fluid throughout the cycle. Therefore, the work absorbed by the engine during these motoring tests was due to fluid friction and mechanical friction, the thermodynamic work being made essentially negligible by virtue of the large dead volume. Tests were run with this engine at 1200, 1400, 1600 and 1800 rpm. At each speed the engine performance was observed with cylinder head temperatures of 600, 700, 800 and 900°C with mean working space pressures of 4.14, 5.52, 6.90, 8.28, 9.66 and 12.41 bar. In the motoring tests measurements were made at 800, 1000, 1200 and 1400 rpm. Mean working space pressures of 1.00, 5.28, 11.03 and 12.41 bar were made with the engine in all cases at ambient temperature. The results of some engine power tests are shown in Figures 4-5 and 4-6. The maximum power observed during these tests was approximately 0.48 KW. The specific fuel consumption was based upon the combustion of "Calor-Gas" with a lower heating value of 46,500 KJ/KG. A specific fuel consumption of 1 Kg/KW-hr is equivalent to an efficiency of 7.75 percent. It was claimed by the authors that at high cylinder head temperature, high working space pressure and low operating speed, an efficiency of about 10 percent was obtained. This efficiency was obtained with no attempt to preheat the incoming air with the hot exhaust gases. They felt that in many applications for small engines, efficiency is rarely as important as size, weight, reliability or capital costs.

The results of the motoring tests are given in Figure 4-7. This shows the motoring power required to drive the engine as a function of operating pressure at four different speeds. Figure 4-8 separates the data into mechanical friction loss, which is taken to be that at 0 operating pressure, and gaseous pumping power loss, which is seen to be proportional to gas pressure and only mildly dependent upon engine speed. By separating the losses in this way much of the seal drag which is dependent upon engine pressure is lumped with gaseous pumping power. Since the flow friction of the gas is proportional to the engine speed for laminar flow and to the engine speed squared for turbulent flow, much of the so-called gaseous pumping power is seal drag.

Tests of an even earlier Philips air engine are reported by Schrader of the U. S. Naval Experimenting Station (51 r). The engine is identified as a Philips model 1/4D external combustion engine, equipped as a portable generator set rated at 124.5 W or more. The engine was operated as continuously as possible for 1,015 hours. The engine had a bore of 2.5" and a stroke of the power piston of 1-7/32" and of the displacer 3/4". This gives a displacement of 98 cm³ for the power piston (the same as the later Philips 1-98 engine.) An external belt-operated air compressor was utilized. Sealing was with cast iron piston rings. Average specific fuel consumption was 4.66 lb/KW-hr (2.12Kg/KW-hr). The fuel was lead-free gasoline and the crank case was oil lubricated. The engine operated almost silently. A microphone installed 24 feet directly above
Figure 4-5. Brake Power and Brake Specific Fuel Consumption of Stirling Air Engine as a Function of Mean Operating Pressure at Four Different Cylinder Head Temperatures and a Constant Engine Speed of 1800 Revolutions per Minute.
Figure 4-6. Brake Power and Brake Specific Fuel Consumption of Stirling Air Engine as a Function of Engine Speed at Different Mean Operating Pressures and a Constant Cylinder Head Temperature of 800°C.
Figure 4-7. Required Motoring Power of Stirling Air Engine as a Function of Mean Operating Pressure at Four Different Speeds and With Engine Cylinder at Ambient Temperature.
the engine gave a rating of 58.9 db with the engine operating under load and 54.4 db with the engine off. The engine design was, as far as could be determined, similar to the one previously described in that the heat exchangers were multi-finned pressure vessels with many fins on the outside of the pressure vessel as well as on the inside. During the 1,015 hour endurance test the oil was scheduled to be changed and was changed every 150 hours. Chrome-plated piston rings were used for the 1,000 hour test. However, unplated rings had been used for a 600-hour test earlier and were also in good shape at the end of that period. Immediately prior to the post-trial disassembly inspection, a measurement of maximum power output was made. The heater head temperature was increased to 1150 F (nominal 1050 to 1075) and the crank case pressure was raised to 108 psi (nominal 85 to 88 psi). Under these conditions, the engine developed 185 W output as compared to the nominal 124.5 W rating. This was considered to be proof of the excellent condition of the engine at the time of the post-trial inspection. During the 1,015 hour test the engine had to be secured (stopped) many times for minor problems. Problems detailed in Reference 51 r were heater head flameout, burner pressure cutout, air leaks, gasoline tube breakage, compressor suction valve failure, compressor discharge valve failure, crank case pressure regulator failure. These are all normal shake-down problems that could be fairly well eliminated with experience. The important thing to note is that the internal parts did not foul with decomposed oil deposits. Possibly these deposits burned off because of the pressurized air working fluid.
4.4 The P75 Engine

United Stirling of Sweden (USS) plans to initiate limited production of their 75 kilowatt P-75 engine by 1981-82. They plan to reach production of 15,000 engines per year by the late 1980's (79 i). Figure 4-9 shows this engine. This engine has been installed in a light truck (78 aa). (See Figure 4-10.) The installation has been successful.

4.5 The P40 Engine

USS is planning a group of related engines--the P40, a 40 kw four cylinder double acting engine; the P75 (just mentioned), and the P150 which is a double P75. The P40 is not now scheduled for serial production; however, production of at least five is part of the DOE sponsored automobile engine programs administered by NASA-Lewis. Figure 4-11 shows the first one of these engines. Figure 4-12 shows this engine as it was installed in an Opel (78 cu). It has been a success as an initial demonstrator. Its drivability is good. It is quiet, but it shows no advantage in fuel economy because the engine, transmission and vehicle were not designed for one another (78 dt).

The second P40 engine has been tested by NASA-Lewis.

The third P40 is installed in a 1979 AMC Concord sedan. The sedan was modified by AMC. Installation of the engine was done by USS. The fourth P40 has been delivered to MTI for familiarization and evaluation. The fifth P40 is a spare.

Figure 4-9. The United Stirling P75 Engine.
Figure 4-10. The P75 Engine Installed in a Light Truck.

Figure 4-11. The P40 Engine.

Figure 4-12. The P40 Engine Installed in an Opel.
5. REVIEW OF STIRLING ENGINE DESIGN METHODS

Other sections in this design manual describe what is going on in Stirling engines today. This section outlines the mathematics behind the Stirling engine process itself. Stirling engine cycle analysis will first be discussed. This subsection discusses what really goes on inside a Stirling engine starting out with the most simple assumptions and then progressing to more and more realistic assumptions. This subsection is the basis for the subsequent three subsections that discuss first-order design methods, second-order design methods and third-order design methods.

First-order design methods start with limited information and calculate power output and efficiency for a particular size engine. Use of the first-order method assumes that others have or will actually design the Stirling engine. First-order analysis is for systems engineers who want to quickly get a feeling for the capability of a Stirling engine.

Second-order design methods take all aspects of the Stirling engine into account and are for those who intend to design a new Stirling engine. A wide spectrum of methods falls under the heading of second-order analysis. In second-order analysis it is assumed that a relatively simple Stirling engine cycle analysis can be used to calculate the basic power output and heat input. It further assumes that various power losses can be deducted from the power output. These power losses are assumed to be calculable by simple formulas and do not interact with other processes. It is further assumed that the separate heat losses can be calculated by simple formula and are addable to the basic heat input. It is further assumed that each one of these heat losses is independent of the others and there is no interaction.

Third-order design analysis is what is generally called nodal analysis. The engine is simulated by dividing it up into a number of sections, called nodes. Equations are written which express the conservation of heat, mass, momentum for each node. These equations are programmed into a digital computer and the engine is simulated starting with an arbitrary initial condition and going until the cycle repeats with a desired degree of accuracy. For those designers who are embarking on the original design of a Stirling engine, the choice must be made between second- and third-order design methods. Generally, as the complexity and therefore the cost of computation increases, the accuracy and general applicability of the result should also increase. However, the state of information on Stirling engine design is still highly incomplete. One cannot draw a graph of computation costs versus accuracy of result and place the different computation methods upon it.
5.1 Stirling Engine Cycle Analysis

In this subsection on cycle analysis the basic thermodynamics of a Stirling engine will be explained and the effect of some necessary complications will be assessed. The thermodynamic definition of a Stirling cycle is isothermal compression and expansion and constant volume heating and cooling, 1, 2, 3, 4, 1 in Figure 5-1.

The thermodynamic definition of an Ericsson cycle is isothermal compression and expansion and constant pressure heating and cooling, 1, 2', 3, 4', 1 in Figure 5-1. This Ericsson cycle encompasses more area than the Stirling cycle and therefore produces more work. However, the volumetric displacement is larger, therefore, the engine is larger. There is a modern pumping engine concept which approximates this cycle (73 p). The early machines built by John Ericsson used valving to attain constant pressure heating and cooling (59 c), thus the cycle name.

The thermodynamic definition of the Otto cycle is adiabatic compression and expansion and constant volume heating and cooling, 1, 2'', 3, 4'', 1 in Figure 5-1. The reason this cycle is mentioned is that the variable volume spaces in a Stirling engine are usually of such size and shape that their compression and expansion is essentially adiabatic since little heat can be transferred to the walls during the process of compression or expansion. An internal combustion engine approximates the Otto cycle. In real Stirling machines, a large portion of the gas is in the dead volume which is compressed and expanded nearly isothermally so the loss of work per cycle is not as great as shown.

![Figure 5-1. Theoretical Stirling, Ericsson and Otto Cycles.](image-url)
In Section 5.1 discrete processes of compression, heating, expansion and cooling will be considered first. Numerical examples will be used to make the processes clearer. The section starts with the simplest case and proceeds through some of the more complicated cases. In the later parts of Section 5.1 cycles will be considered where the discrete processes overlap as they do in a real engine.

5.1.1 Stirling Cycle, Zero Dead Volume, Perfect Regeneration

The Stirling cycle is defined as a heat power cycle using isothermal compression and expansion and constant volume heating and cooling. Figure 5-2 shows such a process. Specific numbers are being used to make the explanations easier to follow and allow the reader to check to see if he is really getting the idea. Let us take 100 cm$^3$ of hydrogen at 10 MPa (~100 atm) and compress it isothermally to 50 cm$^3$. The path taken by the compression is easily plotted because $(P(N))(V(N))$ is a constant. Thus, at 50 cm$^3$ the pressure is 20 MPa (~200 atm). The area under this curve is the work required to compress the gas and it is also the heat output from the gas for the cycle. If the pressure is expressed in Pascals (Newton/sq. meter) (1 atm = $10^5$ N/m$^2$) and if the volume is expressed in m$^3$, then the units of work are $(N/m^2)(m^3) = N\cdot m = Joules = watt \ seconds$. For convenience, megapascals (MPa) and cm$^3$ will be used to avoid very large and very small numbers.*

The equation of the line is

$$(P(N))(V(N)) = 100 \times 10^5 \text{ Pa} \ (100 \times 10^{-6} \text{ m}^3) = 1000 \text{ Joules}$$

$$= 10 \text{ MPa} \ (100 \text{ cm}^3) = 1000 \text{ Joules}$$

The work increment is

$$d(W(N)) = P(N)(d(V(N))) = \frac{1000}{V(N)} d(V(N))$$

Integrating

$$W(1) = 1000 \int \frac{V(2)}{V(1)} d(V(N)) = 1000 \left[ \ln \frac{V(2)}{V(1)} \right]$$

Thus

$$W(1) = 1000 \ln \left( \frac{50}{100} \right) = -693.14 \text{ Joules}$$

The answer is negative because work is being supplied. Also by the perfect gas law,

$$P(N)(V(N)) = M(R)(T(N))$$

*Note that the nomenclature is defined as it is introduced. A full list of nomenclature is given in Appendix B.
Figure 5-2. Theoretical Cycles.
Thus

\[ P(N) = \text{gas pressure at point } N, \text{N/m}^2 \text{ or MPa} \]
\[ V(N) = \text{gas volume at point } N, \text{m}^3 \text{ or cm}^3 \]
\[ M = \text{number of moles, g mol} \]
\[ R = \text{universal gas constant} \]
\[ = 8.134 \text{ Joule/K (g mol)} \]
\[ TC(N) = \text{cold side temperature at point } N, \text{K} \]

\[ (10 \text{ MPa})(100 \text{ cm}^3) = M(8.314)(300) \]
\[ M = 0.4009 \text{ g mol} \]

Therefore, the formula for work normally given in textbooks is:

\[ W(1) = (M)(R)(TC(1)) \times \ln \left( \frac{V(2)}{V(1)} \right) = -693.14 \text{ Joules} \quad (5-3) \]

This quantity is also the negative of heat of the compression of the gas or the heat removed from the cycle.

Next from state 2 to 3 the gas is heated at constant volume from 300 to, say, 900 K. Assume for the moment that the regenerator that supplies this heat has no dead volume and is 100% effective. The heat that must be supplied to the gas by the regenerator matrix is:

\[ QR(2) = M(CV)(TH(3) - TC(2)) \quad (5-4) \]

where

\[ CV = \text{heat capacity at constant volume, j/K (g mol)} \]

For hydrogen

\[ CV = 21.030 \text{ at 600 K average temperature} \]

Therefore

\[ QR(2) = 0.4009(21.030)(900 - 300) \]
\[ = 5059 \text{ Joules} \]

Note that the heat transfer required in the regenerator is 7.3 times more than the heat rejected as the gas is compressed.

The pressure at state 3 after all gas has attained 900 K is:

\[ P(3) = M(R)(TH(3))/V(2) \]
\[ = 0.4009(8.314)(900)/50 \]
\[ = 60 \text{ MPa} \]

*Sometimes for clarity the asterisk (*) is used for multiplication as it is in FORTRAN and BASIC.*
Isothermal expansion of the gas from state 3 to state 4 (Figure 6-1) is governed by the same laws as the compression.

\[ W(3) = M(R)(TH(3)) \ln\left(\frac{V(1)}{V(2)}\right) \]

\[ = 0.4009(8.314)(900) \ln\left(\frac{100}{50}\right) = 2079.4 \text{ Joules} \]

This quantity is also the heat input to the engine. The expansion line is easily plotted when it is noted that \( P(N)(V(N)) = (60 \text{ MPa})(50 \text{ cm}^3) \)

\[ = 3000.0 \text{ Joules} \]

Finally the return of the expanded gas from state 4 to state 1 back through the regenerator finishes the cycle. The same formula applies as for heating.

\[ QR(4) = M(CV)(TC(1) - TH(4)) \]

\[ = 0.4009(21.030)(-900 + 300) \text{ Joules} \]

\[ = -5059 \text{ Joules} \]

Note that since heat capacity of the gas is not dependent on pressure and since the average temperature is the same, the heat transferred to and from the regenerator cancel.

The net work generated per cycle is:

\[ W_1 = W(1) + W(3) \]

\[ = W(\text{in}) + W(\text{out}) = -693.14 + 2079.4 \]

\[ = 1386.3 \text{ Joules} \]

The efficiency of the cycle therefore is:

\[ EF = \frac{\text{net work}}{\text{heat in}} = \frac{W_1}{W(3)} = \frac{1386.3}{2079.4} = 0.6667 \]

In general the efficiency is:

\[ EF = \frac{\text{work in} + \text{work out}}{\text{heat in}} = \frac{M(R)(TC(1))\ln\left(\frac{V(1)}{V(2)}\right) + M(R)(TH(3))\ln\left(\frac{V(1)}{V(2)}\right)}{M(R)(TH(3))\ln\left(\frac{V(1)}{V(2)}\right)} \]

\[ EF = \frac{TH(3) - TC(1)}{TH(3)} = \frac{900 - 300}{900} = 0.6667 \]

This efficiency formula is recognized as the Carnot efficiency formula. Therefore, the limiting efficiency of the Stirling cycle is as high as is possible. We will consider the other cycles represented on Figure 5-2 after considering the effect of the regenerator.
5.1.2 Stirling Cycle, Zero Dead Volume, Imperfect Regenerator

Stirling engines require highly efficient regenerators. Consider an annular gap around the displacer which acts as gas heater, regenerator and cooler (see Figure 5-3). Assume that this engine operates in a stepwise manner and that this annular gap has negligible dead volume. Let $E$ be the regenerator effectiveness during the transfer. For the transfer from cold space to hot space:

Let
- $T_L =$ temperature of gas leaving regenerator
- $T_C = T_C(N)$ for any $N$
- $T_H = T_H(N)$ for any $N$

$$E = \frac{T_L - T_C}{T_H - T_C} \quad (5-7)$$

Now during transfer the heat from the regenerator is:

$$Q_R = M(CV)(T_L - T_C) \quad (5-8)$$

and the heat from the gas heater is:

$$Q_B = M(CV)(T_H - T_L) \quad (5-9)$$

Therefore, the efficiency becomes:

$$E_F = \frac{M(R)(T_H)\ln\left(\frac{V_2}{V_1}\right) - M(R)(T_C)\ln\left(\frac{v(1)}{v(2)}\right)}{M(R)(T_H)\ln\left(\frac{V_2}{V_1}\right) + M(CV)(T_H - T_L)} \quad (5-10)$$

which reduces to:
\[
\text{EF} = \frac{\text{TH} - \text{TC}}{\text{TH} + \frac{\text{CV}}{\text{R}} \left( \frac{(\text{TH} - \text{TC})(1 - E)}{\ln(V(1)/V(2))} \right)}
\]

For the numerical example being used here:

\[
\text{EF} = \frac{900 - 300}{900 + \frac{21.030 (900-300)}{8.314 \ln \frac{100}{50}} (1 - E)} = \frac{600}{900 + 2189.5 (1 - E)}
\]

Figure 5-4 shows how the engine efficiency is affected by regenerator effectiveness for this numerical example. Some of the early Stirling engines worked with the regenerator removed. Figure 5-4 shows that at low regenerator effectiveness, the efficiency is still reasonable. How close it pays to approach 100% effectiveness depends on a trade-off which will be discussed under Section 5.3.

Rallis (77 ay) has worked out a generalized cycle analysis in which the compression and expansion is isothermal but the heating and cooling can be at constant volume or at constant pressure or a combination. The heating process does not need to be the same as the cooling process. He assumes no dead volume, but allows for imperfect regeneration. For a Stirling cycle he derives the formula:
\[ EF = \frac{(KK - 1)(TA - 1) \ln VR}{(1 - E)(TA - 1) + TA(KK - 1) \ln VR} \] (5-12)

where

- \( EF \) = cycle efficiency
- \( KK = \frac{CP}{CV} \)
- \( TA = \frac{TH}{TC} \)
- \( VR = \frac{V(1)}{V(2)} \)

Equations 5-12 and 5-11 are the same, just different nomenclature. Note that for \( E = 1 \), both Equations 5-11 and 5-12 reduce to the Carnot equation, Equation 5-6.

Rallis (77 ay) also derived a formula for the Ericsson cycle efficiency:

\[ EF = \frac{(KK - 1)(TA - 1) \ln VR}{KK(1 - E)(TA - 1) + TA(KK - 1) \ln VR} \] (5-13)

Equation 5-13 also reduces to Equation 5-6 when \( E = 1 \), that is, for perfect regeneration. To attain Carnot efficiency, the compression and expansion ratio must be the same. Rallis shows this using cycles which will not be treated here.

Rallis also gives a useful formula for the net work per cycle for the Stirling cycle:

\[ WI = \frac{VR(TA - 1) \ln VR}{(V(1)) - V(2))(P(1))} \] (5-14)

For instance, for the numerical example being used here:

\[ WI = (50 \text{ cc})(10 \text{ MPa})2(3 - 1) \ln(2/(2 - 1)) \]

\[ = 1386.3 \text{ Joules} \]

which is the same as obtained previously.

### 5.1.3 Otto Cycle, Zero Dead Volume, Perfect or Imperfect Regeneration

The variable volume spaces in Stirling engines are usually shaped so that there is little heat transfer possible between the gas and the walls during the time the gas is expanded or compressed. Analyses have been made by Rallis (77 az) and also by Martini (69 a) which assume adiabatic compression and expansion with the starting points being the same as for the Stirling cycle. For instance for the numerical example in Figure 5-2, compression goes from 1 to 2" instead of from 1 to 2. Expansion goes from 3 to 4" instead of from 3 to 4. It appears that considerable area and therefore work per cycle is lost.

However, this process is not correct because the pressure at point 3 is not the same as for the isothermal case. For the numerical example after compression to point 2" the pressure of the gas is 26.39 MPa and the gas temperature is 396 K. As this gas moves into the hot space through a cooler, regenerator and heater, all of negligible dead volume, it is cooled to 300 K in the cooler, heated to 900 K in the heater. As the gas is transferred at zero total volume
change from the cold space to the hot space the pressure rises. This pressure
rise results in a temperature increase in the gas due to adiabatic compression.
Therefore, at the end of the transfer process the mixed mean gas temperature
in the hot space will be higher than 900 K. Point 3 is calculated for all the
gas to be exactly 900 K. Adiabatic expansion then takes place. Then by the
same process as just described, the transfer of the expanded gas back into the
cold space results in a lower gas temperature than 300 K at the end of this
stroke. The computational process must be carried through for a few cycles
until this process repeats accurately enough. This effect will be discussed
further in Section 5.1.6.

5.1.4 Stirling Cycle, Dead Volume, Perfect or Imperfect Regeneration

An inefficient regenerator backed up by an adequate gas heater and gas cooler
will not change the work realized per cycle but will increase the heat required
per cycle. It will now be shown that addition of dead volume which must be
present in any real engine decreases the work available per cycle.

Assume that the annulus between displacer and cylinder wall (see Figure 5-3)
has a dead volume of 50 cm$^3$, that the temperature gradient from one end of
the displacer to the other is uniform and that the pressure is essentially
constant. The gas contained in this annulus is:

$$
M = \frac{P(1)}{R} \int_{X=0}^{X=LR} \frac{d(VA)}{TZ}
$$

(5-15)

where

- $M =$ moles of gas
- $VA =$ total volume of annulus
- $d(VA) = (VA) \ dX =$ differential volume of the annulus
- $X =$ distance along annulus
- $LR =$ total length of annular regenerator
- $TZ =$ temperature along regenerator

Now

$$
TZ = TH - \frac{X}{LR} (TH - TC)
$$

(5-16)

By substituting and integrating one obtains:

$$
M = \frac{P(1)(VA)}{R} \ln(TH/TC) (TH - TC)
$$

(5-17)

Thus the effective gas temperature of the regenerator dead volume is:

$$
TR = (TH - TC)/\ln(TH/TC)
$$

(5-18)

which is the log mean temperature. Thus for the numerical example:

$$
TR = \frac{900 - 300}{\ln \frac{900}{300}} = 546.1 \ K
$$
Quite often it is assumed that TR = \( \frac{TH + TC}{2} = \frac{900 + 300}{2} = 600 \) K.

For the large dead volumes which will almost always result, it is important to have the right gas temperatures for the regenerator and heat exchangers.

Assume for the moment that the hot and cold gas spaces can be maintained at 900 K and 300 K and that the pressure at the end of the expansion stroke, (Point 4 of Figure 5-2) 30 MPa (~300 atm), is maintained. The gas inventory must be increased. It now is:

\[
M = \frac{P(4)}{R} \left[ \frac{VH + VR}{TH + TR} \right] \\
M = \frac{30}{8.314} \left[ \frac{100 + 50}{900 + 546.1} \right] \\
= 0.7313 \text{ g mol.} 
\]

The equation for the gas expansion is:

\[
P(N) = \frac{(M)(R)}{HL(N) + VR} = \frac{(0.7313)(8.314)}{HL(N) + 50} \\
P(N) = \frac{A}{HL(N) + B} \text{ where } A = 5472; \ B = 82.4 \\
\text{where} \\
HL(N) = \text{hot live volumes at point N} \\
\text{The work output by expanding from } HL(1) = 50 \text{ cm}^3 \text{ to } HL(2) = 100 \text{ cm}^3 \text{ is:}
\]

\[
W(3) = \int_{HL(1)}^{HL(2)} P(N)d(HL(N)) = \int_{HL(1)}^{HL(2)} \frac{A}{HL(N) + B}d(HL(N)) \\
= A \ln \left( \frac{HL(2) + B}{HL(1) + B} \right) \\
= 5472 \ln \left( \frac{100 + 82.4}{50 + 82.4} \right) \\
= 1753 \text{ Joules}
\]

The equation for gas compression is:

\[
P(N) = \frac{(M)(R)}{CL(N) + VR} = \frac{(0.7313)(8.314)}{CL(N) + 50} \\
= \frac{CL(N)}{300 + 546.1}
\]
where \( CL(N) = \) cold live volume at point \( N \)

\[
P(N) = \frac{C}{CL(N) + D} \quad \text{where} \quad C = 1824.02, \quad D = 27.4
\]

Analogously, the work of compression is:

\[
W(1) = C \ln \left( \frac{CL(2) + D}{CL(1) + D} \right)
\]

\[
= 1824.02 \ln \left( \frac{50 + 27.4}{100 + 27.4} \right)
\]

\[
= -908.37 \text{ Joules}
\]

Therefore the net work is:

\[
W1 = W(3) + W(1)
\]

\[
= 1753.08 - 908.37 = 844.71 \text{ Joules}
\]

Figure 5-5 shows how dead volume as \% of maximum total gas volume affects the work per cycle. For more generality the work per cycle is expressed as a \% of the work per cycle at zero dead volume. Note that the relationship is almost linear. This curve differs from that published by Martini (77 h) in that in Figure 5-5 the pressure at the end of the expansion stroke was made the same (average pressure). In the previous Figure 2 of reference 77 h, the minimum pressure was made the same. This caused the average pressure to decrease more rapidly as dead volume increased. Figure 5-5 is more truly representative of the effect of dead volume on work per cycle.

5.1.5 Schmidt Cycle

The Schmidt cycle is defined here as a Stirling cycle in which the displacer and the power piston or the two power pistons move sinusoidally. It is the most complicated case that can be solved analytically. All cases with less restrictive assumptions have had to be solved numerically. The cycle gets its name from Gustaf Schmidt (1871 a) who first published the solution.

The assumptions upon which the Schmidt analysis is based are as follows:

1. Sinusoidal motion of parts.
2. Known and constant gas temperatures in all parts of the engine.
3. No gas leakage.
5. At each instant in the cycle the gas pressure is the same throughout the working gas.

Since Gustaf Schmidt did the analysis, a number of others have checked it through and re-derived it for specific cases. A more accessible paper for those who want to delve into the mathematics was written by Finkelstein (60 j). In this manual the Schmidt cycle will first be evaluated numerically because it is easier to understand this way. Also, the numerical method is easy to generalize to more nearly fit what a machine is actually doing. Piston-displacer engines will be discussed first and then dual-piston engines.
5.1.5.1 Piston-Displacer Engines

5.1.5.1.1 Engine Definition

The nomenclature for engine internal volumes and motions is described in Figures 5-6 and 5-7. The following equations describe the volumes and pressures:

The maximum hot, live volume is:

\[ VL = 2(RC)(DB)^2(\pi/4) \]  \hspace{1cm} (5-22)

The maximum cold, live volume associated with the displacer is:

\[ VK = 2(RC)[(DB)^2 - (DD)^2](\pi/4) \]  \hspace{1cm} (5-23)

*In Equations 5-20 and 5-21, \( HL(N) \) is defined as an array of hot live volumes at \( N \) points during the cycle. \( VL \) is the maximum hot live volume.
The maximum cold, live volume associated with the power piston is:

\[ VP = 2(R2) \left[ (DC)^2 - (DD)^2 \right] (\pi/4) \]  

(5-23a)

For any angle \( F \), the array of hot volumes is:

\[ H(N) = \frac{VL}{2} \left[ 1 - \cos(F) \right] + HD \]  

(5-24)

For any angle \( F \), the array of cold volumes is:

\[ C(N) = \frac{VK}{2} \left[ 1 + \cos(F) \right] + CD + \frac{VP}{2} \left[ 1 - \cos(F - AL) \right] \]  

(5-25)

Therefore, the total gas volume at any crank angle is:

\[ V(N) = H(N) + C(N) + RD \]  

(5-26)

Therefore, by the perfect gas law the pressure at any crank angle is:

\[ P(N) = \frac{M(R)}{\frac{TH}{TH} + \frac{C(N)}{TC} + \frac{RD}{TR}} \]  

(5-27)
Figure 5.7. Phasing of Displacer and Power Piston.

The volume CD includes the dead volume in the cooler as well as the dead volume between the strokes of the displacer and the power piston. According to the classification of engines given in Figure 2-6, the gamma type machine must have some volume between the strokes to allow for clearance and the flow passages between. In the beta type engine the strokes of the displacer and the power piston should overlap so that they almost touch at one point in the cycle. This overlap volume is subtracted from the dead volume in the cold heat exchanger. For a beta type engine with this type of stroke overlap and AL = 90° and VP = VK, then CD = VM - (VP/2)(2 - \sqrt{2}) = VM - VP(1 - (\sqrt{2}/2))

where VM = cold dead volume in heat exchanger and clearances and ducts. For the more general case, one should determine the clearance between the displacer and power piston and adjust it to be as small as practical.
5.1.5.1.2 Sample Engine Specifications

In order to check equations which look quite different, it was decided to specify a particular engine and then determine if the work integral checks. The specification decided upon was:

\[ M(R) = 10.518 \text{ J/K} \]
\[ TH = 600 \text{ K} \]
\[ TC = 300 \text{ K} \]
\[ VL = VK = VP = RD = 40 \text{ cm}^3 \]
\[ HD = CD = 0 \]
\[ AL = 90^\circ \]

TR is defined a number of ways, depending on how it is defined in the analytical equation that is being checked. It may be:

1. Arithmetic mean (Walker)
\[ TR = (TH + TC)/2 = 450 \text{ K} \]

2. Log mean, most realistic
\[ TR = (TH - TC)/\ln(TH/TC) = 432.8 \text{ K} \]

3. Half volume hot, half volume cold (Mayer)
\[ \frac{1}{TR} = \frac{1}{2(TH)} + \frac{1}{2(TC)} \]
\[ TR = 400 \text{ K} \]

The above sample engine specification is for a gamma engine. For a beta engine assume in addition that \( VM = 0 \). Then:
\[ CD = 0 - 40(1 - \frac{\sqrt{2}}{2}) = -11.715 \text{ cm} \]

5.1.5.1.3 Numerical Analysis

Using the numbers given in Section 5.1.5.1.2, Equations 5-22 to 5-27 can be evaluated for \( F = 0, 30, 60 \ldots 360 \), \( P(N) \) can be plotted against \( V(N) \) and the resultant closed curve can be integrated graphically and the maximum and minimum gas pressure can be noted. The author's experience with a number of different examples gives a result which is 4.5% low when compared with valid analytical equations and with numerical calculations with very small crank angle increments. If the reader has access to a programmable calculator or a computer then the computation can be made with any degree of precision desired. Figure 5.8 shows the flow diagram which was used for programming. The author has used both an HP-65 and an HP-67 for this purpose. He has also used this method as part of a larger second-order calculation written in FORTRAN and in BASIC.

Using the 400 K effective regenerator temperature the following results were obtained for the numerical example.

<table>
<thead>
<tr>
<th>Angle Increment, ND, degrees</th>
<th>Work Integral ( \int P(N)dV(N) )</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>314.36 Joules</td>
<td>-4.5</td>
</tr>
<tr>
<td>20</td>
<td>322.56</td>
<td>-2.0</td>
</tr>
<tr>
<td>10</td>
<td>327.53</td>
<td>-0.50</td>
</tr>
<tr>
<td>5</td>
<td>328.78</td>
<td>-0.13</td>
</tr>
<tr>
<td>0.25</td>
<td>329.1994570</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Mayer Equation</td>
<td>329.2005026</td>
<td>0</td>
</tr>
</tbody>
</table>
START

INPUT DIMENSIONS

CALCULATE EQUATION CONSTANTS

INITIALIZE STORAGE REGISTERS

DISPLAY F (OPTIONAL)

CALC AND STORE C(N), H(N), V(N)

DISPLAY V(N) (OPTIONAL)

CALCULATE AND STORE P(N)

DISPLAY P(N) (OPTIONAL)

F = F + ND

PUT V(N) AND P(N) IN SECOND STORAGE REGISTERS

YES

F = ND?

NO

CALCULATE + ACCUMULATE WORK INTEGRAL

FIND PX AND F AT PX

YES

F ≤ 360°?

NO

DISPLAY WORK INTEGRAL PX AND F AT PX

STOP

Figure 5-8. Flow Diagram for Work Integral Analysis.
The Mayer equation will be given in Section 5.1.5.1.4 and discussed more fully there. It uses the same assumptions as were employed in the numerical analysis. One can see from the above table that the result by numerical analysis approaches the Mayer equation result as ND approaches zero. The two check.

If the arithmetic average is used TR = 450 K, then:

<table>
<thead>
<tr>
<th>ND</th>
<th>$\int PdV$</th>
<th>Maximum Pressure, PX</th>
<th>Crank Angle F at PX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 degree</td>
<td>360.45 Joules</td>
<td>58.10 MPa</td>
<td>117 deg.</td>
</tr>
</tbody>
</table>

If the log mean average is used TR = 432.8 K, then:

<table>
<thead>
<tr>
<th>ND</th>
<th>$\int PdV$</th>
<th>PX</th>
<th>F at PX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 degree</td>
<td>350.04 Joules</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the case of the beta engine with essentially touching displacer and power piston at one point in the cycle, CD = -11.715 cm³. For the arithmetic average dead volume temperature TR = 450 K, then:

<table>
<thead>
<tr>
<th>ND</th>
<th>$\int PdV$</th>
<th>PX</th>
<th>F at PX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 degree</td>
<td>516.32 Joules</td>
<td>74.0862 MPa</td>
<td>117 deg.</td>
</tr>
</tbody>
</table>

Precision in calculating this work integral is mainly of academic interest because the result will be multiplied in first-order analysis by an experience factor like 0.5 or 0.6 (one figure precision). Even in second- or third-order analysis, no more than two figure accuracy in the final power output and efficiency should ever be expected. Thus errors less than 1% should be considered insignificant. Therefore, ND = 15° would be adequate for all practical purposes. This error in evaluating the work integral by using large angle increments seems to be insensitive to other engine dimensions. Therefore, one could evaluate the work integral using 30° increments and then make a correction of 4.5%.

5.1.5.1.4 Schmidt Equations

The literature was searched to find all the different Schmidt equations. Quite a large number were found which looked to be different. In this section and in Section 5.1.5.2.3 for the dual piston case these equations will be given and evaluated by determining whether they agree with the numerical analysis just described.

At McDonnell Douglas, Mort Mayer reduced the Schmidt equation to the following relatively simple form (68 c):

$$ W_1 = \frac{M(R)(TC)(\pi)(VP)}{Y^2 + Z^2} \left[ \frac{X}{(X^2 - Y^2 - Z^2)^{1/z}} - 1 \right] $$  (5-28)

where:
\[ W_I = \text{work per cycle, J} \]
\[ M = \text{gas inventory, g mol} \]
\[ R = \text{gas constant} = 8.314 \text{ J/g mol} \cdot \text{K} \]
\[ TC = \text{effective cold gas temperature, K} \]
\[ TH = \text{effective hot gas temperature, K} \]
\[ X = XX + \frac{TC}{TH} (XY) \]
\[ XX = \frac{VP}{2} + CD + \frac{VK}{2} + \frac{RD}{2} \]
\[ XY = \frac{VL}{2} + \frac{RD}{2} \]
\[ Y = \frac{VL}{2} \left(1 - \frac{TC}{TH}\right) \sin(AL) \]
\[ Z = \left[ VP - VL \left(1 - \frac{TC}{TH}\right) \cos(AL) \right] / 2 \]
\[ AL = \text{phase angle between displacer and power piston, normally 90°} \]

From the sample engine specifications:
\[ XX = \frac{40}{2} + 0 + \frac{40}{2} + \frac{40}{2} = 60 \text{ cm}^3 = 60 \times 10^{-6} \text{ m}^3 \]
\[ XY = 0 + \frac{40}{2} + \frac{40}{2} = 40 \text{ cm}^3 = 40 \times 10^{-6} \text{ m}^3 \]
\[ X = 60 \times 10^{-6} + \frac{300}{600} \left(40 \times 10^{-6}\right) = 8 \times 10^{-5} \text{ m}^3 \]
\[ Y = \frac{40 \times 10^{-6}}{2} \left(1 - \frac{300}{600}\right) = 1 \times 10^{-5} \text{ m}^3 \]
\[ Z = \frac{40 \times 10^{-6}}{2} = 2 \times 10^{-5} \text{ m}^3 \]

Using these inputs the Mayer equation gives:
\[ W = 329.2005026 \text{ Joules} \]

The Mayer equation evaluates the integral exactly given the assumptions that were used in its derivation, like sinusoidal motion and half the dead space at hot temperature and half at cold temperature. The numerical method (Section 5.1.5.1.3) approaches this same value as the angle increment approaches zero. The Mayer equation must have \( VP = VK \).

J. R. Senft (76 n) presents a Schmidt equation for finding the energy generated per cycle. He assumes that the temperature of the dead space gas has the arithmetic mean between the hot and cold gas spaces. This equation is for a beta type engine with the displacer and power piston essentially touching at one point during the cycle. His equation is:

\[ W_I = \frac{\pi(1 - AU)PVL(XY) \sin(AL)}{Y + \left(Y^2 - X^2\right)^{1/2} \left[ Y - X \right]^{1/2}} \]  

(5-29)

where:
\[ X = \left[ (AU - 1)^2 + 2(AU - 1)(XY) \cos(AL) + (XY)^2 \right]^{1/2} \]

\[ Y = AU + 4(XX)(AU)/(1 + AU) + Z \]

\[ Z = (1 + (XY)^2 - 2(XY) \cos(AL))^{1/2} \]

\[ AU = TC/TH \]

\[ XX = \frac{RD + HD + CD}{VL} \]

\[ VL = VK \]

\[ XY = VP/VL \]

In order to illustrate and check this equation it is evaluated for a specific case previously computed by numerical methods. (See Section 5.1.5.1.3 for \( TR = 450 \) K and \( CK = -11.715 \) cm\(^3\).

\[ AU = \frac{300}{600} = 0.5 \]

\[ XX = \frac{40}{40} = 1 \]

\[ XY = \frac{40}{40} = 1 \]

\[ AL = 90^\circ \]

\[ PX = \text{maximum pressure attained during each cycle} = 74.0862 \text{ MPa} \]

\[ Z = (1 + 1 - 2(1) \cos 90^\circ)^{1/2} = \sqrt{2} \]

\[ Y = 0.5 + \frac{4(1)(0.5)}{1.5} + \sqrt{2} = 3.247547 \]

\[ X = \left[ (0.5 - 1)^2 + 2(0.5 - 1)(1)(\cos 90^\circ) + 1 \right]^{1/2} = 1.118034 \]

\[ \frac{Y - X}{Y + X} = 0.698424 \]

\[ Y + (Y^2 - X^2)^{1/2} = 6.296573 \]

\[ WI = \frac{\pi(1 - 0.5)(74.0826)(40)(1) \sin (90)(0.698424)}{6.296573} \]

\[ = 516.33 \text{ Joules} \]

This answer agrees very well with results obtained by numerical methods of 516.32 Joules. Senft (77 ak) also has adapted his equation for a gamma type engine (without stroke overlap). In this case the equations for \( WI \) and \( X \) are the same and the equation for \( Y \) is:

\[ Y = \frac{4(XX)(AU)}{(1 + AU)} + 1 + AU + XY \]
Therefore:
\[
Y = \frac{4(1)(0.5)}{1.5} + 1 + 0.5 + 1 = 3.833333
\]
\[
\left[\frac{Y - \frac{Y}{Y + X}}{Y + X}\right] = 0.740518
\]
\[Y + (Y^2 - X^2)^{0.5} = 7.5000.
\]

To agree with the numerical analysis of Section 5.1.5.1.3 for TR = 450 K, PX = 58.10 MPa.

Thus:
\[
W_1 = \pi(1 - 0.5)(58.10)(40) \sin (90^\circ)(0.740518) \quad \frac{7.50000}{7.50000}
\]
\[W_1 = 360.45 \text{ Joules}
\]

This result agrees exactly with the numerical analysis for ND = 1°, TD = 450 K and PX = 58.10 MPa. (See Section 5.1.5.1.3.)

This new Senft equation is also correct.

Cooke-Yarborough (74 i) has published a simplified expression for power output which makes the approximation that not only the volume changes but also the pressure changes are sinusoidal. The regenerator is treated as being half at the hot volume temperature and half at the cold volume temperature. His equation is:

\[
W_1 = \frac{\bar{P}^2}{4} \frac{(VL)(VP)(TH - TC) \sin (AL)}{XX[TC + \frac{XY}{XX}(TH - TC)]} \quad (5-31)
\]

where:

\[\bar{P} = \text{mean pressure of working gas, or pressure with both displacer and power piston at mid-stroke.} \] (With the approximations used, these two pressures can be regarded as identical.) If the mean pressure is known, it can be used directly in Equation 5-31. Otherwise, the mid-stroke pressure can be calculated as follows:

\[
\bar{P} = \frac{(M)(R)}{2(TH) + TR + 2(TC) + 2(VC)}
\]

Substituting the assumed values,

\[
\bar{P} = \frac{10.518}{20} \frac{600 + 40}{600 + 432.8 + 300 + 300} = \frac{10.518}{600 + 432.8 + 300 + 300}
\]
\[ P = 40.59 \text{ MPa} \]
\[ VL = 40 \text{ cm}^3 \]
\[ VP = 40 \text{ cm}^3 \]
\[ XX = \text{total gas volume of system when output piston is at midstroke} \]
\[ = VL + RD + (VP/2) \]
\[ = 40 + 40 + 20 = 100 \text{ cm}^3 \]
\[ TH - TC = 600 - 300 = 300 \text{ K} \]
\[ AL = 90^\circ \]
\[ XY = \text{cold gas volume with both piston and displacer at midstroke} \]
\[ \text{and regenerator volume split between hot and cold volumes} \]
\[ = \frac{RD}{2} + \frac{VK}{2} + \frac{VP}{2} \]
\[ = \frac{40}{2} + \frac{40}{2} + \frac{40}{2} = 60 \text{ cm}^3 \]

Therefore, substituting into Equation 6-31 we have:

\[ W1 = \frac{40.59(40)}{4} \frac{40(40)}{100} \frac{300(1)}{300 + 60(300)} \]
\[ = 318.79 \text{ Joules} \]

Because of how XY is determined this result should be compared to the Mayer equation, that is, to 329.20 Joules. Therefore, the Cooke-Yarborough equation appears to be a reasonably good approximation (3.2% error). The accuracy improves as the dead volume is increased because the pressure waveform is then more nearly sinusoidal.

5.1.5.2 Dual Piston Engines

5.1.5.2.1 Engine Definition and Sample Engine Specifications

The nomenclature for engine internal volumes and motions are described in Figure 5-9. Also given in Figure 5-9 are the assumed values for the sample case. The following equations describe the volumes and pressures.

Hot Volume
\[ H(N) = \frac{VL}{2} [1 - \sin (F)] + HD \] (5-32)

Cold Volume
\[ C(N) = \frac{VK}{2} [1 - \sin (F - AL)] + CD \] (5-33)

Total Volume
\[ V(N) = H(N) + C(N) + RD \] (5-34)
### Dual Piston Engine Nomenclature and Assumptions for Sample Case

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
<th>Assumed Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD</td>
<td>hot dead volume</td>
<td>cm³</td>
<td>0</td>
</tr>
<tr>
<td>RD</td>
<td>regenerator dead volume</td>
<td>cm³</td>
<td>40</td>
</tr>
<tr>
<td>CD</td>
<td>cold dead volume</td>
<td>cm³</td>
<td>0</td>
</tr>
<tr>
<td>VL</td>
<td>hot piston live volume</td>
<td>cm³</td>
<td>40</td>
</tr>
<tr>
<td>VK</td>
<td>cold piston live volume</td>
<td>cm³</td>
<td>40</td>
</tr>
<tr>
<td>TH</td>
<td>effective hot gas temperature</td>
<td>K</td>
<td>600</td>
</tr>
<tr>
<td>TC</td>
<td>effective cold gas temperature</td>
<td>K</td>
<td>300</td>
</tr>
<tr>
<td>TR</td>
<td>effective regenerator gas temp.</td>
<td>K</td>
<td>450</td>
</tr>
<tr>
<td>M</td>
<td>engine gas inventory</td>
<td>g mol⁻¹</td>
<td>1.265</td>
</tr>
<tr>
<td>R</td>
<td>gas constant</td>
<td>J/g mol⁻¹·K</td>
<td>8.314</td>
</tr>
<tr>
<td>M(R)</td>
<td>common gas pressure</td>
<td>MPa</td>
<td>10.518</td>
</tr>
<tr>
<td>P(N)</td>
<td>crank angles</td>
<td>degrees</td>
<td>(ND)(N) = 360</td>
</tr>
<tr>
<td>F</td>
<td>crank angle increment</td>
<td>degrees</td>
<td>N = integer</td>
</tr>
</tbody>
</table>

Figure 5-9. Dual Piston Engine Nomenclature and Assumptions for Sample Case.
5.1.5.2.2 Numerical Analysis

Using the assumed values given in Figure 5-9, Equations 5-32 to 5-35 were evaluated for F = 0, 30, 60 ... 360. The results were:

<table>
<thead>
<tr>
<th>F Degrees</th>
<th>V(N) cm³</th>
<th>P(N) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.0</td>
<td>41.2</td>
</tr>
<tr>
<td>30</td>
<td>87.3</td>
<td>45.7</td>
</tr>
<tr>
<td>60</td>
<td>72.7</td>
<td>54.4</td>
</tr>
<tr>
<td>90</td>
<td>60.0</td>
<td>67.6</td>
</tr>
<tr>
<td>120</td>
<td>52.7</td>
<td>83.0</td>
</tr>
<tr>
<td>150</td>
<td>52.7</td>
<td>91.9</td>
</tr>
<tr>
<td>180</td>
<td>60.0</td>
<td>86.1</td>
</tr>
<tr>
<td>210</td>
<td>72.7</td>
<td>71.2</td>
</tr>
<tr>
<td>240</td>
<td>87.3</td>
<td>57.0</td>
</tr>
<tr>
<td>270</td>
<td>100.0</td>
<td>47.3</td>
</tr>
<tr>
<td>300</td>
<td>107.3</td>
<td>41.9</td>
</tr>
<tr>
<td>330</td>
<td>107.3</td>
<td>39.9</td>
</tr>
<tr>
<td>360</td>
<td>100.0</td>
<td>41.2</td>
</tr>
</tbody>
</table>

These data were graphed in Figure 5-10 and graphically integrated. A value of 695.3 J was obtained. As before, a numerical integration was carried along as the points were calculated. This was 668.8 Joules, a 3.8% error which indicates the accuracy of the graphical integration procedure. To approach the answer that should be obtained by valid Schmidt equations, ND should be reduced toward zero. The results obtained were:

<table>
<thead>
<tr>
<th>Angle Increment, degrees</th>
<th>Work Integral, Joules</th>
<th>Maximum Pressure, MPa</th>
<th>Effective Regen. Temp. K</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>668.8</td>
<td>91.87</td>
<td>450</td>
<td>-4.5</td>
</tr>
<tr>
<td>10</td>
<td>696.8</td>
<td>91.98</td>
<td>450</td>
<td>-0.5</td>
</tr>
<tr>
<td>1</td>
<td>700.324</td>
<td>91.98</td>
<td>450</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>641.284</td>
<td>89.121</td>
<td>432.8</td>
<td>-4.5</td>
</tr>
<tr>
<td>1</td>
<td>671.517</td>
<td>89.220</td>
<td>432.8</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>587.9</td>
<td>80.0</td>
<td>400</td>
<td>-4.5</td>
</tr>
<tr>
<td>1</td>
<td>615.619</td>
<td>83.831</td>
<td>400</td>
<td>0</td>
</tr>
</tbody>
</table>

Note the difference in the result depending on what is used for the effective temperature of the gas in the regenerator. If the regenerator has a uniform temperature gradient from hot to cold, which it usually does, then the log mean temperature (TR = 432.8 K) is correct. The arithmetic mean (TR = 450 K) gives a result for this numerical example 4.3% high. The assumption that the regenerator is half hot and half cold (TR = 400 K) gives a result 9.1% low.

83
5.1.5.2.3 Schmidt Equations

Walker (73 j, 78 dc) gives a Schmidt equation most adaptable to the two piston engine.

\[
W_1 = (P_X)(V_T)\frac{(A_U - 1)}{(K + 1)} \left( \frac{1 - K L}{1 + K L} \right)^{1/2} \frac{D L \sin(E_T)}{1 + (1 - (D L)^2)^{1/2}} \tag{5-36}
\]

where:

- \(W_1\) = work per cycle, Joules
- \(P_X\) = maximum pressure during cycle, MPa
- \(V_T = V_L + V_K = (1 + K)V_L\)
- \(V_L\) = swept volume in expansion space
- \(V_K\) = swept volume in compression space
- \(K\) = swept volume ratio = \((V_K)/(V_L)\)
- \(A_U = T_C/T_H\)
- \(T_C\) = compression space gas temperature
- \(T_H\) = expansion space gas temperature

Figure 5-10. Work Diagram for Dual Piston Sample Case (ND = 30°).
\[ TR = \text{dead space gas temperature} = \frac{(TC + TH)}{2} \]
\[ DL = \frac{((AU)^2 + 2(AU)(K) \cos (AL) + K^2)^{1/2}}{(AU + K + 2S)} \]
\[ AL = \text{angle by which volume variations in expansion space lead those in compression space, degrees} \]
\[ S = 2(RV)(AU)/(AU + 1) \] (This is where the arithmetic average temperature for the regenerator enters.)
\[ RV = VD/VL, \text{ dead volume ratio} \]
\[ VD = \text{total dead volume, cm}^3 = HD + RD + CD \]
\[ ET = \tan^{-1} \left( \frac{K \sin (AL)}{(AU + K \cos (AL))} \right) \] (Note that ET is defined incorrectly in Walker's table of nomenclature and on page 36, but is right on page 28 of reference 73 j.)

