EXPERIMENTAL VERIFICATION OF THE NUMBER RELATION AT ROOM AND ELEVATED TEMPERATURES*

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ABSTRACT

The accuracy of the Neuber equation for predicting notch root stress-strain behavior at room temperature and at 650°C was experimentally investigated. Strains on notched specimens were measured with a non-contacting, interferometric technique and stresses were simulated with smooth specimens. Predictions of notch root stress-strain response were made from the Neuber Equation and smooth specimen behavior. Neuber predictions gave very accurate results at room temperature. However, the predicted interaction of creep and stress relaxation differed from experimentally measured behavior at 650°C.

INTRODUCTION

There has been a demand in recent years for the aircraft industry to provide a more energy efficient turbine propulsion system. Part of this task involves trying to understand the limitations of the current materials and structures being used, especially in the "hot section" of the engine [1]. The hot section components include the turbine blades, vanes, and combustors which operate under

*This work was performed under NASA Grant NAG 3-51.

severe stresses and temperatures. To make improvements in these parts it is first necessary to compile test data which describe the events leading up to failure. Theoretical models can then be developed and compared with experimental data until the failure modes and component lives may be predicted.

The combustor, fabricated from the alloy Hastelloy X, is one component which has gone through the initial testing phase and is now being examined from a theoretical stand-point. Failures in the combustor liner have been attributed to thermal-mechanical fatigue which causes cracking and buckling [2]. A number of constitutive theories have been proposed for predicting the nonlinear stress-strain behavior near holes which serve as cracking sites in the liner [3]. When these theories are incorporated into finite element codes, the final package becomes very complex and requires a large computer facility.

The purpose of this study is to examine a more basic theory, namely the Neuber relation, to see how well it can predict local stress-strain behavior in notched specimens of Hastelloy X. For cyclic loading the Neuber equation is written,

$$(\Delta\sigma)(\Delta\varepsilon) = (K_t')^2 (\Delta S)(\Delta e) \qquad (1)$$

where: $\Delta\sigma$ and $\Delta\varepsilon$ are the notch root stress and strain ranges, respectively;

ΔS and Δe are the remote stress and strain ranges, respectively;

K_t' is the elastic stress concentration factor.

Much of the work involving Neuber's relation has focused on stress redistribution near a notch [4] and the accompanying variation in the stress and strain concentration factors throughout fatigue life [5,6,7]. One of these researchers, Guillot [6], evaluated Neuber's equation at moderately elevated temperatures (260°C) and found that conservative results were obtained for life predictions in 1018 steel and 7475 aluminum. Both Bofferding [5] and Guillot [6] used an Interferometric Strain Gage (I.S.G.) [8-11] to measure notch root strains.

Equation (1) by itself is indeterminate. Knowing the remote stress or strain range leaves three unknowns. The The relationship between stress and strain at both the remote and local locations is needed. Crews and Hardrath [12] assumed that the notch stress could be found by reproducing measured notch strains in smooth samples. This assumption was upheld by Stadnick [13] and other researchers [14,15] who showed that the smooth specimen simulation gave good results in predicting fatigue lives of notched specimens. For this study it was assumed that smooth specimens could be used to supply the needed stress-strain relationship.

Stadnick and Morrow [16] worked on automating the techniques for performing tests on smooth specimens that were controlled according to the Neuber Equation. They evaluated various approaches for subjecting a smooth specimen to the same stresses and strains that theoretically exist at a notch. This theoretical testing technique has been called "Neuber Control". These methods consisted of manual control, and analog or digital computer control.

Separate research efforts have been devoted to using smooth specimens to simulate notch root response, developing laser based measurement devices and estblishing high temperature testing techniques. This study utilized all of these tools to determine the accuracy of Neuber's equation for cyclic loading of notches specimens at temperatures up to 650°C.

EXPERIMENTAL METHODS

1. Interferometric Strain Measurement Technique

The Interferometric Strain Gage (I.S.G.) is described in detail in References [8-11]. This device was used to measure strains both at the local and remote locations in notched specimens. The I.S.G. is a non-contacting laser based device capable of measuring strains over a very short gage length, typically 50-200 microns. The gage length is formed by making two small pyramidal shaped indentations on a sample with a Vicker's hardness tester. These indentations form the actual gage on the specimen and the distance between them constitutes the gage length. Laser light which reflects off the two indentations interacts to form two interference patterns. Each pattern is composed of bright and dark bands of light. The position of each bright fringe is a function of the wavelength of the laser light, the distance between indentations, and the reflecting angle of the indentations.

The I.S.G. functions by using scanning mirrors to reflect the interference bands onto photomultiplier tubes. When a load is applied to the specimen, the distance between the indentations changes, thereby causing the position of each bright fringe to move. Since the change in position of the fringes is proportional to the change in distance between the indentations, a computer can be used to calculate the strain in the specimen.

Two basic requirements must be met in order to utilize this strain measurement technique. First of all, the path of the incoming laser beam and the reflected fringe patterns must not be obstructed. To accomplish this, specimens were heated by an induction method for the elevated temperature tests. Heating coils which surrounded the specimen were designed so that they would not interfere with the laser beam. Another problem was that the specimen surface had to remain smooth and free of excess oxidation during high temperature testing. This problem was solved by depositing a 0.14 micron layer of 40% gold-60% palladium onto the specimen after the indentations had been made.

The I.S.G. was used to measure strains at both the local and the remote region of the notched specimen shown in Figure 1. The local indentations for creating fringe patterns were placed 50 microns from the edge of the notch. This was as near to the edge of the notch as the indentations could be consistently made with the Vicker's hardness tester. The remote indentations were made at a distance of 6.025 mm from the notch edge.

