DESIGN AND ANALYSIS OF COMPOSITE STRUCTURES
WITH STRESS CONCENTRATIONS

S. P. Garbo
McDonnell Aircraft Co., McDonnell Douglas Corporation
St. Louis, Missouri
INTRODUCTION

This presentation provides an overview of an analytic procedure which can be used to provide comprehensive stress and strength analysis of composite structures with stress concentrations. The methodology provides designer/analysts with a user-oriented procedure which, within acceptable engineering accuracy, accounts for the effects of a wide range of application design variables. The procedure permits the strength of arbitrary laminate constructions under general bearing/bypass load conditions to be predicted with only unnotched unidirectional strength and stiffness input data required.

In particular, the presentation includes:

(1) A brief discussion of the relevancy of this analysis to the design of primary aircraft structure
(2) An overview of the analytic procedure with theory/test correlations
(3) An example of the use and interaction of this strength analysis relative to the design of high-load transfer bolted composite joints

Many of the results presented were taken from two recently completed MCAIR programs funded by the U.S. Air Force and the U.S. Navy (refs. 1 and 2).
DESIGN/ANALYSIS ITERATIONS

The usual project sequence of design/analysis iterations is illustrated. The emphasis of methodology discussed in this presentation is in the areas of (1) detailed stress analysis about a structural discontinuity or stress riser which results in local stress concentrations and (2) strength analysis performed on the stress (strain) solutions to determine load capability.
The planview of the F/A-18A wing structure is presented to illustrate the prevalence of stress risers in typical primary aircraft structure. This production structure is comprised of monolithic composite skins mechanically fastened to aluminum spar and rib substructure. Evident are numerous stress risers due to required access doors, cutouts, and pylon post holes required for attachment of external equipment. Additionally, fastener holes are required along all spars and ribs to mechanically attach wingskins to substructure, and local load introductions (e.g., leading- and trailing-edge flap attachments) and major structural splices (e.g., wingfold) result in many required high-load transfer joints. The point to be made is that for current and near-term aircraft, accurate analysis of the effects of stress risers on structural efficiency is of prime importance.
A visual overview of typical coupon, element, and subcomponent specimens tested during the Harrier composite wing development program illustrates again the concern of structural designer/analysts with the effects of stress risers on structural integrity. In almost all structural tests depicted, the presence of a stress riser is evident. For composite materials which possess little of the ductility (forgiveness) of metals, the presence of these geometric details results in structure designed to static strength requirements. This emphasizes the need and importance of comprehensive, accurate, and detailed internal load, stress, and static strength analyses. Without such analysis, costly and extensive test programs are required which are often repeated when changes in load conditions, stress riser geometry, and layup occur. The result is a project dependence on empirically derived allowables which significantly limit full utilization of laminate tailoring design options.
STIFFNESS AND STRENGTH DESIGN DATA

The items listed under lamina (unidirectional ply) and laminate indicate variables which affect stiffness and strength. Fundamental differences between the mechanical behavior of metals and composites are due to the composite stiffness and strength anisotropy and its material inhomogeneity. These factors complicate considerably the complete characterization of lamina mechanical properties relative to metals and introduce the necessity of additional strength characterization requirements at the laminate level for each layup variation. The objective of our analysis development has been to provide a comprehensive and general laminate strength analysis based primarily on lamina stiffness and strength data, thus minimizing the level of laminate testing currently required for applications.
AS/3501-6 GRAPHITE-EPOXY LAMINA PROPERTIES

This table summarizes stiffness and strength data along the principal material axes of the AS/3501-6 graphite-epoxy material system. These data are usually generated by testing unnotched specimens and most data are routinely obtained during material quality assurance testing performed by companies at the time material shipments are received. These data are the basis upon which general laminate stiffness, stress, and strength analyses are performed using the procedures discussed in this presentation.

| MECHANICAL PROPERTY | RTD  
\(^{\Delta}\) | RTW  
\(^{\Delta}\) | ETW  
\(^{\Delta}\) |
<table>
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<tr>
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<td>18.85</td>
<td>18.85</td>
<td>18.54</td>
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<td>18.20</td>
<td>17.80</td>
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<td>(\nu_{12})</td>
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<td>236</td>
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<tr>
<td>(F_{1}^{cu}) (KSI)</td>
<td>321</td>
<td>179</td>
<td>111</td>
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<td>9.5</td>
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<td>(F_{12}) (KSI)</td>
<td>17.3</td>
<td>19.8</td>
<td>11.9</td>
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\(^{\Delta}\) Room temperature dry (as manufactured)