Now in order to check this equation against numerical analysis, it should give a work per cycle of slightly greater than 700.324 Joules when 91.98 MPa is used as the maximum pressure. TR = 450 K is the same assumption for both (see Section 5.1.5.2.2).

Therefore to evaluate:

\[ VT = 40 + 40 = 80 \text{ cm}^3 \]
\[ K = VK/VL = 40/40 = 1 \]
\[ PX = 91.98 \text{ MPa} \]
\[ AU = TC/TH = 300/600 = 0.5 \]
\[ RV = VD/VL = 40/40 = 1 \]
\[ S = 2(1)(0.5)/(0.5 + 1) = 2/3 \]
\[ DL = (0.5^2 + 1^2)^{1/2}/(0.5 + 1 + 2(2/3)) = 0.39460 \]
\[ ET = \tan^{-1} (1/0.5) = 63.43^\circ \]
\[ W1 = -700.37 \text{ Joules} \]

Thus the formula checks to 4 figure accuracy except for the sign.

Walker obtained the above equation along with most of the nomenclature from the published Philips literature. Meijer's thesis contains the same formula (see page 12 of reference 60 c), except Meijer uses (1 - AU) instead of (AU - 1) and a positive result would therefore be obtained.

In Meijer's thesis (60 c), the quantity S is defined so that dead spaces in heaters, regenerator and coolers and clearance spaces in the compression and expansion spaces, all of which have different temperatures associated with them, can be accommodated.

Thus:

\[ S = \sum_{s=1}^{s=n} \frac{V(S) \ TC}{VL \ T(S)} \]  

(5-37)

where \( V(S) \) and \( T(S) \) are the volumes and absolute temperatures of the dead spaces. Using this formula it would be possible to use the more correct log mean temperature for the regenerator. Thus:
The above equation then evaluates to:

\[ P = 671.537 \text{ Joules} \]

This is within 0.003% of the value of 671.517 computed numerically for 1 degree increments (see Section 5.1.5.2.2).

Finkelstein (61 e, 60 j) independently of Meijer derived the following formula for the work per cycle:

\[
W_1 = \frac{(2\pi)(K)(1 - AU)(\sin(AL))(M)(R)(TC)}{(AU + K + (2)(S))\sqrt{1 - (DL)^2(1 + \sqrt{1 - (DL)^2})}} \tag{5-38}
\]

This equation looks quite different from Equation 5-36. It is somewhat simpler but requires the amount of gas in the engine to be specified instead of the maximum pressure.

Using the last numerical example:

\[
S = \frac{40(300)}{40(321.8)} = 0.693
\]

\[ AU = 0.5 \]

\[ K = 1 \]

\[ AL = 90^\circ \]

\[ (M)(R)(TC) = 10.518(300) = 3155.4 \]

\[ DL = \sqrt{1.25}/(1.5 + 2S) = 0.38735 \]

Therefore, the work per cycle is:

\[ W_1 = 671.55 \text{ Joules} \]

This result compares with 671.537 by the Meijer formula and with 671.517 by numerical analysis with 1 degree increments. Therefore, the above formula is correct and is also useful in computing the work output per cycle.
5.1.6 Finkelstein Adiabatic Cycle

The next step toward reality in cycle analysis beyond the Schmidt cycle is to assume that the hot and cold spaces of the engine have no heat transfer capability at all. That is, they are assumed to be adiabatic. For all but miniature engines this is a better assumption than assuming they are isothermal as the Schmidt analysis does. It is still assumed that the heat exchangers and the regenerator are perfect. The cycle has been named by Walker (78 dc) the Finkelstein adiabatic cycle because it was first calculated by Finkelstein (60 v) who was the first to compute it using a mechanical calculator (one case took 6 weeks). The assumptions Finkelstein used are as follows:

1. The working fluid is a perfect gas and the expression \( pV = nRT \) applies.
2. The mass of the working fluid taking part in the cycle remains constant, i.e., there is no leakage.
3. The instantaneous pressure is the same throughout the system, i.e., pressure drops due to aerodynamic friction can be neglected.
4. The volume variations of the compression and expansion spaces are sinusoidal, and the clearances at top dead center are included in the constant volume of the adjacent heat exchangers.
5. The regenerator has a heat capacity which is large compared with that of the working fluid per pass, so that the local temperatures of the matrix remain unaltered. Its surface area and heat transfer coefficient are also assumed to be large enough to change the temperature of the working fluid passing through to the terminal value. Longitudinal and transverse heat conduction are zero.
6. The temperature of the boundary walls of each heat exchanger is constant and equal to one of the temperature limits. The heat exchangers are efficient enough to change the temperature of the working fluid to that of the boundary walls in the course of one complete transit.
7. The temperature of the internal surfaces of the cylinder walls and cylinder and piston heads associated with each working space is constant, and equal to one of the temperature limits. The overall heat transfer coefficient of these surfaces is also constant.
8. Local temperature variations inside the compression and expansion spaces are neglected—this assumes perfect mixing of cylinder contents at each instant.
9. The temperature of the respective portions of the working fluid in each of the ancillary spaces, such as heat exchangers, regenerators, ducts and clearances, is assumed to remain at one particular mean value in each case.
10. The rotational speed of the engine is constant.
11. Steady state conditions are assumed for the overall operation of the engine, so that pressures, temperatures, etc. are subject to cyclic variations only.

The analysis outlined by Finkelstein is very complicated (60 v). The results of this pioneering analysis are given below because they give some understanding of the effect the nearly adiabatic spaces of a real engine has on engine performance.
Finkelstein evaluated a specific case which happened to be a heat pump with a two-piston configuration (see Figure 5-9). The specific parameters were specified in dimensionless form as follows:

\[ K = \frac{V_K}{V_L} = \text{swept volume ratio} \]
\[ 2S = 1 = \text{temperature corrected clearance ratio} \]
\[ \alpha_L = 90^\circ = \text{phase angle} \]
\[ \phi_U = 2 = \frac{\text{temperature of heat rejection}}{\text{temperature of heat reception}} \]

Finkelstein gives results based upon a dimensionless heat transfer coefficient which is also called a number of transfer units. Where:

\[ TU = \frac{(HY)(AH)}{(OM)(M)(MW)(CP)} \]  \hspace{1cm} (5-40)

where

- \( HY \) = heat transfer coefficient, watts/cm\(^2\)K
- \( AH \) = area of heat transfer, cm\(^2\)
- \( OM \) = speed of engine, radians/sec
- \( (M)(MW) \) = mass of working gas, grams
- \( CP \) = heat capacity at constant pressure, j/g K

Real engines can be built where \( TU \) in the hot and cold space is very low all the time. Also real engines can be built where \( TU \) is very high all the time. However, real engines can probably not be built where \( TU \) has a constant intermediate value during the cycle. Nevertheless, the results at these intermediate values calculated by Finkelstein are instructive to show where the breakpoint is between adiabatic-like and isothermal-like operation. Table 5-1 shows the results of this analysis. All the mechanical and heat energies are non-dimensionalized by dividing each by \( M(MW)(R)(TH) \). Note that for this particular numerical example the adiabatic cycle is only about half as efficient as the isothermal cycle in pumping heat. However, this example is for a lower than usual temperature corrected clearance ratio, \( S \), of \( \frac{1}{2} \). It is not uncommon for \( S \) to be much larger. For instance, in the GPU-3 engine, \( S \) could be evaluated as follows: (see Table 3-2)

\[ S = \frac{TC}{VL} \left( \frac{HD}{TH} \frac{RD}{TR} \frac{CD}{TC} \right) \]
\[ = \frac{330}{120.4} \left( \frac{93.3}{1000} \frac{55.5}{604.3} \frac{34.3}{300} \right) \]
\[ = 0.84 \]

The larger \( S \) is, the less dramatic the effect of the adiabatic spaces.

Note that a small amount of heat transfer in the hot and cold space is worse than none at all. This gas spring hysteresis effect has been noted by others (78 as, 78 at). It also shows that if you want to gain all the advantages of heat transfer in the variable volume spaces, the heat transfer coefficient must be high.
<table>
<thead>
<tr>
<th>Dimensionless Quantities</th>
<th>Isothermal Regime</th>
<th>Limited Heat Transfer</th>
<th>Adiabatic Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer units, TU</td>
<td>$\infty$</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Mechanical Energy Input to Expansion Space</td>
<td>-0.518</td>
<td>-0.455</td>
<td>-0.435</td>
</tr>
<tr>
<td>Mechanical Energy Input to Compression Space</td>
<td>1.036</td>
<td>1.107</td>
<td>1.166</td>
</tr>
<tr>
<td>Net Mechanical Energy Input</td>
<td>0.518</td>
<td>0.652</td>
<td>0.731</td>
</tr>
<tr>
<td>Heat to Gas in Expansion Space</td>
<td>0.518</td>
<td>0.478</td>
<td>0.438</td>
</tr>
<tr>
<td>Heat to Gas in Heat Exchanger Next to Expansion Space</td>
<td>0</td>
<td>-0.023</td>
<td>-0.003</td>
</tr>
<tr>
<td>Total Heat In</td>
<td>0.518</td>
<td>0.455</td>
<td>0.435</td>
</tr>
<tr>
<td>Heat from Gas in Compression Space</td>
<td>1.036</td>
<td>0.998</td>
<td>0.880</td>
</tr>
<tr>
<td>Heat from Gas in Heat Exchanger Next to Compression Space</td>
<td>0</td>
<td>0.109</td>
<td>0.278</td>
</tr>
<tr>
<td>Total Heat Out</td>
<td>1.036</td>
<td>1.107</td>
<td>1.158</td>
</tr>
</tbody>
</table>

Finkelstein also shows how the engine pressure changes during the cycle for the cases shown in Table 5-1. (See Figure 5-11.) Note that the swing is largest as would be expected for the adiabatic case and least for the isothermal case and the other cases are in between. Figure 5-12 shows how the expansion space gas temperature varies during the cycle. The bottom curve is for $\eta$ or TU = 0. The labeling on the left-hand side of curve 5-12 is incorrect. Note that as the heat transfer increases, the temperature generally gets close to the infinite heat transfer case which does not vary from 1; that is, the expansion space temperature remains infinitesimally close to the heat source temperature. For zero heat transfer in the expansion space there has to be a discontinuity at a crank angle of 180° because this is the point when the expansion space becomes zero in volume. After 180° the expansion space begins to fill again with gas which is, by definition, at the heat source temperature. In Figure 5-13 the
Figure 5-11. Pressure Variation for Cases Given in Table 5-1 (60 v).

Figure 5-12. Expansion Space Gas Temperature Relative to the Heat Source Temperature in the Expansion Space for the Cases Given in Table 5-1 (60 v).
same calculated information is given for the compression space. Here again the more the number of heat transfer units, $n$, or TU, the closer the gas temperature curve approaches to the perfect heat transfer curve which stays at a temperature ratio of 1. Here the compression space volume becomes zero at $270^\circ$ crank angle. Thus, the discontinuity at this point for an entirely adiabatic case.

In reality the heat transfer coefficient in the compression space and the expansion space will get to be quite large when these spaces almost disappear each cycle. Then the number of transfer units will smoothly get to be very small during the rest of the cycle providing the engine is built in the conventional way.

Most of the design methods of first-, second- and third-order designs start out with some sort of cycle analysis to determine the basic power output and basic heat input and then make the necessary corrections to get the final prediction. One highly regarded method of doing this was published by Rios (69 am). The author spent a considerable amount of time getting this program which originally was supplied in punch card form to the author by Professor J. L. Smith of MIT into working order on his own computer. The Rios analysis uses the same assumptions as Finkelstein did but he does not require that the two pistons move in sinusoidal motion. He starts with arbitrary initial conditions and finds that the second cycle is convergent, that is, it starts at the same point that it ends at, providing the dead volumes are defined so that the clearance volume in the hot and cold spaces is lumped with the heat exchangers. Therefore, these volumes in these spaces go to zero at which point
the gas temperature in these spaces can be re-initialized. Appendix D presents the Rios program which has been modified by the author to be for a heat engine instead of a heat pump as the original thesis gave it. By the nature of the assumptions the temperature of the gases in all parts of the engine except the hot and cold spaces is known in advance and it is also assumed that the pressure is uniform throughout the engine each instant of time. As in the Finkelstein solution just described the temperatures of both the hot and cold spaces are allowed to float. Also, similar to the Finkelstein analysis there are four possible cases. Each case requires a separate set of equations. The four cases are: 1) mass increasing in both hot and cold spaces, 2) mass decreasing in both hot and cold spaces, 3) mass decreasing in cold space and increasing in hot space and 4) mass increasing in cold space and decreasing in hot space. The program employs a simplified Runge-Kutta integration approach. For each of the four cases it calculates a pressure change based upon the conditions at the beginning of the increment. Based upon this pressure change it calculates the pressure at the middle of the increment and using this pressure, it calculates a better approximation of the pressure change for the increment using volumes that are true for the middle of the increment. This final pressure change is used to determine the pressure at the end of the increment and the mass changes during the increment. Based upon these mass changes the decision matrix is set up so that for the next increment the proper option will be selected of the four that are available. The analysis in Appendix D was done for one degree increments. Many modifications to the program would be necessary to do anything different than one degree increments.

Martini has checked the Finkelstein adiabatic analysis for the particular case published by Finkelstein (60 v). The computation procedure is quite different than any others and is explained in detail in Appendix E. It was found that the pressure wave as shown in Figures 5-11 and 5-14 could be duplicated for the adiabatic case with fairly large time steps, as large as 30°. However, at the point of maximum curvature the curve is not really too well defined. Using the Martini method the adiabatic curve from Figure 5-12 is duplicated on a larger scale in Figure 5-15. The calculated points for 15°, 30° and 2° angle increments are plotted. Note that degree increments of 15° and 30°, although adequate for determining the pressure-volume relationship, are not adequate for determining the temperature in the expansion space of the engine. However, 2° angle increments do determine the temperature almost exactly, probably as closely and as accurately as Figure 5-12 was drawn. Figure 5-16 gives a similar evaluation for the adiabatic temperature curve duplicate from Figure 5-13. Note that 15° angle increments and 30° angle increments give substantial errors in comparison to the more exact 2° angle increments. Appendix E gives the method of calculation and shows how accurate it is.

5.1.7 Philips Semi-Adiabatic Cycle

Extremely little has been published by the Philips Company on how they calculate their engines. However, one of their licensees, MAN/MWM, discussed quite generally their process in a lecture at the Von Karmen Institute for Fluid Dynamics (73 aw). Mr. Feurer discloses that one of the Philips processes for calculating a Stirling engine starts out with a semi-adiabatic cycle and then adds additional corrections in a second-order design method. This second-order method will be discussed in Section 5.3 and the semi-adiabatic cycle it
Figure 5-14. Dimensionless Pressure vs Crank Angle Show Accuracy of Martini Method for Various Angle Increments.

Conditions:
- \( k = K = 1 \)
- \( v = 25 \)
- \( a = AL = 90^\circ \)
- \( e = AU = 2 \)
- \( \eta = TU = 0 \)

Read from Fig. 5-11

- \( \times \) For isothermal spaces
- \( + \) For adiabatic spaces
- \( \square \) Isothermal calc.
- \( \bigcirc \) 30° increment calc. adia.
- \( \bigtriangleup \) 15° increment calc. adia.
- \( \bullet \) 2° increment calc. adia.
Figure 5-15. Expansion Space Temperature Ratio vs. Crank Angle Showing Accuracy of Martini Method for Various Angle Increments.
Figure 5-16. Compression Space Temperature Ratio vs. Crank Angle Showing Accuracy of Martini Method for Various Angle Increments.
is dependent upon will be discussed here. As opposed to the more ideal Finkelstein adiabatic cycle, the Philips semi-adiabatic cycle is an adiabatic process that allows for the fact that the gas properties and the heat transfer are not ideal, that is, 1) the compressibility factor must be taken into account and 2) both the heat exchangers and the cylinders have finite heat transfer coefficients. These heat transfer coefficients result in different gas temperatures throughout the cycle than were calculated in the Finkelstein adiabatic cycle. Taking these effects into account the Philips licensee people arrive at what they call the semi-adiabatic cycle. Feurer (73 aw) presents a number of efficiencies and power outputs for the cycle for the conditions given in Table 5-2. In addition he varied the phase angle from zero to 180° and gave results for additional dead volumes of 40, 100 and 200 cm and diameters for the connecting spaces which these additional dead volumes represented of 100, 50 and 20 mm. However, this information is not judged to be of general utility because the description of the heat exchangers and cylinders are not given and the heat transfer coefficients that pertain to these parts of the engine are not given. All of this information along with the compressibility factor which is known for a particular gas is needed to calculate the Philips semi-adiabatic cycle results.

It was surmised by Walker (78 dc, p. 4.16-4.17) that the Philips semi-adiabatic cycle is the same as the Finkelstein adiabatic cycle. Further investigation by Martini presented herein shows that that is not the case. The Martini formulation of the Finkelstein adiabatic cycle given in Appendix E was used to generate the information shown on Figure 5-17. Note that the indicated power or the indicated efficiency is plotted versus the phase angle between the two pistons of a dual piston Stirling engine. The Schmidt power given by Feurer is the same as that calculated by Martini using the applicable computer program. Also, the ideal efficiency is, of course, checked. Note that the Philips semi-adiabatic

Table 5-2
ENGINE CONDITIONS FOR THE NUMERICAL EXAMPLE OF FEURER (73 aw)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium working gas</td>
<td></td>
</tr>
<tr>
<td>1500 rpm</td>
<td></td>
</tr>
<tr>
<td>120 atm mean pressure</td>
<td></td>
</tr>
<tr>
<td>75 °C inside cooler tubes</td>
<td></td>
</tr>
<tr>
<td>750 °C inside heater tubes</td>
<td></td>
</tr>
<tr>
<td>120.5 cm³ heater tube gas volume</td>
<td></td>
</tr>
<tr>
<td>56.5 cm³ cooler tube gas volume</td>
<td></td>
</tr>
<tr>
<td>145.3 cm³ regenerator gas volume</td>
<td></td>
</tr>
<tr>
<td>0 cm³ additional dead volume</td>
<td></td>
</tr>
<tr>
<td>100 mm pistons diameter</td>
<td></td>
</tr>
<tr>
<td>50 mm stroke</td>
<td></td>
</tr>
<tr>
<td>100 mm connecting rod length</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5-17. Comparison of Cycles using the Feurer Example (73 aw).
efficiency is the same as the ideal efficiency at a phase angle of \(0^\circ\) and \(180^\circ\), but drops down to only 50\% instead of the ideal 67\% at about \(70^\circ\) phase angle. The cycle efficiency using the Finkelstein adiabatic analysis cycle is given by the squares on Figure 5-17. There is a small difference depending upon whether purely sinusoidal motion is assumed or whether the crank motion specified in Table 5-2 is employed. It is interesting to note that the Philips semi-adiabatic efficiency and the Finkelstein adiabatic efficiency agree in the region from \(80^\circ\) to \(130^\circ\) in phase angle. Beyond this region of agreement, which may be fortuitous, the Philips semi-adiabatic efficiency tends toward the ideal efficiency and the Finkelstein adiabatic efficiency tends toward zero efficiency.

Concerning the power, Figure 5-17 shows that the Finkelstein adiabatic power is usually less than the Schmidt power. In both cases the crank geometry tends to have the power peak at a lower phase angle than for the sinusoidal geometry. However, the effect at this particular crank ratio is not pronounced. Note that the Philips semi-adiabatic power is lower generally than the Finkelstein adiabatic power and that the Philips power goes to 0 at \(0^\circ\) and \(180^\circ\) phase angle, whereas the Finkelstein adiabatic power for this particular case goes to 0 at \(10^\circ\) and \(180^\circ\) phase angle.

It should be emphasized that this is not by any means a full disclosure of the Philips semi-adiabatic cycle, but it does give all the information that is available on it in the open literature.

5.2 First-Order Design Methods

5.2.1 Definition

A first-order design method is a simple method that can literally be done on the back of an envelope. It relates the power output and efficiency of a machine to the heater and cooler temperature, the engine displacement and the speed. There is no need to specify the engine in any more detail than this. Therefore, this method is good for preliminary system analysis. It is assumed that an experienced Stirling engine design and manufacture team will execute the engine. First-order methods are used to predict the efficiency as well as the power output.

5.2.2 Efficiency Prediction

Efficiency of a Stirling engine is related to the cycle efficiency of a Stirling engine which is the same as the Carnot efficiency, which of course is related to the heat source and heat sink temperatures specified. Section 4 gives all the information available on well-designed Stirling engines which have not been fully disclosed and shows how the quoted efficiencies of these engines relate to the Carnot efficiency.

Carlqvist, et. al (77 al) give the following formula for well optimized engines operating on hydrogen at their maximum efficiency points.
\[
\eta_{\text{eff}} = \frac{P_{\text{net}}}{E_F} = \left(1 - \frac{T_C}{T_H}\right) \cdot C \cdot \eta_H \cdot \eta_M \cdot f_A
\]  

(5-42)

where

- \(\eta_{\text{eff}}\) = overall thermal or effective efficiency
- \(P_{\text{net}}\) = net shaft power with all auxiliaries driven
- \(E_F\) = fuel energy flow
- \(T_C, T_H\) = compression - expansion gas temperature, K
- \(C\) = Carnot efficiency ratio of indicated efficiency to Carnot efficiency, normally from 0.65 to 0.75. Under special conditions 0.80 can be reached.
- \(\eta_H\) = heater efficiency, ratio between the energy flow to the heater and the fuel energy flow. Normally between 0.85 and 0.90.
- \(\eta_M\) = mechanical efficiency, ratio of indicated to brake power. Now about 0.85 should go to 0.90.
- \(f_A\) = auxiliary ratio. At maximum efficiency point \(f_A = 0.95\).

Thus the most optimistic figures:

\[
\eta_{\text{eff}} = \left(1 - \frac{T_C}{T_H}\right)(0.75)(0.90)(0.90)(0.95) = \left(1 - \frac{T_C}{T_H}\right)(0.58)
\]

5.2.3 Power Estimation by First-Order Design Methods

Some attempts have been made to relate the power actually realized in a Stirling engine to the power calculated from the dimensions and operating conditions of the engine using the applicable Schmidt equation. Usually, the actual power realized has been quoted to be 30-40% of the Schmidt power (78 ad, p.100). However, the recommended way of estimating the Stirling engine power output is to use the Beale number method as described by Walker (79 y). To quote from Walker, "William Beale of Sunpower, Inc. in Athens, Ohio, observed several years ago that the power output of many Stirling engines conformed approximately to the simple equation:

\[
P = 0.015 \ p \times f \times V_o
\]

where

- \(P\) = engine power, watts
- \(p\) = mean cycle pressure, bar
- \(f\) = cycle frequency of engine speed, hertz
- \(V_o\) = displacement of power piston, cm\(^3\)

"This can be rearranged as \(P/(pfV_o) = \text{constant}\). The equation was found by Beale to be true approximately for all types and sizes of Stirling engines for which data were available including free piston machines and those with crank mechanisms. In most instances the engines operated with heater temperatures of 650 C and cooler temperatures of 65 C."
"The combination \( P/(pfV_0) \) is a dimensionless group that may be called the Beale number. It is self-evident that the Beale number will be a function of both heater and cooler temperatures. Recent work suggests the relationship of Beale number to heater temperature may be of the form shown in Figure 5-18 by the full line. Although for the sake of clarity the relationship is shown as a single line, it must of course be understood that the relationship is a gross approximation and particular examples of engines that depart widely may be cited. Nevertheless, a surprisingly large number of engines will be found to lie within the bounds of the confidence limits (broken lines) drawn on either side of the proposed relationship. Well designed, high efficiency units with low cooler temperatures will be concentrated near the upper bound. Less well designed units of moderate efficiency with high cooler temperatures will be located at the lower extremity.

"It should be carefully noted that the abcissa of Figure 5-18 is absolute temperature, degrees Kelvin; engines with the hot parts made of conventional stainless steels (say 18-8) will be confined to operate at temperatures limited to the region indicated by the line A-A. High alloy steels for the hot parts will permit the elevation of heater temperature to the limit of B-B. Above this temperature ceramic components would likely be used in the heater assembly."

Figure 5-18 is the best information generated by Walker and his students based upon information available to them, both proprietary and non-proprietary.

Figure 5-18. Beale Number as a Function of Heater Temperature.

5.2.4 Conclusion for First-Order Methods

First-order design methods are recommended for those who would like to evaluate the possibility of the use of a Stirling engine.
5.3 Second-Order Design Methods

5.3.1 Definition

Second-order design methods are relatively simple computational procedures that are particularly useful for optimizing the design of a Stirling engine from scratch. An equation or brief computational procedure is used to determine the basic power output and heat input. The basic power output is then degraded by various identifiable loss terms and the heat input is added to by evaluating a variety of additional heat losses that are known to exist in real engines. Consequently, an estimate is made of the real power output and real heat input using relatively simple means and not resorting to full-blown engine simulations which are the domain of third-order design methods. In second-order analysis one of the Stirling engine cycles described in Section 5.1 is used as a basis.

What is known about the Philips second-order analysis (73 aw) will be given because although very little is known about this analysis procedure, very much has been done with it. Because of the practical successes of the Philips engines, any information that is known about their engine design methods is of importance. Next the equations that have been used to evaluate power losses and heat losses will be given in two separate subsections. It will be left for the designer to decide what power losses and what heat losses pertain to his particular design and to add them to the cycle analysis which is most realistic for this engine to come up with his own second-order design method.

5.3.2 Philips Second-Order Design Method

This method starts with the Philips semi-adiabatic cycle as its basic power output and efficiency and then makes corrections. The corrections in the order that they are applied are shown in Table 5-3. Feurer (73 aw) shows the effect of the non-sinusoidal motion of the crank by Figure 5-19. Note that this is essentially identical to a portion of Figure 5-17 for the white and black triangles. In Figure 5-20 the line labeled "0" is for the power output of the semi-adiabatic cycle. The curve labeled "I" is not drawn because it is so close to the curve labeled "0" and this is for the power output based on the semi-adiabatic cycle less the correction due to the crank motion. The curve labeled "II" has the additional correction of adiabatic residual losses. Note that this has a very large correction at low phase angles but none at phase angles approaching 180°. The final curve labeled "III" in Figure 5-20 shows the additional correction due to flow losses. Note that this correction is small at low phase angle and maximum at a phase angle of 180°. Note that for this case the phase angle of 90° is not necessarily optimum, but is reasonably close. Figure 5-21 shows the adiabatic residual losses that are subtracted from curve I in Figure 5-20 to get curve II. Figure 5-21 also shows the flow losses which are subtracted from curve II in Figure 5-20 to get curve III. In Figure 5-21 it is shown what happens to the efficiency of the engine as the various losses are considered. At the top of Figure 5-21 is the Carnot efficiency which of course only depends on the temperature input and output of the machine. By going from a strictly Schmidt cycle to a semi-adiabatic cycle the bow-shaped curve labeled "I" which has a minimum at 50% efficiency is obtained. Going from sinusoidal to crank motion apparently has little effect.
Figure 5-19. Effect of Two Harmonics on the Schmidt Cycle Power (Based upon Crank Specified In Table 5-2).
Figure 5-20. Power Output Based Upon Conditions for Table 5-2 (73 aw).
Figure 5-21. Engine Efficiencies Based upon Conditions Given in Table 5-2 (73 aw).
Table 5-3

OUTLINE OF PHILIPS SECOND-ORDER
POWER OUTPUT CALCULATION

Start with basic power output computed by semi-adiabatic cycle (Section 5.1.7).

Less: loss due to non-sinusoidal motion of cranks.

Less: adiabatic residual losses which is the difference between the ideal temperature in the cylinders, heat exchangers and connecting spaces on the one hand and the actual temperature in these components on the other which results in an additional power loss.

Less: flow losses due to flow friction and entrance and exit losses and additional losses.

Equals: indicated output.

Less: mechanical losses, seals, bearings, etc.

Less: power for auxiliaries.

Equals: net shaft output

on the efficiency. However, in adding in the effect of the adiabatic residual losses the efficiency curve becomes the one labeled "II" which is much different in shape which peaks at about 150° phase angle. (Compare curve II with the Finkelstein adiabatic efficiency shown in Figure 5-17.) Curve III is the efficiency after the addition of flow losses and curve IV is the final efficiency after the addition of heat conduction losses. Note that the maximum efficiency point when all losses are considered is at a larger phase angle than is the maximum power point. It would seem reasonable for this machine to settle on a phase angle of about 120° because this would be nearly the high point of the power curve as well as nearly the high point of the efficiency curve.

This gives about all that is known about the workings of the Philips second-order design program. There is probably a number of good second-order as well as third-order design programs available to Philips as well as speciality programs for particular parts of the machine. It should be pointed out that all this information is from one paper by Feuer of MAN/MWM, a Philips licensee. Nothing like this has been published directly from Philips.

5.3.3 Power Losses

It would seem reasonable that when isolated groups wrestle with the problem of analyzing a Stirling engine in a practical way, they would consider the various identifiable losses in different orders. The work that follows is chiefly
the result of the United States Air Force-sponsored work on cooling engines (70 ac, 75 ac) as well as HEW-sponsored work on the artificial heart machine (68 c). This work starts out usually with a Schmidt cycle analysis and then applies a number of corrections. Some work has started out with a Finkelstein adiabatic analysis and then applies the corrections to that. (See Section 5.3.5.) This section identifies a number of power losses and presents the published equations which describe them. Power losses fall under two headings: flow friction and mechanical friction. The adiabatic residual losses which were so important in the Philips second-order method described just previously have been either included in this cycle analysis at the start of the evaluation or have been added on the end as an experience factor.

5.3.3.1 Flow Friction Losses

The basic power is computed as if there is no fluid friction. Energy loss due to fluid friction is deducted from the basic power as a small perturbation on the main engine process. If fluid friction consumes a large fraction of the basic power the following methods will not be accurate but then one would not choose a design to be built unless the fluid friction were less than 10% of the basic power.

Fluid friction inside the engine can be computed by published correlations for fluid flow through porous media and in tubes. These flow friction correlations are applicable for steady, fully developed flow. If the fraction of the gas inventory found in the hot spaces and in the cold spaces is plotted against crank angle, it is apparent that to a good approximation this periodic flow can be approximated by (1) steady flow, in one direction, (2) no flow for a period of time, (3) then steady flow back in the other direction and (4) then no flow to complete the cycle. The mass flow into and out of the regenerator is not quite in phase due to accumulation and depletion of mass in the regenerator. Note that the mass flow at the cold end is much more than the mass flow at the hot end mostly due to gas density change. The average mass flow rate and the average fraction of the total cycle time that gas is flowing in one direction at the hot end of the regenerator is used for the heater flow friction and heat transfer calculations. The average mass flow rate and the average fraction of the total cycle time flowing in one direction at the cold end of the regenerator is used for the cooler flow friction and heat transfer calculations. For the regenerator the mean of the above two flows and of the above two fractions has been used successfully. (See Appendix C and 79 ad, 79 o.)

Although the above approximation has been found to work, in each case graph the fractions of the mass of gas in the hot and the cold space during the cycle to determine if the approximations listed above of a constant flow rate, a stationary time and another constant flow rate are really approximated. One should also be certain that the computer algorithm for determining the flow rates and the times of the assumed constant flows are properly evaluated.

It would be more certain to divide the regenerator and even the heater and cooler spaces into a number of sections and evaluate the mass flow rates and the temperatures in each one of these sections for each time step. Then if one can assume that steady-flow friction coefficients apply, the pressure drop and finally the flow loss in each element can be computed and summed to find the
total flow loss for that increment. The flow friction correlations for each part of the engine taking into account the different geometries will now be given. The regenerator will be given first since it is the most important in terms of pressure drop and then the heat exchangers second.

5.3.3.1.1 Regenerator Pressure Drop -- Screens

Kays and London (64 l, p. 33) give the formula for pressure drop through a matrix as would be used for a regenerator:

\[
DP = \frac{(GR)^2}{2(G1)(RO(1))} \left[ \left( 1 + \frac{(AF)^2}{AM} \frac{RO(1)}{RO(2)} - 1 \right) + \frac{CW(LR)(RO(1))}{(HR)(RM)} \right]
\]  

Flow Acceleration Core Friction

where

- \( DP \) = pressure difference, MPa
- \( GR \) = velocity, mass, in regenerator, g/sec cm\(^2\)
- \( G1 \) = constant of conversion = 10\(^7\) g/(MPa·sec\(^2\)·cm)
- \( RO(1), RO(2) \) = gas densities at entrance and exit, g/cm\(^3\)
- \( AF \) = area of flow, cm\(^2\)
- \( AM \) = area of face of matrix, cm\(^2\)
- \( CW \) = factor of friction for matrix
- \( LR \) = length of regenerator, cm
- \( HR \) = radius, hydraulic, of matrix = \( PO/AS \)
- \( RM \) = density of gas at regenerator, g/cm\(^3\)
- \( PO \) = porosity of matrix
- \( AS \) = ratio of heat transfer area to volume for matrix, cm\(^{-1}\)

The flow acceleration term can be ignored in computing windage loss for the full cycle because the flow acceleration for flow into the hot space very nearly cancels the flow acceleration for flow out of the hot space. However, the difference may be significant. One should really leave in the flow acceleration term until experience shows that it does not make any difference. Nevertheless, with this simplifying assumption, the pressure drop due to regenerator friction is:

\[
DP = \frac{CW(XGR)^2(LR)}{2(G1)(HR)(RM)}
\]  

(5-44)

In the above equation the friction factor \( CW \) is a function of the Reynolds number \( RR = 4(HR)(GR)/MU \). Figure A4 shows the correlation for stacked screens usually used in Stirling engines. Note that the relationship is dependent somewhat upon the porosity. Since this calculation is already an approximation, it is recommended that a simpler relationship be used more adapted to use in simple computer programs (see Figure A4). To use this correlation the Reynolds number must be evaluated correctly.

\[
HR = \frac{PO}{AS}
\]  

(5-45)

= hydraulic radius for matrix, cm

\( PO \) = porosity of matrix

\( AS \) = heat transfer area per unit volume, cm\(^{-1}\)
Also,

\[
GR = \frac{WR}{(P0)(AM)}
\]

\[
= \text{mass velocity in matrix, g/sec cm}^2
\]

\[
WR = \text{flow through matrix, g/sec}
\]

\[
AM = \text{frontal area of matrix, cm}^2
\]

Finally, the viscosity is evaluated at the gas temperature in the matrix. (See Table A-6 for data on working gas viscosities.)

5.3.3.1.2 Heater and Cooler Pressure Drop

5.3.3.1.2.1 Tubular

Heater and cooler pressure drops are usually small in comparison with the regenerator. Heaters and coolers are usually small diameter, round tubes although an annular gap is practical for small engines. Pressure drop through these heaters and coolers is determined by Equations 5-47 or 5-48 with CW determined from the Fanning friction factor plot (see Figure A5) and densities DH or DK being evaluated at heat source or heat sink temperature and at average pressure. The length to diameter ratio is usually very large so for simple programs the equations shown with Figure A5 are:

\[
DP = \frac{2(CW)(GH)^2(LH)}{(G1)(IH)(DH)} \quad \text{for heater}
\]

\[
DP = \frac{2(CW)(GC)^2(LC)}{(G1)(IC)(DK)} \quad \text{for cooler}
\]

where in addition

\[
CW = \text{factor of frictions for tubes}
\]
\[
GH = \text{velocity, mass, in heater, g/sec cm}^2
\]
\[
GC = \text{velocity, mass, in cooler, g/sec cm}^2
\]
\[
LH = \text{length of heater tubes, cm}
\]
\[
LC = \text{length of cooler tubes, cm}
\]
\[
IH = \text{diameter, inside, of heater tubes, cm}
\]
\[
IC = \text{diameter, inside, of cooler tubes, cm}
\]
\[
DH = \text{density of gas in heater, g/cm}^3
\]
\[
DK = \text{density of gas in cooler, g/cm}^3
\]

5.3.3.1.2.2 Interleaving Fins (See Reference 77 h)

One of the advantages of this type of heat exchanger is that the gas flows into it rather than through it. Also, it is rather complicated because the flow passage area changes with the stroke. Experimental data are needed. One of the best types of interleaving fins is the nesting cone because the cone like the tube can have a thin wall and heat can be added and removed directly from the outside of the cone. In this type of filling and emptying process the flow
goes from maximum at the entrance to zero at the farthest point. This situation is equivalent to having all the flow flow half the distance volume-wise. Note that the equivalent diameter for this geometry is two times the separation distance between the cone surfaces. If the cone surfaces come close together and if the equivalent length along the cone is quite large, the flow resistance in a nesting cone isothermalizer can be large. There is no sure way of designing a Stirling engine. Each design concept has its good and bad points.

5.3.3.1.3 Heater, Cooler and Regenerator Windage Loss

Once the pressure drops are calculated, it should be noted that the product of the pressure drop in MPa and the volumetric flow rate in cm³/sec is the flow loss in watts. Increment by increment, as the engine is calculated, the instantaneous flow loss as well as the average for the cycle should be calculated. A peak in the flow loss during the cycle may slow down or stop the engine depending upon the size of the effective flywheel.

5.3.3.2 Mechanical Friction Loss

Mechanical friction due to the seals and the bearings is hard to compute reliably. It essentially must be measured. However, if the engine itself were used, the losses due to mechanical friction would be combined with power required or delivered by the engine. If indicated and brake power are determined, then mechanical friction loss is the difference. The friction loss should be measured directly by having the engine operate at the design average pressure with a very large dead volume so that very little engine action is possible. The engine need not be heated but the seals and bearing need to be at design temperature.

5.3.4 Heat Losses

Power losses which need to be subtracted from the basic power output have just been discussed. In this next section heat losses are defined which must be added to the basic heat input. These are: reheat, shuttle, pumping, temperature swing, internal temperature swing and flow friction credit.

5.3.4.1 Reheat Loss

One way that extra heat is required at the heat source is due to the inefficiency of the regenerator. The regenerator reheat the gas as it returns to the hot space. The reheat not supplied by the regenerator must be supplied by the heater as extra heat input. Figure 5-22 shows how the gas temperatures vary in the heater, regenerator and cooler during flow out of the hot space as well as flow into it. Note that at inflow, the gas attains cooler temperature, then is heated up in the regenerator part-way. The temperature difference, A, between the heat source temperature and the gas entering from the regenerator is then multiplied by the heat capacity, the effective flow rate and the fraction of time that this gas is flowing to obtain the reheat loss. The methods derived from the literature and from the author's own practice are given below. The formula for reheat once used by the author is:
Each element in Equation 5-49 is a type of an approximation. The fraction of time flowing into the hot space is estimated by extrapolating the maximum cycle time that this process would occupy if the flow rate were always at its maximum value. This fraction, FR, turns out to be about one-third. FR will be taken as \( \frac{1}{3} \) if an analytical Schmidt equation is used. If a numerical procedure is used, FR may be computed when the flow resistances are calculated providing the approximation is found valid that regenerator flows can be approximated by two steady flows interspersed by two periods of no flow. The effective flow rate then is determined by the flow through the regenerator, WR. If these two periods of constant flow approximation are not used, then for every time step when flow is from the regenerator to the heater a partial reheat loss must be calculated for each such increment and summed for the cycle.

Figure 5-22. Reheat Loss.
Neither heat capacity $CV$ or $CP$ is strictly correct. More complicated analyses can take into account more rigorously the effect of pressure change during gas flow through the regenerator (75 ag, 77 bl). The rationale for using $CV$ in Equation 5-49 is that the transfer of gas takes place when the total volume is relatively constant. However only a small amount of the total volume is in the regenerator at any one time. An equation suggested by Tew of LeRC (78 ad, p. 123) is:

$$RH = \left[ FR(WR)(CP)(TH - TC) - RD(CV)(PX - PN)(NU)(MW) \right] \left( \frac{2}{NT + 2} \right) \ (5-50)$$

where

- $RH = \text{loss, reheat, watts}$
- $FR = \text{fraction of cycle time flow is into hot space}$
- $WR = \text{flow, mass, through regenerator, g/sec}$
- $CP = \text{capacity of heat of gas at constant pressure, j/g K}$
- $TH = \text{temperature, effective, of hot space, K}$
- $TC = \text{temperature, effective, of cold space, K}$
- $RD = \text{Volume, regenerator, dead, cm}^3$
- $CV = \text{capacity of heat of gas at constant volume, j/g K}$
- $PX = \text{maximum pressure, MPa}$
- $PN = \text{minimum pressure, MPa}$
- $NU = \text{frequency of engine, Hz}$
- $MW = \text{molecular weight of gas, g/g mol}$
- $R = \text{constant, gas, universal = 8.314 j/g mol K}$
- $NT = \text{number of transfer units in regenerator}$
- $HY = \text{coefficient of heat transfer, watts/cm}^2\text{K}$
- $AH = \text{area of heat transfer, cm}^2$

In Equation 5-50, the flow heat is watts needed on a continuous basis to raise the temperature of the gas passing into the hot space. The pressure change heat recognizes the fact that some of the heat required to raise the gas temperature can come from increasing the gas pressure which happens at nearly the same time. However, it can happen that the pressure change heat can be larger than the flow heat. In this case a more exact analysis should be employed. The net of the flow heat and the pressure change heat is multiplied by the ineffectiveness of the regenerator to obtain the reheat loss. Equation 5-50 is used in Appendix C to calculate reheat loss.

The temperature difference $A$ in Figure 5-22 is represented by the total temperature difference between the hot metal and the cold metal times the regenerator ineffectiveness. This ineffectiveness is one minus the effectiveness of the regenerator material (see Equation 5-7). This formula for ineffectiveness agrees with the simple equations in earlier standard references on regenerators such as Saunders and Smoleniec (51 q).

The idea of separating power output and the heat losses into a number of superimposed processes has been used by a number of investigators of the Vuilleumier cycle. The details of this analysis have been given in a number of government reports. The Vuilleumier cycle is a heat operated refrigeration machine which
uses helium gas and regenerators very similar to the way the Stirling engine is constructed. This superposition analysis has worked well in VM cycle machines. In an RCA report (69 aa, pp. 3-37) the measured cooling power using this method of analysis was found to be within 8.9% of that calculated. Crouthamel and Shelpuk (75 ac) give the following formula for the reheating loss after it is translated into the nomenclature used in this section.

\[ RH = \left( \frac{1}{4} \right) (WR)(CP)(TM - TW)(\frac{2}{NT + 2}) \]  

Equation 5-51 is written in the same order as Equation 5-49 and therefore can be directly compared. The first term, one quarter, is specific for their particular machine and therefore needs to be evaluated for another type of machine. The flow rate is evaluated in the same way, but the heat capacity is different. Probably this can be justified to be CP instead of CV because the VM cycle machine undergoes a relatively small change in pressure during its cycle. Also, the distinction between metal temperatures and gas temperatures is also relatively small at this stage of analysis.

More elaborate equations for the calculation of reheating loss have been given in the literature. These are at least 10 times more complicated than those already given and no studies have yet been made to show that they are better. Bjorn Qvale (69 n, 78 ad, pp. 126-127) developed a formula which takes the pressure wave into account. He tested his equation against some experimental results from Rea (66 h) and found it to agree within ±20%.

Rios (69 ar, 69 am) employed quite a different formulation to calculate reheating loss. It is also very complicated. It is included in the listing of the Rios program in Appendix D. The reheating loss is calculated on Line 430, but many lines preceding this line are required to calculate values leading up to this line.

5.3.4.2 Shuttle Conduction

Figure 5-23 shows how shuttle conduction works. Shuttle conduction happens anytime a displacer or a hot cap oscillates across a temperature gradient. It is usually not frequency-dependent for the speeds and materials used in Stirling engines. The displacer absorbs heat during the hot end of its stroke and gives off heat during the cold end of its stroke. Usually neither the displacer nor the cylinder wall change temperatures appreciably during the process. Shuttle conduction depends upon the area involved, the thickness of the gas filled gap, G, the temperature gradient \((TH - TW)/LB\), the gas thermal conductivity, KG, and the displacer stroke, SD. It is also dependent on the wave form of the motion and in some cases, upon the thermal properties of the displacer and of the cylinder wall. All formulas in the literature are of the form:

\[ QS = \frac{(YK)(ZK)(SD)^2(KG)(TH - TW)(DC)}{(G)(LB)} \]  

\[ (5-52) \]
where

\[ QS = \text{shuttle heat loss (in this case for one cylinder)} \]
\[ YK = \text{wall properties and frequency factor} \]
\[ ZK = \text{wave form factor} \]
\[ SD = \text{stroke of displacer or hot cap, cm} \]
\[ KG = \text{gas thermal conductivity, } \text{w/cm K} \]
\[ TH = \text{effective temperature of hot space, K} \]
\[ TW = \text{temperature of inlet cooling water, K} \]
\[ DC = \text{inside diameter of engine cylinder} \]
\[ G = \text{clearance around hot cap or displacer, cm} \]
\[ LB = \text{length of displacer or hot cap, cm} \]

The quantity \( ZK \) depends upon the type of displacer or hot cap motion, and \( YK \) depends upon the thermal properties of the walls and the frequency of operation. Table 5-4 shows the results of a literature survey for \( ZK \). Note that there is a substantial disagreement about what \( ZK \) should be for the sinusoidal case. The author has derived the lower value and he would recommend it. This value, \( \pi/8 \), agrees with Rios but does not agree with Zimmerman. However, there are no data that would lay the matter to rest.

![Diagram](image)

Figure 5-23. Shuttle Conduction.
### Table 5-4

**COEFFICIENT FOR SHUTTLE HEAT CONDUCTION EQUATION**
*(Ignoring Effect of Walls)*

<table>
<thead>
<tr>
<th>Motion</th>
<th>Investigator</th>
<th>Ref.</th>
<th>ZK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square wave (\frac{1}{4})</td>
<td>Zimmerman</td>
<td>71 be</td>
<td>(\pi/4 = 0.785)</td>
</tr>
<tr>
<td>time at one end, (\frac{1}{4}) time at other</td>
<td>Crouthamel &amp; Shelpuk</td>
<td>75 ac</td>
<td>(\pi/4 = 0.785)</td>
</tr>
<tr>
<td>Sinusoidal (effect of walls ignored)</td>
<td>Martini</td>
<td>(1)</td>
<td>(\pi/8 = 0.393)</td>
</tr>
<tr>
<td></td>
<td>Zimmerman</td>
<td>71 be</td>
<td>(\pi/5.4 = 0.582)</td>
</tr>
<tr>
<td></td>
<td>Rios</td>
<td>71 an</td>
<td>(\pi/8 = 0.393)</td>
</tr>
<tr>
<td></td>
<td>White</td>
<td>71 ll</td>
<td>(0.186\pi = 0.584)</td>
</tr>
<tr>
<td></td>
<td>--</td>
<td>69 aa</td>
<td>(0.186\pi = 0.584)</td>
</tr>
</tbody>
</table>

(1) McDonnell Douglas Reports, never published.

Rios has published values for \(Y_K\) to take into account the effect of frequency or wall thermal properties which are sometimes important. The most general Rios theory takes into account the thermal properties of the cylinder wall as well as the displacer or hot cap wall (71 an). His new theory gives:

\[
Y_K = \frac{1 + XB}{1 + (XB)^2} \tag{5-53}
\]

where in addition:

\[
XB = 1 + \frac{1}{2\pi} \frac{KG}{G} \left( \frac{L_4}{K_1} + \frac{L_5}{K_2} \right)
\]

\[L_4 = \text{temperature wavelength in displacer, cm}\]

\[L_4 = 2\pi \sqrt{\frac{2(D_4)}{OM}}\]

\[D_4 = \text{thermal diffusivity in displacer, cm}^2/\text{sec}\]

\[OM = \text{engine speed, radians/sec}\]

\[D_4 = \frac{K_1}{(E_4)(M_4)}\]

\[E_4 = \text{density of displacer wall, g/cm}^3\]

\[M_4 = \text{heat capacity of displacer wall, J/g K}\]

\[K_1 = \text{thermal conductivity of displacer, W/cm K}\]

\[L_5 = \text{temperature wavelength in cylinder wall, cm}\]

\[L_5 = 2\pi \sqrt{\frac{2(D_5)}{OM}}\]
The above factor applies for simple harmonic motion and for engines in which D4 is smaller than the thickness of the displacer wall and D5 is smaller than the thickness of the cylinder wall. Rios gives equations for solving the problem for any periodic motion by using Fourier series expansion. To help determine whether the above factor applies, Rios gives some typical values of temperature wavelength at room temperature (see Table 5-5).