When evaluating the Neuber equation, other investigators [5,6] have restricted loading levels to insure that the remote region remained linearly elastic. This allowed the remote strain to be calculated by knowing the stress in the net section and the modulus of elasticity. During these experiments, the complications of defining a net section stress were avoided by measuring the remote strains directly. There were also no limitations on plasticity in the remote region. This allowed the Neuber relation to be evaluated for a greater range of loading conditions.

The loading pattern for this experiment consisted of completely reversed loading with 100 second hold times in both tension and compression. A servo controlled, electro-hydraulic, closed-loop testing machine was used to perform the tests. Many of the details concerning the experimental procedure have not been included in this paper so that the results and conclusions could be emphasized.

2. Stress Simulation

To determine the stresses that existed in notched specimens, smooth specimens were subjected to the strain histories which had been measured at the local and remote locations. Strains that had been measured with the I.S.G. were recorded in real time so that they could be played back on a smooth specimen at the same strain rate that existed on the notched specimen. From this technique, stress-strain hysterisis loops at different locations on a notched specimen were produced.

This method of simulating stresses worked very well from an experimental point of view. All parameters such as strain rate, creep, and total strain were reproduced in the smooth specimen just as they had occurred in the notched plate. The plots of local notch root stress versus strain were considered direct experimental data to which the Neuber predictions could be compared.

3. Neuber Prediction

The Neuber equation, (Eqn. 1), allows local behavior in a notched specimen to be determined as a function of remote stress and strain. In these experiments, a smooth specimen was manually controlled in real time according to Eqn. 1 with remote stress and strain as input parameters.

The measured remote strains and simulated stresses had been recorded on a time scale. These stress and strain values were multiplied together at various points in time and their product was then multiplied by the stress concentration factor squared. These values were replotted on the same time scale and constituted the Neuber prediction curves.

The Neuber relation is evaluated on a reversal by reversal basis. Therefore, the Neuber prediction curves were actually the product of the changes in stress and strain which occurred starting from the beginning of each reversal. By following this procedure, a plot such as the one shown in Figure 2 could be constructed. This plot would allow six reversals of local behavior to be predicted. The time scale was set at 5 sec/cm when the loads were applied and then slowed to 50 sec/cm for the 100 second hold periods. During each test, the values of stress and strain from a smooth specimen were multiplied together on-line with an analog computer to represent the quantity $(\Delta\sigma)(\Delta\varepsilon)$. The specimen was manually controlled in the MTS system so that the product of stress and strain would follow the Neuber prediction curve for each reversal. The illustration in Figure 2 shows how closely the original plot was followed during such a test. An additional analog circuit was designed so that the changes in stress and strain could be multiplied together starting from zero at the beginning of each reversal.

By plotting $(K_t')^2(\Delta s)(\Delta e)$ and imposing the product $(\Delta\sigma)(\Delta\varepsilon)$, the Neuber equation was satisfied for each reversal. The resulting stress and strain values constituted the predicted notch root behavior. The Neuber predictions were then compared to the measured strain vs. simulated stress data for local response.

RESULTS AND DISCUSSION

1. Determination of Stress Concentration Factor

An elastic stress concentration factor for the circular notched specimens was found experimentally using the ISG.

From Peterson (17), the stress concentration factor, K_t was given as 2.37. The experimentally determined stress concentration factor which is defined here as K_t' was found to be equal to 2.27. Figure 3 shows where five sets of indentations for the ISG were placed across the width of a notched specimen. Room temperature strain measurements were recorded at each of these locations while the specimen was cycled well below the proportional limit. The actual strain data and the calculated strain profile are both shown in the figure. By taking the ratio of strains at location #5 and location #1, the strain concentration factor was determined. For elastic strains, the stress and strain concentration factors are equal, therefore, K_t' was also determined ($K_t' = 2.27$). This experimentally determined value of K_t' as well as the designation for the local and remote areas (locations #1 and #5) were used throughout the test program.

2. Interferometric Strain Measurements

At room temperature, strains for a notched specimen were recorded for the initial behavior and also for a cyclically stable condition, i.e. when the material at notch root was stable. Figure 4 shows I.S.G measurements of strain vs. applied load for a notched specimen during the first three cycles of constant amplitude completely reversed loading between $\pm$ 14 KN. The most noticeable effects in notch root behavior were caused by cyclic hardening. The tensile peaks showed a large decrease in strain for each successive cycle due to strain hardening. The compressive strains experienced much less variation during the three cycle period. Creep effects were also present in the room temperature data. The largest amount of creep took place during the first 100 second hold time and then diminished with each successive reversal.

For remote behavior, which is also shown in Fig. 4, cyclic hardening again caused the total strain to decrease for each plotted loop. The effects of creep were minimal for the remote location. The amount of creep at both locations in the specimen decreased as the material stabilized.

When a sufficient number of cycles had been applied to stabilize the material, the I.S.G. was used to record data at the four cyclic load levels which are listed:

LEVEL #	LOAD (KN)
1	± 14.0
2	± 14.5
3	± 15.5
4	± 16.0

Strain measurements were obtained at each of five locations across the notched specimen as indicated in Figure 3. Figures 5 and 6 show results for the lowest load amplitude (Load Level 1) and the highest amplitude (Load Level 4). These figures illustrate the effects of cyclic loading at various distances from the notch. The plastic strain diminished significantly as the distance from the notch increased. Also, when the load was raised from Level 1 to

Level 4, the strain at the remote location (#5) increased by 21% while the local strain (#1) experienced a 50% increase. This gives an indication of the strain concentration near the notch.