\(^{\Delta}\) Room temperature with 1.05% moisture

\(^{\Delta}\) Elevated temperature \(\sim 250^\circ F\) and 1.05% moisture
INTERNAL LOADS ANALYSIS

While this is not the main subject of this presentation, it is important to realize that the needed detailed stress and strength analyses, especially for bolted composite joints, are directly related to internal load distribution analysis. Should local laminate failures occur due to fastener loadings, joint-member and fastener-to-joint-member flexibilities (spring rates) will be altered and internal load paths will change. The variables listed are important to accurate internal load analysis and have direct effects on laminate point strength analysis.

- GEOMETRY
  - IN-PLANE
  - THRU-THE-THICKNESS
- MEMBER STIFFNESSES
- FASTENER STIFFNESSES (SPRING RATES)
- MANUFACTURING TOLERANCES
- MULTIPLE FASTENERS
- LOADING COMPLEXITIES
  - TENSION
  - COMPRESSION
  - SHEAR
  - VARIATIONS
- MATERIAL NONLINEARITIES
DETAILED STRESS DISTRIBUTION ANALYSIS

This list represents some of the more important variables typical of general aircraft structure. The initial thrust of our strength analysis development was to provide a user-oriented procedure which would account for the effects of fastener holes on laminate strength for arbitrary laminates under general biaxial and fastener bearing loads. Currently we are extending these procedures to account for the general class of stress risers, through-the-thickness variables, and finite-geometry effects.

- **GENERAL LOADINGS**
  - Biaxial
  - Bearing/Bypass
  - Bending/Pressure

- **GEOMETRY**
  - Hole size
  - Cutout shape
  - Width (external boundaries)
  - Multiple stress risers
  - Hole tolerances

- **THROUGH-THE-THICKNESS**
  - Stacking sequences
  - Clamp-up (torque-up)
  - Joint eccentricity
  - Fastener types (CSK)
  - Foundation stiffness
  - Orthotropy

- **MATERIAL NONLINEARITIES**

- **ENVIRONMENT**
FAILURE PREDICTION OF BOLTED COMPOSITE JOINTS

An overview of the MCAIR analytic approach is idealized in this illustration. The principle of superposition is used to obtain laminate stress and strain distributions due to combined bearing and bypass loads. Once laminate stresses and strains are known, failure analysis is performed at a characteristic dimension \( R_c \) away from the hole. This bolted joint stress field model (BJSFM) generalizes Whitney and Nuismer's postulated characteristic dimension concept to apply to anisotropic laminates under general biaxial loadings (ref. 3). Failure predictions are made on a ply-by-ply basis by comparing material lamina allowables with elastic point stresses calculated for each ply at a characteristic dimension from the edge of the hole. Various material failure criteria (Tsai-Hill, maximum stress, etc.) available for user selection can then be used to compare stresses with lamina allowables.

This program is available to the industry through the Air Force Flight Dynamics Laboratory and is detailed in reference 1. This user-oriented program can be used to generate complete stress (strain) solutions at or about the hole, or just the failure load, critical ply, and failure location.
The need for detailed distributions in orthotropic laminates is illustrated by using the BJSPM program to determine circumferential stress distributions about a fastener hole loaded by pure bearing. A 50/40/10 layup and a 70/20/10 layup from the 0°, ±45°, and 90° family of ply orientations were analyzed. Solutions for both of these layups are compared with solutions obtained for an isotropic material. In the first two comparisons, bolt bearing was directed along the 0° ply orientation (principal material axis). In comparison with the dashed line representing isotropic material stress solutions, orthotropy in the two composite layups causes a marked change in maximum stress and locations. This directional dependence increases with the degree of orthotropy (higher percentage of 0° plies relative to load direction), as can be observed by comparing the solutions of the 70/20/10 layup with the 50/40/10 layup. Additionally, if bolt bearing is directed off the principal material axis, the circumferential stresses change markedly. The dashed line illustrates the fact that solutions for an isotropic (metal) material are unaffected by change in bolt bearing direction.