Table 5-5

TYPICAL TEMPERATURE WAVELENGTHS
AT ROOM TEMPERATURE CONDITIONS

<table>
<thead>
<tr>
<th>Material</th>
<th>Frequency, HZ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Mild Steel</td>
<td>1.21</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>0.74</td>
</tr>
<tr>
<td>Phenolic</td>
<td>0.85</td>
</tr>
<tr>
<td>Pyrex Glass</td>
<td>0.26</td>
</tr>
</tbody>
</table>

If the wall thickness is considerably smaller than the temperature wavelength, then it may be assumed that radial temperature distribution in the walls is uniform. Rios (71 an) proposes the following definition of YK for this case:

\[
YK = \frac{1}{1 + (SG)^2}
\]

(5-54)

where

\[
SG = \frac{KG}{(G)(OM)} \left[ \frac{1}{(E4)(M4)(SC)} + \frac{1}{(E5)(M5)(SE)} \right]
\]

and

E4 = density of displacer wall, g/cm³
E5 = density of cylinder wall, g/cm³
SC = wall thickness of displacers, cm
SE = wall thickness of cylinder wall, cm
M4 = heat capacity of displacer wall, j/g K
M5 = heat capacity of cylinder wall, j/g K
Note that when the thermal properties of the wall do not matter, $Y_K$, whether evaluated by Equation 5-53 or 5-54, would evaluate to nearly 1. There is not any published formula that treats the case of cylinder and displacer wall thickness on the order of the temperature wavelength. There are also no published formulas for the case of a thick cylinder wall and a thin displacer or visa-versa. For horsepower size engines Equation 5-53 will apply. For model engines or artificial heart engines Equation 5-54 will apply. Therefore, for horsepower size, high pressure engines the recommended equation for shuttle heat conduction is:

$$\begin{align*}
QS &= \frac{1 + XB}{1 + (XB)^2} \pi \frac{(SD)^2(KG)(TH - TC)(DC)}{8 G(LB)}.
\end{align*}$$

(5-55)

For model size engines using low gas pressure and very thin walls:

$$\begin{align*}
QS &= \frac{1}{1 + (SG)^2} \pi \frac{(SD)^2(KG)(TH - TC)(DC)}{8 G(LB)}
\end{align*}$$

(5-56)

It also should be emphasized that Equation 5-55 and 5-56 are for nearly sinusoidal motion of the displacer or hot cap. Square wave motion would double this result. Ramp motion should reduce this result some.

5.3.4.3 Gas and Solid Conduction

This heat loss continues while the engine is hot, independent of engine speed. It is simply the heat transferred through the different gas and solid members between the hot portion and the cold portion of the engine. Heat can be transferred by conduction or radiation. In the regenerator the gas moves, but under this heading the heat loss is computed as if the gas were stagnant. In Section 5.3.4.1, the reheat loss is computed assuming there is no longitudinal conduction.

The uncertainty about what thermal conductivities and what emissivities to use to evaluate this loss makes its measurement with the engine desirable. In some engines the hot and cold spaces are heated and cooled directly. In this case measuring the heat absorbed by the cooling water with the engine heated to temperature but stopped will give this heat loss. However, all the horsepower-size engines described in Sections 3 and 4 have indirectly heated and cooled hot and cold gas spaces. For this case the sum of the gas and solid conduction and the shuttle conduction can be determined by measuring the heat absorbed by the cooling water for a number of slow engine speeds with the engine heater at temperature and then extrapolating to zero engine speed.

Usually the following conduction paths are identified and should be evaluated for each engine:

<table>
<thead>
<tr>
<th>Path No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Engine cylinder wall.</td>
</tr>
<tr>
<td>2.</td>
<td>Displacer or hot cap wall.</td>
</tr>
<tr>
<td>3.</td>
<td>Gas annulus between cylinder and hot cap.</td>
</tr>
<tr>
<td>4.</td>
<td>Gas space inside displacer or hot cap.</td>
</tr>
<tr>
<td></td>
<td>a. gas conduction</td>
</tr>
<tr>
<td></td>
<td>b. radiation</td>
</tr>
<tr>
<td>5.</td>
<td>Regenerator cylinders.</td>
</tr>
<tr>
<td>6.</td>
<td>Regenerator packing.</td>
</tr>
</tbody>
</table>
The engine cylinder, the displacer and regenerator cylinders must be designed strong enough to withstand the gas pressure for the life of the engine without changing dimension appreciably. However, extra wall thickness contributes unnecessarily to the heat loss. For this reason the cylinder walls of most high powered engines are much thinner at the cold end where the creep strength is high than they are at the hot end. This, of course, complicates evaluation of this type of heat loss.

The following types of heat transfer problems need to be solved to evaluate these heat losses:

1. Steady, one dimensional conduction, constant area, variable thermal conductivity.
2. Steady, one dimensional conduction, variable area, variable thermal conductivity.
3. Steady, one dimensional conduction through a composite material (wire screens).
4. Radiation along a cylinder with radiation shields.

Solutions to each one of these problems will now be given.

5.3.4.3.1 Constant Area Conduction

Heat loss by conduction of this type is computed by the formula:

\[ Q = \frac{K_A H (T_H - T_C)}{L_B} \]  

(5-57)

where the thermal conductivities areas and lengths are germane to Path 3 and 4a above, KG is evaluated at mid-point temperature. (See Table A2.)

5.3.4.3.2 Variable Area, Variable Thermal Conductivity

For one dimensional heat conduction where the heat transfer area varies continually and the thermal conductivity changes importantly, the heat conduction path is divided into a number of zones. The average heat conduction area for each zone is calculated. The temperature in each zone is estimated and from this estimate a thermal conductivity is assigned. Figure A-2 gives the thermal conductivities for some probable construction materials in the units used in this manual. It should be noted that there is quite a variability in some common materials like low carbon steel. Measured thermal conductivity different by a factor of 3 is shown. Differences are due to heat treatment and the exact composition. With commercial materials having considerable variability, it is strongly recommended that the static heat loss be checked by extrapolating the heat requirement for the engine to zero speed. This number would then need to be analyzed to determine how much shuttle heat loss is also being measured and how much is static heat loss.

For purposes of illustration, assume 3 zones are chosen along a tapered cylinder wall. (See Figure 5-24.) Temperatures MT(2) and MT(3) must be estimated between MT(1) and MT(4) to start. MT(1) is the hot metal temperature and MT(4)
Figure 5-24. Computation of Tapered Cylinder Wall Conduction.

is the cold metal temperature. The heat transfer areas AT(1) to AT(4) are computed based upon engine dimensions. The heat through each segment is the same. Thus:

\[ CQ = \left( \frac{AK(1) + AK(2)}{2} \right) \left( \frac{AT(1) + AT(2)}{2} \right) \left( \frac{MT(1) - MT(2)}{X(2) - X(1)} \right) \] (5-58)

\[ = \left( \frac{AK(2) + AK(3)}{2} \right) \left( \frac{AT(2) + AT(3)}{2} \right) \left( \frac{MT(2) - MT(3)}{X(3) - X(2)} \right) \] (5-59)

\[ = \left( \frac{AK(3) + AK(4)}{2} \right) \left( \frac{AT(3) + AT(4)}{2} \right) \left( \frac{MT(3) - MT(4)}{X(4) - X(3)} \right) \] (5-60)

Let:

\[ Y(1) = \frac{(X(2) - X(1))}{\left( \frac{AK(1) + AK(2)}{2} \right) \left( \frac{AT(1) + AT(2)}{2} \right)} \] (5-59)

\[ Y(2) = \frac{(X(3) - X(2))}{\left( \frac{AK(2) + AK(3)}{2} \right) \left( \frac{AT(2) + AT(3)}{2} \right)} \] (5-60)

\[ Y(3) = \frac{(X(4) - X(3))}{\left( \frac{AK(3) + AK(4)}{2} \right) \left( \frac{AT(3) + AT(4)}{2} \right)} \] (5-61)
Then:

\[ CQ = \frac{MT(1) - MT(4)}{Y(1) + Y(2) + Y(3)} \]  

(5-62)

Once \( CQ \) is computed then:

\[ MT(2) = MT(1) - (Y(1))(CQ) \]  

(5-63)

\[ MT(3) = MT(2) - (Y(2))(CQ) \]  

(5-64)

\( MT(2) \) and \( MT(3) \) are compared with the original guesses. If they are appreciably different so that the thermal conductivities would be different, then new thermal conductivities based upon these computed values of \( MT(2) \) and \( MT(3) \) would be determined and the process repeated. Once more is usually sufficient.

The same procedure is used for the engine cylinder and the displacer if the walls are tapered.

### 5.3.4.3.3 Conduction Through Regenerator Matrices

Usually the regenerator of a Stirling engine is made from many layers of fine screen that are lightly sintered together. The degree of sintering would have a big bearing on the thermal conductivity of the screen stack since the controlling resistance is the contact between adjacent wires. Some cryogenic regenerators use a bed of lead spheres.

In the absence of data, Gorring (61 n) gives the following formula for conduction through a square array of uniformly sized cylinders.

\[ KX = KG \left( \frac{1 + \frac{KM}{KG}}{1 - \frac{KM}{KG}} \right) \left( \frac{1 + \frac{KM}{KG}}{1 - \frac{KM}{KG}} + FF \right) \]  

(5-65)

where

- \( KX \) = thermal conductivity of the matrix, w/cm K
- \( KG \) = thermal conductivity of the gas in the matrix, w/cm K
- \( KM \) = thermal conductivity of the metal in the matrix, w/cm K
- \( FF \) = fraction of matrix volume filled with solid

The thermal conductivity of the gas \( KG \) and the metal \( KM \) are evaluated at \( TR \). The heat loss through the screens is then determined using an equation like Equation 5-57.

Sometimes the regenerator is made from slots in which metal foils run continuously from hot to cold ends. The conductivity of the matrix in this case is:

\[ KX = \frac{(KG)(G) + (KM)(DW)}{G + DW} \]  

(5-66)

Then the heat loss through the matrix is then determined using an equation like Equation 5-57.
5.3.4.3.4 Radiation Along a Cylinder with Radiation Shields

The engine displacers or the hot cap for a dual piston machine is usually hollow. Heat transport across this gas space is by gas conduction and by radiation. Radiation heat transport follows the standard formula:

\[ CQ = (FA)(FM)(FN)(\pi/4)(DB)^2(SI)((TH)^4 - TC)^4) \]  

(5-67)

where

- \( CQ \) = heat loss by radiation, watts
- \( FA \) = area factor
- \( FM \) = emissivity factor
- \( FN \) = radiation shield factor
- \( DB \) = diameter of cylinder, cm
- \( LB \) = length of cylinder, cm
- \( SI \) = Stefan-Boltzmann constant
  \[ = 5.67 \times 10^{-12} \text{ w/cm}^2 \text{ K}^4 \]
- \( TH \) = hot surface temperature, K
- \( TC \) = cold surface temperature, K

The area factor, \( FA \), is usually determined by a graph computed by Hottel (McAdams, Heat Transmission, 3rd Ed., p. 69). For the case of two discs separated by non-conducting but reradiating walls, his curve is correlated by the simple formula:

\[ FA = 0.50 + 0.20 \ln \frac{DB}{LB} \]  

(5-68)

Equation 5-68 is good for values of \( DB/LB \) from 0.2 to 7. For \( (DB/LB) < 0.2 \) use:

\[ FA = \frac{DB}{LB} \]  

(5-69)

Emissivity factor, \( FM \), is the product of the emissivity at the hot end and at the cold end. Thus:

\[ FM = (EH)(EK) \]  

(5-70)

The hot and cold emissivities can be obtained from any standard text on heat transfer. This emissivity depends upon the surface finish, the temperature and the material. There is a large uncertainty in handbook values.

If the emissivity of the radiation shields is intermediate between the emissivity of the hot and cold surfaces, then from the number of radiation shields, \( NS \), the radiation shield factor, \( FN \), is calculated approximately.

\[ FN = 1/(1 + NS) \]  

(5-71)

5.3.4.4 Pumping Loss

A displacer or a hot cap has a radial gap between the ID of the engine cylinder and the OD of the displacer. The gap is sealed at the cold end. As the engine is pressurized and depressurized, gas flows into and out of this gap. Since the closed end of the gap is cold, extra heat must be added to the gas as it comes back from this gap. Leo (70 ac) gives the formula:
where

QP = pumping heat loss, watts (one cylinder)
DC = diameter of cylinder, cm
LB = length of hot cap, cm
PX = maximum pressure, MPa
PN = minimum pressure, MPa
NU = engine frequency, Hz
CP = heat capacity of gas at constant pressure, j/g K
TH = effective temperature of hot space, K
TC = effective temperature of cold space, K
G = clearance around hot cap, cm
Z1 = compressibility factor of gas
R = universal gas constant = 8.314 j/g mol K
MW = molecular weight
KG = thermal conductivity of the gas, j/g K

5.3.4.5 Temperature Swing Loss

In computing the reheat loss (see Section 5.3.4.1) it was assumed that the regenerator matrix temperature oscillates during the cycle a negligible amount. In some cases the temperature oscillation of the matrix will not be negligible. The temperature swing loss is this additional heat that must be added by the gas heater due to the finite heat capacity of the regenerator. The temperature drop in the regenerator matrix temperature from one end to the other due to a single flow of gas into the hot space is:

\[ TS = \frac{WR(CV)FR(TH - TC)}{NU(MX)(M6)} \]  

where

TS = matrix temperature swing during one cycle, K
WR = mass flow through regenerator, g/sec
CV = gas heat capacity at constant volume, j/g K
FR = fraction of cycle time flow is into hot space
TH = effective hot space temperature, K
TC = effective cold space temperature, K
NU = engine frequency, Hz
MX = mass of regenerator matrix, g
M6 = heat capacity of regenerator metal, j/g K

Half of this, \((TS)/2\), is equivalent to \(A\) in Equation 5-49 and Figure 5-22 since TS starts at zero at the start of the flow and grows to TS. Thus the temperature swing loss is:

\[ SL = FR(WR)(CV)(TS)/2 \]  

Crouthamel and Shelpuk (75 ac) point out this loss but their equation is:

\[ SL = FR(WR)(CP)(TS) \]
Their equation substitutes \( CP \) for \( CV \) as was done also in Section 5.3.4.1. The reason for division by 2 seems to be recognized in their text but is not reflected in their formula. Based upon the discussion in Section 5.3.4.1, it is now recommended that an effective gas heat capacity based upon Equation 5-50 be used in Equations 5-73 and 5-74.

5.3.4.6 Internal Temperature Swing Loss

Some types of regenerator matrices could have such low thermal conductivity (for example, glass rods) that all the mass of the matrix would not undergo the same temperature swing. The interior would undergo less swing and the outside additional swing would result in an additional heat loss. Crouthamel and Shepuk (75 ac) give this loss as:

\[
Q_1 = SL \left[ C_3 \frac{(E_6)(M_6)}{K_M} \frac{(DW/2)^2}{N_U} \right]
\]

(5-76)

where

- \( Q_1 \) = internal temperature swing loss, watts
- \( SL \) = temperature swing loss, watts
- \( C_3 \) = geometry constant (see below)
- \( E_6 \) = density of matrix solid material, g/cm³
- \( M_6 \) = heat capacity of regenerator metal, J/g K
- \( K_M \) = thermal conductivity of regenerator metal, watts/cm K
- \( DW \) = diameter of wire or thickness of foil in regenerator, cm
- \( N_U \) = engine frequency, Hz
- \( F_R \) = fraction of cycle time flow is into hot space

The geometry constant \( C_3 \) is given as 0.32 by Crouthamel and Shepuk (75 ac) who refer to page 112 of Carslaw and Jaeger (59 o). This constant is for a slab. The constant for a cylinder or a wire is 0.25 (59 o, p. 203).

5.3.4.7 Flow Friction Credit

The flow friction in the hot part of the engine is returned to this part of the engine as heat. It is assumed that

\[
F_Z = RW + \frac{HW}{2}
\]

(5-76a)

where

- \( F_Z \) = flow friction credit, watts
- \( RW \) = flow friction in regenerator, watts
- \( HW \) = flow friction in heater, watts

5.3.5 First Round Engine Performance Summary

At this point it is necessary to take stock of the first estimate of the net power out and the total heat in based upon the first estimate of the effective hot and cold gas temperature. The total heat requirement will be used along with the characteristics of the heat exchangers to compute the effective hot
and cold gas temperatures. These new computed temperatures will be used to determine a better estimate of the basic output power and basic heat input. Heat losses and power losses will remain the same. The net power output is:

\[ NP = BP - CF - HW - RW \]  \hspace{1cm} (5-77)

The net heat input is:

\[ QN = BH + RH + QS + CQ + QP + TS + QI - FZ \]  \hspace{1cm} (5-78)

5.3.6 Heat Exchanger Evaluation

Once the first estimate of the net heat input, \( QN \), is computed, the duty of the gas heater and gas cooler are determined:

\[ QB = QN \]  \hspace{1cm} (5-79)

\[ QC = QN - NP \]  \hspace{1cm} (5-80)

Next, the heat transfer coefficient for the gas heater and gas cooler is computed. The most common type is the tubular heat exchanger. Small machines can use an annular gap heat exchanger. Isothermalizer heat exchangers are possible.

5.3.7 Martini Isothermal Second-Order Analysis

So far in Sections 5.1.5 and 5.1.6, means for calculating the basic power output, \( BP \), and the basic heat input, \( BH \), have been given. Means for calculating flow losses \( CF \), \( HW \), and \( RW \) in the cooler, heater and regenerator are reviewed in Sections 5.3.3. Means for calculating heat losses which add to the basic heat input have been discussed in Section 5.3.4. Section 5.3.5 shows how the net heat input and power outputs are calculated, and Section 5.3.6 shows how the amount of heat that must be transferred by the heat exchangers is determined.

To bring this all together there must be a calculation procedure that will allow the performance of a particular engine design to be predicted. The Martini isothermal analysis uses the following method:

1. Using the given heat source and heat sink temperatures and the engine dimensions, find the basic power using a Schmidt cycle analysis.
2. Using the heat source and heat sink temperatures, calculate the basic heat input from the power output using the Carnot efficiency.
3. Evaluate net power, \( NP \), by Equation 5-77, net heat input, \( QN \), by Equation 5-78, gas heater duty by Equation 5-79, and gas cooler duty by Equation 5-80.
4. Using the flow rate and duration during the cycle of gas flowing through the heater, determine the temperature drop needed to allow the gas heater duty to be transferred. Deduct a percentage of this temperature drop based upon experience from the heat source temperature to obtain a first estimate of the effective hot space gas temperature.
5. Using the flow rate and duration during the cycle of gas flowing through the cooler, determine the temperature drop needed to allow the gas cooler duty to be transferred. Add a percentage of this temperature drop based upon experience to the heat sink temperature to obtain the effective cold space gas temperature.
6. Recalculate steps 1, 2, 3, 4 and 5 using the effective hot space temperature for the heat source temperature and the effective cold space temperature for the heat sink temperature. Do this several times till there is no appreciable change in these effective temperatures.

This method is very similar to that published previously by Martini (78 o, 78 ad, 79 ad). A FORTRAN computer program of this method is given in Appendix C.

5.3.8 Rios Adiabatic Second-Order Analysis

P.A. Rios (69 am) developed a computer code for cryogenic coolers which is highly regarded. This has been adapted to heat engine analysis. A full discussion and a FORTRAN listing are included as Appendix D. An outline of this method is now given.

1. Using the given heat source and heat sink temperatures and the engine dimensions, find the basic power using a Finkelstein adiabatic analysis. (The Rios equations are different and more general than Finkelstein used but the assumptions are the same.)
2. Use the adiabatic analysis to calculate basic heat input.
3. Evaluate net power, NP, by Equation 5-77, net heat input, QN, by Equation 5-78, gas heater duty by Equation 5-79 and gas cooler duty by Equation 5-80.
4. Calculate heater and cooler ineffectiveness. Based upon these, modify heat source and heat sink temperatures. Re-do steps 1, 2, 3 and 4 with new temperatures. Three iterations were always found to be enough for convergence.

5.3.9 Conclusion for Second-Order Methods

Second-order methods have the ability to take all engine dimensions and operating conditions into account in a realistic way without getting involved in much more laborious computer simulation routines employed in third-order analysis. The principles employed in second-order analysis have been described. Whether these principles are useful in real life design depends upon their accuracy over a broad range of applications.

5.4 Third-Order Design Methods

Third-order design methods start with the premise that the many different processes assumed to be going on simultaneously and independently in the second-order design method (see Section 5.3) do in reality importantly interact. Whether this premise is true or not is not known and no papers have been published in the open literature which will definitely answer the question. Qvale (68 m, 69 n) and Rios (70 z) have both published papers claiming good agreement between their advanced second-order design procedures and experimental measurements. Third-order design methods are an attempt to compute the complex process going on in a Stirling engine all of a piece. Finkelstein
pioneered this development (62 a, 64 b, 67 d, 75 a) and in the last year or so a number of other people have taken up the work. If the third-order method is experimentally validated, then much can be learned about the workings of the machine that cannot be measured reliably.

Third-order design methods start by writing down the differential equations which express the ideas of conservation of energy, mass and momentum. These equations are too complex for a general analytical solution so they are solved numerically. The differential equations are reduced to their one dimensional form. Then depending on just what author's formulation is being used, additional simplifications are employed.

In this design manual the non-proprietary third-order design methods will be discussed. In this section it will not be possible to describe these methods in detail. However, the basic assumptions that go into each calculation procedure will be given.

5.4.1 Basic Design Method

In broad outline the basic design method is as follows (see Figure 5-25):
1. Specify dimensions and operating conditions, i.e., temperatures, charge pressure, motion of parts, etc. Divide engine into control volumes.
2. Convert the differential equations expressing the conservation of mass, momentum and energy into difference equations. Include the kinetic energy of gas. Include empirical formulas for the friction factor and the heat transfer coefficient.
3. Find a mathematically stable method of solution of the engine parameters after one time step given the conditions at the beginning of that time step.
4. Start at an arbitrary initial condition and proceed through several engine cycles until steady state is reached by noting that the work output per cycle does not change.
5. Calculate heat input.

5.4.2 Fundamental Differential Equations

Following the explanation of Urieli (77 d), there are 4 equations that must be satisfied for each element. They are:
1. Continuity
2. Momentum
3. Energy
4. Equation of state

These relationships will be given in words and then in the symbols used by Urieli using the generalized control volume shown on Figure 5-26.
Figure 5-26. The control volume for Third-Order Analysis.

Figure 5-25. Sample Division of Engine Working Gas Space into Control Volumes.
The continuity equation merely expresses the fact that matter can neither be created nor destroyed. Thus:

\[
\frac{\text{rate of decrease of mass in control volume}}{\text{mass in control volume}} = \frac{\text{net mass flux convected outwards through surface of control volume}}{}
\]

Urieli (77 d) expresses this relationship as:

\[
\frac{\partial m}{\partial t} + V \frac{\partial q}{\partial x} = 0
\]

where:

- \( m = \dot{m}/M \)
- \( \dot{m} \) = mass of gas in control volume, Kg
- \( M \) = mass of gas in engine, Kg
- \( t \) = time, seconds
- \( V = \bar{V}/V_s \)
- \( \bar{V} \) = volume of control volume, m³
- \( V_s \) = total power stroke volume of machine, m³
- \( g = \dot{g}/M(R(T_k)/V_s) \)
- \( \dot{g} \) = mass flux density, kg/m²·sec
- \( R \) = gas constant for working gas, J/Kg·K
- \( T_k \) = cold sink absolute temperature, K
- \( x = \dot{x}/(V_s)^{1/3} \)
- \( \dot{x} \) = distance, meters

5.4.2.2 Momentum Equation

\[
\frac{\partial}{\partial t} (gV) + V \frac{\partial}{\partial x} (g^2V) + V \frac{\partial p}{\partial x} + F = 0
\]

Urieli (77 d) expresses this relationship as:

\[
\frac{\partial}{\partial t} (g\bar{V}) + \bar{V} \frac{\partial}{\partial x} (g^2\bar{V}) + \bar{V} \frac{\partial p}{\partial x} + \mathcal{F} = 0
\]

where in addition:

- \( v = \bar{V}/(V_s/M) \)
- \( \bar{V} \) = specific volume, m³/Kg
- \( p = \bar{p}/(M(R(T_k)/V_s)) \)
- \( \bar{p} \) = pressure, N/m²
- \( F = \mathcal{F}/(M(R(T_k)/(V_s)^{1/3}) \)
- \( \mathcal{F} \) = frictional drag force, N
5.4.2.3 Energy Equation

\[
\left\{ \begin{array}{l}
\text{Rate of heat transfer} \\
\text{to the working gas} \\
\text{from the environment} \\
\text{through control surface } A
\end{array} \right\} = \left\{ \begin{array}{l}
\text{Rate of energy} \\
\text{accumulation} \\
\text{within the control} \\
\text{volume } V
\end{array} \right\}
\]

\[
+ \left\{ \begin{array}{l}
\text{Net energy flux convected} \\
\text{outwards by the working} \\
\text{gas crossing the control} \\
\text{surface } A
\end{array} \right\} + \left\{ \begin{array}{l}
\text{Net rate of flow work} \\
\text{in pushing the mass of} \\
\text{working gas through} \\
\text{the control surface } A
\end{array} \right\}
\]

\[
+ \left\{ \begin{array}{l}
\text{Net rate of mechanical work done by} \\
\text{the working gas on the environment} \\
\text{by virtue of the rate of change of} \\
\text{the magnitude of the control volume } V
\end{array} \right\}
\]

Urieli (77 d) expresses this relationship finally as:

\[
\frac{dQ}{dt} = \frac{3}{\gamma} \left( \frac{mT}{\gamma - 1} \right) + \frac{3}{\gamma} \gamma \left( T(Tk) \right) - g(v) \left( V \frac{\partial p}{\partial x} + F \right) + \frac{DW}{dt}
\]

(5-86)

where in addition:

\[
\begin{align*}
Q &= \frac{\dot{Q}}{(MRTk)} \\
\dot{Q} &= \text{heat transferred, J} \\
\gamma &= \text{ratio of specific heat capacity of working gas} = \frac{CP}{CV} \\
T &= \frac{T}{Tk} \\
\bar{T} &= \text{working gas temperature in control volume, K} \\
W &= \frac{\dot{W}}{(MRTk)} \\
\dot{W} &= \text{mechanical work done, J}
\end{align*}
\]

5.4.2.4 Equation of State

Due to the normalizing parameters Urieli uses the equation of state merely as:

\[
p(V) = m(T)
\]

(5-87)

5.4.3 Comparison of Third-Order Design Methods

A number of third-order design methods will be described briefly.

5.4.3.1 Urieli

This design method is described fully in Israel Urieli's thesis (77 af). A good short explanation is given in his IECEC paper (77 d). He applies his method to an experimental Stirling engine of the two-piston type. The hot cylinder is connected to the cold cylinder by a number of tubes in parallel. Sections of each one of these tubes are heated, cooled or allowed to seek their
own temperature level in the regenerator part. This type of engine was chosen because of ease in programming, and because heat transfer and fluid flow correlations for tubes are well known. Also, an engine like this is built and is operating at the University of Witwatersrand in Johannesburg, South Africa. The intention is to obtain experimental confirmation of this design method. Urieli converts the above partial differential equations to a system of ordinary differential equations by converting all differentials to difference quotients except for the time variable. (See Appendix A.) Then he solves these ordinary differential equations using the fourth order Runge-Kutta method starting from a stationary initial condition. The thesis contains the FORTRAN program. The first copies of this thesis has three errors in the main program. Urieli applied this program to the JPL test engine (78 ar). However, no data have yet come out to compare it with. The program is further discussed in general (79 ac).

5.4.3.2 Schock

Al Schock, Fairchild Industries, Germantown, Maryland, presented some results of calculations using his third-order design procedure at the Stirling Engine Seminar at the Joint Center for Graduate Study in Richland, Washington, August 1977. His calculation started with the same differential equations as Urieli but his method of computer modeling was different but undefined. He confirmed what Urieli had said at the same meeting that the time step must be smaller than the time it takes for sound to travel from one node to the next through the gas. Al Schock's assignment was to develop an improved computer program for the free displacer, free piston Stirling engine built by Sunpower for DOE. The engine had a very porous regenerator. Although the pressures in the expansion and compression space of the engine were different, they were not visibly different when the gas pressure versus time was plotted.

This program is as yet not publicly documented. Schock is awaiting good experimental data with which to correlate the model. Many results were presented at the 1978 IEECE (78 ag) and in the Journal of Energy (79 eh). Schock makes good use of computer-drawn graphics to show what is going on in a free piston machine that was simulated. The last reference states that a listing can be obtained by contacting Al Schock. The author has contacted Dr. Schock but has yet to receive the listing. The program is fully rigorous, but for economy it can be cut down to not include the effect of gas acceleration.

5.4.3.3 Vanderbrug

In reference 77 ae, Finegold and Vanderbrug present a general purpose Stirling engine systems and analysis program. The program is explained and listed in a 42-page appendix.
One paper (79 aa) presents some additional information on this program and shows how SCAM agrees with one experimental point so far published. Table 5-6 shows the comparison. Note that the simple Schmidt cycle predicts almost as well as the SCAM program. Many more data points are needed before SCAM will have a fair evaluation.

5.4.3.4 Finkelstein

Ted Finkelstein has made his computer analysis program (75 al) available through Cybernet. Instructions and directions for use are obtainable from TCA, P. O. Box 643, Beverly Hills, California 90213. One must become skilled in the use of this program since as the engine is optimized it is important to adjust the temperature of some of the metal parts so that the metal temperature at the end of the cycle is nearly the same as at the beginning.

Table 5-6

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<td>1300</td>
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<tr>
<td>SCAM</td>
<td>105</td>
<td>1300</td>
<td>326</td>
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</table>

* Test number 8 16-10
**Dynamometer measurement

Urieli and Finkelstein use the same method in handling the regenerator nodes in that the flow conductance from one node to the next depends upon the direction of flow. Finkelstein solves the same equations as Urieli presents but he neglects the kinetic energy of the flowing gas. By so doing, he is able to increase his time step substantially. Neglecting kinetic energy will cause errors in predicting pressures during the cycle. However, it is not clear what effect this simplifying assumption has upon power output and efficiency calculations. To make a comparison one would have to use the same correlations for friction factor and heat transfer coefficient and be certain that the geometries are identical.

Finkelstein claims that his program has been validated experimentally but the results are proprietary.
5.4.3.5 Lewis Research Center (LeRC)

The author has attempted to formulate a design procedure based upon some computation concepts originally used by M. Mayer at McDonnell Douglas. A simplified version was presented (75 ac). However, an attempt failed to extend the method to include a real regenerator with dead volume and heat transfer as a function of fluid flow. The procedure was computationally stable and approached a limiting value as the time step decreased. But when the heat transfer coefficients were set very high, there should have been no heat loss through the regenerator, but the computation procedure did not allow this to happen because gas was always entering the hot space at the temperature of the hottest regenerator element. There was also the problem of finding the proper metal temperature for the regenerator elements.

Parallel and independently of the author, Roy Tew, Kent Jefferies and Dave Miao at LeRC have developed a computer program which is very similar to the author's (77 b). In addition, they have found a way of handling the regenerator which gets around the problem the author encountered.

The LeRC method assumes that the momentum equation need not be considered along with the equations for continuity, energy and equation of state. They assume that the pressure is uniform throughout the engine and varies with time during the engine cycle. LeRC combines the continuity, energy equation and equation of state into one equation.

\[
\frac{dT}{dt} = \frac{hA}{mc_p}(T_w - T) + \frac{w_f}{m}(T_i - T) + \frac{w_o}{m}(T_o - T) + \frac{V}{mc_p}\frac{dp}{dt}
\]

This equation indicates that the temperature change in a control volume depends upon heat transfer, flow in and out and pressure change. Equation 5-88 could be solved by first-order numerical integration or by higher order techniques such as 4th order Runge Kutta. LeRC did not use this approach.

LeRC used an approach of separating the three effects and considering them successively instead of simultaneously. From a previous time step they have the masses, temperature and volumes for all 13 gas nodes used. From this they calculate a new common pressure. Using this new pressure and the old pressure and assuming no heat transfer during this stage, they calculate a new temperature for each gas node using the familiar adiabatic compression formula. Next, the volumes of nodes 1 and 13, the expansion and compression space, are changed to the new value based upon the rhombic drive. New masses are calculated for each control volume. Once the new mass distribution is known, the new flow rates between nodes are calculated from the old and new mass distributions. The new gas temperature is now modified to take into account the gas flow into and out of the control volumes during the time step. During this calculation it is assumed that each regenerator control volume has a temperature gradient across it equal to the parallel metal temperature gradient and that the temperature of the fluid that flows across the boundary is equal to the average temperature of the fluid before it crossed the boundary; heater and cooler control volumes are at the bulk or average temperature throughout. Next, local heat transfer coefficients are calculated based upon the flows. Temperature equilibration with
the metal walls and matrix is now calculated for the time of one time step and at constant pressure. An exponential equation is used so that no matter how large the heat transfer coefficient, the gas temperature cannot change more than the $\Delta T$ between the wall and the gas. Heat transfer during this equilibration is calculated. In the regenerator nodes heat transfer is used to change the temperature of the metal according to its heat capacity. In the other nodes where the temperature is controlled, the heat transfers are summed to give the basic heat input and heat output. This final temperature set after temperature equilibration along with the new masses and volumes calculated during this time step are now set to be the old ones to start the process for the next time step.

The model is set up to take into account leakage between the buffer space and the working gas volume. LeRC has developed an elaborate method of accelerating convergence of the metal nodes in the regenerator to the steady state temperature.

On the final cycle LeRC considers the effect of flow friction to make the pressure in the compression and expansion space different from each other in a way to reduce indicated work per cycle.

To quote Tew (77 b1):

Typically it takes about 10 cycles with regenerator temperature correction before the regenerator metal temperatures steady out. Due to the leakage between the working and buffer spaces, a number of cycles are required for the mass distribution between working and buffer space to settle out. The smaller the leakage rate, the longer the time required for the mass distribution to reach steady-state. For the range of leakage rates considered thus far it takes longer for the mass distribution to steady out than for the regenerator metal temperatures to settle out. Current procedure is to turn the metal temperature convergence scheme on at the 5th cycle and off at the 15th cycle. The model is then allowed to run for 15 to 25 more cycles to allow the mass distribution to settle out. When a sufficient number of cycles have been completed for steady operation to be achieved, the run is terminated.

Current computing time is about 5 minutes for 50 cycles on a UNIVAC 1100 or 0.1 minute per cycle. This is based on 1000 iterations per cycle or a time increment of $2 \times 10^{-5}$ seconds when the engine frequency is 50 Hz. The number of iterations per cycle (and therefore computing time) can be reduced by at least a factor of 5 at the expense of accuracy of solution. On the order of 10% increase in power and efficiency results when iterations per cycle are reduced to 200 from 1000.

The agreement between the NASA-Lewis model and experiment is discussed in (79a). They got agreement between calculated results and measurements only after they multiplied the computed friction factor for the regenerator by a factor of 4 for hydrogen and by a factor of 2.6 for helium. In a different way this is the same order of magnitude correction that the best second-order analysis requires.
5.4.4 Conclusions on Third-Order Design Methods

1. A number of well constructed third-order design methods are available.
2. A choice is available between rigorous third-order (Urieli, Schock, Vanderbrug), third-order ignoring fluid inertia (Finkelstein), third-order assuming a common pressure (LeRC).
3. There is a spectrum of design methods reaching from the simplest first-order through simple and complex second-order culminating in rigorous third-order analysis. However, all these methods depend upon heat transfer and fluid flow correlations based upon steady flow instead of periodic flow, because correlations of periodic flow heat transfer and flow friction which should be used have not been generated.
4. Third-order analysis can be used to compute flows and temperatures inside the engine which cannot be measured in practice.
5. Third-order analysis can be used to develop simple equations to be used in second-order analysis.
6. Eventually when all calculation procedures are perfected to agree as well as possible with valid tests of Stirling engines, third-order design methods will be the most accurate and also the longest. The most rigorous formulations of third-order will be much longer and more accurate than the least rigorous formulations.
6. REFERENCES

6.1 Introduction

The references in this section are revised and extended from the first edition (78 ed). The authors own accumulation has been cataloged. Also extensive bibliographies by Walker (78 dc) and Aun (78 eb) were checked for additional references. Cataloging of references continues. The following list is as of April 1980.*

Each entry in the following reference list corresponds to a file folder in the author's file. If the author has an abstract or a copy of the paper an asterisk (*) appears at the end of the reference.

All personal authors are indexed (see Section 7).

All known corporate authors are indexed (see Section 8).

The subject index included in the first edition has been deleted because it was found not to be very useful. Possibly some day an index to the Stirling engine literature can be written.

6.2 Interest in Stirling Engines

Because of the way Stirling engine references are cataloged in this section it is easy to plot the rise in interest in Stirling engines by the number of references each year in the literature. Figure 6-1 shows the references per year for the last few years.

6.3 References


1826 a Ericsson, J., British Patent No. 5398, 1826. *


*Note in final preparation: The completion date of the second edition was July 1979. At the request of H. Valentine the references were updated to April 1980. A further update to October 1981 is now available.
Figure 6-1. Stirling Engine References
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<th>Author(s)</th>
<th>Title</th>
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1871 c The Roper Hot Air or Caloric Engine Co. Catalog., reprinted by Alan G. Phillips, P.O. Box 20511, Orlando, FL 32814.


1876 a Ericsson, J., Contributions to the Centennial Exhibition, 1876.

1878 b Bourne, J., "Examples of Steam, Air and Gas Engines of the Most Recent Approved Type," Longmans, Green and Co., London, 1878


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1919 a L'Air Liquide Societe Anonyme, "Improvements in or Relating to Heat Engines," British Patent 126,940 - Complete Not Accepted, January 6, 1919. *


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<th>Volume/Publication No.</th>
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<td>Malone, J.F.J.</td>
<td>&quot;A New Prime Mover,&quot;</td>
<td>The Engineer</td>
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75, pp. 779-787, 1953. *


Imperial College, London, 1953.

691,781, May 20, 1953. *

1953 e  Schultz, B.H., "Approximate Formulae in the Theory of Thermal

1953 f  Philips Co., "Improvements in or Relating to Hot-Gas Reciprocating
Engines and Engines Operating on the Reversed Hot-Gas Engine

1953 g  Philips Co., "Improvements in Heat Regenerator Filling Material

1953 h  Philips Co., "Improvements in or Relating to Hot-Gas Engines,
British Patent 697,082, September 16, 1953.*

1953 i  Philips Co., "Improvements in Hot-Gas Reciprocating Engines,
British Patent 691,785, May 20, 1953. *

1953 j  Philips Co., "Improvements in Hot-Gas Engines," British Patent
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ORIGINAL PAGE IS OF POOR QUALITY


1962 d  Vacant


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<tr>
<td>van der Sluys, Willem L. N.</td>
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<td>van der Ster, J.</td>
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<td>van Eekelen, J. A. M.</td>
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<td>van Nederveen, H. B.</td>
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<td>van Weenen, F. L.</td>
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<td>van Witteveen, R.</td>
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<td>Varney, Frederick M.</td>
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<td>Vedin, B. A.</td>
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<td>Ver Beek, H. J.</td>
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<td>Vesely, E. J., Jr.</td>
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<td>Vicklund, G. D.</td>
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Zykov, V. M., 74 am, 74 bs
8. CORPORATE AUTHOR INDEX

A corporate author is the organization the personal author works for and the organizations that sponsored the work. A reference may have several corporate authors. The references (Section 7) and the reports themselves were searched for corporate authors.

Advanced Technology Lab

69 aa

AERE-Harwell

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Aerojet Energy Conversion Co.

78 ca

Aerojet Liquid Rocket Co.

68 e, 68 h, 68 j, 69 i, 69 ak, 70 r, 71 az, 72 d, 72 k, 73 r, 73 at, 74 x, 74 au, 75 p, 75 au, 76 al, 76 aq, 77 cd

Aerospace Corp.

71 as, 71 bb, 74 at, 74 bp, 74 bq, 75 ae

AFFDL

67 e, 68 x, 69 l, 70 ac, 71 bf, 72 t, 72 z, 74 l, 75 a, 75 b, 76 l, 78 do, 78 dz, 78 ea

Air Product & Chemicals, Inc.

71 j, 71 be, 74 as

Air Systems Command

63 j

Aisin Seiki Company, Ltd.

79 t, 79 aw, 79 bh

American Gas Association

62 m, 77 r, 77 ck

American Industrial Systems, Inc.

76 bq

American Machine Co.

08 a

Amtech Incorp.

78 g, 78 bc

Argonne National Laboratories

78 m, 78 ac, 78 cx, 79 ai, 79 aj, 79 ay, 79 bi, 79 bk, 79 cy

Arthur D. Little, Inc.

59 j, 60 d, 61 h, 63 s, 64 f, 78 dz, 79 aj

Atomic Energy Commission

71 ay, 72 e, 72 f, 72 al, 73 ax

Battelle

61 e, 62 m, 65 a, 65 u, 66 c, 67 b, 68 o, 68 y, 69 d, 73 ay

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79 bu
Boeing
77 au, 78 ah, 78 cy

Booz-Allen Applied Research Inc.
70 w, 72 ao

Bremen Mfg, Co.
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Brigham Young University
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Bucknell University
77 ad

Burns and McDonnell Engineering Co.
78 di

Carnegie-Mellon University
78 cz, 80 w

Chevron Research Co.
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Coleman Co., Inc.
57 f

Combustion Utilities Corp.
29 b

Commissariat A L'Energitc Atomique
73 am

Consolidated National Gas Service Co
72 y

Control Data Corp.
71 n

Corning Glass Works
77 o

Cummins Engine Co.
74 aa

D-Cycle Power Systems, Inc.
77 q

Defense Research Establishment
78 bx

DeLamater Iron Works
1887 b, 1888 b, 1888 c, 1890 b

Department of Commerce
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Department of Defense
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79 cl, 79 cj, 79 ck, 79 cm, 79 cn,
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80 v, 80 w

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Ebasco Services Incorp.
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Ecole Polytechnique de Varsovie
74 cc

English Electric Co.
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Electro-Optical Systems
60 w

ERDA
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European Nuclear Energy Agency
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Fairchild Space & Electronics Co.
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FFV Company
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Florida International University
76 ba

Ford Aerospace & Communications Corp.
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Ford Motor Co.
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George Mason University
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73 ac
Goddard Space Flight Center
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Hague International
75 am

Hartford National Bank and Trust Co.
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52 g, 52 h, 52 i, 55 d, 56 b,
56 d, 56 e, 56 f, 57 c, 57 h,
57 i, 57 j

HEW
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Honeywell Radiation Center
74 l

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75 a, 78 do, 78 ea

IBM
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IIT Research Institute
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Indian Institute of Science
54 a, 55 a, 59 k

Institute of Gas Technology
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Institute of Nuclear Physics-USSR
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Intermediate Technology Development Group
72 az

International Research and Technology Corp.
73 af

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78 au, 78 bu, 78 bw, 78 cr, 78 cs,
78 ei, 79 n, 79 aa, 79 bn, 79 bp,
79 cb, 79 cv

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77 ci, 78 1, 78 o, 78 p, 78 ad,
78 bt, 78 bz, 78 cb, 78 ck, 78 db,
78 dx, 79 c, 79 h, 79 q, 79 ad,
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Kaiser Engineers
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63 f, 63 g, 75 i, 76 h, 77 z,
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72 c, 72 aq, 73 a, 73 y, 73 ad,
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79 bq, 80 f, 80 g, 80 h, 80 i

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69 ac, 69 al, 70 v, 70 x, 71 i,
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73 al, 73 an, 74 n, 74 a, 74 p,
74 av, 75 r, 75 be, 76 r, 76 v,
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Mechanical Technology Inc.
72 v, 76 az, 77 b, 77 m, 77 s,
77 ar, 78 i, 78 x, 78 ba, 78 bd,
78 cf, 78 cl, 78 cw, 78 dt, 79 e,
79 o, 79 p, 79 r, 79 an, 79 ba,
79 bc, 79 bm, 79 bz, 79 cn, 79 co,
79 ct, 80 f, 80 q

Medtronics, Inc.
73 w

Minot State College
77 ak, 77 bf

62 b, 62 c, 65 v, 66 h, 66 p,
67 l, 67 n, 68 g, 68 m, 68 r,
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71 an, 72 ar, 73 ay, 75 am, 75 bb,
75 bf, 76 bm, 78 at

Motorola, Inc.
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NASA-Lewis
55 b, 61 i, 64 k, 65 k, 65 n, 65 y,
71 am, 74 bc, 75 ah, 76 ai, 76 ap,
77 p, 77 ab, 77 ae, 77 ao, 77 aq,
77 ar, 77 as, 77 at, 77 au, 77 av,
77 bd, 77 bp, 77 cj, 77 cs, 78 b,
78 l, 78 q, 78 v, 78 w, 78 x, 78 y,
78 z, 78 ab, 78 ad, 78 af, 78 ag,
78 ah, 78 ai, 78 aj, 78 ak, 78 au,
78 bu, 78 cc, 78 cd, 78 ce, 78 cf,
78 cg, 78 ck, 78 cm, 78 ch, 78 co,
78 cp, 78 cq, 78 ct, 78 cv, 78 cw,
78 cy, 78 cz, 78 da, 78 dl, 78 dn,
78 dt, 79 a, 79 l, 79 n, 79 o, 79 p,
79 bl, 79 bm, 79 br, 79 bv, 79 bw,
79 bz, 79 ca, 79 cb, 79 cc, 79 co,
79 cp, 79 cr, 79 ct, 79 cw, 80 f,
80 q, 80 r, 80 s, 80 t, 80 v, 80 w
Philips, North American
57 g, 57 k, 58 c, 58 h, 58 i, 59 d, 59 h, 59 l, 59 m, 60 o, 60 p, 60 q, 60 r, 60 t, 63 r, 65 v, 66 l, 67 e, 70 h, 70 p, 70 ah, 71 l, 71 p, 71 v, 73 x, 73 ap, 74 b, 74 w, 74 bj, 75 b, 75 m, 75 ab, 76 e, 76 am, 77 f, 77 v, 77 y, 77 ax, 77 bx, 78 bb, 79 aq, 79 av, 79 az, 79 bc

Purdue University
68 m, 68 r, 69 n, 70 m, 71 aj, 71 ak, 74 br

RCA
72 af, 74 y, 75 ac

R & D Associates
79 be

Reactor Centrum Nederland
66 d

Reading University - U.K.
75 k, 78 ay

Recold Corp.
60 s

Research Corp.
38 b, 39 a, 71 aq, 72 x

Rider-Ericsson Engine Co.
06 a, 06 c

Rocketdyne
64 c, 65 c, 67 c, 67 d

Roesel Lab
74 s

Royal Naval Engineering College
78 ap, 79 z, 79 ab, 79 ae, 79 cf

Shaker Research Corp.
78 v, 78 da, 79 l

Sigma Research Inc.
78 m, 79 ay, 79 bk

Space Power Systems, Corp.
60 b, 60 f

Stanford University
50 a, 52 a, 53 a, 76 ak

Stirling Technology Inc.
80 x

Stirling Power Systems
78 ci, 78 cj, 79 ap, 80 p

Solar Energy Research Institute
79 cu

Stone & Webster Engineering Corp.
71 ak

Sunpower
75 n, 75 s, 75 cf, 76 bd, 78 e, 78 as, 78 dr, 78 du, 79 ar, 79 bf

Syracuse University
64 d, 65 d, 66 i

TCA Stirling Engine Research and Development Co.
70 f, 70 g, 72 u, 75 al, 78 al

Technical University of Denmark
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Texas Instruments, Inc.  67 1, 72 am

Thermo Electron Corp.  71 b, 72 d, 74 ba, 75 ai, 76 bc, 78 ac, 78 cx, 79 cy

Thermo-Mechanical Systems Co.  72 ap

Tokyo Gas Company, Ltd.  78 ed, 79 t

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United States Congress, OTA  78 n

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United States Environmental Protection Agency  73 ak, 74 an

United States Naval Post-Graduate-School  64 a, 64 e

United Stirling of Sweden  70 o, 71 m, 71 ah, 73 a, 73 s, 74 z, 75 j, 75 az, 75 bk, 75 by, 77 i, 77 j, 77 al, 77 am, 77 bj, 77 cl, 78 aa, 78 cu, 79 r, 79 bv, 80 t, 80 v

United Technologies Research Center  79 s

Universite Paris X  74 cc

University of Bath  68 af, 71 ae, 72 aj, 72 ax, 73 bd, 74 bu, 78 f, 78 bs, 79 ao

University of Birmingham  70 k, 71 u

University of Calgary  68 n, 68 ad, 69 p, 69 q, 70 g, 71 k, 71 n, 71 o, 72 j, 73 i, 73 j, 73 m, 73 u, 73 v, 74 ao, 74 bx, 76 ax, 76 bl, 77 cg, 78 f, 78 bs, 78 dc, 79 y, 79 ao, 80 c, 80 d, 80 n, 80 o

University of California at Berkeley  75 am

University of California at Los Angeles  79 m

University of California at San Diego  79 bx, 79 by

University of Dakar - Senegal  77 cu

University of Florida  69 o, 70 q

University of London  52 b, 53 c, 61 q, 67 f

University of Michigan  61 n, 68 b

University of Texas  74 bt

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University of Toledo
78 ai

University of Utah
75 ba, 76 au

University of Wisconsin
60 j, 60 v, 60 x, 61 b, 71 h

University of Witwatersrand
75 w, 76 i, 76 x, 76 y, 77 c, 77 d, 77 e, 77 g, 77 af, 77 bq, 78 s, 78 am, 79 g, 79 ah, 79 bb, 79 bg, 79 bt, 79 cx

Utah University
74 az

Washington State University, Medical College
77 x, 78 bz, 79 an

Wayne State University
71 q, 72 r, 73 ar

Westinghouse
73 ax, 74 w, 74 ax, 74 ay, 75 ab, 75 cb, 76 am, 76 ao, 76 ap, 77 cb

West Pakistan University of Engineering and Technology
65 i

Winnebago Industries, Inc.
78 ch

Wright Patterson AFB
62 o, 73 au, 73 av, 74 l

Wolfe & Holland, Ltd.
79 ae

Zagreb University
68 k
This section gives as complete list as possibly of the people and organizations involved in Stirling engines in 1979. Eighty-two organizations responded to the questionnaire that was sent out or are mentioned in the recent literature as being currently active in Stirling engines. These questionnaires are given in Section 9.5 in alphabetical order by company. For the convenience of the reader, the questionnaires were analyzed to obtain as far as possible a ready index to this information. The following indexes are given:

1. Company
2. Contact Person
3. Country and Persons Working
4. Service or Product

9.1 **Company List**

Even though the questionnaires in Section 9.5 are given in alphabetical order by organization, it is sometimes difficult to be consistent about the organization. Therefore, for the convenience of the reader, the organizations are given with the entry number in Table 9-1.