Notch root and remote strains were also measured at 650°C in a specimen which had been cyclically stabilized. Four load levels were again used which are as follows:

LEVEL #	LOAD (KN)
1	± 10.5
2	± 11.3
3	± 12.3
4	± 13.3

Hysteresis loops showing applied load vs. local notch root strain at four different load levels are shown in Figures 7 and 8. At this temperature, small increases in load produced large strains, especially strain due to creep. During the 100 second hold time, the amount of creep strain at each level of loading was as follows:

Level 1: 0.05% creep strain
Level 2: 0.10% creep strain
Level 3: 0.13% creep strain
Level 4: 0.18% creep strain

These values were approximately equal for tension and compression.

3. Stress Simulation & Neuber Predictions

Smooth specimen stress simulations produced the experimental stress-strain behavior at both the remote and local regions. Neuber predictions were also made. The first three cycles of notch root stress-strain behavior at room temperature were plotted in Figure 9. Included in this figure are both the experimental results and the Neuber predictions. During the first cycle, the Neuber prediction was slightly high on stress which caused lower strain peaks. Actually, the tensile and compressive strains were only 9% low for the first cycle. The predicted tensile strain on the second cycle was low by 8% while the compressive strain was 13% lower than the stress simulation.

The Neuber relation was also used to predict the notch root response after the material had reached the cyclically stable condition. In Figure 10, the room temperature results from tne Neuber prediction and the stress simulation have been superimposed for comparison. For the stabilized notch root response at Load Levels 1 and 2, the Neuber method was approximately 6% high in predicting tensile and compressive strain. Load Levels 3 and 4 show nearly a perfect correlation between the two sets of curves.

Neuber's rule was also studied at 650°C for the cyclically stable condition. Figure 11 and 12 show these stable results. The most noticeable trend at all four levels was the amount of stress relaxation predicted by the Neuber relation. For Load Levels 1 and 2, the stresses at the end of the 100 second hold times were low by 23% and 27%, respectively. The stresses were predicted more accurately at the higher load levels. At Load Level 3 the stresses were 22% low and at Level 4 the stresses were 15% lower than the stress simulation. In terms of strain range, the error in predicting Level 1 strains was 20% low while the Level 4 strains were predicted within 10%.

The tendency of the Neuber relation to predict stress relaxation rather than predominant creep during the hold times was caused by the remote information which was used to construct the Neuber plots. The remote location had experienced almost no creep for the Load Levels 1 thru 3. At Load Level 4, the creep accounted for about 12% of the total strain. This caused the Neuber prediction to become more accurate at the highest load level.

CONCLUSIONS

Neuber control of a smooth specimen predicted the notch root stress-strain behavior of a circular center notched plate that was made of Hastelloy X with excellent agreement to direct experimentally measured notch root strains and simulated stresses at room temperature. The agreeement was good for initial behavior during cyclic hardening and for the stable condition at four different load levels. At 650°C and for the stable condition, agreement with experimental data were acceptable with the maximum error at 20%. At this higher temperature, the direct experimental data showed primarily creep strain during hold times. The Neuber prediction showed both creep and stress relaxation. This difference in the general behavior resulted in significantly larger errors at this elevated temperature than those for room temperature.

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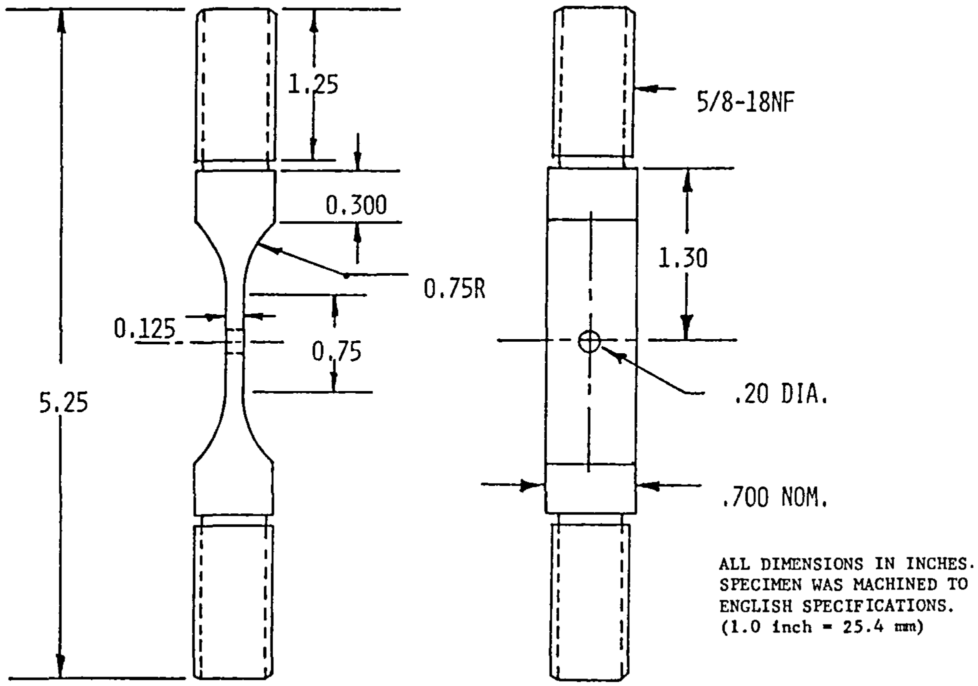