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<table>
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<th>Layup</th>
<th>Orientation</th>
<th>Stress Distribution</th>
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<tr>
<td>50/40/10</td>
<td>θ = 0°</td>
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<tr>
<td>70/20/10</td>
<td>θ = 0°</td>
<td>Isotropic</td>
</tr>
<tr>
<td>70/20/10</td>
<td>θ = 45°</td>
<td>Isotropic</td>
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isotropic
EFFECTS OF FINITE GEOMETRY

To evaluate the effects of finite geometry on stress solutions and establish limits on the accuracy of infinite plate solutions, a comparison of BJSFM solutions with finite element solutions was performed. In this illustration, circumferential stresses normalized to the average bolt bearing stress are plotted about the half-circle from directly in front of the neat-fit bolt (0°) to directly behind the bolt (180°). The dashed line indicates the infinite plate BJSFM solutions, the solid line represents the BJSFM solution corrected using the DeJong finite-width approximation method, and the triangular symbols represent the finite element solution. Results indicated that for an e/D of 9, the BJSFM solutions, corrected for finite width, correlate extremely well with finite element solutions where W/D ratios were greater than 4. However, as indicated in this graph, at an e/D value of 3, BJSFM infinite-plate solutions that were approximately corrected for finite widths (long-dash line), while significantly improved over the original BJSFM infinite-plate solution (short-dash line), still differ from the correct finite geometry solution (triangle symbol line). Full correction for finite geometry is anticipated using boundary collocation procedures currently under development at MCAIR.

SMALL EDGE DISTANCE

\[ \frac{\sigma_\theta}{\sigma_{BR}} \]

\( \theta \) LOCATION ABOUT HOLE - DEG

Finite element model
BJSFM - finite width
BJSFM - infinite plate

Layup: 50/40/10
The previous results ignore thickness-direction variables. These variables can be important and additional procedures are required to account for their effects. Some of the related through-the-thickness variables are listed here. New procedures using beam-on-elastic foundation theory are currently being developed at MCAIR to examine the effect of these design variables on fastener-to-joint-member spring rates and on bearing stress distributions in the thickness direction. Preliminary results of one parametric study are shown that relate the peaking of bearing stresses (relative to an average bearing distribution) to fastener head fixity.

- Joint eccentricity
- Interlaminar stresses
- Stacking effects
- Thickness effects
- Orthotropy effects
- Fastener fit
- Inter-ply reinforcements

![Diagram showing peaking factor and head fixity](image-url)
STRENGTH ANALYSIS

Given a detailed ply-by-ply stress analysis, strength predictions using the characteristic dimension failure hypothesis can be made. Point material failure criteria (Tsai-Hill, maximum strain, etc.) are used to predict ply matrix, fiber, or shear failure about the fastener hole at a distance \( R_c \) away from the hole boundary. This is automatically performed by the BJSPM program by comparing the complete stress (strain) solution about the fastener hole boundary with user-selected material failure criteria. Critical plies, failure loads, and locations are identified. However, in addition to failure at a point, the "patterns" of failure must be evaluated to predict overall joint modes of failure such as bearing, shearout, or net section. Examples of these predictions and correlations with data are presented in the following illustrations.

- LOCAL AND GLOBAL...
  - LOAD LOCATION CRITICAL PLIES
  - MATRIX CRACKING
  - FIBER RUPTURE
  - SHEAR
  - COMPRESSION
  - INTERLAMINAR
  - FIRST PLY
  - PROGRESSIVE

- GENERAL LOAD CONDITIONS
- ENVIRONMENTAL FACTORS
- MATERIAL NONLINEARITIES
Examples of laminate strength predictions under general bearing and bypass loadings are presented for a wide range of layups. The higher stress predictions of each of the solid and dashed line sets represent predictions at the characteristic dimension away from the hole boundary. The solid line represents predicted ply fiber or shear failure and correlates within ±10 percent with the test data for this wide range of layups. The dashed lines represent predicted matrix failure (i.e., matrix cracking due to tensile load transverse to fibers) but no correlation of this predicted failure with changes in mechanical properties was observed, and this type of ply failure was ignored in all subsequent analyses. The lower strength predictions of each of the solid and dashed line sets represent application of failure criteria on the hole boundary. The difference between predictions on the hole boundary \( R_c = 0.0 \) inch or away from the hole boundary \( R_c = 0.02 \) inch correlates with the conservatism inherent in strictly using elastic stress concentration factors to predict strength.