9.2 **Contact Person**

The person or persons mentioned in the questionnaires as the contact person are given in alphabetical order in Table 9-2.

9.3 **Country and Persons Working**

This information is not as informative as was hoped as many of the large efforts in Stirling engines like Phillips and United Stirling did not answer this question.* Table 9-3 shows the country, gives the number used in Section 9-5 and in Tables 9-1 and 9-2, and gives the number of workers if it was given. Otherwise a number is estimated. The number is preceeded by an approximation sign ( ). The total number of organizations and workers for each country is given in Table 9-4.

9.4 **Service or Product**

In order for the information contained in this survey to be of maximum use, Table 9-5 has been prepared which gives the service or product offered or being developed. The numbers in Table 9-5 refer to entry numbers in Section 9-5.

9.5 **Transcription of Questionnaires**

The Questionnaire set out was somewhat ambiguous so the answers came back in different ways. Also to keep from repeating the questions the following format is followed:

*However, estimates were made from other sources.*
Table 9-1
ORGANIZATIONS ACTIVE IN STIRLING ENGINES

1. Advanced Mechanical Technology, Inc.
2. Advanced Energy Systems Division, Westinghouse Electric Corporation
3. Aerojet Energy Conversion Company
4. AGA Navigation Aids Ltd.
5. AiResearch Company
6. Aisin Seiki Company, Ltd.
7. All-Union Correspondence Polytechnical Institute
8. Argonne National Laboratory
9. Boeing Commercial Airplane Company
10. British Oxygen Company
11. Cambridge University, Engineering Department
13. CMC Aktiebolag
14. Cryomeck, Inc.
15. CTI-Cryogenics
16. G. Cussons, Ltd.
17. Daihatsu Diesel Company
18. Eco Motor Industries Ltd.
20. Fairchild Industries
21. Far Infra Red Laboratory
22. F. F. V. Industrial Products
23. Foster-Miller Associates
24. General Electric Space Division
25. Hughes Aircraft Company
26. Japan Automobile Research Institute, Inc.
27. Jet Propulsion Laboratory
28. Joint Center for Graduate Study
29. Josam Manufacturing Company
30. Leybold Heraeus
31. M.A.N. - AG
32. Martini Engineering
33. Martin Marietta Inc.
34. Massachusetts Institute of Technology
35. Mechanical Engineering Institute
36. Mechanical Technology Incorporated
37. Meiji University
38. Mitsubishi Heavy Industries
39. N. V. Philips Industries
40. N. V. Philips Research Laboratories
41. National Bureau of Standards
42. National Bureau of Standards Cryogenics Laboratory
43. NASA-Lewis Research Center
44. Nippon Piston Ring Company, Ltd.
45. Nissan Motor Company, Ltd.
46. North American Philips Corporation
47. Wm. Olds and Sons
48. Ormat Turbines
49. Alan G. Phillips
50. Radan Associates Ltd.
51. Ross Enterprises
52. Royal Naval Engineering College
53. Schuman, Mark
54. Shaker Research Corporation
55. Shipbuilding Research Association of Japan
56. Ship Research Institute
57. Solar Engines
58. Starodubtsev Physicotechnical Institute
59. Stirling Engine Consortium
60. Stirling Power Systems Corporation
61. Sunpower Inc.
62. TCA Stirling Engine Research and Development Company
63. Technical University of Denmark
64. Texas Instruments
65. Thermacore, Inc.
66. Tokyo Gas Company
67. Tokyo Institute of Technology
68. United Kingdom Atomic Energy Authority
69. United States Department of Energy
70. United Stirling
71. Urwick, W. David
72. University of Calgary
73. University of California, San Diego
74. University of Tokyo
75. University of Tokyo, Department of Mechanical Engineering
76. University of Tokyo, Faculty of Engineering
77. University of Witwatersrand
78. Weizmann Institute of Science
79. West, C. D.
80. Yanmar Diesel Company
81. Zagreb University
Late insertions:
82. Thomas, F. Brian
83. Clark Power Systems Inc.
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<td>Allen, Paul C.</td>
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<td>Anderson, Niels Elmo</td>
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Table 9-3: Country and Persons Working
Table 9-3. COUNTRY AND PERSONS WORKING (continued)

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<td>7</td>
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Table 9-4  
WORLDWIDE BREAKDOWN IN STIRLING ENGINE INDUSTRY

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<th>Nation</th>
<th>Number of Organizations</th>
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<td>Japan</td>
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<td>~44</td>
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<td>United Kingdom</td>
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<td>~176</td>
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<td>Canada</td>
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<td>TOTAL</td>
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Table 9-5
STIRLING ENGINE PRODUCTS AND SERVICES
(Numbers refer to entry numbers in Section 9.5)

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<tr>
<td>Demonstration (Model) Engines</td>
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<td>Diesel-Stirling Combined Cycle</td>
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<td>Electric Generator Engines</td>
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<td>Free Piston Engines</td>
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<td>Fuel Emissions</td>
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<td>Gas Compressors</td>
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<td>General Consulting Services</td>
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<td>Heat Pumps</td>
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<td>Remote, Super-reliable Power</td>
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</table>
Advanced Mechanical Technology Inc. (AMTI) (3)
141 California St.
Newton, Mass. 02158
Attn: Dr. Lawrence C. Hoagland or Dr. Walter D. Syniuta
Telephone: (617) 965-3660


AMTI is prime contractor for DOE program and United Stirling (Malmo, Sweden) is subcontractor on Stirling engine design/development. Ricardo Consulting Engineers Ltd. (England) will serve as consultants to USS. Emphasis is on burning coal and coal-derived fuels and biomass in large engines for ICES. Program is just getting underway. We are under contract for phase I only which is an 8-month conceptual design study.

Advanced Energy Systems Div., Westinghouse Electric Corp. (0)
P. O. Box 10864
Pittsburgh, Pa. 15236
Attn: W. D. Pouchot

Had worked on System Integration for artificial heart power using a Stirling engine. Program was phased out in 1978. No current activity.

Aerojet Energy Conversion Co. (5)
P. O. Box 13222
Sacramento, Ca. 95813
Attn: John Moise or Bill Blubaugh
Telephone: (916) 355-2018

Have developed thermocompressor with potential for 10-year high reliability life for driving fully implantable left heart assist system. The unit has demonstrated over 17 percent efficiency with 20 watts input, weighs 0.94 kg and has a volume of 0.43 liters. Over 120,000 hours of endurance testing has been accomplished on thermocompressors for heart assist application.

AGA Navigation Aids Ltd. (3)
Brentford, Middlesex, TW 80 AB, England
Attn: K. C. Sutton-Jones
Telephone: 01-560 6465 Telex: 935956

We have reached the stage of preparing production drawings following full evaluation of the prototype thermo-mechanical generator. It is our intention to commence production early in 1980 and expect to have this machine on the market by the middle of next year (viz. June 1980.) It is anticipated
that the selling price for this unit will be approximately $11,000 and the unit we provide will be capable of delivering 60W 24V continuously into a battery for the consumption of approximately 450 KG. of pure propane gas per annum.

We hope to undertake further development to ascertain that the machine will also operate from less refined fuel, but this will take some time yet to perfect.

(5) AiResearch Co.
Cryo/Cooler Div.
Murray Hill, N. J.

No Response

(6) Aisin Seiki Co., Ltd.
1, Asahi-machi 2-chome
Kariya City, Aichi Pref., Japan
Attn: Shigenori Haramura
Telephone: 0566 24 8337
Telex: 4545-714 AISIN J

The development of the Stirling engine has been started from October, 1975, by Aisin Seiki Co., Ltd., a member of Toyota Motor Group of Companies. We are at present developing a 50 KW Stirling engine for automobile and generator use. This is in cooperation with Tokyo University and under a grant from M.I.T.I. We are trying to achieve the max shaft power of 50KW/3000 rpm and the thermal efficiency of 30 percent/1500 rpm. We have recently achieved 41 KW/2000 rpm and 27.80 percent/1000 rpm. Furthermore we are also developing a 10 hp engine and are conducting research into heat pump systems in cooperation with Tokyo Gas Co.

(7) All-Union Correspondence Polytechnical Institute
USSR, Moscow, 129278
ul, Pavla Korchagina, 22
Attn: Docent Beilin V. I.
Telephone: 283 43 87

Developing of highly effective device with the 20 KW power engine, using gaslike hydroge as fuel.
(Martini comment: This probably means hydrogen working gas.)

(8) Argonne National Laboratory
Components Technology Division
Building 330
Argonne, Illinois 60439
Attn: Robert E. Holtz
Telephone: (312) 972-4465
Telex: 910-258-3285

The goal of this program is to develop and demonstrate large stationary Stirling engines, in the 500 to 3000 hp range, that can be employed with solid coal, coal-derived fuels, and other alternate fuels. Included in this effort are engine design, integration of the heat source with the engine, component testing, prototype construction and testing, and implementation. Accomplishments: Three industrial teams have initiated a conceptual design study of alternate engine configurations. This effort will be followed by the industrial based final design and construction efforts. Studies concerned...
with the integration of the engine with various combustor options are underway. Also, experimental efforts dealing with both seals testing and the measurement of the heat transfer and fluid mechanics during oscillating flow conditions are underway.

(9) Boeing Commercial Airplane Co.
P. O. Box 3707 M.S. 4203
Seattle, Wa. 98124
Attn: John F. Curulla
Telephone: (206) 655-8219

Evaluation of Reciprocating seals concepts has shown that no seal to date (1) Footseal, (2) NASA Polyimide Chevron Seal, (3) Bell Seal or (4) Quad Seal can meet the stringent requirements of 1500 fpm surface speed with 1750 psig gas pressure and 275°F ambient.

(10) British Oxygen Co.
Cryocoole Division
No Response

(11) Cambridge University Engineering Department
Trumpington St.
Cambridge CB2 1PZ
U. K.
Attn: Allan J. Organ
Telephone: Cambridge 66466 Telex: 81239

Development of computer simulations of Stirling cycle machines. Design of miniature Stirling cryogenic coolers. Design of Stirling engines 1/4 - 5 KW. Preparation of facsimile manufacturing drawings of Stirling engines no longer commercially available (KYKO, Philips 200 Watt (1947) etc.)

(12) Carnegie-Mellon University
Pittsburgh, Pa. 15213
Attn: William F. Hughes
Telephone: (412) 578-2507

Study of seals for Stirling engine (reciprocating dry and lubricated.)

We have been interested in temperature calculations and development of criteria for operation below deleterious temperatures.

Presently we have been able to estimate temperature rises in these seals and hope to extend work to include elasto-hydrodynamic and pumping effects. This program is sponsored by NASA.

(13) CMC Aktiebolag
Sanekullavagen 43
S-21774 Malmo
Sweden
Attn: Stig G. Carlqvist
Telephone: 040-918602 Telegrams: Cemotor

Engineering consulting activity based on 30 years of development experience on advanced heat engines; 12 years on turbo-charged Diesel engines and 12 years on Stirling engines. Current program on Stirling engines is in the power range of 10 - 3000 HP, direct as well as indirect heat transfer and is mainly based on a new simplified engine concept and on improved components.
Accomplished in earlier activity the build-up of major Stirling engine company in Sweden (including advanced Stirling engine R & D laboratory.)

(14) Cryomeck, Inc.
Syracuse, New York, Attn. Dr. William Gifford (~5)
No response
(Martini comment: Dr. Gifford is also Professor Mechanical Engineering at the University of Syracuse. Cryomeck is a cooling engine company.

(15) CTI-Cryogenics (~20)
266 Second Ave.
Waltham, MA 02154
Attn: Fred F. Chellis
Telephone: (617) 890-9400
Design, development and manufacture of cryogenic coolers operating on the Stirling cycle, Vuilleumier cycle, and other regenerative cycles. Presently in production manufacture of the Stirling cycle Army Common Module Cooler. We are the American builder and supplier for the Philips designed Model B Stirling cycle machines for production of liquid nitrogen or liquid oxygen at about 25 liters per hour.

(16) G. Cussons Ltd. (~2)
102 Great Clowes Street
Manchester, M7 9RH
England
Attn: B. A. Fuller
Telephone: Telex: 667279
Supply of Stirling cycle hot air engine to universities, technical colleges and vocational training centres worldwide.

(17) Daihatsu Diesel Co. - Japan (~2)
Mr. H. Goto
No response
Involved in design and construction of an 800 hp Stirling engine for a sea craft (79a, 79bj).

(18) Eco Motor Industries Ltd
P. O. Box 934
Guelph N1H 6M6
Ontario, Canada
Attn: J. Pronovost or H. Derderian
Telephone: (519) 823-1470
1/4 HP instrument test bed. Wood fired commercial model under development. 1/2 and 1 KVA. commercial generating set propane fired under development.

(19) Energy Research & Generation, Inc. (10)
Lowell & 57th Street
Oakland, Ca. 94608
Attn: G. M. Benson
Telephone: (415) 658-9785
ERG has been developing for over ten years resonant free-piston Stirling type machines (Thermoscillators) including hydrostatic drives, linear alternators, heat pumps, cryogenic refrigerators and gas compressors. In addition, development has continued on a cruciform variable displacement crank-type Stirling engine having a Rinia arrangement. ERG is performing R & D on heat exchangers, heat pipes, isothermalizers, regenerators, gas springs, gas bearings, seals, materials (including silicon nitride and silicon carbide), and computer modeling as well as on linear motors and alternators, hydraulic drive components and external heat exchangers and heat sources (including combustors and solar collectors.) ERG has built and tested several test engines and presently has separate electro-mechanical, hydraulic, engine and heat exchanger test cells. ERG sells heat exchangers, regenerators, linear motor/alternators, linear motoring dynamometer test stands, gas springs/bearings, dynamic seals and hydraulic components. ERG plans to sell soon an oil-free isothermal compressor with linear motor drive and small Thermoscillators and laboratory demonstrators. The current status on ERG Stirling engines is given in references 77 a and u.

Current work involves both corporately funded and Government sponsored R and D programs. The Government contracts include: Advanced Stirling Engine Heat Exchangers (LeRC DEN-3-166); 15 KW(e) Free-Piston Stirling Engine Driven Linear Alternator (JPL 955468); Free-Piston Stirling Cryogenic Cooler (GSFC NAS 5-25344); Free-Piston Stirling Powered, Accumulator Buffered, Hydrostatic Drive (LeRC NAS 3-21483), Duocel, Foilfin and Thermizer Heat Exchangers (ONR N00014-78-C-0271), Hydrogen/Hydridge Storage (Argonne 7-895451). Pending contracts include Reciproseals, Large Linear Alternators, and Hydrostatic Drive Components.

(20) Fairchild Industries
Germantown, Md.
Attn: Mr. A. Schock
No response
Martini comment: Al Schock has written a fully rigorous Stirling engine computer program under DOE sponsorship.

(21) Far Infra Red Laboratory
U. S. Army Engineer Research and Development Lab.
Fort Belvoir, Virginia
No response

(22) F. F. V. Industrial Products
Linkoping, Sweden
No response
Martini comment: FFV makes the engine the Stirling Power Systems uses. They also are 50 percent owner of United Stirling. They are a Swedish National Company.

(23) Foster-Miller Associates
350 Second Avenue
Waltham, Mass. 01254
Attn: Dr. William M. Toscano
Telephone: (617) 890-3200
"Design and Development of Stirling Engines for Stationary Power Generation Applications in the 500 to 3000 Horsepower Range". Program funded by DOE/ANL. FMA has been Phase I entitled Conceptual Design. Work has just been initiated; no accomplishments to date.

(24) General Electric Space Division
P. O. Box 8661
Philadelphia, Pa. 19101
Attn: Mr. J. A. Bledsoe

No response
Martini comment: G. E. has been building in cooperation with North American Philips a Stirling engine originally designed for radioisotope space power (79 aq). G. E. has also been building a free-piston Stirling engine for powering a three-ton capacity heat pump. (79 as). G. E. has also designed with North American Philips a test engine for LeRC.

(25) Hughes Aircraft Company
Cryogenics and Thermal Controls Department
Culver City, Ca. 90230
Attn: Dr. Bruno Leo or Mr. Richard Doody
Telephone: (213) 391-0711 Telex: 67222

Hughes Aircraft Company is continuing its research and development work on Stirling and Vuilleumier cryogenic refrigerators. Currently, emphasis is being placed upon various modified designs of these units for special applications where maintenance-free life is the most important parameter.

(26) Japan Automobile Research Institute Inc. (79 u).

(27) Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, Ca. 91103
Attn: Frank W. Hoehn
Telephone: (213) 354-6274 Telex, etc: FTS 792-6274

The Jet Propulsion Laboratory is currently working on a program to develop a Stirling Laboratory Research Engine which can eventually be produced commercially and be made available to researchers in academic, industrial, and government laboratories. A first generation 10 KW engine has been designed, fabricated, and assembled. The prototype engine is classified as a horizontally-opposed, two-piston, single-acting machine with a dual crankshaft drive mechanism. The test engine, which is designed for maximum modularity, is coupled to a universal dynamometer. Individual component and engine performance data will be obtained in support of a wide range of analytical modeling activities.

(28) Joint Center for Graduate Study/University of Washington
100 Sprout Road
Richland, Wa. 99352
Attn: Richard P. Johnston or Maurice A. White
Telephone: (509) 375-3176
Fully implantable power source for an artificial heart. Accomplishments:

1. Demonstrated engine lifetime of four years without maintenance before heater lead failure. 2. Current engine performance: Up to 7.7 watts hydraulic power output with 20.1 percent overall efficiency at 5 watts output from 200 cc engine volume. 3. Engine concept produces pumped hydraulic output with no mechanical linkages or dynamic seals. Capable of total hermetic seal welding for long term containment of working fluids.

A working model has been built to demonstrate the self-starting, torque control. It is on display at the Ontario Science Centre in Toronto. Controlled heating, cooling with hydrogen as working fluid was added by Dr. William Martini who made preliminary studies.

An optimized design has been made for a car and a testing prototype for power and efficiency testing. A proposal is being made for funding. Component suppliers and a consultant have been found.

Stirling engine offered by Leybold is a demonstration engine, permitting students and researchers to perform basic efficiency tests, and to observe through the glass cylinder the function. Pressure/vacuum relationships can be dynamically measured and indicated, or the machine may be mechanically driven as a heat/refrigerator pump. In the U.S. & N. America, contact Mr. Tufts--Internationally, production and sales from Bonnerstrasse 504, Postfach 510 760, 5000 Koln (Cologne), W. Germany. Over 400 sold.

Comment by Martini: M.A.N. is a licensee to Philips. They have worked for many years in Stirling engine developments, some of it sponsored by the German government and related to military hardware. Publications from this company are very few. The latest is 1977 bt. They seem to be developing four-cylinder Siemans engines like United Stirling but differing in the arrangement of parts. They have agreed to assist Foster-Miller Associates in designing a 500 to 2000 HP Stirling engine for Argonne National Laboratory.
(32) Martini Engineering
2303 Harris
Richland, Wa. 99352
Attn: W. R. Martini
Telephone: (509) 375-0115
- Preparation of First and Second Edition of Stirling Engine Design
- Publish Quarterly Stirling Engine Newsletter.
- Evaluate isothermalized Stirling engines for Argonne National Lab.
- Offers Stirling engine computation service for all types of Stirling engines.

(33) Martin Marietta Inc.
Cryogenics Division
Orlando, Florida
No response

(34) Massachusetts Institute of Technology
Room 41-204
Cambridge Mass. 02139
Attn: Joseph L. Smith, Jr.
Telephone: (617) 253-2296
Ph.D. Thesis research on heat transfer inside reciprocating expander and
compressor cylinders as in Stirling engines. Special emphasis on the thermo-
dynamic losses resulting from periodic heat transfer between the gas and the
walls of the cylinder

(35) Mechanical Engineering Institute
Agency of Industrial Science and Technology
Japan
Mr. I. Yamashita
No response
Martini comment: Involved in cryo-engine development (79 u).

(36) Mechanical Technology Incorporated
Stirling Engine Systems Division
968 Albany-Shaker Road
Latham, New York 12110
Attn: Tom Marusak
Telephone: (518) 456-4142 ex. 255  Telex, etc. Telecopier (518) 785-2420
TWX 710-443-8150
Automotive Stirling Engine Development Program development of United
Stirling, Sweden, kinematic engines for automotive applications; Free-piston
Development Engine Programs include: (1) 1 Kwe Fossil-Fueled Stationary
Electric Generator (Hardware), (2) 1 Kwe Solar Thermal Electric Generator
(Hardware), (3) 3 Kwe Fossil-Fueled Heat Pump (Hardware), (4) 5 Kwe Fossil-
Fueled Hybrid Electric Vehicle Propulsion System (Design), and (5) 15 Kwe.
Advanced Solar Engine Generator (Design). In addition to these engine programs
MTI is developing linear machinery loading devices for free-piston engines.
Included are linear alternators, hydraulic and pneumatic motor systems, and
resonant piston compressors.
(37) Meiji University
I-I, Kanda-Surugadai
Chiyoda-Ku
Tokyo: 101
Japan
Mr. H. Miyabe
Involved in experimental analysis and regenerator research for the
800 hp seacraft engine (79 u, 79bj).

(38) Mitsubishi Heavy Industries
5-1 Maronouchi 2 Chrome
Chiyoda-Ku
Tokyo, Japan
Mr. T. Kushiyama
No response
Involved in heat exchanger and combustor work on an 800 hp Stirling
engine for a seacraft (79 u, 79 bj).

(39) N.V. Philips Industries
Cryogenic Department
Building TQ III-3
Eindhoven - The Netherlands
Attn: M. A. Stultiens
Telephone: ++31 40 7.83774
Telex, etc.: 51121 phtc n1/nphetq
- Minicooler MC80/1W at 80K
- Liquid Air Generator PLAI07S/7-8 1/hr.
- Liquid Nitrogen plants PLN105S and PLN430S, resp. 7 and 30 1/hr.
- Liquifiers (80 - 200 K) PPG102S and PPG400S/0, 8kW and 3,2kW at 80K.
- Two stage cryogenerator K20 for Cryopumping 10W/20K + 80W/80K.
- Two stage recondensors PPH110 and PPH440/10 1. and 40 1. H2 recondensation.
- Two stage transfermachines PGH105S and PGH420 for targetcooling,
cryopumping, etc.
- Helium liquefier 10-12 1/hr.
Physical Lab., where much research is being done with regard to Stirling
engines, heat-pumps and solar energy systems.

(40) N.V. Philips Company
Philips Research Laboratories
Eindhoven, The Netherlands
Attn: C. L. Spigt
Telephone: 040-43958
Free piston Cryogenerator
Free piston Stirling engine
3kW Stirling engine as heat pump driver
Vuilleumier Cycle
Comments by Martini: This organization is the pioneer of all modern
Stirling engine technology. All the leading companies in Stirling engines
have licenses from this company.

(41) National Bureau of Standards
Room B126, Blg. 226
Active program terminated

Comments by Martini: NBS did obtain a 1-98 engine from Philips and did test it as the prime mover in a heat pump-air conditioning system. The tests were generally successful. (See 1977 ad).

(42) National Bureau of Standards
Cryogenics Laboratory
Boulder, Colorado

No response

(43) NASA - Lewis Research Center
Stirling Engine Project Office
Lewis Research Center
21000 Brookpark Rd.
Cleveland, Ohio 44135
Attn: Robert Ragsdale
Telephone: (216) 433-4000

No response

Comments by Martini: NASA -Lewis administers most of the DOE program on automotive Stirling engines. The major program is with MTI and United Stirling. Many much smaller programs are sponsored including this design manual. Internally, NASA-Lewis has developed a third order analysis (79a) and has tested the GPU-3 engine (79 bl). Testing is now proceeding on the United Stirling P-40 engine.

(44) Nippon Piston Ring Co., Ltd.
No. 1-18, 2-Chome
Uchisaiwaicho, Chiyoda-ku
Tokyo, Japan
Attn: Mr. E. Sugawara
Telephone: Tokyo 503-3311 Telex, etc.: (0222) 2555 NPRT TOJ
Cable address: NPRT TOKYO

1. Development of material capable sliding under absence of lube oil.
2. Basic test and analysis of various piston rings and piston rod seals for pressure, sliding speed, selection of suitable gas, determination of number of seals required, and leakage of gas.
3. Analysis of frictional behaviour during sealing.
4. Development of gas recirculation system.
5. Development of liquid seal and of sealing-liquid recirculation system.
6. Design and manufacturing of piston ring and piston rod seal system for Stirling Engine of 800 PS (HP).

(45) New Power Source Research Dept. Central Engineering Laboratories
1 Natsushima-cho
Yokosuka 237 Japan
Attn: Yasunari Hoshino
Telephone: (0468) 65-1123 Telex: TOK 252-3011
Purpose: To evaluate the characters of Stirling Engine
Actual State: An experimental two-piston single acting engine was
trial made and the fundamental study is being carried out using helium as
working gas. Recently gas leakage analysis across piston rings and regenera-
tor tests are mainly conducted. Also a comparison between our test results
and the calculated data by means of yours Manual (The first edition of the
Stirling Engine Design Manual) is being tried.

(46) North American Philips Corp.  (2)
Philips Laboratories
345 Scarborough Rd.
Briarcliff Manor, N. Y. 10510
Attn: Alexander Daniels
Telephone: (914) 762-0300
.SIPS (Stirling Isotope Power System) - 1 KW electric output engine
was designed, fabricated and assembled; currently awaiting performance tests.
.In-house studies of Stirling cycle.

(47) Wm. Olds and Sons  (1)
Ferry Street
Maryborough, Queensland
Australia
Attn: Peter Olds
Production Model - Horizontal type.
Detachable piston, reversible lever. Production model is approximately
15 inches long and 6 inches high.

(48) Ormat Turbines  (1)
P. O. Box 68
Yaune, Israel
Attn: Dr. Israel Urielli
Comments by Martini: Dr. Urielli continues his interest in Stirling
eengines started in his important Ph.D. thesis (77 af) which fully discloses
and explains an entirely rigorous third order analysis method.

(49) Alan G. Phillips  (0)
P. O. Box 20511
Orlando, Florida 32814
Attn: Alan G. Phillips
Research and History of Pre 1930 Hot Air Engines. Reprinting of Cata-
logs on Hot Air Water Pumping Engines from 1871 to 1929. List of Available
Publications on Request.

(50) Radan Associates Ltd.  (1)
19 Belmont, Lansdown Road
Bath, United Kingdom BA 1 6DZ
Attn: Mr. R. A. Billett
Comments by Martini: Mr. Billett teaches at the School of Engineering,
University of Bath and is involved in Demonstration Stirling engines and
teaching aids. He conducts a Stirling engine course each year.

ORIGINAL PAGE IS
OF POOR QUALITY
(51) Ross Enterprises
37 W. Broad St. #630
Columbus, Ohio 43215
Attn: Andrew Ross
Telephone: (614) 224-9403

Current work includes development of two fractional horsepower Stirling engines; one of medium pressure, and one of low pressure. The low pressure engine is part of a small DOE appropriate technology grant. The aim on the medium pressure engine is to provide, in time, a source of small (100 to 200 watts) Stirling engines for the independent researcher, graduate student, hobbyist, etc.

(52) Royal Naval Engineering College
RNEC Manadon, Stirling Engine Research Facility
Crownhill, Plymouth
Devon, England PL53AQ
Telephone: Plymouth 553740 Ext. RNEL 365

The Royal Naval Engineering College are part of an industrial-university consortium investigating the design and manufacture of Stirling engines. An assessment of Stirling cycle machines in a naval environment is also in hand. Although some experimental work has been done the main effort at present is the development of a general design and simulation algorithm. It is envisaged that a 15-20 KW twin-cylinder engine employing a sodium heat pipe will be on test by December 1979.

Work on the Fluidyne and a tidal flow regenerator test rig is also in progress.

(53) Schuman, Mark
101 G Street S.W. #516
Washington, D. C. 20024
Attn: Mark Schuman
Telephone: (202) 554-8466

Free piston, modified Stirling cycle heat engine invention available for licensing and development. U. S. and foreign patent protection. Two thermally driven partial models demonstrate key novel features.

(54) Shaker Research Corporation
Northway 10 Executive Park
Bellston Lake, N. Y. 12019
Attn: Allan I. Krauter
Telephone: (518) 877-8581

This work, which started in February 1978, is directed at applying hydrodynamic and elastohydrodynamic theory to a sliding elastomeric rod seal for the Stirling engine. The work is also concerned with the experimental determination of film thickness, fluid leakage, and power loss. Finally, the work entails correlating the experimental and theoretical results.

The analytical effort consists of two analyses: an approximate analysis of rod seal behavior at the four extreme piston position / piston velocity points and a detailed temporal analysis of the seal behavior during a complete piston cycle.
The experimental effort involves designing, constructing, and running an apparatus. The apparatus contains a moving transparent cylinder and the stationary elastomeric seal. A pressure gradient of 100 psi can be applied across the seal. Frequencies from 10 Hz to 50 Hz with a one inch total stroke can be employed. Film thickness will be measured with interferometry, fluid leakage by level and pressure changes, and power loss by force cells.

At present, the approximate and detailed analyses are complete, and the experimental apparatus is starting to produce quantitative results.

(55) The Shipbuilding Research Association of Japan (JSBA) (~2)
Senpaku Shinko Bldg., 1-15-16
Toranomon, Minato-ku
Tokyo, Japan
Attn: Mr. H. Fujita

We are researching and developing the marine Stirling engine (double acting 4 cylinders 800 ps) on a six years project from 1976.

Items of basic research are cycle simulation, heat exchangers, burner, sealing apparatus, and control system. Performance test of a 2 cylinders experimental engine will be also carried out.

These researches and tests are performed cooperatively by Research Panel No. 173 (SR173) which is consisted of universities, institutes, and companies.

(56) Ship Research Institute (5)
6-38-1, Shinkawa, Mitaka
Tokyo 181, Japan
Attn: Mr. Shigeji Tsukahara
Telephone: 0422-45-5171

(1) The effect of engine elements such as materials in the regenerator and the dimensions of piston rings on the Stirling engine performance was studied using the Inverted-T type Stirling engine.

It was obtained that the effect of these elements was apparently great. Especially, the effect of the dimension of the piston ring on the net output was very remarkable. For example, the net output was improved in 2.5 times when 15 thin (1 mm) piston rings for a piston were employed instead of 4 thicker (6 mm) piston rings.

In future, amount of leakage of working fluids through piston rings and friction force by piston rings will be measured using the testing machine for Stirling engine elements.

(2) A dynamic mathematical model simulating a Stirling engine is now under development.

(57) Solar Engines (~15)
2937 W. Indian School Rd.
Phoenix, Arizona 85017
Attn: Mr. John Griffin
Telephone: (602) 274-3541

No response

Comments by Martini: Solar Engines has built 20,000 of their Model 1 engine and 7000 of their Model 2 (See Figure 2-7). Solar Engines plans to build six models of their demonstration scale engines.
(58) Starodubtsev Physicotechnical Institute
UL. Observatorskaya 85
Tashkent
Uzbek SSR, U.S.S.R.
Attn: G. G. Ya Umarov

No response
Comments by Martini: Mr. Umarov and his group are very regular contributors to the Soviet Solar Energy Magazine. Quite often the subject is Stirling engines. Mr. Umarov either does not receive or does not answer his mail.

(59) Stirling Engine Consortium
Department of Engineering
University of Reading
Whiteknights, Reading, Berkshire, RG6 2AY, United Kingdom
Attn: Dr. Graham Rice
Telephone: Reading 85123 Ext. 7325

1. Design of 20 kW helium charged research (Consortium) Engine
2. Re-building of 200 watt Air Charged engine with integral heat pipe cylinder heater head
3. Gas flow test rigs for steady-state and dynamic testing of consortium engine components, namely: heater, regenerator and cooler
4. Cycle analysis

(60) Stirling Power Systems Corporation
7101 Jackson Road
Ann Arbor, Michigan 48103
Attn: William B. Lampert
Telephone: (313) 665-6767 Telex: 810-223-6010

SPS is responsible for market development on the Stirling engine being produced by FFV in Sweden. The Recreational Vehicle market is the first market being addressed, as the attributes of the Stirling cycle engine are important, i.e., quiet, low vibration, low emissions, etc. The Stirling engine generator set and system installed in a Winnebago Motor Home was introduced to the RV Industry at the National RVIA Show in November, 1978. The innovative system was very well received. Winnebago Industries is planning on limited production beginning in Spring, 1980. The product consists of a 6.5 KW Stirling engine generator set with an integrated total system to provide electricity, hydronic heating and air conditioning that is automatic in operation; thus, providing home-like comfort for the customer.

(61) Sunpower Inc.
6 Byard St.
Athens, Ohio 45701
Attn: William T. Beale
Telephone: (614) 594-2221

Small electric output free piston engines --100-1000 watt--solar and solid fuel heat-water pumps in same power range using free cylinder mode of the free piston engine, hermetically sealed.
Sunpower sells both the alternator and the water pump with full guarantee for one year.
Sunpower does analysis, computer simulation design, construction and test on all types and sizes of Stirling engines, but specializes in free piston engines.

Late Information: The Sunpower SD 100 engine produced 62 w(e) at an overall fuel-to-electric energy efficiency of 7.5 percent. Hot end temperature was 425°C, cold 40°C. At 475°C hot end temperature power was 80 w(e) and heat-to-electric efficiency was 13 percent.

(62) TCA Stirling Engine Research & Development Company
POB 643
Beverly Hills, Ca. 90213
Attn: Ted Finkelstein
Telephone: (213) 279-1186, 474-8711
1. Development of a gas-fired heat pump and air conditioner.
2. Development of an oilwell gas liquefier.
3. Maintenance and support of TCA Stirling Analyzer Program.

(63) Laboratory for Energetics
Technical University of Denmark
Building 403, DK-2800
Lyngby, Denmark
Attn: Niels Elmo Andersen or Bjorn Qvale

Development of a total energy system composed of a Stirling engine and a Stirling heat pump. The prototype is designed to produce 2 kW of electricity and 8 kW of heat. The total energy utilization is expected to vary from 100 percent at maximum power output to 90 percent at maximum heat output.

Development of a third-order analysis program for Stirling machines. The model is composed of separate models for each of the components of the machine. The cylinder spaces are assumed adiabatic. The heat exchangers and the regenerator models take into account both heat transfer and flow friction.

(64) Texas Instruments
Cryogenics Division
Dallas, Texas
No response

(65) Thermacore, Inc.
780 Eden Road
Lancaster, Pa. 17601
Attn: Donald M. Ernst
Telephone: 569-6551

At the present time, Thermacore is negotiating a contract for a supporting role in the Argonne National Laboratory Program for the Design and Development of Stirling Engines for Stationary Power Generation Applications in the 500-3000 horsepower range. This effort is directed at the use of liquid metal heat pipes for integrating the heat source with the engine heater-head.

Thermacore's personnel are credited with the current state-of-the-art in terms of life for liquid metal heat pipes: 41,000 hours @ 600°C for nickel-potassium; 35,000 hours @ 800°C for Hastelloy X-sodium.
No response
Involved in a feasibility study of a Stirling engine heat pump.

(67) Tokyo Institute of Technology
Naotsugu ISSHIKI (Laboratory)
2-12-1 Ookayama Meguroku
Tokyo 152 Japan
Attn: Naotsugu Isshiki
Telephone: 03 420 7677

1. Experimental study of Stirling engines using several test engines of small size, such as (1) 20 mm diameter & 14 mm stroke swash plate type two cylinder engine of 1/3 kW; (2) the same type of 40 mm diameter and 26 mm stroke engine intended power of 2 kW. The results will be reported in the future.

2. Experimental and theoretical study to know the smallest temperature difference by which the Stirling engine can operate, for future power recovery from waste heat from industry and conventional engines.

(68) United Kingdom Atomic Energy Authority
AERE Harwell
Oxfordshire OX11 ORA
England
Attn: E. H. Cook-Yarborough
Telephone: (0235) 24141 Telex: 83135

Three development and four field-trial thermo-mechanical generators (TMG) constructed. Radio-isotope heated development TMG has run continuously since Nov. 1974. UK National Data Buoy has been powered by propane-heated 25 w TMG (while at sea) since first installation in 1975. Major lighthouse off Irish coast powered by 60 w TMG since Aug. 1978. Fluidyne liquid-piston Stirling engine originally invented at Harwell.

(69) United States Department of Energy
P. O. Box 1398
Bartlesville, OK. 74003
Attn: R. W. Hurn or W. F. Marshall
Telephone: (918) 336-2400

Fuels tolerance, emissions, and power delivery characteristics of 10 hp Philips Stirling.

(70) United Stirling (Sweden) AB & Co.
Box 856
S-201 80 Malmo
Sweden
Attn: Mr. Bengt Hallare (also Mr. Worth Percival, Washington D. C. office)
Telephone: (202) 466-7286 in Washington, D. C.

No response
Comments by Martini: United Stirling is a licensee of N. V. Philips and is the world leader in producing automotive scale Stirling engines. They have a 40 kw, 75 kw and 150 kw machine. They have installed one in a truck and several in automobiles. They plan serial production of the P-75 (75 kw) engine. They are sub-contractor to MTI on the DOE sponsored automobile program through NASA-Lewis. They are sub-contractor to Advanced Mechanical Technology on the 500-3000 hp design study contract let by Argonne National Laboratories.

(71) Urwick, W. David
85/2 St. Anthony St.
Attard, Malta
Attn: W. David Urwick
Telephone: 40986

Retired engineer living in Malta since 1970. Since that date I have built in my small workshop a series of model Stirling engines, as a piece of amateur research, and I take an intense interest in Stirling engine developments throughout the world. I have had two articles published in "Model Engineer" describing what I have done. Last year at the M.E.E. exhibition in London I was awarded a trophy for a 12-cylinder wobble plate Rider engine of unusual design. A further article is now awaiting publication, which will describe this machine.

(72) University of Calgary
Department of Mechanical Engineering
Alberta, Canada
Attn: G. Walker
Telephone: (403) 284-5772

Energy Flow in Regenerative Systems
Stirling Cycle Cryocoolers
Heat Exchangers for Stirling Cycle Systems

(73) University of California, San Diego
Physics B-019
U.C.S.D.
La Jolla, California 92039
Attn: John Wheatley or Paul C. Allen or Douglas N. Paulson
Telephone: (714) 452-2490

Scientific, non-hardware oriented, studies of Malone type heat engines and appropriate working fluids.

(74) University of Tokyo
Dept. of Mechanical Engineering
HONGO 7-3-1, BUNKYO-KU
Tokyo, 113 Japan
Attn: Naomasa Nakajima
Telephone: (03) 812-2111 ext. 6138

1. Measurements of unsteady flow heat transfer rate at heat exchangers Stirling engines.
2. Development of computer simulation programs for Stirling engine design.
3. Design of Stirling engine driven with wood fuel.
I. Diesel-Stirling combined cycle analysis
2. Artificial heart
3. Computer simulation of Stirling cycle

The University of Tokyo, Faculty of Engineering, Dept. Nuclear Eng.
7-3-1, Hongo, Bunkyo-ku
Tokyo, Japan 113
Attn: Yoshihiro Ishizaki
Telephone: (03) 812-2111, ext. 3163, 7565
  .Rotary Stirling engine and rotary Stirling refrigerator.
  .Multi-phase Stirling refrigerator.
  .Cryogenic Stirling engine for the LNG power station.
  .Conceptual design for the application of the Stirling cycle machines.

University of Witwatersrand
Dept. of Mechanical Eng.
1 Jan Smuts Ave.
Johannesburg 2001, South Africa
Attn: Prof. C. Rallis
Telephone: 39-4011
Telex: 8-7330 SA
No response
Comments by Martini: Programs: Have built and tested a Stirling engine experiment (78 s). Have developed a rigorous third order computer code (77 af). Have evaluated liquid piston engines (79 af).

Weizmann Institute of Science
Dept. of Electronics
Rehovot, Israel
Attn: Professor S. Shtrikman
Telephone: (054) 82614
Telex: 31900
Studies of second order design methods.

West, C. D.
114 Garnet Lane
Oak Ridge, Tennessee 37830
Attn: C. D. West
Telephone: (615) 483-0637
Theoretical and experimental investigations of liquid piston engines. Past accomplishments include invention and development of "Fluidyne" liquid piston energy.

YAN MAR Diesel Co. - Japan
Mr. T. Yamada
No response
Involved in a Stirling test engine (79 u).
The current program on the Stirling engine is developed under the general title which may be called: The new performance of the Stirling cycle. It includes two main lines of improvement on kinematic and thermodynamic field. The work continues beginning with the first experimental engine from 1972 having new working mechanism which produces a more appropriate movements of both pistons. That leads to the new indicator diagram closer to Stirling than to the Schmidt cycle. The further program is conceived in such a way as to connect the advantages of improved working mechanism with the new methods of heat transfer. That is now the main line for the future experimental and theoretical research in this field.

Late Insertions:

(82) F. Brian Thomas
Putson Manor
Hereford HR2 6BN
United Kingdom
Attn: F. Brian Thomas
Telephone: Hereford 65220

My opposed twin rhombic drive motor won first prize at Model Engineer Hot Air Engine Competition Jan. 1979. Butane gas fired. 15cc pistons swept volume. Pressurized to 40 psia. Developed 8 watts (mechanical) at 3,000 rpm. Drives its own water cooling circulation pump and a bicycle dynamo!

Currently engaged in building the second of a series of "Swing Beam Engines."

(83) Clark Power Systems, Inc.
916 West 25th Street
Norfolk, VA. 23517
U.S.A.
Attn: David A. Clark
Telephone: (804) 625-5917

Doing design work on a new form of Stirling cycle engine which will be used to generate hydraulic or electric power.
Appendix A

PROPERTY VALUES

Property values for the gases and the solids and liquids used in designing Stirling engines are given in this appendix, both in the form of tables and charts as well as equations which are used as subroutines in computer programs. Also included are heat transfer and fluid flow correlations commonly used in Stirling engine design.