FIGURE 1 NOTCHED SPECIMEN GEOMETRY

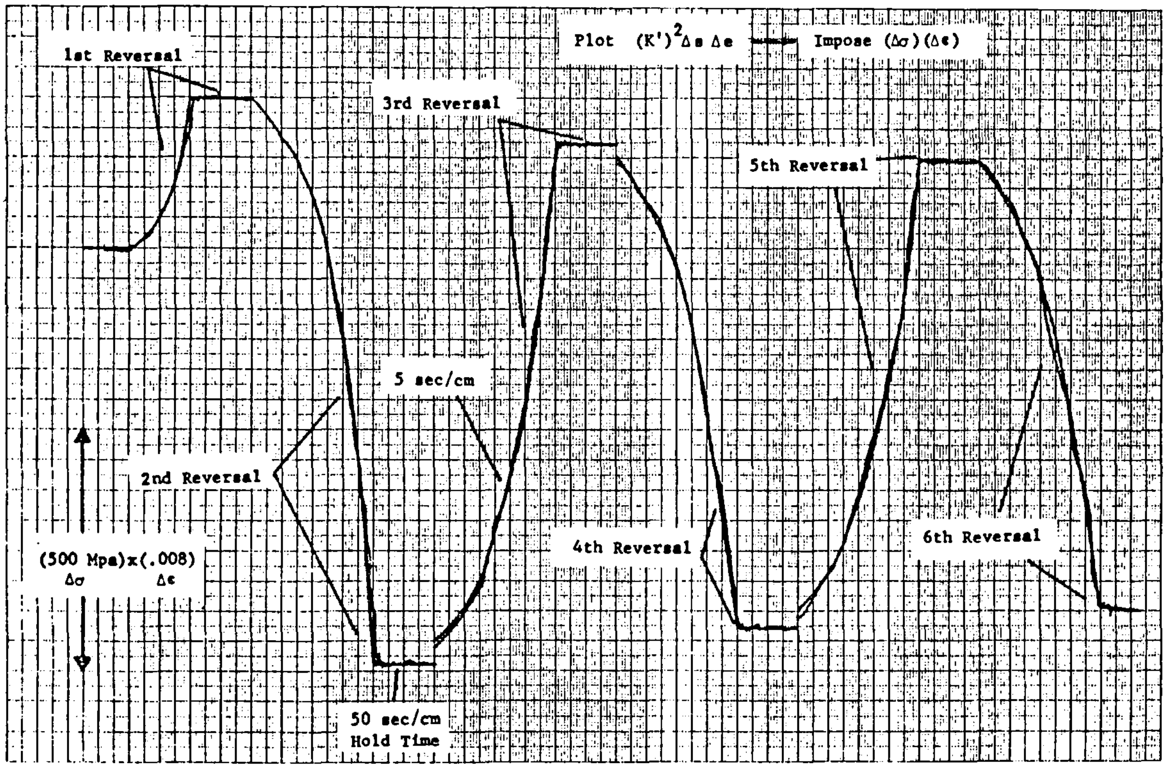

FIGURE 2 NEUBER PREDICTION CURVES

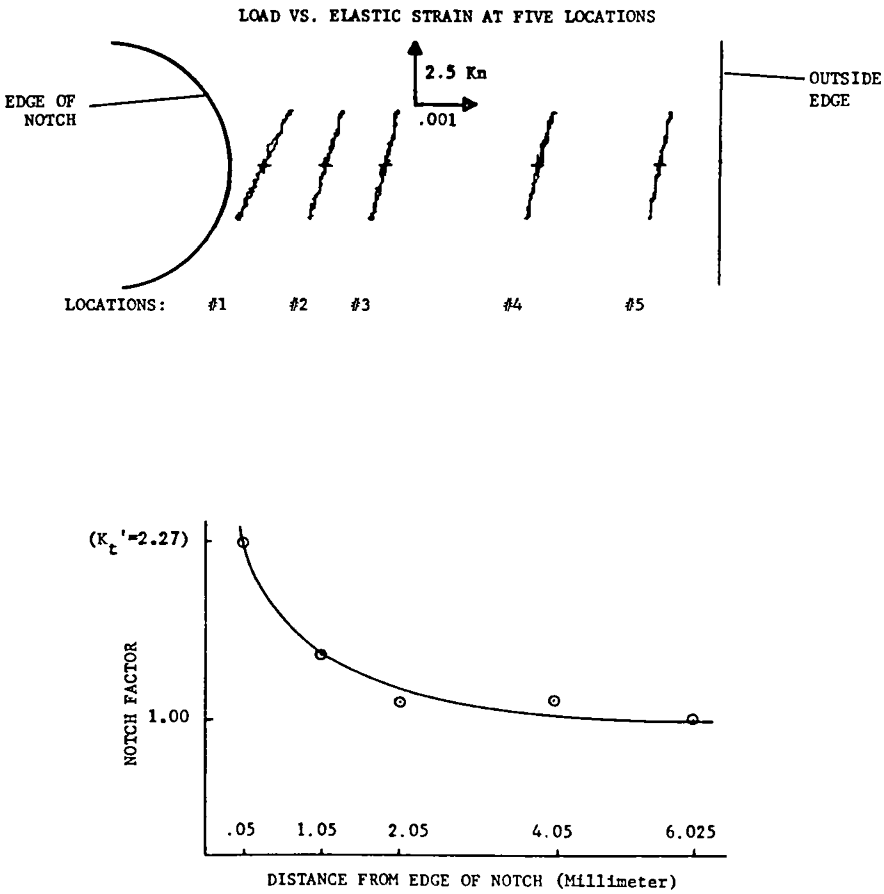

FIGURE 3 DETERMINATION OF STRAIN PROFILE

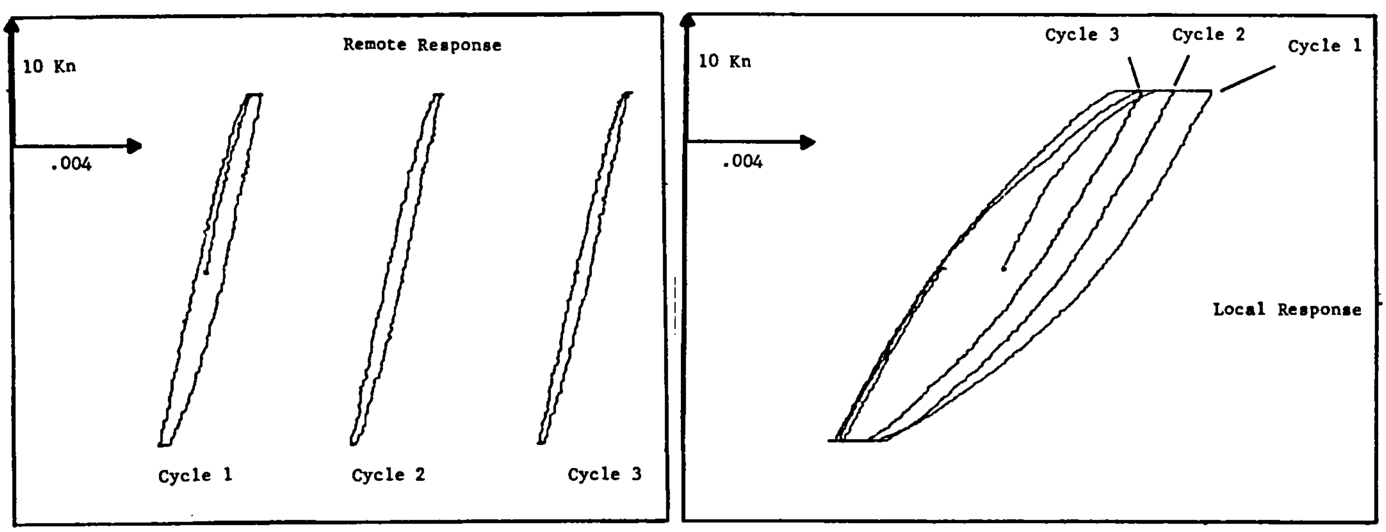