**PREDICTED PLY MATRIX, FIBER, AND SHEAR FAILURES**
HOLE SIZE AND LAYUP EFFECTS

Hole size effects are accounted for analytically. Stress gradients at the edge of the hole vary with hole size and laminate orientation. Smaller diameter holes produce steeper stress gradients which decay rapidly as the distance from the hole is increased. This confines maximum stress to a smaller area in the laminate; thus a laminate has greater load-carrying capability with a small hole than with a larger hole. Analytical correlation is shown with strength data for various hole sizes and laminates loaded in tension and compression.

![Graphs showing stiffness data for different hole sizes and laminates.](image-url)
OFF-AXIS TESTING

Test data are compared with predictions for a laminate with a countersunk hole that is loaded to failure uniaxially at various angles relative to the principal material axis. Countersunk fastener holes were analyzed as being equivalent in diameter to a constant-diameter hole which displaces the same volume of material as the countersunk hole. The correlation shows that the BJSFM procedure can account for countersunk hole configurations, strength anisotropy, and off-axis loadings while using a constant characteristic dimension ($R_C$).
The previous theory-test correlations presented strength data for laminates with an unloaded hole. As bearing stresses are applied to the hole boundary, bypass strength decreases and noncontinuous or nonlinear load-deflection behavior is observed prior to specimen ultimate failure. At low bearing stress conditions, strength predictions based on first-ply fiber failure correlate very closely with the ultimate specimen strength. However, as bearing loads increase, first-ply fiber and shear failures correlate with the occurrence of initial nonlinear or discontinuous load-deflection behavior.
The local nature of failure due to pure bearing loads is examined analytically using the BJSFM procedure. The results shown indicate the critical plies and contours of equal strain levels for an 80-ksi bearing load when first ply failure is predicted. Ply failures occur at 0° and 45° due to excessive shear. The contours indicate where failure patterns in specimens would likely occur. The C-scan documents the occurrence of local damage at this bearing stress level, and qualitatively agrees with predicted failure contours.
The analysis of the previous pure bearing case was extended to include the added effect of a bypass load equivalent to a stress of 30 ksi. The critical plies and contours are indicated. The patterns have shifted to the net section and now the $0^\circ$ ply has become fiber critical (before it was shear critical); the $45^\circ$ ply is still shear critical, but this is now at the net-section region as well as in front of the bolt. The same specimen previously loaded to 80 ksi of bearing only was loaded to that same bearing load and then further loaded to a bypass stress of 30 ksi. The C-scan again qualitatively verified predicted failure patterns.
In the off-the-principal-material-axis study, the BJSFM procedure was used to examine the directional dependence of pure bearing strength in a 50/40/10 laminate. Test data with scatter bands are compared with predictions. Onset of joint nonlinear load deflection behavior is indicated by open triangle symbols. Specimen ultimate bearing strengths are indicated by open circles and occurred at approximately 50 percent higher loads. Specimens failed predominantly in bearing shearout at off-axis load directions ranging from 0° to 22-1/2°, tension-cleavage at the 45° off-axis load direction, and net-section failures at the 67-1/2° to 90° off-axis load directions. Solid symbols indicate specimens tested with no fastener torque-up; all others were torqued to 50 in.-lb. Strengths were predicted based on ply fiber or shear failure using the 0.02-in. characteristic dimension and the maximum strain failure criterion. Compared to off-axis loading of unloaded hole specimens, two important characteristics are predicted: (1) laminate strength is relatively constant over the entire range of off-axis load directions, and (2) ±45° and 90° plies are equally critical within 10 percent. This is in contrast to the unloaded hole case in which a pronounced sensitivity to off-axis bypass loading was observed and singular ply orientations were usually critical.

Unlike loads at onset of nonlinear joint load deflection behavior, test results indicate that ultimate bearing strengths are insensitive to off-axis loading effects at all but the 0° and 90° loading directions. In all cases, specimen failures occurred after permanent hole elongations had developed. The onset of the associated transition from linear to nonlinear failure modes was predictable using the linear-elastic BJSFM procedure.