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<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>Common Conversion Factors</td>
<td>313</td>
</tr>
<tr>
<td></td>
<td>Thermal Conductivity Equations</td>
<td></td>
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<tr>
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<td>A-3</td>
<td>For liquids</td>
<td>314</td>
</tr>
<tr>
<td>A-4</td>
<td>For solids</td>
<td>315</td>
</tr>
<tr>
<td></td>
<td>Graphs</td>
<td></td>
</tr>
<tr>
<td>A-1</td>
<td>For gases and liquids</td>
<td>316</td>
</tr>
<tr>
<td>A-2</td>
<td>For solids</td>
<td>317</td>
</tr>
<tr>
<td>A-3</td>
<td>Various</td>
<td>318</td>
</tr>
<tr>
<td></td>
<td>Specific Heats</td>
<td></td>
</tr>
<tr>
<td>A-5</td>
<td>Heat Capacities of Working Gases</td>
<td>319</td>
</tr>
<tr>
<td></td>
<td>Viscosity</td>
<td></td>
</tr>
<tr>
<td>A-6</td>
<td>Viscosity of Working Gases</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td>Prandtl Number</td>
<td></td>
</tr>
<tr>
<td>A-7</td>
<td>Prandtl Numbers for Working Gases</td>
<td>321</td>
</tr>
<tr>
<td></td>
<td>Heat Transfer and Fluid Flow</td>
<td></td>
</tr>
<tr>
<td>A-4</td>
<td>Flow Friction Coefficient for Screens with Recommended Relationship</td>
<td>322</td>
</tr>
<tr>
<td>A-5</td>
<td>Friction Factor and Heat Transfer Correlation for Flow Inside Circular Tubes with Recommended Relationships</td>
<td>323</td>
</tr>
<tr>
<td>A-6</td>
<td>Heat Transfer Coefficient for Screens with Recommended Relationship</td>
<td>324</td>
</tr>
</tbody>
</table>
Table A-1
Common Conversion Factors
(Standard Units for this Manual are Underlined)

<table>
<thead>
<tr>
<th>To Convert</th>
<th>To</th>
<th>Multiply By</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>centimeters</td>
<td>2.540</td>
</tr>
<tr>
<td>pounds/sq. in.</td>
<td>megapascals (MPa)</td>
<td>0.006894</td>
</tr>
<tr>
<td>atmospheres</td>
<td>megapascals (MPa)</td>
<td>0.1013</td>
</tr>
<tr>
<td>megapascals (MPa)</td>
<td>atmospheres</td>
<td>9.872</td>
</tr>
<tr>
<td>megapascals (MPa)</td>
<td>psia</td>
<td>145.05</td>
</tr>
<tr>
<td>centimeters</td>
<td>inches</td>
<td>0.3937</td>
</tr>
<tr>
<td>BTU/hr</td>
<td>watts</td>
<td>0.2931</td>
</tr>
<tr>
<td>calories</td>
<td>joules</td>
<td>4.1868</td>
</tr>
<tr>
<td>BTU</td>
<td>joules</td>
<td>1055</td>
</tr>
<tr>
<td>watts</td>
<td>BTU/hr</td>
<td>3.412</td>
</tr>
<tr>
<td>joules</td>
<td>calories</td>
<td>0.2388</td>
</tr>
<tr>
<td>joules</td>
<td>BTU</td>
<td>9.479 E-4</td>
</tr>
<tr>
<td>Viscosity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g/cm·sec</td>
<td>poise</td>
<td>1</td>
</tr>
<tr>
<td>centipoise</td>
<td>g/cm·sec</td>
<td>0.01</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>watts</td>
<td>BTU/hr ft°F</td>
<td>57.79</td>
</tr>
<tr>
<td>cm °K</td>
<td>w/cm·°K</td>
<td>0.01731</td>
</tr>
<tr>
<td>BTU/hr ft°F</td>
<td>w/cm·°K</td>
<td>1.443 E-3</td>
</tr>
<tr>
<td>BTU/hr ft²(°F/in)</td>
<td>w/cm²·°K</td>
<td></td>
</tr>
<tr>
<td>Heat Transfer Coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/cm²·K</td>
<td>BTU/hr ft² F</td>
<td>1761</td>
</tr>
<tr>
<td>BTU/hr ft² F</td>
<td>w/cm²·K</td>
<td>5.678 E-4</td>
</tr>
</tbody>
</table>
### Table A-2
Thermal Conductivity of Gases

\[ KG = \exp(A + B \ln(T)) \]
\[ KG = \text{Thermal Conductivity of gas, w/cmK} \]
\[ T = \text{Temperature, K} \]

<table>
<thead>
<tr>
<th>Gas</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium, 1 atm</td>
<td>-10.1309</td>
<td>+0.6335</td>
</tr>
<tr>
<td>Hydrogen, 1 atm</td>
<td>-11.0004</td>
<td>+0.8130</td>
</tr>
<tr>
<td>Water vapor, 1 atm</td>
<td>-15.3304</td>
<td>+1.1818</td>
</tr>
<tr>
<td>Carbon dioxide, 1 atm</td>
<td>-16.5718</td>
<td>+1.3792</td>
</tr>
<tr>
<td>Air, 1 atm</td>
<td>-12.6824</td>
<td>+0.7820</td>
</tr>
</tbody>
</table>

### Table A-3
Thermal Conductivity of Liquids

Equation
\[ KL = \exp(A + B \ln(T)) \]
\[ KL = \text{Thermal Conductivity of Liquid, w/cm K} \]
\[ T = \text{Temperature, K} \]

<table>
<thead>
<tr>
<th>Liquid</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium</td>
<td>2.3348</td>
<td>-0.4113</td>
</tr>
<tr>
<td>Engine Oil</td>
<td>-5.2136</td>
<td>-0.2333</td>
</tr>
<tr>
<td>Freon, CCl₂F₂</td>
<td>-7.3082</td>
<td>0</td>
</tr>
</tbody>
</table>
Table A-4
Thermal Conductivity of Solids

Equations

\[ KM = \text{Thermal Conductivity w/cmK} \]
\[ T = \text{Temperature, K} \]
\[ KM = \exp(A + B \ln T) \]

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 series Stainless Steel</td>
<td>-4.565</td>
<td>+0.4684</td>
</tr>
<tr>
<td>Lucalox Alumina</td>
<td>12.45</td>
<td>-2.440</td>
</tr>
<tr>
<td>Commercial Silicon Carbide</td>
<td>2.661</td>
<td>-0.6557</td>
</tr>
<tr>
<td>Pyrex Glass</td>
<td>-7.207</td>
<td>+0.4713</td>
</tr>
<tr>
<td>Low Carbon Steel</td>
<td>1.836</td>
<td>-0.4581</td>
</tr>
<tr>
<td>70 w/o Mo, 30 w/o W</td>
<td>4.990</td>
<td>-0.7425</td>
</tr>
<tr>
<td>Rene 41</td>
<td>5.472</td>
<td>+0.5662</td>
</tr>
</tbody>
</table>
Figure A-1 Thermal Conductivity of Liquids and Gases usable in Stirling Engines
Figure A-2. Thermal Conductivities of Probable Construction Materials for Stirling Engines.
Figure A-3. Typical Curves Showing Temperature Dependence of Thermal Conductivity (From American Institute of Physics Handbook, 2nd Ed., pp. 4-79).
<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Hydrogen(^1)</th>
<th>Helium(^1)</th>
<th>Air(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CP</td>
<td>CV</td>
<td>CP</td>
</tr>
<tr>
<td>298.15</td>
<td>14.31</td>
<td>10.18</td>
<td>5.20</td>
</tr>
<tr>
<td>400</td>
<td>14.50</td>
<td>10.37</td>
<td>5.20</td>
</tr>
<tr>
<td>500</td>
<td>14.52</td>
<td>10.39</td>
<td>5.20</td>
</tr>
<tr>
<td>600</td>
<td>14.56</td>
<td>10.43</td>
<td>5.20</td>
</tr>
<tr>
<td>700</td>
<td>14.62</td>
<td>10.49</td>
<td>5.20</td>
</tr>
<tr>
<td>800</td>
<td>14.70</td>
<td>10.57</td>
<td>5.20</td>
</tr>
<tr>
<td>1000</td>
<td>14.99</td>
<td>10.86</td>
<td>5.20</td>
</tr>
<tr>
<td>1200</td>
<td>15.43</td>
<td>11.30</td>
<td>5.20</td>
</tr>
<tr>
<td>1500</td>
<td>16.03</td>
<td>11.90</td>
<td>5.20</td>
</tr>
<tr>
<td>2000</td>
<td>17.03</td>
<td>12.90</td>
<td>5.20</td>
</tr>
<tr>
<td>2500</td>
<td>17.86</td>
<td>13.73</td>
<td>5.20</td>
</tr>
<tr>
<td>3000</td>
<td>18.40</td>
<td>14.27</td>
<td>5.20</td>
</tr>
</tbody>
</table>

### Table A-6

**Viscosity of Working Gases**

*g mass/cm sec at PAVG = 10 MPa*

<table>
<thead>
<tr>
<th>TR K</th>
<th>Hydrogen MU</th>
<th>Helium MU</th>
<th>Air MU</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$9.131 \times 10^{-5}$</td>
<td>$1.984 \times 10^{-4}$</td>
<td>$1.979 \times 10^{-4}$</td>
</tr>
<tr>
<td>400</td>
<td>$1.113 \times 10^{-4}$</td>
<td>$2.498 \times 10^{-4}$</td>
<td>$2.515 \times 10^{-4}$</td>
</tr>
<tr>
<td>500</td>
<td>$1.313 \times 10^{-4}$</td>
<td>$2.913 \times 10^{-4}$</td>
<td>$3.051 \times 10^{-4}$</td>
</tr>
<tr>
<td>600</td>
<td>$1.513 \times 10^{-4}$</td>
<td>$3.377 \times 10^{-4}$</td>
<td>$3.587 \times 10^{-4}$</td>
</tr>
<tr>
<td>700</td>
<td>$1.713 \times 10^{-4}$</td>
<td>$3.840 \times 10^{-4}$</td>
<td>$4.123 \times 10^{-4}$</td>
</tr>
<tr>
<td>800</td>
<td>$1.913 \times 10^{-4}$</td>
<td>$4.304 \times 10^{-4}$</td>
<td>$4.659 \times 10^{-4}$</td>
</tr>
<tr>
<td>1000</td>
<td>$2.313 \times 10^{-4}$</td>
<td>$5.232 \times 10^{-4}$</td>
<td>$5.731 \times 10^{-4}$</td>
</tr>
<tr>
<td>1200</td>
<td>$2.713 \times 10^{-4}$</td>
<td>$6.160 \times 10^{-4}$</td>
<td>$6.803 \times 10^{-4}$</td>
</tr>
<tr>
<td>1500</td>
<td>$3.313 \times 10^{-4}$</td>
<td>$7.552 \times 10^{-4}$</td>
<td>$8.411 \times 10^{-4}$</td>
</tr>
<tr>
<td>2000</td>
<td>$4.313 \times 10^{-4}$</td>
<td>$9.872 \times 10^{-4}$</td>
<td>$1.109 \times 10^{-3}$</td>
</tr>
<tr>
<td>2500</td>
<td>$5.313 \times 10^{-4}$</td>
<td>$1.219 \times 10^{-3}$</td>
<td>$1.377 \times 10^{-3}$</td>
</tr>
<tr>
<td>3000</td>
<td>$6.313 \times 10^{-4}$</td>
<td>$1.451 \times 10^{-3}$</td>
<td>$1.645 \times 10^{-3}$</td>
</tr>
</tbody>
</table>


The above data are based upon the following equations:

**For hydrogen:**

$$MU = 88.73 \times 10^{-6} + 0.200 \times 10^{-6}(TR - 293) + 0.118 \times 10^{-6}(PAVG)$$

**For helium:**

$$MU = 196.14 \times 10^{-6} + 0.464 \times 10^{-6}(TR - 293) - 0.093 \times 10^{-6}(PAVG)$$

**For air:**

$$MU = 181.94 \times 10^{-6} + 0.536 \times 10^{-6}(TR - 293) + 1.22 \times 10^{-6}(PAVG)$$
Table A-7
Prandtl Numbers for Working Gases
Prandtl Number, PR, dimensionless
(101 atm pressure)

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Hydrogen PR</th>
<th>Helium PR</th>
<th>Air PR</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.720</td>
<td>0.688</td>
<td>0.761</td>
</tr>
<tr>
<td>400</td>
<td>0.730</td>
<td>0.709</td>
<td>0.772</td>
</tr>
<tr>
<td>500</td>
<td>0.744</td>
<td>0.717</td>
<td>0.795</td>
</tr>
<tr>
<td>600</td>
<td>0.757</td>
<td>0.711</td>
<td>0.830</td>
</tr>
<tr>
<td>700</td>
<td>0.771</td>
<td>0.718</td>
<td>0.864</td>
</tr>
<tr>
<td>800</td>
<td>0.781</td>
<td>0.729</td>
<td>0.899</td>
</tr>
<tr>
<td>1000</td>
<td>0.810</td>
<td>0.749</td>
<td>0.974</td>
</tr>
<tr>
<td>1200</td>
<td>0.846</td>
<td>0.770</td>
<td>1.057</td>
</tr>
<tr>
<td>1500</td>
<td>0.890</td>
<td>0.795</td>
<td>1.189</td>
</tr>
<tr>
<td>2000</td>
<td>0.923</td>
<td>0.828</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td>0.858</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td></td>
<td>0.837</td>
<td></td>
</tr>
</tbody>
</table>
Figure A-4. Flow Through an Infinite Randomly Stacked Woven-Screen Matrix. Flow Friction Characteristics: a Correlation of Experimental Data from Wire Screens and Crossed Rods Simulating Wire Screens. Perfect Stacking, i.e., Screens Touching, is Assumed. (64 1, p. 130)

The dotted line is the recommended relationship. Its equation is:

For RR ≤ 60 let:
\[ \log CW = 1.73 - 0.93 \log(RR) \]

For 60 < RR ≤ 1000:
\[ \log CW = 0.714 - 0.365 \log(RR) \]

For RR > 1000:
\[ \log CW = 0.015 - 0.125 \log(RR) \]
For Friction factor the recommended correlation is:

For $RE \leq 2000$:

$$CW = \frac{16}{RE}$$

For $RE > 2000$:

$$\log(CW) = -1.34 - 0.20 \log(RE)$$

For heat transfer coefficient the recommended correlation is:

if $RE < 3000$ then $ST = \exp(.337 - .812 \ln(RE))$
if $3000 < RE < 4000$ then $ST = 0.0021$
if $4000 < RE < 7000$ then $ST = \exp(-13.31 + .861 \ln(RE))$
if $7000 < RE < 10000$ then $ST = 0.0034$
if $10000 < RE$ then $ST = \exp(-3.37 - .229 \ln(RE))$

where $ST = N_{ST} \left(\frac{N}{Pr}\right)^{2/3}$
Figure A-6. Gas Flow Through an Infinite Randomly Stacked Woven-Screen Matrix, Heat Transfer Characteristics; a Correlation of Experimental Data from Wire Screens and Crossed Rods Simulating Wire Screens. Perfect Stacking, i.e., Screens Touching, is Assumed (64 1, p. 129).

The recommended equation to use for this correlation is:

\[
\begin{align*}
\log \left( \frac{H}{G(CP)} \right) &= -0.13 - 0.412 \log (RR) \\
\ln \left( \frac{H}{G(CP)} \right)^{2/3} &= -0.13 - \frac{0.412}{2.303} \ln (RE) \\
ST &= \left( \frac{H(PR)^{2/3}}{G(CP)} \right) = \exp(-0.299 - 0.412 \ln(RE))
\end{align*}
\]
In this design manual it was decided to use a nomenclature that would be compatible with all computers right from the start so that there would be no need for translating the nomenclature later on. This means that Greek letters and subscripts which have traditionally been part of engineering notation will not be used because no computer can handle them. All computers employ variable names with no distinction between capital letters and small letters. Restrictions for the three main engineering languages are:

FORTRAN - First character must be a letter. Other characters may be letters or numbers. Limit is usually six.

PASCAL - Same as FORTRAN but usually there is no limit to the length of the variable name as long as letters and numbers are used with no punctuation or spaces.

BASIC - First character must be a letter. Second character may be a letter or number. Additional characters may be carried along but are ignored in differentiating variables.

In order to be compatible with all these computer languages and in order to use a reasonably compact nomenclature, the restrictions imposed by the BASIC language will be adopted. This limits the number of variables to 936, which is adequate. Those who program in PASCAL or FORTRAN might want to add to the two letter variable name given here to make it more descriptive.

In PASCAL the type of each variable must be declared in advance. The categories are:

- integer
- real
- character (string)
- boolean

Arrays are also declared in advance.

In FORTRAN there are only real or integer variables. Without specific type declaration variables beginning with I, J, K, L, M and N are integers and the rest are real. This convention is not supported in this nomenclature table.

In programming in FORTRAN one should declare all the variables real or integer at the start. If a variable name is used to identify an array (i.e. A(X,Y,Z)) it cannot also be used to identify a variable (i.e. A). Words are handled with format statements.

In BASIC variables beginning with any letter can be declared integers. Otherwise, all variables are assumed to be real numbers. For instance, if I is declared an integer all variables such as IN, IX, IA etc. are made integers also. If a statement evaluates IA as 3.7, the computer will use it as 3, the
This nomenclature does not group the integers. Therefore, all numbers in the nomenclature are assumed real.

BASIC uses suffixes to identify what type of variable or degree of precision is desired. The suffixes are:

- `%` integer
- `!` single precision
- `#` double precision
- `$` string (letters, number, punctuation, spaces)

Although integers compute faster than single precision numbers, all variables in this nomenclature are presented as single precision real numbers. BASIC assumes this if no suffix is given.

BASIC handles arrays as an additional suffix. For instance, `AX` can be used as a variable. In addition, `AX(A, B, C)` can be used as a three-dimensional array without being confused with `AX`. Since FORTRAN cannot do this, a variable name in this nomenclature will be either an array name or single real number, but not both.

String variables are useful in BASIC or PASCAL programs but will not be defined in this nomenclature.

The explanation of each variable starts out with a noun. For instance, "heat transfer area" becomes "area of heat transfer". This is done so that when the meanings are alphabetized, similar meanings will be together. Table B-1 gives the nomenclature alphabetized by symbol. Table B-2 gives the nomenclature alphabetized by meaning.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Counter for finding right average pressure.</td>
</tr>
<tr>
<td>AA</td>
<td>Factor of correlation, power with pressure.</td>
</tr>
<tr>
<td>AB</td>
<td>$\sqrt{(CR)^2 - (EE - RC)^2}$</td>
</tr>
<tr>
<td>AC</td>
<td>Area of heat transfer for cooler, cm².</td>
</tr>
<tr>
<td>AF</td>
<td>Area of flow, cm².</td>
</tr>
<tr>
<td>AH</td>
<td>Area of heat transfer for heater, cm² (or in general).</td>
</tr>
<tr>
<td>AK</td>
<td>Array of thermal conductivities, W/cm K.</td>
</tr>
<tr>
<td>AL</td>
<td>Angle of phase, degrees.</td>
</tr>
<tr>
<td>AM</td>
<td>Area of face of matrix, cm².</td>
</tr>
<tr>
<td>AS</td>
<td>Ratio of heat transfer area to volume for matrix, cm⁻¹.</td>
</tr>
<tr>
<td>AT</td>
<td>Array of area of metal for heat conduction.</td>
</tr>
<tr>
<td>AU</td>
<td>Ratio to TC to TH = TC/TH, commonly call $\tau$.</td>
</tr>
<tr>
<td>B</td>
<td>Constant for Table Spacing</td>
</tr>
<tr>
<td>B1</td>
<td>$\sqrt{(CR)^2 - (EE + RC)^2}$</td>
</tr>
<tr>
<td>BA</td>
<td>Exponent of correlation of power with pressure.</td>
</tr>
<tr>
<td>BF</td>
<td>Factor of correlation of power with standard.</td>
</tr>
<tr>
<td>BH</td>
<td>Heat, basic input, watts.</td>
</tr>
<tr>
<td>BP</td>
<td>Power, basic, watts.</td>
</tr>
<tr>
<td>C</td>
<td>Array of cold volumes, cm³.</td>
</tr>
<tr>
<td>C3</td>
<td>Constant in internal temperature swing loss equation.</td>
</tr>
<tr>
<td>C4</td>
<td>Length of connecting rod to cold space, cm.</td>
</tr>
<tr>
<td>CA</td>
<td>Option on cooler type: 1 = tubes, 2 = annulus, 3 = fins.</td>
</tr>
<tr>
<td>CC</td>
<td>$\sqrt{(CR-RC)^2 - EE^2}$</td>
</tr>
<tr>
<td>CD</td>
<td>Volume, cold, dead, cm³.</td>
</tr>
<tr>
<td>CF</td>
<td>Loss, flow, cooler, watts.</td>
</tr>
<tr>
<td>CL</td>
<td>Array of cold space live positions.</td>
</tr>
<tr>
<td>CM</td>
<td>Factor, conversion = 2.54 cm/inch</td>
</tr>
<tr>
<td>CN</td>
<td>Minimum of array, FC( ).</td>
</tr>
<tr>
<td>CP</td>
<td>Capacity of heat of gas at constant pressure, J/gK.</td>
</tr>
<tr>
<td>CQ</td>
<td>Loss of heat by conduction, watts, individually and collectively.</td>
</tr>
</tbody>
</table>
CR  Length of connecting rod, cm (if two cranks to hot space).
CV  Capacity of heat of gas at constant volume, J/gK.
CW  Factor of friction for matrix or tubes.
CX  Volume, cold, dead outside, cooler tubes.
CY  Maximum of Array FC().
D1  Diameter, effective or real, of power duct, cm.
D2  Diameter of power piston in gamma engine, cm.
D3  Diameter of power piston drive rod if in working space, cm.
D4  Diffusivity, thermal in displacer, cm²/sec.
D5  Diffusivity, thermal in cylinder wall, cm²/sec.
DB  Diameter at seal in cold space or diameter of displacer, cm.
DC  Diameter inside of engine cylinder, cm.
DD  Diameter of displacer or piston rod (if in working space), cm.
DH  Density of gas in heater g/cm³.
DI  Diameter, inside of annular regenerator, cm.
DK  Density of gas in cooler, g/cm³.
DL  Factor in Schmidt equation = √((AU)² + 2(AU)(K) cos(AL) + K²) / (AU + K + 2S)
DM  Diameter of hot space manifold tubes, cm.
DN  Diameter of heater manifold tubes, cm.
DP  Pressure, difference of, MPa.
DR  Diameter of each regenerator or OD of annular regenerator, cm.
DT  Temperature, increase of in cooling water, K.
DU  Temperature, increase of in cold space, K.
DV  Temperature, increase of in hot space, K.
DW  Diameter of wire or sphere in matrix, or thickness of foils, cm.
E  Effectiveness of regenerator, fraction.
E2  Clearance, end in gamma type power piston, cm.
E4  Density of displacer wall g/cm³.
E5  Density of cylinder wall, g/cm³.
E6  Density of matrix solid material, g/cm³.
EC  Clearance, piston end, cm.
EE  Eccentricity in a rhombic drive, cm.
EF  Efficiency of cycle, fraction.
EH  Emissivity of hot surface.
EK  Emissivity of cold surface.
Emissivity of radiation shields.

Angle used in Schmidt equation (see equation 6-36).

Angle of crank, degrees.

Fraction of cycle time gas is assumed to leave hot space at constant rate.

Fraction of cycle time gas is assumed to enter hot space at constant rate.

Fraction of cycle time that flow out of the cold space is assumed to occur at constant rate.

Fraction of cycle time gas is assumed to enter cold space at constant rate.

Factor for area effect in radiation heat transfer.

Array of gas mass fractions in cold space.

Efficiency of furnace, %.

Fraction of matrix volume filled with solid.

Array of gas mass fractions in hot space.

Factor for emissivity effect in radiation.

Factor for number of radiation shields in radiation.

Factor, conversion = 60 Hz/RPM.

Fraction of cycle time flow is into hot space.

Loss, mechanical due to seal friction, watts.

Flow of cooling water, g/sec.

Flow of cooling water per cylinder, GPM or liters/minute.

Credit for flow friction, watts.

Clearance around hot cap, cm.

Constant of conversion = $10^7$ g/(MPa \cdot sec$^2$ \cdot cm$).

Velocity, mass, in cooler, g/sec cm$^2$.

Velocity, mass, in connecting duct, g/sec cm$^2$.

Velocity, mass in heater, g/sec cm$^2$.

Velocity, mass, in regenerator, g/sec cm$^2$.

Array of hot volumes, cm$^3$.

Option for heater, 1 = tubes, 2 = fins, 3 = single annulus heated one side.

Coefficient of heat transfer at cooler, w/cm$^2$K.

Volume, hot dead, cm$^3$.

Coefficient of heat transfer in heater, w/cm$^2$K.

Array of hot space live positions, cm.
Minimum of array FH ( ).
Factor, conversion = 1.341E-3 HP/watt.
Radius, hydraulic, of matrix = PO/AS.
Loss, flow in heater, watts.
Maximum of array FH ( ).
Coefficient of heat transfer, watts/cm^2K.
Counter for iterations.
Diameter inside of cooler tubes of space between fins or annular clearance, cm.
Diameter, inside of cold duct, cm.
Diameter, inside, of heater tubes or space between fins or gap in annulus, cm.
Power, indicated, watts.
Swept volume ratio = VK/VL
Constant in reheat loss equation.
Coefficient in gas thermal conductivity formula.
Coefficient in gas thermal conductivity formula.
Conductivity, thermal, gas, w/cmK.
CP/CV
Conductivity, thermal, metal, w/cmK.
Option for enclosed gas inside of hot cap, 1 = H2, 2 = He, 3 = air.
Conductivity, thermal, composite of matrix.
Array of gas inventories times gas constant at each increment during cycle.
Length of Power Duct, cm.
Length of temperature wave in displacer.
Length of temperature wave in cylinder wall.
Length of hot cap, cm.
Length of cooler tubes, cm. (total).
Length, cooled, of cooler tubes, cm.
Length of cold duct (pressure drop), cm.
Length of cold duct (dead volume), cm.
Length of heater tube or heater fin, cm.
Length, heated, of heater tubes, cm.
Coefficient of leakage of gas, frac/MPa sec.
Length of regenerator, cm.
LM Length of hot space manifold tubes (for dead volume), cm.
LN Length of heater manifold tubes (for dead volume), cm.
LO Length of hot space manifold tubes (for press drop), cm.
LP Length of heater manifold tubes (for pressure drop), cm.
LR Length of regenerator, cm.
LX Coefficient of gas charge leaking per time increment per pressure difference, frac/MPa.
LY Summation of M*R.

M Moles of working fluid, g mol.
M1 Coefficient to calculate gas viscosity.
M2 Coefficient to calculate gas viscosity.
M3 Coefficient to calculate gas viscosity.
M4 Capacity of heat of displacer wall, J/gK.
M5 Capacity of heat of cylinder wall, J/gK.
M6 Capacity of heat of regenerator metal, J/gK.
MD(X,Y,Z) Array for efficiency data, %.
ME Efficiency, mechanical, %.
MF Loss due to mechanical friction in seals, watts.
ML () Array of compression space live positions for gamma engine, cm.
MP(X,Y,Z) Array for power data, HP.
MR Product of gas inventory and gas constant, J/K.
MS Mesh of screen or foils, number/length.
MT () Array of metal temperatures, K.
MU Viscosity of gas, g/cm sec.
MW Weight, molecular, of gas, g/g mol.
MX Mass of regenerator matrix, g.

N Number of cylinders per engine.
N1 Number of power ducts per cylinder.
N3 Option for engine cylinder material: 1 = glass or alumina, 2 = stainless steel, 3 = iron, 4 = brass, 5 = aluminum, 6 = copper.
N4 Option on regenerator matrix material (see N3).
N5 Option on regenerator wall material (see N3).
NC Number of cooler tubes per cylinder or spaces between fins.
ND Angle of increment, degrees.
NE Number of cold space manifold tubes per cylinder.
NH  Number of heater tubes or fin spaces per cylinder.
NM  Number of hot space manifold tubes per cylinder.
NN  Number of tubes per cylinder in heater tube manifold.
NO  Number of cold ducts per cylinder.
NP  Power, net, watts.
NR  Number of regenerators per cylinder.
NS  Number of internal radiation shields in displacer or hot cap.
NT  Number of transfer units in regenerator.
NU  Frequency of engine, Hz.
OC  Diameter, outside of cooler tubes or fin height, cm.
OD  Diameter, outside, of cold space manifold, cm.
OG  Option of operating gas - 1= hydrogen, 2 = helium, 3 = air.
OH  Diameter, outside of heater tube or height of fins, cm.
OM  Speed of engine, radians/sec.

P ( ) Array of pressure during cycle first with MR = 1, then at average pressure.
P4  \( \frac{\pi}{4} = 0.785398 \)
PG  Pressure, average gas, MPa.
PI  \( 3.14159 = \pi \)
PM  Pressure, mean, for all P's, MPa or dimensionless.
PN  Minimum of P( ).
PU  Porosity of matrix.
PP  Factor, conversion = 0.006894 MPa/psia.
PR  Prandtl Number of the 2/3 power = \( (Pr)^{2/3} \).
PX  Maximum of P( ).

QB  Heat supplied by heater, watts.
QC  Heat absorbed by cooler, watts.
QI  Loss due to internal temperature swing, watts.
QN  Heat, net required, watts.
QP  Loss, pumping for all N cylinder, watts.
QR ( ) Array of heat transferred in regenerator, joules.
QS  Loss, shuttle, for all N cylinders, watts.

R  Constant, gas, universal = 8.314 J/(g mol (K)).
R1  Option on regenerator type - 1 = screen, 2 = foam metal, 3 = spheres, 4 = slots.
R2  Radius of crank to cold space, cm.
RA  Factor, conversion = 0.0174533 radians/degree.
RC  Radius of crank (if two cranks to hot space), cm.
RD  Volume, regenerator, dead, cm³.
RE  Reynolds number, heater or cooler.
RH  Loss, reheat, watts.
RM  Density of gas at regenerator, g/cm³.
RO  Array of gas density, g/cm³.
RR  Reynolds number for regenerator.
RT  Reynolds number, heater.
RV  Ratio of dead volume to expansion space volume = VD/VL.
RW  Loss, flow in all regenerators of engine, watts.
RZ  Reynolds number, cooler.
S  Ratio of dead volume mass to maximum expansion space mass.
SC  Thickness of hot cap wall, cm.
SD  Stroke of displacer or hot cap = 2RC, cm.
SG  Factor in shuttle heat loss.
SI  Constant, Stefan - Boltzman = 5.67 x 10⁻¹² w/cm² K⁴
SL  Loss due to matrix temperature swing, watts.
SP  Speed of engine, RPM.
SR  Thickness of wall of regenerator housing, cm.
SS  Thickness of inside regenerator wall if annular regenerator, cm.
ST  Stanton number times (Pr)²/₃
TA  TH/TC
TC  Temperature, effective, of cold space, K.
TF  Temperature of inside heater tube wall, F.
TH  Temperature, effective, of hot space, K.
TL  Temperature of gas leaving regenerator, K.
TM  Temperature of inside heater tube wall, K.
TR  Temperature of regenerator, K.
TS  Temperature, swing of, in matrix, K.
TU  Number of transfer units.
TW  Temperature of inlet cooling water, K.
TX  Temperature of cooler tube metal, average, K.
TY  Temperature of inlet cooling water, F.
.Z  Temperature along regenerator, K.
V ( ) Array of total gas volume at each increment during cycle.
V1 Number of velocity heads due to entrance, exits and bends in hot space manifold.
V2 Number of velocity heads due to entrance, exits and bends in heater tubes or fins.
V3 Number of velocity heads due to entrance, exits and bends in heater manifold.
V4 Number of velocity heads due to entrance, exits and bends in cooler.
V5 Number of velocity heads due to entrance, exits and bends in cold duct.
V6 Number of velocity heads due to entrance, exit and bends in power duct.
VA Volume, total of annulus.
VC Velocity through gas cooler or connecting duct, cm/sec.
VD Volume, total dead, cm³.
VH Velocity of gas through gas heater, cm/sec.
VK Volume, cold, live (associated with displacer), cm³.
VL Volume, hot live, cm.
VM Volume, cold dead, actually measured in beta engine, cm³.
VN Minimum of V().
VP Volume, live, associated with the power piston, cm³.
VR Ratio of volumes, maximum/minimum.
VT Volume, total, sum of compression and expansion space live volumes, cm³.
VX Maximum of V().

W ( ) Array of works, joules.
W1 Work for 1 cycle and one cylinder, joules.
WC Flow, mass, into or out of cold space, g/sec.
WH Flow, mass, into or out of hot space, g/sec.
WR Flow, mass, through regenerator, g/sec.

X Temporary variable.
XB Factor to calculate shuttle heat loss.
XX Factor, correction, for large angle increments.

Y Temporary variable.
YK Factor in shuttle heat loss equation relating to wall properties and frequency.
YY Temporary variable.
Temporary variable.

Factor of compressibility of gas.

Flag for iteration method, 0 for rapid iteration, 1 for slower method that is sure.

Counter for number of iterations.

Loss, static, heat conductor, specified, watts.

Factor in shuttle heat loss equation relating to wave-form of motion.

Flag for heat conduction method, 0 for specified, 1 for calculated.
<table>
<thead>
<tr>
<th>NOMENCLATURE FOR BODY OF DESIGN MANUAL</th>
<th>ORIGINAL PAGE IS OF POOR QUALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Alphabetized by Meaning)</td>
<td>(Alphabetized by Meaning)</td>
</tr>
<tr>
<td>Angle of crank</td>
<td>degrees</td>
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<tr>
<td>Angle of increment per time step</td>
<td>degrees</td>
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<tr>
<td>Angle of phase</td>
<td>degrees</td>
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<tr>
<td>Angle used in Schmidt equation (6-36)</td>
<td>degrees</td>
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<tr>
<td>Area of flow</td>
<td>cm²</td>
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<tr>
<td>Area, frontal, of matrix</td>
<td>cm²</td>
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<tr>
<td>Area of heat transfer for cooler</td>
<td>cm²</td>
</tr>
<tr>
<td>Area of heat transfer for heater or in general</td>
<td>cm²</td>
</tr>
<tr>
<td>Array of areas of metal for heat cond.</td>
<td>cm²</td>
</tr>
<tr>
<td>Array of cold space live positions</td>
<td>cm</td>
</tr>
<tr>
<td>Array of cold volumes</td>
<td>cm³</td>
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<tr>
<td>Array of compression space live positions for gamma engine</td>
<td>cm</td>
</tr>
<tr>
<td>Array for efficiency data</td>
<td>%</td>
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<tr>
<td>Array of fraction of gas mass to the total in the cold space</td>
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</tr>
<tr>
<td>Array of gas densities</td>
<td>g/cm³</td>
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<tr>
<td>Array of gas inventories x gas constant at each increment during cycle</td>
<td>j/K</td>
</tr>
<tr>
<td>Array of gas mass fractions in hot space</td>
<td>--</td>
</tr>
<tr>
<td>Array of heats transferred between gas and solid in regenerator</td>
<td>joules</td>
</tr>
<tr>
<td>Array of hot space live positions</td>
<td>cm</td>
</tr>
<tr>
<td>Array of hot volumes</td>
<td>cm³</td>
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<td>Array of metal temperatures</td>
<td>K</td>
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<tr>
<td>Array for power data</td>
<td>HP</td>
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<tr>
<td>Array of pressures during cycle, first at M * R = 1, then at average pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>Array of thermal conductivities</td>
<td>w/cmK</td>
</tr>
<tr>
<td>Array of total gas volumes during cycle</td>
<td>cm³</td>
</tr>
<tr>
<td>Array of works</td>
<td>joules</td>
</tr>
</tbody>
</table>

Capacity of heat of cylinder wall     | j/gK    | M5 |
Capacity of heat of displacer wall     | j/gK    | M4 |
<table>
<thead>
<tr>
<th>Term</th>
<th>Units</th>
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<tbody>
<tr>
<td>Capacity of heat of gas at constant pressure</td>
<td>J/gK CP</td>
</tr>
<tr>
<td>Capacity of heat of gas at constant volume</td>
<td>J/gK CV</td>
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<tr>
<td>Capacity of heat of regenerator metal</td>
<td>J/gK M6</td>
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<tr>
<td>Clearance around displacer in annular gap heater</td>
<td>cm IH</td>
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<tr>
<td>Clearance around displacer in annular gap cooler</td>
<td>cm IC</td>
</tr>
<tr>
<td>Clearance around hot cap</td>
<td>cm G</td>
</tr>
<tr>
<td>Clearance end, in gamma type power piston</td>
<td>cm E2</td>
</tr>
<tr>
<td>Clearance piston end</td>
<td>cm EC</td>
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<tr>
<td>Coefficient to calculate gas viscosity</td>
<td>-- M1</td>
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<tr>
<td>Coefficient to calculate gas viscosity</td>
<td>-- M2</td>
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<tr>
<td>Coefficient to calculate gas viscosity</td>
<td>-- M3</td>
</tr>
<tr>
<td>Coefficient of gas leakage</td>
<td>frac/MPa sec LK</td>
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<tr>
<td>Coefficient of gas leakage</td>
<td>frac/ LX (increment)(MPa)</td>
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<td>Coefficient in gas thermal conductivity formula</td>
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<td>-- KB</td>
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<tr>
<td>Coefficient of heat transfer</td>
<td>watt/cm²K HY</td>
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<tr>
<td>Coefficient of heat transfer at cooler</td>
<td>w/cm²K HC</td>
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<tr>
<td>Coefficient of heat transfer in heater</td>
<td>w/cm²K HH</td>
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<td>w/cmK KX</td>
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<td>Constant of conversion - 10⁷</td>
<td>g/(MPa·sec²·cm) G1</td>
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<td>Constant in internal temperature swing loss equation</td>
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<tr>
<td>Constant in reheat loss equation</td>
<td>-- K3</td>
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<tr>
<td>Constant Stefan-Boltzmann - 5.67 x 10⁻¹²</td>
<td>w/cm²K⁴ S1</td>
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<td>Constant for table spacing</td>
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<tr>
<td>Counter for finding right average pressure</td>
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<td>Counter for iterations</td>
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</tr>
<tr>
<td>Counter for number of iterations</td>
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<tr>
<td>Credit of heat for flow friction</td>
<td>watts F2</td>
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<tr>
<td>Density of cylinder wall</td>
<td>g/cm³ E5</td>
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<tr>
<td>Density of displacer wall</td>
<td>g/cm³ E4</td>
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<td>Density of gas in cooler</td>
<td>g/cm³ DK</td>
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<td>Density of gas in heater</td>
<td>g/cm³ DH</td>
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<tr>
<td>Density of gas regenerator</td>
<td>g/cm³ RM</td>
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<tr>
<td>Description</td>
<td>Unit</td>
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<tr>
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<tr>
<td>Density of matrix material</td>
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<tr>
<td>Diameter of displacer</td>
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<td>Diameter of displacer drive rod</td>
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<tr>
<td>Diameter, effective or real of power duct</td>
<td>cm</td>
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<tr>
<td>Diameter of hot space manifold tubes</td>
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<tr>
<td>Diameter, inside of annular regenerator</td>
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<tr>
<td>Diameter of inside of cold duct</td>
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<tr>
<td>Diameter, inside of cooler tubes</td>
<td>cm</td>
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<tr>
<td>Diameter, inside of engine cylinder</td>
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<tr>
<td>Diameter, inside of heater manifold tubes</td>
<td>cm</td>
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<tr>
<td>Diameter, inside of heater tubes</td>
<td>cm</td>
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<tr>
<td>Diameter, outside of annular regenerator</td>
<td>cm</td>
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<td>Diameter, outside of cold space manifold</td>
<td>cm</td>
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<tr>
<td>Diameter, outside of cooler tubes</td>
<td>cm</td>
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<tr>
<td>Diameter, outside of heater tube</td>
<td>cm</td>
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<tr>
<td>Diameter of power piston drive rod if in working space (gamma engine)</td>
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<tr>
<td>Diameter of power piston in gamma engine</td>
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<tr>
<td>Diameter of each regenerator</td>
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<tr>
<td>Diameter of wire or sphere in matrix</td>
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<tr>
<td>Diffusivity, thermal in displacer</td>
<td>cm²/sec</td>
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<td>Diffusivity, thermal in cylinder wall</td>
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<td>Eccentricity in a rhombic drive</td>
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<td>Efficiency of furnace</td>
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<td>Emissivity of cold surface</td>
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<tr>
<td>Emissivity of hot surface</td>
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<tr>
<td>Emissivity of radiation shields</td>
<td>--</td>
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<tr>
<td>Exponent of correlation of power with pressure</td>
<td>--</td>
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<tr>
<td>Factor to calculate shuttle heat loss</td>
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<tr>
<td>Factor to calculate shuttle heat loss</td>
<td>--</td>
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<tr>
<td>Factor of compressibility of gas</td>
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</tr>
<tr>
<td>Term</td>
<td>Unit</td>
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<td>Factor, conversion = 2.54</td>
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<td>Factor, correction to work diagram for large angle increments</td>
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<td>Factor of correlation, power with pressure</td>
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<td>Factor of correlation of power with standard</td>
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<tr>
<td>Factor for effect of areas in radiation</td>
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<td>Factor for emissivity effect in radiation</td>
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<td>Factor of friction for matrix or tubes</td>
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<tr>
<td>Factor for number of radiation shields in radiation</td>
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<tr>
<td>Factor in Schmidt Equation (see Eq. 6-36)</td>
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<td>Factor in shuttle heat loss equation</td>
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<td>Flag for heat conduction method</td>
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<td>Flag for iteration method</td>
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<tr>
<td>Flow of cooling water per cylinder</td>
<td>GPM or liter/</td>
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<tr>
<td>Flow of cooling water</td>
<td>g/sec</td>
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<tr>
<td>Flow, mass into or out of cold space</td>
<td>g/sec</td>
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<tr>
<td>Flow, mass into or out of hot space</td>
<td>g/sec</td>
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<tr>
<td>Flow, mass through regenerator</td>
<td>g/sec</td>
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<tr>
<td>Fraction of cycle time gas is assumed to enter hot space at constant rate</td>
<td></td>
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</tr>
<tr>
<td>Fraction of cycle time gas is assumed to enter cold space at constant rate</td>
<td></td>
</tr>
<tr>
<td>Fraction of matrix volume filled with solid</td>
<td></td>
</tr>
<tr>
<td>Fraction of time flow is into hot space</td>
<td></td>
</tr>
<tr>
<td>Frequency of engine</td>
<td>Hz</td>
</tr>
<tr>
<td>Heat absorbed by cooler</td>
<td>watts</td>
</tr>
<tr>
<td>Heat, basic input</td>
<td>watts</td>
</tr>
<tr>
<td>Heat, net required</td>
<td>watts</td>
</tr>
</tbody>
</table>
Heat supplied by heater: joules QB
Height of fins in cooler: cm OC
Height of fins in heater: cm OH
Length of regenerator: cm LR
Length of cold duct (dead volume): cm LF
Length of cold duct (pressure drop): cm LE
Length of connecting rod: cm CR
Length of connecting rod to cold space: cm C4
Length, cooled, of cooler tubes: cm LD
Length, of cooler tubes, total: cm LC
Length, heated, of heater tubes: cm LI
Length of heater manifold tubes (for dead volume): cm LN
Length of heater manifold tubes (for pressure drop): cm LP
Length of heater tube or heater fin: cm LH
Length of hot cap or displacer: cm LB
Length of hot space manifold tubes (dead volume): cm IM
Length of hot space manifold tubes (pressure drop): cm LO
Length of power duct: cm L1
Length of temperature wave in cylinder wall: cm L5
Length of temperature wave in displacer: cm L4
Loss, flow, cooler: watts CF
Loss, flow in heater: watts HW
Loss, flow in all regenerators of engine: watts RW
Loss of heat due to conduction, calculated: watts CQ
Loss due to internal temperature swing: watts QI
Loss due to matrix temperature swing: watts SL
Loss due to mechanical friction except seals: watts MF
Loss, mechanical, due to seal friction: watts FS
Loss, pumping, for all N cylinders: watts QP
Loss, reheat: watts RH
Loss, shuttle, for all N cylinders: watts QS
Loss, static heat conduction, specified: Watts ZH

Mass of regenerator matrix: g MX
Maximum of array FC( ): -- CY
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum of array FH( )</td>
<td></td>
</tr>
<tr>
<td>Maximum of P( )</td>
<td>MPa</td>
</tr>
<tr>
<td>Maximum of V( )</td>
<td>cm³</td>
</tr>
<tr>
<td>Mesh of screen or foils</td>
<td>number/cm</td>
</tr>
<tr>
<td>Minimum of array FC( )</td>
<td></td>
</tr>
<tr>
<td>Minimum of array FH( )</td>
<td></td>
</tr>
<tr>
<td>Minimum of P( )</td>
<td>MPa</td>
</tr>
<tr>
<td>Minimum of V( )</td>
<td>cm³</td>
</tr>
<tr>
<td>Moles of working fluid</td>
<td>g mol</td>
</tr>
<tr>
<td>Number of cold ducts per cylinder</td>
<td>--</td>
</tr>
<tr>
<td>Number of cold space manifold tubes per cylinder</td>
<td>--</td>
</tr>
<tr>
<td>Number of cooler tubes per cylinder or spaces between fins</td>
<td>--</td>
</tr>
<tr>
<td>Number of cylinders per engine</td>
<td>--</td>
</tr>
<tr>
<td>Number of heater tubes or fin spaces per cylinder</td>
<td>--</td>
</tr>
<tr>
<td>Number of hot space manifold tubes per cylinder</td>
<td>--</td>
</tr>
<tr>
<td>Number of internal radiation shields in displacer or hot cap</td>
<td>--</td>
</tr>
<tr>
<td>Number of power ducts per cylinder</td>
<td></td>
</tr>
<tr>
<td>Number of regenerators per cylinder</td>
<td>--</td>
</tr>
<tr>
<td>Number of transfer units</td>
<td>--</td>
</tr>
<tr>
<td>Number of transfer units in regenerator</td>
<td>--</td>
</tr>
<tr>
<td>Number of tubes per cylinder in heater tube manifold</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in cold duct</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in cooler</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in heater tube</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in heater</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in hot space</td>
<td>--</td>
</tr>
<tr>
<td>Number of velocity heads due to entrance, exit and bends in power duct</td>
<td>--</td>
</tr>
</tbody>
</table>

323
<table>
<thead>
<tr>
<th>Option on cooler type:</th>
<th>1 = tubes</th>
<th>2 = annulus, cooled one side</th>
<th>3 = fins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option for enclosed gas inside of hot cap:</td>
<td>1 = H₂</td>
<td>2 = H₂</td>
<td>3 = air</td>
</tr>
<tr>
<td>Option for engine cylinder material:</td>
<td>1 = glass or alumina</td>
<td>2 = stainless steel, super alloy or SiC</td>
<td>3 = cast iron or carbon steel</td>
</tr>
<tr>
<td>Option for heater:</td>
<td>1 = tubes</td>
<td>2 = fins</td>
<td>3 = single annulus heated one side</td>
</tr>
<tr>
<td>Option of operating gas:</td>
<td>1 = hydrogen</td>
<td>2 = helium</td>
<td>3 = air</td>
</tr>
<tr>
<td>Option on regenerator matrix material (Same as N3):</td>
<td>1 = screens</td>
<td>2 = foam metal</td>
<td>3 = spheres</td>
</tr>
<tr>
<td>Option on regenerator wall material (Same as N3):</td>
<td>1 = screens</td>
<td>2 = foam metal</td>
<td>3 = spheres</td>
</tr>
</tbody>
</table>

- Porosity of matrix
- Power, basic: watts
- Power, indicated: watts
- Power, net: watts
- Prandtl, number to 2/3 power
- Pressure, average: psia
- Pressure, average gas: MPa
- Pressure, difference of: MPa
- Pressure, mean
- Product of gas inventory and gas constant: j/K
- Radius of crank to cold space: cm
- Radius of crank (if 2 cranks then to hot space): cm
- Radius, hydraulic, of regenerator matrix: cm
- Ratio of dead volume to expansion space volume
- Ratio of dead volume mass to expansion space mass

CA
KS
N3
N4
R1
N5
PO
BP
IP
NP
PR
PS
PG
DP
PM
MR
R2
RC
HR
RV
S
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of heat transfer area to volume of matrix</td>
<td>cm⁻¹</td>
<td>AS</td>
</tr>
<tr>
<td>Ratio of TH to TC</td>
<td></td>
<td>TA</td>
</tr>
<tr>
<td>Ratio of TC to TH</td>
<td></td>
<td>AU</td>
</tr>
<tr>
<td>Ratio of volumes, maximum/minimum</td>
<td></td>
<td>VR</td>
</tr>
<tr>
<td>Reynolds number, cooler</td>
<td></td>
<td>RZ</td>
</tr>
<tr>
<td>Reynolds number, heater</td>
<td></td>
<td>RT</td>
</tr>
<tr>
<td>Reynolds number, heater or cooler</td>
<td></td>
<td>RE</td>
</tr>
<tr>
<td>Reynolds number, regenerator</td>
<td></td>
<td>RR</td>
</tr>
<tr>
<td>Space between fins in cooler</td>
<td>cm</td>
<td>IC</td>
</tr>
<tr>
<td>Space between fins in heater</td>
<td>cm</td>
<td>IH</td>
</tr>
<tr>
<td>Speed of engine</td>
<td>Radians/sec</td>
<td>OM</td>
</tr>
<tr>
<td>Speed of engine</td>
<td>RPM</td>
<td>SP</td>
</tr>
<tr>
<td>Stanton, number x (Pr)²/³</td>
<td></td>
<td>ST</td>
</tr>
<tr>
<td>Stroke of displacer or hot cap</td>
<td>cm</td>
<td>SD</td>
</tr>
<tr>
<td>Summation of M * R</td>
<td>j/K</td>
<td>LY</td>
</tr>
<tr>
<td>Temperature of cooler tube metal, average</td>
<td>K</td>
<td>TX</td>
</tr>
<tr>
<td>Temperature, effective, of cold space</td>
<td>K</td>
<td>TC</td>
</tr>
<tr>
<td>Temperature of gas leaving regenerator</td>
<td>K</td>
<td>TL</td>
</tr>
<tr>
<td>Temperature, effective, of hot space</td>
<td>K</td>
<td>TH</td>
</tr>
<tr>
<td>Temperature of inlet cooling water</td>
<td>K</td>
<td>TW</td>
</tr>
<tr>
<td>Temperature of inlet cooling water</td>
<td>F or C</td>
<td>TY</td>
</tr>
<tr>
<td>Temperature of inside heater tube wall</td>
<td>F or C</td>
<td>TF</td>
</tr>
<tr>
<td>Temperature of inside heater tube wall</td>
<td>K</td>
<td>TM</td>
</tr>
<tr>
<td>Temperature, increase of, in cold space</td>
<td>K</td>
<td>DU</td>
</tr>
<tr>
<td>Temperature, increase of, in cooling water</td>
<td>K</td>
<td>DT</td>
</tr>
<tr>
<td>Temperature, increase of, in hot space</td>
<td>K</td>
<td>DV</td>
</tr>
<tr>
<td>Temperature along regenerator</td>
<td>K</td>
<td>TZ</td>
</tr>
<tr>
<td>Temperature of regenerator, effective</td>
<td>K</td>
<td>TR</td>
</tr>
<tr>
<td>Temperature, swing of, in matrix</td>
<td>K</td>
<td>TS</td>
</tr>
<tr>
<td>Thickness of expansion cylinder wall</td>
<td>cm</td>
<td>SE</td>
</tr>
<tr>
<td>Thickness of foils in slot type regenerator</td>
<td>cm</td>
<td>DW</td>
</tr>
<tr>
<td>Thickness of hot cap wall</td>
<td>cm</td>
<td>SC</td>
</tr>
<tr>
<td>Thickness of inside regenerator wall if annular regenerator</td>
<td>cm</td>
<td>SS</td>
</tr>
<tr>
<td>Thickness of wall of regenerator housing</td>
<td>cm</td>
<td>SR</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physical Property</td>
<td>Unit</td>
<td>Symbol</td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
<td>---------------</td>
<td>--------</td>
</tr>
<tr>
<td>Velocity of gas through gas cooler or connecting duct</td>
<td>cm/sec</td>
<td>VC</td>
</tr>
<tr>
<td>Velocity of gas through gas heater</td>
<td>cm/sec</td>
<td>VH</td>
</tr>
<tr>
<td>Velocity, mass, in connecting duct</td>
<td>g/sec cm²</td>
<td>GD</td>
</tr>
<tr>
<td>Velocity, mass, through cooler</td>
<td>g/sec cm²</td>
<td>GC</td>
</tr>
<tr>
<td>Velocity, mass, in heater</td>
<td>g/sec cm²</td>
<td>GH</td>
</tr>
<tr>
<td>Velocity, mass, in regenerator</td>
<td>g/sec cm²</td>
<td>GR</td>
</tr>
<tr>
<td>Viscosity of gas</td>
<td>g cm sec</td>
<td>MU</td>
</tr>
<tr>
<td>Volume, cold, dead</td>
<td>cm³</td>
<td>CD</td>
</tr>
<tr>
<td>Volume, cold, dead actually measured in beta engine</td>
<td>cm³</td>
<td>VM</td>
</tr>
<tr>
<td>Volume, cold, dead outside cooler tubes</td>
<td>cm³</td>
<td>CX</td>
</tr>
<tr>
<td>Volume, cold, live (with displacer)</td>
<td>cm³</td>
<td>VK</td>
</tr>
<tr>
<td>Volume, hot, dead</td>
<td>cm³</td>
<td>HD</td>
</tr>
<tr>
<td>Volume, hot, live</td>
<td>cm³</td>
<td>VL</td>
</tr>
<tr>
<td>Volume, live (with power piston)</td>
<td>cm³</td>
<td>VP</td>
</tr>
<tr>
<td>Volume, regenerator, dead</td>
<td>cm³</td>
<td>RD</td>
</tr>
<tr>
<td>Volume, total, of annulus</td>
<td>cm³</td>
<td>VA</td>
</tr>
<tr>
<td>Volume, total, dead = HD + RD + CD</td>
<td>cm³</td>
<td>VD</td>
</tr>
<tr>
<td>Volume, total, live = VL + VK</td>
<td>cm³</td>
<td>VT</td>
</tr>
<tr>
<td>Weight, molecular of gas</td>
<td>g/g mol</td>
<td>MW</td>
</tr>
<tr>
<td>Work for one cycle and one cylinder</td>
<td>joules</td>
<td>W1</td>
</tr>
</tbody>
</table>
APPENDIX C
Isothermal Second Order Design Program

In this appendix the Isothermal Second Order Design Program is explained. A nomenclature is given which pertains only to Appendix C. Two BASIC programs were prepared—one for design purposes and one to compare the General Motors data with predictions. From the design program written in BASIC, a program written in FORTRAN was prepared and validated. A listing of the FORTRAN program is given in this appendix. This program takes a file of data for input, and prints the input quantities and the results. Finally, a sample of the design program output and the final results of the comparison program are presented.