FIGURE 4 INITIAL ROOM TEMPERATURE DATA AT THE LOCAL AND REMOTE LOCATIONS

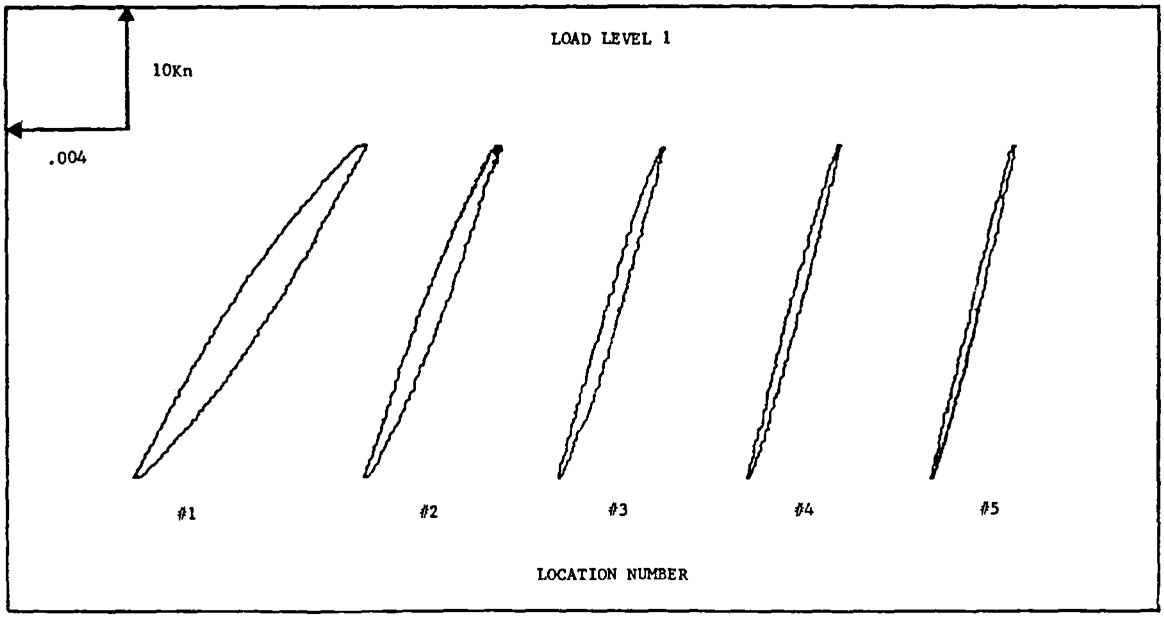

FIGURE 5 STRAIN VS. LOAD AT ROOM TEMPERATURE
FOR A STABLE CONDITION

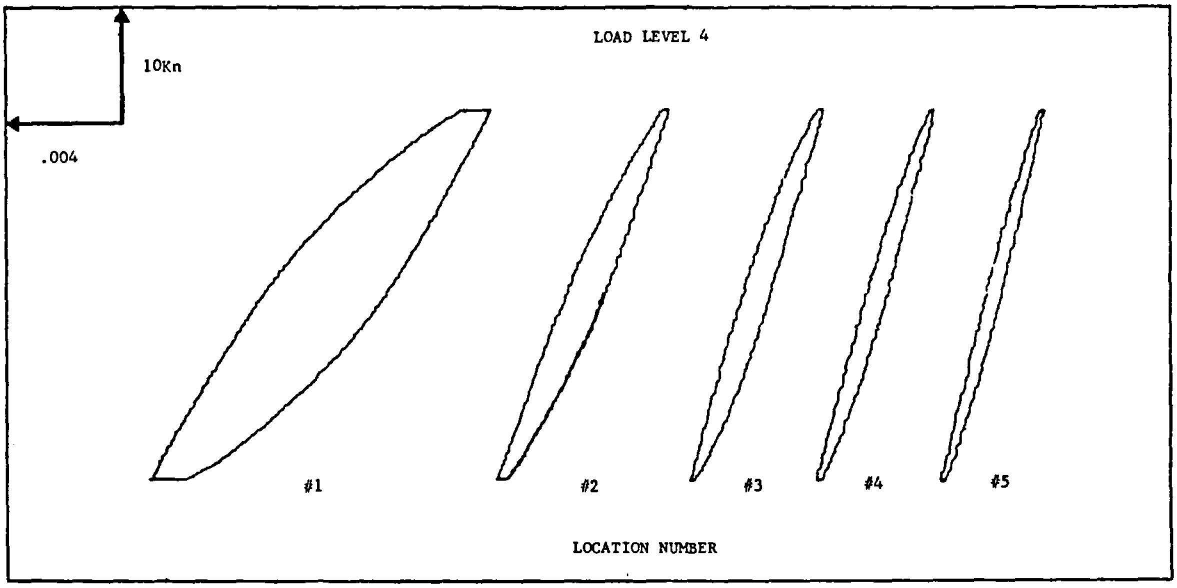