(50/40/10 LAYUP)
A further verification of the BJSFM strength procedure was performed using a boundary collocation procedure to account for finite geometry effects. The 1.50-in.-thick specimen was designed to contain two different lug sizes that would be loaded in pure bearing. The lower left photograph indicates the failed left end, and the lower right C-scan documents the close-to-failure damage of the right end of the specimen. Failure modes of both ends were as predicted.

\[ P_{\text{TEST}} = 150 \text{ kips} \]

\[ P_{\text{NET-SECTION}} = 159 \text{ kips} \]

\[ P_{\text{SHEAR FAILURE}} = 154 \text{ kips} \]
The utility of the previous analytic procedures was recently demonstrated in a MCAIR/NADC-funded program (ref. 2) to design high-load-transfer metal-to-composite bolted joints applicable to a fighter wing root area. This graph indicates predicted strength envelopes used to design the required joint for either a monolithic 55/44/4 laminate (lower curve) or a locally softened 52/44/4 laminate (horizontal, upper curve) where 0° plies were replaced by ±45° plies local to fastener hole areas. For the monolithic case, the limit cutoff represents the predicted first-ply shear failures under pure bearing loads.
JOINT THICKNESS VERSUS NUMBER OF BOLTS

Based on a design ultimate load requirement of 30,000 lb/in., the previous strength envelope for the monolithic laminate was used to predict the number of in-line bolts required versus overall joint thickness. Cutoffs based on fastener shear strength are also indicated.
Based on a seven-bolt-in-tandem joint configuration, the BJSFM laminate strength procedure was used to efficiently evaluate the effect of layup variations on joint thickness. The results of this study are plotted for layups having 4 to 25 percent of their 90° plies aligned with the chord direction of the baseline wing; the remaining plies were oriented in the spanwise (0°) direction.

\[ P = 30,000 \text{ LB/IN.} \]
Based on the results of the previous two analytic studies, scarfed monolithic-laminate and scarfed softened-laminate specimens were fabricated to verify predictions. The graphite-epoxy and titanium members are shown with the 0.50-in.-diameter fasteners used in the monolithic-laminate design. The specimen is representative of a single column of a wing root splice that would be spaced every 2.50 in. across the chord of the wing root.
The scarfed softened-laminate splice is shown assembled and strain gaged prior to test. The softened-laminate splice test specimen represents a single spanwise column of the wing root design which would be repeated approximately every 10 in. across the chord of the baseline wing root area. The taper of the planform was designed to simulate the gradual increase of spanwise loading due to increased wing bending moments encountered closer to the root area.
The scarfed monolithic-laminate splice (upper photo) and softened-laminate splice (lower photo) are pictured after static failure. As predicted, both specimens failed in the net-section area of that fastener location having the highest bypass load. The static joint failure load for the monolithic-laminate was 68 kips; the softened-laminate failed at 125 kips. Both loads were within 10 percent of prediction.
Correlations between predicted bolt load distributions and test are indicated for the scarfed monolithic-laminate splice. The predicted distribution is based on neat-fit fasteners with member and fastener flexibilities accounted for.
EFFECT OF HOLE CLEARANCES ON BOLT LOADS

To investigate the cause of scatter in bolt load distributions, the effect of a 0.003-in. and 0.006-in. clearance at each of the fasteners was analyzed. The results illustrated in this figure offer a possible explanation of the scatter and indicate the importance of this variable.
The occurrence of a 0.003-in. clearance on each of the fasteners was evaluated relative to its effect on joint member loads. Unlike its effect on bolt loadings, when the area of the joint members is taken into account, the effect of clearance on joint member loading is relatively small.
CONCLUSIONS

As a backdrop for conclusions, this sketch idealizes the evolution of structural testing typical of project hardware development programs. Usually testing proceeds from simple coupon specimens to more structurally detailed element specimens, then to subcomponent box beams, and finally to full-scale static and fatigue-test articles. For the application of composites the testing requirements are extensive and costly, especially for establishing material allowables, because of loading possibilities and geometry effects which change for each laminate variation.

The BJSFM analysis procedure should considerably reduce the need for coupon testing relative to laminate variations, hole size, general bearing/bypass load interactions, and environmental test conditions.

Analysis and experimental data were compared to verify that the BJSFM procedure accurately predicts laminate strength under general tension and compression bearing/bypass loadings. Theoretical studies were reported which demonstrated method utility and the need for detailed stress solutions. Correlations between the closed-form BJSFM stress solutions and finite element solutions were reported which defined current program finite geometry limitations.

Three benefits are provided by using the BJSFM procedure. (1) Test data requirements are minimized, requiring only knowledge of mechanical properties for the unidirectional-ply and one-laminate configuration. (2) Design conservatisms and unconservatisms can be reduced through this accurate, comprehensive, and inexpensive analysis. (3) Parametric evaluations of failure mechanisms can easily be conducted to provide guidance for the design of efficient bolted composite joints.
REFERENCES