C.1 Description

The program described in this appendix is an outgrowth of the calculation procedure presented at the 1978 IECEC (78 o) and also in the authors 1979 IECEC paper (79 ad). The following major changes have been made over the previous publications.

1. Corrections have been made to the program particularly the effect of multiple cylinders had not been taken into account consistently.

2. Property values for hydrogen, helium, or air can be used. In addition, the effect of temperature on thermoconductivity has been taken into account when previously only the effect of temperature on viscosity was written into the program.

3. For the cases that are non-convergent, the program adopts a more cautious method so that the process would be convergent no matter what design had been chosen. The process shown in reference 78 o for selecting the effective hot gas and cold gas temperature was found to be non-convergent in some cases.

4. All flow resistance including losses due to bends and entrances and exits are included.

5. Temperature difference between the effective gas temperature and the adjacent heat exchanger can be set at any specified fraction of the log mean temperature difference.

6. Static heat leak can be calculated from dimensions or specified in advance.

The basic assumption in the isothermal second order design program described herein is that there exists an effective hot space and cold space constant temperature that can be used to compute the power output per cycle for a Stirling engine. This effective gas temperature is assumed not to change during the cycle, although, in fact, it really does to an important degree. It is assumed that the effective temperature can be calculated by determining the
amount of heat that must be transferred through the heat exchanger during a particular cycle and this should determine the offset between metal temperature and the effective gas temperature. For instance, the hot space temperature is less than the heat source temperature by a fraction of the log mean temperature difference in the gas heater that is needed to transfer the heat to the hot space from the heat source. In the same way, the effective cold space temperature is hotter than the heat sink water temperature by a fraction of the log mean temperature difference for that heat exchanger.

The method of zeroing in on the effective hot and cold gas temperatures is most critical in determining how long the calculation takes per case. The original computational procedure determines the temperature difference required from the present heat requirement and the heat transfer capabilities of the heat exchanger. For well designed engines, with large heat exchangers, this iteration method for the effective temperatures is rapidly convergent. However, when only a small amount of heat exchange surface is specified in the engine the original method leads to completely uncontrolled oscillations or very slow damping of the solution. For these cases the program switches to a more cautious iteration procedure. In the first iteration, the effective hot space temperature is assumed to be the same as the hot metal temperature and the effective cold space temperature is assumed to be the same as the inlet cold water temperature. Then the error between the amount of heat that must be transferred in the gas heater compared with the amount of heat that is transferred due to the temperature difference is computed. Another error is computed for the amount of heat that must be transferred in the gas cooler compared to the amount of heat that can be transferred due to the temperature difference. Next, these two temperature differences are changed by an amount input into the program, in this case, 64° K, that is the hot space temperature is decreased by 64 degrees and the cold space temperature is increased by 64 degrees. The calculation is repeated and the heat transfer errors for both the hot and the cold space are again computed. This error is usually less because the heat required is somewhat less but the heat that can be transferred is a lot more and they are beginning to get into balance. At this point, we have two temperatures and two errors for the hot space and two temperatures and two errors for the cold space. It would seem reasonable then to apply a secant method to extrapolate what the temperature would be for zero error in both the hot and cold space. This was tried and found to be calculationally unstable because the two iteration processes strongly interact. Therefore, it was found necessary to be more cautious about approaching the roots of these two equations. The procedure used here makes successive corrections of 64 degrees until the heat transfer error changed sign. Then it makes successive corrections of 16 degrees until another sign change is noted, and then 4 degrees, and then 1 degree and so on. This iteration procedure has been found to be unconditionally stable for all cases that have been tried, but it is time consuming. For very small heat transfer areas and a specified constant heat leak the calculated effective gas temperatures can be wrong. The program stops and the error is indicated. If static heat losses are calculated from the dimensions then this problem does not occur.

The first convergence method requires 45 sec/case. The second method requires between six and seven minutes to compute using the Radio Shack TRS-80 and the Microsoft BASIC computer program. Using the Prime Interim 750 CPU computer with FORTRAN, the first convergence method requires two seconds per case to compute.

Note in editing: This program is valid for four cylinder engines only.
C.2 Nomenclature

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>N/RM</td>
</tr>
<tr>
<td>A1</td>
<td>Counter for finding right average pressure</td>
</tr>
<tr>
<td>AA</td>
<td>.435 correlation of power with pressure</td>
</tr>
<tr>
<td>AC</td>
<td>Heat transfer area for cooler, cm²</td>
</tr>
<tr>
<td>AF</td>
<td>Area of flow, cm²</td>
</tr>
<tr>
<td>AH</td>
<td>Heat transfer area of heater, cm²</td>
</tr>
<tr>
<td>AL</td>
<td>Phase angle alpha = 90 degrees</td>
</tr>
<tr>
<td>AS</td>
<td>Area to volume ratio for regenerator matrix = 179 cm²/cm³ for Met Net 0.05-0.20</td>
</tr>
<tr>
<td>B</td>
<td>Table spacing constant</td>
</tr>
<tr>
<td>BA</td>
<td>.1532 = exponent of correlation of power with pressure</td>
</tr>
<tr>
<td>BF</td>
<td>Bugger factor to convert power outputs to nearly what GM says they should be</td>
</tr>
<tr>
<td>BH</td>
<td>Basic heat input, watts (BHI)</td>
</tr>
<tr>
<td>BP</td>
<td>Basic power, watts</td>
</tr>
<tr>
<td>C()</td>
<td>Cold volumes at 360/ND Points/cycle</td>
</tr>
<tr>
<td>CD</td>
<td>Cold dead volume, cm³</td>
</tr>
<tr>
<td>CF</td>
<td>Cooler windage, watts</td>
</tr>
<tr>
<td>CM</td>
<td>2.54 cm/inch</td>
</tr>
<tr>
<td>CN</td>
<td>Minimum FC( )</td>
</tr>
<tr>
<td>CP</td>
<td>Heat capacity of hydrogen at constant P = 14.62 j/g K @ 700 K (assumed not to vary importantly with temperature)</td>
</tr>
<tr>
<td>CR</td>
<td>Length of connecting rod, cm</td>
</tr>
<tr>
<td>CRT</td>
<td>Logical Unit no. for input file</td>
</tr>
<tr>
<td>CV</td>
<td>Heat capacity of hydrogen at constant volume = 10.49 j/g K @ 700 K</td>
</tr>
<tr>
<td>CW</td>
<td>Friction factor for Met Net and others</td>
</tr>
<tr>
<td>CX</td>
<td>Cold dead volume outside cooler tubes, cm³</td>
</tr>
<tr>
<td>CY</td>
<td>Maximum FC( )</td>
</tr>
<tr>
<td>DC</td>
<td>Diameter engine cylinder, cm</td>
</tr>
<tr>
<td>DD</td>
<td>Diameter of piston drive rod, cm</td>
</tr>
<tr>
<td>DN</td>
<td>360/ND</td>
</tr>
<tr>
<td>DP</td>
<td>Pressure drop, MPa</td>
</tr>
<tr>
<td>DR</td>
<td>Diameter of regenerator, cm</td>
</tr>
<tr>
<td>DT</td>
<td>Temperature rise in cooling water, K</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>DU</td>
<td>Temperature change for cold space, K</td>
</tr>
<tr>
<td>DV</td>
<td>Temperature change for hot space, K</td>
</tr>
<tr>
<td>DW</td>
<td>Diameter of &quot;wire&quot; in regenerator, cm = 0.017(2.54) = 0.00432 cm</td>
</tr>
<tr>
<td>EC</td>
<td>Piston end clearance, cm</td>
</tr>
<tr>
<td>F</td>
<td>Crank angle, degrees</td>
</tr>
<tr>
<td>FC1</td>
<td>((F3 + F4)/2)</td>
</tr>
<tr>
<td>FC(</td>
<td>Fraction of gas mass in cold spaces at 360/ND Points/cycle</td>
</tr>
<tr>
<td>FE</td>
<td>Furnace efficiency, %</td>
</tr>
<tr>
<td>FF</td>
<td>Filler factor, fraction of regenerator volume filled with solid</td>
</tr>
<tr>
<td>FH1</td>
<td>((F1 + F2)/2)</td>
</tr>
<tr>
<td>FH(</td>
<td>Fraction of gas mass in hot spaces at 360/ND Points/cycle</td>
</tr>
<tr>
<td>FQ</td>
<td>60 Hz/rpm</td>
</tr>
<tr>
<td>FR</td>
<td>((FH + FC)/2)</td>
</tr>
<tr>
<td>FW</td>
<td>Flow of cooling water, g/sec</td>
</tr>
<tr>
<td>FX</td>
<td>Cooling water flow GPM @ 2000 rpm per cylinder</td>
</tr>
<tr>
<td>F1</td>
<td>Fraction of cycle time gas is assumed to leave hot space at constant rate</td>
</tr>
<tr>
<td>F2</td>
<td>Fraction of cycle time gas is assumed to enter hot space at constant rate</td>
</tr>
<tr>
<td>F3</td>
<td>Fraction of cycle time that flow out of cold space is assumed to occur at constant rate</td>
</tr>
<tr>
<td>F4</td>
<td>Fraction of cycle time that flow into cold space is assumed to occur at constant rate</td>
</tr>
<tr>
<td>G</td>
<td>Gap in hot cap, cm = 0.56 cm</td>
</tr>
<tr>
<td>GC</td>
<td>Mass velocity through cooler, g/sec cm²</td>
</tr>
<tr>
<td>GD</td>
<td>Mass velocity in connecting duct, g/sec cm²</td>
</tr>
<tr>
<td>GH</td>
<td>Mass velocity in heater, g/sec cm²</td>
</tr>
<tr>
<td>GR</td>
<td>Mass velocity in regenerator, g/sec cm²</td>
</tr>
<tr>
<td>H(</td>
<td>Hot volumes at 360/ND Points/cycle</td>
</tr>
<tr>
<td>HC</td>
<td>Heat transfer coefficient at cooler, w/cm² K</td>
</tr>
<tr>
<td>HD</td>
<td>Hot dead volume, cm³</td>
</tr>
<tr>
<td>HH</td>
<td>Heat transfer coefficient in heater, w/cm² K</td>
</tr>
<tr>
<td>HN</td>
<td>Minimum FH( )</td>
</tr>
<tr>
<td>HP</td>
<td>1.341E-3 HP/watt</td>
</tr>
<tr>
<td>HX</td>
<td>Maximum FH( )</td>
</tr>
<tr>
<td>I</td>
<td>Iteration counter</td>
</tr>
<tr>
<td>IC</td>
<td>ID of cooler tube, cm</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>ID</td>
<td>Inside diameter of connecting duct, cm</td>
</tr>
<tr>
<td>IH</td>
<td>ID of heater tubes, cm</td>
</tr>
<tr>
<td>IP</td>
<td>Indicated power, watts</td>
</tr>
<tr>
<td>J</td>
<td>Iteration counter</td>
</tr>
<tr>
<td>KA</td>
<td>Coefficient for gas thermal conductivity calculation</td>
</tr>
<tr>
<td>KB</td>
<td>Coefficient for gas thermal conductivity calculation</td>
</tr>
<tr>
<td>KG</td>
<td>Gas thermal conductivity, watts/cm K</td>
</tr>
<tr>
<td>KM</td>
<td>Metal thermal conductivity, w/cm K</td>
</tr>
<tr>
<td>K3</td>
<td>Constant in reheat loss equation</td>
</tr>
<tr>
<td>L1</td>
<td>Fraction of total gas charge leaking per MPa P per second</td>
</tr>
<tr>
<td>L( )</td>
<td>Gas inventory x gas constant, j/K (changes due to leak)</td>
</tr>
<tr>
<td>LB</td>
<td>Length of hot cap, cm</td>
</tr>
<tr>
<td>LC</td>
<td>Length of cooler tube, cm</td>
</tr>
<tr>
<td>LD</td>
<td>Heat transfer length of cooler tube, cm</td>
</tr>
<tr>
<td>LE</td>
<td>Length of connecting duct, cm</td>
</tr>
<tr>
<td>LH</td>
<td>Heater tube length, cm</td>
</tr>
<tr>
<td>LI</td>
<td>Heater tube heat transfer length, cm</td>
</tr>
<tr>
<td>LP</td>
<td>Logical unit No. for output file</td>
</tr>
<tr>
<td>LR</td>
<td>Length of regenerator, cm</td>
</tr>
<tr>
<td>LX</td>
<td>Fraction of gas charge leaking per time increment per ΔP</td>
</tr>
<tr>
<td>LY</td>
<td>Accumulation of MR's</td>
</tr>
<tr>
<td>M</td>
<td>Number of moles of gas in working fluid, g mol</td>
</tr>
<tr>
<td>ME</td>
<td>Mechanical efficiency, %</td>
</tr>
<tr>
<td>MF</td>
<td>Mechanical friction loss</td>
</tr>
<tr>
<td>MR</td>
<td>Gas inventory times gas constant, j/K</td>
</tr>
<tr>
<td>MU</td>
<td>Gas viscosity, g/cm sec</td>
</tr>
<tr>
<td>MW</td>
<td>Molecular weight, g/g mol</td>
</tr>
<tr>
<td>MX</td>
<td>Mass of regenerator matrix</td>
</tr>
<tr>
<td>M1</td>
<td>Coefficients in viscosity equation</td>
</tr>
<tr>
<td>M2</td>
<td>Coefficients in viscosity equation</td>
</tr>
<tr>
<td>M3</td>
<td>Coefficients in viscosity equation</td>
</tr>
<tr>
<td>N</td>
<td>Number of cylinders per engine</td>
</tr>
<tr>
<td>NC</td>
<td>Number of cooler tubes per cylinder</td>
</tr>
<tr>
<td>ND</td>
<td>Degree increment in time step (normally 30 degrees)</td>
</tr>
<tr>
<td>NE</td>
<td>Number of connecting ducts per cylinder</td>
</tr>
<tr>
<td>NH</td>
<td>Number of heater tubes per cylinder</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>NP</td>
<td>Net power, watts</td>
</tr>
<tr>
<td>NR</td>
<td>Number of regenerators per cylinder</td>
</tr>
<tr>
<td>NT</td>
<td>Number of transfer units in regenerator, NTUP</td>
</tr>
<tr>
<td>NU</td>
<td>Engine frequency, Hz</td>
</tr>
<tr>
<td>NS</td>
<td>&quot;Name&quot;</td>
</tr>
<tr>
<td>OC</td>
<td>OD of cooler tubes, cm</td>
</tr>
<tr>
<td>OD</td>
<td>Outside diameter of connecting duct, cm</td>
</tr>
<tr>
<td>OG</td>
<td>Operating gas, 1 = hydrogen, 2 = helium, 3 = air</td>
</tr>
<tr>
<td>OH</td>
<td>Heater tube OD, cm</td>
</tr>
<tr>
<td>P( )</td>
<td>Pressures first with MR = 1, later at average pressure</td>
</tr>
<tr>
<td>PG</td>
<td>Average gas pressure, MPa</td>
</tr>
<tr>
<td>PI</td>
<td>3.14159</td>
</tr>
<tr>
<td>PM</td>
<td>Mean Pressure, of all P's</td>
</tr>
<tr>
<td>PN</td>
<td>Minimum pressure, MPs</td>
</tr>
<tr>
<td>PP</td>
<td>0.006894 MPa/psia</td>
</tr>
<tr>
<td>PR</td>
<td>Prandtl number to the 2/3 power = (Pr)^2/3</td>
</tr>
<tr>
<td>PS</td>
<td>Average pressure, psia</td>
</tr>
<tr>
<td>PX</td>
<td>Maximum pressure, MPa</td>
</tr>
<tr>
<td>P4</td>
<td>π/4 = .785398</td>
</tr>
<tr>
<td>QC</td>
<td>Heat absorbed by cooler, watts</td>
</tr>
<tr>
<td>QN</td>
<td>Net heat required, watts</td>
</tr>
<tr>
<td>QP</td>
<td>Pumping loss for all N cylinders</td>
</tr>
<tr>
<td>QS</td>
<td>Shuttle loss, watts</td>
</tr>
<tr>
<td>R</td>
<td>Gas constant, 8.314 J/g mol K</td>
</tr>
<tr>
<td>RA</td>
<td>0.0174533 radians/degree</td>
</tr>
<tr>
<td>RC</td>
<td>Crank radius, cm</td>
</tr>
<tr>
<td>RD</td>
<td>Regenerator dead volume, cm^3</td>
</tr>
<tr>
<td>RE</td>
<td>Reynolds number, heater or cooler</td>
</tr>
<tr>
<td>RH</td>
<td>Reheat loss, watts</td>
</tr>
<tr>
<td>RM</td>
<td>Gas density for regenerator, g/cm^2</td>
</tr>
<tr>
<td>RP</td>
<td>Sum and average of power ratios</td>
</tr>
<tr>
<td>RQ</td>
<td>Sum and average of efficiency ratios</td>
</tr>
<tr>
<td>RR</td>
<td>Regenerator Reynolds number</td>
</tr>
<tr>
<td>RT</td>
<td>Reynolds number, heater</td>
</tr>
<tr>
<td>RW</td>
<td>Regenerator windage, watts, for all cylinders in engine</td>
</tr>
<tr>
<td>RZ</td>
<td>Reynolds number, cooler</td>
</tr>
</tbody>
</table>
Wall thickness of hot cap, cm
Wall thickness of expansion cylinder wall, cm
Temp swing loss, watts = QTS
Engine speed, RPM
Wall thickness of regenerator housing, cm
Stanton number x(Pr)^2/3
Effective cold space temperature, K
Inside heater tube wall temperature, F
Effective Hot space temperature, K
Inside heater tube wall temperature, K
Regenerator temperature, K
Matrix temp swing, K = DELTMX
Inlet cooling water, K
Cooler tube metal temperature average, K
Inlet cooling water temperature, F
Total gas volume at 360/ND Points/cycle
Velocity through gas cooler or connecting duct, cm/sec
Velocity through gas heater, cm/sec
Minimum total volume, cm^3
Maximum total volume, cm^3
"Value"
Flow rate into or out of cold space, g/sec
Flow rate into or out of hot space, g/sec
(WH + WC)/2 = g/sec through regenerator = WRS
Work for one cycle and one cylinder, joules
Temporary variable
Correction factor to work diagram for large angle increments
Temporary variable
Temporary variable
Temporary variable
0 for rapid iteration method, = 1 for slower iteration method when rapid method does not work
Iteration counter
Specified static heat conduction loss, watts
0 for specified static conduction, 1 for calculated static conduction
**ISOTHERMAL SECOND ORDER CALCULATION**

**PROGRAM ISO -10 OCT 1979 -**

**WRITTEN BY WILLIAM R. MARTINI**

**PROGRAM WRITTEN WITH THE PRIMOS OPERATING SYSTEM**

**PROGRAM MUST HAVE ACCESS TO BOTH THE INPUT FILE AND AN OUTPUT FILE**

**SEE ATTACHED REFERENCE FOR LIST AND DESCRIPTION OF NOMENCLATURE**

C SETS UP ARRAYS (DIMENSIONS)

- DIMENSION H(13), C(13), P(13), FH(13), FC(13), V(14)

C SETS UP INTEGERS

- INTEGER A1,0G, ZA, ZB, ZZ, CRT, TRM

C SETS UP REAL NUMBERS

- REAL IC, ID, IH, IP, KA, KB, KG, KM, K3, L1, LB, LD, LE, LI, LR, LX, LY, M, ME, MF,
  1MR, MU, MW, MX, M1, M2, M3, NP, NU, LC, LH, L(14), NT, ND

C SETS UP LOGICAL UNIT NUMBERS. 'CRT' IS THE LOGICAL UNIT NUMBER FOR THE INPUT FILE, AND 'LP' IS THE LOGICAL UNIT NUMBER FOR THE OUTPUT FILE.

- DATA CRT/5/, LP/6/

C PROGRAM READS IN ENGINE DIMENSIONS, OPERATING CONDITIONS, AND CONVERSION CONSTANTS FROM THE INPUT FILE. ALSO THIS IS THE RETURN POINT AFTER A CASE HAS BEEN COMPLETED. IF THERE ARE NO MORE CASES TO RUN (I.E. AN END OF FILE OCCURS), THE PROGRAM CALLS EXIT.

C THE DEGREE INCREMENT IS SET AT 30 DEGREES.

- ND = 30

C A CORRECTION FACTOR IS CALCULATED WHICH INCREASES THE ACCURACY IN CALCULATING THE WORK INTEGRALS WITH 30 DEGREE INCREMENTS.

- XX = 1.5321E-5 ND**1.9797

C TEMPERATURE CHANGE FOR COLD SPACE (DU) AND TEMPERATURE CHANGE FOR HOT SPACE (DV) ARE SET.

- DU = 64
\[ DV=64. \]

C THE FIRST THING THE PROGRAM DOES IS TO COMPUTE A LIST OF ENGINE
C VOLUMES.
C
C CONVERSION TO KELVIN DEGREES FROM INPUT FAHRENHEIT DEGREES.
\[ TM=(TF+460.)/1.8 \]
\[ TW=(TY+460.)/1.8 \]
C CONVERSION TO HERTZ AND TO MPA.
\[ NU=SP/60. \]
\[ PG=.006894*PS \]
C DETERMINES GAS PROPERTY VALUES FROM 'OG' (IF 'OG' = 1, THE PROPERTY
C VALUES FOR HYDROGEN ARE USED. IF 'OG' = 2, THE PROPERTY VALUES FOR
C OXYGEN ARE USED. IF 'OG' = 3, THE PROPERTY VALUES FOR AIR ARE USED.)
C PROPERTY VALUES FOR ADDITIONAL GASES MAY BE ADDED IF DESIRED.
\[ IF(OG.EQ.1) \text{ GOTO 20} \]
\[ IF(OG.EQ.2) \text{ GOTO 21} \]
\[ KA=-12.6824 \]
\[ KB=.7820 \]
\[ CP=1.0752 \]
\[ CV=.7883 \]
\[ M1=1.8194E-4 \]
\[ M2=5.36E-7 \]
\[ M3=1.22E-6 \]
\[ MW=29. \]
\[ PR=.9071 \]
\[ \text{GOTO 22} \]
\[ 20 \]
\[ KA=-11.0004 \]
\[ KB=.8130 \]
\[ CP=14.62 \]
\[ CV=10.49 \]
\[ M1=8.873E-5 \]
\[ M2=2.0E-7 \]
\[ M3=1.18E-7 \]
\[ MW=2.02 \]
\[ PR=.8408 \]
\[ \text{GOTO 22} \]
\[ 21 \]
\[ KA=-10.1309 \]
\[ KB=.6335 \]
\[ CP=5.2 \]
CV=3.12
M1=1.6614E-4
M2=4.63E-7
M3=-9.3E8
MW=4.
PR=.8018

C CONVERSION OF COOLING WATER FLOW TO GRAMS/SECOND. INITIALLY COOLER
C TUBE METAL TEMPERATURE IS MADE THE SAME AS THE INLET COOLING WATER
C TEMPERATURE. THE TOTAL HEAT TRANSFER AREAS FOR ALL THE ENGINES
C COOLERS AND ALL THE ENGINES HEATERS ARE CALCULATED.
22   FW=63.12*FX
     TX=TW
     AC=PI*IC*L+NC*N
     AH=PI*IH*LI*N

C CALCULATES ENGINE DEAD VOLUMES AND INITIALIZES PRESSURES AND VOLUMES.
C INITIALIZES FOR DETERMINATION OF AVERAGE PRESSURE AND MAXIMUM AND
C MINIMUM VOLUMES.
     HD=P4*IH*IH*NH+HC*DC**2*P4
     CX=P4*ID*LE*N
     RD=(1.-FF)*P4*DR**2.*LR*NR+PI*DC*G*L
     CD=CX+P4*IC*2.*LC*NC*EC*P4*(DC**2.-DD**2.)
     PM=0.
     VX=0.
     UN=1.E30

C INITIALLY SETS THE EFFECTIVE HOT SPACE TEMPERATURE TO THE HOT METAL
C TEMPERATURE AND THE EFFECTIVE COLD SPACE TEMPERATURE TO THE COOLING
C WATER TEMPERATURE FOR THE FIRST TIME AROUND. CALCULATES THE LOG MEAN
C TEMPERATURE FOR THE REGENERATOR. CALCULATES THE LEAKAGE COEFFICIENT
C FOR 30 DEGREE INCREMENTS.
     TH=TM
     TC=TW
     TR=(TM-TW)/ALOG(TM/TW)
     LX=L1*ND/(360.*NU)

C SINCE THE THERMOCONDUCTIVITY ENTERS THE CALCULATION ONLY AT THE
C REGENERATOR TEMPERATURE IT CAN BE CALCULATED BEFORE THE MAIN
C ITERATION LOOP.
     KG=EXP(KA+KB*ALOG(TR))

C START OF DO LOOP 23 TO CALCULATE ENGINE VOLUMES.
DO 23 I=1,13
C CALCULATES THE HOT VOLUME AND COLD VOLUME FOR EACH ANGLE INCREMENT FOR
C CRANK OPERATED PISTONS. SINCE A DOUBLE ACTING MACHINE HAS A PISTON
C DRIVE ROD (DD) AND A SINGLE ACTING MACHINE DOES NOT, "DD" IS USED AS
C AN INDICATOR OF WHETHER THE COLD VOLUME OF THE ENGINE IS ABOVE THE
C PISTON OR BELOW IT.
   X=30.*(I-1)*RA
   J=I
   IF(DD.EQ.0) GOTO 24
   Y=(30.*(I-1)+AL)*RA
   GOTO 25
24   Y=(30.*(I-1)-AL)*RA
25   H(J)=P4*DC**2*(RC-SQRT(CR**2-(RC*SIN(X))**2)+RC*COS(X)+CR)+HD
   IF(DD.EQ.0) GOTO 26
   C(J)=P4*(DC**2-DD**2)*(SQRT(CR**2-(RC*SIN(Y))**2)-RC*COS(Y)-CR+RC)
   1+CD
   GOTO 27
26   C(J)=P4*DC**2*(RC-SQRT(CR**2-(RC*SIN(Y))**2)+RC*COS(Y)+CR)+CD
C CALCULATES THE TOTAL GAS VOLUME AND FINDS THE MAXIMUM VOLUME.
27   V(J)=H(J)+RD+C(J)
   IF(V(J).GT.VX) VX=V(J)
C FINDS THE MINIMUM VOLUME.
   IF(V(J).LT.VN) VN=V(J)
C CALCULATES THE INITIAL GAS INVENTORY.
   IF(J.EQ.3) L(1)=PG*(H(J)/TH+RD/TR+C(J)/TC)
C END OF LOOP TO CALCULATE ENGINE VOLUMES
   CONTINUE
C 'ZA' IS SET AT ZERO SO THAT THE FASTEST WAY OF ARRIVING AT THE PROPER
C EFFECTIVE HOT SPACE AND COLD SPACE TEMPERATURE WILL BE TRIED FIRST.
C ALSO A COUNTER, 'ZB', IS SET AT ZERO.
   ZA=0
   ZB=0
C INITIALIZATION
   200   A=0
   29   PM=0
   LY=0
C START OF DO LOOP 28 (TO CALCULATE PRESSURES).
   DO 28 I=1,13
   CONTINUE
C CALCULATE PRESSURE
   P(I) = L(I)/(H(I)/TH+RD/TR+C(I)/TC)
C CALCULATE GAS INVENTORY FOR NEXT INCREMENT DUE TO LEAKAGE
   L(I+1) = L(I)*(1.0-LX*(P(I)-PG))
C ACCUMULATE VALUES, MEAN PRESSURE AND MEAN GAS INVENTORY.
   IF(I.EQ.1) GOTO 28
   PM=PM+P(I)
   LY=LY+L(I)
C END OF DO LOOP 28 (TO CALCULATE PRESSURES FOR ONE ENGINE CYCLE)
28 CONTINUE
C INDEXES CYCLE COUNTER, CALCULATES MEAN PRESSURE, READJUSTS GAS
C INVENTORY TAKING INTO ACCOUNT GAS LEAKAGE.
   A=A+1
   PM=PM/12.
   IF(A.LT.3) GOTO 30
   L(1)=L(13)
   GOTO 31
30  L(1)=L(13)*PG/PM
C CONVERGENCE CRITERIA: PRESSURE FROM BEGINNING TO THE END OF CYCLE
C MUST NOT CHANGE BY MORE THAN ONE HUNDRETH OF A PERCENT AND THE MEAN
C PRESSURE MUST BE WITHIN ONE PERCENT OF THE DESIRED GAS PRESSURE.
C USUALLY ONE OR TWO CYCLES ARE REQUIRED TO MEET THIS CRITERIA.
31  X=ABS(P(1)-P(13))
    Z=ABS(PM-PG)
    IF(X.GT.0.0001.OR.Z.GT.0.01) GOTO 29
C INITIALIZING
   W1=0
   PX=0
   PN=10000.
   MR=LY*ND/360
C START OF DO LOOP 32 (FINDS THE MAXIMUM AND MINIMUM PRESSURE).
   DO 32 I=1,13
      IF(P(I).GT.PX) PX=P(I)
      IF(P(I).LT.PN) PN=P(I)
32 CONTINUE
C START OF DO LOOP 33 (FINDS THE WORK PER CYCLE BY INTEGRATING THE
C PRESSURE VOLUME LOOP).
   DO 33 I=1,12
      WI=WI*(P(I)/P(I+1))*(V(I+1)-V(I))/2.
 33 CONTINUE
C BASIC POWER FOR THE WHOLE ENGINE IS CALCULATED FROM THE INTEGRATED
C POWER USING THE CORRECTION FACTOR XX WHICH COMPENSATES FOR THE
C TRUNCATION ERROR OF USING ONLY A SMALL NUMBER OF POINTS TO INTEGRATE.
BP=NU*XX*W1*N
C INITIALIZING
   HX=0
   CY=0
   HN=1
   CN=1
C CALCULATES AN ARRAY GIVING THE FRACTION OF THE TOTAL GAS INVENTORY IN
C THE HOT SPACE AND IN THE COLD SPACE FOR EACH POINT DURING THE CYCLE.
   DO 34 I=1,13
      FH(I)=P(I)*H(I)/(MR*TH)
      IF(FH(I).GT.HX) HX=FH(I)
      IF(FH(I).LT.HN) HN=FH(I)
      FC(I)=P(I)*C(I)/(MR*TC)
      IF(FC(I).GT.CY) CY=FC(I)
      IF(FC(I).LT.CN) CN=FC(I)
 34 CONTINUE
C IF FH(I) AND FC(I) ARE GRAPHED AS A FUNCTION OF THE ANGLE, IT IS SEEN
C THAT A GOOD APPROXIMATION OF THE GRAPH IS TO HAVE TWO PERIODS PER
C CYCLE OF CONSTANT MASS FLOW INTERSPERSED WITH PERIODS OF NO FLOW AT
C ALL. F1 TO F4 ARE THE FRACTIONS OF THE TOTAL CYCLE TIME WHEN
C DIFFERENT FLOWS ARE ASSUMED TO OCCUR (SEE NOMENCLATURE).
C WHEN 'FH1' AND 'FC1' ARE CALCULATED, THE AVERAGE CYCLE TIME, WHEN FLOW
C IS ASSUMED TO OCCUR EITHER INTO OR OUT OF THE HOT SPACE AND EITHER
C INTO OR OUT OF THE COLD SPACE, IS CALCULATED.
   F1=(HX-HN)/(6*(FH(1)-FH(3)))/2
   F2=(HX-HN)/(6*(FH(10)-FH(8)))/2
   F3=(CY-CN)/(6*(FC(8)-FC(10)))/2
   F4=(CY-CN)/(6*(FC(3)-FC(1)))/2
   FH1=(F1+F2)/2
   FC1=(F3+F4)/2
C EFFECTIVE MASS FLOW INTO OR OUT OF THE HOT SPACE IS CALCULATED.
\[ M = \frac{MR}{R} \]
\[ WH = \frac{(HX-HN)M*MW*NU}{FH} \]
C EFFECTIVE MASS FLOW INTO OR OUT OF THE COLD SPACE IS CALCULATED.
\[ WC = \frac{(CY-CN)M*MW*NU}{FC1} \]
C FRACTION OF THE TIME THE FLOW IS ASSUMED TO PASS THROUGH THE
C REGENERATOR AND THE FLOW RATE OF THE REGENERATOR IS CALCULATED AS THE
C AVERAGE BETWEEN THE HOT AND COLD FLOWS.
\[ FR = \frac{(FH1+FC1)}{2} \]
\[ WR = \frac{(WH+WC)}{2} \]
C REGENERATOR GAS DENSITY.
\[ RN = 0.1202*MW*PG/TR \]
C CALCULATES REGENERATOR WINDAGE LOSS.
\[ MU = M1+M2*(TR-293.)+M3*PG \]
\[ GR = WR/(P4*DR**2*NR) \]
\[ RR = DW*GR/MU \]
\[ CW = 2.7312*(1+10.397/RR) \]
\[ DP = CW*GR**2*LR/(2E7*DW*RM) \]
\[ A = N/RM \]
\[ RW = DP*WR*2.*FR*A \]
C CALCULATES HEATER WINDAGE LOSS. IN THIS CALCULATION THE VISCOSITY FOR
C THE INPUT TEMPERATURE AND SUBROUTINE 'REST' RETURNS THE FRICTION
C FACTOR FOR THE INPUT REYNOLDS NUMBER. THE CALCULATION TAKES INTO
C ACCOUNT FRICTIONAL LOSSES, AS WELL AS 4.4 VELOCITY HEADS FOR AN
C ENTRANCE AND AN EXIT LOSS, ONE 180 DEGREE BEND, AND TWO 90 DEGREE
C BENDS.
\[ MU = M1+M2*(TM-293.)+M3*PG \]
\[ RM = 0.1202*MW*PG/TM \]
\[ A = N/RM \]
\[ GH = WH/(P4*IH**2*NH) \]
\[ RE = IH*GH/MU \]
\[ RT = RE \]
\[ IF(RE.LT.2000.) GOTO 35 \]
\[ X = ALOG(RE) \]
\[ X = -3.09-.2*X \]
\[ CW = EXP(X) \]
GOTO 36
35  \[ CW = 16./RE \]
36  \[ AF = P4*IH**2*NH \]
$VH = WH / (RM \times AF)$

$DP = 2 \times CW \times GC \times GH \times 2 \times LH / (1E7 \times IH \times RM) + VH \times 2 \times 4.4 \times RM / 2E7$

$HW = DP \times WH \times 2 \times FH1 \times A$

This calculates the windage loss through the gas cooler and the connecting tube. The same comments for the gas heating windage loss apply here as well. The velocity heads charge to the gas cooler is 1.5 for a simple entrance and exit loss. In the connecting head line, three velocity heads are changed to account for entrance and exit loss plus two 90 degree bends.

$\mu = M1 + M2 \times (TX-293.) + M3 \times PG$

$RM = .1202 \times Hw \times PG / TX$

$A = N / RM$

$GC = WC / (P4 \times IC \times 2 \times NC)$

$RE = IC \times GC / HU$

$RZ = RE$

IF (RE.LT.2000.) GOTO 37

$X = ALOG(RE)$

$X = -3.09 - 2.2 \times X$

$CW = EXP(X)$

GOTO 38

37

$CW = 16 / RE$

38

$AF = P4 \times IC \times 2 \times NC$

$VC = WC / (RM \times AF)$

$DP = 2 \times CW \times GC \times 2 \times LC / (1E7 \times IC \times RM) + VC \times 2 \times 1.5 \times RM / 2E7$

$GD = WC / (P4 \times ID \times 2 \times NE)$

$RE = ID \times GD / HU$

IF (RE.LT.2000.) GOTO 39

$X = ALOG(RE)$

$X = -3.09 - 2.2 \times X$

$CW = EXP(X)$

GOTO 40

39

$CW = 16 / RE$

40

$AF = P4 \times ID \times 2 \times NE$

$VC = WC / (RM \times AF)$

$DP = DP + 2 \times CW \times GD \times 2 \times LE / (1E7 \times ID \times RM) + VC \times 2 \times 3.0 \times RM / 2E7$

$CF = DP \times WC \times 2 \times FC1 \times A$

This calculates indicated power.

$IP = BP - HW - RW - CF$
C CALCULATES MECHANICAL FRICTION LOSS.
   MF=(1.-ME/100.)*IP
C CALCULATES NET POWER.
   NP=IP-MF
C CALCULATES BASIC HEAT INPUT.
   BH=BP/(1.-TC/TH)
C CALCULATES REHEAT LOSS FOR MET NET .05-.20 WHICH IS USED IN THE 4L23
C MACHINE. THIS SECTION IS SPECIFIC FOR THIS TYPE OF REGENERATOR
C MATERIAL.
   IF(RR.LT.42.) GOTO 41
   IF(RR.LT.140.) GOTO 42
   X=EXP(1.78-.5044*ALOG(RR))
   GOTO 43
41  X=EXP(-1.826-.0583*ALOG(RR))
   GOTO 43
42  X=EXP(.5078-.2435*ALOG(RR))
43  NT=X*LR/DW
   X=WR*CP*(TH-TW)
   Y=RDC*(PX-PN)*NU*MW/(R*FR)
   K3=FR*(X-Y)
   RH=K3/(NT+2)*N*2
C CALCULATES TEMPERATURE SWING LOSS.
   MX=NR*P4*DR**2*LR*FF*7.5
   TS=K3/(NU*MX**1.05)
   SL=K3*TS*N/(2*(TH-TX))
C CALCULATES PUMPING OR APPENDIX LOSS.
   X=(PI*DC/KG)**.6
   Y=((PX-PN)*HW*NU*CP*2/((TH+TX)*R))**1.6
   Z=G**2.6
   QP=N*X**2*LB*(TH-TX)*Y/Z/1.5
C CALCULATES SHUTTLE HEAT LOSS.
   QS=2*P4*RC*RC*KG*(TH-TC)*DC/(G*LB)**N
C CALCULATES STATIC HEAT LOSS. THIS CAN BE EITHER SPECIFIED OR
C CALCULATED FROM THE BASIC DIMENSIONS.
   IF(ZZ.EQ.1) ZH=(TH-TC)*((KM*((DR**2*P4*FF+PI*DR*SR)/LR+
   1PI*DC*(SC+SE)/LB)+KG*(DR**2*P4*(1-FF)/LR+DC**2*P4/LB))
C SUMS ALL LOSSES TO CALCULATE NET HEAT DEMAND.
   QN=BH+ZH+SL+RH-HW-RW/2+QS+QP
C CALCULATES COOLER HEAT LOAD.
OC=ON-NP
C TEMPERATURE RISE IN COOLING WATER.
DT=OC/(FW*4.185)
C EFFECTIVE COLD METAL TEMPERATURE.
TX=TW+DT/2
C CALCULATES HEAT TRANSFER COEFFICIENT IN THE COLD HEAT EXCHANGER.
C
RE=RZ
J=1
C GOTO SUBROUTINE REST
GOTO 100
44 HC=ST*CP*GC/PR
C
C TWO DIFFERENT METHODS OF ARRIVING AT THE PROPER EFFECTIVE HOT SPACE
C AND COLD SPACE TEMPERATURE ARE INTERSPERSED. THE FASTEST WAY,
C WHICH IS USUALLY TRIED FIRST, INVOLVES CALCULATING WHAT THE
C TEMPERATURE DIFFERENCE HAS TO BE BETWEEN THE METAL TEMPERATURE AND
C THE EFFECTIVE GAS TEMPERATURE CONSIDERING THE HEAT TRANSFER
C CAPABILITY OF THE HEAT EXCHANGER AND THE CORRECTION FACTOR.
C HOWEVER, IF THE HEAT EXCHANGER IS TOO SMALL, THE FIRST ITERATION
C METHOD GOES UNSTABLE AND A SECOND, MORE CAUTIOUS, METHOD MUST BE
C EMPLOYED. THE 'ZA' IS THE FLAG WHICH SHOWS THAT THE SECOND
C METHOD IS CALLED IN.
IF(ZA.EQ.1) GOTO 46
C
C 'X' IS USED AS A TEMPORARY VARIABLE FOR THE PREVIOUS COLD
C TEMPERATURE. THE COLD TEMPERATURE IS CALCULATED, ASSUMING THERE IS
C NO ERROR BETWEEN THE HEAT THAT CAN BE TRANSFERRED AND THE HEAT THAT
C SHOULD BE TRANSFERRED. CONTER 'ZB' IS INDEXED. A TEST IS NOW MADE
C OF THE 'TC' VALUE JUST CALCULATED. IF THE EFFECTIVE COLD GAS
C TEMPERATURE IS GREATER THAN THE EFFECTIVE HOT GAS TEMPERATURE OR
C LESS THAN THE COOLING WATER TEMPERATURE THIS ITERATION METHOD HAS
C GONE UNSTABLE AND THE SECOND, MORE CAUTIOUS, METHOD IS BROUGHT IN.
C ALSO IF THE FIRST ITERATION METHOD HAS NOT COME TO AN ANSWER WITHIN
C 10 ITERATIONS, ('ZB' GREATER THAN 10), THE SECOND ITERATION METHOD
C IS BROUGHT IN. THE INITIAL CHANGE IN THE HOT GAS TEMPERATURE, 'DV',
C AND IN THE COLD GAS TEMPERATURE, 'DU', ARE BOTH SET AT 64 DEGREES.
C THE FLAG 'ZA' IS SET AT 1 AND 'TC' AND 'TH' ARE SET AT THE INITIAL
C VALUES. CONTROL PASS TO 46 WHERE THE SECOND APPROACH BEGINS. IF
C THE VALUE OF 'TC' DOES NOT INDICATE THE SECOND APPROACH IS NEEDED
C CONTROL PASSES TO 48 TO START CALCULATION OF THE EFFECTIVE
C TEMPERATURE IN THE HOT SPACE.

X=TC
YY=HC*FC1*AC*N*BF
TC=QC/YY+TX
E2=QC-YY*(TC-TX)
ZB=ZB+1
IF(TC.GT.TH.OR.TC.LT.TX.OR.ZB.GT.10.) GOTO 47
GOTO 48

C ON THE FIRST TIME THROUGH 'TC' = 'TW' AND THE ERROR IN THE COLD SPACE,
C E2, IS MADE EQUAL TO THE REQUIRED HEAT TRANSFER THROUGH THE GAS
C COOLERS, 'QC'. THEN THE NEXT ESTIMATE FOR 'TC' IS MADE BY ADDING
C 'DU', 64 DEGREES, TO 'TX', THE AVERAGE TEMPERATURE OF THE GAS
C COOLER METAL. THE PROGRAM THEN GOES TO 48, SKIPPING OVER THE REST OF
C THE ADJUSTMENT PROGRAM FOR THE COLD SPACE.

46 IF(TC.EQ.TW) GOTO 49
C IF 'TC' IS NOT EQUAL TO 'TW', AS IT WILL BE FOR ANYTHING EXCEPT
C FOR THE FIRST TIME THROUGH, THE PREVIOUS ERROR IS SAVED AS 'E1'.
C THEN 'E2' IS CALCULATED AS THE DIFFERENCE BETWEEN THE HEAT THAT
C SHOULD BE TRANSFERRED AND THE HEAT THAT CAN BE TRANSFERRED BY THE
C CAPABILITIES OF THE HEAT EXCHANGER.
E1=E2
E2=QC-FC1*AC*N*(TC-TX)*BF
C IF THIS ERROR IS POSITIVE, THEN THE CORRECTION NUMBER, 'DU', IS
C ADDED TO THE COLD TEMPERATURE, 'TC', AND THE PROGRAM GOES ON TO THE
C HOT SPACE ANALYSIS.
C IF(E2.GT.0) GOTO 50
C IF THIS ERROR IS NEGATIVE AND THE PREVIOUS ERROR WAS POSITIVE,
C THEN THE DEGREE INCREMENT, 'DU', IS JUST DIVIDED BY 4, FOR FUTURE
C CORRECTIONS.
C IF(E2.LT.0.AND.E1.GT.0) DU=DU/4
C THE DEGREE INCREMENT IS SUBTRACTED FROM 'TC'. IF 'TC' BECOMES
C GREATER THAN 'TH', THE HOT METAL TEMPERATURE, OBVIOUSLY THERE IS
C INSUFFICIENT COOLER HEAT TRANSFER AREA AND THE PROGRAM STOPS FOR
C THIS CASE. THIS CAN OCCUR FOR SMALL COOLER AREAS AND SPECIFIED HEAT
C LEAKS.
   TC=TC-DU
   IF(TC.GT.TM) GOTO 51
C CALCULATES HEAT TRANSFER COEFFICIENT FOR GAS HEATER. FLAG 'ZA'
C INDICATES WHETHER THE FAST METHOD OF CONVERGENCE AT 59 OR THE SLOW
C METHOD AT 52 SHOULD BE USED.
48   RE=RT
   J=2
C GOTO SUBROUTINE REST
   GOTO 100
  59   HH=ST*CP*GH/PR
       IF(ZA.EQ.1) GOTO 52
C THIS IS ANALOGOUS TO THE COMMENT MADE AFTER 44 ON THE COLD SPACE,
C EXCEPT THIS IS FOR THE HOT SPACE.
       Y=TH
       YY=HH*FH1*AH*N*BF
       TH=TM-QN/YY
       E4=QN-YY*(TM-TH)
       IF(TH.GT.TM.OR.TH.LT.TC) GOTO 47
       GOTO 53
C THIS IS ANALOGOUS TO 46 TO 48, EXCEPT THIS IS FOR THE HOT SPACE.
  52   IF(TH.EQ.TM) GOTO 54
      E3=E4
      E4=QN-HH*FH1*AH*N*(TM-TH)*BF
      IF(E4.GT.0) GOTO 55
      IF(E4.LT.0.AND.E3.GT.0) DU=DV/4
      TH=TH+DV
      IF(TH.LT.TW) GOTO 56
      GOTO 55
C CONVERGENCE CRITERIA FOR THE FIRST ITERATION METHOD. THE ITERATION
C IS COMPLETE WHEN CHANGE IN THE EFFECTIVE HOT SPACE AND COLD SPACE
C TEMPERATURE IS LESS THAN ONE DEGREE KELVIN PER ITERATION.
  53   XI=ABS(TH-Y)
      X2=ABS(TC-X)
      IF(X1.GT.1.0X2.GT.1.0) GOTO 200
      GOTO 57
C CONVERGENCE CRITERIA FOR THE SLOWER, SECOND METHOD OF ITERATION.
C CONVERGENCE IS COMPLETE WHEN THE AIR IN THE HOT SPACE AND THE AIR IN
C THE COLD SPACE ARE BOTH LESS THAN 1% OF THE HEAT TRANSFERRED THROUGH
C THE HEAT EXCHANGERS.
58  X1=ABS(E4)
    X2=ABS(E2)
    X3=QN/100
    X4=QC/100
    IF(X1.GT.X3.OR.X2.GT.X4) GOTO 200
C COMPLETES PREPARATION FOR OUTPUT
57  A=-HW-RW/2
    B=100.*IP/QN
    C1=QN*(100./FE-1.)
    D=FE*Np/QN
    E=100.*QN/FE
C REINITIALIZING
I=I+1
ZA=0
ZB=0
GOTO 60
C LOCATION OF CONTROL FOR THE SECOND ITERATION METHOD.
47  DU=64
    ZA=1
    TC=TW
    TH=TM
    GOTO 46
C LOCATION OF CONTROL IF 'TC' EQUALS 'TW'.
49  E2=QC
    TC=TX+DU
    GOTO 48
C LOCATION OF CONTROL IF 'E2' IS GREATER THAN 0.
50  TC=TC+DU
    GOTO 48
C BECAUSE OF INSUFFICIENT COOLER AREA THE PROGRAM IS TERMINATED FOR
C THIS CASE.
51  WRITE(LP,1)
    GOTO 300
C LOCATION OF CONTROL IF 'TH' EQUALS 'TM'.
54  E4=QN
    TH=TM-DV
    GOTO 58
C LOCATION OF CONTROL IF 'TH' IS NOT LESS THAN 'TW'.
55    TH=TH-DV
GOTO 58
C BECAUSE OF INSUFFICIENT HEATER AREA THE PROGRAM IS TERMINATED FOR
C THIS CASE.
56    WRITE(LP,2)
GOTO 300
C THIS IS WHERE THE PRINTING OF THE OUTPUT STARTS. TO COMPRESS OUTPUT
C THE OPERATING CONDITIONS AND ENGINE DIMENSIONS ARE IDENTIFIED ONLY BY
C THEIR FORTRAN SYMBOL.
C
C PRINTS PROGRAM HEADING
60    WRITE(LP,10)
C PRINTS CURRENT OPERATING CONDITIONS
   WRITE(LP,3) SP,PS,ND,TF,L1,TY,FX,OG
C PRINTS CURRENT DIMENSIONS
   WRITE(LP,4) DC,DR,IC,OC,DW,DD,IH,DH,GB,LR,CR,LC,LD,LH
   WRITE(LP,5) LI,NC,NR,NH,FF,AL,CX,ME,FE,EC,SC,SE,SR,ZZ,ZH,KM,ID,
   1LE,NE,BF
C PRINTS POWER OUTPUTS AND HEAT INPUTS
   WRITE(LP,6) BP,BH,HW,RH,RW,QS,CF,GP,IP,SL,MF,ZH,NP,A
   WRITE(LP,7) QN,B,C1,D,E
   WRITE(LP,8) TM,TW,TH,TC
C PRINTS WORK DIAGRAM FROM DATA
   WRITE(LP,9)
   DO 61 I=1,13
   F=ND*I-30,
   G=L(I)/R
   WRITE(LP,11) F,H(I),C(I),V(I),P(I),G
61    CONTINUE
GOTO 300
C END OF MAIN PROGRAM
45    CALL EXIT
C
C SUBROUTIN REST
C CALCULATES STANTON NUMBER FROM REYNOLDS NUMBER
100   IF(RE.GE.10000.) ST=EXP(-3.57024-.22946*ALOG(RE))
     IF(RE.LT.10000.) ST=.0034
     IF(RE.LT.7000.) ST=EXP(-13.3071+.861016*ALOG(RE))
IF(RE.LT.4000.) ST=.0021
IF(RE.LT.3000.) ST=EXP(.337046-.812212*ALOG(RE))
IF(J.EQ.1) GOTO 59
GOTO 59

C OUTPUT FORMAT:
1 FORMAT(IO('t'),'INSUFFICIENT COOLER AREA',IO('_'))
2 FORMAT(IO('t'),'INSUFFICIENT HEATER AREA',IO('_'))
3 FORMAT('CURRENT OPERATING CONDITIONS ARE:'/'SP=',F10.2,T17,'PS=',
1T10.2,T33,'ND=',F10.2,T49,'TF=',F10.2,'/','L1=',F10.4,T17,'TY=',
2T10.4,T33,'FX='F10.4,T49,'DG=',I2//)
4 FORMAT('CURRENT DIMENSIONS ARE:'/'DC=',F10.4,T17,'DR=',F10.4,T33,
1'I=',F10.4,T17,'DW=',F10.4,T33,'G='F11.5,T17,'LB=',F10.4,T33,'LR=',
3F10.4,T49,'CR=',F10.4,'RC=',F10.4,'LC=',F10.4,T33,'LD=',F10.4,
4T49,'LH='F10.4)
5 FORMAT('LI=',F10.4,T17,'NC=',I5,T33,'NR=',I3,T49,'N=',I3,'/','NH=',I4,
1T17,'FF='F10.4,T33,'AL='F10.2,T49,'CX='F10.4,'/','ME='F10.4,T17,
2'FE='F10.4,T33,'EC='F10.5,T49,'SC='F10.5,'SE='F10.5,T17,'SR=',
3F10.5,T33,'ZZ='I3,T49,'ZH='F10.2,'/','KM='F10.4,T17,'ID='F10.4,
4T33,'LE='F10.4,T49,'NE='I3,'/','BF='F10.4//)
6 FORMAT('POWER, WATTS',T34,'HEAT REQUIREMENT, WATTS'/,2X,'BASIC',
1T20,'/','BASIC',T55,F13.4,'/','HEATER F.L.',T20,F13.4,'T36',
2'REHEAT',T55,F13.4,'/','REGEN F.L.',T20,F13.4,'T36',
3'SHUTTLE',T55,F13.4,'/','COOLER F.L.',T20,F13.4,'T36',
4'PUMPING',T55,F13.4,'/','MECH FRICT.',T20,F13.4,
5'T55',T36,'CONDUCTION',T55,F13.4,'/','BRAKE',T20,F13.4,'T36',
6'FLOW FRICT.',T55,F13.4,'/','EDIT',T55,F13.4)
7 FORMAT(34('-'),T34,'HEAT TO ENGINE',T55,F13.4,'/','INDICATED EFF.Z=',
1T10.4,'/','FURNACE LOSS',T55,F13.4,'/','OVERALL EFF.Z=',
2F10.4,'/','FUEL INPUT',T55,F13.4)
8 FORMAT(54('-')/,'HOT METAL TEMP. K=',F10.4,T34,'COLD METAL TEMP. K=',F10.4,T34,'/','INLET TEMP. K=',F10.4,'/','EFFEC HOT SP.TEMP.K=',F10.4,T34,'/','EFFEC COLD SP.TEMP.K=',F10.4/)
9 FORMAT('FINAL WORK DIAGRAM:'//,'ANGLE',T11,'/','HOT VOL.',T23,'COLD VOL.
1','/','T36','/','TOT. VOL.',T50,'PRESSURE',T63,'GAS INV.')
10 FORMAT(///,'ISOTHERMAL SECOND ORDER CALCULATION--'/ PROG. ISO'
1'/ 10 OCT 1977'/ 'WRITTEN BY WILLIAM R. MARTINI//)
11 FORMAT(1X,I4,T8,F11.4,T21,F11.4,T34,F11.4,T47,F11.4,T60,F11.4)
END
C.4 Sample of Input File for FORTRAN Program

```

10.16, 12.9, 12.02, 115, 167, 312, 3.14159
785398, 0.0432, 25.0, 90.0, 80.0, 1.0
9680, 41.8, 25.58, 472, 640, 36.4, 4.06
0.0174533, 0.0406, 6.40, 1400, 2.0, 0.635, 1.016
0.0510, 2.500, 3.500, 6, 2.13, 65, 2.325
4, 90., 1200, 135., 2000, 1, 435, 1532
0.76, 71.6, 4, 0.0689, 2.54, 60.
8.314, 1.341E-3, 0.0406, 0., 179.