FIGURE 6 STRAIN VS. LOAD AT ROOM TEMPERATURE
FOR A STABLE CONDITION

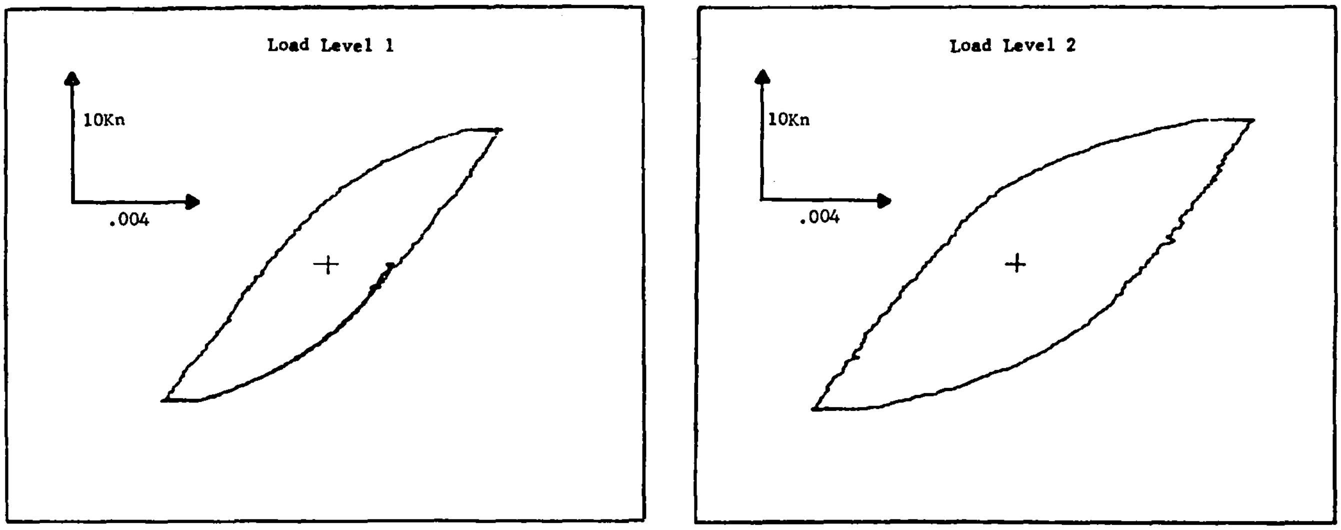

FIGURE 7 LOCALLY MEASURED STRAIN VS. LOAD AT 650°C

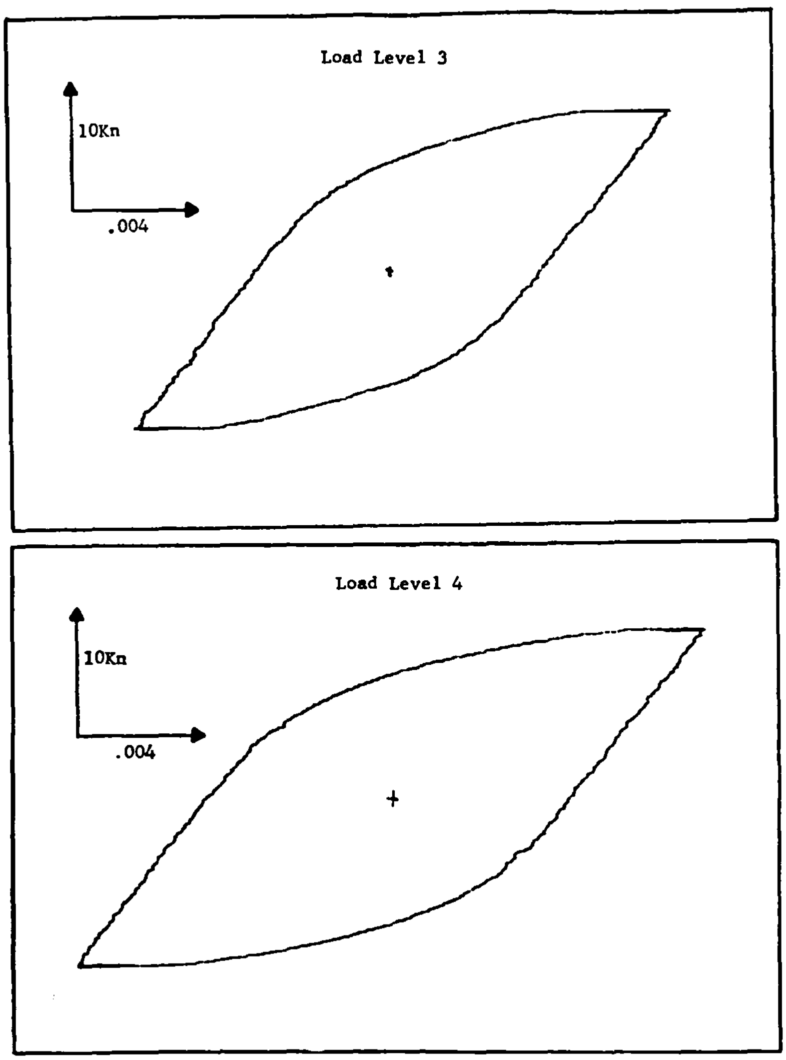

FIGURE 8 LOCALLY MEASURED STRAIN VS. LOAD AT 650°C

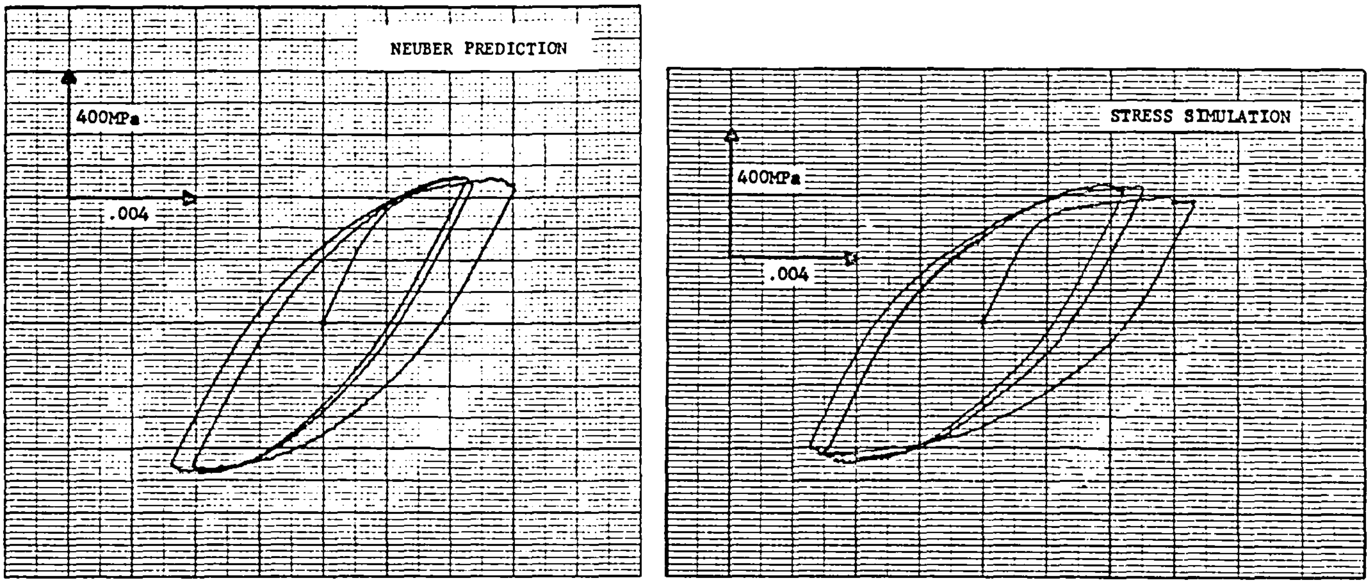

FIGURE 9 EXPERIMENTAL AND PREDICTED LOCAL RESPONSE AT ROOM TEMPERATURE

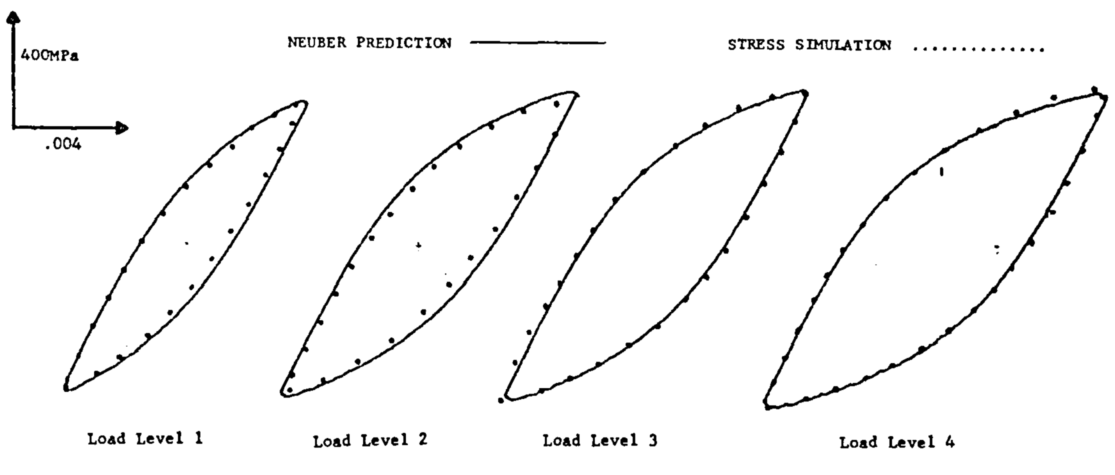

FIGURE 10 EXPERIMENTAL AND PREDICTED LOCAL RESPONSE AT ROOM TEMPERATURE