```

C.5 Sample of Output File Produced by FORTRAN Program

```

ISOTHERMAL SECOND ORDER CALCULATION--
PROG. 150
10 DEC 1979
WRITTEN BY WILLIAM R. FORTNIE

CURRENT OPERATING CONDITIONS ARE:
SP = 7000.00 TS = 1400.00 ND = 30.00 TF = 1200.00
LL = 0.0000 TY = 135.0000 FX = 25.0000 DI = 1

CURRENT DIMENSIONS ARE:
SB = 10.1600 DP = 3.5000 TC = 0.1150 OC = 0.1670
MR = 0.0012 BP = 4.0600 TH = 0.4726 OH = 0.6400
BS = 0.04000 LR = 6.4000 LR = 2.5000 CR = 13.6500
RC = 2.3256 IR = 12.9000 ID = 12.0200 LH = 41.6000
LH = 25.5800 MC = 312 NR = 6 N = 4
KH = 36 FF = 0.2000 AL = 90.00 CX = 254.2804
KH = 90.0000 FL = 80.0000 UC = 0.04060 SC = 0.06350
SF = 0.010160 SR = 0.05100 ZZ = 0 7H = 9680.00
KM = 0.2000 ID = 0.2600 LE = 71.0000 NE = 6
KE = 0.4000

```
### C.5 (Continued)

**POWER, WATTS**

<table>
<thead>
<tr>
<th></th>
<th>BASIC</th>
<th>HEATER F.I.</th>
<th>REGENERATOR F.I.</th>
<th>COOLER F.I.</th>
<th>NET</th>
<th>MECHANICAL FRIC.</th>
<th>BRAKE</th>
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**HEAT REQUIREMENT, WATTS**

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<tr>
<th></th>
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<th>REHEAT</th>
<th>SHUTTLE</th>
<th>PUMPING</th>
<th>TEMP. SWING</th>
<th>CONDUCTION</th>
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**INDICATED EFF. %**

- 41.5837

**OVERALL EFF. %**

- 29.9403

**FUEL INPUT**

- 243382.7188

**FINAL WORK DIAGRAM: ANGLE**

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>HOT VOL.</th>
<th>COLD VOL.</th>
<th>TOT. VOL.</th>
<th>PRESSURE</th>
<th>GAS INV.</th>
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**HOT METAL TEMPERATURE, K**

- 922.2222

**COOLING WATER INLET TEMPERATURE, K**

- 330.5555

**EFF EFFICIENT HOT SP. TEMP. K**

- 824.2458

**EFF EFFICIENT COLD SP. TEMP. K**

- 370.1363
C.6 Comparison Program Results

Table C-1 gives the final comparison between the isothermal second order analysis with a corrections factor of 0.4 and the General Motors validated predictions of the performance of their 4L23 engine. Figures 3-1 to 3-3 show the graphs from R. Diepenhorst "Calculated 4L23 Stirling Engine Performance", 19 Jan. 1970, Section 2.115 of GMR-2690 (reference 78 bh). These graphs were read as accurately as possible with dividing calipers to obtain the power outputs and efficiencies quoted in column 5 and 8 of Table C-1.
Table C-1 Comparison of Isothermal Second Order Analysis of the 4L23 Engine with the experimentally validated analysis by General Motors

**Correction Factor is 15.4**

<table>
<thead>
<tr>
<th>TEMP.</th>
<th>ENGINE</th>
<th>AVERAGE</th>
<th>CALC.</th>
<th>GN'S</th>
<th>CALC.</th>
<th>CALC.</th>
<th>GN'S</th>
<th>CALC.</th>
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<td>BHP</td>
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<tr>
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<tr>
<td>TUBES</td>
<td>PRESSURE</td>
<td>POWER</td>
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<td>GN'S</td>
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<td>%</td>
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<table>
<thead>
<tr>
<th>TEMPERATURE</th>
<th>ENGINE</th>
<th>AVERAGE</th>
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**AVERAGE RATIO**

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APPENDIX D
ADIABATIC SECOND ORDER
DESIGN PROGRAM (RIOS)

D.1 Description

D.1.1 Introduction

As was stated in the first edition of the design manual the Rios method for Stirling engine design is highly regarded by engineers at the Philips Company as being almost equivalent to their proprietary codes. Dr. Glendon Benson has stated that it is the basis for his proprietary code.

In his 1969 thesis, P.A. Rios published a computer code for a Stirling refrigerator. This code was somewhat verified through experimental data obtained from his two piston-two cylinder Stirling refrigerator.

Prof. J.L. Smith, Jr., of M.I.T. stated that this program was found to be reliable and useful by North American Philips engineers for designing cooling engines. At the time the Philips engineers used this program they had no program of their own but could get performance predictions for specific designs from N.V. Philips, Eindhoven, Netherlands. Other comments made at a panel discussion on Stirling engines at the 1977 Intersociety Energy Conversion Engineering Conference in Washington D.C. indicated that the Rios program is as good as the proprietary Philips program.

In order to verify these claims we obtained a card deck from Prof. Smith containing a listing of the Rios program as found in his thesis. Then we added to the Rios program equations to calculate the dimensionless numbers required by the Rios program from engine dimensions. We also added equations to the end of the program to calculate the losses for a real engine. These equations are given in the Rios thesis but are not part of the Rios program. The program was installed on the Amdahl 470/6 - II computer at Washington State University. It is accessed from the Joint Center for Graduate Study using a computer terminal connected to the WYLBER system. The program executes in 0.91 seconds. Compiling and linking requires 2.76 seconds.

Although the original Rios program is for a refrigerator, the program given in Section D.3 has been modified to apply to an engine. The author decided to apply it to the General Motors 4L23 engine, a four cylinder, double acting crank operated engine with tubular heat exchangers since this engine is most similar to present day automobile engines.

This appendix contains a complete nomenclature list which Rios did not have. Next is a listing of the FORTRAN program with many comments that make the program understandable. The full numerical results of 18 test cases summarized in Table D-1 are on file at Martini Engineering. The comparison on Table D-1 shows that the pumping or appendix loss predicted by the Rios program is an order
### Table D-1

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<td>205.5</td>
<td>1.25</td>
<td>36.19</td>
<td>30.65</td>
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</table>
of magnitude larger than the same loss predicted by the isothermal second order program. The equations used are entirely different for the two cases. The equation used in the isothermal second order analysis was checked with the original source and was found to be correct. Rios derives his appendix loss equation in his thesis. Then in other parts of the thesis the equation is quoted differently. Although the author does not understand the reasons for many assumptions Rios makes, it is clear that the equation must be substantially modified for a heat engine. Rios ignores the temperature swing loss which for the 4L23 engine is quite large. The program presented in Appendix D should be modified to use the correct appendix loss equation and include the temperature swing loss equation. However since these two errors compensate and since they are relatively small corrections it was not considered worthwhile repeating the 18 production cases.

D.1.2 The Rios Calculation Method Rios starts by calculating a perfect engine and then makes corrections. His perfect engine obeys the following assumptions. (69 am, pp. 24-26)

1. At each instant in time the pressure throughout the engine is uniform.
2. Hot and cold gas spaces are adiabatic - no heat transfer to or from either the expansion or the compression space.
3. Heat transfer in the heater, cooler, and regenerator is perfect - zero temperature difference between gas and neighboring wall.
4. The temperature at any point in a heat-exchange component is constant with time.
5. Uniform temperature exists at any cross section perpendicular to the direction of flow.
6. The gas in the cylinders is perfectly mixed.
7. The Ideal Gas Laws apply.

In broad outline the Rios calculation method proceeds as follows:

1. Calculate dimensionless quantities from the engine dimensions and operating conditions.
2. Calculate engine volumes for the angle increment selected.
3. Calculate engine pressure to go with the volumes and given operating conditions. Start with an arbitrary initial pressure and traverse the cycle twice. The second cycle will be correct.
4. Calculate power losses:
   a. heater windage
   b. regenerator windage
   c. cooler windage
5. Calculate heat losses:
   a. reheat
   b. shuttle
   c. pumping
   d. heater ineffectiveness
   e. cooler ineffectiveness

6. If 5d or 5e are appreciable, modify the heat source and heat sink temperature then re-do parts 1, 3, 4, and 5. Three iterations has been found adequate for convergence.

D.2 Nomenclature for Appendix D

Rios did not give a nomenclature so the one given below has been tabulated to the best of the authors knowledge and understanding.

\[ \begin{align*}
\text{AFC} &= \text{Cooler free flow area, cm}^2 \\
\text{AFH} &= \text{Heater free flow area, cm}^2 \\
\text{AFR} &= \text{Regenerator free flow area, cm}^2 \\
\text{ALF} &= 4.7123889 \ (270 \text{ degrees}) \\
\text{ARG} &= \sin \ (\text{PV angle}) \\
\text{BDR} &= \text{Regenerator diameter, cm} \\
\text{BEC} &= \text{Piston end clearance, cm} \\
\text{BPD} &= \text{Piston diameter, cm} \\
\text{BPL} &= \text{Hot cap length, cm} \\
\text{BRC} &= \text{Piston gap, cm} \\
\text{BRL} &= \text{Regenerator length, cm} \\
\text{BRO} &= \text{Regenerator density factor} \\
\text{BST} &= \text{Piston stroke, cm} \\
\text{BTC} &= \text{Effective cold temperature, K} \\
\text{BTCI} &= \text{Cold metal temperature, K} \\
\text{BTR} &= \text{Regenerator temperature, K} \\
\text{BTW} &= \text{Hot effective gas temperature, K} \\
\text{BTW1} &= \text{Hot metal temperature, K} \\
\text{BWD} &= \text{Effective regenerator wire diameter, cm} \\
\text{C(\theta)} &= \text{Cold space as fraction of the stroke amplitude at mid-increment} \\
\text{C(\theta)} &= \text{varies from 0 to 2 and back.} \\
\text{CALF(\theta)} &= \sin \text{ change per radian increment} \\
\text{CALFP} &= \text{CALF(\theta)} \\
\text{CF1} &= \cos \text{ of phase angle}
\end{align*} \]
CI() = Same as C() for beginning of increment
CMMAX = Largest cold dimensionless mass
CMU = Cold hydrogen viscosity
CNTU = Number of heat transfer units in cold space
COFI() = Cos values for cold space
CON = Conduction loss, watts
CPI = Hydrogen heat capacity
CRC = \sqrt{ZZC^2 - CALF()}^2
CRW = CRC in hot space
CTD = Cooler tube inside diameter, cm
CTLL = Total cooler tube length, cm
CTLS = Cooled cool tube length, cm
CVI = Hydrogen heat capacity
DALF = 2r/NDIV
DC() = Angle derivative of C()
DCI() = Angle derivative of CI()
DDD = Cooler duct diameter, cm
DLL = Cooler duct length, cm
DM = Sum of changes in mass (DMRE)
DMC = Cold dimensionless mass change XDMC()
DMRE = Sum of changes in mass (DM)
DMW = Hot dimensionless mass change XDMW()
DMX = Dimensionless change in mass relating to X, the fraction from the cold end
DP = Change in pressure
DPR = DP array
DTC = Cooler metal temperature - effective temperature
DTH = Delta TH
DV = Dead volume, cm³
DVC = DC()
DVCI = DCI()
DVW = DW()
DVWI = DWI()
DW() = Angle derivative of W()
DWI() = Angle derivative of WI()
DX = 1/XNDS
EX1 = 1 - XNHT
EX2 = 2 - XNHT
FC = Cold friction factor
FFF = Friction flow credit, watts
FH = Hot friction factor
FR() = Regenerator friction factors (3 pts.)
FI = Phase angle, rad.
FII = Phase angle in deg.
FIPV = PV angle (output) arcsin (ARG)
FR() = Regenerator friction factor
G1 = Y value subplot
G2 = Y value subplot
GDMS() = Calculated mass flow values
GGV = Dead volume at side of hot cap, cm³
GINT() = Flow loss variable
GI2() = Pressure drop value
GI3() = Pressure drop value
GLH = Heater pressure drop integral
GLR = Regenerator pressure drop integral
GLS = Cooler pressure drop integral
H(1) = Fraction of total reduced dead volume from cold end to midway in cooler
H(2) = Fraction of total reduced dead volume from the cold end through the cooler
H(3) = Fraction of total reduced dead volume from the cold end through half the regenerator
H(4) = Fraction of total reduced dead volume through the regenerator
H(5) = Fraction of total reduced dead volume through the middle of the gas heater (1-H(5) includes the rest of the heater and clearance on the end and sides of the hot cap)
HAC = Cold active volume amplitude, cm³
HAV = Hot active volume amplitude, cm³
HCV = Reduced cooler and cold ducting dead volume, dimensionless
HEC = Reduced cold end clearance dead volume, dimensionless
HGV = Reduced hot cap gap dead volume, dimensionless
HHC = Reduced hot clearance dead volume, dimensionless
HHV = Reduced heater dead volume, dimensionless
HMU = Hot hydrogen viscosity
HRV = Reduced regenerator dead volume, dimensionless
HT = Basic heat input, watts
HTD = Heater tube inside diameter, cm
HTE = Heat to engine, watts
HTLL = Total heater tube length, cm
HTLS = Heated heater tube length, cm
HTW = Hot end heat transfer integral, dimensionless
IND() = Array that shows if mass change is positive, or negative in warm and cold sides
J = Temporary angle variable, radians
K = 1 if warm mass change is positive, 2 if negative
L = 1 if cold mass change is positive, 2 if negative
LUP = Iterational counter
M = X value for plot calculation
MBR = Number of regenerators
MCT = Number of cooler tubes per cylinder
MHT = Number of heater tubes for cylinder
MW = Dimensionless mass in hot space = (mass, grams)(R)(BTW)/(PMXI(HAV))
N = NDIV or X value for plot subroutine
NN = 1 up to phase angle, 2 after
NDIV = Number of divisions per crank rotation (must be a multiple of 4 so that the phase angle at 90 degrees can be an even number of divisions) (Program must be revised if NDIV is not 360)
NDIV1 = NDIV + 1
NDS = Number of divisions in dead space
NE = NDIV/4 + 1
NET = Regenerator filler option +5 = metnet
NF = NDIV/4
NFF = NF + 1
NFI = (phase angle)(NDIV)/360
NFIN = Main loop final counter, for first part = phase angle, for second part = end of cycle
NIN = NDS + 1
NITE = Cycle counter (counts to 15)
NL = (NDIV/2) + 1
NLOP = Option counter limits changes in options to 7 (removed in final version)
NO = IND(K,L) - 1, 2, 3, or 4 starts as 1
NOC = Number of cylinders
NS = (NDIV/4) + 2
NST = Main loop initial counter, for the first part = 1, for second part = phase angle
NT = (NDIV/4) + 2
NWR = Governs printout, zero for overall results only, different from zero added PV data
P = Pressure, dimensionless
PALF = Thermal diffusivity of piston
PDR = Piston rod diameter, cm
PI4 = /4 = .78539816
PAVG = Dimensionless average pressure
PMAX = Maximum pressure, dimensionless
PMIN = Minimum pressure, dimensionless
PMX = Maximum pressure (MPa)
PMXI = Avg. pressure MPa
PR() = Pressure, dimensionless, fraction of maximum pressure
PO = Basic power, watts
POT = Net power, watts
PS = Dimensionless pressure from end of previous cycle
PW = Pressure at halfway point for increment
QB = Beta for shuttle heat loss calculation
QCP = Cooler windage, watts
QDK = Reheat factor
QFS = Pumping loss factor
QHC = Shuttle loss, watts
QHG = Pumping loss, watts
QHP = Heater windage, watts
QHR = Reheat loss, watts
QLM = Reheat factor, λ₁
QL1 = Shuttle factor, λ₁
QNPH = Reheat pressurization effect
QNTU = Regenerator transfer units, dimensionless
QP = Windage factor
QR() = Regenerator windage loss values, watts
QRP = Average regenerator windage, watts
R = Gas constant, joules/(gm)(K)
R2 = Constant = R(g_c)2
RE() = Regenerator Reynolds number in cold, middle, and hot part
REC = Cold Reynolds number
REH = Hot Reynolds number
RER = Regenerator Reynolds factor
RMU = Regenerator hydrogen viscosity, g/cm sec.
RNTU = Regenerator heat transfer units
RP = Maximum pressure/minimum pressure
RVT = Displaced mass ratio
S = Pressure at halfway point, dimensionless
SALF() = Sin values for cold space
SALFP = Average sin values for cold space
SFI = Sin of phase angle
SHR = Specific heat ratio for working gas
SIFI() = SALF()
SIFIP = SALF(1)
SMC = Cold mass + ½ change in mass
SMW = Hot mass + ½ change in mass
SPD = Engine speed, rad/sec
TEC() = Dimensionless cold gas temperature
TEST = Ensures that difference in dimensionless mass < .001
TEST1 = Ensures that difference in dimensionless pressure < .005
TEW() = Dimensionless hot gas temperature
TMPC = Average TEC()
TMPW = Average TEW()
TCDM = Dimensionless average cold temperature for entire cycle
TWDM = Dimensionless average warm temperature for entire cycle
UDM() = Critical mass flow values from subplot
UIN() = Critical pressure drop integral values from subplot
UI23, 24, 33, 34 = Critical pressure drop values
UPA = Power piston area, cm^2
UTR = Temperature ratio = hot metal temp, K
            cold metal temp, K
VC = C()
VCC = Cold volume cm$^3$
VCD = Cold dead volume, cm$^3$
VCI = CI()
VD = Reduced dead volume, dimensionless
VH = Hot volume, cm$^3$
VHD = Hot dead volume, cm$^3$
VRC = Regenerator dead volume, cm$^3$
VT = Total volume, cm$^3$
VW = W()
VWI = WI()
W() = Hot space as fraction of the stroke amplitude, calculated at mid increment
WC = Dimensionless cold work
WI() = Same as W() for beginning of increment
WMMAX = Largest hot dimensionless mass
WW = Hot work, dimensionless
X = Short term variable
XDMC() = Change in cold mass, grams
XDMW() = Change in hot mass, grams
XI1 = Pressure drop integral - accounts for the relationship between the shapes of mass and pressure fluctuations
XI2 = Influence of mass flow time variation on the heat transfer
XI3 = XI1/XI2
XINT = Basic pressure drop integral - for windage
XMC = Cold gas mass, relative to total inventory
XMCC() = Cold gas mass, grams
XMT() = Total mass, grams
XMW = Hot gas mass, relative to total inventory
XMWS = Hot dimensionless gas mass from previous cycle
XMWX() = Hot gas mass, grams
XND = NDIV
XHDS = NDS
XHNT = Value for exponent in heat transfer relation of regenerator matrix
XX = Short term variable
Y = |DMX|
ZEF = Indicated efficiency, %
ZZC = Connecting rod length/½ stroke for cold piston
ZZW = Connecting rod length/½ stroke for hot piston

D.3 FORTRAN Listing with Full Comments

1. //KIG5423 JOB ()+KOSOK+NOTIFY=WRMJC,REGION=120K
2. // EXEC FORTGCL
3. //FORT.SYSIN DD *

4. C IF MEMORY IS A PROBLEM ALL DIMENSIONS TO 720 CAN BE changed TO 361
5. 0 DIMENSION XHNC(720),XNHU(720),IND(2,2),FR(720),XMX(720),XMX(720)
6. 1 IEC(720),TEN(T72),XHT(720)
7. 0 DIMENSION W(720),C(720),WI(720),CI(720),DW(720),DC(720),DWI(720),
8. 1 UCI(720),DPR(720)
9. D DIMENSION SIFI(720),COFI(720),CALF(720),CALF(720),H(5),UIN(5),
10. 1 UCH(5),SBN(20),GIN(20),G12(20),G13(20),RE(3),FR(3),OR(3)

11. C SPECIFIED ENGINE DIMENSIONS AND OPERATING CONDITIONS ARE READ
12. C IN FROM DATA CARDS

13. 2 READ(5,5) ZZC,ZZW,XNH,FMI,SPD,NET,NOC
14. C SCREEN IF NEXT DATA SET IS BLANK

15. IF(ZZC) 4000,511,4000
16. 4000 READ(S1,1) SHP,WF,SMDNG,FMT,NOS,DG,BLL
17. READ(S1,1710) BTU,BTH,BPT,BAT,BPL,BRC,BEC
18. READ(S1,1720) PRI,BRI,BRI,RBI,BRI,BRI,MBR,MBT
19. READ(S1,1730) CTU,CTI,CTL,HT,HLL,HTL,HHT,HTH

20. C CONSTANTS ARE NOW SETUP
21. PI=3.1415926
22. PTI=79539816

23. C HOT AND COLD METAL TEMPERATURES ARE SET AT GIVEN HOT AND COLD

24. GAS TEMPERATURES
25. CTW1 = BTU
26. BTH = BTU
27. C POWER PISTON AREA
28. UFA = BBF*PR*PI

29. C TEMPERATURE RATIO; INVERSE OF NORMAL
30. HR = CTW/BC

31. C HOT CYLINDER; VOLUME AMPLITUDE IS REFERENCE VOLUME FOR ALL

32. C DIMENSIONLESS VOLUMES;
33. HR = UFA*PSI/2
34. GTP = PI*BBF*BF*PL*BR
35. C INITIALIZE UP COUNTER; SET FOR 3 ITERATIONS
36. CUP = 1

37. C ***RETURN POINT AFTER 1 CYCLE****
THE FOLLOWING ARE THE VOLUMES IN CUBIC FEET: 

CANDIDATE AND DEBRIS EFFECTIVE GAS TEMPS. AND VACUUMS

1. HDC (HEC+HCV/2)/VD
2. HCV (HEC+HCV/2)/VTR
3. HEC (HEC+HCV/2)/VTR
4. HAV (HEC+HCV/2)/VTR
5. HAV (HEC+HCV/2)/VTR
6. HAV (HEC+HCV/2)/VTR
7. HAV (HEC+HCV/2)/VTR
8. HAV (HEC+HCV/2)/VTR

THE FOLLOWING ARE NOT REGENERATOR AND COLD DEAD VOLUMES IN CUBIC FEET:

1. VHC = HAV*HFTL/HAV
2. VHC = HAV*HFTL/HAV
3. VHC = HAV*HFTL/HAV

ON THE FIRST ITERATION ONLY THE INPUT VALUES ARE WRITTEN OUT

IF (LUP-1) 343, 343, 346

333 HDC = HAV*PI4*PDR*BST/2

THE FOLLOWING ARE THE REDUCED VOLUMES DIVIDED INTO FRACTIONS, RE-EVALUATED

4. H(1) = (HEC+HCV/2)/VD
5. H(2) = (HEC+HCV/2)/VD
6. H(3) = (HEC+HCV/2)/VD
7. H(4) = (HEC+HCV/2)/VD
8. H(5) = (HEC+HCV/2)/VD

CALCULATIONS FROM 349 TO 295 ARE ON FIRST ITERATION ONLY. ENGINE

VOLUMES AND VOLUME DERIVATIVES ARE CALCULATED AND DECISION MATRIX

IS DEFINED

IF (LUP-1) 349, 349, 295

349 XND = NDIV
349 NDIV1 = NDIV + 1
C  DALF=0.0174 RADIANS/INCREMENT
81. C  DALF = 6.2831853/XND
82. C  NT = NDIV/442
83. C  NE = NT - 1
84. C  IF(PDR) 4060,4060,4070
85. C  4070 ALF=1.5707963
86. C  4060 ALF = 4.7123889
87. C  4080 NF = NDIV/4
88. C  CALL SUBROUTINE TO CALCULATE DUPLICABLE SPACE ABOVE OR UNDER COLD
89. C  PISTON AT THE MIDPOINT AND AT THE BEGINNING OF EACH ANGLE
90. C  INCREMENT AS A FRACTION OF THE PISTON STROKE AMPLITUDE.
91. C  CALL VOLC(DALF,NF,C,CI,DC,DCI,ZZC,NDIV,SIFI,COFI,SALF,CAFL,
92. C  1PDR,ALF)
93. C  THE PHASE ANGLE MUST BE 90 DEG FOR A HEAT ENGINE AND 270 DEG FOR
94. C  A HEAT PUMP
95. C  100 FI = CALC*HFL
96. C  101  FX1 = FI*180./PIE
97. C  102  WRITE(6,11) CHR,NDIV,FI1,ANG,DDE,DLL
98. C  103  SFI = SIN(FI1)
99. C  104  CFI = COS(FI1)
100. C  105  CALL VOLC(U,WF,0,WFI,CFI,SFI,ZZW,NDIV,SIFI,COFI,SALF,CAFL,
101. C  106  1D08,1D09,
102. C  107  DGAMMA:F
103. C  108  109  IF(1,1) = 1
110. C  111  IF(1,2) = 2
112. C  113  IF(2,1) = 3
114. C  115  IF(2,2) = 4
116. C  117  C  FIRST GUESS AT INITIAL QUANTITIES.
118. C  119  C  COUNTER NW IS 1 AT START OF CYCLE WHEN LIVE COLD VOLUME IS 2,50.
120. C  121  C  IT BECOMES 2 AFTER LIVE HOT VOLUME BECOMES ZERO. PHASE ANGLE IS
122. C  123  C  270 DEG FOR A HEAT PUMP AND 90 DEG FOR A HEAT ENGINE
124. C  125
295 NH = 1
296 C NONE OF THE GAS MASS IS ASSUMED TO BE IN THE COLD SPACE TO START,
297 C VOLUME IS ZERO.
298 C XMC = 0.
299 C RELATIVE PRESSURE IS ASSUMED TO BE MAXIMUM, APPROXIMATION.
300 P = 1.
301 C AT START ALL GAS MASS IS ASSUMED TO BE IN THE HOT SPACE. IGNORES
302 C DEAD VOLUME.
303 C XHW = 1 - CFI
304 C MASS FROM PREVIOUS CYCLE IS ASSUMED AT START TO BE ZERO. TO ASSURE
305 C AT LEAST 2 CYCLES TO CONVERGENCE.
306 C XMWS = 0.
307 C PREVIOUS CYCLE PRESSURE
308 F = 1.
309 C INITIAL ASSUMPTION FOR VALUE OF IND(K,L) IS CORRECT FOR HEAT
310 C ENGINE WRONG FOR HEAT PUMP.
311 C NO = 1
312 C INITIALIZE DIMENSIONLESS WORKS
313 WW = 0.
314 WC = 0.
315 NITE = 1
316 NST = 1
317 NFINT = NFINT
318 C DISPLACED MASS RATIO
319 C RVT = HAC*UTR/HAV
320 CI(NDIVI) = CI(I)
321 WI(NDIVI) = WI(I)
322 C **************************************************************
323 C START OF MAIN DO LOOP; RETURN POINT AFTER EACH INCRIMENT
324 404 DO 102 I = NST, NFINT
325 C TRANSFERS VOLUMES AND DERIVATIVES FROM STORAGE
326 WW = WI(I)
327 WC = CI(I)
328 WWI = WI(I)
329 VC = CI(I)
330 DWW = DW(I)
331 DVC = DC(I)
332 DWWI = DWI(I)
333 DVC = DC(I)
334 C SPLITS TO 4 OPTIONS
GO TO (201, 202, 203, 204); NO
182. C INTEGRATION PROGRAM FOR MASS INCREASING IN BOTH HOT AND COLD SPACES,
183. C NO-1 (SEE NOTE 15)
184. C COMPUTES PRESSURE CHANGE BASED UPON INITIAL CONDITIONS
185. 201 IP = -SHR*P*(RVT*DVC1+DUWI)/(RVT*VC1+DUWI+SHR*VD)*DALF
186. C FINDS PRESSURE AT MID INCREMENT
187. 5 = P+DP/2.
188. C CALCULATES PRESSURE CHANGE BASIC UPON MID POINT VALUES
189. DP = -SHR*P*(RVT*DVC1+DUWI)/(RVT*VC1+DUWI+SHR*VD)*DALF
190. C CALCULATES MASS CHANGES
191. DMW = S*DVM*DALF+VW*DP/SHR
192. DMC = -(DMW+VD*DP)/RVT
193. C DETERMINS CHOICE MATRIX
194. IF(DMW) 302, 301, 301
195. 301 K = 1
196. GO TO 303
197. 302 K = 2
198. 303 IF(OMC) 304, 305, 305
199. 304 L = 1
200. GO TO 306
201. 305 L = 2
202. 306 NC = IND(K,L)
203. C IF CHOICE IS CHANGED NEXT ITERATION WILL BE THROUGH A DIFFERENT
204. C OPTION
205. GO TO 400
206. C INTEGRATION PROGRAM FOR MASS DECREASING IN BOTH HOT AND COLD SPACES,
207. C NO-2 (SEE OPTION 1 FOR DETAILED EXPLANATION)
208. 202 IF(XMC) 803, 801, 801
209. 803 XMC = 0.0
210. 801 IF(XMW) 805, 802, 802
211. 805 XMW = 0.0
212. 8029DP = -SHR*(XMC*RVT*DVC1+VCI+XMW*DUWI)/UWI/)
213. 1 (XMC*RVT/P+XMW/P+SHR*VD)*DALF
214. DMC = XMC*(DVC1*DALF/VCI+DP/SHR/P)
215. DMW = -RVT*DMC-VD*DP
216. S = P+DP/2.
217. SMC = XMC+DMC/2.
218. SMW = XMW+DMW/2.
219. ODF = -SHR*(SMC*RVT*DVC1+VCI+SMW+DVW/VW)/UW/)
220. 1 (SMC*RVT/P+SMW/P+SHR*VD)*DALF
C INTEGRATION PROGRAM FOR MASS DECREASING IN COLD SPACE AND INCREASING IN HOT SPACE, NO=3 (SEE OPTION 1 FOR DETAILED EXPLANATION)

201. \[ DMC = \text{SMC} \times (\text{DVC} + \text{DALF} / \text{VCI} \times \text{DP} / \text{SHR} / S) \]
202. \[ DMW = -\text{RVT} \times \text{DMC} - \text{VD} \times \text{DP} \]
203. IF (DMW) 312, 312, 307
204. 312 \[ K = 2 \]
205. GO TO 308
206. 307 \[ K = 1 \]
207. 308 IF (DMC) 309, 309, 310
208. 309 \[ L = 2 \]
209. GO TO 311
210. 310 \[ L = 1 \]
211. 311 NO = IND(K, L)
212. GO TO 409
213. C INTEGRATION PROGRAM FOR MASS DECREASING IN COLD SPACE AND INCREASING IN HOT SPACE, NO=4 (SEE OPTION 1 FOR DETAILED EXPLANATION)

215. 203 IF (XMC) 704, 703, 703
216. 704 \[ XMC = 0. \]
217. 703 \[ DOP = -\text{SHR} \times (\text{P} \times \text{VW} + \text{XMC} \times \text{RVT} \times \text{DVC} / \text{VCI}) / (\text{VWI} + \text{XMC} \times \text{RV}) \]
218. \[ 1 \]
219. \[ \text{DMC} = \text{XMC} \times (\text{DVC} \times \text{DALF} / \text{VCI} \times \text{DP} / \text{SHR} / \text{P}) \]
220. \[ \text{DMW} = -\text{RVT} \times \text{DMC} - \text{VD} \times \text{DP} \]
221. \[ S = \text{P} \times \text{DP} / 2. \]
222. \[ \text{SMC} = \text{XMC} \times \text{DMC} / 2. \]
223. \[ \text{SMW} = \text{XMC} \times \text{DMW} / 2. \]
224. \[ ODF = -\text{SHR} \times (S \times \text{VW} \times \text{XMC} \times \text{RVT} \times \text{DVC} / \text{VCI}) / (\text{VWI} \times \text{SP} \times \text{RVT} \times \text{SHR}) \]
225. \[ 1 \]
226. \[ \text{DMC} = \text{SMC} \times (\text{DVC} \times \text{DALF} / \text{VCI} \times \text{DP} / \text{SHR} / S) \]
227. \[ \text{DMW} = -\text{RVT} \times \text{DMC} - \text{VD} \times \text{DP} \]
228. IF (DMW) 313, 314, 314
229. 314 \[ K = 1 \]
230. GO TO 315
231. 313 \[ K = 2 \]
232. 315 IF (DMC) 316, 316, 317
233. 316 \[ L = 2 \]
234. GO TO 318
235. 317 \[ L = 1 \]
236. 318 NO = IND(K, L)
237. GO TO 400
238. C INTEGRATION PROGRAM FOR MASS DECREASING IN COLD SPACE AND DECREASING IN HOT SPACE, NO=4 (SEE OPTION 1 FOR DETAILED EXPLANATION)

239. 204 IF (XMW) 705, 702, 702
705 XMW = 0.
7020DP = -SHR*(P*RVT*DVCI+XMW*DVWI/VWI)/(RVT*VC
1 +XMW/F+SHR*VD)*DALF
244. DMW = XMID*DVWI/DALF/VWI+DP/SH/P
245. DMC = -(DMW+VD*DP)/RVT
246. S = P+DP/2.
247. SMC = XMC+DMC/2.
248. SHW = XMID+DMW/2.
249. ODP = -SHR*(S*RVT*DVC+SMW*DVW/VW)/(RVT*VC
1 +SMW/F+SHR*VD)*DALF
250. DMW = SHW*(DVWI/DALF/VWI+DP/SH/S)
251. DMC = -(DMW+VD*DP)/RVT
253. IF(DMW) 319,319,320
319 K = 2
320 K = 1
321 IF(DMC) 322,323,323
323 L = 1
324 NO = IMD(K,L)
325. C INCRIMENTS PRESSURE AND MASS
326. 400 P = P+DP
327. XMC = XMC+DMC
328. XMID = XMID+DMW
329. CALCULATES WORKS
330. PW = P-DP/2.
331. WC = WC+PW*DVC*DALF
332. WW = WW+PW*DVW*DALF
333. RECORDS RESULTS INTO ARRAYS
334. FR(I) = P
335. IPRI(I) = DP
336. XMCI(I) = XMC
337. XMID(I) = XMID
338. XDMC(I) = DMC
339. XDMW(I) = DMW
340. C *****END OF MAIN DO LOOP*****
102 CONTINUE
341. GO TO (401,402),NN
342. C RESET MAIN DO LOOP FOR LAST PART OF CYCLE
401 NST = NFI+1
402 NFIV = NDIV
403 NN = 2
404 GO TO 404
405 C TESTS FOR CONVERGENCE AT END OF CYCLE. THE CHANGE IN THE FRACTION
406 C OF MASS IN THE HOT SPACE FROM ONE CYCLE TO THE NEXT MUST BE LESS
407 C THAN 0.1%, AND THE CHANGE IN PRESSURE FROM ONE CYCLE TO THE NEXT
408 C MUST BE LESS THAN 0.5%. HOWEVER, NO MORE THAN 15 CYCLES ARE
409 C ALLOWED.
410 C 402 TEST = SORT(((XMWS-XMW)**2)
411 TEST1 = SORT((PS-P)**2)
412 IF(NITE-15) 471,471,406
413 471 IF(TFST-.001)473,473,405
414 473 IF(TEST1-.005)406,406,405
415 C REINITIALIZE FOR NEXT CYCLE
416 405 NN = 1
417 XMC = 0.
418 PS = P
419 XMWS = XMW
420 WW = 0.
421 WC = 0.
422 NIT = 1
423 NFIV = NFI
424 NITE = NITE+1
425 NO = 4
426 GO TO 404
427 C THE DIMENSIONLESS PRESSURES AND WORKS HAVE BEEN CALCULATED FOR ONE
428 C CYCLE, NOW THE ADDITIONAL HEAT AND POWER LOSSES WILL BE CALCULATED.
429 C CALCULATE AVERAGE DIMENSIONLESS PRESSURE.
430 406 PAVG=0
431 DO 3000 I=1, NDIV
432 3000 PAVG=PAVG+PR(I)
433 PAVG=PAVG/NDIV
434 C DETERMINE MAXIMUM AND MINIMUM DIMENSIONLESS PRESSURE
435 PMAX = XLARGE(PR,NDIV)
436 PMIN = SMALL(PR,NDIV)
437 C ADJUST DIMENSIONLESS WORKS TO RELATE TO NEWLY DETERMINED MAXIMUM
438 C PRESSURE
439 WC = WC/PMAX
440 WW = WW/PMAX
C PRESSURE RATIO
RP = PMAX/PHIN.

C FIND MAXIMUM RACSES AND ADJUST THEM TO MAXIMUM PRESSURE.
CMAX = XLARGE(XMIX*NDIV)
WMAX = XLARGE(XMIX*NDIV)
CHMAX = CMAX/FMAX
WMAX = WMAX/FMAX

C CALC. MAX. PRESSURE, MPA
PMAX=PHMAX*PMX1/PAYG

C CALCULATES ANGLE BETWEEN PRESSURE WAVE AND VOLUME WAVE FOR A HEAT ENGINE.
ARG = 2.*RP/(RP-1.)*WM/3.1416
IF(1.-ARG**2) 1507,1608,1608

1608 FIPV = ARSIN(ARG)
XNDS = NDS

C CALCULATES VALUES USED IN FLOW LOSS CALCULATIONS AND FLOW INTEGRALS
X = 0.
DX = 1./XNDS
NIN = NDS + 1
COR = PMAX**2*(XNHT-2.)*DHALF**2*(XNHT-1.)
DO 854 I=1,NIN
CALL PINT(X,DMW,XMIX,RVT,DC,NDIV,DMRE,PR,XINT,DPF,FIX1,XII,XI2,XNHT)
XINT = XINT/DHALF/FMAX
DMRE = DMRE/FMAX/6.2832
XI1 = XII*COR/(1.5708*DMRE)**2*(1.-XNHT)
XI2 = XI2*COR/(1.5708*DMRE)**2*(2.-XNHT)
XI3 = XII/XI2
GOMS(I) = DMRE
GINT(I) = XINT
GI2(I) = XI2
GI3(I) = XI3
X = X+DX
854 CONTINUE

C INTERPOLATES FLOW INTEGRALS
DO 910 I=1,5
UIN(I) = PLOT(GINT,H(I))
UDM(I) = PLOT(GOMS,H(I))
910 CONTINUE

UI23 = PLOT(GI2,H(2))
UI24 = PLOT(GI2,H(4))
UI33 = PLOT(GI3,H(2))  
UI34 = PLOT(GI3,H(4))

C  *****CALCULATION OF CONSTANTS*****  
C SPECIFIC FOR HYDROGEN GAS  
HMU = .8873E-04+2E-06*(BTW-293.)  
CMU = .8873E-04+2E-06*(BTC-293.)  
BTR = (BTW-BTC)/ALOG(BTW/BTC)  
RMU = .8873E-04+2E-06*(BTR-293.)  
CP1 = 14.6  
CV1 = 10.46  
R2 = 82.3168E6  
R = 4.116

C  *****COLD EXCHANGER PRESSURE DROP*****  
RE = UDM(1)*PMX*SPD*HAC*CTD/(BTC*AFH*CMU*R)  
1985 FC = 16./REC  
GO TO 1987  
1986 FC = EXP(-1.34-.2*ALOG(REC))  
1987 GLS = CTTL*SPD*SPD*HAC*HAC*FC*UIN(1)/(CTD*AFH*AFH*BTC*R2)  
QP = NOC*SPD*PMX*HAC/(2.*PIE)  
QCP = QP*GLS  
C  *****HOT EXCHANGER PRESSURE DROP*****  
REH = UDM(5)*PMX*SPD*HAC*HTD/(BTC*AFH*HMU*R)  
1988 FH = 16./REH  
GO TO 1993  
1989 FH = EXP(-1.34-.2*ALOG(REH))  
1993 GLH = HTLL*SPD*SPD*HAC*HAC*BTW*FH*UIN(5)/(HTD*AFH*AFH*BTC*BTC*R2)  
QHP = QP*GLH  
C  *****SCREEN--HETNET OPTION*****  
RER = PMX*HAC*SPD*BWD/(AFR*R)  
RE(1) = RER*UDM(2)/(BTC*CMU)  
RE(2) = RER*UDM(3)/(BTR*RMU)  
RE(3) = RER*UDM(4)/(BTW*HMU)  
DO 2030 I=1,3  
IF(NET) 2015,2015,2022  
2015 IF(RE(1)-60,) 2017,2017,2018  
2017 FR(I) = EXP(1.73-.93*ALOG(RE(I)))  
2017 GO TO 2030  
2021 IF(RE(I)-1000,) 2019,2019,2021
2019 FR(I) = EXP(.714-.365*ALOG(RE(I)))
GO TO 2030
2021 FR(I) = EXP(.015-.125*ALOG(RE(I)))
GO TO 2030
2022 FR(I) = 2.73*(1+.10397/RE(I))
2030 CONTINUE

C *****REGENERATOR PRESSURE DROP*****
GLR = BRL*SPD*SPD*HAC/HAC/(BWD*AFR*AFR*R2*BTC)
QR1 = QP*GLR*UIN(2)/FR(1)
QR2 = QP*GLR*UIN(3)/FR(2)*BTR/BTC
QR3 = QP*GLR*UIN(4)/FR(3)*UTR
QP = (QR1+QR3+QR2)/6.

C CALCULATES EFFECTIVE HOT AND COLD GAS TEMPERATURES BASED UPON THE
C NUMBER OF TRANSFER UNITS IN THE HEAT EXCHANGERS, SPECIFIC FOR
C HYDROGEN
C
CNTU = .112*CTLS/(CTD*REC**.2)
DTC = WC*(SHR-1.)/(2.*UDM(1)*SHR*(EXP(2.*CNTU)-1.))
BTC = BTC1*1.-DTC)
HNTU = .1044*HTLS/(HTD*REH**.2)
DTH = WW*(SHR-1.)/(2.*UDM(5)*SHR*(EXP(2.*HNTU)-1.))

C NOTE, TEMPERATURES TIO IS REDEFINED FOR NEXT ITERATION
UTR = BTW/BTC
C
C *****REHEAT LOSS*****
RNTU = BRL*4.37/(BWB*SQRT(F'I4*2.*RE(1)))
QNTU = BRL*4.031/(BWB*SQRT(F'I4*2.*RE(3)))
QNH = AFR*BRL*.1950/(PI4*HAC*UDM(2)*UF-1.))
QDK = QNH*(UI33+UI34*UDM(2)*UDM(4))/2,
QLM = (1.+QDK)/(RNTU/UI23+QNTU*UDM(2)/(UDM(4)*UI24))
QHR = UDM(2)*CP1*(BTW-BTC)*SPD*PMX*HAC*QLM*NOC/(BWC*BTC**2.)

C *****SHUTTLE LOSS*****
QL1 = 231.2*SQF(T(SPD*BRC*BRC)
QB = (2.*QL1*QL1-QL1)/(2.*QL1*QL1-1.)
QHC = .00146*BST*(BTW-BTC)*PI4*BP*BST*QE*NOC/(BRC*BPL)

C *****PUMPING LOSS*****
QFS = (RP/(BTW/(BTW-2.*BTC)-BST/BPL))/((1./1./((BTW-2.*BTC)/BST/
QHG = ABS(SPD*PMX*GGV*BST*SHR-QFS*ARG*NOC/((SHR-1.)*BPL*RP*B))

C *****BASIC POWER*****
PD = (WW*HAV+WC*HAC)*(+.50)*PMX*SPD*NOC/PIE
C NET POWER

POF = PO-OCF-QRF

C GET READY TO REPORT ON ONE ITERATION AND PREPARE FOR THE NEXT.
C RESET HOT END DIMENSIONLESS HEAT TRANSFER INTEGRAL

HTW = 0.

C THE PROGRAM TRIES TO KEEP PMAX=1. THIS ADJUSTMENT OF THE PRESSURE
C AND MASSES DOES THIS

509 I=1,NDIV

PR(I) = PR(I)/PMA
XMCX(I) = XMCX(I)/PMA
509 XMX(I) = XMX(I)/PMA

512 DIMENSIONLESS HOT AND COLD GAS TEMPERATURES FOR EACH INCREMENT.
513 C IF THEY ARE LESS THAN ZERO CORRECT TO ZERO

514 WI(NDIV) = WI(1)
515 CI(NDIV) = CI(1)
516 DO 1003 I=1,NDIV
517 IF(XMCX(I)) 1003,1003,1002
518 1002 TEC(I) = PR(I)*CI(I+1)/XMCX(I)

GO TO 1006

1003 TEC(I) = 0.

1006 IF(XMX(I)) 1004,1004,1005
1005 TEW(I) = PR(I)*WI(I+1)/XMX(I)

GO TO 1001

1004 TEW(I) = 0.

1001 CONTINUE

516 DIMENSIONLESS AVERAGE HOT AND COLD GAS TEMPERATURES FOR FULL CYCLE.

517 TEW(NDIV) = TEW(1)
518 TEC(NDIV) = TEC(1)
519 PR(NDIV) = PR(1)
520 XMCX(NDIV) = XMCX(1)
521 XMX(NDIV) = XMX(1)
522 TWDM = 0.
523 TCDM = 0.