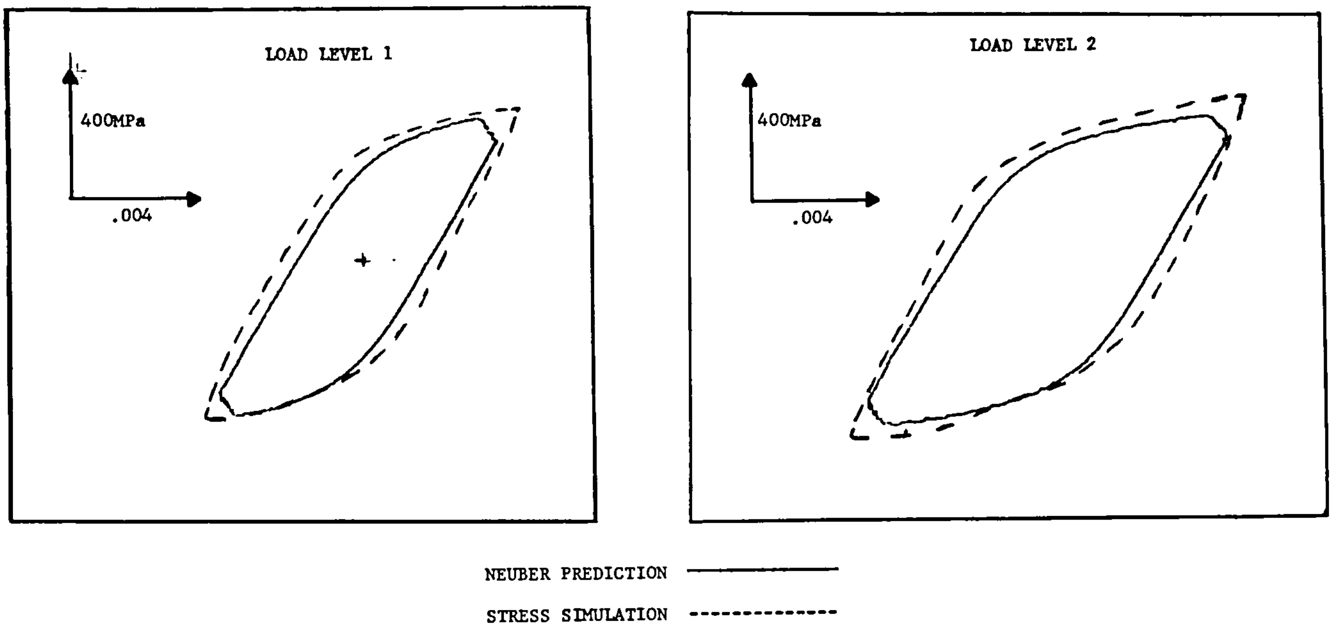

FIGURE 11 EXPERIMENTAL AND PREDICTED LOCAL RESPONSE AT 650°C

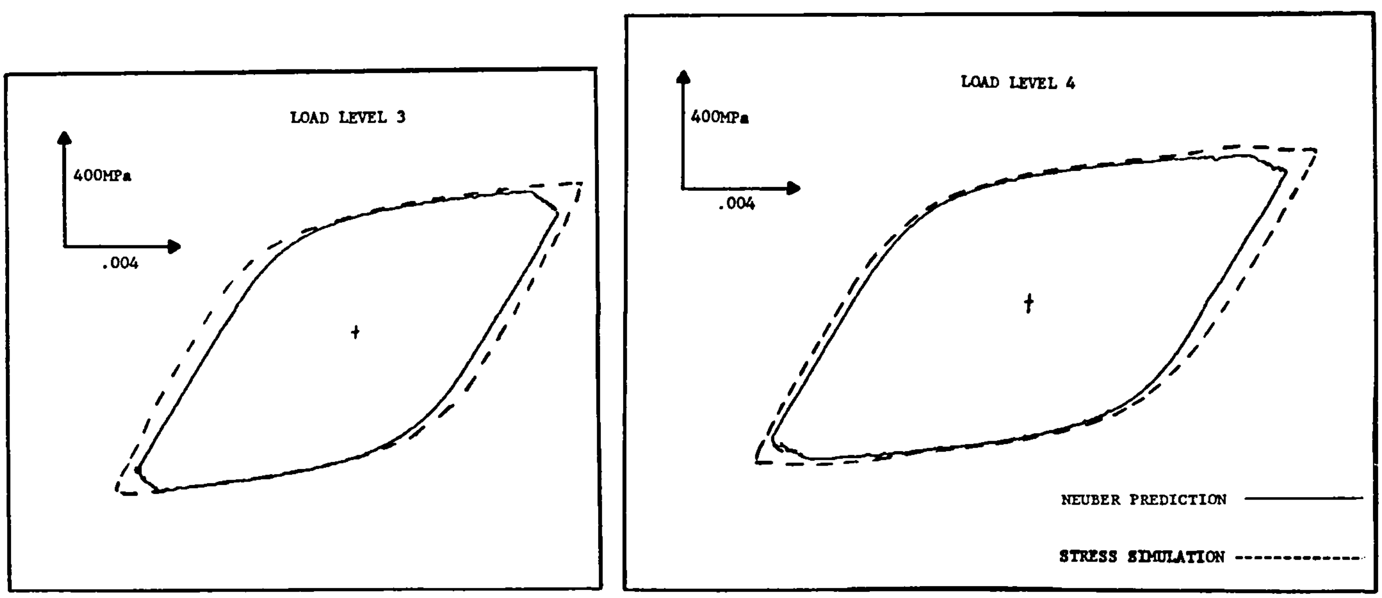

FIGURE 12 EXPERIMENTAL AND PREDICTED LOCAL RESPONSE AT 650°C