524 DO 573 I=1,NDIV
525 DMW = XMX(I+1)-XMX(I)
526 IF(DMW) 574,574,575
527 574 TMPW = (TEW(I)+TEW(I+1))/2.
528 575 DMW = TWDH*(TMPW-1.)*DMW
529 575 DMC = XMCX(I+1)-XMCX(I)

576 IF(1MC) 576,573,573
576 \[ T_{MPC} = \frac{(T_{EC}(I)+T_{EC}(I+1))}{2}. \]
577 \[ TCDM = TCDM + (T_{MPC} - T_{DMC}) \]
578 CONTINUE
579
580 TMPC = TEC(I)+TEC(I+1)/2.
581 TCDM = TCDM+(T_{MPC}-T_{DMC})
582 CONTINUE
583
584 TWDM = TWDM*SHR/(SHR-1.)
585 TCDM = TCDM*SHR/(SHR-1.)
586 C HOT END HEAT TRANSFER INTEGRAL FOR FULL CYCLE AND TOTAL GAS MASS AT
587 C EACH POINT IN THE CYCLE. TOTAL MASS SHOULD NOT CHANGE.
588 C
589 DO 1021 I=1,NDIV
590 HTW = HTW+(WI(I+1)-WI(I))*(PR(I)+PR(I+1))/2.
591 1021 XMT(I) = XMCX(I)*RVT+XMXW(I)+PR(I)*VD
592 C BASIC HEAT INPUT, WATTS
593 HT = HTW/SHP*PMX_HAV_NOC/(2*PIE)
594 C SPECIFIC STATIC CONDUCTION HEAT LOSS FOR THE 4L23 ENGINE
595 CON = 9680.
596 C FLOW FRICTION CREDIT, WATTS
597 FFF = (QHP5-QRP)_(-1)
598 C HEAT TO ENGINE, WATTS
599 HTE = HT+QHR+QHC+QHG+CON+FFF
600 C INDICATED EFFICIENCY, %
601 ZEF = 100.*POF/HTE
602 C PRINT OUT RESULTS OF ONE ITERATION
603 WRITE(6,12) LUP
604 WRITE(6,3010) HT
605 WRITE(6,3020) QHP,QHR
606 WRITE(6,1925) ORP,QHC,QCP,OHG,POF,CON,ZEF,FFF,HTE
607 C AFTER ALL LOSSES ARE TAKEN INTO ACCOUNT LUP IS INDEXED. THE PROGRAM
608 C DOES 3 ITERATIONS WITH PRINTOUTS BEFORE GOING INTO A SUMMARY.
609 LUP = LUP+1
610 IF(LUP-3) 339,339,1607
611 C IF INPUT VALUE NWR IS OTHER THAN ZERO THE FOLLOWING SUMMARY
612 C INFORMATION IS PRINTED AT THE END OF THE COMPUTATION
613 1607 IF(NWR) 1613,606,1613
614 1613 WRITE(6,51) TWDM,TCDM
615 C PRINT OUT EACH 10 DEGREES, ANGLE, HOT VOLUME, HOT GAS TEMPERATURE,
616 C COLD VOLUME, COLD GAS TEMPERATURE, TOTAL VOLUME, PRESSURE
617 1149 WRITE(6,20)
618 DO 3001 I=10,NDIV,10
619 X=PR(I)*PMX
620 VH=VHD+HAV*WI(I)
STARTS OVER WITH NEXT DATA SET.
606 GO TO 2
511 CALL EXIT
1 FORMAT (F10.4,4F10.4,2F10.4)
5 FORMAT (5F10.4,210)
11 FORMAT (21H SPECIFIC HEAT RATIO=F10.4,10X,16H DIV. PER CYCLE=
15/1X,20H PHASE ANGLE(DEG.) =F10.4,9X,9X- INCR. IN DF INT. ='-15,'
23X,DUCK DIAMETER(CH)="F10.4,10X,"DUCK LENGTH(CH)="F10.4" OUTPUT
3T:'//)
12 FORMAT(' ITERATION ',',I2)
3010 FORMAT(8X,' BASIC POWER(WATTS)='F10.1,9X,' BASIC HEAT(WATTS)='
1F10.1)
3020 FORMAT(5X,' HEATER WINDBAG(WATTS)='F10.1,8X,' REHEAT LOSS(WATTS)='
1*F10.1)
510 FORMAT(' DIMENSIONLESS ,VG. GAS TEMP/' HOT END ','F10.4,10X,
1'COLD END ','F10.4)
2010 FORMAT (2X,19H COLD CRANK RATIO =
1F10.4,7X,' HOT CRANK RATIO =',F10.4)
1710 FORMAT (7F10.4)
1720 FORMAT (5F10.4,2110)
1730 FORMAT (6F10.4,110)
1805 FORMAT(18H INPUT DIMENSIONS:
1810 FORMAT(21H COLD MET TEMP(K)='F10.4,6X,18H HOT MET TEMP(CM)=
1F10.4,5X,16H PISTON DIA(CH)=F10.4,7X,19H PISTON STROKE(CH)=
2F10.4)
1820 FORMAT(21H HOT CAP LENGTH(CH)=F10.4,10X,16H PISTON GAP(CH)= F10.4
1/21H PIST END CLR(CH)= F10.4,8X,18H PIST ROD DIA(CH)= F10.4)
1830 FORMAT(5X,16H REGEN POROSITY='F10.4,11X,15H REGEN DIA(CH)=
1F10.4,21H REG. WIRE DIA(CH)= F10.4,8X,18H REGEN LENGTH(CH)=
2F10.4)
1840 FORMAT(7X,14H NUM OF REGEN= 15,19X,12H NUM OF CT = 15,
1/21H COOL TUBE DIA(CH)= F10.4,10X,
216H TOT CT LEN(CH)= F10.4)
1850 FORMAT(4X,17H COOL CT LEN(CH)= F10.4,5X,21H HEATER TUBE DIA(CH)=
1F10.4,5X,16H TOT HT LEN(CH)= F10.4,9X,17H HEAT HT LEN(CH)= F10.4/
29X,12H NUM OF HT = 15,13X,18H HEAT TRAN. EXP. = F10.4)
1860 FORMAT(1X,20H AVG. PRESSURE(MPA)= F10.4,3X
123H ENGINE SPEED (RAD/SEC) = F10.4/5X, 16H NETMID OPTION = I5,
215X, 16H NUMBER OF CYL. = I5)
1921 FORMAT(6X, 20H EFFEC. HOT TEMP(K) = F10.1, 6X,
121H EFFEC. COLD TEMP(K) = F10.1/5X, 23H DISPLACED MASS RATIO = F10.4,
28X, 19H REDUCED DEAD VOL = R10.4/)
1925 FORMAT(28H REGENERATOR WINDAGE (WATTS) = F10.1, 6X,
121H SHUTLE LOSS (WATTS) = F10.1/5X, 23H COOLER WINDAGE (WATTS) = F10.1
2H, 5X, 22H APPENDIX LOSS (WATTS) = F10.1/10X, 18H NET POWER (WATTS) = F10.1,
56X, 19H CONDUCTION (WATTS) = F10.1/9Y, 19H INDICATED EFF. (Z) = F10.1,
45X, 22H FLOW FRICTION (WATTS) = F10.1/42X, 23H HEAT TO ENGINE (WATTS) =
5F10.1)
20 FORMAT(' PRESSURE, MPA-VOLUME, CM3 DATA FOR ONE CYLINDER/' ANGLE IN
16', 5X, 'HOT VOL', 6X, 'COLD VOL', 5X, 'TOTAL VOL', 7X, 'PRESSURE')
21 FORMAT(5X, I5, 3X, F10.4, 5X, F10.4, 5X, F10.4, 5X, F10.4)
END

C SUBROUTINE TO FIND LARGEST OF A LIST
FUNCTION XLARGE(X, NDIV)
DIMENSION X(720)
XLARGE = X(I)
DO 505 I = 2, NDIV
IF(XLARGE = X(I)) 506, 505, 505
506 XLARGE = X(I)
505 CONTINUE
RETURN
END

C SUBROUTINE TO FIND SMALLEST OF A LIST
FUNCTION SMALL(X, NDIV)
DIMENSION X(720)
SMALL = X(I)
DO 507 I = 2, NDIV
IF(SMALL = X(I)) 508, 507, 508
508 SMALL = X(I)
507 CONTINUE
RETURN
END

C SUBROUTINE TO INTERPOLATE
FUNCTION PREDICATE
DIMENSION X(20)
N = 10.*H
Z = H-H/10
M = H+1
G1 = X(N)
N = M+1
G2 = X(M)
FLOT = Z*G2+(1.-Z)*G1
RETURN
END

C SUBROUTINE TO LIST COLD VOLUMES AND DERIVATIVES
SUBROUTINE VOLC(BALF,RF,C,CI,DC,DCI,ZZC,NDIV,SIFI,COFI,SALF,CALF,
1PDR,ALF)
DIMENSION C(720),CI(720),DC(720),DCI(720)
DIMENSION SIFI(720),COFI(720),SALF(720),CALF(720)
NDIV1 = NDIV
DO 852 I=1,NDIV1
COFI(I)=COS(ALF)
SALF(I)=SIN(ALF)
ALF=ALF+ALF
DO 855 I=1,NDIV
855 ALF=ALF+ALF
852 ALF=ALF+ALF
ALF=ALF+ALF
DO 851 I=1,NDIV
851 SALF(I) = (SALF(I+1)-SALF(I))/DALF
850 DALF(NDIV1) = DALF(1)
DO 851 I=1,NDIV
850 SIFI(I) = SALF(I)
850 COFI(NDIV1) = COFI(1)
850 SIFI(NDIV1) = SIFI(I)
850 N = NF**4
DO 302 I = 1,N
302 CRC = SQRT(ZZC**2-CALF(I)**2)
301 CRC = SQRT(ZZC**2-CALF(I)**2)
C SEE NOTE 11
IF(PDR) 7010,7010,7020
7020 C(I)=1.-SALF(I)+CRC-ZZC
7010 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
7000 C(I)=1.-SALF(I)+CRC-ZZC
SUBROUTINE FOR PRESSURE DROP INTEGRAL CALCULATION

SUBROUTINE PINT (X, DMW, DMC, RVT, DVC, NDIV, DM, PR, XINT)

DIMENSION DMW(720), DMC(720), DVC(720), PR(720), DFR(720)

DM = 0,
XINT = 0,
XI1 = 0,
EX1 = 1.0-XNHT
XI2 = 0,
EX2 = 2.0-XNHT

DO 101 I = 1, NDIV

DMX = DMC(I)-X+(DMW(I)/RVT+DMC(I))
Y = ABS(DMX)
DM = DM+Y

A = DFR(I)*Y**EX1
IF(DMX) 201, 202, 202

201 A = -A
202 XI1 = XI1+A
XI2 = XI2+Y**EX2

XINT = XINT+Y*DMX/PR(I)*DVC(I)

XNDIV = NDIV

RETURN

END

SUBROUTINE TO LIST HOT VOLUMES AND DERIVATIVES

SUBROUTINE VOLW(W, WI, DW, DWI, CFI, SFI, ZZW, NDIV, SIFI, COFI, SALF, CALF, DALF)

DIMENSION SIFI(720), COFI(720), SALF(720), CALF(720)

SIFI = SIFI(I)*CFI-COFI(I)*SFI

DO 101 I = 1, NDIV

SALF1 = SIFI(I+1)*CFI-COFI(I+1)*SFI
SALFP = (SIFIP+SALF1)/2.
CALFP = (SALFP1-SIFIP)/DALF
CRW = SQRT(ZZW**2-CALFP**2)
W(I)=1.0+SALFP-CRW+ZZW
WI(I)=1.0+SIFIP-CRW+ZZW
DW(I)=CALFP*(1.0-SALFP/CRW)
DWI(I)=CALFP*(1.0-SIFIP/CRW)
101 SIFIP = SALF1
RETURN
END

//GO SYSIN DD *
5.874 5.874 .204 9.6516 209.44 5.4
1.39 90 360 1 10 .26 71.0
330. 1033. 10.16 4.65 6.4 .0406 .0406
.060 .8 3.5 .0043 2.5 6 312
.115 12.9 12.02 .472 41.8 25.58 36
0.0

C XXXXXXXXX XXXXXXXXX XXXXXXXXX XXXXXXXXX
C INTEGER DATA MUST BE RIGHT JUSTIFIED. THERE ARE 7 DATA FIELDS PER
C LINE OF 7 COLUMNS EACH.
C DATA FIELD LAYOUT IS AS FOLLOWS:
C********ZZC ********ZXZ ********NHT ********PMX1 ********SPD ********NMT ********NOC
C********SHR ********NFI ********NDIV ********NHF ********NDS ********NDD ********NLL
C********BTC ********BFD ********BET ********BFN ********BFC ********BFC
C********PDR ********BRO ********BND ********BID ********BRL ********HER ********NHT
C********CTD ********CTDL ********CTLS ********CSDL ********CTLL ********CTLS ********INT
C NEXT DATA SET REPEATS LAST.
C CHANGE ACCORDING TO DATA PRINTOUT.
C DATA MUST BE WITHIN CONTROL CARDS(/).
COMMAND?
D.4 Evaluation of Appendix Loss as Calculated by Rios

In his 1969 series (69 am), Rios calculated the appendix loss in a Stirling refrigerator. He refers to this loss as the loss due to gas motion in the radial clearance. The appendix loss calculated by Rios is more than an order of magnitude higher than that calculated by the second order method. It was decided to evaluate the derivation of Rios more closely (69 am, pp. 136-138) to determine the cause of such a large discrepancy. Many steps taken by Rios were not understood by this author, but when the adaptation from refrigerator to heat engine was carefully analyzed, some changes were made that resulted in an appendix loss comparable to that given by the second order code.

D.4.1 Rios Appendix Loss Adapted to a Heat Engine

The pumping or appendix loss is the loss due to gas flow into and out of the radial clearance between the piston and displacer. The following assumptions are made:

1. The radial clearance is small, so it can be assumed that the gas entering and leaving the radial clearance volume is at the adjacent cylinder wall temperature.

2. The temperature gradient at the stroked part of the cylinder is smaller than that of the unstroked part and is approximated by Rios to be:

\[
\frac{dT}{dx} = \frac{\Delta T}{2BPL} \tag{D-1}
\]

Where:
- \(dT\) = the temperature gradient
- \(dx\) = distance along the stroked part of the cylinder
- \(\Delta T\) = the temperature difference from one end of the gap to the other
- \(BPL\) = the hot cap or gap length
3. Variations in piston motion, pressure and gas flow may be approximated by sinusoids.

The highest average pressure and temperature in the gap is reached near top dead center, after the hot cap has compressed the hot gases into the gap. The lowest average pressure and temperature is reached near bottom dead center, after the expansion stroke of the hot cap (where the total engine volume is maximum).

Considering assumption 2, Rios calculates the space-average temperature fluctuation of the stroked and unstroked parts of the gap is:

\[
\bar{T} = \frac{BTL + BTR + (BTR - BSL) BST}{BPL} \sin (SPD(t))
\]

so

\[
\bar{T}_{\text{min}} = \frac{BTL + BTR - (BTR - BSL) BST}{BPL}
\]

and

\[
\bar{T}_{\text{max}} = \frac{BTL + BTR + (BTR - BSL) BST}{BPL}
\]

where \( \bar{T} \) = the space-average temperature fluctuation

\( \bar{T}_{\text{min}} \) = the minimum space average temperature

\( \bar{T}_{\text{max}} \) = the maximum space average temperature

\( BTL \) = the cold metal temperature

\( BTR \) = the hot gas temperature

\( BST \) = the hot cap stroke

\( SPD \) = engine speed, rad/sec

\( t \) = time, seconds

The pressure is:

\[
P = \frac{PMX + PMN}{2} + \frac{PMX - PMN}{2} \sin ((SPD)t - \phi)
\]

where \( P \) = the pressure fluctuation

\( PMX \) = the maximum pressure (MPa)

\( PMN \) = the minimum pressure

\( \phi \) = the angle between the pressure and volume variations

A small error is introduced if it is assumed that the maximum temperature and pressure occur simultaneously, and that the minimum pressure and temperature occur simultaneously. The mass difference is assumed to be the difference between the mass of each of these points and is calculated by Rios to be:

\[
MG(\text{max}) - MG(\text{min}) = \frac{GGV}{R} \left[ \frac{PMX}{T_{\text{max}}} + \frac{PMN}{T_{\text{min}}} \right]
\]

where \( MG(\text{max}) \) = the maximum mass in the gap
\[ M_G \text{ (min)} = \text{the minimum mass in the gap} \]
\[ GGV = \text{the dead volume in the gap} \]
\[ R = \text{the gas constant} \]

The mass fluctuation amplitude is defined to be:

\[ M_{AG} = \frac{1}{2} \frac{GGV}{R} \left( \frac{P_{MIN}}{T_{min}} - \frac{P_{MAX}}{T_{max}} \right) \quad (D-7) \]

And the gap mass fluctuation is approximated by:

\[ M_G = M_{MG} + M_{AG} \sin \left( (SPD)t - \theta' \right) \quad (D-8) \]

where \( M_{MG} \) is the average mass in the gap.

Rios assumes that:

\[ \theta' \approx 0 \quad \text{because both are close to 180°} \quad (D-9) \]

From equation D-1 the temperature of the gas moving in and out of the radial clearance is given by Rios as:

\[ T = \frac{T}{4} - \frac{(BTW - DTC)(BST)}{BPL} \sin \left( (SPD)t \right) \quad (D-10) \]

The enthalpy flow into the cylinder is given by:

\[ dH_G = -CP1 \frac{T}{4} \frac{dM}{BPL} \]
\[ = -CP1 \left( \frac{T}{4} - \frac{(BTW - BTC)(BST)}{BPL} \sin (SPD \times t) \right) SPD M_{AG} \cos (SPD \times t - \theta) \, \mathrm{dt} \quad (D-12) \]

where \( CP1 \) = the heat capacity of the gas at constant pressure

\( dH_G \) = the enthalpy flow into the cylinder

\( dM \) = the mass flow into the gap

Net enthalpy flow per cycle is integrated by Rios to be:

\[ H_G = \int dH_G = \frac{\text{PIE}}{4} CP1 M_{AG} \Delta T \left( \frac{S}{L} \right) \sin \theta \quad (D-13) \]
\[ = \left( \frac{\text{PIE}}{4} \right) \left( \frac{\text{SHR}}{\text{SHR} - 1} \right) \left( \frac{\text{BST}}{\text{BPC}} \right) \left( \frac{1}{\text{RP}} \right) (PMX)(GGV) \sin \theta (QFS) \quad (D-14) \]
where:

\[
QFS = \left[ \frac{1}{BTW + BTC - BST} - \frac{RP}{BTW - BTCT + BPL} \right] - \frac{BTW + BTC + BST}{BTW - BTCT - BPL}
\]

\[
PIE = 3.14159
\]

\[
SHR = \text{the specific heat ratio of the gas}
\]

So total enthalpy flow is given by:

\[
QHG = \left( H_G x SPD \right) \left( 2 x PIE \right)
\]

\[
= \frac{PMX x GGV x BST x SHR x Sin \theta x SPD x NOC x QFS}{RP x 8 x BPL x (SHR - 1)}
\]

where

- QHG = the appendix loss
- NOC = the number of cylinders

**D.5.2 Results**

Some major errors were found. In a refrigerator, maximum pressure and minimum temperature occur almost simultaneously in the gap while in a heat engine the maximum pressure and maximum temperature occur almost simultaneously. The correction is shown in Equation D-6.

The second error had resulted from a confusion of signs in Rios thesis. In his derivation (69 am, 136-138) the mass difference correctly contains a subtraction sign, while on page 57 and in his sample calculation (Appendix I, page 178) the sign is incorrectly changed to a plus sign.

The computer program in Section D.3 gives the pumping loss as:

(See lines 435-438)

C *****Pumping loss*****

\[
QFS = \left( \frac{RP}{BTW/(BTW - 2 x BTC) - BST/BPL} \right) + \left( \frac{1}{BTW/((BTW - 2 x BTC) + BST/BPL)} \right)
\]

\[
QHG = ABS(SPD x PMX x GGV x BST x SHR x QFS x ARG x NOC/((SHR - 1) x BPL x RP x 8.))
\]

Based upon the analysis given above it should be:

\[
X = \frac{BTW + BTC1}{BTW - BTC1}
\]

\[
Y = \frac{BST/BPL}{RP/(X + Y)} + \frac{1}{(X - Y)}
\]

\[
QHG = ABS(SPD x PMX x GGV x BST x SHR x QFS x ARG x NOC/((SHR - 1) x BPL x RP x 8.))
\]

The formula for QFS is quite different. The formula for QHG is unchanged.
Let \( I = \frac{BTW}{(BTW - 2 \times BIC)} \)

Then the ratio of the new pumping loss to the old pumping loss, RATIO, is:

\[
\text{RATIO} = \frac{RP/(X + Y) + 1/(X - Y)}{RP/(Y + Z) - Y/(Y + Z)}
\]

For case 17 which is compared in detail in Section 7

PMX = 12.86 MPa, PMIN = 6.95 MPa

from the pressure - volume data for every 10°. Therefore

\[
RP = 12.86/6.95 = 1.860
\]

\[
BTW = 1033 \text{ K}
\]

\[
BIC = BIC1 = 330 \text{ K}
\]

Therefore:

\[
X = \frac{1033 + 330}{1033} = 1.939
\]

\[
Y = \frac{4.65}{0.727} = 6.4
\]

\[
Z = \frac{1033}{2(330)} = 2.769
\]

\[
\text{RATIO} = 0.211
\]

Therefore the true pumping (appendix) loss for case 17 is 14162.7(0.211) = 2988.8.

Now it only disagrees by a factor of 3 rather than 14.
APPENDIX E
ADIABATIC CYCLE ANALYSIS BY THE MARTINI METHOD

The method given below is a small extension of the work published earlier (75 af). It does not require the solution of a differential equation, but instead requires the solution at each time step of an algebraic equation that is implicit in the unknown pressure.

E 1 Nomenclature for Appendix E

A = initial temperature multiplier for expansion space
AD = phase angle, degrees
AR = phase angle, radians
B = initial temperature multiplier for compression space
C () = compression space volumes, cm³
CP = heat capacity of helium at constant pressure
   = 5.20 j/gk
CR = nondimensional, temperature corrected clearance ratio
    = 2*E*T / [ (DE/DR) + (DC/TE) ]
CS = CR*V/(2*E*T)
DA = angle increment, radians
DC = dead volume with compression space, cm³
DE = dead volume with expansion space, cm³
DR = Regenerator dead volume, cm³
DT = time increment, seconds
E = ratio between absolute temperature of heat rejection and heat reception
E () = expansion space volumes, cm³
F = crank angle measured from the minimum volume in the expansion space, radians
GA = (k-1)/k where k = Cp/Cv
   = .286 for hydrogen
   = 0.400 for helium
I = integer counter
I2 = counter to indicate which temperature will be solved for in Finkelstein equations.
IN = number of time increments per revolution
IM = IN
IX = iteration counter

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NOMENCLATURE (continued)

K = swept volume in expansion space/swept volume in compression space
K1 = V*CR/(R*2*E*T)
K2 = V/(2*E*W*R*T)
MC = mass flow into compression space g/sec.
ME = mass flow into expansion space, g/sec.
MH = measured heat input j/cycle
MR = gas inventory time gas constant, j/k
MW = measured work j/cycle
NC = nondimensional heat transfer coefficient for compression space
NE = nondimensional heat transfer coefficient for expansion space
OM = angular velocity, radians/sec
P( ) = common gas pressure, MPa
PI = 3.14159
PM = mean pressure
PQ = (P(I+1)/P(I))\up GA
R = gas constant for helium
= 2.0785 j/gk
SP = sum of the pressures
T = temperature of cylinder walls and heat exchange associated with
the expansion space, K.
T( ) = bulk gas temperature in the expansion space
TR = effective temperature of gas in regenerator, K
U = step function for expansion space; if ME >0 then U = 1 if not U = 0
U( ) = bulk gas temperature in the compression space
V = total swept volume of expansion space, cm³
VM = maximum VT(I)
VT(I)= E(I) + C(I)
W = total hydrogen gas inventory, grams
WC( )= mass of gas in compression space, grams
WE( )= mass of gas in expansion space, grams
WR = W*R
X = temporary variable
= step function for compression space
Y1 = trial expansion space temperature K
Z = counter to tell which gas
Z1 = trial compression space temperature K
E 2 Derivation of Equations

In general the total gas inventory at time increment I is:

\[ W = \frac{P(I) \cdot E(I)}{R \cdot T(I)} + \frac{P(I) \cdot C(I)}{R \cdot U(I)} + \frac{P(I) \cdot V \cdot C_R}{R \cdot 2 \cdot E \cdot T} \]  \hspace{1cm} (E1)

- mass in expansion space
- mass in compression space
- mass in dead spaces

\[ W = WE(I) + WC(I) + P(I) \cdot K_I \]  \hspace{1cm} (E2)

at time increment I + 1 the gas inventory is

\[ W = \frac{P(I+1) \cdot E(I+1)}{R \cdot T(I+1)} + \frac{P(I+1) \cdot C(I+1)}{R \cdot U(I+1)} + \frac{P(I+1) \cdot V \cdot C_R}{R \cdot 2 \cdot E \cdot T} \]  \hspace{1cm} (E3)

\[ W = WE(I+1) + WC(I+1) + P(I+1) \cdot K_I \]  \hspace{1cm} (E4)

In Equations E1 and E3 the knowns are W, E(I), E(I+1), R, C(I), C(I+1), V, CR, E, T. The unknowns are T(I), U(I) AND P(I) in Equation E1 and T(I+1), U(I+1) and P(I+1) in Equation E3. One must start by assuming T(∅) = T and U(∅) = E*T and then P(∅) can be calculated from Equation E1. Equation E3 still has three unknowns. To find a solution we must use the adiabatic compression law.

That is:

\[ \frac{T(I+1)}{T(I)} = \left( \frac{P(I+1)}{P(I)} \right)^{\frac{k-1}{k}} = PQ \]  \hspace{1cm} (E5)

where \( k = C_p/C_v = 1.40 \) for hydrogen. So \( (k-1)/k = 0.286 \). Also

\[ \frac{U(I+1)}{U(I)} = \left( \frac{P(I+1)}{P(I)} \right)^{0.286} = PQ \]  \hspace{1cm} (E6)

Equation E5 and E6 do not depend upon the mass of gas being considered. The mass may change. It does not matter. If WE(I+1) < WE(I) then gas is leaving the expansion space. For the gas in the expansion space Equation E5 applies. Thus by combining Equations E3, E4, and E5

\[ WE(I+1) = \frac{P(I+1) \cdot E(I+1)}{R \cdot T(I) \cdot PQ} \]  \hspace{1cm} (E7)
In the first edition of the Design Manual (78 ad, pp. 65-71) it was assumed that the masses of gas are proportional to volumes. However, this is not strictly true. For instance the volume of the expansion space may be decreasing so gas would be expected to be flowing out. However, if the total volume of gas is decreasing at a higher rate, gas may be flowing into this space instead of out of it. In consideration of this possibility a more exact formulation is given here than was used in the first edition of the Design Manual.

If $WE_{(I+1)} > WE_{(I)}$ gas is entering the expansion space. In this case we have two kinds of gas, the gas that was in there the whole time and the gas that entered.

For this case, the volume of the gas space at the end of the increment $E(I+1)$ is divided into two parts.

$$E(I+1) = ES(I+1) + EE(I+1)$$  \hspace{1cm} (EB)

The original gas volume shrinks to

$$ES(I+1) = \frac{WE(I) \cdot R \cdot TS(I+1)}{P(I+1)}$$  \hspace{1cm} (E9)

where $TS(I+1)$ is the new temperature of the original gas.

Substituting in Equation E5

$$ES(I+1) = \frac{WE(I) \cdot R \cdot T(I) \cdot PQ}{P(I+1)}$$  \hspace{1cm} (E10)

The new gas volume is calculated by:

$$EE(I+1) = \frac{(WE(I+1) - WE(I)) \cdot R \cdot TE(I+1)}{P(I+1)}$$  \hspace{1cm} (E11)

where $TE(I+1)$ is the new temperature of the entering gas. Since this gas starts at temperature $T$, application of Equation E5 gives

$$EE(I+1) = \frac{(WE(I+1) - WE(I)) \cdot R \cdot T \cdot PQ}{P(I+1)}$$  \hspace{1cm} (E12)
Combining Equation E8 with E10 and E12 gives

\[
E(I+1) = \frac{WE(I) * R * T(I) * PQ}{P(I+1)} + \frac{(WE(I+1) - WE(I)) * R * T * PQ}{P(I+1)}
\] (E13)

which reduces to

\[
WE(I+1) = \frac{(E(I+1) * P(I+1))}{R * PQ} - WE(I) * (T(I) - T) T
\] (E14)

Similarly for the compression space, if gas is flowing out, that is if \(WC(I+1) < WC(I)\) then

\[
WC(I+1) = \frac{P(I+1) * C(I+1)}{R * U(I) * PQ}
\] (E15)

If gas is flowing in, that is \(WC(I+1) > WC(I)\) then

\[
C(I+1) = \frac{WC(I) * R * U(I) * PQ}{P(I+1)} + \frac{(WC(I+1) - WC(I+1)) * R * E * T * PQ}{P(I+1)}
\] (E16)

which reduces to

\[
WC(I+1) = \frac{C(I+1) * P(I+1) - WC(I) [U(I) - E * T]}{R * PQ}
\] (E17)

To calculate \(WE(I+1)\) and \(WC(I+1)\) one does not need to calculate the next temperatures, \(T(I+1)\) and \(U(I+1)\) because they are worked into Equations E7 to E17. However, these temperatures will be used in the next increment and must be calculated. If \(WE(I+1) > WE(I)\) then gas is entering the expansion space. The temperature of the gas already in this space becomes:

\[
T(I+1) = T(I) * PQ
\] (E18)

and the temperature of the gas entering the expansion space is:

\[
T(I+1)^1 = T * PQ
\] (E19)

The average gas temperature is the mass average of these two gas masses so

\[
T(I+1) = \frac{T(I) * PQ * WE(I) + T * PQ * (WE(I+1) - WE(I))}{WE(I+1)}
\] (E20)
If $WE(I+1) < WE(I)$ then $T(I+1)$ is calculated by Equation E18.

The temperatures in the compression space are treated in a similar way. If $WC(I+1) > WC(I)$ then

$$U(I+1) = \frac{U(I)*PQ*WC(I) + T*E*PQ*(WC(I+1) - WC(I))}{WC(I+1)}$$  \hspace{1cm} (E21)

If $WC(I+1) < WC(I)$ then

$$U(I+1) = U(I)*PQ$$  \hspace{1cm} (E22)

The calculation proceeds in the following order:

1. Pick $P(\emptyset)$ from the known initial conditions given a measured pressure or a pressure computed assuming gas spaces have surrounding metal temperature.

2. For the next time step choose $P(I+1)$ the same as $P(I)$, $P(0)$ the first time around.

3. If $E(I+1) > E(I)$ calculate $WE(I+1)$ by Equation E14 if not by Equation E7.

4. If $C(I+1) > C(I)$ calculate $WC(I+1)$ by Equation E15 if not by Equation E17.

5. Calculate the mass balance error $EE$ by:

$$EE = WE(I+1) + WC(I+1) + P(I+1)*K1 - W$$ \hspace{1cm} (23)

6. Choose another $P(I+1)$ 1% greater than $P(I)$.

7. If the already calculated $WE(I+1) > WE(I)$ then calculate $WE(I+1)$ by Equation E14; if not then by Equation E7 (Using $P(I+1)$ from Step 6).

8. If the already calculated $WC(I+1) > WC(I)$ then use Equation E15; if not, Equation E17 (Using $P(I+1)$ from Step 6.)


10. By the secant method estimate what $P(I+1)$ should be by extrapolation or interpolation of the two errors and the two pressures to determine what pressure would give zero error.

11. Repeat steps 7, 8, 9, and 10 until convergence is obtained at an error in mass balance of less than one part per million.

12. Accumulate integral of $VT(I)$ vs. $P(I)$ curve to obtain work output per cycle.

13. Accumulate integral of $E(I)$ vs. $P(I)$ curve to obtain heat input per cycle.

14. If $WE(I+1) > WE(I)$ then calculate $T(I+1)$ by Equation E20; if not then by Equation E18.

15. If $WC(I+1) > WC(I)$ then calculate $U(I+1)$ by Equation E21; if not then by Equation E22.
16. Index to the next set of expansion and compression space volumes and start over with step 2.

17. After one full revolution, print out the value of the integrals accumulated and compare the pressure at 360° with the pressure at 0°. If the error is greater than 0.1%, then repeat the cycle.

The above calculation procedure has been programmed using a TRS-80 computer in the Basic language.

Martini Adiabatic Cycle Results

The first thing to show is that this calculation procedure gives exactly the same results as the Finkelstein-Lee method (60 v, 76 bl). Table E1 compares the results. Time steps from 12 per cycle to 240 per cycle (30° increment to 1.5° increment) are shown. The 240 per cycle was as large as the 16K storage TRS-80 computer available at the time could handle with the computer formulation which saves all results in arrays. Figure E1 shows how the numerical results extrapolate to zero angle increment. The extrapolation (Figure E1, Table E1) is in all cases extremely close to what Finkelstein said it would be. The agreement is amazing since Ted Finkelstein performed these calculations without benefit of computer. One important thing to note is that relatively large angle increments can be used still with reasonable accuracy. For instance, for a 15° angle increment the errors are:

<table>
<thead>
<tr>
<th></th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Ratio</td>
<td>-1.05</td>
</tr>
<tr>
<td>Work Required</td>
<td>+0.88</td>
</tr>
<tr>
<td>Heat Input</td>
<td>-2.37</td>
</tr>
<tr>
<td>Coefficient of Performance</td>
<td>-3.30</td>
</tr>
</tbody>
</table>
Table E.1

COMPARISON OF FINKELSTEIN ADIABATIC CYCLE CALCULATIONS AND MARTINI ADIABATIC CYCLE CALCULATIONS

Sinusoidal Motion, $K = 1$, $E = 2$, $CR = 1$, $AD = 90^\circ$

<table>
<thead>
<tr>
<th>This Report Degree Increment</th>
<th>Steps Cycle</th>
<th>Maximum Press joules cycle</th>
<th>Minimum Press joules cycle</th>
<th>Energy Output joules cycle</th>
<th>Heat Input joules cycle</th>
<th>Coefficient of Performance</th>
<th>Iterations Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>12</td>
<td>5.198</td>
<td>-0.87831 WRT</td>
<td>0.453119 WRT</td>
<td>0.515899</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>24</td>
<td>5.2140</td>
<td>-0.894804 WRT</td>
<td>0.471572 WRT</td>
<td>0.527012</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>5.1930</td>
<td>-0.890696 WRT</td>
<td>0.480606 WRT</td>
<td>0.539584</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>5.178</td>
<td>-0.888513 WRT</td>
<td>0.481783 WRT</td>
<td>0.542235</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>240</td>
<td>5.1742</td>
<td>-0.887832 WRT</td>
<td>0.482141 WRT</td>
<td>0.543054</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$\infty$</td>
<td>5.162</td>
<td>-0.8865 WRT</td>
<td>0.483 WRT</td>
<td>0.545</td>
<td>Extrapolation</td>
<td></td>
</tr>
</tbody>
</table>

Finkelstein (Ref. 60) Not Given 5.16 -0.886 WRT 0.481 WRT 0.543
Figure E-1. Extrapolation of Results to Zero Angle Increment.
APPENDIX F
NON-AUTOMOTIVE PRESENT APPLICATIONS AND FUTURE APPLICATIONS OF STIRLING ENGINES

In this appendix "present applications" will be defined as products that are for sale on the open market as well as products that are in limited production and are for sale even if the sale is restricted or at a very high price.

F1 Present Applications

F1.1 Demonstration Engines

Small, inexpensive demonstration engines are excellent educational tools and serve well to inform the general public and the technical community of new technical possibilities. Two Stirling engines made by Solar Engines of Phoenix, Arizona, (Figure F1) have been widely advertised and sold. Model 1 sells with a book on Stirling engines by Andy Ross. Model 2 comes assembled with a parabolic mirror for solar heating.

From the author's own experience, both of these engines work reliably and have a high no-load speed, but can produce very little power. However, tests have shown that they produce about 60 percent of the maximum possible indicated power, considering the temperature applied, the speed and the displacement of one atmosphere air.

Two handsome models are offered by ECO Motor Industries Ltd., Guelph, Ontario, Canada (See Figure F-2). These engines are fired with methyl alcohol. The "Stirling" hot air engine uses a unique linkage devised by Mr. Pronovost, the proprietor. The "Ericsson" engine models the linkage of the improved Ericsson pumping engine of 1890. Both engines come with assembly and operating instructions and working drawings.

A model Stirling engine designed especially as a classroom demonstration of a heat engine and a cooling engine is available from Leybold-Heraus, Koln, Germany (See Figure F-3). It produces measurable power (about 10 watts). The engine has glass walls so the movement of both the piston and the displacer can be observed.

Sunpower has offered for sale a classroom demonstrator for a number of years. So far about 50 of these demonstrators have been sold. In the fall of 1976 I was asked to analyze one that had been modified for laser heat input. In its original condition I calculated this engine could produce about 7 watts indicated power at an indicated efficiency of 15 percent. This engine operated at 2.5 atm average pressure and 20 Hz with helium. The rub was (literally) that the measured combined mechanical efficiency and alternator efficiency was only 12.4 percent. The presently reported characteristics are: 41 cm high, 23 cm square base, 4 Kg, 2-10 watts output. Prices were (Aug. 1978):
a. Model 1 - Flame heated engine. (77 br)

b. Model 2 - Solar heated engine. (79k)

Figure F-1. Stirling Engines by Solar Engines.
Figure F-2. ECQ Motor Industries engines.

- "Ericsson" Hot Air Engine.
- "Stirling" Hot Air Engine.
Figure F-3. The Leybold-Heraus Model Hot Air Engine.

Figure F-4. The Model SD-100 Sunpower 70 w Electric Power Source.
Figure F-6. Stirring Power Systems & kW Stirring Electrode Combined with a
6.5 kW, 60 Hz, 110/240 Volt Generator.

Figure F-5. The Harwell Thermo-Mechanised Generator.
Model 10B with factory installed water pump $500
Alternator to fit 10B engine $400
Fresnel lens with mount and clock drive $640
Propane heater to replace 100 w electric heater $100
Cooler $50
Refrigerant pump with inertia compressor $200

This engine is still a reasonable starting point to learn first-hand about Stirling engines of intermediate efficiency. With intelligent improvements one can show up to 20 percent overall efficiency from this engine.

F1.2 Electric Power Generators

Stirling electric power generators are beginning to be applied because they have been shown to be very reliable and quiet.

Sunpower’s Model SD-100 generator produces 70 w (e) of 12 VDC electric power (See Figure F-4). It operates at 35 hz with helium at 16 bar. Propane heats the engine to 650 °C. It operates silently. It has operated an electric trolling motor at full power. Current developmental price is $5,000 each!

AGA Navigation Aids Ltd. is selling the thermo-mechanical generator (TMG) developed at Harwell, England (77 t.) Their 25 watt machine when operating on propane uses only 27 percent of the fuel required by a 25 w (e) thermo-electric generator. In addition, the TMG shows no power degradation after over four years of operation. Two models are available: a 25 watt, 10 percent efficient machine; and a 60 watt, 9 percent efficient machine. Generators up to 250 watts are planned. Two are in actual use. Figure F-5 shows a developmental TMG before it was installed in the National Data Buoy off Land’s End, England.

Stirling Power Systems of Ann Arbor, Michigan, has eight 8 kw Stirling engines from FFV of Sweden built into automatic total power systems for Winnebago motor homes (79 ap). Figure F-6 shows the power system ready for installation into the side of the vehicle. The power system is entirely automatic. It starts from cold in 15 seconds. Electricity is supplied to the electric refrigerator, stove and air conditioner and lights. Waste heat from the engine is supplied to convectors in the motor home if heat is needed or to the radiator on the roof if it is not.

This development incorporates improvements in the full system much of which is not related to the Stirling engine. However, in this system two prime features of the Stirling engine are demonstrated—quietness and reliability. Table F-1 compares the measured sound level at various points of a Stirling engine equipped motor home with the same home equipped with a gasoline engine. Note that the conventional powered system is 250 percent more noisy than the Stirling-powered machine. To calibrate the dBA sound rating, 62 dBA is a kitchen exhaust fan and 59 dBA is a bathroom exhaust fan as used on a motor home. Reliability is as yet not proven because none of them are in the hands of the average customer. The life of a Stirling engine is estimated at 5,000 to 10,000 hours compared with 2,000 hours for an Otto cycle engine. Projected maintenance requirements (Table F-2) are speculative, but indicate that the motor home owner who will probably not care for the gasoline engine as well as he should would be much better off with the Stirling engine.

Present models operate on unleaded gasoline to use the same fuel as the motor home engine. Later models will be equipped to operate on various types of fuels including diesel oil, fuel oil, and kerosene.
Table F-1. Sound Level Measurements (78 cl)

<table>
<thead>
<tr>
<th></th>
<th>STIRLING ENGINE</th>
<th>OTTO-CYCLE ENGINE</th>
<th>% Higher Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>A weighted scale,</td>
<td>55 dBA</td>
<td>80 dBA</td>
<td>250%</td>
</tr>
<tr>
<td>one meter from</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>source, outside</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kitchen, inside</td>
<td>51 dBA</td>
<td>56 dBA</td>
<td>50%</td>
</tr>
<tr>
<td>Rear Seats, inside</td>
<td>48 dBA</td>
<td>58 dBA</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table F-2. Projected Maintenance Requirements

<table>
<thead>
<tr>
<th></th>
<th>STIRLING ENGINE</th>
<th>OTTO-CYCLE ENGINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check Oil</td>
<td>N/A</td>
<td>20 hours</td>
</tr>
<tr>
<td>Change Oil</td>
<td>N/A</td>
<td>150 hours</td>
</tr>
<tr>
<td>Change Oil Filter</td>
<td>N/A</td>
<td>300 hours</td>
</tr>
<tr>
<td>Change Spark Plugs</td>
<td>N/A</td>
<td>500 hours</td>
</tr>
<tr>
<td>Tune-Up</td>
<td>N/A</td>
<td>500 hours</td>
</tr>
<tr>
<td>Add Helium Bottle</td>
<td>2,000 hours</td>
<td>N/A</td>
</tr>
<tr>
<td>Change Igniter</td>
<td>2,000 hours</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Fuel economy, a major advantage in other Stirling engines, is not true here. It is reported that the Stirling system uses slightly less fuel than its conventional counterpart. Designers of the engine purposely traded off efficiency for lower manufacturing costs.

F1.3 Pumping Engines

The old hot air engines were used almost entirely for pumping water. Today only one is known to be almost ready for sale. Metal Box India has been developing a fluid piston engine. According to Dr. Colin West, they have one that will pump water ten feet high at an efficiency of 7 percent using propane gas as fuel. They plan to market a coal-fired machine in India.

F2 Future Applications

For this manual, "future applications" are defined as one-of-a-kind engines on out through just an idea. Treatment in this section will be brief with the reference being given if possible.
F2.1 Solar Heated Engines

Solar heated Stirling engines are not new. John Ericsson built one in 1872 (77 br). Now they are seriously being considered. Pons showed that system cost of solar Stirling power in mass production is projected at 5¢/kwh (79 dk.) Presently utilities are purchasing new capacity at 5¢/kwh. This study plans an 18.6 m (61 ft) diameter front braced mirror with a P-75 engine at the focus.

Sunpower, Inc. has designed and built a 1 kw free piston Stirling engine directly connected to an alternator (78 ac). Performance (78 as) of 42 percent engine efficiency at 1.25 kw output at 60 Hz from a 10 cm diameter power piston operating with an amplitude of 1 cm and a charge pressure of 25 bar has been predicted for the SPIKE (See Figure F-7.) A different test engine which could be solar heated attained a measured 32 percent efficiency at 1.15 kw output (79 ar). Solar heated engines of 100 kw size operating at 60 Hz are envisioned.

Mechanical Technology Incorporated has been doing the linear generator for the above development. The generator efficiency has hit 90 percent, but because of gas spring losses, engine efficiency of 33 percent is degraded to 19 percent system efficiency. MTI plans a 15 kw, 60 Hz engine-generator for a dispersed mirror solar electric system.

F2.2 Reliable Electric Power

Besides those developments already in the present application category DOE is sponsoring two different developments for isotope-powered electric power generation in remote locations. One uses the Philips Stirling engine (79 aq). The other uses a free-piston engine and linear electric generator (79 aq, 79 am). These developments had been linked to radioisotope heat, but this part was cancelled. These engines use electric heat. Plans are to substitute a combustion system.

F2.3 Heat Pumping Power

Stirling engines in reverse, heat pumps, have enjoyed a good market in the cryogenic industry to produce liquified gases and to cool infrared sensors and the like (77 ax). Stirling engines have also been tested to take the place of the electric motor in a common Rankine cycle heat pump for air conditioning (77 ad, 78ax, 79 at). One free-piston engine pump is being developed for this purpose (77 w). Engine driven heat pumps have the advantage of heating the building with both the waste heat from the engine and the product of the heat pump (77 j). Also being considered and undergoing preliminary testing are Stirling heat engine heat pumps. These could be two conventional Stirling engines connected together (73 x) or free-piston machines which eliminate much of the machinery and the seals (69 h). Using machines of this type it appears possible that the primary fuel needed to heat our buildings can be greatly reduced to less than 25 percent of that now being used (77 h, 78 p). With this type of incentive Stirling engines for house heating and cooling may be very big in the future.
SUNPOWER I KILOWATT ENGINE SPIKE

Figure F-7.
Biomedical Power

Miniature Stirling engines are now being developed to power an artificial heart (72 ak). Indeed this engine appears uniquely suited for this application since it is very reliable and can be made efficient in small sizes. One engine of this size ran continuously for 4.07 years before both electric heaters failed. Most engine parts had operated 6.2 years with no failures. Once the blood pump compatibility with the body is improved to the order of years from the present six months then this application area will open up.

Between the tens to hundreds of horsepower required for automobiles and the few watts required for artificial hearts may be many other applications. For instance, powered wheelchairs now use a cumbersome lead-acid battery and control box between the wheels and an electric motor belt driving each large wheel. With a Stirling engine and thermal energy storage the same performance might be obtained, using a TES-Stirling engine, belt driving each wheel with the speed controlled electrically. The large battery box and controls could be dispensed with and the chair could become truly portable by being collapsible like an unpowered wheelchair. There may be many specialized applications like this.

Central Station Power

Many people have asked if Stirling engines are useful in the field of central station electric power. Very little has been published attempting to answer this question (68 k). R. J. Meijer (77 bc) calculates that Stirling engines can be made up to a capacity of 3,000 HP/cylinder and 500 HP/cylinder Stirling engines have been checked experimentally using part engine experiments (77 bc). Many simple but efficient machines could be used to convert heat to say hydraulic power. Then one large hydraulic motor and electric generator could produce the power. In the field of advanced electric power generation it should be emphasized that the Stirling engine can operate most efficiently over the entire temperature range available and could supplant many more complicated schemes for increasing the efficiency of electric power generation.

Argonne National Laboratory has the charter from DOE to foster 500 to 2,000 HP coal-heated neighborhood electric power total energy systems (78 g, 79 ai, 79 aj). Initial studies show that straightforward scale-up of known Stirling engines and the applications of known materials could lead to considerable improvement in our use of coal.

Third World Power

Stirling engines in some forms are very simple and easy to maintain. They can use available solid fuels more efficiently and attractively than the present alternative. Metal Box India's development of a coal-fired water pump has already been mentioned. Also it has just been demonstrated that 1 atm minimum pressure air engines (79 bj, 79 ar) designed with modern technology can generate 880 watts while an antique engine of the same general size only generated 50 watts. There is probably a very good market for an engine that would fit into a wood stove or something similar and operate a 12 volt generator or a water pump. The waste heat from the engine would still be usable to heat water or warm the room and electricity would be produced as well.
F2.7  Power For Other Uses?

Who is to say whether the above list of uses is complete. As these machines come into use and many people become involved in perfecting them for their own purposes, many presently unforeseen uses may develop. A silent airplane engine may even be possible for small airplanes. The Stirling engine is still a heat engine and is limited to the Carnot efficiency as other heat engines are, but it appears to be able to approach it more closely than the others. Also the machine is inherently silent and uses fewer moving parts than most other engines. What more will inventive humans do with such a machine? Only the future can tell.